E-Companion
On Designing a Fire Emergency Vehicle Fleet

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E1

A.1 Proof of equation (7)*:* For all x we have

$$\sum_{n=0}^{z} n\frac{x^{z-n}}{(z-n)!} = \sum_{n=0}^{z}(z-(z-n))\frac{x^{z-n}}{(z-n)!}$$
$$= \sum_{k=0}^{z}(z-k)\frac{x^k}{k!}$$
$$= z\left(\sum_{k=0}^{z-1}\frac{x^k}{k!}+\frac{x^z}{z!}\right) - x\sum_{k=0}^{z-1}\frac{x^k}{k!}$$
$$= \frac{e^x z\Gamma(z,x)}{(z-1)!} + \frac{x^z}{(z-1)!} - \frac{e^x x\Gamma(z,x)}{(z-1)!} = \frac{x^z - e^x(x-z)\Gamma(z,x)}{\Gamma(z)}.$$

A.2 Proof of equation (8)*:*

$$\sum_{n=0}^{\infty} n\cdot e^{-\rho}\frac{\rho^{z+n}}{(z+n)!} = e^{-\rho}\left(\sum_{n=0}^{\infty}(n+z)\frac{\rho^{z+n}}{(z+n)!} - z\sum_{n=0}^{\infty}\frac{\rho^{z+n}}{(z+n)!}\right)$$
$$= e^{-\rho}\left(\rho\sum_{n=0}^{\infty}\frac{\rho^{(z-1)+n}}{((z-1)+n)!} - z\sum_{n=0}^{\infty}\frac{\rho^{(z-1)+n+1}}{((z-1)+n+1)!}\right)$$
$$= e^{-\rho}\left(\rho\frac{\rho^{z-1}}{(z-1)!} + \rho\sum_{n=1}^{\infty}\frac{\rho^{(z-1)+n}}{((z-1)+n)!} - z\sum_{n=1}^{\infty}\frac{\rho^{(z-1)+n}}{((z-1)+n)!}\right)$$
$$= e^{-\rho}\left(\frac{\rho^z}{(z-1)!} + (\rho-z)\sum_{n=0}^{\infty}\frac{\rho^{z+n}}{(z+n)!}\right) = \frac{e^{-\rho}\rho^z}{\Gamma(z)} + (\rho-z)\left(1-\frac{\Gamma(z,\rho)}{\Gamma(z)}\right).$$

A.3 Proof of Theorem 1: Applying (9) it follows that f satisfies

$$f(z+1)+f(z-1)-2f(z) = \frac{e^{-\rho}\rho^z(u+v)}{\Gamma(z+1)} > 0,$$

for $z = 1,2,\ldots$, and therefore f is *discrete convex* in z and has a minimizer (see e.g., Murota 2003, p. 14). □

A.4 Proof of Theorem 2: The Hessian matrix of $f(\bar{\alpha})$ is positive-semi-definite since its eigenvalues are

$$\overrightarrow{\Lambda} = (0,0,\ldots,\frac{\lambda\rho(\bar{\alpha})^z e^{-\rho(\bar{\alpha})}\left(\sum_{j=1}^{n}\prod_{k\neq j}\mu_k^2\right)(u+v)}{\prod_{j=1}^{n}\mu_j\cdot\left(\sum_{j=1}^{n}\prod_{k\neq j}\alpha_j\mu_k\right)\Gamma(z)}), \tag{23}$$

which are $\Lambda_j \geq 0$ for all j. The domain $\left\{\bar{\alpha}|\sum_j \alpha_j = 1, \alpha_j \geq 0\right\}$ is a compact set and f is continuous with respect to $\bar{\alpha}$. □

E2

Demand and Service Stationarity Testing

We treat each process involving station j as a time series. We use a time unit based on the daily system activity to conduct this analysis. Our null hypothesis is that the arrival and service processes involving a station are stationary.

The following figures, EC.1 - EC.2, and Figures EC.3 - EC.4, represent the results of the ADF and PP tests for time-series stationarity of the arrival and service processes. The black bars in the figures indicate the critical value of -3.41 for a p-value lower than 0.01. The ADF test shows statistical significance for all stations for both the arrival and service processes.

