Appendix for "Favorable Risk Selection in Medicare Advantage: The Effect of Allowing Non-Medical Services"

Appendix A: Technical Results

The following lemma characterizes the equilibrium outcomes. In the proof of Lemma 4, we can see that if $\phi(q_i) \le 0$, then beneficiaries would never choose original Medicare, justifying the assumption that $\phi(q_i) > 0$ in Assumption 2(ii).

LEMMA 4. *The optimal solution x_i^* to (6) and the associated θ_i^* and $E[\Pi_i^*]$ are given as follows.*

(i) For Plan $i \in \{1, 2P\}$,

$$(x_i^*,\, \theta_i^*,\, E[\Pi_i^*]) = \begin{cases} \left(\dfrac{2Z_0\phi(q_i)}{(Z_0+2R)q_i} + \dfrac{(q_0-q_i)s}{q_i},\quad \dfrac{Z_0+2R}{2\phi(q_i)},\quad \dfrac{(Z_0+2R)^2}{8\bar\theta\phi(q_i)} \right), & \text{if } \dfrac{Z_0+2R}{2\phi(q_i)} \le \bar\theta, \\ \left(\dfrac{Z_0+(q_0-q_i)s\bar\theta}{q_i\bar\theta},\quad \bar\theta,\quad \dfrac{1}{2}(Z_0+2R-\bar\theta\phi(q_i)) \right), & \text{otherwise.} \end{cases} \tag{17}$$

(ii) For Plan 2G, if $\Omega = Z_0 + 2R + V_G - 2K_G \le 0$, then there is no feasible solution to problem (6). If $\Omega > 0$, then

$$(x_{2G}^*,\, \theta_{2G}^*,\, E[\Pi_{2G}^*]) = \begin{cases} \left(\dfrac{2k_t(Z_0+V_G)}{\Omega},\quad \dfrac{\Omega}{2k_tq_0},\quad \dfrac{\Omega^2}{8k_tq_0\bar\theta}. \right), & \text{if } \dfrac{\Omega}{2k_tq_0} \le \bar\theta, \\ \left(\dfrac{Z_0+V_G}{q_0\bar\theta},\quad \bar\theta,\quad \dfrac{1}{2}(\Omega - k_tq_0\bar\theta) \right), & \text{otherwise.} \end{cases} \tag{18}$$

The following lemma characterizes trivial equilibria. To exclude those trivial equilibria, Assumption 2(iii) in the main body of the paper assumes that the two conditions (i) and (ii) identified in Lemma 5 hold throughout the paper.

LEMMA 5. *For Plan 2G, the following holds.*

(i) An MA insurer can obtain (weakly) positive profit if and only if $V_G - 2K_G > -2R - Z_0$. Otherwise, the MA insurer always obtains negative profit and thus never chooses Plan 2G.

(ii) An MA insurer finds it optimal to choose Plan $i \in \{1, 2P\}$ over Plan 2G for some capitation payment R if and only if $V_G - 2K_G \le (k_tq_0 - (k_p + k_tq_i))\bar\theta + (q_0 - q_i)s\bar\theta$. Otherwise, the MA insurer always chooses Plan 2G over Plan i regardless of R.

Lemma 5(i) implies that, if the general well-being benefit V_G is too low compared to the cost K_G, then the MA insurer never chooses Plan 2G. Lemma 5(ii) characterizes the opposite case where, if V_G is too high compared to K_G, then the MA insurer always finds it optimal to choose Plan 2G. Note that, in condition in Part (ii), the right-hand side is the sum of the reduction in cost and increase in consumer utility under Plan i compared to the original Medicare for the most vulnerable beneficiary of type $\bar\theta$.

Appendix B: Proofs of All Results

We first prove Lemmas 4 and 5 in Appendix A, which we use in the rest of the proofs.

Proof of Lemma 4. In this proof, we use $\phi(q_i)$ defined in (4), where $\phi(q_i) > 0$ by Assumption 2.

(1) **Plan 1.** We solve problem (6) without the constraint and show that the unconstrained optimal solution satisfies the constraint. A beneficiary joins Plan 1 if and only if $U_1 \ge U_0$, which is equivalent to $((x_1+s)q_1 - sq_0)\theta \le Z_0$. If $(x_1+s)q_1 - sq_0 > 0$, then this holds if and only if $\theta \le \theta_1 = \min\left\{\frac{Z_0}{(x_1+s)q_1 - sq_0}, \bar\theta\right\}$. If $(x_1+s)q_1 - sq_0 \le 0$, then $U_1 \ge U_0$ regardless of θ, and thus we can set $\theta_1 = \bar\theta$. We analyze these two cases separately.

(a) First, suppose that $(x_1+s)q_1 - sq_0 > 0$. If $\frac{Z_0}{(x_1+s)q_1-sq_0} \geq \bar{\theta}$, then $\theta_1 = \bar{\theta}$, and thus

$$E[\Pi_1] = \int_0^{\bar{\theta}} \Pi_1 f(\theta)d\theta = R - \frac{1}{2}\bar{\theta}(k_p + k_t q_1 - x_1 q_1). \tag{19}$$

We observe that $E[\Pi_1]$ is strictly increasing in x_1, and therefore the solution is the maximum x_1 such that $\frac{Z_0}{(x_1+s)q_1-sq_0} \geq \bar{\theta}$. Then, it is immediately clear that any x_1 such that $\frac{Z_0}{(x_1+s)q_1-sq_0} > \bar{\theta}$ cannot be optimal. Therefore, for the optimal solution, we need to consider only the case when $\frac{Z_0}{(x_1+s)q_1-sq_0} \leq \bar{\theta}$.

If $\theta_1 = \frac{Z_0}{(x_1+s)q_1-sq_0} \leq \bar{\theta}$, then $E[\Pi_1] = \int_0^{\theta_1} \Pi_1 f(\theta)d\theta$, and thus

$$\frac{\partial E[\Pi_1]}{\partial x_1} = -\frac{Z_0 q_1(2Z_0(k_p + k_t q_1) + s(q_0 - q_1)(2R - Z_0) - x_1 q_1(Z_0 + 2R))}{2\bar{\theta}(sq_0 - (x_1+s)q_1)^3}.$$

The denominator is strictly negative because we assumed $(x_1+s)q_1 - sq_0 > 0$. Therefore, $\frac{\partial E[\Pi_1]}{\partial x_1} = 0$ if and only if

$$x_1 = x_1^F = \frac{2Z_0\phi(q_1)}{(Z_0+2R)q_1} + \frac{(q_0-q_1)s}{q_1},$$

which generates

$$\theta_1^F = \frac{Z_0+2R}{2\phi(q_1)} \qquad E[\Pi_1^F] = \frac{(Z_0+2R)^2}{8\bar{\theta}\phi(q_1)}. \tag{20}$$

Note that $\frac{\partial E[\Pi_1]}{\partial x_1} > 0$ when $x < x_1^F$ and $\frac{\partial E[\Pi_1]}{\partial x_1} < 0$ when $x > x_1^F$. Thus, if $\theta_1^F \leq \bar{\theta}$, then the unique maximum is achieved at x_1^F, generating θ_1^F and $E[\Pi_1^F]$. If $\theta_1^F > \bar{\theta}$, then we have a boundary solution, satisfying $\theta_1^B = \bar{\theta}$, which generates

$$x_1^B = \frac{Z_0 + (q_0-q_1)s\bar{\theta}}{q_1\bar{\theta}} \qquad \text{and} \qquad E[\Pi_1^B] = \frac{1}{2}(Z_0 + 2R - \bar{\theta}\phi(q_1)).$$

Note that both x_1^F and x_1^B satisfy $(x_1+s)q_1 - sq_0 > 0$ if and only if $\phi(q_i) > 0$, which always holds by Assumption 2. (If $\phi(q_i) \leq 0$, then we can never be in case (a).) Also, $E[\Pi_1^F] \geq 0$ and $E[\Pi_1^B] \geq 0$ and thus the unconstrained solution satisfies the constraint.

(b) Second, suppose that $(x_1+s)q_1 - sq_0 \leq 0$. Then, it is always the case that $U_1 \geq U_0$, and thus $E[\Pi_1]$ is given by (19). Again, $E[\Pi_1]$ is strictly increasing in x_1, and therefore the solution is the maximum x_1 such that $(x_1+s)q_1 - sq_0 \leq 0$. Therefore, the optimal solution is

$$x_1^\dagger = \frac{q_0-q_1}{q_1}s, \quad \theta_1^\dagger = \bar{\theta}, \quad \text{and} \quad E[\Pi_1^\dagger] = R - \frac{1}{2}\bar{\theta}\phi(q_1).$$

Now, we compare the solutions from cases (a) and (b). Note that

$$E[\Pi_1^F] - E[\Pi_1^\dagger] = \frac{1}{8\bar{\theta}\phi(q_1)}\left[Z_0^2 + 4Z_0R + 4(R - \bar{\theta}\phi(q_1))^2\right] > 0,$$
$$E[\Pi_1^B] - E[\Pi_1^\dagger] = \frac{1}{2}Z_0 > 0.$$

Therefore, the optimal solution is from case (a).

(2) **Plan 2P.** The analysis is the same as in Plan 1, except that we change q_1 to q_2. Therefore, the optimal solution from the first order condition is

$$x_{2P}^F = \frac{2Z_0\phi(q_2)}{(Z_0+2R)q_2} + \frac{(q_0-q_2)s}{q_2},$$
$$\theta_{2P}^F = \frac{Z_0+2R}{2\phi(q_2)}, \tag{21}$$
$$E[\Pi_{2P}^F] = \frac{(Z_0+2R)^2}{8\bar{\theta}\phi(q_2)},$$

if $\theta_{2P}^F < \bar{\theta}$. If $\theta_{2P}^F \geq \bar{\theta}$, then the optimal solution is the boundary solution satisfying $\theta_{2P}^B = \bar{\theta}$, which is

$$x_{2P}^B = \frac{Z_0 + (q_0-q_2)s\bar{\theta}}{q_2\bar{\theta}}; \quad E[\Pi_{2P}^B] = \frac{1}{2}(Z_0+2R-\bar{\theta}\phi(q_2)).$$

Note that $E[\Pi_{2P}^F] \geq 0$ and $E[\Pi_{2P}^B] \geq 0$ and thus the unconstrained solution satisfies the constraint.

