Forced Labor in Labor Supply Chains- Contracting and Information Asymmetry

Felix Papier, Christopher Tang, Javaiz Parappathodi

Appendix

A. Summary of Notation

Table 3 summarizes the notation of the paper.

B. Comparison Table of Literature

Table 4 summarizes key features of existing research on unethical supplier activities.

C. Agent Selection in the General Case of $q \geq 2$ Types.

In this section, we analyze the general case for arbitrary $q \geq 2$. We define the optimal menu of contracts and show that it leads the agent strategy $s = (e_1, ...e_q)$ (e_i being the i-th unit row vector) to be a PBE. We also extend the results of Corollary 2 to the general case.

Belief updating: In any round i (i.e., when the i-th contract is proposed), an agent knows that none of the other agents has a type $j < i$ if they play strategy s, because then a contract would already have been awarded to some other agent. Thus, at the beginning of round i, an agent believes that the types of the

Symbol	Description
α_i	Probability of occurrence of the agent of type i
$\widehat{\alpha}(n)$	Support term for the model with selection from multiple agents
a (a_i)	Audit-effort level of the buyer (under contract i)
c_g	Coercion cost incurred by the agent per unit of coercion
c_a	Audit cost for each audit-level unit
δ	Support term for additional profit from using coercion
γ	Effectiveness of coercion
D (D_i)	Labor demand of the buyer (Labor quantity of contract i)
$f(a,g)$	Probability of detecting coercion during pre-contract auditing for an audit-effort level a and coercion level g
g (g_i)	Coercion level of the agent (Coercion level of the agent of type i)
K (K_i)	Labor pool size (size of labor pool available to agent of type i)
η	Level of available pre-contract information about the agent
λ_i	Lagrange multiplier
m	Loss incurred by the agent in case coercion is detected during pre-contract auditing
n	Number of agents
p (p_i)	Price per worker paid to the agent (Price per worker paid to the agent under contract i)
ψ	Probability that forced labor is detected by post-contract scrutiny
Ψ	The buyer's expected reputation damage (when forced labor is detected by post-contract scrutiny)
$\pi(\cdot)$	Agent's profit
$\Pi(\cdot)$	Buyer's profit
Π^c	Part of the buyer's profit made from recruiting workers in-house
q	Number of types of agents
r	Revenue obtained by the buyer for each worker
u	In-house recruitment cost per worker
v	Reservation value of workers, $v \sim U[0,1]$
w (w_i)	Wage paid to the workers (Wage paid to the workers by agent of type i)
w^0	Fair wage

Table 3 Table of symbols.

	Agent's Behavior	Detection Method: Buyer Audit	Detection Method: 3rd Party Scrutiny	Information Structure: Information Asymmetry	Information Structure: Private Information	Workers' Utility
Babich and Tang (2012)	Endogenous	✓	–	✓	Supplier Behavior	–
Rui and Lai (2015)	Endogenous	✓	–	✓	Supplier Behavior	–
Guo et al. (2016)	Exogenous	–	✓	✓	Buyer Behavior	–
Plambeck and Taylor (2016)	Endogenous	✓	–	–	–	–
Chen and Lee (2017)	Endogenous	–	✓	✓	Supplier Behavior	–
Caro et al. (2018)	Endogenous	✓	–	✓	Supplier Behavior	–
Chen et al. (2019)	Exogenous	✓	✓	✓	Supplier Capability	–
Cho et al. (2019)	Endogenous	✓	✓	✓	Buyer Audit	–
Kalkanci and Plambeck (2020)	Exogenous	✓	–	✓	Supplier Behavior	–
Huang et al. (2022)	Endogenous	✓	✓	–	–	–
Lu and Tomlin (2022)	Exogenous	✓	–	✓	Supplier Behavior	–
This Paper	Endogenous	✓	✓	✓	Agent Capability	✓

Table 4 Features of published research literature.

other $n-1$ agents follow a multinomial distribution with $n-1$ trials, $(q+1-i)$ types (all types $j<i$ have zero probability), and type j having success probability of

$$\alpha_j^{(i)} = \frac{\alpha_j}{1-\sum_{l=1}^{i-1}\alpha_l}, j \geq i. \tag{35}$$

If j other agents choose the contract in the same round as the focal agent, the focal agent's probability of winning the contract through the random lottery is $1/(j+1)$. Hence, the probability for an agent of winning contract i, at the beginning of round i, is given by

$$\sum_{j=0}^{n-1}\left(\alpha_i^{(i)}\right)^j (1-\alpha_i^{(i)})^{n-1-j}\binom{n-1}{j}\cdot\frac{1}{j+1} = \sum_{j=1}^{n}(\alpha_i^{(i)})^j\left(1-\alpha_i^{(i)}\right)^{n-j}\binom{n}{j}\cdot\frac{1}{n\alpha_i^{(i)}} = \frac{1-(1-\alpha_i^{(i)})^n}{n\alpha_i^{(i)}}. \tag{36}$$

Similarly, if the same agent would wait until round $j>i$ to accept the contract, their success probability of winning the contract would be

$$\begin{aligned}&\left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1}\left(\sum_{j=0}^{n-1}\alpha_j^{(i)}(1-\alpha_j^{(i)})^{n-1-j}\frac{\binom{n-1}{j}}{j+1}\right)\\ &= \frac{1-(1-\alpha_j^{(i)})^n}{n\alpha_j^{(j)}}\left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1} = \tilde{\alpha}_n(j)\beta_n(i,j),\end{aligned} \tag{37}$$

where $\tilde{\alpha}_n(j) \equiv \frac{1-(1-\alpha_j^{(i)})^n}{n\alpha_j^{(j)}}$ represents the probability, at the beginning of round j, that an agent that accepts a contract j will receive it, while term $\beta_n(i,j) \equiv \left(1-\sum_{l=i}^{j-1}\alpha_l^{(i)}\right)^{n-1}$ represents the probability that no other agent that plays strategy s will be awarded the contract between rounds i and j (excluding round j). We define $\beta_n(j,i) = \beta_n(i,j)^{-1}$.

Necessary constraints for a PBE: The profit of an agent of type j with contract (p,D) and audit level $a_i^*(p,D)$ is given by

$$\pi_j(p,D,a_i^*(p,D)) = pD - \delta_{i,j}\frac{D^2}{K_j}, \tag{38}$$

where $\delta_{i,j} = 1-\frac{\gamma}{4c_g}\left(1-\frac{K_j}{K_i}\right)^2$ if $i<j$ and $\delta_{i,j}=1$ else. Term $1-\delta_{i,j}$ represents the cost reduction that an agent of type j can achieve from using coercion if the buyer uses the audit level that is optimal for an agent

of type i. In order for strategy s to constitute a PBE, an agent should maximize their expected profit by playing strategy s compared to deviating from that strategy. That means that at the beginning of any round i, it should hold for an agent of any type j with $i<j$ that

$$\tilde{\alpha}_n(i)\cdot\pi_j(p_i,D_i,a_i^*(p_i,D_i)) \le \max_{(i'|i<i'\le q)} \tilde{\alpha}_n(i')\cdot\beta_n(i,i')\cdot\pi_j(p_{i'},D_{i'},a_{i'}^*(p_{i'},D_{i'})), \tag{39}$$

and for an agent of any type j with $j=i$

$$\tilde{\alpha}_n(j)\cdot\pi_j(p_j,D_j,a_j^*(p_j,D_j)) \ge \max_{(i'|j<i'\le q)} \tilde{\alpha}_n(i')\cdot\beta_n(j,i')\cdot\pi_j(p_{i'},D_{i'},a_{i'}^*(p_{i'},D_{i'})), \tag{40}$$

where set of constraints (39) prevents agents from accepting a contract 'too early' and the set of constraints (40) prevents agents from accepting contracts 'too late' for their type. Next, we use Lemma 3 to substantially reduce the number of constraints, from $(1/2)q^3$ to $q(q-1)$ IC constraints.

Lemma 3. *The IC constraints of* (39) *and* (40) *can be simplified to*

$$IC(i,j):\ \tilde{\alpha}_n(i)\beta_n(j,i)\pi_j(p_i,D_i,a_i^*(p_i,D_i)) \le \tilde{\alpha}_n(j)\pi_j(p_j,D_j,a_j^*(p_j,D_j)), \forall 1\le i,j\le q. \tag{41}$$

Expected buyer profit under strategy s**.** The buyer awards contract $j=1$ only if there is at least one agent of type $j=1$ in the pool, which occurs with probability $1-(1-\alpha_1)^n$. If there is no agent of type $j=1$, the buyer proceeds to the next round, proposing the contract $j=2$, and so on. Hence, the probability that there is at least one agent of type j (given that no agent has type $i<j$) is given by

$$1-\left(1-\frac{\alpha_j}{1-\sum_{l=1}^{j-1}\alpha_l}\right)^n, \tag{42}$$

while the probability of 'reaching' round j is given by

$$\left(1-\sum_{l=1}^{j-1}\alpha_l\right)^n. \tag{43}$$

Let us write the menu of contracts as $(\mathbf{p},\mathbf{D},\mathbf{a}^*(\mathbf{p},\mathbf{D}))=(p_1,D_1,a_1^*(p_1,D_1),...,p_q,D_q,a_q^*(p_q,D_q))$. Then, we can express expected buyer profit as

$$\Pi(\mathbf{p},\mathbf{D},\mathbf{a}^*(\mathbf{p},\mathbf{D}))=\sum_{j=1}^{q}\left(1-\sum_{l=1}^{j-1}\alpha_l\right)^n\left(1-\left(1-\frac{\alpha_j}{1-\sum_{l=1}^{j-1}\alpha_l}\right)^n\right)\left((r-p_j)D_j-c_a a_j^*(p_j,D_j)+\Pi^c(D_j)\right). \tag{44}$$

The buyer's sequential contract design problem.