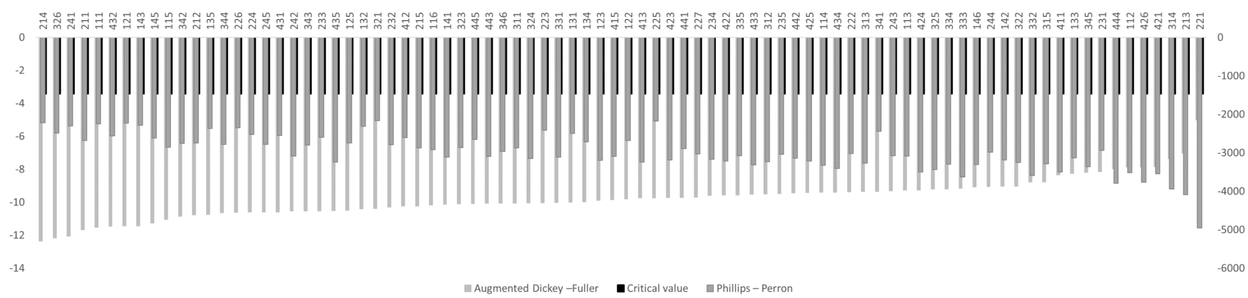

Figure EC.1 Augmented Dickey–Fuller and Phillips–Perron stationarity tests of demand for chiefs - station level

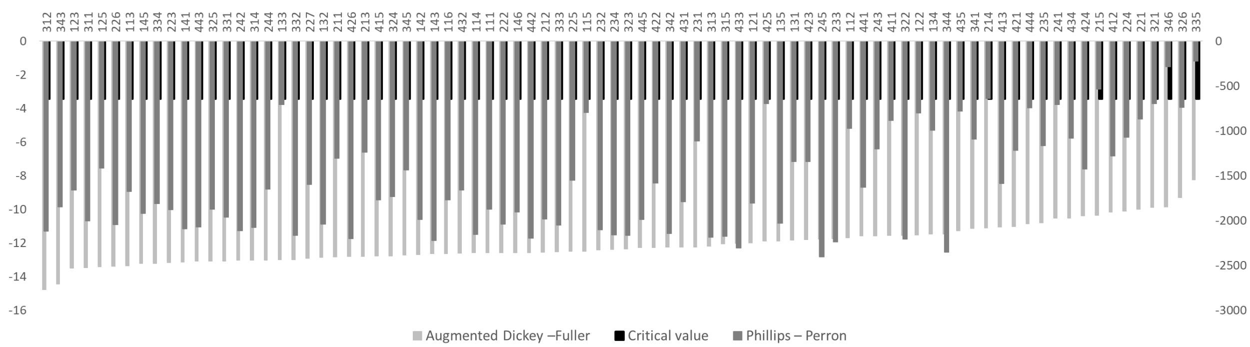

Figure EC.2 Augmented Dickey–Fuller and Phillips–Perron stationarity tests for service of chiefs - station level

Similarly, the PP test exhibited significance for all arrivals and most stations (with failures for Stations 215, 346, and 335, where the test statistic is just below the critical value). Thus, out of a total of 332 tests (84 stations multiplied by four tests), we observed three failures at the 0.01 confidence level.

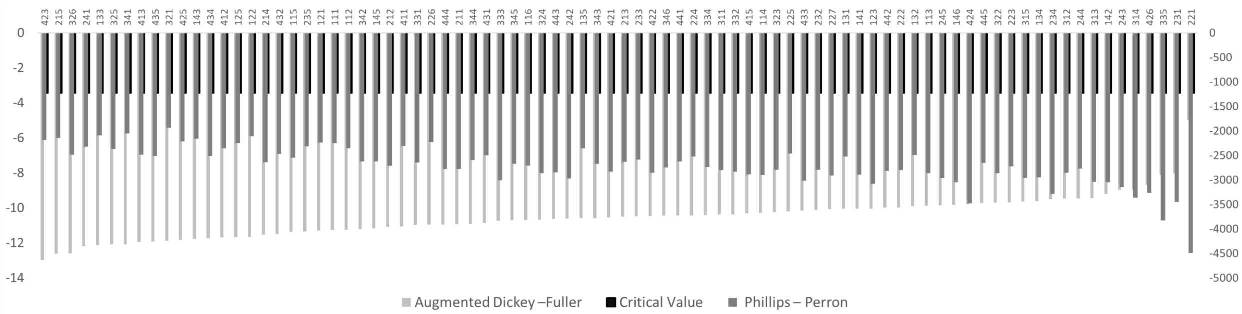

Figure EC.3 **Augmented Dickey–Fuller and Phillips–Perron stationarity tests of demand for pumpers - station level**