(3) **Plan 2G.** We solve problem (6) without the constraint and see if the unconstrained optimal solution satisfies the constraint. A beneficiary joins Plan 2G if and only if $U_{2G} \geq U_0$, which is equivalent to $\theta \leq \theta_{2G} = \min\left\{\frac{Z_0+V_G}{q_0x_{2G}}, \bar{\theta}\right\}$. If $\frac{Z_0+V_G}{q_0x_{2G}} \geq \bar{\theta}$, then $\theta_{2G} = \bar{\theta}$, and thus

$$E[\Pi_{2G}] = \int_0^{\bar{\theta}} \Pi_{2G} f(\theta)d\theta = R - K_G + \frac{q_0\bar{\theta}}{2}(x_{2G} - k_t).$$

We observe that $E[\Pi_{2G}]$ is strictly increasing in x_{2G}, and therefore the solution is the maximum x_{2G} such that $\frac{Z_0+V_G}{q_0x_{2G}} \geq \bar{\theta}$. Then, it is immediately clear that any x_{2G} such that $\frac{Z_0+V_G}{q_0x_{2G}} > \bar{\theta}$ cannot be optimal. Therefore, for the optimal solution, we need to consider only the case when $\frac{Z_0+V_G}{q_0x_{2G}} \leq \bar{\theta}$.

If $\theta_{2G} = \frac{Z_0+V_G}{q_0x_{2G}} \leq \bar{\theta}$, then $E[\Pi_{2G}] = \int_0^{\theta_{2G}} \Pi_{2G} f(\theta)d\theta$, and thus

$$E[\Pi_{2G}] = \frac{(Z_0+V_G)(-k_t(Z_0+V_G) + (2R+Z_0+V_G-2K_G)x_{2G})}{2q_0\bar{\theta}x_{2G}^2}.$$

It is immediately clear that if $2R+Z_0+V_G-2K_G \leq 0$, then $E[\Pi_{2G}] < 0$ for any x_{2G}, violating the constraint. Thus, there is no feasible solution to problem (6).

Now, suppose that $2R+Z_0+V_G-2K_G > 0$. Then, we have

$$\frac{\partial E[\Pi_{2G}]}{\partial x_{2G}} = \frac{(Z_0+V_G)(2k_t(Z_0+V_G) + (2K_G-Z_0-2R-V_G)x_{2G})}{2q_0\bar{\theta}x_{2G}^3}.$$

Therefore, $\frac{\partial E[\Pi_{2G}]}{\partial x_{2G}} = 0$ if and only if $x_{2G} = x_{2G}^F = \frac{2k_t(Z_0+V_G)}{2R+Z_0+V_G-2K_G}$, which generates

$$\theta_{2G}^F = \frac{2R+Z_0+V_G-2K_G}{2k_tq_0}, \tag{22}$$
$$E[\Pi_{2G}^F] = \frac{(2R+Z_0+V_G-2K_G)^2}{8k_tq_0\bar{\theta}}.$$

Note that $\frac{\partial E[\Pi_{2G}]}{\partial x_{2G}} > 0$ when $x < x_{2G}^F$ and $\frac{\partial E[\Pi_{2G}]}{\partial x_{2G}} < 0$ when $x > x_{2G}^F$. Thus, if $\theta_{2G}^F \leq \bar{\theta}$, then the unique maximum is achieved at x_{2G}^F, generating θ_{2G}^F and $E[\Pi_{2G}^F]$. If $\theta_{2G}^F > \bar{\theta}$, then we have a boundary solution, satisfying $\theta_{2G}^B = \bar{\theta}$, which generates

$$x_{2G}^B = \frac{Z_0+V_G}{q_0\bar{\theta}}; \quad E[\Pi_{2G}^B] = \frac{1}{2}(2R+Z_0+V_G-2K_G-k_tq_0\bar{\theta}).$$

Note that $E[\Pi_{2G}^F] \geq 0$ and $E[\Pi_{2G}^B] \geq 0$, and thus this unconstrained solution satisfies the constraint. □

Proof of Lemma 5.

(i) In the proof of Lemma 4, we showed that $E[\Pi_{2G}] < 0$ for any x_{2G} if $V_G - 2K_G \leq -2R - Z_0$ and that $E[\Pi_{2G}^*] \geq 0$ if $V_G - 2K_G > -2R - Z_0$. Therefore, the MA insurer can earn (weakly) positive profit if and only if $V_G - 2K_G > -2R - Z_0$.

(ii) This proof consists of two steps. For Plan $i \in \{1, 2P\}$, in Step 1, we show that $E[\Pi_i^*] \geq E[\Pi_{2G}^*]$ for some R if the condition in (ii) holds. In Step 2, we show that $E[\Pi_i^*] < E[\Pi_{2G}^*]$ for any R if the condition in (ii) does not hold.

Step 1. We first observe that, using (20), (21), and (22) in the proof of Lemma 4, we have $\theta_i^* < \bar{\theta}$ if and only if $R < \bar{R}_i$, where $\bar{R}_i = \phi(q_i)\bar{\theta} - \frac{Z_0}{2}$ for $i \in \{1, 2P\}$ and $\bar{R}_{2G} = k_t q_0 \bar{\theta} - \frac{Z_0 + V_G - 2K_G}{2}$.

When $R \geq \max\{\bar{R}_i, \bar{R}_{2G}\}$, we have $\theta_i^* = \theta_{2G}^* = \bar{\theta}$, and thus using Lemma 4, we have

$$E[\Pi_i^*] - E[\Pi_{2G}^*] = \frac{1}{2}[(k_t q_0 - \phi(q_i))\bar{\theta} - (V_G - 2K_G)] \geq 0, \tag{23}$$

where the inequality holds by the condition in (ii).

Step 2. Now, assume that condition in (ii) does not hold. First, suppose $R \geq \max\{\bar{R}_i, \bar{R}_{2G}\}$. Then, using (23) in Step 1, it is clear that $E[\Pi_i^*] < E[\Pi_{2G}^*]$.

Second, suppose that $R \leq \min\{\bar{R}_i, \bar{R}_{2G}\}$. Then, $\theta_i^* \leq \bar{\theta}$ and $\theta_{2G}^* \leq \bar{\theta}$. Using Lemma 4, we observe that $E[\Pi_i^*] \geq E[\Pi_{2G}^*]$ if and only if $R \geq \hat{R}_i$, where $\hat{R}_i = \frac{(V_G - 2K_G)\sqrt{\phi(q_i)}}{2(\sqrt{k_t q_0} - \sqrt{\phi(q_i)})} - \frac{Z_0}{2}$. Note that $\phi(q_i) > 0$ by Assumption 2, and $\sqrt{k_t q_0} - \sqrt{\phi(q_i)} > 0$ because $k_t q_0 - \phi(q_i) = (s + k_t)(q_0 - q_i) - k_p > 0$ by Assumption 2. We have

$$\begin{aligned}\hat{R}_i - \bar{R}_i &= \frac{(V_G - 2K_G)\sqrt{\phi(q_i)}}{2(\sqrt{k_t q_0} - \sqrt{\phi(q_i)})} - \phi(q_i)\bar{\theta} > \frac{(k_t q_0 - \phi(q_i))\bar{\theta}\sqrt{\phi(q_i)}}{2(\sqrt{k_t q_0} - \sqrt{\phi(q_i)})} - \phi(q_i)\bar{\theta} \\ &= \frac{\bar{\theta}}{2}\sqrt{\phi(q_i)}(\sqrt{k_t q_0} - \sqrt{\phi(q_i)}) > 0,\end{aligned}$$

where the first inequality holds because condition in (ii) does not hold and the second inequality holds because $k_t q_0 - \phi(q_i) = (s + k_t)(q_0 - q_i) - k_p > 0$ by Assumption 2. This means that $\hat{R}_i > \min\{\bar{R}_i, \bar{R}_{2G}\}$, and thus $E[\Pi_i^*] < E[\Pi_{2G}^*]$ when $R \leq \min\{\bar{R}_i, \bar{R}_{2G}\}$.

Finally, suppose that $\min\{\bar{R}_i, \bar{R}_{2G}\} < R < \max\{\bar{R}_i, \bar{R}_{2G}\}$. If $\bar{R}_i < \bar{R}_{2G}$, then $\theta_i^* = \bar{\theta}$ and $\theta_{2G}^* < \bar{\theta}$. Therefore, using Lemma 4, we observe that $\frac{\partial(E[\Pi_i^*] - E[\Pi_{2G}^*])}{\partial R} = 1 - \frac{2R + Z_0 + V_G - 2K_G}{2k_t q_0 \bar{\theta}} > 0$, when $R < \bar{R}_{2G} = k_t q_0 \bar{\theta} - \frac{Z_0 + V_G - 2K_G}{2}$. Similarly, if $\bar{R}_i > \bar{R}_{2G}$, then $\theta_i^* < \bar{\theta}$ and $\theta_{2G}^* = \bar{\theta}$, and thus $\frac{\partial(E[\Pi_i^*] - E[\Pi_{2G}^*])}{\partial R} = \frac{Z_0 + 2R}{2\bar{\theta}\phi(q_i)} - 1 < 0$, when $R < \bar{R}_i = \bar{\theta}\phi(q_i) - \frac{Z_0}{2}$. Therefore, it is clear that $E[\Pi_i^*] < E[\Pi_{2G}^*]$ when $R \in (\min\{\bar{R}_i, \bar{R}_{2G}\}, \max\{\bar{R}_i, \bar{R}_{2G}\})$, because $E[\Pi_i^*] < E[\Pi_{2G}^*]$ when $R = \bar{R}_i$ or $\bar{R}_{2G}$. □

Proof of Lemma 1. A type-θ beneficiary enrolls in Plan $i \in \{1, 2P, 2G\}$ if and only if $U_i \geq U_0$. Using equations (1) and (3), it is straightforward to obtain the result. □

Proof of Proposition 1. First, the optimal marginal treatment charges x_i^* are characterized in Lemma 4. Second, using (20), (21), and (22) in the proof of Lemma 4, we can see that $\theta_i^* < \bar{\theta}$ if and only if $R < \bar{R}_i$, where $\bar{R}_i = \phi(q_i)\bar{\theta} - \frac{Z_0}{2}$ for $i \in \{1, 2P\}$ and $\bar{R}_{2G} = k_t q_0 \bar{\theta} - \frac{Z_0 + V_G - 2K_G}{2}$. □

Proof of Proposition 2. We compare the expected profits of (i) Plans 1 and 2P, and (ii) Plans 2P and 2G, using the results from the proof of Lemma 4.

(i) We show that it is always the case that $E[\Pi_{2P}^*] > E[\Pi_1^*]$, and thus Plan 1 is never chosen. Note that $\phi(q_1) - \phi(q_2) = (s+k_t)(q_1 - q_2) > 0$, and thus $\theta_1^F = \frac{Z_0+2R}{2\phi(q_1)} < \frac{Z_0+2R}{2\phi(q_2)} = \theta_{2P}^F$.