The solution to the following optimization problem

$$\max_{(\mathbf{p},\mathbf{D})} \Pi(\mathbf{p},\mathbf{D},\mathbf{a}^*(\mathbf{p},\mathbf{D})), \tag{45}$$

subject to constraints

$$\text{IC}(i,j):\ \tilde{\alpha}_n(i)\beta_n(j,i)\left(p_iD_i-\delta_{i,j}\frac{D_i^2}{K_j}\right)\le\tilde{\alpha}_n(j)\left(p_jD_j-\frac{D_j^2}{K_j}\right), \forall 1\le i,j\le q, \tag{46}$$

$$\text{IR}(j):\ p_j\ge\frac{D_j}{K_j}, \forall 1\le j\le q, \tag{47}$$

$$\text{Other: } \max(0,D-K)\le D_j\le\min(D,K_j), \forall 1\le j\le q, \tag{48}$$

corresponds to the profit-maximizing sequential menu of contracts, which we formalize in the following proposition.

Proposition 5. *The optimal solution* $(\hat{\mathbf{p}}^*, \hat{\mathbf{D}}^*, \mathbf{a}^*(\hat{\mathbf{p}}^*, \hat{\mathbf{D}}^*))$ *to problem* (45)-(48) *guarantees the selection of the agent with the lowest type* j *in the agent pool and maximizes expected buyer profit* Π *among the feasible menus of contracts that induce a PBE for the game between the* n *agents in the pool under agent strategy* s.

In the following proposition, we extend some of the results of Corollary 2 of the case with $q = 2$ to the general case.

Proposition 6. *Problem* (45)-(48) *has the following characteristics:*

1. *Constraint IR(j) for any* $j < q$ *is redundant with other IC and IR constraints, such that the only IR constraint relevant to the problem is IR(q).*

2. *For every feasible solution, it holds for any pair* $i < j$ *that*

$$D_i \geq \sqrt{\frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j)} \cdot \sqrt{\frac{\frac{1}{K_j} - \frac{1}{K_i}}{\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}}} \cdot D_j > \sqrt{\frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j)} \cdot D_j, \tag{49}$$

and

$$p_i D_i \geq \frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j) \cdot p_j D_j. \tag{50}$$

Part (1) of Proposition 6 shows that the corresponding result of the case with two agent types applies to all agents except for those of the type with the smallest labor pool size. Part (2) allows us to restrict the search space of the optimal solution in a similar way as we could for the model with $q = 2$ types.

D. Agents' Rational Beliefs under the Sequential Menu of Contracts with $q = 2$ Types.

Consider a sequential menu of contracts in which the buyer offers (p_h, D_h) in round 1. If no agent accepts the contract (p_h, D_h) in round 1, then the buyer proceeds to round 2 by offering (p_l, D_l). If at least one agent accepts (p_h, D_h), then the accepting agents must belong to type 1 (type h). If more than one agent accepts the contract, the buyer selects randomly one of the accepting agents of type 1. Because the contract is intended for type h (or type 1) agents to accept in round 1, let us first examine the belief of an agent of type h. This agent believes that there are $j, j = 0, \ldots, (n-1)$, agents of type h in the pool (excluding himself) with probability $\binom{n-1}{j}(1-\alpha)^j \alpha^{(n-1)-j}$, where each agent belongs to type h with probability $(1-\alpha)$. Also, this type h agent knows that, when there are j other agents of type h, the agent's chance of winning the contract is $1/(j+1)$ due to random lottery. Hence, each agent of type h believes that their (ex-ante) probability of winning the contract in the first round is:

$$\sum_{j=0}^{n-1}(1-\alpha)^j \alpha^{n-1-j}\binom{n-1}{j} \cdot \frac{1}{j+1} = \sum_{j=1}^{n}(1-\alpha)^j \alpha^{n-j}\binom{n}{j} \cdot \frac{1}{n(1-\alpha)} = \frac{1-\alpha^n}{n(1-\alpha)}. \tag{51}$$

Next, we determine the belief of an agent of type l. As explained above, the sequential menu of contracts is designed to entice only type h agent(s) to accept the contract in round 1. Hence, an agent of type l knows that only the contract in round 2 is accepted by him and other type l agents. However, all agents know that the process will end after round 1 if there is at least one agent of type h in the pool. Hence, in the event that the buyer proceeds to round 2, each of the n agents will know with certainty that all other agents are of type l. Due to random lottery, an agent with type l will know their winning probability is $1/n$ in round

2. Therefore, prior to the first round, each agent of type l knows that their (ex-ante) winning probability for the contract in the second round is $\alpha^{n-1} \cdot (1/n)$, where $\alpha^{(n-1)}$ is the probability that all other $(n-1)$ agents are of type l. However, if no agent accepts the contract in round 1, it reveals that all agents are of type l. Hence, when the second round takes place, each agent knows their (ex-post) winning probability is $1/n$.

E. Proofs

Proof of Proposition 1. Note that the audit detection probability function is defined in such a way that $f(a, g=0) = 0$. Thus, we can rewrite (2) without the indicator function in the following manner:

$$\pi(p,a,g) = pD \cdot (1 - f(a,g)) - wD - c_g \cdot \gamma g^2 K - f(a,g) \cdot m. \tag{52}$$

The function given in (52) is differentiable in g. Then, taking the derivative of the agent's profit (52) with respect to g and using (1), we obtain:

$$\frac{d}{dg}\pi(p,a,g) = -(pD+m)\frac{d}{dg}f(a,g) + \gamma D - 2c_g\gamma K g. \tag{53}$$

Further, using the form of detection probability $f(a,g) = a \cdot \eta \frac{\gamma g K}{D}$, we obtain

$$\frac{d}{dg}\pi(p,a,g) = -(pD+m) \cdot a \cdot \eta \frac{K\gamma}{D} + \gamma D - 2c_g\gamma K g, \tag{54}$$

which leads to

$$\frac{d}{dg}\pi(p,a,g) = 0 \Leftrightarrow -(pD+m) \cdot a\eta \cdot \frac{K\gamma}{D} + \gamma D = 2c_g\gamma K g \Leftrightarrow g^* = \frac{D}{2c_g \cdot K} - \frac{a \cdot \eta}{2c_g}\left(p + \frac{m}{D}\right). \tag{55}$$

We observe that $\pi(p,a,g)$ is concave in g for given p and a. From (1), we can retrieve w^*. Level g is limited by the upper bound that ensures non-negative wage w, see discussion in Section 4. For wage to be non-negative, $g \leq D/(K\gamma)$ must hold, which is implied by the technical assumption $\gamma < 2c_g$ (see Section 4). □

Proof of Lemma 1. Buyer profit given by (3) can be rewritten as the following, for any given level of $g > 0$:

$$\Pi_{g>0} = ((rD - \psi \cdot \Psi - pD) \cdot (1 - f(a,g)) - c_a \cdot a) + \left(r - \frac{D}{K} - u\right) D \cdot f(a,g). \tag{56}$$

It is easy to observe that the first term in (56) is negative from assumptions $\psi \cdot \Psi > rD$, $a, p \geq 0$, and $f(a,g) \leq 1$. Further, if $\left(r - \frac{D}{K} - u\right) \leq 0$, $\Pi_{g>0} < 0$, but by assumption the buyer can make a positive profit from outsourcing. Or, if $\left(r - \frac{D}{K} - u\right) > 0$, $\Pi_{g>0}$ is less than the profit of sourcing exclusively in-house (see (4)), which in turn implies that outsourcing at coercion level g is less preferable than recruiting the workforce internally. Consequently, the buyer will choose parameters (p,a) such that $g^*(p,a) = 0$. □

Proof of Proposition 2. When $g^* = 0$, we can rewrite the buyer's profit function given by (3) in the following manner:

$$\Pi(p,a,g) = (r-p)D - c_a \cdot a. \tag{57}$$

Observe from (5) that we can express the audit level a in terms of price p so that $a = \frac{D}{K} \cdot \left(\eta \cdot (p + \frac{m}{D})\right)^{-1}$. For the case when $g^* = 0$, the buyer's problem can consequently be rewritten as:

$$\max_{p \geq D/K} \left[(r-p)D - c_a a\right] = \max_{p \geq D/K} \left[(r-p)D - c_a \left(\frac{\frac{D}{\eta K}}{p + \frac{m}{D}}\right)\right]. \tag{58}$$

We obtain as derivative of the buyer profit with respect to p:

$$\frac{d}{dp}\left((r-p)D - c_a\frac{D}{\eta K}\left(p+\frac{m}{D}\right)^{-1}\right) = -D + c_a\frac{D}{\eta K}\left(p+\frac{m}{D}\right)^{-2}, \tag{59}$$

which decreases in p, such that the first-order condition yields:

$$p^* = \max\left\{\frac{D}{K}, \sqrt{\frac{c_a}{\eta K}} - \frac{m}{D}\right\}. \tag{60}$$

Optimal audit level a^* follows from

$$g^*(p^*, a^*) = 0 \Leftrightarrow p = \frac{D}{a\eta K} - \frac{m}{D} \Leftrightarrow a^* = \frac{D}{\eta K(p^* + \frac{m}{D})}.$$

For $c_a \geq \eta K(\frac{D}{K}+\frac{m}{D})^2$, it follows consequently $p^* = \sqrt{\frac{c_a}{\eta K}} - \frac{m}{D}$ and, hence, $a^* = \frac{D}{\sqrt{\eta K c_a}}$. For $c_a < \eta K(\frac{D}{K}+\frac{m}{D})^2$, $p^* = \frac{D}{K}$ and, consequently, $a^* = \frac{D^2}{\eta(D^2+mK)}$. The optimal buyer profit follows from $(r-p^*)D - c_a a^*$ according to (3) with $g^*(p^*, a^*) = 0$. □