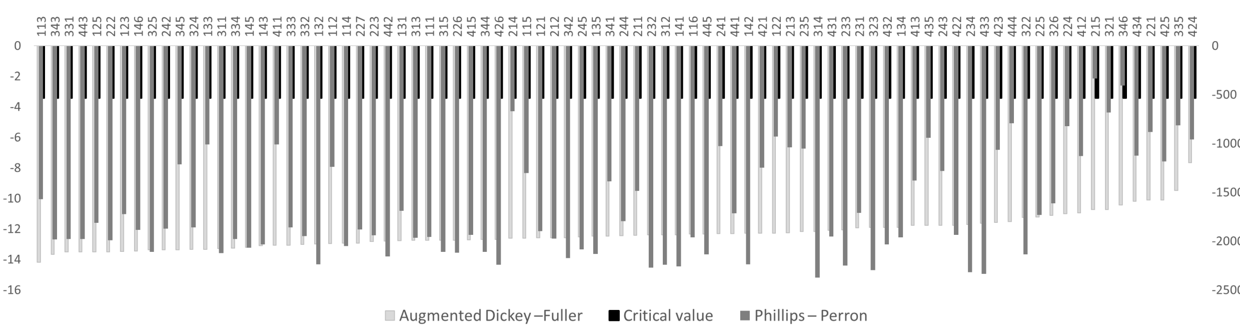

Figure EC.4 **Augmented Dickey–Fuller and Phillips–Perron stationarity tests for service of pumpers - station level**

E3

Table EC.5 compares the number of occupied pumpers for each district in the data (empirical) to the same parameter calculated from our model ($M_i/G_i/\infty$). The level of agreement between these two sets of numbers further supports the usage of our model for solving the EVF problem.

	District 11		District 13		District 14		District 21	
# of occupied pumpers	Empirical	$M_i/G_i/\infty$	Empirical	$M_i/G_i/\infty$	Empirical	$M_i/G_i/\infty$	Empirical	$M_i/G_i/\infty$
0	0.7806	0.75	0.8	0.795533	0.756	0.73	0.936	0.934
1	0.167	0.215	0.156	0.181972	0.18	0.23	0.058	0.0635
2	0.039	0.03	0.029	0.02	0.045	0.037	0.005	0.0022
3	0.01	0.0029	0.0045	0.0016	0.0125	0.0038	≈ 0	≈ 0
4	0.0017	0.00021	0.00047	0.00009	0.002	0.0003	-	-
5	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	-	-
	District 22		District 23		District 24		District 41	
0	0.8	0.795	0.86	0.864	0.873	0.868	0.945	0.947
1	0.156	0.182	0.128	0.126	0.11	0.122	0.054	0.051
2	0.029	0.02	0.01	0.009	0.014	0.0086	-	-
3	0.0047	0.0016	0.0004	0.00045	0.00094	0.0004	-	-
4	0.00046	0.00009	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-
	District 42		District 43		District 44			
0	0.9	0.91	0.872	0.86	0.82	0.81		
1	0.086	0.083	0.108	0.127	0.156	0.163		
2	0.0034	0.0038	0.017	0.0094	0.021	0.016		
3	-	-	-	-	0.0013	0.001		

Table EC.5 The distribution of the number of occupied pumpers in a district is Poisson.

