Sales Information Transparency and Trust in Repeated Vertical Relationships

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Online Supplement

We start this online supplement with a discussion of the main model assumptions. Then, Appendix B provides the proofs of propositions 1 - 7. We then, in Appendix C, show that the retailer's individual rationality constraint in state H and incentive compatibility constraint in state L are not binding in the dynamic game. Appendix D considers an alternative mechanism under past-sales information.

Appendix A: Discussion of Main Model Assumptions

In the main model, a few important assumptions were adopted. In this section, we discuss these assumptions, describe the way they can be interpreted to practical market conditions and highlight which assumptions are critical to our results.

Multiple states of demand

For simplicity, our main model assumes two possible states of demand. Yet, asymmetric information may involve additional states, which can be drawn from a discrete or a continuous distribution. In this section, we discuss the implications of such a setting on the results reported above.

A standard result in a setting with multiple states (i.e., more than two), discrete or continuous, is that the manufacturer offers a menu that specifies more than two options. In a static game, the manufacturer sets the quantity that maximizes the profits of a centralized channel for the highest possible state, which is captured by state H in our model, and distorts the quantities below the monopoly level for all other states of demand, which are captured by state L in our model. Such quantity distortion intensifies as the state of demand decreases. For a formal treatment of the case of multiple states under the static setting see Laffont and Martimort (2002).[9] In particular, when the demand parameter of our model, V, is continuously distributed along the interval $V \sim [V_L, V_H]$ according to a distribution function $f(V)$ with a CDF $F(V)$, the quantity in the static game under asymmetric information is $q_V^S = V/2 - (1 - F(V))/2f(V)$.[10] At the

[9] Pages 86 - 92 cover the case of more than two discrete types and pp. 134 - 140 analyze a continuum of types.

[10] To see why, the marginal information rents, equivalent to $\pi_R(q_L; H) - \pi_R(q_L; L) = (V_H - V_L)q_L$ in our two-states framework, is $\partial\pi_R(q_V; V)/\partial V = q_V$. Hence the expected information rents is: $\int_{V_L}^{V_H} \left(\int_{V_L}^{V} q_V dV \right) f(V)dV = \int_{V_L}^{V_H} \frac{1-F(V)}{f(V)} q_V f(V)dV$.

highest possible state of demand, $q^S_{V_H} = V_H/2$ as in the two-states case analyzed above. At the lowest possible state of demand and for a uniform distribution $q^S_{V_L} = V_L/2 - (V_H - V_L)/2$, which qualitatively corresponds to q^S_L in the two-states model in equation (4). Hence, the results of our model concerning the quantity distortion in state L qualitatively capture the predicted quantity distortion of all states (below the highest possible state) in the context of a model with multiple states. Applying the intuition behind the results of our model to a setting with multiple states, we expect that a repeated game without past sales information sharing reduces the quantity distortion in all states of demand (below the highest state). According to the same logic, past sales information is expected to further reduce the quantity distortion.

As for our main result – that the retailer may benefit or hurt by past-sales information sharing – notice that this result (as illustrated by Figure 1) depends on the *expected* profit of the retailer among all states, and it does not directly depend on the number of states. We therefore expect that the two main effects of past sales information sharing that were identified above: manufacturer's ability to detect a deviation (reduces the retailer's expected information rents) and the manufacture's incentive to raise the quantity in low states (increases the retailer's information rents), should follow to more than two states of demand. Hence, along the lines of our model, the retailer may find it optimal to share past sales information or hold such information, depending on the strength of these two conflicting effects.

The retailer has the market power

In the main model, we assumed that the manufacturer – the uninformed firm – has the market power to design a take-it-or-leave-it menu of contracts to the retailer: the privately informed firm. The assumption regarding the distribution of market power in the channel is crucial to our results. To understand why, consider the opposite extreme case in which the retailer is endowed with both the private information regarding demand and has the market power to make a take-it-or-leave offer to the manufacturer. Then, the retailer can offer the contract that specifies the monopoly quantity and compensates the manufacturer for his production costs. In this case, the retailer extracts the entire surplus from the distribution channel, and leaves zero payoff to the manufacturer. A similar outcome happens if it is the manufacturer that has both the market power and is endowed with the private information. Therefore, our model is relevant in settings in which one firm has superior information, while the other has the market power. Much of the literature that studies models of asymmetric information with contract design adopts similar assumptions (e.g., Ha 2001, Özer and Wei 2006, Lobel and Xiao 2017).