Proof of Lemma 2. For any given (D_l, D_h), we can derive the optimal price p_i^* by considering the first order condition of $\Pi(p_l, D_l, p_h, D_h)$ given in (13) (with respect to p_i) so that:

$$\frac{\partial}{\partial p_i}\Pi(p_l, D_l, p_h, D_h) = -\alpha_i D_i + \alpha_i\frac{c_a D_i}{\eta K_i(p_i + m/D)^2} = 0 \Leftrightarrow p_i^* = \sqrt{\frac{c_a}{\eta K_i}} - \frac{m}{D}. \tag{61}$$

Note that the second derivative of the profit function with respect to p_i is strictly negative, i.e., buyer profit is concave in price p_i. Observe from (19) that p_i^* is independent of D_i. By substituting the expression for p_i^* into the profit function of the buyer given in (12) and by setting the derivative with respect to D_i to zero, we obtain:

$$\frac{\partial}{\partial D_i}\Pi(p_l^*, D_l, p_h^*, D_h) = -\alpha_i(p_i^* - u) - \alpha_i\frac{c_a}{\eta K_i(p_i^* + m/D)} + \alpha_i\frac{2(D-D_i)}{K} = 0$$

$$\Leftrightarrow D_i^* = D - \frac{K}{2}\left(\sqrt{\frac{c_a}{\eta K_i}} - \frac{m}{D} - u + \frac{c_a}{K_i(\sqrt{\frac{c_a}{\eta K_i}})}\right) = D - K\left(\sqrt{\frac{c_a}{\eta K_i}} - \frac{m/D+u}{2}\right). \tag{62}$$

The second derivative of buyer profit with respect to D_i is $-2\alpha_i/K < 0$. Consequently, buyer profit under the optimal price (under any given price p_i) is strictly concave in D_i. □

Proof of Proposition 3. Constraints ICl and ICh are both satisfied for a given solution if and only if it holds that

$$\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq p_h D_h - p_l D_l \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2). \tag{63}$$

Note from (63) that both constraints (ICl and ICh) can be simultaneously satisfied only if

$$\frac{1}{K_h}((D_h)^2 - (D_l)^2) \leq \frac{1}{K_l}(\delta(D_h)^2 - (D_l)^2) \Leftrightarrow \left(\frac{\delta}{K_l} - \frac{1}{K_h}\right)(D_h)^2 \geq \left(\frac{1}{K_l} - \frac{1}{K_h}\right)(D_l)^2$$

$$\Leftrightarrow D_h \geq \sqrt{\left(\frac{1}{K_l} - \frac{1}{K_h}\right)\left(\frac{\delta}{K_l} - \frac{1}{K_h}\right)^{-1}} \cdot D_l > D_l, \tag{64}$$

Thus, D_h has to be greater than the term given in (64) to allow both IC constraints to be respected (except for the case $D_h = D_l = 0$ in which equality holds). Note that the term under the square root is strictly greater than one, because $K_l < K_h$ and $(K_l/K_h) < \delta < 1$.

Under the assumptions of the proposition, we can rewrite problem (13)-(18) as

$$\max_{(y_l,y_h)} \alpha\left(-y_l - \frac{c_a D_l^2}{\eta K_l}\left(y_l + \frac{mD_l}{D}\right)^{-1}\right) + (1-\alpha)\left(-y_h - \frac{c_a D_h^2}{\eta K_h}\left(y_h + \frac{mD_h}{D}\right)^{-1}\right) \tag{65}$$

subject to

$$ICh:\ y_h - y_l \geq \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}, \tag{66}$$

$$ICl:\ y_l - y_h \geq \frac{D_l^2}{K_l} - \delta\frac{D_h^2}{K_l}, \tag{67}$$

$$IRl: y_l \geq \frac{D_l^2}{K_l}, \tag{68}$$

where we substitute $y_i = p_i D_i, i = l,h$. The term $\alpha(rD_l + \Pi^c(D_l)) + (1-\alpha)(rD_h + \Pi^c(D_h))$ in the objective function is constant in p_i, IRh is redundant, and constraints (18) on demand quantities D_i hold by assumption for (D_l, D_h). Note that the objective function is easily seen to be concave in y_l and in y_h and, because it is separable, it is also jointly concave in (y_l, y_h). The constraints are affine-linear in y_l, y_h, such that the optimization problem is concave (see, e.g., Boyd and Vandenberghe 2004).

First, let us assume that $y_l^* \geq D_l^2/K_l$**, i.e., that IRl is satisfied at the unconstrained optimal prices.** It is easy to observe from (19) that condition $y_l^* \geq D_l^2/K_l$, i.e., that IRl is satisfied, can be expressed as

$$\left(\sqrt{c_a/(\eta K_m)} - (m/D)\right) D_l \geq D_l^2/K_l \Leftrightarrow c_a \geq \eta K_m((D_l/K_l) + (m/D))^2.$$

We can write the Lagrangian function as

$$\begin{aligned} L(y_l, y_h, \lambda_h, \lambda_l) = {} & \alpha\left(-y_l - \frac{c_a D_l^2}{\eta K_l}\left(y_l + \frac{mD_l}{D}\right)^{-1}\right) + (1-\alpha)\left(-y_h - \frac{c_a D_h^2}{\eta K_h}\left(y_h + \frac{mD_h}{D}\right)^{-1}\right) \\ & + \lambda_l\left(y_l - y_h - \frac{D_l^2}{K_l} + \delta\frac{D_h^2}{K_l}\right) + \lambda_h\left(y_h - y_l - \frac{D_h^2}{K_h} + \frac{D_l^2}{K_h}\right), \end{aligned} \tag{69}$$

where $\lambda_l, \lambda_h \geq 0$ are the Lagrangian multipliers of the IC constraints. We derive a saddle point for $L(\cdot)$, by solving

$$\frac{\partial L}{\partial y_l} = -\alpha + \frac{\alpha c_a D_l^2}{\eta K_l}\left(y_l + \frac{mD_l}{D}\right)^{-2} + \lambda_l - \lambda_h = 0 \Leftrightarrow \hat{y}_l^* = \sqrt{\frac{\alpha c_a}{\eta K_l(\alpha - \lambda_l + \lambda_h)}} D_l - \frac{mD_l}{D}, \tag{70}$$

$$\frac{\partial L}{\partial y_h} = -(1-\alpha) + \frac{(1-\alpha)c_a D_h^2}{\eta K_h}\left(y_h + \frac{mD_h}{D}\right)^{-2} - \lambda_l + \lambda_h = 0 \Leftrightarrow \hat{y}_h^* = \sqrt{\frac{(1-\alpha)c_a}{\eta K_h(1-\alpha+\lambda_l-\lambda_h)}} D_h - \frac{mD_h}{D} \tag{71}$$

Note that $0 < \alpha - \lambda_l + \lambda_h < 1$ must hold for the optimal constrained values $(\hat{y}_l^*, \hat{y}_h^*)$ to have a solution. By assumption that the demand quantities are feasible, it holds that

$$\frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} \leq \delta\frac{D_h^2}{K_l} - \frac{D_l^2}{K_l}.$$

Thus, only one of the two IC constraints (or none) can hold with equality for given (D_l, D_h), leading to three distinct cases, depending on the location of the difference of the unconstrained optimal values $y_h^* - y_l^*$:

$$(a)\ y_h^* - y_l^* < \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h},\ (b)\ \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} \leq y_h^* - y_l^* \leq \delta\frac{D_h^2}{K_l} - \frac{D_l^2}{K_l},\ (c)\ \delta\frac{D_h^2}{K_l} - \frac{D_l^2}{K_l} < y_h^* - y_l^*. \tag{72}$$

Case (A): For the first case, ICh is violated at the optimal unconstrained values (y_l^*, y_h^*). From the concavity of the buyer profit in (y_l, y_h), ICh will hold tight at its optimal constrained values, and we have $\lambda_h \geq 0, \lambda_l = 0$. We obtain as the equation for solving for λ_h:

$$\hat{y}_h^* - \hat{y}_l^* = \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h} = \sqrt{\frac{(1-\alpha)c_a}{\eta K_h(1-\alpha-\lambda_h)}} D_h - \frac{mD_h}{D} - \sqrt{\frac{\alpha c_a}{\eta K_l(\alpha+\lambda_h)}} D_l + \frac{mD_l}{D}$$

$$\Leftrightarrow \frac{b_h}{\sqrt{1-\alpha-\lambda_h}} - \frac{b_l}{\sqrt{\alpha+\lambda_h}} = b_0, \tag{73}$$

with

$$b_h = \sqrt{\frac{(1-\alpha)c_a}{\eta K_h}} D_h \geq 0,\ b_l = \sqrt{\frac{\alpha c_a}{\eta K_l}} D_l \geq 0,\ b_0 = \frac{1}{K_h}\left(D_h^2 - D_l^2\right) + \frac{m}{D}\left(D_h - D_l\right). \tag{74}$$

Note that (73) must have exactly one root in the interval $[0, 1-\alpha[$, because the left-hand side is strictly increasing in λ_h and goes from a value less than b_0 (by the condition of case A) to $+\infty$.