First, if $\theta_1^F < \theta_{2P}^F \le \bar\theta$, then

$$E[\Pi_1^*] - E[\Pi_{2P}^*] = \frac{(Z_0+2R)^2}{8\bar\theta\phi(q_1)} - \frac{(Z_0+2R)^2}{8\bar\theta\phi(q_2)} < 0.$$

Second, if $\theta_1^F \le \bar\theta < \theta_{2P}^F$, then

$$\begin{aligned} E[\Pi_1^*] - E[\Pi_{2P}^*] &= \frac{(Z_0+2R)^2}{8\bar\theta\phi(q_1)} - \frac{1}{2}(Z_0+2R-\bar\theta\phi(q_2)) \\ &= \frac{(Z_0+2R)}{4\bar\theta} \cdot \frac{(Z_0+2R)}{2\phi(q_1)} - \frac{1}{2}(Z_0+2R-\bar\theta\phi(q_2)) \\ &\le \frac{(Z_0+2R)}{4\bar\theta} \cdot \bar\theta - \frac{1}{2}(Z_0+2R-\bar\theta\phi(q_2)) \\ &= -\frac{\phi(q_2)}{2}\left[\frac{Z_0+2R}{2\phi(q_2)} - \bar\theta\right] < 0, \end{aligned}$$

where the first inequality holds because $\theta_1^F = \frac{Z_0+2R}{2\phi(q_1)} \le \bar\theta$ and the second inequality holds because $\bar\theta < \theta_{2P}^F = \frac{Z_0+2R}{2\phi(q_2)}$.

Finally, if $\bar\theta < \theta_1^F < \theta_{2P}^F$, then

$$E[\Pi_1^*] - E[\Pi_{2P}^*] = \frac{1}{2}(Z_0+2R-\bar\theta\phi(q_1)) - \frac{1}{2}(Z_0+2R-\bar\theta\phi(q_2)) = \frac{1}{2}\bar\theta(\phi(q_2)-\phi(q_1)) < 0.$$

Therefore, it is always the case that $E[\Pi_{2P}^*] > E[\Pi_1^*]$.

(ii) This proof consists of two steps. In Step 1, we compare $E[\Pi_{2P}^*]$ and $E[\Pi_{2G}^*]$ in three different ranges of R. In Step 2, we show that there exists R^T such that $E[\Pi_{2P}^*] \ge E[\Pi_{2G}^*]$ if and only if $R \ge R^T$.

Step 1. Using Proposition 1, we observe that

$$\bar R_{2G} - \bar R_{2P} = \frac{1}{2}[2\bar\theta(k_t q_0 - \phi(q_2)) - (V_G - 2K_G)] > 0,$$

because $\bar\theta(k_t q_0 - \phi(q_2)) - (V_G - 2K_G) \ge 0$ by Lemma 5, where $k_t q_0 - \phi(q_2) = (q_0 - q_2)(k_t + s) - k_p > 0$ by Assumption 2(i). We consider the following three different ranges of R.

First, suppose that $\bar R_{2P} < \bar R_{2G} \le R$. Then, $\theta_{2P}^* = \theta_{2G}^* = \bar\theta$, and using Lemma 4, we have, by Lemma 5,

$$E[\Pi_{2P}^*] - E[\Pi_{2G}^*] = \frac{1}{2}[\bar\theta(k_t q_0 - \phi(q_2)) - (V_G - 2K_G)] \ge 0.$$

Second, suppose that $\bar R_{2P} \le R < \bar R_{2G}$. Then, $\theta_{2P}^* = \bar\theta$ and $\theta_{2G}^* < \bar\theta$. Using Lemma 4, we observe that

$$\frac{\partial(E[\Pi_{2P}^*] - E[\Pi_{2G}^*])}{\partial R} = 1 - \frac{2R + Z_0 + V_G - 2K_G}{2k_t q_0 \bar\theta} > 0,$$

when $R < \bar R_{2G} = k_t q_0 \bar\theta - \frac{Z_0+V_G-2K_G}{2}$. Therefore, $E[\Pi_{2P}^*] - E[\Pi_{2G}^*]$ is strictly increasing in R when $R \in [\bar R_{2P}, \bar R_{2G})$. Using Lemma 4, we observe that $E[\Pi_{2P}^*] - E[\Pi_{2G}^*]$ is a quadratic in R with a negative quadratic coefficient and the maximum is achieved at $R = \bar R_{2G}$. Therefore, $E[\Pi_{2P}^*] \ge E[\Pi_{2G}^*]$ if and only if $R \ge \hat R'$, where

$$\hat R' = k_t q_0 \bar\theta - \frac{1}{2}(V_G - 2K_G + Z_0) - \sqrt{k_t q_0 \bar\theta((k_t q_0 - \phi(q_2))\bar\theta - (V_G - 2K_G))},$$

noting that the expression inside the square root is positive by Lemma 5.

Finally, suppose that $R < \bar{R}_{2P} < \bar{R}_{2G}$. Then, $\theta^*_{2P} < \bar{\theta}$ and $\theta^*_{2G} < \bar{\theta}$. Using Lemma 4, $E[\Pi^*_{2P}] \geq E[\Pi^*_{2G}]$ if and only if $R \geq \hat{R}$, where

$$\hat{R} = \frac{(V_G - 2K_G)\sqrt{\phi(q_2)}}{2(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})} - \frac{Z_0}{2}.$$

Note that $\phi(q_2) > 0$ by Assumption 2, and $\sqrt{k_t q_0} - \sqrt{\phi(q_2)} > 0$ because $k_t q_0 - \phi(q_2) = (s + k_t)(q_0 - q_2) - k_p > 0$ by Assumption 2.

Step 2. Now, we show that there exists R^T such that $E[\Pi^*_{2P}] \geq E[\Pi^*_{2G}]$ if and only if $R \geq R^T$. We consider two cases depending on the value of $\hat{R}$.

First, if $\hat{R} < \bar{R}_{2P}$, then (a) when $R < \bar{R}_{2P}$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}] \geq 0$ if and only if $R \geq \hat{R}$, (b) when $R \in [\bar{R}_{2P}, \bar{R}_{2G})$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}]$ is strictly increasing in R, and (c) when $R \geq \bar{R}_{2G}$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}] \geq 0$. Therefore, $E[\Pi^*_{2P}] \geq E[\Pi^*_{2G}]$ if and only if $R \geq \hat{R} = R^T$.

Second, if $\hat{R} \geq \bar{R}_{2P}$, then (a) when $R < \bar{R}_{2P}$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}] < 0$, (b) when $R \in [\bar{R}_{2P}, \bar{R}_{2G})$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}]$ is strictly increasing in R, and (c) when $R \geq \bar{R}_{2G}$, $E[\Pi^*_{2P}] - E[\Pi^*_{2G}] \geq 0$. Therefore, there exists $R^T = \hat{R}' \in [\bar{R}_{2P}, \bar{R}_{2G}]$ such that $E[\Pi^*_{2P}] \geq E[\Pi^*_{2G}]$ if and only if $R \geq R^T$. □

Proof of Lemma 2.

(i) Using (20), $\theta^*_1 = \theta^F_1$ if $\theta^F_1 \leq \bar{\theta}$, and $\theta^*_1 = \bar{\theta}$ if $\theta^F_1 > \bar{\theta}$. Similarly, using (21), $\theta^*_{2P} = \theta^F_{2P}$ if $\theta^F_{2P} \leq \bar{\theta}$, and $\theta^*_{2P} = \bar{\theta}$ if $\theta^F_{2P} > \bar{\theta}$. Therefore, $\theta^*_{2P} \geq \theta^*_1$ if $\theta^F_{2P} \geq \theta^F_1$. Comparing (20) and (21), we observe that it is always the case that $\theta^F_{2P} \geq \theta^F_1$, because $\phi(q_1) - \phi(q_2) = (s + k_t)(q_1 - q_2) > 0$.

(ii) Using (22), $\theta^*_{2G} = \theta^F_{2G}$ if $\theta^F_{2G} \leq \bar{\theta}$, and $\theta^*_{2G} = \bar{\theta}$ if $\theta^F_{2G} > \bar{\theta}$. Therefore, $\theta^*_{2G} \leq \theta^*_1$ if $\theta^F_{2G} \leq \theta^F_1$. Comparing (20) and (22), $\theta^F_{2G} \leq \theta^F_1$ if and only if $R \geq R^C$, where $R^C = \frac{(V_G - 2K_G)\phi(q_1)}{2(k_t q_0 - \phi(q_1))} - \frac{Z_0}{2}$. Note that $k_t q_0 - \phi(q_1) = (s + k_t)(q_0 - q_1) - k_p > 0$ by Assumption 2. We observe that

$$\bar{R}_1 - R^C = \frac{\phi(q_1)}{2(k_t q_0 - \phi(q_1))}\left[2\bar{\theta}(k_t q_0 - \phi(q_1)) - (V_G - 2K_G)\right] > 0,$$

because $\bar{\theta}(k_t q_0 - \phi(q_1)) \geq (V_G - 2K_G)$ by Lemma 5, where $k_t q_0 - \phi(q_1) > 0$. Also,

$$\bar{R}_{2G} - R^C = \frac{k_t q_0[2\bar{\theta}(\phi(q_1) - k_t q_0) + V_G - 2K_G]}{2(\phi(q_1) - k_t q_0)} > 0,$$

because $\bar{\theta}(\phi(q_1) - k_t q_0) + V_G - 2K_G \leq 0$ by Lemma 5, where $\phi(q_1) - k_t q_0 < 0$. Therefore, $R^C < \bar{R}_1, \bar{R}_{2G}$. This means that, when $R < R^C$, it is always the case that $\theta^F_1 < \theta^F_{2G} < \bar{\theta}$, and thus $\theta^*_1 < \theta^*_{2G}$. When $R \geq R^C$, then $\theta^F_{2G} \leq \theta^F_1$, which necessarily means $\theta^*_{2G} \leq \theta^*_1$. Thus, we have shown that $\theta^*_{2G} \leq \theta^*_1$ if and only if $R \geq R^C$. □

Proof of Proposition 3. If $V_G \leq 2K_G$, then $R^T = \hat{R} < 0$ by Proposition 2, and thus the MA insurer always chooses Plan 2P and enrolls more beneficiaries than Plan 1 by Lemma 2. If $V_G > 2K_G$, then the results trivially follow from Proposition 2 and Lemma 2. Now, we show that (a) $R^T > 0$ when V_G is sufficiently high, (b) $R^T = \hat{R} < \bar{R}_{2P}$ and $\hat{R} < q_0 k^o_t \bar{\theta}/2$ when $\bar{\theta}$ is sufficiently high for a given V_G, and (c) $R^C < \hat{R}$ when q_2 ($< q_1$) is not too low.

(a) It is straightforward to see this from the expression of $\hat{R}$ in (11) and the proof of Proposition 2.

