Information Provision from a Platform to Competing Sellers: The Role of Strategic Ambiguity

E-Companion

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The online supplement is comprised of several parts. In Appendix A we provide a summary of the notations used in this paper. In Appendix B, we study the case of verifiable information that cannot be manipulated. We use this analysis in order to establish the incentives of the platform to convey market information to its sellers. In Appendix C, we explore the robustness of our main results when θ follows a different distribution (specifically, a beta distribution, chosen due to its flexibility in providing various density shapes – from uniform-like to normal-like). Appendix D includes the proofs of all the results presented in this work.

Appendix A. Glossary of Notations

Parameter	Description
n	Number of sellers
a	Deterministic part of the potential demand
γ	Competition intensity, $\gamma > 0$
α	Platform's revenue share, $0 < \alpha < 1$
k	Number of intervals in the partition
Index	Description
i	Seller index, $i = 1, .., n$
j	Interval index, $j = 1, .., k$
Variable	Description
θ	Random part of the potential demand with zero mean and variance σ^2.
Decision variable	Description
p_i	Selling price set by seller i
$m(\theta)$	Message shared by the platform with the sellers upon observing θ
θ_1^*	Partition point under region-forecast equilibrium with two intervals
$\theta_j^{(k)}$	Partition point j under region-forecast equilibrium with k intervals
Function	Description

$f(\theta)$	Probability density function of θ
$F(\theta)$	Cumulative distribution function of θ
$f(\theta\|m)$	Sellers' posterior belief about θ for a given message m (shared by the platform)
$q_i(\overrightarrow{p})$	Demand of seller i (random variable) for a given vector of selling prices
$Q(\overrightarrow{p})$	Total demand in the market (random variable) for a given vector of selling prices
I_j	Interval j under partition equilibrium with two intervals
$f(\theta, I_j)$	Joint probability of the platform to observe θ and report message I_j
$E[\theta\|I_j]$	Expected value of θ given $\theta \in I_j$ under a partition with two intervals
$H(\theta\|I)$	Conditional entropy of θ based on information I under a partition with two intervals
$E[V(\theta\|I)]$	Expected variance of θ based on information I under a partition with two intervals
$I_j^{(k)}$	Interval j in a partition equilibrium with k intervals
$l_j^{(k)}$	Length of interval $I_j^{(k)}$
$E[\theta\|I_j^{(k)}]$	Expected value of θ given $\theta \in I_j^{(k)}$ under a partition with k intervals
$E[V(\theta\|I^{(k)})]$	Expected variance of θ based on information I under a partition with k intervals
$\bar{k}$	Upper bound on the number of intervals in a feasible partition equilibrium
Profit function	Description
π_p	Profit of the platform
π_i	Profit of seller i
$\mathit{\Pi}_p$	Ex-ante payoff of the platform
$\mathit{\Pi}_i$	Ex-ante payoff of seller i
Π_p^{FI}	Ex-ante payoff of the platform when verifiable information is shared
Π_i^{FI}	Ex-ante payoff of seller i when verifiable information is shared (full information)
Π_p^{NI}	Ex-ante payoff of the platform when no information is shared
Π_i^{NI}	Ex-ante payoff of seller i when no information is shared
Π_p^{RF}	Ex-ante payoff of the platform under a region forecast with two intervals
Π_i^{RF}	Ex-ante payoff of seller i under a region forecast with two intervals
$\Pi_p^{(k)}$	Platform's ex-ante payoff under a region forecast with k intervals
$\Pi_i^{(k)}$	Seller i's ex-ante payoff under a region forecast with k intervals

Appendix B. Incentives to Convey Market Information

In this appendix we examine whether the platform has an incentive to provide market information to the sellers, and whether the latter would agree to receive such forecast information from the platform when information is verifiable and cannot be manipulated. We use the superscript FI (Full Information) to denote the outcome when verifiable information is shared, and the superscript NI to denote the outcome when no information is shared. The following lemma demonstrates that in the case where information is verifiable, information sharing benefits both the platform and the sellers.

Lemma 3. ***(full information vs. no information sharing)***

(i) When verifiable information is shared, the selling price is $p_i^{FI} = \frac{a+\theta}{2+\gamma}$, and the ex-ante payoffs of the platform and of each seller are $\Pi_p^{FI} = \frac{\alpha n(1+\gamma)}{(2+\gamma)^2}(a^2+\sigma^2)$ and $\Pi_i^{FI} = \frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}(a^2+\sigma^2)$, respectively.

(ii) When verifiable information is not shared, the selling price is $p_i^{NI} = \frac{a}{2+\gamma}$, and the ex-ante payoffs of the platform and of each seller are $\Pi_p^{NI} = \frac{\alpha n(1+\gamma)}{(2+\gamma)^2}a^2$ and $\Pi_i^{NI} = \frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}a^2$, respectively.

Lemma 3 illustrates that when information is verifiable, information sharing benefits both the platform and the sellers. Two factors that lie at the core of this research, and affect the value of sharing information, are the market uncertainty and the competition level among sellers. First, note that the value of information is proportional to σ^2. Absent information sharing, the sellers set their prices based on the prior belief regarding the value of θ, and as σ^2 increases, this estimate becomes less accurate. This problem is mitigated when information is shared and the sellers set their prices based on the actual market condition. The second factor that determines the value of information sharing is the competition level, captured by γ. As the competition level increases, the value of information decreases, because competition eradicates much of the payoff that the sellers can gain. Because the platform's payoff is proportional to the payoffs of the sellers, the value it obtains from information sharing is also reduced as competition intensifies. Based on Lemma 3, we summarize the incentives of the platform to share verifiable information with the sellers.

Proposition 11. ***(incentives to share verifiable information)***

When information is verifiable, at equilibrium, the platform will share information with the sellers. Such an outcome benefits both the platform and the sellers.

Proof of Lemma 3. In what follows, we prove part (i) of the lemma (i.e., when information is shared truthfully).

When information is shared, based on equation (1) and equation (3), each seller solves the following optimization problem:

$$max_{p_i}\left\{p_i\left(a+\theta-(1+\gamma)p_i+\frac{\gamma}{n-1}\sum_{t\in N_{-i}}p_t\right)\right\}.$$

Since the objective of each seller is a concave parabolic function, the first-order condition of this optimization problem is also the sufficient condition, and is equal to

$$a + \theta - (1+\gamma)p_i + \frac{\gamma}{n-1}\sum_{t \in N_{-i}} p_t - (1+\gamma)p_i = 0.$$

Based on the symmetry between the sellers (implying that $p_i = p_t$), and our assumption that an interior solution exists, the equilibrium price is $p_i^{FI} = \frac{a+\theta}{2+\gamma}$ and the demand each seller faces is $q_i^{FI} = \frac{(a+\theta)(1+\gamma)}{2+\gamma}$. Therefore, by equation (3), seller i's ex-ante payoff is given by $\Pi^{FI} = E\left[(1-\alpha)p_i^{FI}q_i^{FI}\right] = E\left[\frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}(a+\theta)^2\right] = \frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}(a^2+\sigma^2)$, and similarly, by equation (2), the ex-ante payoff of the platform is $\Pi_p^{FI} = \frac{\alpha n(1+\gamma)}{(2+\gamma)^2}(a^2+\sigma^2)$.