We can approximate the solution to (73) by using the Babylonian Method[12] to approximate the square roots, $\sqrt{\alpha+\lambda_h} \approx \sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}} \cdot \lambda_h$ and $\sqrt{1-\alpha-\lambda_h} \approx \sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h$, which leads to:

$$\frac{b_h}{\sqrt{1-\alpha-\lambda_h}} - \frac{b_l}{\sqrt{\alpha+\lambda_h}} - b_0 = 0 \Leftrightarrow b_h\sqrt{\alpha+\lambda_h} - b_l\sqrt{1-\alpha-\lambda_h} - b_0\sqrt{\alpha+\lambda_h}\sqrt{1-\alpha-\lambda_h} = 0$$

$$\approx b_h\left(\sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}}\lambda_h\right) - b_l\left(\sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h\right) - b_0\left(\sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}}\lambda_h\right)\left(\sqrt{1-\alpha} - \frac{1}{2\sqrt{1-\alpha}}\lambda_h\right)$$

$$\Leftrightarrow \frac{b_0}{4\sqrt{\alpha(1-\alpha)}}\lambda_h^2 + \left(\frac{b_0\sqrt{\alpha}+b_l}{2\sqrt{1-\alpha}} - \frac{b_0\sqrt{1-\alpha}-b_h}{2\sqrt{\alpha}}\right)\lambda_h + \left(b_h\sqrt{\alpha} - b_l\sqrt{1-\alpha} - b_0\sqrt{\alpha(1-\alpha)}\right) = 0,$$

$$\Leftrightarrow \lambda_h^2 + 2\left(2\alpha - 1 + \frac{b_l}{b_0}\sqrt{\alpha} + \frac{b_h}{b_0}\sqrt{(1-\alpha)}\right)\lambda_h + 4\left(\frac{b_h}{b_0}\alpha\sqrt{1-\alpha} - \frac{b_l}{b_0}(1-\alpha)\sqrt{\alpha} - \alpha(1-\alpha)\right) = 0,$$

$$\Rightarrow \lambda_{h|1,2} \approx 1 - 2\alpha - \frac{b_l}{b_0}\sqrt{\alpha} - \frac{b_h}{b_0}\sqrt{1-\alpha} \pm \sqrt{\left(1 - 2\alpha - \frac{b_l}{b_0}\sqrt{\alpha} - \frac{b_h}{b_0}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0}\left(\frac{b_h}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0\right)}, \tag{75}$$

where we retain the larger of the two roots (where $\pm$ is $+$) of the above quadratic function for λ_h, because the other root is non-positive (which follows from the condition that ICh is violated), which implies $b_h/\sqrt{1-\alpha} - b_l/\sqrt{\alpha} < b_0$. The Lagrangian multiplier λ_h (and also λ_l, see below) can be derived in closed-form from the Quartic Formula, details can be obtained from the authors upon request.

Because the optimization problem (65)-(67) is concave, the reduced problem with ICh enforced with equality is also concave (see Boyd and Vandenberghe 2004, Section 4.2.4). Hence, if λ_h is such that $\hat{y}_l^* < D_l^2/K_l$, then the optimal solution is $\hat{y}_l^* = D_l^2/K_l$ and $\hat{y}_h^* = \frac{D_l^2}{K_l} + \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}$ such that ICh holds with equality.

Case (B): For the second case, the IC constraints are not binding for the optimal unconstrained values (y_l^*, y_h^*), and we have $\lambda_h = \lambda_l = 0$. Constraint IRl is satisfied by assumption.

Case (C): For the third case, ICl is violated at values (y_l^*, y_h^*). Therefore, ICl will be tight under the constrained optimal prices and $\lambda_h = 0, \lambda_l \geq 0$. Condition $\lambda_h = 0, \lambda_l \geq 0$ leads to $\hat{y}_l^* > y_l^*$, such that assumption $y_l^* > D_l^2/K_l$ implies $\hat{y}_l^* > D_l^2/K_l$ so that IRl is satisfied. Then, we obtain as the equation for solving for λ_l:

$$\hat{y}_h^* - \hat{y}_l^* = \frac{\delta D_h^2}{K_l} - \frac{D_l^2}{K_l} = \sqrt{\frac{(1-\alpha)c_a}{\eta K_h(1-\alpha+\lambda_l)}} D_h - \frac{mD_h}{D} - \sqrt{\frac{\alpha c_a}{\eta K_l(\alpha-\lambda_l)}} D_l + \frac{mD_l}{D}$$

$$\Leftrightarrow \frac{b_h}{\sqrt{1-\alpha+\lambda_l}} - \frac{b_l}{\sqrt{\alpha-\lambda_l}} = b_0', \tag{76}$$

[12] See, e.g., https://www.sciencedirect.com/science/article/pii/S0315086098922091 [13 April 2023].

with b_l, b_h as defined in (74) and

$$b_0' = \frac{1}{K_l}\left(\delta D_h^2 - D_l^2\right) + \frac{m}{D}\left(D_h - D_l\right). \tag{77}$$

Note that (76) must have exactly one root in the interval $[0,\alpha[$, because the left-hand side is strictly decreasing in λ_l and goes from a value greater than b_0' (by the condition of case C) to $-\infty$. As with the first case, we can approximate the solution to (76) by using the Babylonian Method, $\sqrt{\alpha-\lambda_l} \approx \sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}}\cdot\lambda_l$ and $\sqrt{1-\alpha+\lambda_l} \approx \sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l$, which leads to:

$$\frac{b_h}{\sqrt{1-\alpha+\lambda_l}} - \frac{b_l}{\sqrt{\alpha-\lambda_l}} - b_0' = 0 \Leftrightarrow b_h\sqrt{\alpha-\lambda_l} - b_l\sqrt{1-\alpha+\lambda_l} - b_0'\sqrt{\alpha-\lambda_l}\sqrt{1-\alpha+\lambda_l} = 0$$

$$\approx b_h\left(\sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}}\lambda_l\right) - b_l\left(\sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l\right) - b_0'\left(\sqrt{\alpha} - \frac{1}{2\sqrt{\alpha}}\lambda_l\right)\left(\sqrt{1-\alpha} + \frac{1}{2\sqrt{1-\alpha}}\lambda_l\right)$$

$$\Leftrightarrow \lambda_l^2 - 2\left(2\alpha - 1 + \frac{b_l}{b_0'}\sqrt{\alpha} + \frac{b_h}{b_0'}\sqrt{(1-\alpha)}\right)\lambda_l + 4\left(\frac{b_h}{b_0'}\alpha\sqrt{1-\alpha} - \frac{b_l}{b_0'}(1-\alpha)\sqrt{\alpha} - \alpha(1-\alpha)\right) = 0,$$

$$\Rightarrow \lambda_{l|1,2} \approx 2\alpha - 1 + \frac{b_l}{b_0'}\sqrt{\alpha} + \frac{b_h}{b_0'}\sqrt{(1-\alpha)} \pm \sqrt{\left(1 - 2\alpha - \frac{b_l}{b_0'}\sqrt{\alpha} - \frac{b_h}{b_0'}\sqrt{1-\alpha}\right)^2 - 4\frac{\alpha(1-\alpha)}{b_0'}\left(\frac{b_h}{\sqrt{1-\alpha}} - \frac{b_l}{\sqrt{\alpha}} - b_0'\right)}, \tag{78}$$

where we retain the smaller of the two roots (where $\pm$ is $-$) of the above quadratic function for λ_l, which is positive because of the violation of ICl, which implies $b_h/\sqrt{1-\alpha} - b_l/\sqrt{\alpha} > b_0'$. Please note that if the first term in the expression for $\lambda_{l|1,2}$ is negative, both roots are negative, which is not possible.

Next, let us assume that $y_l^* < D_l^2/K_l$**, i.e., that IRl is not satisfied at optimal prices.**

If solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ satisfies ICl and ICh, it is the optimal solution, because the buyer profit is concave and 'separable' in y_l, y_h.

If solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ violates ICh, we know from Case (A) that $\lambda_h \geq 0, \lambda_l = 0$ and hence $\hat{y}_l^*$ without IRl is less than or equal to y_l^* and, by consequence, violates IRl. Because concavity of the problem is preserved when ICh is enforced with equality and because $y_l^* < D_l^2/K_l$, the optimal solution is $\hat{y}_l^* = D_l^2/K_l$ and $\hat{y}_h^* = \frac{D_l^2}{K_l} + \frac{D_h^2}{K_h} - \frac{D_l^2}{K_h}$, such that ICh is enforced.

The solution $y_l = D_l^2/K_l$ and $y_h = y_h^*$ can never violate ICl, which is equivalent to

$$\hat{y}_l^* - \hat{y}_h^* = \frac{D_l^2}{K_l} - \left(\sqrt{\frac{c_a}{\eta K_h}} - \frac{m}{D}\right)D_h < \frac{D_l^2}{K_l} - \delta\frac{D_h^2}{K_l} \Leftrightarrow \sqrt{\frac{c_a}{\eta K_h}} > \delta\frac{D_h}{K_l} + \frac{m}{D}, \tag{79}$$

because assumption $c_a < \eta K_l\left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2$ implies

$$c_a < \eta K_l\left(\frac{D_l}{K_l} + \frac{m}{D}\right)^2 \Leftrightarrow \sqrt{\frac{c_a}{\eta K_l}} < \frac{D_l}{K_l} + \frac{m}{D}$$

$$\Leftrightarrow \sqrt{\frac{c_a}{\eta K_h}} < \sqrt{\frac{K_l}{K_h}}\left(\frac{D_l}{K_l} + \frac{m}{D}\right) \leq \sqrt{\frac{K_l}{K_h}}\cdot\sqrt{\frac{\frac{\delta}{K_l} - \frac{1}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}}}\cdot\frac{D_h}{K_l} + \sqrt{\frac{K_l}{K_h}}\cdot\frac{m}{D} \leq \delta\cdot\frac{D_h}{K_l} + \sqrt{\frac{K_l}{K_h}}\cdot\frac{m}{D} < \delta\cdot\frac{D_h}{K_l} + \frac{m}{D},$$

$$. \tag{80}$$

where the first inequality in the line above is a consequence of (21) and the second inequality is a consequence of the following relation:

$$\sqrt{\frac{K_l}{K_h}}\cdot\sqrt{\frac{\frac{\delta}{K_l} - \frac{1}{K_h}}{\frac{1}{K_l} - \frac{1}{K_h}}} \leq \delta \Leftrightarrow \frac{\delta}{K_l} - \frac{1}{K_h} \leq \delta^2\frac{K_h}{K_l}\left(\frac{1}{K_l} - \frac{1}{K_h}\right) \Leftrightarrow \delta K_l - \frac{K_l^2}{K_h} \leq \delta^2\left(K_h - K_l\right)$$

$$\Leftrightarrow (K_h - K_l)\,\delta^2 - K_l\delta + \frac{K_l^2}{K_h} \geq 0 \Leftrightarrow \left(1 - \frac{K_l}{K_h}\right)\delta^2 - \frac{K_l}{K_h}\delta + \left(\frac{K_l}{K_h}\right)^2 \geq 0$$
$$\Leftrightarrow \left(1 - \frac{K_l}{K_h}\right)\delta^2 - \frac{K_l}{K_h}\delta + \left(\frac{K_l}{K_h}\right)^2 \geq \left(1 - \frac{K_l}{K_h}\right)\frac{K_l}{K_h}\delta - \frac{K_l}{K_h}\delta + \left(\frac{K_l}{K_h}\right)^2$$
$$= -\frac{K_l^2}{K_h^2}\delta + \left(\frac{K_l}{K_h}\right)^2 \geq 0, \tag{81}$$

which follows from assumptions $\frac{K_l}{K_h} < \delta < 1$. □

Proof of Proposition 4. The relation $D_h > D_l$ (except for $D_h = D_l = 0$ in which case equality holds) follows from Proposition 3. Note that this constraint is linear in D_l, D_h and independent of prices p_l, p_h. From the left-side inequality of (63) and $D_h \geq D_l$, it follows that $p_h D_h - p_l D_l \geq \frac{1}{K_h}(D_h^2 - D_l^2) \geq 0$, i.e., $p_h D_h \geq p_l D_l$ for any feasible solution. Next, we show statements (1)-(3) of the proposition.

Statement (1): ICh is violated at the optimal unconstrained solution: $p_h^* D_h^* - p_l^* D_l^* < (1/K_h)((D_h^*)^2 - (D_l^*)^2)$**.**

We prove Statement (1) of the proposition, by showing that, if IRl and (21) hold for the optimal unconstrained solution $(p_l^*, D_l^*, p_h^*, D_h^*)$, for any feasible point (p_l', D_l', p_h', D_h') (i.e., a point that satisfies Eqs. (14)-(18)) for which $p_h^* D_h' - p_l^* D_l' > (1/K_h)((D_h')^2 - (D_l')^2)$, there exists another feasible point with demand quantities (D_l'', D_h'') with greater or equal profit for which either Case (1) or Case (2A) of Proposition 3 applies.

Let us label the directed line segment from (p_l', D_l', p_h', D_h') to $(p_l^*, D_l', p_h^*, D_h')$ by vector $\mathbf{v}_1$ and the vector from $(p_l^*, D_l', p_h^*, D_h')$ to $(p_l^*, D_l^*, p_h^*, D_h^*)$ by $\mathbf{v}_2$. Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are 'profit-increasing', i.e., the closer a point is to the end point of the vector, the greater the buyer profit, and all points on vector $\mathbf{v}_2$ have higher buyer profit than all points on vector $\mathbf{v}_1$. This is obvious from the fact that $\Pi(p_l, D_l, p_h, D_h)$ is concave in (p_l, p_h) for any given (D_l, D_h) with maximum value (p_l^*, p_h^*), and the fact that $\Pi(p_l^*, D_l, p_h^*, D_h)$ is concave in (D_l, D_h) at prices (p_l^*, p_h^*) with maximum value (D_l^*, D_h^*) (see proof of Lemma 2), and approaching the optimal value of a concave function on a direct line increases the objective function. Let us consider three distinct cases:

(i) **Point** $(p_l^*, D_l', p_h^*, D_h')$ **violates IRl, i.e.,** $p_l^* < D_l'/K_l$**.** Then, Case (1) of Proposition 3 applies to demand quantities (D_l', D_h').

(ii) **Point** $(p_l^*, D_l', p_h^*, D_h')$ **satisfies IRl, i.e.,** $p_l^* \geq D_l'/K_l$**, but violates ICh, i.e.,** $p_h^* D_h' - p_l^* D_l' < (1/K_h)((D_h')^2 - (D_l')^2)$**.** Then, the optimal prices for demand quantities (D_l', D_h') are given by Case (2A) of Proposition 3.

(iii) **Point** $(p_l^*, D_l', p_h^*, D_h')$ **satisfies IRl, i.e.,** $p_l^* \geq D_l'/K_l$ **and satisfies ICh, i.e.,** $p_h^* D_h' - p_l^* D_l' \geq (1/K_h)((D_h')^2 - (D_l')^2)$**.** Then, we can derive demand quantities (D_l'', D_h'') that lie on vector $\mathbf{v}_2$ between (D_l', D_h') and (D_l^*, D_h^*).

By assumption, IRl holds at point $(p_l^*, D_l^*, p_h^*, D_h^*)$, i.e., $p_l^* \geq D_l^*/K_l$. Let us choose (D_l'', D_h'') such that $(p_l^*, D_l'', p_h^*, D_h'')$ is the point that lies closest to the end on $\mathbf{v}_2$ and for which holds

$$p_h^* D_h'' - p_l^* D_l'' = \frac{1}{K_h}((D_h'')^2 - (D_l'')^2).$$

There has to be at least one such point, because $p_h^* D_h' - p_l^* D_l' \geq (1/K_h)((D_h')^2 - (D_l')^2)$ and $p_h^* D_h^* - p_l^* D_l^* < (1/K_h)((D_h^*)^2 - (D_l^*)^2)$ by assumption, such that the Intermediate Value Theorem applies. Then, because both points $(p_l^*, D_l^*, p_h^*, D_h^*)$ and $(p_l^*, D_l', p_h^*, D_h')$ satisfy IRl and because the constraint IRl defines a half-space on (D_l, D_h) at given price p_l^*, any point on the direct line between these points also lies in the same half-space, i.e., $(p_l^*, D_l'', p_h^*, D_h'')$ also satisfies IRl. Point $(p_l^*, D_l'', p_h^*, D_h'')$ also satisfies ICl, because ICh holds with equality (see definition of (D_l'', D_h'') above) and the region on (D_l, D_h) on which ICh and ICl cannot both be violated (the region defined by (21)) is a half-space. Demand quantity pairs (D_l^*, D_h^*) and (D_l', D_h') both lie in this half-space by assumption, such that pair (D_l'', D_h'') also lies in the same half-space because it lies on the line segment between (D_l^*, D_h^*) and (D_l', D_h'). We conclude that Case (2A) of Proposition 3 applies.

Statement (2): ICh and ICl are satisfied at the optimal solution. Statement (2) is obvious: If ICh, ICl, and IRl are satisfied at the optimal unconstrained solution, the optimal solutions of the constrained and the unconstrained problems coincide.

Statement (3): ICl is violated for the optimal unconstrained solution: $\frac{1}{K_l}(\delta(D_h^*)^2 - (D_l^*)^2) < p_h^* D_h^* - p_l^* D_l^*$. The proof of statement (3) follows the same line of argumentation as the one for statement (1), by using constraint ICl, $\frac{1}{K_l}(\delta D_h^2 - D_l^2)$, instead of ICh, $\frac{1}{K_h}(D_h^2 - D_l^2)$ (and reverse signs for the inequalities). We do not repeat the arguments here for space considerations. □

Proof of Lemma 3. Note that it easily follows from (37) that $\beta_n(i,i) = 1$ and $\beta_n(i,i') = \beta_n(i,i'') \cdot \beta_n(i'',i')$ for any $i \le i'' \le i'$. For the constraint set of (39), let us write the right-hand side as

$$\begin{aligned}
&\max_{(i'|i<i'\le q)} \tilde{\alpha}_n(i') \cdot \beta_n(i,i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})) \\
&= \max\{\max_{(i'|i<i'<j)} \tilde{\alpha}_n(i')\beta_n(i,i')\pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \beta_n(i,j) \max_{(i'|j\le i'\le q)} \tilde{\alpha}_n(i')\beta_n(j,i')\pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'}))\} \\
&= \max\{\max_{(i'|i<i'<j)} \tilde{\alpha}_n(i')\beta_n(i,i')\pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \beta_n(i,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j))\},
\end{aligned} \tag{82}$$

where the equality in the second line comes from the constraint set of (40). Next, let us consider $i = j-1$. Then, with the above simplification, we can write the constraints as

$$\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a_{j-1}^*(p_{j-1}, D_{j-1})) \le \beta_n(j-1,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j)), \tag{83}$$

and with $i = j-2$, as

$$\begin{aligned}
&\tilde{\alpha}_n(j-2)\pi_j(p_{j-2}, D_{j-2}, a_{j-2}^*(p_{j-2}, D_{j-2})) \\
&\le \max\{\beta_n(j-2,j-1)\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a_{j-1}^*(p_{j-1}, D_{j-1})), \beta_n(j-2,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j))\} \\
&= \beta_n(j-2,j-1) \cdot \max\{\tilde{\alpha}_n(j-1)\pi_j(p_{j-1}, D_{j-1}, a_{j-1}^*(p_{j-1}, D_{j-1})), \beta_n(j-1,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j))\} \\
&= \beta_n(j-2,j-1)\beta_n(j-1,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j)) = \beta_n(j-2,j)\tilde{\alpha}_n(j)\pi_j(p_j, D_j, a_j^*(p_j, D_j)),
\end{aligned} \tag{84}$$

where the equality in the last line follows from (83) for $i = j-1$. We can continue the same reasoning to show that for (39),

$$\begin{aligned}
&\tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)) \le \max_{(i'|i<i'\le q)} \tilde{\alpha}_n(i') \cdot \beta_n(i,i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \forall i < j \\
\Leftrightarrow\ &\tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)) \le \tilde{\alpha}_n(j) \cdot \beta_n(i,j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)), \forall 1 \le i \le j.
\end{aligned} \tag{85}$$

