EC.1. Proofs for Section 3

Proof of Lemma 1 By Watson (1929) and Stirling's formula,

$$\frac{1}{2} \le \sum_{k=0}^{z} \frac{z^k e^{-z}}{k!} \le \frac{1}{2} + \frac{2z^z e^{-z}}{3z!} \le \frac{1}{2} + \frac{2}{3\sqrt{2\pi z}}.$$

Since

$$0 < \sum_{k=z+1}^{\infty} \frac{(z/\beta)^k}{k!} \le \sum_{k=z+1}^{\infty} \frac{(z/\beta)^{z+1}}{(z+1)!} \left(\frac{z/\beta}{z+2}\right)^{k-z-1} = \frac{(z/\beta)^{z+1}}{(z+1)!} \frac{1}{1-\frac{z/\beta}{z+2}} \le \frac{1}{1-1/\beta} \frac{1}{\sqrt{2\pi z}} (1/\beta)^z e^{z(1-2/\beta)},$$

we are able to obtain our bounds.

Since $\bar{\gamma}(\beta, \bar{z}(\beta,\delta)) = \underline{\gamma}(\beta, \underline{z}(\beta,\delta)) = \delta$,

$$\left[1 + \frac{\frac{4}{3} + \frac{2}{1-1/\beta}}{\sqrt{2\pi \bar{z}(\beta,\delta)}}\right] \left[\frac{1}{\beta} e^{(1-1/\beta)}\right]^{\bar{z}(\beta,\delta)-\underline{z}(\beta,\delta)} = 1.$$

Since $0 < \frac{1}{\beta} e^{(1-1/\beta)} < 1$ and $\bar{z}(\beta,\delta)$ is decreasing in δ for $\beta > 1$, $\bar{z}(\beta,\delta) - \underline{z}(\beta,\delta)$ is increasing in δ. □

EC.2. Proofs for Section 4

Proof of Proposition 1 We first claim that $G^*(b)$ must be continuous for $b > \underline{b}$ for a given p^*. Suppose that $G^*(b+) > G^*(b-)$ at some b. Then, for ϵ sufficiently small, the cost difference for bidding at b and $b+\epsilon$ is

$$(1-\delta)b + c(W(b-\epsilon|(p^*\Lambda, G^*), z)) - (1-\delta)(b+\epsilon) - c(W(b+\epsilon|(p^*\Lambda, G^*), z)) > 0.$$

Thus, bidding at $b+\epsilon$ is preferred to bidding at b, and hence $G^*(\cdot)$ must be continuous.

Second, if $\underline{b}$ is the lowest bid allowed, the lowest bid must be $\underline{b}$ as it would otherwise cost users more to bid the lowest bid without lowering the queueing latency otherwise. □

Proof of Theorem 1 By Proposition 1, the support of the equilibrium fee distribution $G^*(\cdot)$ includes $\underline{b}$ and $G^*(\cdot)$ is continuous. Thus, (6) follows as the users' cost is the same for any bid b in the support, i.e., $(1-\delta)\underline{b} + c\left(W_q(p^*\Lambda) + \frac{z}{\mu}\right) = (1-\delta)b + c\left(W_q(p^*\Lambda(1-G^*(b))) + \frac{z}{\mu}\right)$. Since the equilibrium joining probability $p^* \le 1$, $p^* = 1$ if the users' utility $R(\delta, \underline{b}) - c\left(W_q(\Lambda) + \frac{z}{\mu}\right) \ge 0$. Otherwise, p^* is given by $c\left(W_q(p^*\Lambda) + \frac{z}{\mu}\right) = R(\delta, \underline{b})$ and users' utility is 0 in equilibrium. Thus, we have (5).

Since the highest possible bid is the smallest solution to $\bar{G}^*(b)=0$ or $\underline{b}+\frac{c\left(W_q(p^*\Lambda)+\frac{z}{\mu}\right)-c\left(\frac{z+1}{\mu}\right)}{1-\delta}$, (9) holds as

$$
\begin{aligned}
\Phi^*(z) &= (1-\delta)\left(p^*\Lambda\underline{b}+\int_{\underline{b}}^{\underline{b}+\frac{c\left(W_q(p^*\Lambda)+\frac{z}{\mu}\right)-c\left(\frac{z+1}{\mu}\right)}{1-\delta}} p^*\Lambda\bar{G}^*(b)db\right)\\
&= (1-\delta)p^*\Lambda\underline{b}+\int_{c\left(\frac{1+z}{\mu}\right)}^{c\left(W_q(p^*\Lambda)+\frac{z}{\mu}\right)} W_q^{-1}\left(c^{-1}(s)-\frac{z}{\mu}\right)ds\\
&= (1-\delta)p^*\Lambda\underline{b}+\int_{1/\mu}^{W_q(p^*\Lambda)} W_q^{-1}(t)dc\left(t+\frac{z}{\mu}\right)\\
&= (1-\delta)p^*\Lambda\underline{b}+\int_{0}^{p^*\Lambda} \tilde{\lambda}dc\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right) \qquad \text{(EC.1)}\\
&= (1-\delta)p^*\Lambda\underline{b}+p^*\Lambda c\left(W_q(p^*\Lambda)+\frac{z}{\mu}\right)-\int_0^{p^*\Lambda} c\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right)d\tilde{\lambda}\\
&= p^*\Lambda\min\left\{R(\delta,\underline{b}),(1-\delta)\underline{b}+c\left(W_q(\Lambda)+\frac{z}{\mu}\right)\right\}-\int_0^{p^*\Lambda} c\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right)d\tilde{\lambda}
\end{aligned}
$$

and the expected utility of the users are given by (7). □

PROPOSITION EC.1. *The equilibrium solution given in Theorem 1 has the following properties.*

