Online Appendix to "Coordination for Assembly"

Appendix OA.1: Additional Tables and Figures

Table OA.1 and Table OA.2 present summary statistics of cohorts' project effort levels in the two-contractor and four-contractor treatments, respectively.

Table OA.1 Summary Statistics of Cohorts' Project Effort Level (Two-Contractor Treatments)

Treatments	$M2N1$				$M2N2$				$M2N6$			
Cohorts	1	2	3	4	1	2	3	4	1	2	3	4
Mean	125.48	136.69	113.19	111.20	148.17	141.79	147.67	123.44	170.42	161.08	168.66	165.48
Std. Error	1.99	0.98	0.93	0.54	1.17	1.03	1.15	2.03	0.28	1.32	0.87	1.28
Min	115.48	127.16	110.00	110.00	141.71	132.31	140.48	111.64	169.16	148.69	157.76	149.76
Max	144.41	145.64	125.32	119.86	157.63	152.64	159.16	149.56	174.99	169.33	173.99	170.75

Table OA.2 Summary Statistics of Cohorts' Project Effort Level (Four-Contractor Treatments)

Treatments	$M4N1$				$M4N2$				$M4N6$			
Cohorts	1	2	3	4	1	2	3	4	1	2	3	4
Mean	110.36	110.29	110.11	110.27	110.88	110.28	110.68	110.36	135.62	131.32	151.51	136.91
Std. Error	0.38	0.21	0.08	0.22	0.63	0.19	0.38	0.33	1.41	1.12	1.92	1.68
Min	109.98	109.98	109.98	109.98	110.00	110.00	110.00	110.00	121.34	120.68	127.91	122.58
Max	117.65	113.62	111.32	114.47	120.78	113.32	116.98	116.65	143.21	139.39	159.57	146.85

Table OA.3 and Table OA.4 summarize the average per-period effort amounts and their comparisons.

Table OA.3 Experimental Results on Per-Period Effort Amount Decisions

Treatment	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
$M2N2$	25.76 (1.28)	25.42 (1.18)	25.51 (1.10)	23.14 (0.98)	23.04 (0.81)	23.38 (0.69)
$M4N2$	19.10 (0.16)	19.19 (0.21)	19.09 (0.23)	18.78 (0.09)	18.78 (0.09)	18.87 (0.13)
$M2N6$	32.05 (1.61)	31.85 (1.11)	30.05 (1.37)	26.20 (0.67)	24.56 (1.07)	23.86 (0.90)
$M4N6$	27.74 (0.75)	26.09 (0.80)	24.81 (0.83)	23.41 (0.96)	21.94 (0.93)	20.08 (0.67)

Note: The numbers are the average effort amount in each period for different treatments. Standard errors are in parenthesis.

Table OA.4 Experimental Results on Per-Period Effort Amount Decisions

Treatment	$P_1 > P_2$	$P_2 > P_3$	$P_3 > P_4$	$P_4 > P_5$	$P_5 > P_6$
$M2N2$	0.4129	0.4129	0.0045	0.4129	0.2580
$M4N2$	0.1715	0.1715	0.1715	0.4660	0.1715
$M2N6$	0.4227	0.1370	0.1370	0.0925	0.1939
$M4N6$	0.0198	0.0110	0.0085	0.0005	0.0085

Note: $P_i > P_j$ states that the average effort amount in period i is greater than the average effort in period j. All the p-values are generated using paired sample t-test and adjusted using the Benjamini-Hochberg method (Benjamini and Hochberg 1995) for multiple-hypothesis testing. For each pair of comparison samples, the Shapiro Wilk test could not reject the null hypothesis that the paired samples follow a normal distribution.

Table OA.5 presents the results of hypothesis tests in treatment $M4N6$. The hypothesis is that the provisional Lead contractor's effort level is less than the Lead threshold in each period.

Table OA.5 Provisional Lead Contractors' Performances in Treatment M4N6

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Effort Level	32.6	60.32	85.87	103.25	120.72	138.84
Threshold	38.33	70.93	98.65	104.2	121.58	139.05
p-value	0.014	0.014	0.014	0.497	0.497	0.514

Note: The Lead thresholds in period 1 are computed based on achieving (6). Since in period 1, the provisional Leader does not reach the Lead threshold, all the Lead thresholds in periods 2 to 6 are computed using the previous provisional Lead contractor's effort level based on the SPSE. All the p-values are generated using the one-sample t-test and adjusted using the Benjamini-Hochberg method (Benjamini and Hochberg 1995) for multiple-hypothesis testing. Wilcoxon signed-rank test results are consistent with the t-test results at the 10% significance level.

Table OA.6 presents the results of hypothesis tests in treatment $M4N6$. The hypothesis is that the Follow contractor's effort level is greater than the Follow threshold in each period.

Table OA.6 Follow Contractors' Performances in Treatment M4N6

	Period 1	Period 2	Period 3	Period 4	Period 5
Effort Level	22.88	48.21	72.65	96.44	118.68
Threshold	18.33	36.67	55	64.2	101.58
p-value	0.009	0.006	0.006	0.006	0.014

Note: The Follow thresholds in period 1 are computed based on achieving (7). Since in period 1, the provisional Leader does not reach the Lead threshold, all Follow thresholds in periods 2 to 5 are computed based on the SPSE. All the p-values are generated using the one-sample t-test and adjusted using the Benjamini-Hochberg method (Benjamini and Hochberg 1995) for multiple-hypothesis testing. The Follow threshold overlaps with the Lead threshold in the last period, and we omit the hypothesis for period 6 in this context, as it complements the hypothesis tested in period 6 of Table OA.5, and the adjusted p-value is 0.486. Wilcoxon signed-rank test results are consistent with the t-test results at the 10% significance level.

Table OA.7 presents the robustness check for the analysis in Table 8, using treatments theoretically capable of achieving the payoff-dominant project effort level (i.e., Treatments $M2N2$, $M2N6$, and $M4N6$).

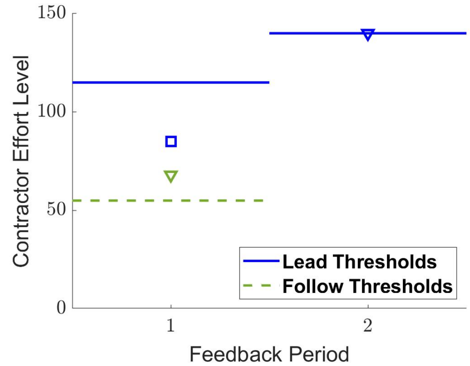
Table OA.7 Determinants of Contractor's Effort Decisions in Period 1

Coefficients	Subject Effort Amount in Period 1
Lead in Period 1 Previous Round	0.850^{***} (0.263)
Profit Previous Round	0.105^{***} (0.023)
Times Being Lead Previous Round	0.121 (0.085)
Round $\in \{1, \cdots, 20\}$	0.071 (0.043)
Group of Four Dummy	-3.303^{**} (1.409)
Six Feedback Periods Dummy	5.608^{**} (1.595)
Constant	21.026^{***} (1.285)
Number of Observations	2,128
Adjusted R^2	0.374

Note: *** p < 0.01, ** p <0.05, * p < 0.1. Standard errors (in parenthesis) are corrected for clustering at the cohort level.

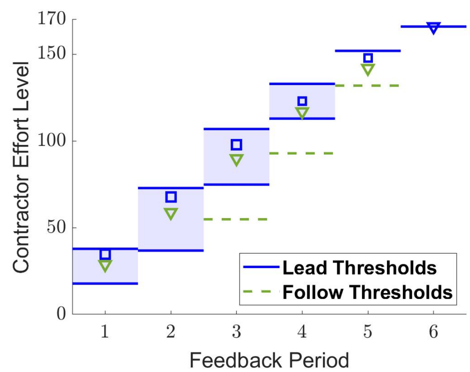
Figures OA.1 and OA.2 display effort levels of the provisional Lead contractor and the Follow contractor in treatments $M2N2$ and $M6N6$, respectively.

Figure OA.1 Effort Progress in Treatment $M2N2$



Note: Figure OA.1 illustrates the results in the $M2N2$ treatment. The x-axis of the figure represents the information feedback period, and the y-axis represents the contractor effort level. The highest and the lowest effort levels are denoted by □ and ∇.
The provisional Lead contractor and the Follow contractor are supposed to meet the Lead threshold (the solid line) and the Follow threshold (the dashed line), respectively.
The Lead and Follow thresholds in period 1 are computed based on achieving (6) and (7). All the thresholds in periods 2 to 6 are computed based on the realized effort levels and the SPSE. The Follow threshold overlaps with the Lead threshold in period 2 and is not plotted for brevity.