Then, by defining $\beta_n(i,j) = \beta_n(j,i)^{-1}$, and by rewriting (40) as

$$\begin{aligned}
&\tilde{\alpha}_n(j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)) \ge \max_{(i'|j<i'\le q)} \tilde{\alpha}_n(i') \cdot \beta_n(j,i') \cdot \pi_j(p_{i'}, D_{i'}, a_{i'}^*(p_{i'}, D_{i'})), \forall j < q \\
\Leftrightarrow\ &\tilde{\alpha}_n(j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)) \ge \tilde{\alpha}_n(i) \cdot \beta_n(j,i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)), \forall j < i \le q \\
\Leftrightarrow\ &\tilde{\alpha}_n(i) \cdot \pi_j(p_i, D_i, a_i^*(p_i, D_i)) \le \tilde{\alpha}_n(j) \cdot \beta_n(i,j) \cdot \pi_j(p_j, D_j, a_j^*(p_j, D_j)), \forall j < i \le q,
\end{aligned} \tag{86}$$

both (39) and (40) can be summarized in the single equation (41), which finalizes the proof. □

Proof of Proposition 5. A strategy induces a PBE if it allows a player to update its belief about the types of the other players in each round, and that, in each round, each player's decisions are expected-profit-maximizing under the player's current belief (Fudenberg and Tirole 1991, p. 326).

The first condition is met, because we have shown with the discussion around (35)-(37) that an agent at the beginning of round i can update the probability of winning the contract of round i with $\tilde{\alpha}_n(i)$, and the probability of winning the contract of round j with $j > i$ with $\tilde{\alpha}_n(j)\cdot\beta_n(i,j)$, exploiting the information that no agent in the pool has type $i' < i$ (because then the sequential process would have already been terminated). The second condition is also met, because (39) and (40) ensure that each agent of type $j = 1,...,q$ maximizes profit when choosing contract j (and only contract j) under belief updating and strategy s, and those constraints are equivalent to (46) according to Lemma 3. Thus, (46) enforces the second condition of a PBE for strategy s.

Since each agent accepts the contract of its corresponding type, and the menu of contracts has a contract for each agent type, the procedure always terminates with the selection of an agent. Since strategy s implies that an agent with a lower type accepts a contract before an agent of a higher type, the selection of the agent with the lowest type (i.e., the one with the largest labor pool) is guaranteed. □

Proof of Proposition 6. For part (1), observe that $\tilde{\alpha}_n(j),\beta_n(i,j) \geq 0$ for all i,j. Constraint $IC(i,j)$ ensures that $\pi_j(p_j,D_j,a_j^*(p_j,D_j)) \geq (\tilde{\alpha}_n(i)\beta_n(j,i)/\tilde{\alpha}_n(j))\left(p_iD_i - \delta_{i,j}\frac{D_i^2}{K_j}\right)$ and $IR(i)$ ensures that $\left(p_iD_i - \frac{D_i^2}{K_i}\right) \geq 0$. For any $j < i$, it further holds $\left(p_iD_i - \delta_{i,j}\frac{D_i^2}{K_j}\right) \geq \left(p_iD_i - \frac{D_i^2}{K_i}\right)$, because $\delta_{i,j} = 1$ and $K_j > K_i$, such that $IC(i,j)$ and $IR(i)$ for any $i < j$ imply $IR(j)$. However, for $j = q$, we cannot find $j < i \leq q$, such that $IR(q)$ is the only IR constraint that is not redundant with other constraints.

For part (2), note that for any pair $i < j$, Eq. (46) leads to two IC constraints

$$\begin{aligned} &\tilde{\alpha}_n(i)\beta_n(j,i)\left(p_iD_i - \delta_{i,j}\frac{D_i^2}{K_j}\right) \leq \tilde{\alpha}_n(j)\left(p_jD_j - \frac{D_j^2}{K_j}\right), \\ \text{and}\quad &\tilde{\alpha}_n(i)\left(p_iD_i - \frac{D_i^2}{K_i}\right) \geq \tilde{\alpha}_n(j)\beta_n(i,j)\left(p_jD_j - \delta_{j,i}\frac{D_j^2}{K_i}\right) \\ &\Leftrightarrow \tilde{\alpha}_n(i)\beta_n(j,i)\left(p_iD_i - \frac{D_i^2}{K_i}\right) \geq \tilde{\alpha}_n(j)\left(p_jD_j - \delta_{j,i}\frac{D_j^2}{K_i}\right). \end{aligned} \tag{87}$$

Then, by combining both constraints, we obtain

$$\begin{aligned} &\tilde{\alpha}_n(i)\beta_n(j,i)\frac{D_i^2}{K_i} - \tilde{\alpha}_n(j)\frac{\delta_{j,i}D_j^2}{K_i} \leq \tilde{\alpha}_n(i)\beta_n(j,i)p_iD_i - \tilde{\alpha}_n(j)p_jD_j \leq \tilde{\alpha}_n(i)\beta_n(j,i)\frac{\delta_{i,j}D_i^2}{K_j} - \tilde{\alpha}_n(j)\frac{D_j^2}{K_j} \\ \Leftrightarrow\ &D_i \geq \sqrt{\frac{\tilde{\alpha}_n(j)\left(\frac{1}{K_j} - \frac{\delta_{j,i}}{K_i}\right)}{\tilde{\alpha}_n(i)\beta_n(j,i)\left(\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}\right)}}\cdot D_j = \sqrt{\frac{\tilde{\alpha}_n(j)\beta_n(i,j)\left(\frac{1}{K_j} - \frac{1}{K_i}\right)}{\tilde{\alpha}_n(i)\left(\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i}\right)}}\cdot D_j, \end{aligned} \tag{88}$$

because $\delta_{j,i} = 1$ and $K_i > K_j$ for $i \leq j$ by assumption and $D_i, D_j \geq 0$. Also, from (87) and $\delta_{j,i} = 1$ follows that it must hold $\frac{\delta_{i,j}}{K_j} - \frac{1}{K_i} > 0 \Leftrightarrow \delta_{i,j} > \frac{K_j}{K_i} \Leftrightarrow \frac{K_j}{K_i} > 1 - \frac{4c_g}{\gamma}$ for the constraints to admit a feasible solution, which is ensured by the assumption $\gamma < 2c_g$ (see Proposition 1). Finally, plugging the last line of (88) into the left-hand-side of the first line of (88) shows that the left-hand side must be non-negative, which leads to

$$\tilde{\alpha}_n(i)\beta_n(j,i)p_iD_i - \tilde{\alpha}_n(j)p_jD_j \geq 0 \Leftrightarrow \tilde{\alpha}_n(i)\beta_n(j,i)p_iD_i \geq \tilde{\alpha}_n(j)p_jD_j \Leftrightarrow p_iD_i \geq \frac{\tilde{\alpha}_n(j)}{\tilde{\alpha}_n(i)}\beta_n(i,j)\cdot p_jD_j. \quad \square$$

F. Robustness Checks

F.1. Inclusion of strictly convex audit cost

In this subsection, we study the effect of allowing for audit cost that are strictly convex in the audit level, compared to audit cost that are linear in the audit level, as we have assumed in the main part of the paper (see Section 4). The assumption of audit cost that are quadratic in the audit level is frequently made in the literature, to model strict convexity (see, e.g., Caro et al. 2018, Chen et al. 2019). For the case of public information, we derive the agent's profit-maximizing response and the buyer's optimal contract, similar to the analysis we presented in Section 5. For the case of private information (information asymmetry), we provide numerical analysis to show that that the insights we generated for the model with linear audit cost also hold (directionally) in the model with quadratic audit cost.

Public information

The analysis for quadratic audit cost is similar to the analysis of the case with linear audit cost. The buyer's profit function is given by:

$$\Pi(p,a,g) = (r-p)D\cdot(1-f(a,g)) + \left(r - \frac{D}{K} - u\right) D\cdot f(a,g) - c_a\cdot a^2 - (1-f(a,g))\cdot\psi\cdot\Psi\cdot\mathbb{1}_{\{g>0\}}. \quad (89)$$

As before, the buyer selects the contract such that the agent's optimal response is $g^*=0$. Note that the agent's profit function does not change. Hence, the results from Proposition 1 and Lemma 1 remain valid also for quadratic audit cost. Next, we proceed to deriving the buyer's optimal contract as given below:

Proposition 7. *Under quadratic audit cost, the buyer's optimal contract (p^*,a^*) satisfies:*

1. **Low audit cost.** *When $c_a < \frac{\eta^2}{2KD^4}(D^2+mK)^3$, then $(p^*,a^*) = \left(\frac{D}{K}, \frac{D^2}{\eta(D^2+mK)}\right)$ and the buyer's profit is $\Pi(p^*,a^*,g^*(p^*,a^*)) = rD - \frac{D^2}{K} - \frac{c_aD^4}{\eta^2(D^2+mK)^2}$.*
2. **High audit cost.** *When $c_a \geq \frac{\eta^2}{2KD^4}(D^2+mK)^3$, then $(p^*,a^*) = \left(\left(\frac{2c_aD}{\eta^2K^2}\right)^{\frac{1}{3}} - \frac{m}{D}, \left(\frac{D^2}{2c_a\eta K}\right)^{1/3}\right)$, where $p^*\geq(D/K)$, and the buyer's profit is $\Pi(p^*,a^*,g^*(p^*,a^*)) = rD - \left(\frac{c_aD^4}{4\eta^2K^2}\right)^{1/3} + m$.*

The buyer's profit $\Pi(p^*,a^*,g^*(p^*,a^*))$ is increasing in K in both cases.