(b) We have $\bar{R}_{2P} - \hat{R} = \phi(q_2)\bar{\theta} - \frac{(V_G - 2K_G)\sqrt{\phi(q_2)}}{2(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})}$. Therefore, if $\bar{\theta}$ is sufficiently high for a given V_G, then $R^T = \hat{R} < \bar{R}_{2P}$ by Proposition 2. Also, $\hat{R}$ is independent of $\bar{\theta}$, and thus $\hat{R}$ is smaller than $q_0 k^o_t \bar{\theta}/2$ if $\bar{\theta}$ is sufficiently high for a given V_G.

(c) Using (11) and (13), we can rewrite $\hat{R}$ and R^C as follows:

$$\hat{R} = \frac{(V_G - 2K_G)}{2}\left(\frac{\sqrt{k_p + k_t q_2 + (q_2 - q_0)s}}{\sqrt{k_t q_0} - \sqrt{k_p + k_t q_2 + (q_2 - q_0)s}}\right) - \frac{Z_0}{2},$$
$$R^C = \frac{(V_G - 2K_G)}{2}\left(\frac{k_p + k_t q_1 + (q_1 - q_0)s}{(s + k_t)(q_0 - q_1) - k_p}\right) - \frac{Z_0}{2}.$$

Although $q_2 < q_1$, suppose that we have $q_2 = q_1$. Then, we observe that

$$\begin{aligned}\frac{\sqrt{k_p + k_t q_2 + (q_2 - q_0)s}}{\sqrt{k_t q_0} - \sqrt{k_p + k_t q_2 + (q_2 - q_0)s}} &= \frac{\sqrt{k_p + k_t q_1 + (q_1 - q_0)s}}{\sqrt{k_t q_0} - \sqrt{k_p + k_t q_1 + (q_1 - q_0)s}} \times \left(\frac{\sqrt{k_t q_0} + \sqrt{k_p + k_t q_1 + (q_1 - q_0)s}}{\sqrt{k_t q_0} + \sqrt{k_p + k_t q_1 + (q_1 - q_0)s}}\right)\\ &= \frac{\sqrt{k_t q_0 (k_p + k_t q_1 + (q_1 - q_0)s)} + (k_p + k_t q_1 + (q_1 - q_0)s)}{(s + k_t)(q_0 - q_1) - k_p}\\ &> \frac{k_p + k_t q_1 + (q_1 - q_0)s}{(s + k_t)(q_0 - q_1) - k_p},\end{aligned}$$

where $k_p + k_t q_1 + (q_1 - q_0)s > 0$ and $(s + k_t)(q_0 - q_1) - k_p > 0$ by Assumption 2. Therefore, if $q_2 = q_1$, then $R^C < \hat{R}$ because $V_G > 2K_G$. It is easy to see that $\hat{R}$ is strictly increasing in q_2. Therefore, there exists $q_c \in [0, q_1)$ such that, if $q_2 \in (q_c, q_1)$, then $R^C < \hat{R}$, and if $q_2 \leq q_c$, then $R^C \geq \hat{R}$. □

Proof of Proposition 4. Let $R^T(k_t, k_p) = \hat{R}(k_t, k_p)$ represent $\hat{R}$ as a function of k_t and k_p. We first obtain the following two partial derivatives of $\hat{R}$. First, we have

$$\frac{\partial \hat{R}(k_t, k_p)}{\partial k_p} = \frac{(V_G - 2K_G)\sqrt{k_t q_0}}{4\sqrt{\phi(q_2)}(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})^2} > 0, \tag{24}$$

because $V_G > 2K_G$ by Assumption 3 and $\phi(q_2) > 0$ by Assumption 2. Second, we have

$$\frac{\partial \hat{R}(k_t, k_p)}{\partial k_t} = -\frac{q_0(V_G - 2K_G)(k_p - (q_0 - q_2)s)}{4\sqrt{k_t q_0}\sqrt{\phi(q_2)}(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})^2}. \tag{25}$$

It is easy to see that $\frac{\partial \hat{R}}{\partial k_t} > 0$ if $k_p < (q_0 - q_2)s$ and $\frac{\partial \hat{R}}{\partial k_t} < 0$ if $k_p > (q_0 - q_2)s$.

Now, we show the following inequalities one by one: $\hat{R}(k_{t1}, k_{p2}) > \hat{R}(k_{t2}, k_{p2}) > \hat{R}(k_{t2}, k_{p1}) > \hat{R}(k_{t1}, k_{p1})$. First, $\hat{R}(k_{t1}, k_{p2}) > \hat{R}(k_{t2}, k_{p2})$ because $\frac{\partial \hat{R}}{\partial k_t} < 0$ noting that $k_{p2} > (q_0 - q_2)s$. Second, $\hat{R}(k_{t2}, k_{p2}) > \hat{R}(k_{t2}, k_{p1})$ because $\frac{\partial \hat{R}}{\partial k_p} > 0$. Last, $\hat{R}(k_{t2}, k_{p1}) > \hat{R}(k_{t1}, k_{p1})$ because $\frac{\partial \hat{R}}{\partial k_t} > 0$ noting that $k_{p1} < (q_0 - q_2)s$. □

Proof of Proposition 5. This proof consists of three steps. We first explore the comparative statics of $R^T = \hat{R}$ and R^C with respect to k_t in Step 1 and with respect to k_p in Step 2. Then, we combine these results in Step 3.

Step 1. We obtain the comparative statics of $\hat{R}$ and R^C with respect to k_t. We first establish the following result that we use in the subsequent proof: if we assume $q_1 = q_2$, then $\left|\frac{\partial \hat{R}}{\partial k_t}\right| \geq 2 \cdot \left|\frac{\partial R^C}{\partial k_t}\right|$. We have

$$\begin{aligned}\frac{\partial R^C}{\partial k_t} &= -\frac{q_0(V_G - 2K_G)(k_p - (q_0 - q_1)s)}{2((q_0 - q_1)(k_t + s) - k_p)^2},\\ \frac{\partial \hat{R}}{\partial k_t} &= -\frac{q_0(V_G - 2K_G)(k_p - (q_0 - q_2)s)}{4\sqrt{k_t q_0}\sqrt{\phi(q_2)}(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})^2}\\ &= -\frac{q_0(V_G - 2K_G)(k_p - (q_0 - q_2)s)(\sqrt{k_t q_0} + \sqrt{\phi(q_2)})^2}{4\sqrt{k_t q_0}\sqrt{\phi(q_2)}(k_t q_0 - \phi(q_2))^2}\\ &= -\frac{q_0(V_G - 2K_G)(k_p - (q_0 - q_2)s)}{2((q_0 - q_2)(k_t + s) - k_p)^2}\left[1 + \frac{1}{2} \cdot \frac{\sqrt{k_t q_0}}{\sqrt{\phi(q_2)}} + \frac{1}{2} \cdot \frac{\sqrt{\phi(q_2)}}{\sqrt{k_t q_0}}\right].\end{aligned}$$

Note that

$$\left|\frac{\partial \hat{R}}{\partial k_t}\right| = \left|-\frac{q_0(V_G-2K_G)(k_p-(q_0-q_2)s)}{2((q_0-q_2)(k_t+s)-k_p)^2}\right| \times \left|1+\frac{1}{2}\cdot\left(\frac{\sqrt{k_t q_0}}{\sqrt{\phi(q_2)}}+\frac{\sqrt{\phi(q_2)}}{\sqrt{k_t q_0}}\right)\right|$$
$$\geq 2\cdot\left|-\frac{q_0(V_G-2K_G)(k_p-(q_0-q_2)s)}{2((q_0-q_2)(k_t+s)-k_p)^2}\right|,$$

because for any $A,B>0$, we have $\left(\frac{\sqrt{A}}{\sqrt{B}}-\frac{\sqrt{B}}{\sqrt{A}}\right)^2=\frac{A}{B}+\frac{B}{A}-2\geq 0$. Thus for $q_1=q_2$ we have $\left|\frac{\partial \hat{R}}{\partial k_t}\right|\geq 2\cdot\left|\frac{\partial R^C}{\partial k_t}\right|$.

(i) Suppose $k_p<(q_0-q_1)s<(q_0-q_2)s$. Then, we have $\frac{\partial \hat{R}}{\partial k_t},\frac{\partial R^C}{\partial k_t}>0$ because $V_G>2K_G$ by Assumption 3. Therefore, if $q_1=q_2$, then $\frac{\partial \hat{R}}{\partial k_t}\geq 2\cdot\frac{\partial R^C}{\partial k_t}$, and thus $\frac{\partial \hat{R}}{\partial k_t}-\frac{\partial R^C}{\partial k_t}\geq\frac{\partial R^C}{\partial k_t}>0$. This means that when q_2 $(<q_1)$ is sufficiently close to q_1, we have $\hat{R}-R^C>0$ by the proof of Proposition 3 and $\frac{\partial \hat{R}}{\partial k_t}-\frac{\partial R^C}{\partial k_t}=\frac{\partial(\hat{R}-R^C)}{\partial k_t}>0$ because $\frac{\partial \hat{R}}{\partial k_t}$ is continuous in q_2 and $\frac{\partial R^C}{\partial k_t}$ is independent of q_2.

(ii) Suppose $(q_0-q_1)s\leq k_p\leq(q_0-q_2)s$. Then, we have $\frac{\partial \hat{R}}{\partial k_t}\geq 0$ and $\frac{\partial R^C}{\partial k_t}\leq 0$, where $\frac{\partial \hat{R}}{\partial k_t}$ and $\frac{\partial R^C}{\partial k_t}$ are not zero at the same time because $q_1>q_2$. Therefore, $\frac{\partial(\hat{R}-R^C)}{\partial k_t}>0$.

(iii) Suppose $(q_0-q_1)s<(q_0-q_2)s<k_p$. Then, we have $\frac{\partial \hat{R}}{\partial k_t},\frac{\partial R^C}{\partial k_t}<0$ because $V_G>2K_G$ by Assumption 3. Therefore, if $q_1=q_2$, then $\frac{\partial \hat{R}}{\partial k_t}\leq 2\cdot\frac{\partial R^C}{\partial k_t}$, and thus $\frac{\partial \hat{R}}{\partial k_t}-\frac{\partial R^C}{\partial k_t}\leq\frac{\partial R^C}{\partial k_t}<0$. This means that when q_2 $(<q_1)$ is sufficiently close to q_1, we have $\hat{R}-R^C>0$ by the proof of Proposition 3 and $\frac{\partial \hat{R}}{\partial k_t}-\frac{\partial R^C}{\partial k_t}=\frac{\partial(\hat{R}-R^C)}{\partial k_t}<0$ because $\frac{\partial \hat{R}}{\partial k_t}$ is continuous in q_2 and $\frac{\partial R^C}{\partial k_t}$ is independent of q_2.

Therefore, there exists $\underline{q}_2\in(0,q_1)$ such that, for all $q_2\in(\underline{q}_2,q_1)$, we have $\Delta^{CR}(k_{t2},k_{p1})>\Delta^{CR}(k_{t1},k_{p1})$ by cases (i) and (ii) and $\Delta^{CR}(k_{t1},k_{p2})>\Delta^{CR}(k_{t2},k_{p2})$ by case (iii).