Demand is correlated between periods

In the main model, it was assumed that the state of demand fluctuates in an i.i.d manner between periods. Under this assumption, past sales information that is shared cannot improve the predictive capabilities of the manufacturer, thus, allowing us to focus on the strategic effects of sharing past sales information and mute the role of such information in forecasting future demand. We now relax this assumption, and explore the other extreme case – when demand is perfectly correlated across periods, and we then discuss the case of imperfect demand correlation between periods.

Suppose that the state of demand in all periods is identical and equals to $V_\theta \in \{V_H, V_L\}$ with probabilities p and $1-p$, respectively. In the first period, the manufacturer offers the retailer a menu $\{(q_H, T_H), (q_L, T_L)\}$ and the retailer chooses a specific contract. When past sales information is not shared, in equilibrium, the chosen contract reveals the demand state.Therefore, in all future periods, the manufacturer offers a contract $(q_{\widetilde{\theta}}, T_{\widetilde{\theta}}) = (q^*_{\widetilde{\theta}}, \pi_R(q^*_{\widetilde{\theta}}; \widetilde{\theta}))= (V_{\widetilde{\theta}}/2, V^2_{\widetilde{\theta}}/4)$, where $\widetilde{\theta}$ is the inference made by the manufacturer based on the retailer's chosen contract during the first period and $q^*_\theta = V_\theta/2$ is the quantity that maximizes the centralized profit given θ. When the retailer shares past sales information, the manufacturer observes the state of demand of all future periods at the end of the first period and offers in each period the contract that implements the centralized outcome given the true state, $(q_\theta, T_\theta) = (q^*_\theta, \pi_R(q^*_\theta; \theta))= (V_\theta/2, V^2_\theta/4)$. Note that in both of these cases, when no-past sales information is shared and when it is shared, in equilibrium the manufacturer earns in the second period onward the centralized profits while the retailer earns a payoff of 0. Therefore, the difference between these two cases stems from the contract offered by the manufacturer during the first period. The following proposition summarizes the preference of the retailer with respect to information sharing in this setting.

Proposition 7. ***(with perfectly correlated demand, the retailer never shares information).*** *Suppose that demand is identical in all periods. Then, under both past sales and no past sales information, in the first period the manufacturer sets the static quantities and then the centralized quantities in all future periods. Moreover, the retailer is always hurt by sharing past sales information while the manufacturer always benefits from such information.*

The main conclusion is that when past sales information provides perfect knowledge regarding future demand, information losses its strategic role, and the retailer will choose not to share it with the manufacturer. This result highlights the main contribution of our paper: identifying the strategic effect of past sales information as a way to form trust and showing that because of this effect, the retailer may want to share such information. Intuitively, recall that sharing past sales information in our model has two effects on the retailer. First, given a fixed quantity, past sales information decreases the retailer's information rents because the manufacturer can detect and punish a retailer's deviation from choosing the correct contract. Second, past sales information motivates the manufacturer to raise the offered quantity during periods of low demand, which increases the overall efficiency of the distribution channel and also the retailer's information rents. When demand is perfectly correlated across periods, the second effect vanishes and the manufacturer does not increase the quantity following information sharing: the offered quantity is identical to the quantity in the static setting (during the first selling period) and is independent of whether there is information sharing or not. Consequently, past sales information always hurts the retailer.

Based on the analysis of the two extreme cases, of i.i.d demand and perfectly correlated demand, we conjecture that for the case of non-perfect (but positive) correlation between demand periods, there is a threshold, based on the level of correlation, that determines whether the retailer will choose to share with its manufacturer past sales information.

Sales information manipulation by the retailer

So far, we have assumed that the retailer cannot misrepresent the sales information: if information is shared, it is truthfully transmitted to the manufacturer, and in Section 5.1 the assumption was that with a certain ex-ante probability information is not transmitted to the manufacturer. In both of these cases, the retailer was unable to manipulate the content of the shared information.