1. $p^*(z)$ *decreases in* z*, from* 1 *if* $\frac{1}{\Lambda}W_q^{-1}\left(c^{-1}\left(R(\delta,\underline{b})\right)\right)>1$.
2. $G^*(\cdot|z)$ *stochastically increases while* $p^*(z)=1$ *and then decreases as* z *increases;*
3. *For a given* z*,* $G^*(b|z)$ *is strictly increasing convex in* b *before it reaches 1.*

Proof of Proposition EC.1 The first statement is clear. To establish the remaining statements, we first show that $W_q(\tilde{\lambda})$ is increasing convex in $\tilde{\lambda}$. By (3), we can express the queueing latency as a function of θ as $W_q(\theta)=\frac{1}{\mu[1-(1+K)\theta^K+K\theta^{K+1}]}$. Taking the derivate of W_q yields that

$$\frac{dW_q(\theta)}{d\tilde{\lambda}}=W_q'(\theta)\frac{d\theta}{d\tilde{\lambda}}=\frac{K(K+1)}{\mu^2}\frac{(1-\theta)^3\theta^{K-1}}{[1-(1+K)\theta^K+K\theta^{K+1}]^3}$$

and θ is increasing in $\tilde{\lambda}$, W_q is increasing in $\tilde{\lambda}$ and it suffices to show that $\frac{(1-\theta)^3\theta^{K-1}}{\eta^3(\theta)}$, where $\eta(\theta)=1-(1+K)\theta^K+K\theta^{K+1}$, is increasing in θ. Since

$$\frac{d}{d\theta}\left(\frac{(1-\theta)^3\theta^{K-1}}{\eta^3(\theta)}\right)=\frac{(1-\theta)^2\theta^{K-2}}{\eta^4(\theta)}[(K-1-(K+2)\theta)\eta(\theta)-3\eta'(\theta)(1-\theta)\theta]\triangleq\frac{(1-\theta)^2\theta^{K-2}}{\eta^4(\theta)}h(\theta),$$

and $h(\theta)>0$ when $K\le 3$, it suffices to show that $h(\theta)>0$ for $\theta\in(0,1)$ and hence $h'(\theta)<0$ given that $h(1)=0$ for $K\ge 4$. We show this by establishing that $h'(\theta)$ has a unique global maximum $\theta=1$ in $(0,1]$ at $h'(1)=0$. Taking the derivatives, we obtain:

$$
\begin{aligned}
h'(\theta)=&-(K+2)+K(K+1)(2K+1)\theta^{K-1}+(K+1)(-4K^2-4K+2)\theta^K+K(K+2)(2K+1)\theta^{K+1},\\
h''(\theta)=&K(K+1)\theta^{K-2}[(K-1)(2K+1)+(-4K^2-4K+2)\theta+(K+2)(2K+1)\theta^2].
\end{aligned}
$$

It can be easily shown that the term in "[ ]" in $h''(\theta)$ is quadratic with exactly two roots $\theta_1 < \theta_2$ in $(0,1)$. Thus, $h'(\theta)$ must achieve its global maxima at either θ_1 or 1. Since θ_1 is a root of $h''(\theta)=0$, it satisfies $(K-1)(2K+1)+(-4K^2-4K+2)\theta_1 = -(K+2)(2K+1)\theta_1^2$ and $h'(\theta_1)$ can be reduced to

$$h'(\theta_1) = -(K+2)+2K(2K+1)\theta_1^{K-1}-(4K^2+4K-2)\theta_1^K,$$

which is bounded from above by $\left\{\frac{4K+2}{K+2}\left[\frac{(2K+1)(K-1)}{2K^2+2K-1}\right]^{K-1}-1\right\}(K+2)$. Applying $\ln(1-x) < -x - \frac{x^2}{2}$, we have:

$$\begin{aligned}
&\frac{d}{dK}\ln\left[\frac{4K+2}{K+2}\left(\frac{(2K+1)(K-1)}{2K^2+2K-1}\right)^{K-1}\right]\\
=&\frac{6K^3+18K^2+9K+3+(K+2)(2K+1)(2K^2+2K-1)\ln(1-\frac{3K}{2K^2+2K-1})}{(K+2)(2K+1)(2K^2+2K-1)}\\
<&\frac{6K^3+18K^2+9K+3+(K+2)(2K+1)(-3K-\frac{9K^2}{2K^2+2K-1})}{(K+2)(2K+1)(2K^2+2K-1)}\\
=&\frac{-3(4K^4+11K^3+3K^2-K+1)}{(K+2)(2K+1)(2K^2+2K-1)^2} < 0
\end{aligned}$$

for $K \geq 1$. Since $\frac{4K+2}{K+2}(\frac{(2K+1)(K-1)}{2K^2+2K-1})^{K-1}|_{K=4} < 1$, $h'(\theta) < 0$ $\theta \in [0,1]$ for $K \geq 4$.