(ii) When information is not shared, based on equation (1) and equation (3), each seller solves the following optimization problem:

$$max_{p_i}\left\{E\left[p_i\left(a+\theta-(1+\gamma)p_i+\frac{\gamma}{n-1}\sum_{t\in N_{-i}}p_t\right)\right] = p_i\left(a-(1+\gamma)p_i+\frac{\gamma}{n-1}\sum_{t\in N_{-i}}p_t\right)\right\}.$$

Since the objective of each seller is a concave parabolic function, the first-order condition of this optimization problem is also the sufficient condition, and is equal to

$$a - (1+\gamma)p_i + \frac{\gamma}{n-1}\sum_{t \in N_{-i}} p_t - (1+\gamma)p_i = 0.$$

Based on the symmetry between the sellers (implying that $p_i = p_t$), and our assumption that an interior solution exists, the equilibrium price is given by $p_i^{NI} = \frac{a}{2+\gamma}$ and the demand each seller faces is $q_i^{NI} = a + \theta - \frac{a}{2+\gamma}$. Therefore, by equation (3), seller i's ex-ante payoff is given by $\Pi^{NI} = E\left[(1-\alpha)p_i^{NI}q_i^{NI}\right] = E\left[\frac{(1-\alpha)a}{2+\gamma}\left(\frac{1+\gamma}{2+\gamma}a+\theta\right)\right] = \frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}a^2$, and similarly, by equation (2), the ex-ante payoff of the platform is $\Pi_p^{NI} = \frac{\alpha n(1+\gamma)}{(2+\gamma)^2}a^2$. ■

Proof of Proposition 11. The claim is proved since $\Pi_p^{FI} - \Pi_p^{NI} = \frac{\alpha n(1+\gamma)}{(2+\gamma)^2}\sigma^2 > 0$ and $\Pi_i^{FI} - \Pi_i^{NI} = \frac{(1-\alpha)(1+\gamma)}{(2+\gamma)^2}\sigma^2 > 0$. ■

Appendix C. Numerical Analysis Using Beta Distribution

In the main model we assumed that θ follows a uniform distribution. This assumption is adopted for ease of exposition and mathematical tractability. In this appendix, we explore the robustness of our main results when θ follows a different distribution. Specifically, in the following analysis we assume that θ follows a beta distribution. The choice of a beta distribution for this analysis is justified by the following desired properties it encompasses: (a) bounded support that corresponds to $[-\epsilon, \epsilon]$ (as in our main model), achieved by using a scaled beta distribution; (b) ability to mimic the commonly-used Gaussian demand distribution by using certain parameter values. To achieve these properties, we use the following formula:

$$f(\theta) = \frac{(\epsilon+\theta)^{\rho-1}(\epsilon-\theta)^{\phi-1}}{(2\epsilon)^{\rho+\phi-1}B(\rho,\phi)} \quad \theta \in \Theta;\ \rho, \phi > 0,$$

where ρ and ϕ are the shape parameters, and $B(\rho, \phi)$ is the beta function:

$$B(\rho,\phi) = \int_0^1 x^{\rho-1}(1-x)^{\phi-1}dx.$$

We further apply the restriction that $\rho = \phi = 4$ to ensure a symmetric bell-shaped distribution and zero mean. For the following numerical study, we use $a = 100$, $n = 10$ and $\alpha = 0.3$, as applied in the numerical examples in the main model, and we focus on the case of a partition equilibrium into two intervals (i.e., equivalent to Section 5). Panel (a) in Figure 9 examines the relationship between θ_1^* and the competition intensity, and illustrates that θ_1^* decreases in γ. This result is aligned with Proposition 2 and illustrated in Panel (b) of Figure 1. In Panel (b) of Figure 9, we examine the way in which the existence of the partition equilibrium depends on the competition intensity among the sellers (γ) and the level of market uncertainty (Ψ). Note that this graph is similar to Panel (a) of Figure 4. Finally, Panels (c) and (d) in Figure 9 portray the platform's payoff in the partition equilibrium in comparison to its payoff under full information sharing and no information sharing. In Panel (c), we examine these payoffs as a function of the competition level, and in Panel (d), as a function of the market uncertainty level. The results are aligned with Proposition 4 and the corresponding plots in Figure 5. We conclude from the analysis that the main results are also valid when the the random element follows a bell-shaped beta distribution.

Appendix D. Proofs

Proof of Lemma 1. Similar to the proof of Lemma 3(i), upon receiving message m, and when the sellers use the value of this message to infer the precise value of θ, the sellers set a price $p_i = \frac{a+m}{2+\gamma}$. In this case, by equation (1), the demand each seller faces is $q_i = \frac{a(1+\gamma)-m}{2+\gamma} + \theta$. Therefore, by equation (2), the payoff of the platform is

$$\pi_p = \frac{\alpha n(a+m)\left((a+\theta)(1+\gamma)+\theta-m\right)}{(2+\gamma)^2}.$$

Because information is non-verifiable, the platform chooses m so as to maximize its payoff. Since π_p is a quadratic concave function of m, it is maximized at $m = \theta + \frac{\gamma(a+\theta)}{2}$, which in turn leads to $p_i = \frac{a+\theta}{2}$. ■

Proof of Proposition 1. The claim is true due to the information manipulation outlined in Lemma 1. ■

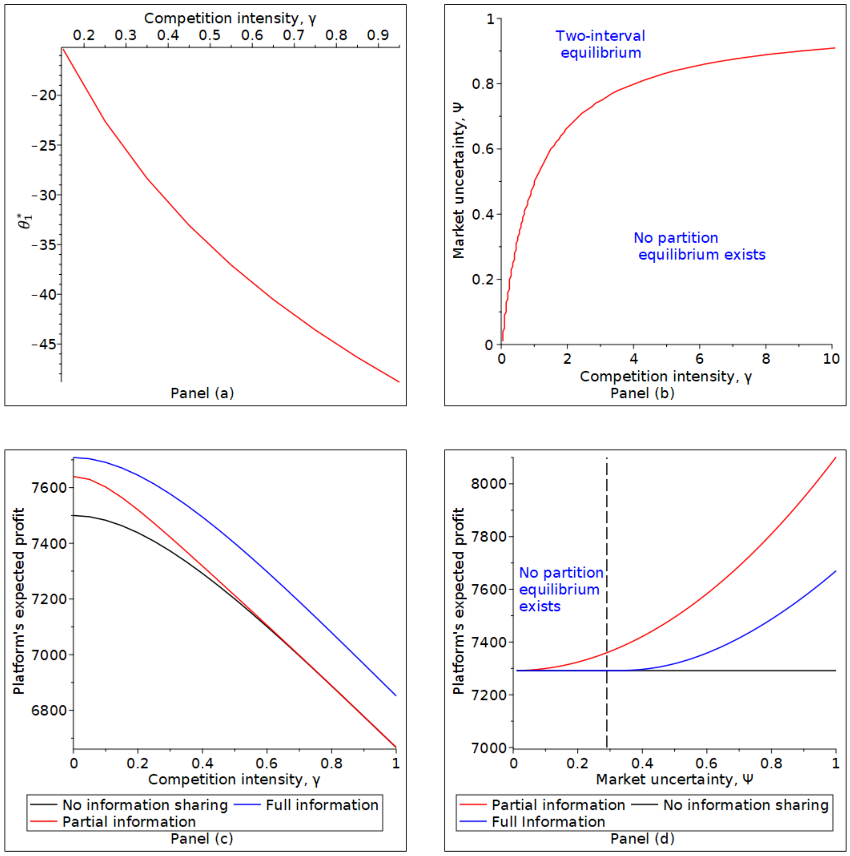

Figure 9: Replication of the main results under a beta distribution with $\rho = \phi = 4$. In Panels (a) and (c), $\epsilon = 50$, and in Panel (d), $\gamma = 0.4$.