Step 2. We now differentiate $\hat{R}$ and R^C with respect to k_p. Using eq. (24) in the proof of Proposition 4, we have

$$\frac{\partial \hat{R}}{\partial k_p}=\frac{(V_G-2K_G)\sqrt{k_t q_0}}{4\sqrt{\phi(q_2)}(\sqrt{k_t q_0}-\sqrt{\phi(q_2)})^2}\times\frac{(\sqrt{k_t q_0}+\sqrt{\phi(q_2)})^2}{(\sqrt{k_t q_0}+\sqrt{\phi(q_2)})^2}=\frac{(V_G-2K_G)\sqrt{k_t q_0}(\sqrt{k_t q_0}+\sqrt{\phi(q_2)})^2}{4\sqrt{\phi(q_2)}(k_t q_0-\phi(q_2))^2}>0.$$

Also, we have

$$\frac{\partial R^C}{\partial k_p}=\frac{k_t q_0(V_G-2K_G)}{2(k_p-(q_0-q_1)(k_t+s))^2}>0.$$

Therefore, if we assume that $q_1=q_2$, then

$$\frac{\partial \hat{R}}{\partial k_p}-\frac{\partial R^C}{\partial k_p}=\frac{(V_G-2K_G)\sqrt{k_t q_0}}{2(k_t q_0-\phi(q_1))^2}\times\left(\frac{(\sqrt{k_t q_0}+\sqrt{\phi(q_1)})^2}{2\sqrt{\phi(q_1)}}-\sqrt{k_t q_0}\right)=\frac{(V_G-2K_G)\sqrt{k_t q_0}}{2(k_t q_0-\phi(q_1))^2}\times\frac{k_t q_0+\phi(q_1)}{2\sqrt{\phi(q_1)}}>0.$$

This means that when q_2 $(<q_1)$ is sufficiently close to q_1, we have $\frac{\partial \hat{R}}{\partial k_p}-\frac{\partial R^C}{\partial k_p}=\frac{\partial(\hat{R}-R^C)}{\partial k_p}>0$ because $\frac{\partial \hat{R}}{\partial k_p}$ is continuous in q_2 and $\frac{\partial R^C}{\partial k_p}$ is independent of q_2. Therefore, $\Delta^{CR}(k_{t2},k_{p2})>\Delta^{CR}(k_{t2},k_{p1})$.

Step 3. Combining results from Steps 1 and 2, we have $\Delta^{CR}(k_{t1},k_{p2})>\Delta^{CR}(k_{t2},k_{p2})>\Delta^{CR}(k_{t2},k_{p1})>\Delta^{CR}(k_{t1},k_{p1})$. □

Proof of Lemma 3. We show the results in two steps. In Step 1, we obtain the optimal enrollment level in the centralized system. In Step 2, we evaluate the social welfare, health outcomes, and healthcare expenditures.

Step 1. In the centralized system, the social planner can determine (θ_i^*,x_i,R) freely to maximize social welfare subject to $E[\Pi_i]\geq 0$, where $E[\Pi_i]$ can be expressed as

$$E[\Pi_i]=\int_0^{\theta_i^*}(R+q_i x_i\theta)f(\theta)d\theta-\int_0^{\theta_i^*}((q_i k_t+k_p\cdot\mathbf{1}_{i\in\{1,2P\}})\theta+K_G\cdot\mathbf{1}_{i\in\{2G\}})\,f(\theta)d\theta. \tag{26}$$

Therefore, $E[\Pi_i]$ is increasing in R and x_i. Then, we observe that, for any given (θ_i^*, x_i), the optimal R is such that $E[\Pi_i] = 0$ because $W = CW - C$ is decreasing in R (specifically, CW is independent of R and C is increasing in R). Thus, the constraint $E[\Pi_i] \geq 0$ always binds at the optimal solution. Then, we have

$$\begin{aligned} W_i &= \int_0^{\theta_i^*} (U_i - R) f(\theta) d\theta + \int_{\theta_i^*}^{\bar{\theta}} (U_0 - q_0 k_t^o \theta) f(\theta) d\theta \\ &= \int_0^{\theta_i^*} (V_H - q_i s\theta + V_G \cdot \mathbf{1}_{i\in\{2G\}}) f(\theta) d\theta - \int_0^{\theta_i^*} (R + q_i x_i \theta) f(\theta) d\theta + \int_{\theta_i^*}^{\bar{\theta}} (U_0 - q_0 k_t^o \theta) f(\theta) d\theta \\ &= \int_0^{\theta_i^*} (V_H - (q_i s + q_i k_t + k_p \cdot \mathbf{1}_{i\in\{1,2P\}})\theta + (V_G - K_G) \cdot \mathbf{1}_{i\in\{2G\}}) f(\theta) d\theta + \int_{\theta_i^*}^{\bar{\theta}} (U_0 - q_0 k_t^o \theta) f(\theta) d\theta, \end{aligned}$$

where we replace the second term in the second line using (26) and $E[\Pi_i] = 0$. Then, W_i becomes a function of θ_i^* alone without R or x_i. Now, we have

$$\frac{\partial W_i}{\partial \theta_i^*} = \frac{1}{\bar{\theta}} \left[Z_0 + (q_0 - q_i) s \theta_i^* + (q_0 k_t^o - q_i k_t) \theta_i^* - k_p \cdot \mathbf{1}_{i\in\{1,2P\}} \theta_i^* + (V_G - K_G) \cdot \mathbf{1}_{i\in\{2G\}} \right],$$

which is strictly positive for all plans, because $k_t^o > k_t$ by Assumption 1, $(q_0 - q_i)k_t > k_p$ for $i \in \{1, 2P\}$ by Assumption 2, and $V_G > 2K_G$ by Assumption 3. It is then optimal to set $\theta_i^* = \bar{\theta}$ for $i \in \{1, 2P, 2G\}$.

Step 2. First, in the last expression of W_i in Step 1, we observe that the integrand of the first term is smaller for Plan 1 than for Plan 2P, because $q_1 > q_2$. Since $\theta_1^* = \theta_{2P}^* = \bar{\theta}$, we have $W_1 < W_{2P}$, and thus Plan 1 never achieves the highest social welfare. Also, we have

$$W_{2P} - W_{2G} = \int_0^{\bar{\theta}} \left[((s + k_t)(q_0 - q_2) - k_p)\theta - (V_G - K_G) \right] f(\theta) d\theta,$$

where $(s + k_t)(q_0 - q_2) - k_p > 0$ by Assumption 2. Thus, if V_G is sufficiently small (and correspondingly if K_G is sufficiently small too because $V_G > 2K_G$ by Assumption 3), then we have $W_{2P} > W_{2G}$.

Second, when $\theta_1^* = \theta_{2P}^* = \theta_{2G}^* = \bar{\theta}$, the health outcomes are $O_1 = q_0 - q_1$, $O_{2P} = q_0 - q_2$, and $O_{2G} = 0$, and thus $O_{2P} > O_1 > O_{2G}$. Therefore, Plan 2P achieves the best health outcome.

Finally, the healthcare expenditures are $E_1 = \int_0^{\bar{\theta}} (q_1 k_t \theta + k_p \theta)\, dF(\theta)$, $E_{2P} = \int_0^{\bar{\theta}} (q_2 k_t \theta + k_p \theta)\, dF(\theta)$, and $E_{2G} = \int_0^{\bar{\theta}} (q_0 k_t \theta + K_G)\, dF(\theta)$. We observe that the integrand is smallest for E_{2P}, followed by E_1 and E_{2G}, because $(q_0 k_t \theta + K_G) - (q_1 k_t \theta + k_p \theta) = K_G + ((q_0 - q_1)k_t - k_p)\theta > 0$ by Assumption 2 and $(q_1 k_t \theta + k_p \theta) - (q_2 k_t \theta + k_p \theta) = (q_1 - q_2) k_t \theta > 0$. Therefore, we have $E_{2P} < E_1 < E_{2G}$ and Plan 2P achieves the lowest healthcare expenditure. □

Proof of Proposition 6. This proof consists of three steps. In Step 1, we show that $\theta_{2P}^*(R_{2P}^*) > \theta_{2G}^*(R_{2G}^*)$. In Step 2, we compare the health outcomes and healthcare expenditures. In Step 3, we obtain the conditions under which $W_{2P}(R_{2P}^*) > W_{2G}(R_{2G}^*)$.

Step 1. When $R = \hat{R}$, we can obtain the following relationships using Lemma 4:

$$\theta_{2P}^*(\hat{R}) = \sqrt{E[\Pi_{2P}^*(\hat{R})]} \cdot \sqrt{\frac{2\bar{\theta}}{\phi(q_2)}} \quad \text{and} \quad \theta_{2G}^*(\hat{R}) = \sqrt{E[\Pi_{2G}^*(\hat{R})]} \cdot \sqrt{\frac{2\bar{\theta}}{k_t q_0}}.$$

Then, $\theta_{2P}^*(\hat{R}) > \theta_{2G}^*(\hat{R})$, because $E[\Pi_{2P}^*(\hat{R})] = E[\Pi_{2G}^*(\hat{R})]$ by Proposition 2 and $k_t q_0 - \phi(q_2) = (k_t + s)(q_0 - q_2) - k_p > 0$ by Assumption 2. Therefore, $\theta_{2P}^*(R_{2P}^*) > \theta_{2G}^*(R_{2G}^*)$, because $R_{2P}^* \geq \hat{R} > R_{2G}^*$ by the (IC2) constraint and both $\theta_{2P}^*(R)$ and $\theta_{2G}^*(R)$ are increasing functions of R by Lemma 4.

Step 2. First, the health outcome is always zero under Plan 2G regardless of θ^*_{2G}, $O_{2G}=0$, but it is strictly positive under Plan 2P, $O_{2P}>0$, because $O_{2P}=q_0-\left(\int_0^{\theta^*_{2P}} q_2\, dF(\theta)+\int_{\theta^*_{2P}}^{\bar\theta} q_0\, dF(\theta)\right)$, where $\theta^*_{2P}>0$ and $q_2<q_0$. Thus, we have $O_{2P}(R^*_{2P})>O_{2G}(R^*_{2G})$.