Figure OA.2 Effort Progress in Treatment $M2N6$



Note: Figure OA.2 illustrates the results in the $M2N6$ treatment. The x-axis of the figure represents the information feedback period, and the y-axis represents the contractor effort level. The highest and the lowest effort levels are denoted by □ and ∇.
The provisional Lead contractor exerts enough effort to stay within the Lead range (the shaded area) marked by the Lead thresholds (solid lines) in periods 1-4 and to meet the Lead thresholds in periods 5 and 6. The Follow contractor is supposed to meet the Follow threshold (the dashed line).
In period 1, the Lead range (the shaded area) is computed based on (4), with the upper and lower bounds of the range marked by solid lines. The Follow threshold characterized by the dashed line is computed based on (5). All the thresholds in periods 2 to 6 and Lead ranges in periods 2 to 4 are computed based on the realized effort levels and the SPSE. The Follow thresholds overlap with the Lead threshold in periods 1, 2, and 6 and are not plotted for brevity.

Appendix OA.2: Proofs and Technical Details

Proof of Proposition 1. Since the contractors only observe their own effort level, the proof is the same as the results in the assembly supply chain literature, e.g., Proposition 1 in Gerchak and Wang (2004). The details are omitted for brevity. ∎

Proof of Corollary 1. For any contractor $i \in \{1, 2, \cdots, M\}$, if $\hat{q}_{-i,N} = \underline{q}$, the best response for contractor i is $\hat{q}_{i,N} = \underline{q}$, which shows $\mathbf{\hat{q}_N} = (\underline{q}, \cdots \underline{q}, \cdots, \underline{q})$ is a Nash equilibrium; namely, $\mathbf{\hat{q}_N} \in \mathbf{\hat{E}_N}$.

(3) also holds true because for any $\hat{q}_{i,N}$, $\pi_i(\hat{q}_{i,N}; \mathbf{\hat{q}_{-i,N}})$ is minimized when $\mathbf{\hat{q}_{-i,N}} = (\underline{q}, \cdots \underline{q}, \cdots, \underline{q})$, and $(\underline{q}, \cdots \underline{q}, \cdots, \underline{q}) \in \mathbf{\hat{E}_{-i,N}}$. Moreover, given $\mathbf{\hat{q}_{-i,N}} = (\underline{q}, \cdots \underline{q}, \cdots, \underline{q})$, we have

$$\underset{\hat{q}_{i,N} \in \mathbf{\hat{E}_{i,N}}}{\arg\max}\ \pi_i(\hat{q}_{i,N}; \underline{q}, \cdots \underline{q}, \cdots, \underline{q}) = \underline{q}. \tag{23}$$

Based on Definition 1, $\mathbf{\hat{q}_N} = (\underline{q}, \cdots, \underline{q})$ is a secure equilibrium.

Finally, the uniqueness follows through because $\hat{q}_{i,N} = \underline{q}$ is the unique maximizer of $\underset{\hat{q}_{i,N} \in \mathbf{\hat{E}_{i,N}}}{\arg\max}\ \pi_i(\hat{q}_{i,N}; \underline{q}, \cdots \underline{q}, \cdots, \underline{q})$ for any $i \in \{1, 2, \ldots, M\}$. ∎

Definition of SPSE. It is worth noting that the Secure equilibrium defined in Definition 1 applies to the N-stage coordination game without information feedback, which is a static game. However, when incorporating the information-feedback mechanism, the N-stage coordination game transforms into a dynamic game with complete information. Therefore, to characterize the dynamic version of the secure equilibrium, we need to refine this concept within the framework of subgame-perfect equilibrium (SPE). To formally define SPSE, we start with a series of necessary definitions and notations.

In a M-contractor N-period coordination game under the information feedback setting with all contractors' initial effort level vector $\mathbf{\hat{q}_0} = (q_{1,0}, \cdots, q_{M,0})$, for any period $n \in \{1, 2, \cdots, N\}$, let the state of period n be the effort level vector of all contractors at the beginning of the period, $\mathbf{\hat{q}_{n-1}} = (\hat{q}_{1,n-1}, \cdots, \hat{q}_{M,n-1})$. Note that, in the main analysis, we assume zero initial effort levels. However, for convenience in the proof of Proposition 2, we relax this assumption and allow for nonzero initial effort levels. This generalization is solely for the convenience of the proof; the case of zero initial effort levels in the main model is a special case of this general setting. Accordingly, under this general setting, the effort level of contractor i at the end of period n includes the initial effort that $\hat{q}_{i,n} = \sum_{t=0}^{n} q_{i,t}$.

For any $n \in \{1, 2, \cdots, N\}$, we denote by $\mathbf{q_{i,n:N}} \triangleq (q_{i,n}, \cdots, q_{i,N})$ contractor i's strategy for periods n to N, and by $\hat{q}_{i,n:N} \triangleq \sum_{t=n}^{N} q_{i,t}$ contractor i's total effort exerted in periods n to N. Then, for any $n \in \{1, 2, \ldots, N\}$, let $(\mathbf{q_{1,n:N}}, \cdots, \mathbf{q_{M,n:N}})|\mathbf{\hat{q}_{n-1}}$ be the restriction of the strategy profile $(\mathbf{q_{1,1:N}}, \cdots, \mathbf{q_{M,1:N}})|\mathbf{\hat{q}_0}$ ($(\mathbf{q_{1,1:N}}, \cdots, \mathbf{q_{M,1:N}})$ for notational simplicity) to the subgame continuing from period n given the state of period n, $\mathbf{\hat{q}_{n-1}}$.

Moreover, given $(\mathbf{q_{1,n:N}}, \cdots, \mathbf{q_{M,n:N}})|\mathbf{\hat{q}_{n-1}}$, the restriction of a strategy profile $(\mathbf{q_{1,1:N}}, \cdots, \mathbf{q_{M,1:N}})$ to the subgame continuing from period n with the state $\mathbf{\hat{q}_{n-1}}$, we denote by $\mathbf{\hat{q}_{n:N}} \triangleq (\hat{q}_{1,n:N}, \cdots, \hat{q}_{M,n:N})$ the vector of all contractors' total effort exerted in periods n to N. For a subgame continuing from period n with the state of period n, $\mathbf{\hat{q}_{n-1}}$, we denote by $\mathbf{\hat{E}_{n:N}}$ the set of the restrictions of all SPEs to the subgame; we denote by $\hat{\mathbf{E}}_{\mathbf{i,n:N}}$ the set of subgame-perfect Nash equilibrium strategies for contractor i, such that each equilibrium strategy is also the i^{th} element of an restriction in $\hat{\mathbf{E}}_{\mathbf{n:N}}$; we denote by $\hat{\mathbf{E}}_{\mathbf{-i,n:N}}$ the set of $M-1$-tuple of subgame-perfect Nash equilibrium strategies for all contractors but i, such that each $M-1$-tuple is a subset of an restriction in $\hat{\mathbf{E}}_{\mathbf{n:N}}$. In the subgame, we use $\pi_i(\hat{\mathbf{q}}_{\mathbf{n:N}};\hat{\mathbf{q}}_{\mathbf{n-1}})$ and $\pi_i(\hat{q}_{i,n:N};\hat{\mathbf{q}}_{\mathbf{-i,n:N}},\hat{\mathbf{q}}_{\mathbf{n-1}})$ instead of $\pi_i(\hat{\mathbf{q}}_{\mathbf{N}})$ and $\pi_i(\hat{q}_{i,N};\hat{\mathbf{q}}_{\mathbf{-i,N}})$ to emphasize that the profit critically depends on contractors' decisions from periods n to N since the state of period n, $\hat{\mathbf{q}}_{\mathbf{n-1}}$, is given. When there is no ambiguity, we drop the dependence on $\hat{\mathbf{q}}_{\mathbf{n-1}}$ and use $\pi_i(\hat{\mathbf{q}}_{\mathbf{n:N}})$ and $\pi_i(\hat{q}_{i,n:N};\hat{\mathbf{q}}_{\mathbf{-i,n:N}})$ for $\pi_i(\hat{\mathbf{q}}_{\mathbf{n:N}};\hat{\mathbf{q}}_{\mathbf{n-1}})$ and $\pi_i(\hat{q}_{i,n:N};\hat{\mathbf{q}}_{\mathbf{-i,n:N}},\hat{\mathbf{q}}_{\mathbf{n-1}})$ for ease of exposition.

We now define SPSE in Definition 2 below.