E4

	$\alpha^*_{i,j}(\%)$						
Run-Area\Station	Station 1	Station 2	Station 3	Station 4	Station 5	$P(RT_i \leq 384)$	
RA - 311	**88**	0	0	12	0	0.915	
RA - 312	0	**0**	50	34	16	0.977	**Case 1**
RA - 313	7	0	**49**	33	11	0.933	$v = 1,\ u = 10$
RA - 314	0	0	0	**22**	78	0.985	
RA - 315	0	0	21	0	**79**	0.931	Mean cost 3.657
Optimal Number of Vehicles z_i^*	1	0	1	1	1		$z^* = 4$
ρ	0.0426	0	0.0528	0.0494	0.093		
RA - 311	**88**	0	0	12	0	0.915	
RA - 312	32	**0**	41	22	5	0.968	**Case 2**
RA - 313	7	0	**49**	33	11	0.933	$v = 1,\ u = 20$
RA - 314	0	0	0	**22**	78	0.985	
RA - 315	0	0	21	0	**79**	0.931	Mean cost 3.67
Optimal number of Vehicles z_i^*	1	0	1	1	1		$z^* = 4$
ρ	0.052	0	0.05	0.045	0.089		
RA - 311	**84**	0	13	2	1	0.933	
RA - 312	25	**0**	60	4	11	0.969	**Case 3**
RA - 313	16	0	**65**	12	7	0.902	$v = 1,\ u = 60$
RA - 314	0	0	38	**23**	39	0.983	
RA - 315	0	0	20	3	**77**	0.932	Mean cost 3.72
Optimal Number of Vehicles z_i^*	1	0	1	1	1		$z^* = 4$
ρ	0.054	0	0.095	0.024	0.063		
RA - 311	**100**	0	0	0	0	0.952	
RA - 312	0	**0**	100	0	0	0.975	**Case 4**
RA - 313	0	0	**100**	0	0	0.965	$v = 1,\ u = 100$
RA - 314	0	0	0	**100**	0	0.978	
RA - 315	0	0	0	0	**100**	0.980	Mean cost 3.765
Optimal Number of Vehicles z_i^*	1	0	1	1	1		$z^* = 4$
ρ	0.0427	0	0.0884	0.0603	0.0387		
RA - 311	**100**	0	0	0	0	0.952	
RA - 312	0	**100**	0	0	0	0.988	**Case 5**
RA - 313	0	0	**100**	0	0	0.965	$v = 1,\ u = 7,000$
RA - 314	0	0	0	**100**	0	0.978	
RA - 315	0	0	0	0	**100**	0.980	Mean cost 5.601
Optimal number of Vehicles z_i^*	1	1	1	1	1		$z^* = 5$
ρ	0.0427	0.031	0.056	0.06	0.038		
RA - 311	**100**	0	0	0	0	0.952	
RA - 312	0	**100**	0	0	0	0.988	**Case 6**
RA - 313	0	0	**100**	0	0	0.965	$v = 1,\ u = 10,000$
RA - 314	0	0	0	**100**	0	0.978	
RA - 315	0	0	0	0	**100**	0.980	Mean cost 5.797
Optimal number of Vehicles z_i^*	1	1	1	1	2		$z^* = 6$
ρ	0.0427	0.031	0.056	0.06	0.019		