Since $W_q(\cdot)$ and $c(\cdot)$ are both strictly increasing and convex, or equivalently $W_q^{-1}(\cdot)$ and $c^{-1}(\cdot)$ are increasing concave, $G^*(b|z)$ is strictly increasing convex in b, i.e., the third statement follows. Furthermore, when $p^*(z)=1$, $c\left(W_q((\Lambda)+\frac{z}{\mu}\right)-c\left(W_q\left((1-G^*(b|z))\Lambda\right)+\frac{z}{\mu}\right)=(1-\delta)(b-\underline{b})$, implying that $G^*(\cdot|z)$ stochastically increases in z. When $p^*(z)<1$, $G^*(b|z) = 1-\frac{W_q^{-1}\left(c^{-1}(R(\delta,b)-\frac{z}{\mu}\right)}{W_q^{-1}(c^{-1}(R(\delta,\underline{b}))-\frac{z}{\mu})}$ is increasing in z, i.e., $G^*(\cdot|z)$ stochastically decreases in z. Thus, the second statement holds. □

Proof of Proposition 2 When $p^*(z)=1$,

$$\begin{aligned}
\Phi^*(z) &= \Lambda\left[(1-\delta)\underline{b}+c\left(W_q(\Lambda)+\frac{z}{\mu}\right)\right]-\int_0^{\Lambda} c\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right)d\tilde{\lambda}\\
&= \Lambda(1-\delta)\underline{b}+\int_0^{\Lambda}\left[c\left(W_q(\Lambda)+\frac{z}{\mu}\right)-c\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right)\right]d\tilde{\lambda},
\end{aligned}$$

which increases in z due to the convexity of $c(\cdot)$. When $p^*(z)<1$, we first establish the log-concavity of $\Phi^*(z)$ for piece-wise linear $c(\cdot)$ functions. That is, for $0=s_0<s_1<\cdots$, $k_1<k_2<\cdots$ and $w\in[s_{i-1},s_i]$,

$$c(w) = d_0+\sum_{j=0}^{i-1}k_j(s_j-s_{j-1})+k_i(w-s_{i-1}). \tag{EC.2}$$

Suppose that $W_q(0)+\frac{z}{\mu}\in[s_{m-1},s_m)$ and $W_q(p^*(z)\Lambda)+\frac{z}{\mu}\in[s_{n-1},s_n)$. By (EC.1),

$$\begin{aligned}
\Phi^*(z) &= (1-\delta)\underline{b}p^*(z)\Lambda+\int_0^{p^*(z)\Lambda}\tilde{\lambda}dc\left(W_q(\tilde{\lambda})+\frac{z}{\mu}\right)\\
&= (1-\delta)\underline{b}p^*(z)\Lambda+\sum_{j=m}^{n-1}(k_{j+1}-k_j)\int_{s_j-\frac{z}{\mu}}^{W_q(p^*(z)\Lambda)}W_q^{-1}(\tilde{\lambda})d\tilde{\lambda}+k_m\int_0^{W_q\left(p^*(z)\Lambda\right)}W_q^{-1}(\tilde{\lambda})d\tilde{\lambda}
\end{aligned}$$

and is differentiable even if $W_q(0)+\frac{z}{\mu}=s_{m-1}$ as

$$\lim_{\epsilon\downarrow 0}\frac{\Phi^*(z+\epsilon)-\Phi^*(z)}{\epsilon}-\lim_{\epsilon\downarrow 0}\frac{\Phi^*(z)-\Phi^*(z-\epsilon)}{\epsilon}=\frac{-(k_m-k_{m-1})W_q^{-1}\left(s_{m-1}-\frac{z}{\mu}\right)}{\mu}=0.$$

Since

$$\begin{aligned}\frac{d\ln\left[\Phi^*(z)\right]}{dz}=\frac{\Phi^{*\prime}(z)}{\Phi^*(z)}&=\frac{-(1-\delta)\underline{b}}{\mu W_q'(p^*(z)\Lambda)\Phi^*(z)}+\frac{-k_m p^*(z)\Lambda}{\mu\Phi^*(z)}\\&\quad+\sum_{j=m}^{n-1}(k_{j+1}-k_j)\frac{W_q^{-1}\left(s_j-\frac{z}{\mu}\right)-W_q^{-1}\left(c^{-1}(R-\underline{b})-\frac{z}{\mu}\right)}{\mu\Phi^*(z)}\leq 0\end{aligned}$$

by applying $p^{*\prime}(z)=-\frac{1}{\Lambda\mu W_q'(p^*(z)\Lambda)}$ from (5), $\Phi^*(z)$ decreases in z. Furthermore, both the first term and the summands in the third term are all decreasing in z, by the concavity of $W_q^{-1}(\cdot)$. Note that $\frac{\Phi^*(z)}{p^*(z)\Lambda}$ is the expected fee paid by a user who joins the system and $G^*(\cdot|z)$ is the fee distribution in equilibrium. Since $G^*(\cdot|z)$ is stochastically decreasing in z, the second term is also decreasing in z. Thus, $\ln\left[\Phi^*(z)\right]$ is concave and, by the Weierstrass' approximation, remains concave for general increasing convex $c(\cdot)$ functions. □