Proof of Proposition 7. When $g^*=0$, observe from (5) that we can express the audit level a in terms of price p so that $a = (\frac{D}{K})/(\eta\cdot(p+\frac{m}{D}))$. Combining this observation with (89), the buyer's problem can be rewritten as:

$$\max_{p\geq D/K} [(r-p)D - c_aa^2] = \max_{p\geq D/K} \left[(r-p)D - c_a\left(\frac{\frac{D}{\eta K}}{p+\frac{m}{D}}\right)^2\right]. \quad (90)$$

We obtain as derivative of the buyer profit with respect to p:

$$\frac{d}{dp}\left((r-p)D - c_a\frac{D^2}{\eta^2K^2}\left(p+\frac{m}{D}\right)^{-2}\right) = -D + 2\cdot c_a\frac{D^2}{\eta^2K^2}\left(p+\frac{m}{D}\right)^{-3}, \quad (91)$$

which decreases in p, such that the first-order condition yields:

$$p^* = \max\left\{\frac{D}{K}, \left(\frac{2c_aD}{\eta^2K^2}\right)^{1/3} - \frac{m}{D}\right\}. \quad (92)$$

Optimal audit level a^* follows from

$$g^*(p^*,a^*) = 0 \Leftrightarrow p = \frac{D}{a\eta K} - \frac{m}{D} \Leftrightarrow a^* = \frac{D}{\eta K(p^*+\frac{m}{D})}. \quad (93)$$

For $c_a \geq \frac{\eta^2}{2KD^4}(D^2+mK)^3$, it follows consequently $p^* = \left(\frac{2c_a D}{\eta^2 K^2}\right)^{1/3} - \frac{m}{D}$ and, hence, $a^* = \left(\frac{D^2}{2\eta K c_a}\right)^{1/3}$. For $c_a < \frac{\eta^2}{2KD^4}(D^2+mK)^3$, $p^* = \frac{D}{K}$ and, consequently, $a^* = \frac{D^2}{\eta(D^2+mK)}$. The optimal buyer profit follows from (89) with $g^*(p^*, a^*) = 0$, as $(r-p^*)D - c_a a^2$. □

Private information

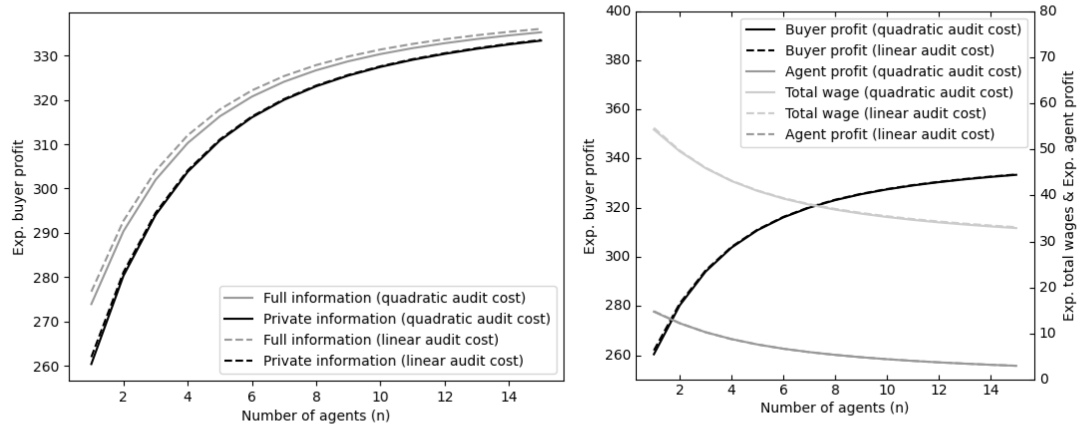
For the analysis of the case with information asymmetry, we first have to derive the expression for the optimal, unconstrained price of the contract of type i, given by

$$p_i^* = \left(\frac{2c_a D_i}{\eta^2 K^2}\right)^{1/3} - \frac{m}{D}, i = l, h. \tag{94}$$

Note that, different from the model with linear audit cost, the optimal price p_i^* is dependent on D_i, which renders the derivation of the analytic results that we presented in Sections 6 and 7 complex. In the following, we therefore use computational experiments to analyse the behavior of the model with quadratic audit cost for the case of information asymmetry.

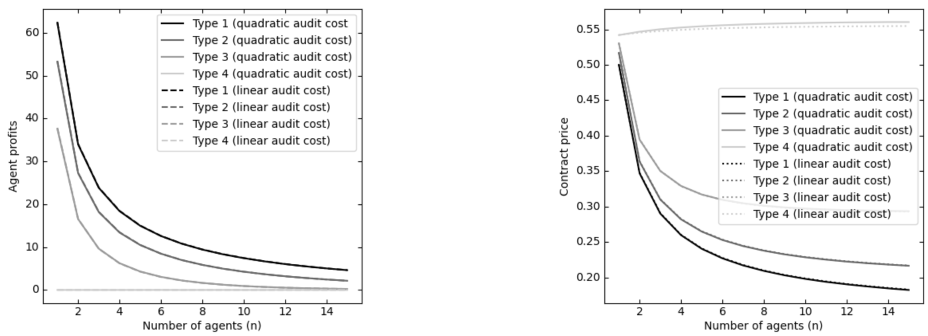
For the numerical analysis, we use the data that we also used in Section 8, but we calibrate the parameter c_a for quadratic audit cost such that total audit cost in the base case with $q = 4$ is identical between the model with linear and the model with quadratic audit cost. We obtain $c_a = 17.88$ for the quadratic audit cost model. The results are shown in Figures 8, 9, and 10.

The results between the model with linear audit cost and the model with quadratic audit cost are quite close and show similar trends (note that the lines indicating the results of the model with linear audit cost and those of the model with quadratic audit cost are often overlapping). More specifically, the value of private information, the expected profit of the agent, and the total wage decrease with an increasing number of agents, while the expected buyer profit increases (see Fig. 8). One difference between both models is that the value of perfect information flattens out quicker with higher α_1 values in the model with quadratic audit cost than in the model with linear audit cost (see Fig. 10b). This is expected, as the need for auditing drops with a higher proportion of large agents, and the drop in the value of perfect information is more pronounced for quadratic audit cost. We also observe that the contract price offered to the smallest agent is (slightly) higher in the model with quadratic audit cost than in the model with linear audit cost, while we observe the opposite for the largest agent type (see Fig. 10c). It is interesting that the *steeper* audit costs lead to *larger* differences in prices between agent types. Furthermore, as in the model with linear audit cost, the smallest agent will not retain any surplus, while those of all other types of agents are positive but decrease in n, caused by the reduction in contract prices that the buyer offers, due to higher competition between the agents (see Fig. 9). In addition, the value of perfect information increases in markets with larger proportion of bigger agents (see Fig. 10a and 10b), which is similar to our observation of the linear audit cost model. Finally, it can be seen from Figure 10c that with more larger agents present in the pool, the contract prices offered to the agents decrease. In summary, we conclude that our findings are similar and directionally robust to changes in the audit cost function.



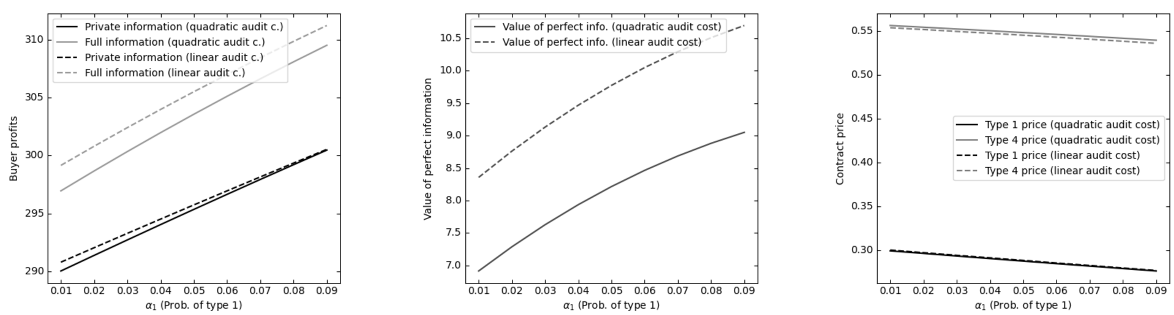
(a) Buyer profit with private vs. full information. (b) Buyer profit, agent profit, and workers' wages.

Figure 8 Sensitivity of profits as a function of the number of agents n.



(a) Agent profit per agent type. (b) Contract price of labor per agent type.

Figure 9 Agent profit and contract price as a function of n.



(a) Expected buyer profit (b) Value of perfect information (c) Contract price of agent type 1 & 4.

Figure 10 Buyer profit, value of information, and contract price as a function of variability α_1.

F.2. Inclusion of an agent penalty for detection of forced labor during post-contract scrutiny

In this section, we include a penalty in the agent's profit function. This penalty is due if the agent uses forced labor and if this forced labor is detected only through post-contract scrutiny.