Definition 2 **(SPSE in N-stage coordination game with feedback)**. *In an M-contractor N-period coordination game under the information feedback setting with all contractors' initial effort level vector $\hat{\mathbf{q}}_{\mathbf{0}}$, a strategy profile $(\mathbf{q}^{\dagger}_{\mathbf{1,1:N}},\cdots,\mathbf{q}^{\dagger}_{\mathbf{M,1:N}})$ is an SPSE if*

(i) $(\mathbf{q}^{\dagger}_{\mathbf{1,1:N}},\cdots,\mathbf{q}^{\dagger}_{\mathbf{M,1:N}})$ *is an SPE, and*

(ii) for every subgame continuing from any period $n\in\{1,2,\cdots,N\}$ with the state of period n, $\hat{\mathbf{q}}^{\dagger}_{\mathbf{n-1}}$, each $\hat{q}^{\dagger}_{i,n:N}$ for $i\in\{1,2,\cdots,M\}$ characterized by the restriction $(\mathbf{q}^{\dagger}_{\mathbf{1,n:N}},\cdots,\mathbf{q}^{\dagger}_{\mathbf{M,n:N}})|\hat{\mathbf{q}}^{\dagger}_{\mathbf{n-1}}$ of the strategy profile $(\mathbf{q}^{\dagger}_{\mathbf{1,1:N}},\cdots,\mathbf{q}^{\dagger}_{\mathbf{M,1:N}})$ satisfies

$$\hat{q}^{\dagger}_{i,n:N}=\underset{\hat{q}_{i,n:N}\in\hat{\mathbf{E}}_{\mathbf{i,n:N}}}{\arg\max}\;\min_{\hat{\mathbf{q}}_{\mathbf{-i,n:N}}\in\hat{\mathbf{E}}_{\mathbf{-i,n:N}}}\pi_i(\hat{q}_{i,n:N};\hat{\mathbf{q}}_{\mathbf{-i,n:N}}).\tag{24}$$

Supporting Lemmas. Lemmas OA1, OA2, and OA3 below are useful in our proof of Proposition 2. The proof of the lemmas will be provided following the proof of Proposition 2.

Lemma OA1. *The condition stated in (24) is equivalent to, for any $i\in\{1,2,\cdots,M\}$,*

$$\pi_i\left(\hat{\mathbf{q}}^{\dagger}_{\mathbf{n:N}}\right)\le\pi_i\left(\hat{\mathbf{q}}_{\mathbf{n:N}}\right)\quad\forall\,\hat{\mathbf{q}}_{\mathbf{n:N}}\in\hat{\mathbf{E}}_{\mathbf{n:N}}.\tag{25}$$

Lemma OA2. *In an M-contractor N-period coordination game under the information feedback setting, the following statements hold.*

(i) If $N\le M-1$, and at least $N+1$ contractors have equal lowest initial effort level that

$$q_{1,0}=q_{2,0}=\cdots=q_{M,0},\tag{26}$$

all contractors exert at the lower boundary $\frac{\underline{q}}{N}$ in each period is an SPE strategy.

(ii) Given the initial effort level vector $\hat{\mathbf{q}}_{\mathbf{0}}=(q_{1,0},\cdots,\ q_{M,0})$, we denote by m the number of contractors whose initial effort levels satisfy

$$0\le\ q_{i,0}<q^{\bullet}-\underline{q},\quad i\in\{1,2,\cdots,M\},\tag{27}$$

where

$$q^{\bullet}=\min\left\{q^{*},\,\overline{q}-(m-1)\frac{\overline{q}-\underline{q}}{N}+q_{(1),0}\right\}.\tag{28}$$

If $N>m-1$, any strategy profile that leads to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}})<q^{\bullet}$ is not an SPE.

(iii) If $N\le m-1$, $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}})<q^{\bullet}$ in any SPSE.

Lemma OA3. *Let m be the same as in Lemma OA2(ii) that satisfies (27). Given all contractors' initial effort level vector $\hat{\mathbf{q}}_\mathbf{0} = (q_{1,0}, \cdots, q_{M,0})$ such that $0 \leq q_{i,0} \leq q^\bullet - \underline{q}, \forall\, i \in \{1, 2, \cdots, M\}$, if the feedback periods $N > m-1$, in period $n = N - m + 1$, contractors' SPSE effort levels satisfy*

$$\hat{q}_{(m),N-m+1} = q^\bullet - (m-1)\frac{\underline{q}}{N}, \text{ and} \tag{29}$$

$$\hat{q}_{(1),N-m+1} \geq \max\left\{(N-m+1)\frac{\underline{q}}{N},\, q^\bullet - (m-1)\frac{\overline{q}}{N}\right\}; \tag{30}$$

in any period $n \geq N + 2 - m$, the SPSE effort for contractors are

$$q_{i,n} = \frac{\underline{q}}{N}, \text{ if } \hat{q}_{i,n-1} \geq \hat{q}_{(N+2-n),n-1}, \tag{31}$$

$$q_{i,n} = q^\bullet - (N-n)\frac{\underline{q}}{N} - \hat{q}_{i,n-1}, \text{ if } \hat{q}_{i,n-1} = \hat{q}_{(N+1-n),n-1}, \text{ and} \tag{32}$$

$$q_{i,n} \in \left[\max\left\{\frac{\underline{q}}{N}, q^\bullet - (N-n)\frac{\underline{q}}{N} - \hat{q}_{i,n-1} - r\frac{\overline{q}-\underline{q}}{N}\right\},\, q^\bullet - (N-n)\frac{\underline{q}}{N} - \hat{q}_{i,n-1}\right],$$
$$\text{if } \hat{q}_{i,n-1} = \hat{q}_{(N+1-n-r),n-1}, \text{ where } r \in \{1, 2, \cdots, N-n\}. \tag{33}$$

Proof of Proposition 2. Note that, based on Lemma OA1, we establish that (24) is satisfied by showing (25) is satisfied throughout this proof.

Step 1. Proposition 2(i) We first prove Proposition 2(i). In this case, $N \leq M-1$ is satisfied. Since (26) is satisfied, Lemma OA2(i) guarantees that $q_{i,n} = \frac{\underline{q}}{N}$ in any period $n = 1, \cdots, N$ is an SPE strategy for all contractors. Moreover, in each period, all contractors exert at the lower boundary, and, thus, producing $\frac{\underline{q}}{N}$ in each period achieves the minimum profit in any subgame; namely, (25) is satisfied. Per Lemma OA1, (24) is satisfied for every subgame. Therefore, based on Definition 2, $q_{i,n} = \frac{\underline{q}}{N}$ in any period $n = 1, \cdots, N$ is an SPSE strategy for all contractors.

To prove the uniqueness of the SPSE strategy, we suppose for the purpose of contradiction that there exists another SPSE equilibrium in which at least one contractor's strategy is different from producing $\frac{\underline{q}}{N}$ in one period. Per Definition 2 and Lemma OA1, this strategy must be an SPSE equilibrium strategy and can achieve lower or the same equilibrium outcome as producing $\frac{\underline{q}}{N}$ in each period. Achieving lower or the same equilibrium outcome indicates that at least one contractor must exert $\frac{\underline{q}}{N}$ in each period. However, given one contractor exerts $\frac{\underline{q}}{N}$ in each period, all other contractors should also exert $\frac{\underline{q}}{N}$ in each period to establish the best response (because exerting any amount of effort higher than $\frac{\underline{q}}{N}$ is sub-optimal), which contradicts the statement that there exists another SPSE that at least one contractor will exert a different effort amount different than $\frac{\underline{q}}{N}$ in one period. This establishes the uniqueness.

Step 2. Proposition 2(ii) We now prove Proposition 2(ii).

Step 2.1 (4) and (5). Note that (4) and (5) are derived based on (6) and (7) minus $(N - M + 1 - n)\frac{\overline{q}}{N}$, respectively. In other words, to achieve (6) and (7), the contractors with maximum and minimum effort levels need to make sure they can catch up with the thresholds in (6) and (7) in the next few periods until period $N - M + 1$.

Step 2.2 Verify there is no incentive to deviate from (4) and (5). For the highest effort level contractor, any strategy that leads to an effort level higher than the upper boundary characterized in

(4) $(q^\circ - (N-n)\frac{\underline{q}}{N})$ will make the final effort level higher than q°. This strategy is suboptimal and, thus, not an SPE strategy. Per Definition 2(i), it is not an SPSE strategy.