Proof of Proposition 3 When $p^*(z)=1$, $\underline{\gamma}\left(\beta^*(z),z\right)$ is decreasing. Thus, it suffices to show that the function is quasi-convex when $p^*(z)<1$, or equivalently, that $\ln\left((z^*)^{-1}(z)\right)-\ln(\beta^*(z))$ is quasi-convex. Since $-\ln(\beta^*(z))$ is convex by Proposition 2 and

$$\left[\ln\left((z^*)^{-1}(z)\right)\right]'=\frac{\left(\beta\left(\log\left(\beta\right)-1\right)+1\right)^2}{\beta\left(\beta-1\right)} \tag{EC.3}$$

decreases in β and hence increases in z, $\ln\left((z^*)^{-1}(z)\right)-\ln(\beta^*(z))$ is quasi-convex. □

Proof of Proposition 4 As shown in the proof of Proposition 3, $\ln\left((z^*)^{-1}(z)\right)$ is increasing convex while $\ln\left(\beta^*(z)\right)$ is increasing concave. Thus, $\{z^n : n=0,1,\dots\}$ is a decreasing sequence when $z^1\in(z_1^*,z_2^*)$ and an increasing sequence when $z^1<z_1^*$ or $z^1>z_2^*$, and hence must converge in both cases. The sequence can only converge to z_1^* when $z^1\in(z_1^*,z_2^*)$ and to ∞ when $z^1>z_2^*$. When $z^1<z_1^*$, by a simple induction, $\beta^n=\beta^*(z^n)>\beta_1^*$ by the monotonicity of $\beta^*(z)$. Hence, $z^n<z_1^*$ for all n and the sequence converges to z_1^*. □

EC.3. Proofs for Section 5

Proof of Lemma 2 Note that

$$\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right)$$
$$=\alpha\int_0^{\min\left\{\Lambda,W_q^{-1}\left(c^{-1}(R(\delta,0))-\frac{zK_m}{\eta}\middle|\frac{\eta}{K_m},K_m\right)\right\}}\left[R(\delta,0)-c\left(W_q\left(\lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]d\lambda.$$

$\int_0^{W_q^{-1}\left(c^{-1}(R(\delta,0))-\frac{zK_m}{\eta}\middle|\frac{\eta}{K_m},K_m\right)}\left[R(\delta,0)-c\left(W_q\left(\lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]d\lambda$ is the total fee as if $\Lambda=\infty$ and is decreasing and log-concave in z by Proposition 2. Since $\int_0^{\Lambda}\left[R(\delta,0)-c\left(W_q\left(\lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]d\lambda$ is also decreasing and log-concave in z, so is $\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right)$, and hence, the left-hand side of (13) is quasi-convex in z by a similar argument as in the proof of Proposition 3. Thus, to establish Lemma 2, it suffices to show that, for a given z, there exists $(\mu,K,\underline{b})$ such that $(z,\mu,K,\underline{b})$ is feasible if and only if

$$\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),z\right)\le\delta. \tag{EC.4}$$

To show the "if" part, assume that (EC.4) holds. Since $\lim_{\underline{b}\to+\infty}\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}\right),z\right)=1$ and $\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}\right),z\right)$ is continuous in $\underline{b}$, there exists $\underline{b}>\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+$ such that $\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}\right),z\right)=\delta$, i.e., $\left(z,\frac{\eta}{K_m},K_m,\underline{b}\right)$ is a feasible solution. To show the "only if" part, assume that $(z,\mu,K,\underline{b})$ is a feasible solution. Then, $\underline{\gamma}(z|\mu,K,\underline{b})=\delta$ and

$$\begin{aligned}\beta^*(z|\mu,K,\underline{b})&\le\beta^*\left(z\middle|\mu,K,\frac{1}{1-\delta}\left[R(\delta,0)+c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+\right)\\&\le\beta^*\left(z\middle|\frac{\eta}{K},K,\frac{1}{1-\delta}\left[R(\delta,0)+c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+\right)\\&\le\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)+c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),\end{aligned}$$

where the first inequality follows as

$$\frac{\partial\beta^*(z|\mu,K,\underline{b})}{\partial\underline{b}}=\begin{cases}\alpha\Lambda>0, & \text{if } \underline{b}<\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+,\\ \alpha\underline{b}\frac{\partial\lambda^*(z|\mu,K,\underline{b})}{\partial\underline{b}}<0, & \text{if } \underline{b}>\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+,\end{cases} \tag{EC.5}$$