Proof of Proposition 2. Assume that upon receiving a message m, the sellers believe that $\theta \in m$. Based on this belief system, each seller solves the problem:

$$max_{p_i}\left\{E[\pi_i|m] = (1-\alpha)p_i\left(a + E[\theta|m] - (1+\gamma)p_i + \frac{\gamma}{n-1}\sum_{t\in N_{-i}} p_t\right)\right\}, i \in N.$$

Because the seller's payoff is a concave function of p_i, it is maximized for $p_i = \frac{a+E[\theta|m]+\frac{\gamma}{n-1}\sum_{t\in N_{-i}} p_t}{2(1+\gamma)}$. Due to the symmetry of the sellers, we set $p_i = p, i \in N$, implying that the symmetric optimal price is $p(m) = \frac{a+E[\theta|m]}{2+\gamma}$. For this price, the payoff of the platform as a function of θ and m is given by:

$$\pi_p(\theta, m) = \frac{\alpha n\left(a(1+\gamma) + \theta(2+\gamma) - E[\theta|m]\right)\left(a + E[\theta|m]\right)}{(2+\gamma)^2}. \tag{11}$$

We seek values of θ_1 that provide a partition satisfying the conditions for truthtelling for any value of θ:

$$\pi_p(\theta, m) \geq \pi_p(\theta, \overline{m}) \quad \forall \theta \in m \; and \; \overline{m} \neq m. \tag{12}$$

These two conditions can be written explicitly, for $j = 1, 2$, as:

$$(a(1+\gamma)+\theta(2+\gamma)-E[\theta|I_j])\,(a+E[\theta|I_j]) \geq (a(1+\gamma)+\theta(2+\gamma)-E[\theta|I_{3-j}])\,(a+E[\theta|I_{3-j}])\,, \forall\theta \in I_j.$$

Since $E[\theta|I_j] = \frac{\int_{\theta\in I_j}\theta f(\theta)d\theta}{\int_{\theta\in I_j}f(\theta)d\theta} = \frac{\theta_1+\epsilon\cdot(-1)^j}{2}$, these two conditions can be simplified to $\theta_1 \geq \gamma a + \theta(\gamma+2)$ $\forall\theta \in (-\epsilon, \theta_1)$ and $\theta_1 \leq \gamma a + \theta(\gamma+2)$ $\forall\theta \in (\theta_1, \epsilon)$. Due to the linearity of the inequalities in θ, we can further simplify them to $\theta_1 \geq \gamma a + \theta_1(\gamma+2)$ and $\theta_1 \leq \gamma a + \theta_1(\gamma+2)$, which can then be written as $\theta_1 \leq -\frac{a\gamma}{1+\gamma}$ and $\theta_1 \geq -\frac{a\gamma}{1+\gamma}$. Therefore, there exists a unique solution $\theta_1^* = -\frac{a\gamma}{1+\gamma}$, which satisfies both inequalities. The condition for existence of a region-forecast equilibrium is derived from the condition $-\epsilon < \theta_1^* < \epsilon$. Since $\theta_1^* < 0$, the existence condition can be reduced to $-\epsilon < \theta_1^*$ and, by using the notation $\Psi(= \epsilon/a < 1)$, it can be further rewritten as $\gamma < \frac{\Psi}{1-\Psi}$. ■

Proof of Lemma 2. In our case, $f(\theta, I_j) = \begin{cases} \frac{1}{2\epsilon} & \text{if } \theta \in I_j \\ 0 & o/w \end{cases}$ is the joint probability of the platform to observe θ and report message I_j, and the conditional p.d.f.s of θ, depending on whether it belongs to I_1 or I_2, are $f(\theta|I_1) = \frac{1}{\epsilon+\theta_1}$ and $f(\theta|I_2) = \frac{1}{\epsilon-\theta_1}$, respectively. By equation (8), $H(\theta|I) = -\int_{-\epsilon}^{\epsilon}\sum_{j=1}^{2} f(\theta, I_j)log f(\theta|I_j)d\theta = -\int_{-\epsilon}^{\theta_1}\frac{1}{2\epsilon}log\left(\frac{1}{\epsilon+\theta_1}\right)d\theta - \int_{\theta_1}^{\epsilon}\frac{1}{2\epsilon}log\left(\frac{1}{\epsilon-\theta_1}\right)d\theta = \frac{\epsilon+\theta_1}{2\epsilon}log(\epsilon+\theta_1) + \frac{\epsilon-\theta_1}{2\epsilon}log(\epsilon-\theta_1)$. Since $\frac{d}{d\theta_1}H(\theta|I) = \frac{1}{2\epsilon}log\left(\frac{\epsilon+\theta_1}{\epsilon-\theta_1}\right)$ and $\frac{d^2}{d\theta_1^2}H(\theta|I) = \frac{1}{(\epsilon-\theta_1)(\epsilon+\theta_1)} > 0$, then $H(\theta|I)$ is a convex function of θ_1 with a minimum at $\theta_1 = 0$, implying that $H(\theta|I)$ decreases in θ_1 for $\theta_1 \in (-\epsilon, 0)$. In addition, by equation (9), $E[V(\theta|I)] = \frac{\epsilon+\theta_1}{2\epsilon}\int_{-\epsilon}^{\theta_1}\left(\theta - \frac{\theta_1-\epsilon}{2}\right)^2\frac{1}{\epsilon+\theta_1}d\theta + \frac{\epsilon-\theta_1}{2\epsilon}\int_{\theta_1}^{\epsilon}\left(\theta - \frac{\theta_1+\epsilon}{2}\right)^2\frac{1}{\epsilon-\theta_1}d\theta = \frac{\theta_1^2}{4} + \frac{\epsilon^2}{12}$. Therefore, $E[V(\theta|I)]$ is also a convex function of θ_1 with a minimum at $\theta_1 = 0$, implying that $E[V(\theta|I)]$ decreases in θ_1 for $\theta_1 \in (-\epsilon, 0)$. By Proposition 2 and using algebraic manipulations, $\theta_1^* = a(\frac{1}{1+\gamma} - 1)$, implying that θ_1 is always negative and decreases in γ for all $\gamma \geq 0$. Thus, when $\theta_1 = \theta_1^*$, both $H(\theta|I)$ and $E[V(\theta|I)]$ increase in γ, with a minimum at $\gamma = 0$. ■

Proof of Proposition 3. The proof follows directly from Lemma 2. The amount of shared information is maximized when the conditional entropy and conditional variance are minimized, and this is achieved when $\gamma = 0$ (based on Lemma 2). Therefore, the amount of shared information decreases with the level of competition.■

Proof of Proposition 4. The proofs concentrate on the platform's profit. Since the sellers' profit is proportional to that of the platform, i.e., $\Pi_i = \frac{1-\alpha}{\alpha n}\Pi_p$, the claims in the proposition regarding the sellers' expected profit are straightforward.