Additionally, for the highest effort level contractor, any strategy that leads to a failure to reach the lower boundary in (4) $(\max\{n\frac{\underline{q}}{N},\quad q^\circ-(M-1)\frac{\underline{q}}{N}-(N-M+1-n)\frac{\bar{q}}{N}\})$ will cause $\hat{q}_{(M),N-M+1} < q^\circ - (M-1)\frac{\underline{q}}{N}$. That is, the subgame continuing from period $n+1$ satisfy the condition in Lemma OA2(ii), and thus lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\circ$. This strategy does not satisfy the SPE subgame perfection (and thus is not an SPSE strategy per Definition 2(i)) because, for the whole game, $N > m-1$ is satisfied, and any strategy leads to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\circ$ is not an SPE strategy per Lemma OA2(ii). Therefore, any strategy that leads to failure to satisfy (32)is not an SPSE strategy.

Similarly, for the lowest effort level contractor, any strategy that leads to failure to satisfy (5) will cause (7) does not hold. This leads to $\hat{q}_{(1),N} < q^\circ$, which causes $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$. Per Lemma OA2(ii), this strategy is not an SPE strategy and thus, per Definition 2(i), is not an SPSE strategy.

Step 2.3 (6) - (10). Note that Proposition 2 is a special case of Lemmas OA2 and OA3, when $\hat{\mathbf{q}}_{\mathbf{0}} = (0,\cdots,\ 0)$ and $m = M$ (which also indicate that $q^\circ = q^\bullet$). Therefore, per Lemma OA3, these strategies will lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\circ$. Per Lemma OA2(ii), these strategies are not SPE strategies and thus not SPSE strategies.

For the expositional convenience, we denote by $Q_{(i),n}$, the contractor whose effort level at the end of period n is the ith smallest among all contractors; we also denote by $Q_{(i:j),n}$, the group of contractors whose effort levels at the end of period n are the ith to jth smallest among all contractors, where $1 \le i < j \le M$. Note that (6) - (10) of Proposition 2 is a special case of Lemma OA3 when $\hat{\mathbf{q}}_{\mathbf{0}} = (0,\cdots,\ 0)$ and $m = M$. Therefore, Lemma OA3 immediately implies (6) - (10) of Proposition 2 hold.

Step 2.4 (11). To show (11), we first note that given $\hat{\mathbf{q}}_{\mathbf{0}} = (0,\cdots,0)$, there exists a scenario that even if all contractors exert at the upper boundary for periods 1 to N-M+1, they still cannot achieve the required project effort level q^* when $(N-M+1)\frac{\bar{q}}{N} < q^* - (M-1)\frac{\underline{q}}{N}$. To include this special scenario, we denote by $\hat{q}_{(M),N-M+1} = \min\{q^* - (M-1)\frac{\underline{q}}{N}, (N-M+1)\frac{\bar{q}}{N}\}$ the SPSE effort level in period $N-M+1$. Finally, based on (8) – (10), the equilibrium effort level is $\hat{q}^*_{i,N} = \min\{q^*,\ \bar{q} - (M-1)\frac{\bar{q}-\underline{q}}{N}\}$. ■

Proof of Lemma OA1. We establish the lemma in two steps.

Step 1. (25)⇒(24). We first show that if (25) is true, (24) also holds true. To that end, we first prove that, for any $\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$

$$\underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\arg\min} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \hat{\mathbf{q}}^\dagger_{\mathbf{-i,n:N}}. \tag{34}$$

Suppose, for the purpose of contradiction, this statement is not true. There must exist $\hat{\mathbf{q}}'_{\mathbf{-i,n:N}} \ne \hat{\mathbf{q}}^\dagger_{\mathbf{-i,n:N}}$ such that, $\underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\arg\min} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \hat{\mathbf{q}}'_{\mathbf{-i,n:N}}$ for any $\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$. Therefore, if we let $\hat{q}_{i,n:N} = \hat{q}^\dagger_{i,n:N}$, then $\underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\arg\min} \pi_i(\hat{q}^\dagger_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \hat{\mathbf{q}}'_{\mathbf{-i,n:N}}$, which implies $\hat{q}'_{-i,N} = \min\{\hat{\mathbf{q}}'_{\mathbf{-i,N}}\} < \hat{q}^\dagger_{i,N}$ and $\hat{q}'_{-i,N} < \min\{\hat{\mathbf{q}}^\dagger_{\mathbf{-i,N}}\}$ by the definition of $\pi_i(\cdot)$ in (2) and $\hat{\mathbf{q}}'_{\mathbf{-i,N}} \ne \hat{\mathbf{q}}^\dagger_{\mathbf{-i,N}}$ (indicated by the assumption $\hat{\mathbf{q}}'_{\mathbf{-i,n:N}} \ne \hat{\mathbf{q}}^\dagger_{\mathbf{-i,n:N}}$). However, this leads to $\pi_i\left(\hat{\mathbf{q}}'_{\mathbf{n:N}}\right) < \pi_i\left(\hat{\mathbf{q}}^\dagger_{\mathbf{n:N}}\right)$, which contradicts to (25). Therefore, (34) must hold true, and therefore,

$$\underset{\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}}{\arg\max} \ \underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\min} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \underset{\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}}{\arg\max} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}^\dagger_{\mathbf{-i,N}}) = \hat{q}^\dagger_{i,n:N}.$$

This proves (24) holds true.

Step 2. (24)⇒ (25). We then show that if (24) is true, (25) also holds true.

We first prove that, if (24) holds true, then for any $\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$

$$\underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\arg\min} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,n:N}}. \tag{35}$$

Suppose, for the purpose of contradiction, this statement is not true. By the definition of $\pi_i(\cdot)$ in (2), there must exist $\hat{\mathbf{q}}'_{\mathbf{-i,n:N}} \neq \hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,n:N}}$ such that, $\underset{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}}{\arg\min} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \hat{\mathbf{q}}'_{\mathbf{-i,n:N}}$ for any $\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$. This indicates that,

$$\underset{\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}}{\arg\max} \min_{\hat{\mathbf{q}}_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}_{\mathbf{-i,n:N}}) = \underset{\hat{q}_{i,n:N} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}}{\arg\max} \pi_i(\hat{q}_{i,n:N}; \hat{\mathbf{q}}'_{\mathbf{-i,N}}) = \hat{q}'_{i,n:N}.$$

We next prove by contradiction that $\hat{q}'_{i,n:N} \neq \hat{q}^{\dagger}_{i,n:N}$ to complete the proof that (35) holds true. Suppose, for the purpose of contradiction, that $\hat{q}'_{i,n:N} = \hat{q}^{\dagger}_{i,n:N}$, but $\hat{\mathbf{q}}'_{\mathbf{n:N}} \neq \hat{\mathbf{q}}^{\dagger}_{\mathbf{n:N}}$. It must hold true that $\hat{\mathbf{q}}'_{\mathbf{-i,n:N}} \neq \hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,n:N}}$. By the definition of $\pi_i(\cdot)$, to maximize the profit in the subgame with the given state of period n, $\hat{\mathbf{q}}^{\dagger}_{\mathbf{n-1}}$, contractor i needs to select $\hat{q}_{i,n:N}$ such that $\hat{q}_{i,N} = \hat{q}^{\dagger}_{i,n-1} + \hat{q}_{i,n:N} = \min\{\hat{\mathbf{q}}_{\mathbf{-i,N}}\}$, and thus $\hat{q}'_{i,n:N} = \hat{q}^{\dagger}_{i,n:N}$ indicates $\min\{\hat{\mathbf{q}}'_{\mathbf{-i,N}}\} = \min\{\hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,N}}\}$.

Note that $\hat{\mathbf{q}}'_{\mathbf{-i,n:N}} \neq \hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,n:N}}$ indicates that, there exists at least a contractor (say contractor $j \neq i$) such that $\hat{q}'_{j,n:N} \neq \hat{q}^{\dagger}_{j,n:N}$. If $\hat{q}^{\dagger}_{j,n:N} > \hat{q}'_{j,n:N}$, this contradicts $\hat{\mathbf{q}}^{\dagger}_{\mathbf{n:N}} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$ because given $\min\{\hat{\mathbf{q}}'_{\mathbf{-i,N}}\} = \min\{\hat{\mathbf{q}}^{\dagger}_{\mathbf{-i,N}}\}$ and $\hat{q}'_{i,n:N} = \hat{q}^{\dagger}_{i,n:N}$, contractor j can achieve higher profit by reducing $\hat{q}^{\dagger}_{j,n:N}$ to $\hat{q}'_{j,n:N}$ per the definition of $\pi_i(\cdot)$. If $\hat{q}'_{j,n:N} > \hat{q}^{\dagger}_{j,n:N}$, we can reach a contraction to $\hat{\mathbf{q}}'_{\mathbf{n:N}} \in \hat{\mathbf{E}}_{\mathbf{i,n:N}}$ following a similar reasoning. Therefore, these contradictions show that $\hat{q}'_{i,n:N} \neq \hat{q}^{\dagger}_{i,n:N}$. This leads to the conclusion that (35) holds true.