i.e., $\beta^*(z|\mu,K,\underline{b})$ is maximized at $\underline{b}=\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+$ for any fixed (z,μ,K), the second inequality follows as

$$\beta^*\left(z\middle|\mu,K,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+\right)$$
$$=\int_0^{\lambda^*\left(z\middle|\mu,K,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+\right)}\left[R(\delta,0)-c\left(W_q(\tilde{\lambda}|\mu,K)+\frac{z}{\mu}\right)\right]d\tilde{\lambda},$$

which increases in μ since both the upper limit and integrand are non-negative and increasing in μ, and the last inequality follows if $W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)$ is increasing in K, which is shown below. Thus,

$$\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),z\right)\le\underline{\gamma}\left(\beta^*\left(z|\mu,K,\underline{b}\right),z\right)=\delta,$$

i.e., (EC.4) holds.

It remains to show that $W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)$ is increasing in K. Letting $\theta\in(0,1)$ be the unique solution to $\frac{\theta-\theta^{K+1}}{1-\theta}=K\frac{\Lambda}{\eta}$, we have $W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)=\frac{K}{\eta[1-(1+K)\theta^K+K\theta^{K+1}]}=\frac{\theta}{(1-\theta)[\Lambda(1+K)-(\eta+\Lambda K)\theta]}$. It is clear that $W_q(\Lambda|\eta,1)<W_q\left(\Lambda\middle|\frac{\eta}{2},2\right)$ and it suffices to show that $\frac{d}{dK}W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)>0$ for $K\in[2,\infty)$, which is equivalent to

$$\frac{(\eta+2\Lambda K+\Lambda)\,\theta}{\Lambda(1+K)-(\eta+\Lambda K)\,\theta^2}>\frac{-\eta\theta^{K+1}\ln(\theta)}{\Lambda K(1-\theta)}+1-\frac{1}{K}\tag{EC.6}$$

as $\frac{d\theta}{dK}=\frac{\theta\left[\Lambda(1-\theta)+\eta\theta^{K+1}\ln(\theta)\right]}{K[\Lambda(1+K)-(\eta+\Lambda K)\theta]}$. Since $1+\frac{\Lambda}{\eta}K-\frac{\Lambda K}{\theta\eta}=\theta^K\in\left(0,\frac{\Lambda}{\eta}\right)$, $\theta\in\left(\frac{\Lambda K}{\eta+\Lambda K},\frac{\Lambda K}{\eta-\Lambda+\Lambda K}\right)$ and hence,

$$\begin{aligned}\frac{(\eta+2\Lambda K+\Lambda)\,\theta}{\Lambda(1+K)-(\eta+\Lambda K)\,\theta^2}&>\frac{K\,(\eta+2\Lambda K+\Lambda)}{(1+K)\eta+\Lambda K}\\&\ge\frac{2\Lambda}{\left[\Lambda^2+\eta^2\right]K-(\eta-\Lambda)^2}-\frac{1}{K}+1\\&>\frac{-\eta\theta^{K+1}\ln(\theta)}{\Lambda K(1-\theta)}+1-\frac{1}{K},\end{aligned}$$

where the last inequality holds as $\theta^K<\left(\frac{\Lambda K}{\eta-\Lambda+\Lambda K}\right)^K\le\frac{2\Lambda^2K}{\left[\Lambda^2+\eta^2\right]K-(\eta-\Lambda)^2}$ and $-\frac{\ln(\theta)}{1-\theta}\le\frac{1}{\theta}$. Thus, (EC.6) holds and hence, $W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)$ is increasing in K. □

Proof of Proposition 5 Denote the throughput rate as $\lambda^*(z|\mu,K,\underline{b})=p^*(z|\mu,K,\underline{b})\Lambda$. Suppose that $(z,\mu,K,\underline{b})$ is an optimal solution. Note that

$$\begin{aligned}\lambda^*(z|\mu,K,\underline{b})&\le\lambda^*\left(z\middle|\mu,K,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+\right)=\min\left\{W_q^{-1}\left(c^{-1}(R(\delta,0))-\frac{z}{\mu}\middle|\mu,K\right),\Lambda\right\}\\&\le\lambda^*\left(z\middle|\frac{\eta}{K},K,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K},K\right)+\frac{zK}{\eta}\right)\right]^+\right)\\&\le\lambda^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{zK_m}{\eta}\right)\right]^+\right)=\lambda^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right)\\&\le\lambda^*(z|\mu,K,\underline{b}),\end{aligned}\tag{EC.7}$$