(i) Suppose $\Psi \leq \frac{\gamma}{1+\gamma}$; then by Proposition 2, a meaningful cheap-talk region-forecast information-sharing equilibrium does not exist. Thus, the parties interact as in the case of no information sharing, implying that $\Pi_p^{RF} = \Pi_p^{NI}$. Otherwise, if $\Psi > \frac{\gamma}{1+\gamma}$, then by equation (11),

$$\pi_p(\theta, m) = \frac{\alpha n \left(a(1+\gamma) + \theta(2+\gamma) - E[\theta|m]\right)\left(a + E[\theta|m]\right)}{(2+\gamma)^2}. \tag{13}$$

Therefore,

$$\Pi_p^{RF} \equiv E[\pi_p(\theta, m)] = \frac{\alpha n}{2\epsilon(2+\gamma)^2}\left[\left(a + \frac{\theta_1 - \epsilon}{2}\right)\int_{-\epsilon}^{\theta_1}\left(a(1+\gamma) + \theta(2+\gamma) - \frac{\theta_1 - \epsilon}{2}\right)d\theta\right.$$
$$\left. + \left(a + \frac{\theta_1 + \epsilon}{2}\right)\int_{\theta_1}^{\epsilon}\left(a(1+\gamma) + \theta(2+\gamma) - \frac{\theta_1 + \epsilon}{2}\right)d\theta\right],$$

which can be written explicitly as $\Pi_p^{RF} = \frac{\alpha n\left((1+\gamma)^2(3a^2+\epsilon^2)+(1+2\gamma)a^2\right)}{4(1+\gamma)(2+\gamma)^2}$.

(ii) It is clear that Π_p^{RF} increases in ϵ, which means that the expected profit of the platform increases when the support of θ is larger (implying, in our case, a larger uncertainty). In addition, since $\frac{d}{d\gamma}\Pi_p^{RF} = -\frac{\alpha n\gamma\left((1+\gamma)^2(3a^2+\epsilon^2)+(5+4\gamma)a^2\right)}{4(1+\gamma)^2(2+\gamma)^3} < 0$, the expected profit of the platform decreases when the competition between the sellers is more intense.

(iii) By Lemma 3, $\Pi_p^{FI} - \Pi_p^{RF} = \frac{\alpha n\left(3\gamma^2a^2+(1+\gamma)^2\epsilon^2\right)}{12(1+\gamma)(2+\gamma)^2} > 0$ and $\Pi_p^{RF} - \Pi_p^{NI} = \frac{\alpha n\left((1+\gamma)^2\epsilon^2-\gamma^2a^2\right)}{4(1+\gamma)(2+\gamma)^2}$, which is positive because $\Psi = \frac{\epsilon}{a} > \frac{\gamma}{1+\gamma}$.

(iv) $\frac{d}{d\gamma}\left(\Pi_p^{RF} - \Pi_p^{NI}\right) = -\frac{\alpha n\gamma\left((1+\gamma)^2\epsilon^2-\gamma^2a^2+2(2+\gamma)a^2\right)}{4(1+\gamma)(2+\gamma)^3}$, which is negative because $\Psi = \frac{\epsilon}{a} > \frac{\gamma}{1+\gamma}$. ■

Proof of Proposition 5. (i) In what follows, we provide the explicit set of equations to find $(\theta_1^{(k)}, \theta_2^{(k)}, ..., \theta_{k-1}^{(k)})$, which divides the support $[-\epsilon, \epsilon]$ into k intervals, such that the platform will always convey the correct message (i.e., $m^*(\theta) = I_j^{(k)}$ if and only if $\theta \in I_j^{(k)}$). The strategy of this proof is as follows. First, we characterize the conditions that ensure that a platform observing the value θ will not choose to report an adjacent interval instead of the one containing θ. We then characterize the set of equations that ensures that no platform type wishes to deviate to an adjacent interval. This set of equations characterizes a unique partition for the entire support. Finally, we show that in this partition, no platform type wishes to deviate to any other interval (not only to an adjacent one).

We start by seeking values of $\theta_j^{(k)}$ satisfying the two conditions for truthtelling when the platform considers the possibility of reporting an adjacent interval instead of the correct one (i.e., reporting $(\theta_{j-1}^{(k)}, \theta_j^{(k)})$ when $\theta \in (\theta_j^{(k)}, \theta_{j+1}^{(k)})$, $j = 1, ..., k-1$, or vice versa). Following the proof of Proposition 2, these conditions are

$$\left(a(1+\gamma) + \theta(2+\gamma) - E[\theta|I_j^{(k)}]\right)\left(a + E[\theta|I_j^{(k)}]\right) \geq$$

$$\left(a(1+\gamma) + \theta(2+\gamma) - E[\theta|I_{j+1}^{(k)}]\right)\left(a + E[\theta|I_{j+1}^{(k)}]\right),\ \forall\theta \in I_j^{(k)}$$

$$\left(a(1+\gamma) + \theta(2+\gamma) - E[\theta|I_j^{(k)}]\right)\left(a + E[\theta|I_j^{(k)}]\right) \leq$$

$$\left(a(1+\gamma)+\theta(2+\gamma)-E[\theta|I_{j+1}^{(k)}]\right)\left(a+E[\theta|I_{j+1}^{(k)}]\right),\ \forall\theta\in I_{j+1}^{(k)}$$

where $E[\theta|I_s^{(k)}]=\frac{\int_{\theta\in I_s^{(k)}}\theta f(\theta)d\theta}{\int_{\theta\in I_s^{(k)}}f(\theta)d\theta}=\frac{\theta_{s-1}^{(k)}+\theta_s^{(k)}}{2}$, which can be simplified to $\theta_j^{(k)}\geq\theta(2+\gamma)+a\gamma-\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}}{2}$, $\forall\theta\in(\theta_{j-1}^{(k)},\theta_j^{(k)})$ and $\theta_j^{(k)}\leq\theta(2+\gamma)+a\gamma-\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}}{2}$, $\forall\theta\in(\theta_j^{(k)},\theta_{j+1}^{(k)})$. Due to the linearity of the inequalities in θ, we can further simplify them to $\theta_j^{(k)}\geq\theta_j^{(k)}(2+\gamma)+a\gamma-\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}}{2}$ and $\theta_j^{(k)}\leq\theta_j^{(k)}(2+\gamma)+a\gamma-\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}}{2}$, which can then be written as $\theta_j^{(k)}\leq\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}-2a\gamma}{1+\gamma}$ and $\theta_j^{(k)}\geq\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}-2a\gamma}{1+\gamma}$. Therefore, there exists a unique solution $\theta_j^{(k)}=\frac{\theta_{j-1}^{(k)}+\theta_{j+1}^{(k)}-2a\gamma}{1+\gamma}$, which satisfies both inequalities, and it can be written as

$$\theta_{j-1}^{(k)}-2(1+\gamma)\theta_j^{(k)}+\theta_{j+1}^{(k)}=2a\gamma. \tag{14}$$

Since $\theta_0^{(k)}=-\epsilon$ and $\theta_k^{(k)}=\epsilon$, the set of $k-1$ partition points in the region-forecast equilibrium can be calculated by $(\theta_1^{(k)},\theta_2^{(k)},...,\theta_{k-1}^{(k)})^T=A^{-1}\cdot(2a\gamma+\epsilon,2a\gamma,...,2a\gamma,2a\gamma-\epsilon)^T$, where matrix A is $(a_{i,j})_{(k-1)\times(k-1)}$ with $a_{i,i}=-2(1+\gamma)$, $a_{i+1,i}=a_{i,i+1}=1$, $i=1,...,k-1$, and all other entries are zero.