Table EC.6 Optimal capacity, deployment and mutual aid

E5

Actual deployment

Station	114	132	141	212	225	231	242	312	323	332	345	415	423	435	445
111	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
112	87	5	7	0	0	0	0	0	0	0	0	0	0	0	0
113	91	0	0	0	0	0	9	0	0	0	0	0	0	0	0
114	91	7	2	0	0	0	0	0	0	0	0	0	0	0	0
115	92	0	0	0	0	0	8	0	0	0	0	0	0	0	0
116	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
121	35	65	0	0	0	0	0	0	0	0	0	0	0	0	0
122	79	21	0	0	0	0	0	0	0	0	0	0	0	0	0
123	86	14	0	0	0	0	0	0	0	0	0	0	0	0	0
125	0	9	0	0	0	0	0	0	91	0	0	0	0	0	0
131	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	97	0	0	0	0	0	0	0	0	3	0	0	0	0
133	0	85	8	0	0	0	0	0	0	0	0	0	7	0	0
134	0	86	0	0	0	0	0	14	0	0	0	0	0	0	0
135	0	85	0	0	0	0	0	9	0	0	6	0	0	0	0
141	3	0	97	0	0	0	0	0	0	0	0	0	0	0	0
142	0	0	98	0	0	0	0	0	0	0	0	2	0	0	0
143	7	50	43	0	0	0	0	0	0	0	0	0	0	0	0
145	0	82	18	0	0	0	0	0	0	0	0	0	0	0	0
146	0	3	93	0	0	0	0	0	0	0	0	4	0	0	0
211	0	0	0	28	0	0	72	0	0	0	0	0	0	0	0
212	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
213	0	0	0	59	0	38	3	0	0	0	0	0	0	0	0
214	0	0	0	95	0	5	0	0	0	0	0	0	0	0	0
215	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
221	0	0	0	0	54	46	0	0	0	0	0	0	0	0	0
222	0	0	0	0	95	5	0	0	0	0	0	0	0	0	0
223	0	0	0	0	38	62	0	0	0	0	0	0	0	0	0
224	0	0	0	0	42	0	0	0	58	0	0	0	0	0	0
225	0	0	0	0	97	0	0	0	3	0	0	0	0	0	0
226	0	0	0	0	80	0	0	0	20	0	0	0	0	0	0
227	0	0	0	0	32	0	0	0	68	0	0	0	0	0	0
231	0	0	0	2	0	96	2	0	0	0	0	0	0	0	0
232	0	0	0	0	9	82	9	0	0	0	0	0	0	0	0
233	0	0	0	0	73	27	0	0	0	0	0	0	0	0	0
234	0	0	0	8	0	92	0	0	0	0	0	0	0	0	0
235	0	0	0	0	54	0	0	0	46	0	0	0	0	0	0
241	5	0	0	0	0	0	95	0	0	0	0	0	0	0	0
242	0	0	0	3	0	0	97	0	0	0	0	0	0	0	0
243	0	0	0	3	0	56	41	0	0	0	0	0	0	0	0
244	4	0	0	0	0	0	96	0	0	0	0	0	0	0	0
245	0	0	0	0	0	38	63	0	0	0	0	0	0	0	0
311	0	4	0	0	0	0	0	91	0	0	5	0	0	0	0
312	0	0	0	0	0	0	0	92	0	6	2	0	0	0	0
313	0	0	0	0	0	0	0	83	11	6	0	0	0	0	0
314	0	0	0	0	0	0	0	54	1	44	1	0	0	0	0
315	0	0	0	0	0	0	0	34	0	56	10	0	0	0	0
321	0	22	0	0	0	0	0	18	60	0	0	0	0	0	0
322	0	0	0	0	0	0	0	4	96	0	0	0	0	0	0
323	0	0	0	0	0	0	0	4	96	0	0	0	0	0	0
324	0	0	0	0	0	0	0	4	96	0	0	0	0	0	0
325	0	0	0	0	0	0	0	22	11	66	0	0	0	0	0
326	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
331	0	0	0	0	0	0	0	4	0	90	6	0	0	0	0
332	0	0	0	0	0	0	0	8	1	90	1	0	0	0	0
333	0	0	0	0	0	0	0	9	3	88	0	0	0	0	0
334	0	0	0	0	0	0	0	7	0	91	2	0	0	0	0
335	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
341	0	41	0	0	0	0	0	0	0	0	53	0	6	0	0
342	0	0	0	0	0	0	0	0	0	0	69	0	31	0	0
343	0	0	0	0	0	0	0	21	0	0	79	0	0	0	0
344	0	0	0	0	0	0	0	60	0	2	38	0	0	0	0
345	0	0	0	0	0	0	0	0	0	0	92	0	8	0	0
346	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
411	0	0	71	0	0	0	0	0	0	0	0	29	0	0	0
412	0	0	0	0	0	0	0	0	0	0	0	97	0	0	3
413	0	0	3	0	0	0	0	0	0	0	0	97	0	0	0
415	0	0	3	0	0	0	0	0	0	0	0	94	0	0	4
421	0	0	0	0	0	0	0	0	0	0	8	0	92	0	0
422	0	0	0	0	0	0	0	0	0	0	4	0	96	0	0
423	0	0	0	0	0	0	0	0	0	0	11	0	89	0	0
424	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0
425	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0
426	0	0	0	0	0	0	0	0	0	10	64	0	26	0	0
431	0	0	0	0	0	0	0	0	0	0	0	0	5	11	84
432	0	0	0	0	0	0	0	0	0	0	0	0	0	91	9
433	0	0	0	0	0	0	0	0	0	0	0	0	5	88	7
434	0	0	0	0	0	0	0	0	0	0	0	0	0	93	7
435	0	0	0	0	0	0	0	0	0	0	0	0	0	95	5
441	0	0	0	0	0	0	0	0	0	0	0	65	0	1	33
442	0	4	22	0	0	0	0	0	0	0	0	31	35	0	8
443	0	0	0	0	0	0	0	0	0	0	0	37	0	0	63
444	0	0	0	0	0	0	0	0	0	0	0	8	0	7	85
445	0	0	0	0	0	0	0	0	0	0	0	0	0	10	90
Vehicles	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Optimal deployment