where the first inequality follows as λ^* is maximized at $\underline{b}=\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q(\Lambda|\mu,K)+\frac{z}{\mu}\right)\right]^+$ for given (μ,K,z) by (5), the second inequality follows as $W_q(\lambda|\mu,K)$ is decreasing in μ for a given (λ,K), the third inequality follows as $W_q\left(\lambda\middle|\frac{\eta}{K},K\right)$ is increasing in K for a given λ, shown in the proof of Lemma 2, and the last inequality follows from the optimality of $(z,\mu,K,\underline{b})$. Thus, $\lambda^*(z|\mu,K,\underline{b})=\lambda^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right)=\lambda^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right)$, implying that either $(\mu,K,\underline{b})=\left(\frac{\eta}{K_m},K_m,0\right)$ or $\lambda^*(z|\mu,K,\underline{b})=\Lambda$. Furthermore, since $\beta^*\left(z\middle|\tilde{\mu},\tilde{K},\underline{\tilde{b}}\right)$ is minimized at $\left(\tilde{\mu},\tilde{K},\underline{\tilde{b}}\right)=\left(\frac{\eta}{K_m},K_m,0\right)$ when $\lambda^*\left(z\middle|\tilde{\mu},\tilde{K},\underline{\tilde{b}}\right)=\Lambda$ by (9), $\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right)\le\beta^*\left(z|\mu,K,\underline{b}\right)$ and hence, $\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right),z\right)\ge\underline{\gamma}\left(\beta^*\left(z|\mu,K,\underline{b}\right),z\right)=\delta$ and the throughput optimization problem can be reduced to the one that determines the confirmation latency z as

$$\max_z \quad \lambda^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right) \tag{EC.8}$$

$$\text{s.t.}\quad \underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),z\right)\le\delta, \tag{EC.9}$$

$$\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right),z\right)\ge\delta. \tag{EC.10}$$

Since the left-hand sides of both (EC.9) and (EC.10) are quasi-convex by the proof of Lemma 2 and Proposition 3, respectively, if the feasible region defined by (EC.9) and (EC.10) is non-empty, there exist $\underline{z}=z_\lambda$ and $\bar{z}\ge z_\lambda$ such that (EC.9) is binding and the structure of optimal solutions can be summarized as follows.

1. If $R(\delta,0)<c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{z_\lambda K_m}{\eta}\right)$, $(z,\mu,K,\underline{b})=\left(z_\lambda,\frac{\eta}{K_m},K_m,0\right)$ is the unique optimal solution and the optimal $\lambda^*\left(z_\lambda\middle|\frac{\eta}{K_m},K_m,0\right)<\Lambda$.
2. Otherwise, the optimal solution is not unique and the optimal throughput $\lambda^*\left(z|\mu,K,\underline{b}\right)=\Lambda$. Let $z_R=\frac{\eta}{K_m}\left[c^{-1}\left(R(\delta,0)\right)-W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)\right]$.
   (a) If there exist $\underline{z}'$ and $\bar{z}'$, $\underline{z}'\le\bar{z}'$, such that (EC.10) is binding, then the set of optimal z is $[z_\lambda,\min\{z_R,\underline{z}'\}]$.
   (b) Otherwise, the set of optimal z is $[z_\lambda,\min\{z_R,\bar{z}\}]$.

Thus, Proposition 5 holds. □

Proof of Proposition 6 Suppose that $(z,\mu,K,\underline{b})$ is an optimal solution. Then,

$$\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),z\right)\le\delta\le\underline{\gamma}\left(\beta^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right),z\right)$$

by the proof of Proposition 5, and hence, there exists $\underline{b}^*(z)\in\left[0,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right]$ such that $\underline{\gamma}\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)=\delta$ and $\Phi^*(z|\mu,K,\underline{b})=\Phi^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)$. Furthermore, users' expected total utility

$$\begin{aligned}U^*(z|\mu,K,\underline{b})&=\Lambda R(\delta,0)-\Phi^*(z|\mu,K,\underline{b})-\int_0^\Lambda c\left(W_q(\lambda|\mu,K)+\frac{z}{\mu}\right)d\lambda\\&\le\Lambda R(\delta,0)-\Phi^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)-\int_0^\Lambda c\left(W_q\left(\lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{K_m}{\eta}\right)d\lambda\\&=U^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right).\end{aligned}$$

Thus, we must have $(\mu,K)=\left(\frac{\eta}{K_m},K_m\right)$ and the problem to optimize users' total utility can be reduced to $\max_{z\in[z_\lambda,\min\{z_R,\underline{z}'\}]}\left\{U^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)\right\}$ if (EC.10) can be binding and to

$\max_{z\in[z_\lambda,\min\{z_R,\bar{z}_\lambda\}]} \left\{U^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)\right\}$ otherwise. Furthermore, by equations (9) at $p^*(z)=1$ and (2), $U^*\left(z\middle|\frac{\eta}{K_m},K_m,\underline{b}^*(z)\right)$ can be written as a function of $\beta^*=\alpha\Phi^*$ as

$$U^*(\beta^*)=\Lambda R(\delta,0)-\frac{\beta^*}{\alpha}-\int_0^\Lambda c\left(W_q\left(\tilde{\lambda}\middle|\frac{\eta}{K_m},K_m\right)+\frac{\ln(2\delta)}{\frac{\eta}{K_m}\left[1-\frac{1}{\beta^*}-\ln(\beta^*)\right]}\right)d\tilde{\lambda}. \tag{EC.11}$$