We now discuss the implications of information manipulation. Suppose that the retailer can choose to transmit past-sales information in a manipulative manner such that when demand is high, the retailer can transmit the wrong information as to create the impression that demand is actually low. In this case, if the retailer wishes to choose a contract not according to the actual demand state, he will manipulate the shared information such that observing past sales cannot allow the manufacturer to infer deviation. Anticipating this, the manufacturer ignores past-sales information. Since such information cannot be used by the manufacturer, the model will be identical to the one without information sharing. We summarize this discussion using the following Corollary.

Corollary 7. *When past sales information can be manipulated, the model is identical to the one without past sales information sharing.*

It is also worth noting that in such a case, a signaling game may emerge in which the retailer signals the true state of demand to the manufacturer. We leave the analysis of such a setting for future research.

Retailer's ability to carry inventory between periods

Another aspect that was muted in the main model is the ability of the retailer to carry inventory between periods. In the main model, we have assumed that units that are not sold during a specific period cannot be used in future periods.

The ability of the retailer to carry inventory between periods changes the solution outlined above. Due to the complexity of this issue, such an extension deserves a separate paper. Below we explain the way such an ability influences the dynamics between the retailer and the manufacturer. Consider a retailer observing a low market demand; in the main model, we show that in the optimal solution such a retailer will be strictly better-off not mimicking a retailer observing a high demand state; thus, the incentive compatibility constraint of the low-type retailer is non-binding. However, when the low type can carry inventory it is possible that for the solution outlined in the main model and when no-information is shared, he will prefer to mimic a retailer observing a high demand state. In this case, the retailer receives a high number of units from the manufacturer and sells in the market only partial quantity (based on the low market demand). In a future period, when demand is high, the retailer may choose the contract designed for the low market demand, and supplement the quantity offered for this contract with the units held in inventory. Moreover, when demand is low, the retailer may choose to reject the contract and sell only the units that he already holds in inventory. Therefore, the contract outlined above for the case with no-information sharing may not constitute an equilibrium when units can be held in inventory and sold in future periods. Yet, note that when past sales information is shared, such an outcome cannot happen because the manufacturer will observe a deviation in the form of discrepancy between the sold quantity and the purchased quantity. While in this paper we do not analyze the case of carrying inventory between periods, the strategic role of inventory in dynamic models has been recognized before. Some examples include Anand et al. (2008), Guan et al. (2019) and Roy et al. (2020), where the last paper also discusses the effect of information transparency on the ability to use inventory in a strategic manner.

Appendix B: Proofs

Below are the proofs of propositions 1 - 7.

Proof of Proposition 1:

Differentiating (10) with respect to q_L and q_H yields the following first-order conditions:

$$\frac{\partial \Pi_M^{D,ne}(q_H, q_L)}{\partial q_H} = p\pi'_R(q_H; H) = 0.$$

$$\frac{\partial \Pi_M^{D,ne}(q_H, q_L)}{\partial q_L} = -p\pi'_R(q_L; H) + \pi'_R(q_L; L) + \delta p \Delta'(q_L) = 0. \tag{26}$$

Substituting $\pi'_R(q_{\widetilde{\theta}}; \theta) = V_\theta - 2q_{\widetilde{\theta}}$ (where $\theta, \widetilde{\theta} \in \{H, L\}$) and $\Delta'(q_L) = V_H - V_L$ into the first-order conditions and solving for q_L and q_H yields $q_H^{D,ne} = q_H^S = q_H^*$ and $q_L^{D,ne}$ as defined in (11). Turning to $T_L^{D,ne}$ and $T_H^{D,ne}$, solving IR_L^D (in (6)) and IC_H^S (in (2)) for T_H and T_L given q_L and q_H yields:

$$T_L^{D,ne} = \pi_R(q_L; L) + \delta p\big(\Delta(q_L) - (\Delta(q_L^S)\big), \tag{27}$$

$$T_H^{D,ne} = \pi_R(q_H; H) - \Delta(q_L)$$

$$+\delta p\big(\Delta(q_L) - \Delta(q_L^S)\big).$$

Substituting $\pi_R(q_{\widetilde{\theta}}; \theta) = (V_\theta - q_{\widetilde{\theta}})q_{\widetilde{\theta}}$, $q_H = q_H^{D,ne}$, $q_L = q_L^{D,ne}$ and rearranging yields (12).