We prove now, by contradiction, that there is a unique set of $k-1$ partition points $(\theta_1^{(k)},...,\theta_{k-1}^{(k)})$ that solves the above set of linear equations. Suppose $(\theta_1^{(k)},...,\theta_{k-1}^{(k)})$ is a legitimate partition, and suppose there exists an additional legitimate partition in which the first partition point, denoted by $\bar{\theta}_1$, satisfies $\bar{\theta}_1-\theta_1^{(k)}=\delta>0$. Then, by applying equation (14) on the first three points of the two partitions, we obtain: $-\epsilon-2(1+\gamma)\theta_1^{(k)}+\theta_2^{(k)}=2a\gamma$ and $-\epsilon-2(1+\gamma)\bar{\theta}_1+\bar{\theta}_2=2a\gamma$, so that the second partition point in the additional partition satisfies $\bar{\theta}_2-\theta_2^{(k)}=2(1+\gamma)\delta$. Note that $\bar{\theta}_2-\theta_2^{(k)}$ is proportional to δ and is also larger than δ. Similarly, suppose that $\bar{\theta}_{j-1}-\theta_{j-1}^{(k)}=\omega_{j-1}\delta$ and $\bar{\theta}_j-\theta_j^{(k)}=\omega_j\delta$, where $1\leq\omega_{j-1}<\omega_j$; then, by applying equation (14) in a similar manner as above, we obtain $\bar{\theta}_{j+1}-\theta_{j+1}^{(k)}=(2(1+\gamma)\omega_j-\omega_{j-1})\,\delta=((1+2\gamma)\omega_j+(\omega_j-\omega_{j-1}))\,\delta>(1+2\gamma)\omega_j\delta$, which is proportional to δ and is also larger than $\omega_j\delta$. Then, by induction, the gaps $\bar{\theta}_j-\theta_j^{(k)}, j=1,2,...,k$, are an increasing series, where all elements are proportional to δ, so that $\bar{\theta}_k>\theta_k^{(k)}=\epsilon$. Therefore, the additional partition is not legitimate.

Now, suppose that $\theta_1^{(k)}-\bar{\theta}_1=\bar{\delta}>0$. Then, by applying equation (14) on the first three points of the two partitions, we obtain: $-\epsilon-2(1+\gamma)\theta_1^{(k)}+\theta_2^{(k)}=2a\gamma$ and $-\epsilon-2(1+\gamma)\bar{\theta}_1+\bar{\theta}_2=2a\gamma$, so that the second partition point in the additional partition satisfies $\theta_2^{(k)}-\bar{\theta}_2=2(1+\gamma)\bar{\delta}$. Note that $\theta_2^{(k)}-\bar{\theta}_2$ is proportional to $\bar{\delta}$ and is also larger than $\bar{\delta}$. Similarly, suppose that $\theta_{j-1}^{(k)}-\bar{\theta}_{j-1}=\omega_{j-1}\bar{\delta}$ and $\theta_j^{(k)}-\bar{\theta}_j=\omega_j\bar{\delta}$, where $1\leq\omega_{j-1}<\omega_j$; then, by applying equation (14) in a similar manner as above, we obtain $\theta_{j+1}^{(k)}-\bar{\theta}_{j+1}=(2(1+\gamma)\omega_j-\omega_{j-1})\,\bar{\delta}=((1+2\gamma)\omega_j+(\omega_j-\omega_{j-1}))\,\bar{\delta}>(1+2\gamma)\omega_j\bar{\delta}$, which is proportional to $\bar{\delta}$ and is also larger than $\omega_j\bar{\delta}$. Then, by induction, the gaps $\theta_j^{(k)}-\bar{\theta}_j, j=1,2,...,k$, are an increasing series, where all elements are proportional to $\bar{\delta}$, so that $\epsilon=\theta_k^{(k)}>\bar{\theta}_k$. Therefore, the additional partition is not legitimate.

Finally, we prove that if it is more profitable to report $m(\theta)=I_j^{(k)}$ rather than $m(\theta)=I_{j-1}^{(k)}$ or

$m(\theta) = I_{j+1}^{(k)}$ when $\theta \in I_j^{(k)}$, then it is also more profitable to report $m(\theta) = I_j^{(k)}$ than to report any other interval. The payoff of the platform is a concave quadratic function of $E[\theta|I_j^{(k)}]$, and the value of $E[\theta|I_j^{(k)}]$ increases monotonically in j for a given partition. Therefore, if $E[\theta|I_j^{(k)}]$ provides a higher payoff to the platform than $E[\theta|I_{j-1}^{(k)}]$ and $E[\theta|I_{j+1}^{(k)}]$, then it also provides a higher payoff than any other interval in the partition.

(ii) In order to find properties of the partition, we rewrite equation (14) as follows: $\theta_{j+1}^{(k)} - \theta_j^{(k)} = \theta_j^{(k)} - \theta_{j-1}^{(k)} + 2\gamma(a + \theta_j^{(k)})$. This expression shows that each interval is larger than its predecessor (i.e., $l_{j+1}^{(k)} = \theta_{j+1}^{(k)} - \theta_j^{(k)} > \theta_j^{(k)} - \theta_{j-1}^{(k)} = l_j^{(k)}$), because $a + \theta_j^{(k)} > 0$. Furthermore, because $\theta_j^{(k)}$ increases in j, each interval is longer than its predecessor. ■

Proof of Proposition 6. (i) Based on the proof of Proposition 5(i), if $\delta > 0$, then the gaps $\bar{\theta}_j - \theta_j^{(k)}, j = 1, 2, ..., k$, are an increasing series, where all elements are proportional to δ, so there exists a value of δ such that $\bar{\theta}_{k-1} = \epsilon$. This result proves that if an equilibrium partition into k intervals exists, then a partition into $k-1$ intervals can always be achieved for some value of δ.