Second, the healthcare expenditures under the two plans are

$$E_{2P}=\int_0^{\theta^*_{2P}}(q_2k_t\theta+k_p\theta)\, dF(\theta)+\int_{\theta^*_{2P}}^{\bar\theta} q_0k_t^o\theta\, dF(\theta), \quad \text{and}$$
$$E_{2G}=\int_0^{\theta^*_{2G}}(q_0k_t\theta+K_G)\, dF(\theta)+\int_{\theta^*_{2G}}^{\bar\theta} q_0k_t^o\theta\, dF(\theta).$$

We observe that the integrand of the first term of E_{2P} is smaller than the integrands of any other terms of E_{2P} and E_{2G}. This is because $(q_0k_t\theta+K_G)-(q_2k_t\theta+k_p\theta)=K_G+((q_0-q_2)k_t-k_p)\theta>0$ by Assumption 2. Also, $q_0k_t^o\theta-(q_2k_t\theta+k_p\theta)>((q_0-q_2)k_t-k_p)\theta>0$ by Assumption 1 and Assumption 2. We have shown that $\theta^*_{2P}(R^*_{2P})>\theta^*_{2G}(R^*_{2G})$ in Step 1, and thus we have $E_{2P}(R^*_{2P})<E_{2G}(R^*_{2G})$.

Step 3. In this step, we obtain the conditions under which $W_{2P}(R^*_{2P})>W_{2G}(R^*_{2G})$. We can easily see that $R^T=\hat R=0$ when $V_G=\tilde V_G(K_G)=2K_G+Z_0\left(\frac{\sqrt{k_tq_0}}{\sqrt{\phi(q_2)}}-1\right)$, and $\hat R\ge 0$ if and only if $V_G\ge\tilde V_G(K_G)$.

First, we show that $W_{2P}(0)>W_{2G}(0)$ when $R=0$, $V_G=\tilde V_G(K_G)$, and V_G (and correspondingly K_G) is sufficiently small. When $R=0$ and $V_G=\tilde V_G(K_G)$, we have

$$W_{2P}(0)-W_{2G}(0)=\frac{Z_0[k_t^oZ_0q_0(k_tq_0-\phi(q_2))-4K_G\cdot\sqrt{k_tq_0}\sqrt{\phi(q_2)}\phi(q_2)]}{8k_tq_0\phi(q_2)^2\bar\theta},$$

where $k_tq_0-\phi(q_2)=(k_t+s)(q_0-q_2)-k_p>0$ by Assumption 2. Thus, if V_G ($>2K_G$ by Assumption 3) is sufficiently small, then K_G is sufficiently small, and we have $W_{2P}(0)-W_{2G}(0)>0$.

Second, we show that when $V_G=\tilde V_G(K_G)$, we have $W_{2P}(R^*_{2P})>W_{2P}(0)$. When the social planner wants to induce Plan 2P by solving (14), the whole set of constraints reduces to $R\ge\hat R=0$, and we show in the proof of Proposition 7 that the optimal capitation payment is always strictly positive, $R^*_{2P}>0$. That is, the constraint does not bind and thus $W_{2P}(R^*_{2P})>W_{2P}(0)$.

Third, we show that $\lim_{V_G\to\tilde V_G(K_G)^+}W_{2G}(R^*_{2G})=W_{2G}(0)$. When inducing Plan 2G by solving problem (14), the constraint becomes $R<\hat R$, and thus the feasible solution is $R\in[0,\hat R)$. When $V_G\to\tilde V_G(K_G)^+$, we have $\hat R\to 0^+$. Thus, $\lim_{V_G\to\tilde V_G(K_G)^+}W_{2G}(R^*_{2G})=W_{2G}(0)$.

Combining the above results, if V_G ($>2K_G$ by Assumption 3) is sufficiently small, K_G is sufficiently small, and as $V_G\to\tilde V_G(K_G)^+$, we have $W_{2P}(R^*_{2P})>W_{2P}(0)>W_{2G}(0)=W_{2G}(R^*_{2G})$. By continuity, there exists $\epsilon>0$ such that, if $V_G\in(\tilde V_G(K_G),\tilde V_G(K_G)+\epsilon)$, then $W_{2P}(R^*_{2P})>W_{2G}(R^*_{2G})$. This condition on V_G is equivalent to $V_G-2K_G\in\left(Z_0\left(\frac{\sqrt{k_tq_0}}{\sqrt{\phi(q_2)}}-1\right),Z_0\left(\frac{\sqrt{k_tq_0}}{\sqrt{\phi(q_2)}}-1\right)+\epsilon\right)$. Note that if V_G is too small, we may have $V_G-2K_G<Z_0\left(\frac{\sqrt{k_tq_0}}{\sqrt{\phi(q_2)}}-1\right)$. But in this case, we have $V_G<\tilde V_G(K_G)$ and the feasible region is empty when inducing Plan 2G. Thus, we ignore this case. □

Proof of Proposition 7. According to the discussion in Section 7.3, we can simplify problem (14) as follows.

$$\max_{R\in[0,\bar R_{2P}]} \quad W_{2P}(R)=CW_{2P}(R)-C_{2P}(R),$$
$$s.t. \quad R\ge\hat R.$$

We show the result in two steps. In Step 1, we obtain the optimal solution R^*_{2P}. In Step 2, we obtain the comparative statics of the optimal solution with respect to k_t.

Step 1. We constrained R to be within $[0, \bar{R}_{2P}]$, but it is easy to see that, even without this constraint, any $R \in (\bar{R}_{2P}, \infty)$ cannot be optimal. Therefore, we only consider $R \in [0, \bar{R}_{2P}]$. We observe that $W_{2P}(R)$ is a quadratic function of R, the coefficient of the quadratic term is

$$\frac{k_t^o q_0 - 2\phi(q_2)}{2\bar{\theta}\phi(q_2)^2}, \tag{27}$$

and the first-order condition, $W'_{2P}(R^F_{2P}) = 0$, is satisfied at

$$R^F_{2P} = \frac{k_t^o Z_0 q_0}{2(2\phi(q_2) - k_t^o q_0)}. \tag{28}$$

We can divide the problem into three cases.

(i) If $k_t^o q_0 > 2\phi(q_2)$, then (27) is strictly positive and $W_{2P}(R)$ achieves its minimum at $R^F_{2P} < 0$. Therefore, $W_{2P}(R)$ is strictly increasing in $R \in [0, \bar{R}_{2P}]$, and thus the optimal solution is $R^*_{2P} = \bar{R}_{2P}$. This solution satisfies the constraint, because $\bar{R}_{2P} > \hat{R}$.

(ii) If $k_t^o q_0 = 2\phi(q_2)$, then we observe that $W_{2P}(R)$ becomes a linearly increasing function. Therefore, again, the optimal solution is $R^*_{2P} = \bar{R}_{2P}$, which satisfies the constraint.

(iii) If $k_t^o q_0 < 2\phi(q_2)$, then (27) is strictly negative, and $W_{2P}(R)$ achieves its maximum at $R^F_{2P} > 0$ and strictly increases in $R \in [0, R^F_{2P}]$. Therefore, if $R^F_{2P} \geq \bar{R}_{2P}$, then the optimal solution is $R^*_{2P} = \bar{R}_{2P}$. If $R^F_{2P} < \bar{R}_{2P}$, then $W_{2P}(R)$ achieves its maximum at $R = R^F_{2P}$ and is strictly decreasing in $R \in [R^F_{2P}, \bar{R}_{2P}]$. Thus, considering the constraint, the optimal solution is $R^*_{2P} = \max\{R^F_{2P}, \hat{R}\}$.

Step 2. We compare R^*_{2P} with $R^*_{(2P)}$, where $R^*_{(2P)}$ is the solution without the constraint. We observe from Step 1 that the constraint binds if and only if $R^F_{2P} < \hat{R}$ in case (iii). When the constraint does not bind, we have $R^*_{2P} = R^*_{(2P)}$, and when the constraint binds, we have $R^*_{2P} = \hat{R} > R^*_{(2P)} = R^F_{2P}$. Therefore, in the rest of the proof, we show that there exist $\bar{k}_t^\dagger \leq \bar{k}_t \leq k_t^o$ such that the constraint does not bind when $k_t \leq \bar{k}_t^\dagger$ and the constraint binds when $k_t > \bar{k}_t$. (By Assumption 1, we have $k_t < k_t^o$, and thus we set $\bar{k}_t = k_t^o$ when we cannot show the existence of the constraint-binding region analytically.)

We observe that $\phi(q_2) = k_p + k_t q_2 - (q_0 - q_2)s$ is strictly increasing in k_t and thus there exists $\bar{k}_t'' \geq 0$ such that $2\phi(q_2) > k_t^o q_0$ if and only if $k_t > \bar{k}_t''$. (If $\bar{k}_t'' = 0$, then $2\phi(q_2) > k_t^o q_0$ for all $k_t > 0$.) Then, we are in case (iii) of Step 1 if and only if $k_t > \bar{k}_t''$. In case (iii), we observe that R^F_{2P}, expressed in (28), is strictly decreasing in $k_t \in (\bar{k}_t'', \infty)$, where $\lim_{k_t \to \infty} R^F_{2P} = 0$. Also, $\bar{R}_{2P}$, expressed in (9), is strictly increasing in k_t, where $\lim_{k_t \to \infty} \bar{R}_{2P} = \infty$. This means that there exists $\bar{k}_t' \geq \bar{k}_t''$ such that $R^F_{2P} < \bar{R}_{2P}$ if and only if $k_t > \bar{k}_t'$.

Therefore, $R^*_{2P} = \bar{R}_{2P}$ when $k_t \leq \bar{k}_t'$ and $R^*_{2P} = \max\{R^F_{2P}, \hat{R}\}$ when $k_t > \bar{k}_t'$. Here, if $\bar{k}_t' \geq k_t^o$, then the constraint never binds, because $R^*_{2P} = \bar{R}_{2P}$ for all $k_t < k_t^o$ and, by assumption, $\bar{R}_{2P} > \hat{R}$ for all $k_t < k_t^o$. Therefore, we now need to consider only the case when $\bar{k}_t' < k_t^o$. We consider two cases depending on the value of k_p:

(a) Suppose $k_p \leq (q_0 - q_2)s$. Then, when $k_t = 0$, we have $\phi(q_2) = k_p - (q_0 - q_2)s \leq 0$, and thus it must be that $\bar{k}_t'' > 0$. Moreover, $\lim_{k_t \to \bar{k}_t''^+} R^F_{2P} = \infty$, and thus $\bar{k}_t' > \bar{k}_t''$. Then, when $k_t = \bar{k}_t'$, we are in case (iii) of Step 1 and we have $R^F_{2P} = \bar{R}_{2P}$ $(> \hat{R})$. As we have shown, R^F_{2P} is strictly decreasing in $k_t \in (\bar{k}_t'', \infty)$, where $\lim_{k_t \to \infty} R^F_{2P} = 0$.