Station	114	132	141	212	225	231	242	312	323	332	345	415	423	435	445
111	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
112	34	31	34	0	0	0	0	0	0	0	0	0	0	0	0
113	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
114	36	28	35	0	0	0	0	0	0	0	0	0	0	0	0
115	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
116	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
121	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0
122	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0
123	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0
125	0	50	0	0	0	0	0	0	50	0	0	0	0	0	0
131	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	51	0	0	0	0	0	0	0	0	49	0	0	0	0
133	0	35	33	0	0	0	0	0	0	0	0	0	32	0	0
134	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0
135	0	56	0	0	0	0	0	0	0	0	44	0	0	0	0
141	45	0	55	0	0	0	0	0	0	0	0	0	0	0	0
142	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0
143	84	8	8	0	0	0	0	0	0	0	0	0	0	0	0
145	0	50	50	0	0	0	0	0	0	0	0	0	0	0	0
146	0	44	56	0	0	0	0	0	0	0	0	0	0	0	0
211	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
212	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
213	0	0	0	51	0	49	0	0	0	0	0	0	0	0	0
214	0	0	0	50	0	50	0	0	0	0	0	0	0	0	0
215	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
221	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
222	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
223	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
224	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
225	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
226	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
227	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
231	0	0	0	46	0	54	0	0	0	0	0	0	0	0	0
232	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
233	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
234	0	0	0	50	0	50	0	0	0	0	0	0	0	0	0
235	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
241	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
242	0	0	0	100	0	0	0	0	0	0	0	0	0	0	0
243	0	0	0	48	0	52	0	0	0	0	0	0	0	0	0
244	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
245	0	0	0	0	0	100	0	0	0	0	0	0	0	0	0
311	0	49	0	0	0	0	0	0	0	0	51	0	0	0	0
312	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0
313	0	0	0	0	0	0	0	0	56	44	0	0	0	0	0
314	0	0	0	0	0	0	0	0	30	30	40	0	0	0	0
315	0	0	0	0	0	0	0	0	0	51	49	0	0	0	0
321	0	48	0	0	0	0	0	0	52	0	0	0	0	0	0
322	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
323	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
324	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
325	0	0	0	0	0	0	0	0	49	51	0	0	0	0	0
326	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0
331	0	0	0	0	0	0	0	0	0	52	48	0	0	0	0
332	0	0	0	0	0	0	0	0	36	38	27	0	0	0	0
333	0	0	0	0	0	0	0	0	48	52	0	0	0	0	0
334	0	0	0	0	0	0	0	0	0	55	45	0	0	0	0
335	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
341	0	35	0	0	0	0	0	0	0	0	35	0	31	0	0
342	0	0	0	0	0	0	0	0	0	0	50	0	50	0	0
343	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0
344	0	0	0	0	0	0	0	0	0	44	56	0	0	0	0
345	0	0	0	0	0	0	0	0	0	0	50	0	50	0	0
346	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
411	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0
412	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
413	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0
415	0	0	99	0	0	0	0	0	0	0	0	0	0	0	1
421	0	0	0	0	0	0	0	0	0	0	50	0	50	0	0
422	0	0	0	0	0	0	0	0	0	0	50	0	50	0	0
423	0	0	0	0	0	0	0	0	0	0	50	0	50	0	0
424	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0
425	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0
426	0	0	0	0	0	0	0	0	0	31	34	0	34	0	0
431	0	0	0	0	0	0	0	0	0	0	0	0	48	0	52
432	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
433	0	0	0	0	0	0	0	0	0	0	0	0	48	0	52
434	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
435	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
441	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
442	0	25	26	0	0	0	0	0	0	0	0	0	26	0	24
443	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
444	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
445	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
Vehicles	1	1	1	1	0	1	0	0	1	1	1	0	1	0	1

Table EC.7 Actual deployment vs. optimal deployment of chief vehicles for $v=1$ and $u=1,000,000$