(ii) (a) When the support of θ is divided into k intervals, by proof of Proposition 5(ii), we can express the length of interval $j-1\,(j = 2, 3, .., k)$ as the following linear combination of its successive intervals $(j, .., k)$ with positive coefficients (independent of k):

$$l_{j-1}^{(k)} = l_j^{(k)} - 2\gamma(a + \theta_{j-1}^{(k)}) = l_j^{(k)} - 2\gamma\left(a + \theta_k^{(k)} - \sum_{t=j}^{k} l_t^{(k)}\right) = -2\gamma(a + \theta_k^{(k)}) + (1 + 2\gamma) l_j^{(k)} + 2\gamma \sum_{t=j+1}^{k} l_t^{(k)}, \quad (15)$$

where $l_k^{(k)} = \theta_k^{(k)} - \theta_{k-1}^{(k)}$ and $\theta_k^{(k)} = \epsilon$. We prove the claim $l_{j+1}^{(k+1)} < l_j^{(k)}, j = 1, .., k$ by contradiction. Suppose that dividing the support of θ into $k+1$ intervals results in a partition in which $l_{k+1}^{(k+1)} > l_k^{(k)}$. Then, by equation (15), $l_k^{(k+1)} > l_{k-1}^{(k)}$. By combining the last two inequalities with equation (15), we obtain $l_{k-1}^{(k+1)} > l_{k-2}^{(k)}$. Repeating the process iteratively yields $l_{j+1}^{(k+1)} > l_j^{(k)}$ for all $j = 1, .., k$, implying that $\sum_{j=1}^{k} l_j^{(k+1)} > \sum_{j=1}^{k} l_j^{(k)}$. Since $\sum_{j=1}^{k+1} l_j^{(k+1)} = \sum_{j=1}^{k} l_j^{(k)} = 2\epsilon$, the latter inequality cannot exist and the assumption that $l_{k+1}^{(k+1)} > l_k^{(k)}$ is contradicted. Therefore, we conclude that $l_{k+1}^{(k+1)} < l_k^{(k)}$, and thus, by equation (15), $l_{j+1}^{(k+1)} < l_j^{(k)}$ for all $j = 1, ..k$. To complete the proof, recall that, by Proposition 5(ii), $l_j^{(k+1)} < l_{j+1}^{(k+1)}$ for all $j = 1, ..k$.

Since $l_{j+1}^{(k+1)} < l_j^{(k)}$, we conclude that $\sum_{t=1}^{j} l_t^{(k+1)} < \sum_{t=1}^{j} l_t^{(k)}$ implying that $\theta_j^{(k+1)} = -\epsilon + \sum_{t=1}^{j} l_t^{(k+1)} < -\epsilon + \sum_{t=1}^{j} l_t^{(k)} = \theta_j^{(k)}$. Similarly, since $l_{j+1}^{(k+1)} < l_j^{(k)}$, we conclude that $\sum_{t=j+1}^{k} l_{t+1}^{(k+1)} < \sum_{t=j+1}^{k} l_t^{(k)}$, implying that $\theta_j^{(k+1)} = \epsilon - \sum_{t=j+1}^{k} l_{t+1}^{(k+1)} < \epsilon - \sum_{t=j+1}^{k} l_t^{(k)} = \theta_j^{(k)}$.

(ii) (b) In order to prove the claim, we start by stating the following auxiliary lemma regarding the sums of the cubic interval lengths in two equilibrium partitions — one into k intervals and the other into $k+1$ intervals:

Lemma. A1. $\sum_{j=1}^{k+1} \left(l_j^{(k+1)}\right)^3 < \sum_{j=1}^{k} \left(l_j^{(k)}\right)^3$.

Proof. From part (a) of Proposition 6(i), we conclude that $l_1^{(k+1)}$ is the smallest interval in both of the partitions (into k and $k+1$ intervals). For simplicity of calculation, let $\delta_j \equiv l_j^{(k)} - l_{j+1}^{(k+1)}$, so that $\delta_j > 0$ and $\sum_{j=1}^{k} \delta_j = l_j^{(k+1)}$. Thus, we state:

$$\sum_{j=1}^{k} \left(l_j^{(k)}\right)^3 - \sum_{j=1}^{k+1} \left(l_j^{(k+1)}\right)^3 = \sum_{j=1}^{k} \left(\left(l_j^{(k)}\right)^3 - \left(l_{j+1}^{(k+1)}\right)^3\right) - \left(l_1^{(k+1)}\right)^3$$

$$= \sum_{j=1}^{k} \left(\left(l_{j+1}^{(k+1)} + \delta_j\right)^3 - \left(l_{j+1}^{(k+1)}\right)^3\right) - \left(\sum_{j=1}^{k} \delta_j\right)^3$$

$$= \sum_{j=1}^{k} \left(3\left(l_{j+1}^{(k+1)}\right)^2 \delta_j + 3 l_{j+1}^{(k+1)} \delta_j^2 + \delta_j^3\right) - \left(\sum_{j=1}^{k} \delta_j\right)^3 \geqslant \sum_{j=1}^{k} \left(3\left(\sum_{t=1}^{k} \delta_t\right)^2 \delta_j\right) - \left(\sum_{j=1}^{k} \delta_j\right)^3$$

$$= 3\left(\sum_{t=1}^{k} \delta_t\right)^2 \sum_{j=1}^{k} \delta_j - \left(\sum_{j=1}^{k} \delta_j\right)^3 = 2\left(\sum_{j=1}^{k} \delta_j\right)^2 = 2\left(l_1^{(k+1)}\right)^3 > 0.$$

Now, the expected variance of θ for a given partition into k intervals is expressed by $E[V[\theta|I^{(k)}]] = \sum_{j=1}^{k} \left(\frac{(\theta_j - \theta_{j-1})^2}{12} P(\theta \in I_j^{(k)})\right) = \sum_{j=1}^{k} \left(\frac{l_j^2}{12} \cdot \frac{l_j}{2\epsilon}\right) = \frac{1}{24\epsilon} \sum_{j=1}^{k} l_j^3$. Thus, by Lemma A1, $E[V[\theta|I^{(k+1)}]] < E[V[\theta|I^{(k)}]]$.

(ii) (c) By equation (13), $\pi_p(\theta, I^{(k)}) = \frac{\alpha n(a(1+\gamma)+\theta(2+\gamma)-E[\theta|I^{(k)}])(a+E[\theta|I^{(k)}])}{(2+\gamma)^2}$. Therefore, using the following two properties: $E[E[\theta|I^{(k)}]] = E[\theta] = 0$ and

$E[\theta \cdot E[\theta|I^{(k)}]] = E[E[\theta \cdot E[\theta|I^{(k)}]|I^{(k)}] = E[E[\theta|I^{(k)}] \cdot E[\theta|I^{(k)}]] = E[E^2[\theta|I^{(k)}]]$

$= E^2[E[\theta|I^{(k)}]] + V[E[\theta|I^{(k)}]] = V[E[\theta|I^{(k)}]]$,

we obtain

$\Pi_p^{(k)} = E[\pi_p(\theta, I^{(k)})] = \frac{\alpha n\left(a\left(a(1+\gamma)+(2+\gamma)E[\theta]-E[E[\theta|I^{(k)}]\right)+E\left[\left(a(1+\gamma)+\theta(2+\gamma)-E[\theta|I^{(k)}]\right)E[\theta|I^{(k)}]\right]\right)}{(2+\gamma)^2}$

$= \frac{\alpha n\left(a\left(a(1+\gamma)-E[E[\theta|I^{(k)}]\right)+a(1+\gamma)E[E[\theta|I^{(k)}]]+(2+\gamma)E[\theta\cdot E[\theta|I^{(k)}]-E[E^2[\theta|I^{(k)}]]\right)}{(2+\gamma)^2}$

$= \frac{\alpha n\left(a^2(1+\gamma)+(2+\gamma)V[E[\theta|I^{(k)}]]-V[E[\theta|I^{(k)}]]\right)}{(2+\gamma)^2} = \frac{\alpha n(1+\gamma)\left(a^2+V[E[\theta|I^{(k)}]]\right)}{(2+\gamma)^2}$.