To model this penalty, we subtract term $(1-f(a,g))\psi\Psi^s\mathbb{1}_{\{g>0\}}$ with penalty Ψ^s from the agent's profit function $\pi(p,a,g)$ (we use index ^{s} to indicate quantities related to this model extension):

$$\begin{aligned}\pi(p,a,g) &= pD\cdot(1-f(a,g)) - wD - c_g\cdot\gamma g^2 K - f(a,g)\cdot m\cdot\mathbb{1}_{\{g>0\}} - (1-f(a,g))\cdot\psi\cdot\Psi^s\cdot\mathbb{1}_{\{g>0\}}\\ &= (p-w)D - c_g\cdot\gamma g^2 K - f(a,g)\,(pD+m-\psi\cdot\Psi^s) - \psi\cdot\Psi^s\cdot\mathbb{1}_{\{g>0\}},\end{aligned} \tag{95}$$

where we used the relationship $f(a, g=0)=0$. All other modeling assumptions are identical to those described in Section 4. Note that we could also model the case in which the buyer collects the penalty, by suitably adjusting parameter Ψ. Let us set $m^s = m - \psi \cdot \Psi^s$, and note that the agent profit of the extended model differs from the agent profit of the base model only through the modified parameter m^s and the additive term $-\psi \cdot \Psi^s \cdot \mathbb{1}_{\{g>0\}}$. Next, we investigate how the profit-maximizing response function of the agent changes with the introduction of the agent penalty, compared to the base model of Section 4:

Proposition 8. *In the case of an agent penalty for post-contract detection, for any given buyer contract (p, a), the agent's profit-maximizing coercion response $g^*(p, a)$ satisfies:*

$$g^*(p,a) = \begin{cases} \left[\frac{D}{2c_g K} - \frac{a\eta(p+\frac{m^s}{D})}{2c_g}\right]^+, & \text{if } \frac{\gamma}{4c_g K}\left(D - Ka\eta(p+\frac{m^s}{D})\right)^2 > \psi \cdot \Psi^s, \\ 0, & \text{else.} \end{cases} \tag{96}$$

The coercion level $g^*(p, a)$ is decreasing in price p, audit effort a, available pre-contract information η, and coercion cost c_g. The corresponding wage is $w^*(p, a) = D/K - \gamma g^*(p, a)$.

Proof of Proposition 8. The proof for $g > 0$ follows the same lines as the proof of Proposition 1 but with parameter m replaced by m^s. However, because of the additional term $(\psi \cdot \Psi^s) \cdot \mathbb{1}_{\{g>0\}}$, the expected agent profit under the optimal $g > 0$ has to be compared against the agent profit for $g = 0$, which is given by $(p - D/K)D$. The difference in profit between using the $g^* > 0$ and using $g = 0$ can be written as (using $g^* > 0 \Rightarrow a \le \frac{D}{K\eta(p+m^s/D)}$):

$$\pi\left(p, a, \left[\frac{D}{2c_g K} - \frac{a\eta(p+\frac{m^s}{D})}{2c_g}\right]^+\right) - \left(p - \frac{D}{K}\right)D = \frac{\gamma}{4c_g K}\left(D - Ka\eta(p+\frac{m^s}{D})\right)^2 - \psi \cdot \Psi^s, \tag{97}$$

and zero otherwise. Optimal wage w^* can be retrieved in the same way as in Proposition 1. The negative sensitivity of $g^*(p, a)$ with respect to p, a, η, and c_g also holds for the extended model. □

We observe from Proposition 8 that, in addition to the dynamics of the base model, the agent must be able to increase its expected profit from using coercion by more than the expected penalty in case of post-scrutiny detection.

In the extended model, the buyer also chooses a contract that leads to zero coercion, as in the base model (the result of Lemma 1 holds for the extended model, because the proof relies entirely on the profit function of the buyer, which is identical between both models). Because of the agent penalty in the case of post-contract scrutiny, the buyer can reduce the audit level that is required to ensure zero coercion from the agent. Using the condition for the first case of (96), we obtain an 'optimal' audit level of

$$\frac{\gamma}{4c_g K}\left(D - Ka\eta(p+\frac{m^s}{D})\right)^2 = \psi \cdot \Psi^s \Leftrightarrow a^* = \frac{\frac{D}{K} - 4c_g\sqrt{\psi \cdot \Psi^s}}{\eta(p+\frac{m^s}{D})} < \frac{D}{K\eta(p+\frac{m^s}{D})}. \tag{98}$$

On the one hand, with the additional penalty, coercion becomes more costly to the agent. On the other hand, the effect of auditing diminishes, because the difference between pre-contract and post-contract penalties reduces for the agent. Therefore, it is not immediately intuitive how the additional penalty of the extended model affects the optimal audit level. When taking the derivative of the optimal audit level with respect to the penalty, we observe that the optimal audit level only decreases in the penalty $(\psi \cdot \Psi^s)$ if $p > \left(\frac{1}{4c_g K} - \frac{m}{D}\right)$; otherwise, a higher agent penalty may even *increase* the optimal auditing level. We employ the 'revised' optimal audit level to derive the corresponding result of Proposition 2 for the extended model.

Proposition 9. *In the case of an agent penalty for post-contract detection, the buyer's optimal contract* (p^*, a^*) *satisfies:*

1. **Low penalty and high audit cost.** *When* $\psi \cdot \Psi^s \leq \left(\frac{D}{4c_g K}\right)^2$ *and* $c_a \geq \frac{\eta D(\frac{D}{K}+\frac{m^s}{D})^2}{\frac{D}{K}-4c_g\sqrt{\psi\cdot\Psi^s}}$*, the optimal price is* $p^* = \sqrt{\frac{c_a\left(\frac{D}{K}-4c_g\sqrt{\psi\cdot\Psi^s}\right)}{\eta D}} - \frac{m^s}{D}$*.*
2. **Else.** *In all other cases, the optimal price corresponds to the fair wage, i.e.,* $p^* = \frac{D}{K}$*.*

In both cases, the optimal audit level is given by

$$a^* = \left[\frac{\frac{D}{K} - 4c_g\sqrt{\psi \cdot \Psi^s}}{\eta(p^* + \frac{m^s}{D})}\right]^+. \tag{99}$$

Proof of Proposition 9. We know that the buyer chooses $g^* = 0$. When $g^* = 0$, the buyer's profit function is $\Pi(p, a, g) = (r - p)D - c_a \cdot a$. When applying the optimal audit level a^*, we obtain as derivative of the buyer profit with respect to p:

$$\frac{d}{dp}\Pi = -D + \frac{c_a}{\eta}\left(\frac{D}{K} - 4c_g\sqrt{\psi \cdot \Psi^s}\right)\left(p + \frac{m^s}{D}\right)^{-2}, \tag{100}$$

which decreases in p for $\frac{D}{K} > 4c_g\sqrt{\psi \cdot \Psi^s}$, such that the first-order condition yields:

$$p^* = \max\left\{\frac{D}{K}, \sqrt{\frac{c_a\left(\frac{D}{K} - 4c_g\sqrt{\psi\cdot\Psi^s}\right)}{\eta D}} - \frac{m}{D}\right\}. \tag{101}$$

For $c_a \geq \frac{\eta D(\frac{D}{K}+\frac{m^s}{D})^2}{\frac{D}{K}-4c_g\sqrt{\psi\cdot\Psi^s}}$, it follows consequently $p^* = \sqrt{\frac{c_a\left(\frac{D}{K}-4c_g\sqrt{\psi\cdot\Psi^s}\right)}{\eta D}} - \frac{m^s}{D}$, and for $c_a < \frac{\eta D(\frac{D}{K}+\frac{m^s}{D})^2}{\frac{D}{K}-4c_g\sqrt{\psi\cdot\Psi^s}}$, we obtain $p^* = \frac{D}{K}$. Recall that for $\frac{D}{K} - 4c_g\sqrt{\psi \cdot \Psi^s} < 0 \Rightarrow \psi \cdot \Psi^s > \left(\frac{D}{4c_g K}\right)^2$, no auditing is required. □

Proposition 9 indicates that for a sufficiently high post-contract penalty, no auditing is required to rule out coercion, the penalty being sufficiently large to discourage the agent. This case does not exist in our base model. Hence, a large agent penalty for post-contract detection can lead to a situation, in which the agent does not have any incentive to use coercion, even if the buyer does not perform pre-contract auditing.

We conclude that the results for the extended model with an agent penalty for forced labor that is detected only through post-contract scrutiny are structurally similar to those of the base model. The analyses of Sections 6 and 7, however, become more complex, with several cases to be taken care of.

References

Babich V, Tang CS (2012) Managing opportunistic supplier product adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing & Service Operations Management* 14(2):301–314.

Boyd S, Vandenberghe L (2004) *Convex Optimization* (Cambridge: Cambridge University Press).

Caro F, Chintapalli P, Rajaram K, Tang CS (2018) Improving supplier compliance through joint and shared audits with collective penalty. *Manufacturing & Service Operations Management* 20(2):363–380.

Chen L, Lee HL (2017) Sourcing under supplier responsibility risk: The effects of certification, audit, and contingency payment. *Management Science* 63(9):2795–2812.

Chen S, Zhang Q, Zhou Y (2019) Impact of supply chain transparency on sustainability under NGO scrutiny. *Production and Operations Management* 28(12):3002–3022.

Cho SH, Fang X, Tayur S, Xu Y (2019) Combating child labor: Incentives and information disclosure in global supply chains. *Manufacturing & Service Operations Management* 21(3):692–711.

Fudenberg D, Tirole J (1991) *Game Theory* (Boston, MA: The MIT Press), 4th edition.

Guo R, Lee HL, Swinney R (2016) Responsible sourcing in supply chains. *Management Science* 62(9):2722–2744.

Huang L, Song JS, Swinney R (2022) Managing social responsibility in multi-tier supply chains. *Manufacturing & Service Operations Management* 24(6):2843–2862.

Kalkanci B, Plambeck EL (2020) Reveal the supplier list? A trade-off in capacity vs. responsibility. *Manufacturing & Service Operations Management* 22(6):1251–1267.

Lu T, Tomlin B (2022) Sourcing from a self-reporting supplier: Strategic communication of social responsibility in a supply chain. *Manufacturing & Service Operations Management* 24(2):902–920.

Plambeck EL, Taylor TA (2016) Supplier evasion of a buyer's audit: Implications for motivating supplier social and environmental responsibility. *Manufacturing & Service Operations Management* 18(2):184–197.

Rui H, Lai G (2015) Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management* 24(6):934–946.