We also observe that $\hat{R}$ is weakly increasing in k_t when $k_p \le (q_0 - q_2)s$, as we have shown in the proof of Proposition 4. Thus, there exists $\bar{k}_t > \bar{k}_t'$ such that $R_{2P}^F \ge \hat{R}$ when $k_t \in [\bar{k}_t', \bar{k}_t]$ and $R_{2P}^F < \hat{R}$ when $k_t > \bar{k}_t$. (It is possible that $R_{2P}^F \ge \hat{R}$ for all k_t, in which case we set $\bar{k}_t = \infty$.) Therefore, the constraint binds if and only if $k_t > \bar{k}_t$. Note that this implies that $\bar{k}_t^\dagger = \bar{k}_t$. Since k_t is bounded above by k_t^o, if $\bar{k}_t < k_t^o$, then the constraint binds if $k_t \in (\bar{k}_t, k_t^o)$. If $\bar{k}_t \ge k_t^o$, then this constraint-binding region does not exist, in which case we simply set $\bar{k}_t = k_t^o$ to simplify the exposition of the proposition.

(b) Suppose $k_p > (q_0 - q_2)s$. Suppose that, for any given $k_p \le (q_0 - q_2)s$ from case (a), there exists $\bar{k}_t$ such that $\bar{k}_t < k_t^o$. Then, for $k_t \in (\bar{k}_t, k_t^o)$, we already know that we are in case (iii) of Step 1 and $R_{2P}^F < \hat{R}$. Note that $\hat{R}$ strictly increases in k_p by (24) in the proof of Proposition 4. Also, we can easily see that R_{2P}^F, expressed in (28), strictly decreases in k_p. Therefore, we can conclude that, even when $k_p > (q_0 - q_2)s$, we have $R_{2P}^F < \hat{R}$ for $k_t \in (\bar{k}_t, k_t^o)$. We have already shown that $R_{2P}^* = \bar{R}_{2P}$ when $k_t \le \bar{k}_t'$, and thus we can set $\bar{k}_t^\dagger = \bar{k}_t'$. Again, if the constraint-binding region does not exist, we set $\bar{k}_t = k_t^o$. □

Proof of Proposition 8. Suppose that the constraint IC2 binds and the conditions in the proposition hold. Denote $\Delta W := W_{2P}^* - W_1^*$. Solving the two cases, we have

$$
\begin{aligned}
W_1^* =& \frac{Z_0^2 + Z_0(-4\phi(q_1)\overline{\theta} + 2k_t^o q_0\overline{\theta}) + (-2\phi(q_1) + k_t^o q_0)\overline{\theta}(k_t^o q_0\overline{\theta} + q_0 s\overline{\theta} - 2V_H)}{4\phi(q_1)\overline{\theta} - 2k_t^o q_0\overline{\theta}} \\
W_{2P}^* =& \frac{(V_G - 2K_G)^2 d_1 + 4(V_G - 2K_G)d_2 + 4\phi(q_2)(\phi(q_2) - k_t q_0)^2\overline{\theta}(2Z_0 + k_t^o q_0\overline{\theta} + q_0 s\overline{\theta} - 2V_H)}{8\phi(q_2)(\phi(q_2) - k_t q_0)^2\overline{\theta}} \\
d_1 =& (\phi(q_2) + 2/\tau + k_t q_0)(k_t^o q_0 - 2\phi(q_2)) \\
d_2 =& Z_0\phi(q_2)(k_t q_0 - \phi(q_2))(1 + k_t\tau q_0) \\
\tau =& 1/\sqrt{k_t\phi(q_2)q_0}.
\end{aligned}
$$

From these expressions, we can see that ΔW is quadratic in V_G, and it is easy to check that

$$
\frac{\partial^2 \Delta W}{\partial V_G^2} = -\frac{2(\phi(q_2) - k_t^o q_0/2)\left(1 + k_t q_0\sqrt{\frac{1}{k_t q_0(k_p + k_t q_2 - q_0 s + q_2 s)}}\right)}{4\phi(q_2)(k_p - (q_0 - q_2)(k_t + s))^2\overline{\theta}}.
$$

Hence, when $\phi(q_2) - k_t^o q_0/2 > 0$, ΔW is concave in V_G.

We next show that ΔW is first positive and then negative as we increase V_G from $2K_G$. For the first claim, let $V_G = 2K_G$. We then have $\hat{R} < 0$ and hence the IC2-constraint does not bind. As we increase V_G, from the monotonicity of $\hat{R}$ with respect to V_G it is easy to see that eventually the constraint binds, until which point the difference ΔW must be positive.

For the last part, let us increase V_G. Since ΔW is concave in V_G, either it becomes negative or either Assumption 2(iii) (for Plan 1) or Assumption 3(ii) is violated (we can check that Assumption 3(ii) is always stricter than Assumption 2(iii) for Plan 2P). We show that under our assumptions, the value becomes negative such that neither constraint binds.

We first show that Assumption 3(ii) is not violated when q_2 is not too low and $2\phi(q_2) \ge k_t^o q_0$ (and hence $2\phi(q_1) \ge k_t^o q_0$). To that end, denote

$$
\hat{V} := 2(K_G - \phi(q_2)\overline{\theta} + \sqrt{\phi(q_2)k_t q_0}\overline{\theta}).
$$

Then we have $\hat{R} \le \overline{R}_{2P}$ if and only if $V_G \le \hat{V}$. To see that Assumption 3(ii) is not violated, we can check that $\Delta W(V_G = \hat{V}) \le 0$. We have

$$\Delta W(V_G = \hat{V}) = \frac{-Z_0^2 - (2\phi(q_1) - k_t^o q_0)(2\phi(q_2) - k_t^o q_0)\overline{\theta}^2 + 2Z_0\overline{\theta}(2\phi(q_1) - k_t^o q_0)}{2(2\phi(q_1) - k_t^o q_0)\overline{\theta}},$$

where the denominator is positive by our assumption. If $q_2 = q_1$, then the numerator is a quadratic form and always negative, and clearly it is also negative for $q_2 < q_1$ for q_2 not too low.

We will next show that Assumption 2(iii) (for Plan 1) is not violated when $\overline{\theta}$ is high enough. Recall from the assumption that $\overline{V}_G = 2K_G + (k_t q_0 - \phi(q_1))\overline{\theta}$. Substituting this into ΔW, we have

$$\Delta W(V_G = \overline{V}_G) = -\frac{\alpha_1 Z_0^2 + \alpha_2 Z_0 \overline{\theta} + \alpha_3 \overline{\theta}^2}{8\phi(q_2)(2\phi(q_1) - k_t^o q_0)(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})^2 \overline{\theta}}$$

$$\alpha_1 = 4\phi(q_2)(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})^2$$

$$\alpha_2 = -4\sqrt{\phi(q_2)}(k_t q_0 - \phi(q_1))(2\phi(q_1) - k_t^o q_0)(\sqrt{k_t q_0} - \sqrt{\phi(q_2)})$$

$$\alpha_3 = (\phi(q_1) - k_t q_0)^2 (2\phi(q_1) - k_t^o q_0)(2\phi(q_2) - k_t^o q_0).$$

The denominator is positive for $2\phi(q_1) - k_t^o q_0$. Differentiating for $\overline{\theta}$, and noting we must have $k_t q_0 > \phi(q_1)$ and $k_t q_0 > \phi(q_2)$ in Assumption 2(iii), we can check that the numerator is positive for high enough values of $\overline{\theta}$. □

Proof of Proposition 9. We first observe that we can drop the fraction of j-type beneficiaries, α^j, in the objective function and in the second constraint without altering the problem. Then, the objective function Π_i^j increases in the treatment charge x_i^j, and the threshold $\theta_i^j(x_i^j)$ (weakly) decreases in x_i^j. Therefore, the treatment charge that maximizes Π_i^j, denoted by $\bar{x}_i^j$, must satisfy $\theta^j = \theta_i^j(\bar{x}_i^j)$ because of the first constraint. Thus,

$$\bar{x}_i^j = \frac{(q_0^j - q_i^j)s^j}{q_i^j} + \frac{Z_0^j + V_G \cdot \mathbf{1}_{i\in\{2G\}}}{\theta^j q_i^j}.$$

We verify that $(\bar{x}_i^j + s^j)q_i^j - s^j q_0^j > 0$. When $x_i^j = \bar{x}_i^j$, we have

$$\Pi_i^j(\bar{x}_i^j) = R + Z_0^j + (V_G - K_G)\cdot \mathbf{1}_{i\in\{2G\}} - \left(k_p^j \cdot \mathbf{1}_{i\in\{1,2P\}} + k_t^j q_i^j - (q_0^j - q_i^j)s^j\right)\theta^j. \tag{29}$$

Then, $\Pi_i^j(\bar{x}^j) \ge 0$ if and only if

$$\theta^j \le \hat{\theta}_i^j = \frac{R + Z_0^j + (V_G - K_G)\cdot \mathbf{1}_{i\in\{2G\}}}{k_p^j \cdot \mathbf{1}_{i\in\{1,2P\}} + k_t^j q_i^j - (q_0^j - q_i^j)s^j}, \tag{30}$$

where the denominator is strictly positive for all $i = \{1, 2P, 2G\}$ by Assumption 2. If $\theta^j \le \hat{\theta}_i^j$, then the MA insurer finds it profitable to enroll the j-type beneficiaries, and the optimal solution is $x_i^{j*} = \bar{x}_i^j$. If $\theta^j > \hat{\theta}_i^j$, then the treatment charge $\bar{x}_i^j$ generates a strictly negative profit and the MA insurer is better off not enrolling the j-type. The MA insurer can prevent j-type beneficiaries from enrolling by setting x_i^{j*} sufficiently high so that $\theta^j > \theta_i^j(x_i^{j*})$. □

Proof of Proposition 10. In equation (29) in the proof of Proposition 9, we observe that $\Pi_{2P}^j(\bar{x}_{2P}^j) > \Pi_1^j(\bar{x}_1^j)$, for any $j \in \{l, h\}$, because $q_1^j > q_2^j$. Thus, the MA insurer never chooses Plan 1. Between Plans 2P, and 2G, we have $\Pi_{2G}^j(\bar{x}_{2G}^j) > \Pi_{2P}^j(\bar{x}_{2P}^j)$ if and only if

$$\theta^j < \tilde{\theta}^j = \frac{V_G - K_G}{(q_0^j - q_2^j)(k_t^j + s^j) - k_p^j}.$$

Let $A_1 = k_t^j q_0^j$, $A_2 = k_p^j + k_t^j q_2^j - (q_0^j - q_2^j)s^j$, $A_3 = V_G - K_G$, and $A_4 = R + Z_0^j$. Then, we have $\tilde{\theta}^j = \frac{A_3}{A_1 - A_2}$, $\hat{\theta}_{2P}^j = \frac{A_4}{A_2}$, and $\hat{\theta}_{2G}^j = \frac{A_3 + A_4}{A_1}$. We observe that

$$\hat{\theta}_{2G}^j - \hat{\theta}_{2P}^j = \frac{A_2 A_3 - (A_1 - A_2)A_4}{A_1 A_2} \qquad \text{and} \qquad \tilde{\theta}^j - \hat{\theta}_{2G}^j = \frac{A_2 A_3 - (A_1 - A_2)A_4}{A_1(A_1 - A_2)}.$$