Finally, we prove by contradiction that (35) holds true indicates (25) also holds true. Suppose, for the purpose of contradiction, (25) does hold true. There must exists a contractor i and $\hat{\mathbf{q}}''_{\mathbf{n:N}} \neq \hat{\mathbf{q}}^{\dagger}_{\mathbf{n:N}}$ with $\hat{\mathbf{q}}''_{\mathbf{n:N}} \in \hat{\mathbf{E}}_{\mathbf{n:N}}$ such that, for that contractor i, $\pi_i\left(\hat{\mathbf{q}}^{\dagger}_{\mathbf{n:N}}\right) > \pi_i\left(\hat{\mathbf{q}}''_{\mathbf{n:N}}\right)$, which indicates

$$\pi_i\left(\hat{q}^{\dagger}_{i;n:N}; \hat{\mathbf{q}}^{\dagger}_{\mathbf{-i;n:N}}\right) > \pi_i\left(\hat{q}''_{i;n:N}; \hat{\mathbf{q}}''_{\mathbf{-i;n:N}}\right) > \pi_i\left(\hat{q}^{\dagger}_{i;n:N}; \hat{\mathbf{q}}''_{\mathbf{-i;n:N}}\right), \tag{36}$$

where the second inequality holds true because $\left(\hat{q}''_{i;n:N}; \hat{\mathbf{q}}''_{\mathbf{-i;n:N}}\right) \in \hat{\mathbf{E}}_{\mathbf{n:N}}$, but $\left(\hat{q}^{\dagger}_{i;n:N}; \hat{\mathbf{q}}''_{\mathbf{-i;n:N}}\right) \notin \hat{\mathbf{E}}_{\mathbf{n:N}}$ since $\hat{q}^{\dagger}_{i;n:N} \neq \hat{q}''_{i;n:N}$, which can be proved following the same proof of $\hat{q}'_{i,n:N} \neq \hat{q}^{\dagger}_{i,n:N}$ above. Since $\hat{\mathbf{q}}''_{\mathbf{-i,n:N}} \in \hat{\mathbf{E}}_{\mathbf{-i,n:N}}$, (36) contradicts (35). Therefore, (25) holds true.

Combining steps 1 and 2, we complete the proof of Lemma OA1. ■

Proof of Lemma OA2. Note that m is the number of contractors who do not have enough initial effort level that they could join the Lead group by only producing at the lower boundary $\frac{q}{N}$ in each round, and therefore, these m contractors need to strategically coordinate to achieve $q^{\bullet}_n$. We prove Lemma 2 by induction on the number of periods N.

Step 1. The induction basis for Lemma 2(i). We start with the inductional basis for Lemma OA2(i). Note that when $N = 1$, and at least 2 contractors (say contractors i and j) have equal lowest initial effort level, all contractors exert at the lower boundary $\frac{q}{N}$ is a Nash equilibrium. This is because given contractor i will exert $\frac{q}{N}$, the best response for contractor j is to exert $\frac{q}{N}$ regardless of how many other contractors will exert, and given contractors i and j will exert $\frac{q}{N}$, the best response for all other contractors is to exert $\frac{q}{N}$.

Since an SPE needs to be established in sequential games, we consider $N=2$. In this case, we have $M \geq 3$ to satisfy $N \leq M-1$, and at least 3 contractors (say contractors i, j, and k) have equal lowest initial effort level. All contractors exert $\frac{q}{N}$ in each period can be easily verified as a Nash equilibrium, and we next verify the subgame perfection. In the subgame continuing from period 2 (the last period) with the state $\hat{\mathbf{q}}_{\mathbf{1}}$ (which follows through strategies in period 1 that all contractors exert $\frac{q}{N}$), since contractors i, j, and k still have the lowest effort level at the beginning of period 2, all contractors exert $\frac{q}{N}$ in period 2 is a Nash equilibrium, as shown above when $N=1$. To complete the subgame perfection verification, we next investigate if any contractor has the incentive to deviate from producing $\frac{q}{N}$ in period 1, given all other contractors will exert $\frac{q}{N}$ in both periods. Note that even if a randomly selected contractor (say contractor k from contractors i, j, and k) exerts $q > \frac{q}{N}$ while all other contractors exert $\frac{q}{N}$ in period 1, all contractors exert $\frac{q}{N}$ is still a Nash equilibrium for the subgame continuing from period 2. This is because contractors i and j still have the equal lowest effort level at the beginning of period 2, and therefore, they should exert $\frac{q}{N}$ in period 1 based on our arguments when $N=1$. Therefore, all contracts exert $\frac{q}{N}$ is an SPE in this scenario, and we established the induction basis for Lemma OA2(i).

Step 2. The induction basis for Lemma 2(ii). We now consider the induction basis for Lemma OA2(ii). When $N=2$, we mainly focus on $m=2$ since when $m=0$ or $m=1$ (i.e., none or only one contractor is not guaranteed to join the Lead group), Lemma OA2(ii) holds. This is because when all contractors are in the Lead group, they all only need to exert at the lower boundary to achieve $q^{\bullet}$; when only one contractor is not in the Lead group, that contractor rationally should match the Lead group to achieve $q^{\bullet}$, under which that contractor is better off compared to any $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^{\bullet}$. Note that since when $m=2$, all contractors (except for contractors i and j) have initial effort levels of at least $\min\{q^{*} - N\frac{q}{N}, (N-m+1)\frac{\overline{q}-q}{N} + q_{(1),0}\}$, then the best response for them is to exert $\frac{q}{N}$ in both periods, and their total effort levels are all at least $q^{\bullet}$. Therefore, both contractors i and j exert at the lower boundary $\frac{q}{N}$ in each period is not an SPE equilibrium strategy because of failing the subgame perfection. Particularly in period 1, given all other contractors (except for contractors i and j) will achieve at least $q^{\bullet}$ in the end, even if contractor j only exerts $\frac{q}{N}$, contractor i should achieve $\hat{q}_{i,1} = q^{\bullet} - \frac{q}{N}$. This is because given contractor i has achieved an effort level $q^{\bullet} - \frac{q}{N}$ at the end of period 1 and will exert $\frac{q}{N}$ in period 2 to achieve $\hat{q}_{i,2} = q^{\bullet}$, contractor j's best response is to exert $q^{\bullet} - \frac{q}{N} - q_{j,0}$ instead of $\frac{q}{N}$ to match $\hat{q}_{j,2} = q^{\bullet}$ in period 2. This statement holds for any strategy that leads to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^{\bullet}$ because given contractor j exerts any amount of effort q to make $\hat{q}_{j,1} < q^{\bullet} - \frac{q}{N}$ in period 1, the best response for contractor i is still to achieve $\hat{q}_{i,1} = q^{\bullet} - \frac{q}{N}$ in period 1 since in period 2, contractor j's best response is to make $\hat{q}_{j,2} = q^{\bullet}$ given contractor i exerts $\frac{q}{N}$ to achieve $\hat{q}_{i,2} = q^{\bullet}$. This establishes the induction basis for Lemma OA2(ii).

Step 3. The induction basis for Lemma 2(iii). When $N=1$, $N \leq m-1$ indicates $m \geq 2$. We focus on $m=2$ (contractors i and j), and the scenarios that $m>2$ can be shown similarly. Without loss of generality, we assume that $q_{i,0} > q_{j,0}$. Note that it is rational for contractors who are already in the Lead group (not including contractors i and j) to exert at the lower boundary to achieve an effort level of $q^{\bullet}$. For contractors i and j, any strategies that make their effort levels in the final period N equal are SPE strategies. However, it is easy to verify that the only strategies that satisfy (25) in Lemma OA1 are contractor i exerts $\frac{q}{N}$ and contractor j exerts $q_{i,0} + \frac{q}{N} - q_{j,0}$, under which $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) = q_{i,0} + \frac{q}{N} < q^{\bullet}$ ($m = 2$ implies $q_{i,0} + \frac{q}{N} < q^{\bullet}$ per the definition of m).