Note that

$$\frac{dU^*(\beta^*)}{d\beta^*}=-\frac{1}{\alpha}+\frac{\ln(2\delta)}{\frac{\eta}{K_m}}\frac{\left(\frac{1}{\beta^*}\right)^2-\frac{1}{\beta^*}}{\left[1-\frac{1}{\beta^*}-\ln(\beta^*)\right]^2}\cdot\int_0^\Lambda c'\left(W_q\left(\lambda\middle|\frac{\eta}{K_m},1\right)+\frac{\ln(2\delta)}{\frac{\eta}{K_m}}\frac{1}{1-\frac{1}{\beta^*}-\ln(\beta^*)}\right)d\lambda,$$

which decreases from ∞ to $-\frac{1}{\alpha}$ as β^* increases from 1 to ∞, i.e., there exists a unique maximizer $\hat{\beta}^*$. Furthermore, since $U^*\left(\beta^*\left(z_\lambda\middle|\frac{\eta}{K_m},K_m,0\right)\right)=0$, $\left.\frac{dU^*(\beta^*)}{d\beta^*}\right|_{\beta^*=\beta^*\left(z_\lambda\middle|\frac{\eta}{K_m},K_m,0\right)}\le 0$ and hence $z^*(\hat{\beta}^*)\ge z_\lambda$. Thus, there is a unique optimal $z=\min\left\{z_R,\underline{z}',z^*\left(\hat{\beta}^*\right)\right\}$ if (EC.10) can be binding and $z=\min\left\{z_R,\bar{z},z^*\left(\hat{\beta}^*\right)\right\}$ otherwise. □

EC.4. Proofs for Section 6

Proof of Proposition 7 The constraints of the reduced optimization problem analogous to (EC.8)-(EC.10) can be written as

$$\underline{\gamma}\left(\alpha B_0+\alpha\Phi^*\left(z\middle|\frac{\eta}{K_m},K_m,\frac{1}{1-\delta}\left[R(\delta,0)-c\left(W_q\left(\Lambda\middle|\frac{\eta}{K_m},K_m\right)+\frac{zK_m}{\eta}\right)\right]^+\right),z\right)\le\delta, \tag{EC.12}$$

$$\underline{\gamma}\left(\alpha B_0+\alpha\Phi^*\left(z\middle|\frac{\eta}{K_m},K_m,0\right),z\right)\ge\delta. \tag{EC.13}$$

Proposition 7 can be established by the same procedures for solving the corresponding optimization problems under the basic model. □

Proof of Proposition 8 Following a similar argument as that provided in the proof of Theorem 1, we can show that G^* is continuous for any given p^*. Next, we show that b increases in C. Suppose that $c_1<c_2$ but $b_1>b_2$. As a result, it is more costly for users with c_1 to bid at b_2 than at b_1, i.e.,

$$(1-\delta)b_1+c_1W_q\left((1-G^*(b_1)p^*\Lambda)\right)\le(1-\delta)b_2+c_1W_q((1-G^*(b_2)p^*\Lambda))$$

or

$$W_q\left((1-G^*(b_2))p^*\Lambda\right)-W_q((1-G^*(b_2))p^*\Lambda)\ge\frac{(1-\delta)(b_1-b_2)}{c_1}.$$

Hence, it is more costly for users with c_2 to bid at b_2 than at b_1 as

$$[(1-\delta)b_2+c_2W_q((1-G^*(b_2))p^*\Lambda)]-[(1-\delta)b_1+c_2W_q((1-G^*(b_1))p^*\Lambda)]\ge(1-\delta)\left[b_2-b_1+\frac{c_2}{c_1}(b_1-b_2)\right]>0,$$

which contradicts with the definition of an equilibrium.

The monotonicity of b depending on C implies that users with a waiting cost $C(qp^*)$ bid at $b(q)$ in equilibrium, i.e., $b(q)$ is a minimizer of the total cost $(1-\delta)b + C(qp^*)\, W_q((1-G^*(b))p^*\Lambda)$. By the first-order optimality condition, we have:

$$(1-\delta)\frac{db(q)}{dq} = C(qp^*)\, W_q'((1-q)p^*\Lambda).$$

Solving the above differential equation with the boundary condition $b(0) = \underline{b}$, we obtain the desired result for $b(q)$. □

Proof of Proposition 9 Again we can show that G^* is continuous for any given p^*. We first show that b increases in δ. Suppose that $\delta_1 > \delta_2$ but $b_1 < b_2$. Then, it is more costly for users with z_2 to bid at b_1 than at b_2, i.e.,

$$(1-\delta_2)b_2 + cW_q\left((1-G^*(b_2)p^*\Lambda)\right) \le (1-\delta_2)b_1 + cW_q((1-G^*(b_1)p^*\Lambda))$$

or

$$cW_q((1-G^*(b_1)p^*\Lambda)) - cW_q\left((1-G^*(b_1)p^*\Lambda)\right) \ge (1-\delta_2)(b_2-b_1).$$

Then,

$$\begin{aligned}&[(1-\delta_1)b_1 + cW_q((1-G^*(b_1))p^*\Lambda)] - [(1-\delta_1)b_2 + cW_q((1-G^*(b_1))p^*\Lambda)]\\ &> (1-\delta_1)(b_1-b_2) + (1-\delta_2)(b_2-b_1) = (\delta_1-\delta_2)(b_2-b_1) > 0,\end{aligned}$$

i.e., it is more costly for users with z_1 to bid at b_1 than at b_2, which contradicts with the definition of an equilibrium.