Note that $A_1 - A_2 = (q_0^j - q_2^j)(k_t^j + s^j) - k_p^j > 0$ by Assumption 2. First, if $\hat{\theta}_{2P}^j < \tilde{\theta}^j$, then $A_2 A_3 > (A_1 - A_2)A_4$. Thus, we have $\hat{\theta}_{2P}^j < \hat{\theta}_{2G}^j < \tilde{\theta}^j$. Second, if $\hat{\theta}_{2P}^j \geq \tilde{\theta}^j$, then $A_2 A_3 \leq (A_1 - A_2)A_4$. Thus, we have $\tilde{\theta}^j \leq \hat{\theta}_{2G}^j \leq \hat{\theta}_{2P}^j$. In each case, the result follows from the definitions of $\hat{\theta}_i^j$ and $\tilde{\theta}^j$. □

Proof of Proposition 11. Note that $R^T = \hat{R}$ by Assumption 3, and both $\hat{R}$ and R^C are independent of $\bar{\theta}$. In this proof, we use the following results that we have already shown in the proofs of Proposition 4 and Proposition 5: (1) $\frac{\partial \hat{R}}{\partial k_p} > 0$ and $\frac{\partial R^C}{\partial k_p} > 0$, (2) $\frac{\partial \hat{R}}{\partial k_t} > 0$ if $k_p < (q_0 - q_2)s$ and $\frac{\partial \hat{R}}{\partial k_t} < 0$ if $k_p > (q_0 - q_2)s$, and (3) $\frac{\partial R^C}{\partial k_t} > 0$ if $k_p < (q_0 - q_1)s$ and $\frac{\partial R^C}{\partial k_t} < 0$ if $k_p > (q_0 - q_1)s$.

(*i*) We can express $\hat{R}$ and R^C as functions of q_0 by substituting $q_1 = q_0 - \delta_1$ and $q_2 = q_0 - \delta_2$, eliminating q_1 and q_2. Then, we have

$$\frac{\partial \hat{R}}{\partial q_0} = \frac{k_t(V_G - 2K_G)(\sqrt{k_t q_0} + \sqrt{k_p + k_t q_0 - \delta_2(k_t + s)})}{4\sqrt{k_t q_0}\sqrt{k_p + k_t q_0 - \delta_2(k_t + s)}(\sqrt{k_t q_0} - \sqrt{k_p + k_t q_0 - \delta_2(k_t + s)})} > 0,$$

where $k_p + k_t q_0 - \delta_2(k_t + s) = k_p + k_t q_2 - (q_0 - q_2)s > 0$ and $\sqrt{k_t q_0} > \sqrt{k_p + k_t q_0 - \delta_2(k_t + s)}$ because $k_t q_0 - (k_p + k_t q_0 - \delta_2(k_t + s)) = (k_t + s)(q_0 - q_2) - k_p > 0$ by Assumption 2. Also, $\frac{\partial R^C}{\partial q_0} = \frac{k_t(V_G - 2K_G)}{2(\delta_1(k_t + s) - k_p)} > 0$, where $\delta_1(k_t + s) - k_p = (q_0 - q_1)(k_t + s) - k_p > 0$ by Assumption 2. Noting that $(q_0 - q_1)s < (q_0 - q_2)s$, we have the following results.

(a) We have $\frac{\partial \hat{R}}{\partial k_p} > 0$, $\frac{\partial R^C}{\partial k_p} > 0$, $\frac{\partial \hat{R}}{\partial q_0} > 0$, and $\frac{\partial R^C}{\partial q_0} > 0$. Also, if $k_p < (q_0 - q_1)s$, we have $\frac{\partial \hat{R}}{\partial k_t} > 0$ and $\frac{\partial R^C}{\partial k_t} > 0$. Then, it follows that $(R^C)^L \leq (R^C)^H$ and $(R^T)^L \leq (R^T)^H$.

(b) If we assume $k_t^L = k_t^H$, then it naturally follows that $(R^C)^L \leq (R^C)^H$ and $(R^T)^L \leq (R^T)^H$.

(*ii*) We can express $\hat{R}$ and R^C as functions of q_0 alone by substituting $q_1 = \delta_1 q_0$ and $q_2 = \delta_2 q_0$. Then, we have

$$\frac{\partial \hat{R}}{\partial q_0} = \frac{-k_p k_t(V_G - 2K_G)}{4\sqrt{k_t q_0}\sqrt{k_p - q_0 s + \delta_2 q_0(k_t + s)}(\sqrt{k_t q_0} - \sqrt{k_p - q_0 s + \delta_2 q_0(k_t + s)})^2} < 0,$$

where $k_p - q_0 s + \delta_2 q_0(k_t + s) = k_p + k_t q_2 - (q_0 - q_2)s > 0$ by Assumption 2. Also, $\frac{\partial R^C}{\partial q_0} = \frac{-k_p k_t(V_G - 2K_G)}{2(k_p + q_0(\delta_1 - 1)(k_t + s))^2} < 0$. If $k_p > (q_0 - q_2)s$, then we have $\frac{\partial \hat{R}}{\partial k_t} < 0$ and $\frac{\partial R^C}{\partial k_t} < 0$. Thus, if we assume $k_p^L = k_p^H > (q_0 - q_2)s$, then it follows that $(R^C)^L \geq (R^C)^H$ and $(R^T)^L \geq (R^T)^H$. □

Appendix C: Parameter Calibration

To draw figures in the main body, we roughly estimate the parameter values as follows using diabetes as an example, based on readily available information in the literature. We by no means claim that these are accurate estimates. Rather, we want the parameter values in our figures to be within reasonable orders of magnitude.

- q_i (episode risk): According to the 2020 National Diabetes Statistics Report, 7.8 million hospital discharges were reported with diabetes as listed diagnosis in 2016 (Report available at https://diabetesresearch.org/wp-content/uploads/2022/05/national-diabetes-statistics-report-2020.pdf). This corresponds to 339 per 1000 adults with diabetes, or 33.9%. Note that this value may include multiple episodes of a single individual, but at the same time does not count minor episodes that did not require hospitalization. Considering this, we roughly set $q_0 = 0.34$ with q_1 and q_2 offering some 10%-20% reduction from q_0.

- R (capitation payment): According to Zuckerman et al. (2017)[1], the typical annual capitation payment was between \$8616 and \$9814 in 2015. However, our model considers only the costs related to the treatment of diabetes (instead of all medical conditions), and thus an appropriate value for R may be less than this amount. We suppose that the reasonable range of R is roughly between \$5,000 and \$10,000. For most figures, we do not need to set the value of R, because they either have R on the vertical axis or endogenously set the optimal R.
- $\bar{\theta}$ (maximum vulnerability): This is a theoretical parameter that does not affect the numerical analysis itself as long as the values of $k_t^o\bar{\theta}$, $k_t\bar{\theta}$, $k_p\bar{\theta}$, and $s\bar{\theta}$ are kept fixed, as we can see in the definitions of the MA insurer's profit in (2) and beneficiaries' utilities in (1) and (3). We choose $\bar{\theta} = 100$ for simplicity and choose appropriate values of k_t^o, k_t, k_p, and s accordingly.
- k_t^o and k_t (marginal treatment cost in OM and MA plans): According to the American Diabetes Association[2], the average annual medical expenditure attributed to diabetes in 2017 was \$9,601. If we assume that this value is roughly equal to the expected treatment cost for a beneficiary with an episode risk of q_0, then $E[k_t q_0 \theta] = k_t q_0 \bar{\theta}/2 = \$9,601$. Using $q_0 = 0.34$ and $\bar{\theta} = 100$, we have $k_t = 565$. We explore the range $k_t \in [450, 600]$ with $k_t^o = 610$ in our numerical analysis, which satisfies $k_t < k_t^o$ in Assumption 1.
- k_p (marginal preventive-benefit cost): According to a report by KFF, the average annual budget used for MA plans' supplemental services was \$996 in 2019 (See https://www.kff.org/medicare/issue-brief/higher-and-faster-growing-spending-per-medicare-advantage-enrollee-adds-to-medicares-solvency-and-affordability-challenges/). Assuming that this budget is roughly equal to the average cost of preventive services for the entire population of beneficiaries, we have $E[k_p\theta] = k_p\bar{\theta}/2 = \$996$. Using $\bar{\theta} = 100$, we get $k_p = 19.92$. In practice, k_p may be less than this value, because the supplemental-service budget is also used for services such as dental, vision, and hearing. Thus, in our numerical analysis, we pick values from the range $k_p \in [10, 20]$.
- K_G (general well-being supplementary services cost): Similar to the estimation of k_p, we suppose that the average annual budget for MA plans' supplemental services, \$996 in 2019 according to KFF, is roughly equal to the cost of general well-being services under Plan 2G. We set $K_G = 1000$.
- V_G (value of general well-being benefit): In order to satisfy the assumption $V_G > 2K_G$ in Assumption 3, we choose values in the following range: $V_G \in [2.5K_G, 3.5K_G] = [2500, 3500]$, which can generate figures with different equilibrium outcomes.
- Z_0 (annual premium under Plan 0): According to an article by Forbes, the most popular Medigap plan is plan F, which provides the most comprehensive supplemental coverage (See https://www.forbes.com/health/medicare/medicare-plan-f/). The article says "People who are eligible for Plan F enrollment can expect to pay a monthly premium between \$150 and \$400, with the average hovering around \$230." Using the average of \$230, the annual premium is $12 \times \$230 = \$2760$. Thus, we roughly set $Z_0 = 3000$.

[1] Zuckerman S, Skopec L, Guterman S (2017) Do Medicare Advantage plans minimize costs? Investigating the relationship between benchmarks, costs, and rebates. *The Commonwealth Fund*.

[2] American Diabetes Association (2018) Economic costs of diabetes in the U.S. in 2017. *Diabetes Care* 41(5):917–928.

- V_H (valuation of the absence of an adverse episode): This is a difficult parameter to calibrate, but we use Quality-Adjusted Life Year (QALY) to come up with a rough number. QALY measures the value of health outcomes and is used to estimate the value of medical interventions. According to U.S. Department of Veterans Affairs, the U.S. healthcare system uses roughly \$50,000-\$100,000 for the cost per QALY threshold when evaluating the cost effectiveness of medical interventions (See https://www.herc.research.va.gov/include/page.asp?id=cost-effectiveness-analysis). Using this as a proxy, we set $V_H = 75{,}000$.
- s (beneficiaries' marginal disutility of an episode of illness): In our model, this parameter includes both mental and physical disutility, and thus is difficult to quantify. Therefore, we choose a value that can generate rich figures with different equilibrium outcomes. We use one of the following values: $s \in \{60, 240\}$, which are 10% and 40% of the maximum marginal treatment cost $k_t = 600$ that we use.