Next, to show this is also true in sequential games, we consider $N = 2$ (which indicates $m \geq 3$). We only focus on $m = 3$ (contractors i, j, and k), and the scenarios of $m > 3$ can be shown similarly. Without loss of generality, we assume that $q_{i,0} > q_{j,0}$ and $q_{i,0} > q_{k,0}$. Note that any strategies that let all contractors i, j, and k reach the same total effort level are SPE strategies. More importantly, once $N \leq m-1$ (which is opposite to the condition in Step 5.2), not all SPE strategies can lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^{\bullet}_N$. It is easy to verify that strategies that satisfy (25) in Lemma OA1 are contractor i exerts $\frac{q}{N}$ in both periods, and contractors j and k reach the same total effort level as contractor i in the final period N. All these strategies lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) = q_{i,0} + 2\frac{q}{N} < q^{\bullet}$ ($m = 3$ implies $q_{i,0} + 2\frac{q}{N} < q^{\bullet}$ per the definition of m). This establishes the induction basis for Lemma OA2(iii).

Step 4. The induction step for Lemma 2(i). In the induction step, we assume Lemma OA2 holds for any $N \in \{1, 2, \cdots, t\}$ and next show it is also true for $N = t+1$. For Lemma OA2(i), we note that when $N = t+1 \leq M-1 (i.e., M \geq N+1 = t+2)$, and at least $t+2$ contractors have equal lowest initial effort level. Given all contractors exert $\frac{q}{N}$ in period 1, the number of contractors who have equal lowest effort level at the beginning of period 2 is strictly larger than $t+1$. Per our induction hypothesis for Lemma OA2(i), all contractors exert at the lower boundary $\frac{q}{N}$ in each period is an SPE strategy for the subgame continuing from period 2 with the state $\hat{\mathbf{q}}_{\mathbf{1}}$. Given this subgame SPE, we arrive at the conclusion that all contractors exert $\frac{q}{N}$ in period 1 is a Nash Equilibrium. Also, the subgame perfection follows through for all subgames continuing period 2 with the state $\hat{\mathbf{q}}_{\mathbf{1}}$. To show it is also the SPE strategy for the whole game, we need to verify no contractor has the incentive to deviate from producing $\frac{q}{N}$ in period 1. This is true since even if there is a contractor (regardless the contractor has the lowest initial effort level or not) exerts $q > \frac{q}{N}$ while all other contractors exert $\frac{q}{N}$ in period 1, the best response for all contractors is still to exert $\frac{q}{N}$ in each of the rest periods, since there are still $t+1$ contractors have the lowest effort level in the subgame continuing from period 2, which has t periods in total, and per our induction hypothesis for Lemma OA2(i), producing $\frac{q}{N}$ in each of the rest periods is an SPE strategy. Therefore, the contractor should not deviate from producing $\frac{q}{N}$ in period 1, and Lemma OA2(i) holds.

Step 5. The induction step for Lemma 2(ii). For Lemma OA2(ii), $N > m-1$ indicates $m \leq N = t+1$. We next discuss based on $m \leq N-1$ and $m = N$. If $m \leq N-1 = t$, for the subgame continuing from period 2, the number of contractors whose effort levels at the beginning of period 2 are smaller than $\min\{q^* - N\frac{q}{N}, (N-m+1)\frac{\overline{q}-q}{N} + q_{(1),0}\} + \frac{q}{N}$ is at most m. Because the total periods of the subgame continuing from period 2 is $N-1$, which satisfies $N-1 \geq m > m-1$, per the induction hypothesis, any strategy profiles that lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^{\bullet}$ is not an SPE for the subgame continuing from period 2, and thus is not an SPE for the whole game.

In the other case, when $m = N = t+1$, $N - m + 1 = 1$, and a randomly selected contractor i among the m contractors needs to decide the effort amount for period 1 based on the SPE of the subgame continuing from period 2 (following the backward induction). If contractor i achieves $\hat{q}_{i,1} = \min\{q^* - N\frac{q}{N}, (N-m+1)\frac{\overline{q}-q}{N} + q_{(1),0}\} + \frac{q}{N}$ (which is achievable because

$\min\{q^* - N\frac{\underline{q}}{N}, (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{(1),0}\} + \frac{\underline{q}}{N} \leq \frac{\overline{q}-\underline{q}}{N} + q_{(1),0} + \frac{\underline{q}}{N} = \frac{\overline{q}}{N} + q_{(1),0})$, then for the subgame continuing from period 2, the number of contractors whose effort levels at the beginning of period 2 are smaller than $\min\{q^* - \underline{q}, (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{i,0}\} + \frac{\underline{q}}{N}$ is at most $m-1 = t$. Per the induction hypothesis, all the contractors could achieve $q^\bullet$ as the project effort level at the end of the game. If contractor i does not achieve $\hat{q}_{i,1} = min\left\{q^* - N\frac{\underline{q}}{N},\ (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{(1),0}\right\} + \frac{\underline{q}}{N}$, then for the subgame continuing from period 2, the number of contractors whose effort levels at the beginning of period 2 are smaller than $min\left\{q^* - \underline{q},\ (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{i,0}\right\} + \frac{\underline{q}}{N}$ is $m > N-1 = t$. All the contractors could only have $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$ as the project effort level at the end of the game. Rationally, contractor i will choose the strategy to achieve $q^\bullet$. Therefore, any strategy profiles that lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$ will be dominated and thus is not an SPE. This completes the proof for Lemma OA2(ii).

Step 6. The induction step for Lemma 2(iii). For Lemma OA2(iii), $N \leq m-1$ indicates $m \geq N+1 = t+2$. For the contractor with the highest initial effort level (say contractor i) among the m contractors, even if he exerts enough effort to join the Lead group in period 1, for the subgame continuing from period 2, the number of contractors who are not in the Lead group can still be $m-1$ if none of the other contractors among the m contractors exert enough effort to join the Lead group in period 1. Since $m-1 \geq N = N-1+1 = t+1$, per the induction hypothesis, $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$. Therefore, contractor i's best response (i.e., the SPE strategy) to other contractors' strategies in this scenario is to exert less effort and not join the Lead group. Under this SPE strategy, $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$. It is worth noting that if another contractor (or other contractors) from the m contractors and contractor i can both exert enough effort in period 1 to make the number of contractors who are not in the Lead group be strictly less than N, then, base on Lemma OA2(ii), $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) = q^\bullet$. However, since we have already identified SPE strategies that lead to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) < q^\bullet$, any strategy that achieves $q_{\min}(\hat{\mathbf{q}}_{\mathbf{N}}) = q^\bullet$ does not satisfy (25) in Lemma OA1 and thus is not an SPSE strategy. This completes the proof for Lemma OA2(iii) and also completes the whole proof for Lemma OA2. ■

Proof of Lemma OA3. Note that, different from the constraint in the definition of m, the constraint on the initial effort level vector includes the equality of the upper boundary. We use this constraint to guarantee that $\hat{q}_{(M),N-M+1}$ in (29) is feasible. That is, the initial effort levels are not too large that contractor $\hat{Q}_{(m),N-m+1}$ is forced to overexert an amount of effort that $\hat{q}_{(m),N-m+1} > min\{q^* - (m-1)\frac{\underline{q}}{N}, (N-m+1)\frac{\overline{q}}{N} + q_{(1),0}\}$ stated in (29). Based on the definition of m, given $0 \leq q_{i,0} \leq \min\{q^* - N\frac{\underline{q}}{N}, (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{(1),0}\}, \forall i \in \{1, 2, \cdots, M\}$, there are $M-m$ contractors whose initial effort level is exactly $\min\{q^* - N\frac{\underline{q}}{N}, (N-m+1)\frac{\overline{q}-\underline{q}}{N} + q_{(1),0}\}$. For these contractors, the dominant strategy is to exert $\frac{\underline{q}}{N}$ in each period, and their project effort levels are guaranteed to be $q^\bullet$.

Step 1. The induction basis for (29) – (32) of Lemma OA3. We prove by induction on periods and start with $N = 2$. A few remarks are in order. First, $N = 2$ indicates that $m \leq 2$, and we focus on $m = 2$ (say contractors i and j) since when $m = 1$ or $m = 0$, the results hold trivially. Second, $m = 2$ implies that only (29) - (32) will be used to characterize the SPSE strategy, while (33) will not be used because there is no feasible r in this scenario. Third, we assume without loss of generality that contractor i will take the leading position.