The claim $\Pi_p^{(k+1)} > \Pi_p^{(k)}$ is proved by applying part (b) of Proposition 6(ii) , whereas the claim $\Pi_i^{(k+1)} > \Pi_i^{(k)}$ is straightforward from the proportionality between the sellers' and the platform's expected profits. ■

Proof of Proposition 7. From part (a) in the proof of Proposition 6(ii), $l_j^{(k)} - l_{j-1}^{(k)} = 2\gamma(a + \theta_{j-1}^{(k)}) > 2\gamma(a-\epsilon)$. Therefore, and because $l_1^{(k)} > 0$, we infer that $l_2^{(k)} > 2\gamma(a-\epsilon)$, $l_3^{(k)} > l_2^{(k)} + 2\gamma(a-\epsilon) > 4\gamma(a-\epsilon)$,..., $l_k^{(k)} > 2\gamma(a-\epsilon)(k-1)$; thus, $\sum_{j=1}^{k} l_j^{(k)} > 2\gamma(a-\epsilon)\sum_{j=1}^{k-1} j = \gamma(a-\epsilon)k(k-1)$. Hence, and since $\Theta = [-\epsilon, \epsilon]$, a necessary condition for the existence of a partition with k intervals is $\gamma(a-\epsilon)k(k-1) < 2\epsilon$. Thus, any k that does not satisfy this condition (i.e., $k \geq \bar{k} = \frac{1}{2} + \sqrt{\frac{2\Psi}{\gamma(1-\Psi)} + \frac{1}{4}}$) cannot result in a partition equilibrium. ■

Proof of Proposition 8. Assume that upon receiving a message m, the sellers believe that $\theta \in m$. Based on this belief system, each seller solves the problem:

$$max_{p_i}\left\{E[\pi_i|m]=(1-\alpha)p_i\left(a+E[\theta|m]-(1+\gamma_i)p_i+\gamma_{3-i}p_{3-i}\right)\right\}, i\in\{1,2\}$$

where $\gamma_1=\gamma-\Delta$, $\gamma_2=\gamma+\Delta$ and $\Delta\in(0,\gamma)$. Because the payoff of seller i is a concave function of p_i, we solve the set of the two first-order conditions associated with the optimization problems above, resulting in $p_i=\frac{(a+E[\theta|m])\left(2+3(\gamma-\Delta\cdot(-1)^i)\right)}{4(1+2\gamma)+3(\gamma^2-\Delta^2)}, i\in\{1,2\}$. For these prices, the payoff of the platform as a function of θ and m is given by:

$$\pi_p(\theta,m)=\frac{2\alpha\left(Aa+B\theta-CE[\theta|m]\right)\left(a+E[\theta|m]\right)}{\left(4+8\gamma+3(\gamma^2-\Delta^2)\right)^2}, \tag{16}$$

where $A\equiv 4(1+4\gamma)+3(1+3\gamma)(\gamma^2-\Delta^2)+18\gamma^2$, $B\equiv(2+3\gamma)\left(4(1+2\gamma)+3(\gamma^2-\Delta^2)\right)$ and $C\equiv 2(2+6\gamma+3\gamma^2)+3(\gamma^2-\Delta^2)$. We seek values of θ_1 that provide a partition satisfying the conditions for truthtelling for any value of $\theta\in$:

$$\pi_p(\theta,m)\geq\pi_p(\theta,\overline{m})\quad\forall\theta\in m\ and\ \overline{m}\neq m. \tag{17}$$

These two conditions can be written explicitly as:

$$\left(Aa+B\theta-CE[\theta|I_j]\right)\left(a+E[\theta|I_j]\right)\geq\left(Aa+B\theta-CE[\theta|I_{3-j}]\right)\left(a+E[\theta|I_{3-j}]\right), \forall\theta\in I_j, j=1,2.$$

Since $E[\theta|I_j]=\frac{\int_{\theta\in I_j}\theta f(\theta)d\theta}{\int_{\theta\in I_j}f(\theta)d\theta}=\frac{\theta_1+\epsilon\cdot(-1)^j}{2}$, the two conditions above can be rewritten as

$$\left(Aa+B\theta-C\frac{\theta_1-\epsilon}{2}\right)\left(a+\frac{\theta_1-\epsilon}{2}\right)\geq\left(Aa+B\theta-C\frac{\theta_1+\epsilon}{2}\right)\left(a+\frac{\theta_1+\epsilon}{2}\right), \forall\theta\in I_j, j=1,2,$$

which can be simplified to $\theta_1\geq\frac{(A-C)a+B\theta}{C}$ $\forall\theta\in(-\epsilon,\theta_1)$ and $\theta_1\leq\frac{(A-C)a+B\theta}{C}$ $\forall\theta\in(\theta_1,\epsilon)$. Due to the linearity of the inequalities in θ, we can further simplify them to $\theta_1\geq\frac{(A-C)a+B\theta_1}{C}$ and $\theta_1\leq\frac{(A-C)a+B\theta_1}{C}$. Therefore, there exists a unique solution $\theta_1^A=-a\frac{A-C}{B-C}=-\frac{a\gamma\left(4(1+3\gamma)+9(\gamma^2-\Delta^2)\right)}{4(1+4\gamma)+3(1+3\gamma)(\gamma^2-\Delta^2)+18\gamma^2}<0$, which satisfies both inequalities. The condition for existence of a region-forecast equilibrium is derived from the condition $\theta_1^A>-\epsilon$. In addition, the partition point θ_1^A decreases with γ since $\frac{\partial}{\partial\gamma}\theta_1^A=-\frac{(2+3\gamma)\left(8+36\gamma+30\gamma^2+27\gamma^3+24(\gamma^2-\Delta^2)\right)+27\Delta^4}{\left(4(1+4\gamma)+3(1+3\gamma)(\gamma^2-\Delta^2)+18\gamma^2\right)^2}<0$. ■

Proof of Corollary 2. The claim is proved since $\frac{\partial}{\partial\Delta}\theta_1^A=\frac{12\gamma\Delta(2+3\gamma)^2}{\left(4(1+4\gamma)+3(1+3\gamma)(\gamma^2-\Delta^2)+18\gamma^2\right)^2}>0$. ■

Proof of Proposition 9. (i) By substituting θ_1^A into eq. 16, we obtain $\Pi_p^A=ua^2+v\epsilon^2$. (ii) Let $\tau\equiv\gamma^2-\Delta^2>0$. Thus, by their definition above Proposition 9, $u=\frac{\alpha(2+3\gamma)(48\gamma^2+27\gamma\tau+36\gamma+6\tau+8)}{2(8\gamma+3\tau+4)(18\gamma^2+9\gamma\tau t+16\gamma+3\tau+4)}$ and