Suppose that all users join the system. The monotonicity of b depending on δ implies that users with a security requirement $\delta(q)$ bid at $b(q)$ in equilibrium, i.e., $b(q)$ is a minimizer of the total cost $(1-\delta(q))b + c\left(W_q((1-G^*(b))p^*\Lambda) + \frac{\ln[2\delta(q)]}{\mu\left[1-\frac{1}{\beta}+\ln\left(\frac{1}{\beta}\right)\right]}\right)$. By the first-order optimality condition, we have:

$$(1-\delta(q))\frac{db(q)}{dq} = cW_q'((1-q)p^*\Lambda).$$

Solving the above differential equation with the boundary condition $b(0) = \underline{b}$, we obtain the desired result for $b(q)$. □

Proof of Lemma 3 Let $N(t)$ be the difference in length between the longest chain and the fork after $\mathcal{B}$ has survived for t amount of time after it is mined. Note that $N(t)$ is a continuous time Markov Chain with transition rates $q_{i,i+1} = \mu$ and $q_{i,i-1} = \frac{\mu}{\beta}$ for all integer i. Let S_n be its embedded chain. Then, by Theorem 11 of Alm (2002), for any integer $i \geq 0$,

$$\begin{aligned}
P(\tau > t | N(0) = i) &= P(N(s) \geq 0,\, 0 \leq s \leq t | N(0) = i) \\
&= 1 - \frac{1}{\beta^{i+1}} + \sum_{k=0}^{\infty} e^{-\frac{(1+\beta)\mu t}{\beta}} \frac{[(1+\beta)\mu t]^k}{\beta^k k!} \sum_{n=k+1}^{\infty} \frac{i+1}{n} P(S_n = -1 | S_0 = i) \\
&= \sum_{n=0}^{\infty} \frac{(i+1) e^{-\frac{(1+\beta)\mu t}{\beta}}}{n} P(S_n = -1 | S_0 = i) \sum_{k=0}^{n-1} \frac{[(1+\beta)\mu t]^k}{\beta^k k!},
\end{aligned}$$

and $P(\tau > t | N(0) = i) = 0$ for $i < 0$. Furthermore, since the fork starts to grow once the immediate predecessor of $\mathcal{B}$ has been mined, we have $P(N(0) = i) = \frac{\beta}{(\beta+1)^{2-i}}$ for $i \leq 1$. Thus,

$$\begin{aligned}
P(\tau > t) &= P(\tau > t | N(0) = 1) \frac{\beta}{\beta+1} + P(\tau > t | N(0) = 0) \frac{\beta}{(\beta+1)^2} \\
&= \frac{\beta - 1}{\beta} + \sum_{n=1}^{\infty} \binom{2n}{n-1} \left\{ \frac{2\beta^n e^{-\frac{(\beta+1)\mu t}{\beta}}}{2n(\beta+1)^{2n+1}} \sum_{k=0}^{2n-1} \frac{[(1+\beta)\mu t]^k}{\beta^k k!} \right\} \\
&\quad \frac{\beta - 1}{(\beta+1)^2} + \sum_{n=1}^{\infty} \binom{2n-1}{n-1} \left\{ \frac{\beta^n e^{-\frac{(\beta+1)\mu t}{\beta}}}{(2n-1)(\beta+1)^{2n+1}} \sum_{k=0}^{2n-2} \frac{[(1+\beta)\mu t]^k}{\beta^k k!} \right\} \\
&= \frac{\beta - 1}{\beta} + \frac{\beta - 1}{(\beta+1)^2} + \sum_{n=1}^{\infty} \binom{n}{\lceil n/2 \rceil - 1} \left\{ \frac{(3 + (-1)^n)\, \beta^{\lceil n/2 \rceil} e^{-\frac{(\beta+1)\mu t}{\beta}}}{2n(\beta+1)^{2\lceil n/2 \rceil + 1}} \sum_{k=0}^{n-1} \frac{[(\beta+1)\mu t]^k}{\beta^k k!} \right\}.
\end{aligned}$$

□

Proof of Proposition 10 A system equilibrium is a solution (Z^*, β^*) to the two equations $\beta^* = \alpha\Phi^*(Z^*)$ and $Z^* = Z(\beta^*, \delta)$, or equivalently, $\ln(Z^{-1}(Z^*|\delta)) = \ln(\alpha\Phi^*(Z^*))$. The assumption implies that $[\ln(Z^{-1}(Z^*|\delta))]' = \frac{1}{Z^{-1}(Z^*|\delta) Z'(Z^{-1}(Z^*|\delta))}$ is negative and increases in Z^*, i.e., $\ln(Z^{-1}(Z^*|\delta))$ is convex decreasing. By Proposition 2, there can be at most two solutions to $\ln(Z^{-1}(z^*|\delta)) = \ln(\alpha\Phi^*(z^*))$. □