Step 1.1. Verify the SPE condition - Definition 2(i). Contractor i will exert to achieve $\hat{q}_{(2),1} = \hat{q}_{i,1} = min\{q^* - \frac{\underline{q}}{N}, \frac{\overline{q}}{N} + q_{(1),0}\}$ following (29) in period 1 and exert $\frac{\underline{q}}{N}$ following (31) in period 2, contractor j will exert any amount of effort to achieve $\hat{q}_{(1),1} = \hat{q}_{j,1} \geq \max\{\frac{\underline{q}}{N}, \hat{q}_{(2),1} - \frac{\overline{q}-\underline{q}}{N}\}$ following (30) and exert $q^{\bullet} - \hat{q}_{j,1}$ following (32), and all the other $M-2$ contractors will exert $\frac{\underline{q}}{N}$ in each period. Following these strategies, all contractors can achieve $\hat{q}_{i,2} = \hat{q}_{j,2} = \cdots = \hat{q}_{M,2} = q_{\min}(\hat{\mathbf{q}}_{\mathbf{2}}) = q^{\bullet}$, and thus, strategies in (29) – (32) are Nash equilibrium strategies. Moreover, strategies in (31) and (32) can be verified are Nash equilibrium for the subgame in period 2 with the state $\hat{\mathbf{q}}_{\mathbf{1}}$ (which follows through strategies characterized from (29) and (30) in period 1). Thus, (29) – (32) characterize SPE strategies.

Step 1.2. Verify (24) is satisfied - Definition 2(ii). Per Lemma OA2(ii), since $N > m-1$, there does not exist any SPE that leads to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{2}}) < q^{\bullet}$. This indicates that (27) is satisfied, and therefore (29) – (32) satisfy (25). Per Lemma OA1, (24) is satisfied. Thus, based on Definition 2, (29) - (32) characterize an SPSE strategy, and this establishes the induction basis for (29) – (32).

Step 2. The induction basis for (33) of Lemma OA3.

Step 2.1. Verify the SPE condition - Definition 2(i). To establish the induction basis for (33), we focus on $N=3$, where $m=3$ or $m=2$ are feasible numbers that satisfy $N > m-1$ (when $m=1$ or $m=0$, results hold trivially), and the only feasible number for r is $r=1$ when $m=3$. When $m=2$, following a similar process as above, we can show (29) – (32) characterize an SPSE strategy. We now focus on $m=3$ (assume contractors i, j, and k with $q_{i,0} \geq\ q_{j,0} \geq\ q_{k,0}$ without loss of generality) and list all contractors' strategies following (29) – (33) as follows.

(a) Contractor i exerts $q^{\bullet} - 2\frac{\underline{q}}{N} - q_{i,0}$ to achieve $\hat{q}_{(3),1} = \hat{q}_{i,1} = q^{\bullet} - 2\frac{\underline{q}}{N}$ in period 1 following (29) and exerts $\frac{\underline{q}}{N}$ in both periods 2 and 3 following (31).

(b) Contractor j exerts any amount of effort to achieve $\hat{q}_{(2),1} = \hat{q}_{j,1} \in [\max\{\frac{\underline{q}}{N}, q^{\bullet} - 2\frac{\overline{q}}{N}\}, q^{\bullet} - 2\frac{\underline{q}}{N}]$ in period 1, $q^{\bullet} - \frac{\underline{q}}{N} - \hat{q}_{j,1}$ in period 2 following (32), and $\frac{\underline{q}}{N}$ in period 3 following (31).

(c) Contractor k exerts any amount of effort to achieve $\hat{q}_{(1),1} = \hat{q}_{k,1} \geq \max\{\frac{\underline{q}}{N}, q^{\bullet} - 2\frac{\overline{q}}{N}\}$ in period 1 following (30), any amount of effort that satisfies $q_{k,2} \in [q^{\bullet} -\ \hat{q}_{k,1} - \frac{\overline{q}}{N},\ q^{\bullet} - \hat{q}_{k,1} - \frac{\underline{q}}{N}]$ in period 2 following (33), and $q^{\bullet} - \hat{q}_{k,2}$ in period 3 following (32).

Note that following (a), (b), and (c), each of contractors i, j, and k and the rest $M-3$ contractors who exert $\frac{\underline{q}}{N}$ in each period can achieve $\hat{q}_{i,3} = \hat{q}_{j,3} = \hat{q}_{k,3} = \cdots = \hat{q}_{M,3} = q_{\min}(\hat{\mathbf{q}}_{\mathbf{3}}) = q^{\bullet}$. We can also verify that (a), (b), and (c) are the best responses, and the strategies in each subgame satisfy the subgame perfection; namely, (29) – (33) characterize SPE strategies.

Step 2.2. Verify (24) is satisfied - Definition 2(ii). Per Lemma OA2(ii), since $N > m-1$, there does not exist any SPE that leads to $q_{\min}(\hat{\mathbf{q}}_{\mathbf{3}}) < q^{\bullet}$. This indicates that (27) is satisfied, and therefore (29) – (33) satisfy (25). Per Lemma OA1, (24) also holds true. Thus, based on Definition 2, (29) - (33) characterize an SPSE strategy, and this establishes the induction basis for (33).

Step 3. The induction step for Lemma OA3. In the induction step, we assume (29) - (33) characterize SPSE strategies for $N \in \{1, 2, \cdots, t\}$ and show that (29) - (33) also characterize SPSE strategies for $N = t+1$.

Step 3.1. Verify the SPE condition - Definition 2(i). Note that when $m \leq\ N = t+1$, following (29), at the beginning of period $N-m+2$, contractor $Q_{(m),N-m+1}$ will achieve $\hat{q}_{(m),N-m+1} = q^{\bullet} - (m-1)\frac{\underline{q}}{N}$.

Thus, denoted by N'_{N-m+2} the total periods of the subgame continuing from period $N-m+2$, we have $N'_{N-m+2} = m-1 \leq t$. Also, denote by m'_{N-m+2} the number of contractors whose effort levels at the beginning of period $N-m+2$ satisfy $0 \leq \hat{q}_{i,N-m+1} < q^\bullet - N'_{N-m+2}\frac{q}{N}$ $\forall i \in \{1,2,\cdots,M\}$, we have $m'_{N-m+2} = m-1 \leq t$. Since both $N'_{N-m+2} > m'_{N-m+2} - 1$ and $N'_{N-m+2} = m-1 \leq t$ are satisfied, per the induction hypothesis for $N \in \{1,2,\cdots,t\}$, (29) - (33) characterize SPSE strategies for the subgame continuing from period $N-m+2$ with the state $\mathbf{\hat{q}_{N-m+1}}$ (which follows through strategies characterized from (29) and (30) in period 1 to period $N-m+1$). This result, along with the induction hypothesis for $N \in \{1,2,\cdots,n\}$, verifies the subgame perfection. Therefore, (29) - (33) characterize SPE strategies for $N = t+1$.

Step 3.2. Verify (24) is satisfied - Definition 2(ii). Per Lemma OA2(ii), since $N > m-1$, there does not exist any SPE that leads to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$. This indicates that (25) is satisfied. Per Lemma OA1, (24) is satisfied. Thus, based on Definition 2, (29) - (33) characterize SPSE strategies for $N = n+1$, and this completes the induction proof.

Step 4. Verify failure to satisfy (29) – (33).

(29). Any strategy that leads to $\hat{q}_{(m),N-m+1} > q^\bullet - (m-1)\frac{q}{N}$ will cause $\hat{q}_{(m),N} > q^\bullet$. This strategy is not an SPE strategy (because of over-exerting effort that could not generate revenue) and thus not an SPSE strategy per Definition 2(i). If $\hat{q}_{(m),N-m+1} < q^\bullet - (m-1)\frac{q}{N}$, then for the subgame continuing from period $N-m+2$, the number of contractors who are not in the Lead group is m per the definition of m in Lemma OA2, but the number of feedback periods is $m-1$. Therefore, the condition in Lemma OA2(iii) is satisfied, and all SPSE strategies will lead to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$ per Lemma OA2(iii). This indicates that any strategy leads to $\hat{q}_{(m),N-m+1} < q^\bullet - (m-1)\frac{q}{N}$ does not satisfy the SPE subgame perfection (and thus is not an SPSE strategy per Definition 2(i)) because, for the whole game, $N > m-1$ is satisfied, and any strategy leads to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$ is not an SPE strategy per Lemma OA2(ii). Combining these proofs, we conclude that any strategy that leads to failure to satisfy (29) is not an SPSE strategy.

(30). Any failure to satisfy (30) will lead to $\hat{q}_{(1),N} < q^\bullet$, which indicates $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$. Per Lemma OA2(ii), any strategy leads to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$ is not an SPE strategy and thus, per Definition 2(i), not an SPSE strategy.