$v = \frac{\alpha(18\gamma^2+9\gamma\tau+16\gamma+3\tau+4)}{2(8\gamma+3\tau+4)^2}$. The claims $\frac{du}{d\Delta} > 0$ and $\frac{dv}{d\Delta} > 0$ are true since

$$\frac{du}{d\tau} = -\frac{3240(\frac{2}{3}+\gamma)\left(\gamma^4 + (\frac{6}{5}\tau + \frac{8}{5})\gamma^3 + (\frac{27}{80}\tau^2 + \frac{13}{10}\tau + \frac{47}{45})\gamma^2 + (\frac{3}{16}\gamma + \frac{1}{40})(\tau + \frac{4}{3})^2\right)}{(8\gamma+3\tau+4)^2(18\gamma^2+9\gamma\tau+16\gamma+3\tau+4)^2} < 0,$$

$$\frac{dv}{d\tau} = -\frac{3\left(12\gamma^2 + (9\tau+12)\gamma + 3\tau + 4\right)}{2\left(8\gamma+3\tau+4\right)^3} < 0,$$

and $\frac{d\tau}{d\Delta} < 0$. ■

Proof of Proposition 10. The outline of the proof is as follows. We first characterize, in Lemma 4, the inverse demand function. We then, in Lemma 5, outline the partition equilibrium prices and quantities. Based on these two lemmas, we develop the expressions for the consumer surplus for the various information settings. Finally, we compare the consumer surplus and social welfare across these settings. ■

Lemma 4. The inverse demand function of a seller who offers a quantity x given that each of the other sellers offers a quantity q is $p(x,q) = a + \theta - \frac{(n-1+\gamma)x+(n-1)\gamma q}{n(1+\gamma)-1}$. ■

Proof of Lemma 4. We build a set of n symmetrical equations where equation i is a variation of the demand function of seller i (as it appears in equation (1)).

$$\begin{cases} (n-1)(1+\gamma)p_1 - \gamma p_2 - \gamma p_3 - ... - \gamma p_n = (n-1)(a-\theta-q_1) \\ -\gamma p_1 + (n-1)(1+\gamma)p_2 - \gamma p_3 - ... - \gamma p_n = (n-1)(a-\theta-q_2) \\ -\gamma p_1 - \gamma p_2 - (n-1)(1+\gamma)p_3 - ... - \gamma p_n = (n-1)(a-\theta-q_3) \\ \vdots \\ -\gamma p_1 - \gamma p_2 - \gamma p_3 - ... + (n-1)(1+\gamma)p_n = (n-1)(a-\theta-q_n) \end{cases}$$

In order to obtain the inverse demand functions p_i, $i = 1,..,n$, we start by eliminating all p_i except for p_1 by performing algebraic manipulations on the above equations. In particular, we multiply equations $2, 3, ..., n$ by $\frac{\gamma}{n-1+\gamma}$, and then sum all these equations. The result of this summation is $(n-1)\left(1+\gamma-\frac{\gamma^2}{n-1+\gamma}\right)p_1 = (n-1)\left((a+\theta)\left(1+\frac{(n-1)\gamma}{n-1+\gamma}\right) - q_1 - \frac{\gamma}{n-1+\gamma}\sum_{i=2}^n q_i\right)$, from which we extract $p_1 = a + \theta - \frac{(n-1+\gamma)q_1+\gamma\sum_{i=2}^n q_i}{n(1+\gamma)-1}$. Due to the symmetry of the equations, we obtain $p_i = a + \theta - \frac{(n-1+\gamma)q_i+\gamma\sum_{t\in N_{-i}} q_t}{n(1+\gamma)-1}, i = 1,..,n$. Therefore, if one seller offers quantity x given that each of the other sellers offers quantity q, then the inverse demand function of the former seller would be $p(x,q) = a + \theta - \frac{(n-1+\gamma)x+(n-1)\gamma q}{n(1+\gamma)-1}$. ■

Lemma 5. The partition equilibrium price and quantity are, respectively, $p_j^{RF} = \frac{a}{2(1+\gamma)} + \frac{\epsilon\cdot(-1)^j}{2(2+\gamma)}$ and $q_j^{RF} = \frac{1+2\gamma}{2(1+\gamma)}a + \theta - \frac{\epsilon\cdot(-1)^j}{2(2+\gamma)}, j = 1, 2$. ■

Proof of Lemma 5. The expression for p_j^{RF} is obtained by combining the following results: $p_j =$

$\frac{a+E[\theta|I_j]}{2+\gamma}$, $E[\theta|I_j] = \frac{\theta_1^*+\epsilon\cdot(-1)^j}{2}, j = 1,2$ and $\theta_1^* = -\frac{a\gamma}{1+\gamma}$ (see proof of Proposition **2**). The expression for q_j^{RF} is obtained by substituting the expression for p_j^{RF} into equation (1).

We next outline the expressions for the consumer surplus under the various settings. For the case of no information sharing, the consumer surplus is:

$$CS^{NI} = n\int_{\theta\in\Theta}\left[\int_0^{q^{NI}} p(q, q^{NI})dq - p^{NI}q^{NI}\right]\frac{1}{2\epsilon}d\theta = \frac{n(\gamma+n-1)\left(3(1+\gamma)^2a^2+(2+\gamma)^2\epsilon^2\right)}{6(n(1+\gamma)-1)(2+\gamma)^2};$$

for full information sharing, the consumer surplus is:

$$CS^{FI} = n\int_{\theta\in\Theta}\left[\int_0^{q^{FI}} p(q, q^{FI})dq - p^{FI}q^{FI}\right]\frac{1}{2\epsilon}d\theta = \frac{n(1+\gamma)^2(n-1+\gamma)(3a^2+\epsilon^2)}{6(2+\gamma)^2(n(1+\gamma)-1)};$$

and for the case of partial information (i.e., region forecast), the consumer surplus is (based on equation (10)):

$$CS^{RF} = \frac{n(n-1+\gamma)\left(3(2+\gamma)(2+7\gamma+10\gamma^2+4\gamma^3)a^2+(1+\gamma)^2(7+10\gamma+4\gamma^2)\epsilon^2\right)}{24(1+\gamma)^2(2+\gamma)^2(n(1+\gamma)-1)}.$$

Assuming that the partition equilibrium exists (i.e., $|\theta_1^*| < \epsilon$), we now calculate the following differences, which prove the claim:

$$CS^{NI} - CS^{RF} = \frac{n(n-1+\gamma)(3+2\gamma)(\epsilon^2(1+\gamma)^2-a^2\gamma^2)}{8(2+\gamma)^2(1+\gamma)^2(n(1+\gamma)-1)} = \frac{n(n-1+\gamma)(3+2\gamma)(\epsilon^2-(\theta_1^*)^2)}{8(2+\gamma)^2(n(1+\gamma)-1)} \geq 0,$$

$$CS^{RF} - CS^{FI} = \frac{n(n-1+\gamma)(3+2\gamma)(3\gamma^2a^2+(1+\gamma)^2\epsilon^2)}{24(1+\gamma)^2(2+\gamma)^2(n(1+\gamma)-1)} = \frac{n(n-1+\gamma)(3+2\gamma)(\epsilon^2+3(\theta_1^*)^2)}{24(2+\gamma)^2(n(1+\gamma)-1)} > 0.$$

(ii) From the above expressions for the consumer surplus, it is clear that the consumer surplus increases in ϵ. By Proposition 4, both the platform's and the sellers' payoffs (weakly) increase in ϵ. Thus, the social welfare, which is captured by $SW \equiv n\Pi_i + \Pi_p + CS$, also increases in the market uncertainty ϵ. ■