(31). Note that $\frac{q}{N}$ is the lower boundary of the effort amount that one contractor can exert in each period. Therefore, any deviation of the strategy in (31) will cause contractors in the Lead group to have a total effort level higher than $q^\bullet$, which is obviously suboptimal, and therefore is not an SPSE strategy.

(32). Exerting an effort amount higher than $q^\bullet - (N-n)\frac{q}{N} - \hat{q}_{i,n-1}$ will cause the total effort level higher than $q^\bullet$, which is obviously suboptimal, and therefore is not an SPSE strategy. Exerting an effort amount lower than $q^\bullet - (N-n)\frac{q}{N} - \hat{q}_{i,n-1}$ will cause the subgame continuing from period $n+1$ satisfy the condition in Lemma OA2(ii), and thus lead to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$. This strategy does not satisfy the SPE subgame perfection (and thus is not an SPSE strategy per Definition 2(i)) because, for the whole game, $N > m-1$ is satisfied, and any strategy leads to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$ is not an SPE strategy per Lemma OA2(ii). Therefore, any strategy that leads to failure to satisfy (32)is not an SPSE strategy.

(33). Finally, following the same reasoning as above, we can show that any strategy that leads to failure to satisfy (33) is not an SPSE strategy because (1) exerting an effort amount higher than $q^\bullet - (N-$

$n)\frac{\underline{q}}{N} - \hat{q}_{i,n-1}$ will cause the total effort amount higher than $q^\bullet$, and (2) exerting an effort amount lower than $\max\left\{\frac{\underline{q}}{N}, q^\bullet - (N-n)\frac{\underline{q}}{N} - \hat{q}_{i,n-1} - r\frac{\overline{q}-\underline{q}}{N}\right\}$ does not satisfy the SPE subgame perfection due to the satisfaction of the condition stated in Lemma OA2(iii), which leads to $q_{\min}(\mathbf{\hat{q}_N}) < q^\bullet$. ■

Proof of Corollary 2. Note that when $\hat{q}_{i,N-M+1} = \hat{q}_{j,N-M+1}$, $\forall\ i,j\ \in 1,2,\cdots,M$, (12) follows through directly from (6). In periods $N+2-M$ to N, when $\hat{q}_{i,n-1} = \hat{q}_{j,n-1}$, $\forall i,j \in \{1,2,\cdots,M\}$, (8) – (10) indicate that $q_{i,n} = q_{j,n} = \frac{\underline{q}}{N}$. Similarly, when $\hat{q}_{i,N-M+1} \neq \hat{q}_{j,N-M+1}$, $\forall i,j \in \{1,2,\cdots,M\}$, only the contractor with $\hat{q}_{(M),N-M+1}$ follows (6). When $\hat{q}_{i,n-1} \neq \hat{q}_{j,n-1}$ $\forall i,j \in \{1,2,\cdots,M\}$ and $n \in \{N+2-M,\cdots,N\}$, in each period only the contractor with $\hat{q}_{(N+1-n),n-1}$ follows (9). Finally, in period N, there is no feasible r, and therefore, the contractor with $\hat{q}_{(1),n-1}$ will follow (9). ■

Proof of Proposition 3. Note that $N \geq \lceil (M-1)\ \frac{\overline{q}-\underline{q}}{\overline{q}-q^*} \rceil$ implies $q^* - (M-1)\frac{\underline{q}}{N} \leq (N-M+1)\frac{\overline{q}}{N}$. Therefore, $\min\{q^* - (M-1)\frac{\underline{q}}{N}, (N-M+1)\frac{\overline{q}}{N}\} = q^* - (M-1)\frac{\underline{q}}{N}$, and $q^\circ = q^*$ per the definition of q°. Thus, following the SPSE strategies characterized in (4) – (10), all contractors achieve $q_{\min}(\mathbf{\hat{q}_N}) = q^\circ = q^*$. When $N < \lceil (M-1)\frac{\overline{q}-\underline{q}}{\overline{q}-q^*} \rceil$, $\min\{q^* - (M-1)\frac{\underline{q}}{N}, (N-M+1)\frac{\overline{q}}{N}\} = (N-M+1)\frac{\overline{q}}{N}$; that is, $q^\circ = \overline{q} - (M-1)\frac{\overline{q}-\underline{q}}{N} <\ q^*$. This shows that $q_{\min}(\mathbf{\hat{q}_N}) = q^\circ <\ q^*$. Combining the two arguments, we have proved that $N = \lceil (M-1)\frac{\overline{q}-\underline{q}}{\overline{q}-q^*} \rceil$ is the minimum number to achieve $q_{\min}(\mathbf{\hat{q}_N}) = q^*$ in an SPSE. ■

Appendix OA.3: Example of Experiment Instructions[10]

Instruction for information feedback treatment with two contractors (M2N2)

You are about to participate in an experimental study of decision-making. If you follow these instructions carefully and make good decisions, you will earn money that will be paid to you in cash at the end of the session. If you have a question at any time, please raise your hand and the experimenter will come to your station and answer it. We ask that you not talk with one another for the duration of the experiment.

The monetary unit in this experiment is called Experimental Currency Unit (ECU). You will accumulate your earnings throughout periods. Your objective is to earn as much as you can. After the experiment, your earnings in terms of ECU will be converted to US dollars at the rate of 30 ECU for $1 and paid to you in private and in cash, in addition to a $5 show up fee.

How to earn money

The session consists of 20 periods. In each period you will be matched with a different person in the room. Both of you will work on a project which requires each of you to work on two tasks. At the beginning of each period, you and the other player will simultaneously decide how much effort to exert on the first task. You will then simultaneously decide how much effort to exert on the second task without observing each other's effort level in the first task. Similarly, without observing each other's effort level in the second task, you will make your decision on the third task. After completing the third task, you will observe each other's total effort level for the first three tasks, as well as your safety level. Based on the total effort level and safety level, you will make your decision on the fourth task. Following this, you will again make your decisions on the fifth and sixth tasks without observing each other's effort level in the fourth and fifth tasks. [Similarly, without observing each other's effort level in the second task, you will make your decision on the third task, and so on for all six tasks. Note that you only observe your own effort on the pervious tasks, and your effort is only revealed to others after the end of the sixth task.] Your effort level in each task may be any number from 18.33 to 38.33 (up to two decimal places) and costs you 0.75 ECU per unit. Your *total* effort is the sum of your efforts in the six tasks.

The *project progress* is defined as the smaller one of the two *total* efforts chosen by you and the other player. The revenue you and the other player earn from the project depends on the minimum of the *project progress* and 170. So the earnings are determined as follows:

Your Earnings = (minimum of *project progress* and 170) − 0.75 × your *total* effort

Your *safety level*, which is the effort level guaranteed to contribute to the project progress, is determined as follows. For the fourth to sixth tasks:

[10] Underlined sentences are unique to the information feedback treatment ($M2N2$) instruction and should be deleted or replaced with the sentences in the square brackets for the no information feedback treatment instructions ($M2N1$). The instructions are available upon request for all other treatments.

- If your total effort in the first three tasks is *lower* than the other player's in the first three tasks, your *safety level* is the difference between your total effort and the other player's plus the minimum required effort in the three remaining tasks (18.33 X 3 = 54.99).
  - For example, suppose your total effort level after task 3 was 90 and it was the lower than the other player's effort level of 114. In this case your safety level is 54.99 + (114 - 90) = 78.99.
- If your total effort up to that task is *higher* than the other player's up to that task, your *safety level* is the minimum required effort in the three remaining tasks (18.33 X 3 = 54.99).
  - For example, suppose your total effort level after task 3 was 90 and it was higher than the other player's effort level of 80. In this case your safety level is 54.99.

Example:

Suppose at the end of the six tasks, the total effort levels for the two players are:

- Player 1: 150
- Player 2: 175

Because Player 1's total effort level is smaller, the project progress is 150, and the earnings are:

- Player 1: $150 - 0.75 \times 150 = 37.5$
- Player 2: $150 - 0.75 \times 175 = 18.75$

Now, suppose instead Player 1's total effort level was still the smaller one in the group, but it was 171. In this case the project progress is 171, and because $171 > 170$, the earnings are:

- Player 1: $170 - 0.75 \times 171 = 41.75$
- Player 2: $170 - 0.75 \times 175 = 38.75$

How you will be paid

At the end of the session, the computer will sum up your total earnings for all periods, convert them to US dollars, add them to your $5 show up fee, and display your total earnings for the session. Please use this information to fill out the checkout form and wait quietly until the monitor calls you to come to the front of the room to be paid your earnings in private and in cash. After you have been paid, you will be free to leave the laboratory.