Remark: Notice that the first line of $T_H^{D,ne}$ in (27) is identical to T_H^S (evaluated at a given q_L), and the second line is the dynamic element. This additional dynamic term is positive, yet the proposition reveals that $T_H^{D,ne} < T_H^S$. The reason why $T_H^{D,ne} < T_H^S$ even though the second line in $T_H^{D,ne}$ is positive is that $q_L^{D,ne} > q_L^S$. The increase in $q_L^{D,ne}$ above q_L^S decreases $T_H^{D,ne}$ because it increases the static information rents (represented by the term: $-\Delta(q_L)$ of the first line), but increases the dynamic information rents (represented by the second line). As $\delta p < 1$, the second effect is weaker than the first effect, resulting in a decrease in $T_H^{D,ne}$ below T_H^S.■

Proof of Proposition 2:

Differentiating (19) with respect to q_L and q_H yields the following first-order conditions:

$$\frac{\partial \Pi^{D,e}(q_H, q_L)}{\partial q_H} = p\pi'_R(q_H; H) = 0,$$

$$\frac{\partial \Pi_M^{D,e}(q_H, q_L)}{\partial q_L} = -p\pi'_R(q_L; H) + \pi'_R(q_L; L) + \left[\frac{1+p}{1+\delta p}\right]\Delta'(q_L) = 0. \tag{28}$$

Substituting $\pi'_R(q_{\widetilde{\theta}}; \theta) = V_\theta - 2q_{\widetilde{\theta}}$ (where $\theta, \widetilde{\theta} \in \{H, L\}$) and $\Delta'(q_L) = V_H - V_L$ into the first-order conditions and solving for q_L and q_H yields $q_H^{D,e} = q_H^S = q_H^*$ and $q_L^{D,e}$ as defined in (20). Turning to $T_L^{D,e}$ and $T_H^{D,e}$, solving IR_L^D (in (6)) and IC_H^D (in (7)) for T_H and T_L given q_L and q_H yields:

$$T_L^{D,e} = \pi_R(q_L; L) + \frac{\delta p}{1+\delta p}\left(\Delta(q_L) - \Delta(q_L^S)\right), \tag{29}$$

$$T_H^{D,e} = \pi_R(q_H; H) - \Delta(q_L) + \frac{2\delta p}{1+\delta p}\big(\Delta(q_L) - \Delta(q_L^S)\big).$$

Notice that as in the case of no ex-post information, the dynamic element in $T_H^{D,e}$ (the last term in $T_H^{D,e}$) is positive for a given q_L. Yet, $T_H^{D,e} < T_H^S$, because $q_L^{D,e} > q_L^S$ and $\frac{2\delta p}{1+\delta p} < 1$, hence we can apply the same intuition as in the proof of Proposition 1. Finally, substituting $\pi_R(q_{\widetilde{\theta}}; \theta) = (V_\theta - q_{\widetilde{\theta}})q_{\widetilde{\theta}}$, $q_H = q_H^{D,e}$, $q_L = q_L^{D,e}$ and rearranging yields (21).■

Proof of Proposition 3:

The retailer's expected profit in the ex-post information case is: $\Pi_R^{D,e} \equiv p(\pi_R(q_H^{D,e}; H) - T_H^{D,e}) + (1 -$

$p)(\pi_R(q_L^{D,e};L) - T_L^{D,e})$, or:

$$\Pi_R^{D,e} = \frac{p(V_H - V_L)(V_L - pV_H)}{2(1-p)(1+\delta p)^2} \tag{30}$$

$$+\frac{\delta p^2(V_H - V_L)}{2(1-p)(1+\delta p)^2}\left((1-p-\delta(1+p+p^2))V_H + (1+\delta-p+2\delta p)V_L\right).$$

Recalling that $\Pi_R^{D,ne}$ is given by (15), we have that the gap in the retailer's expected profit with and without ex-post information is:

$$\Pi_R^{D,e} - \Pi_R^{D,ne} = \frac{\delta p^3(1-\delta)(V_H - V_L)^2}{2(1-p)(1+\delta p)^2}\left[1 - \delta(2+\delta p)\right]. \tag{31}$$

The first term in (31) is strictly positive, hence the sign of $\Pi_R^{D,e} - \Pi_R^{D,ne}$ is determined according to the sign of the term in the squared brackets in (31). We have that $1 - \delta(2+\delta p) > 0$ iff $\delta < \widetilde{\delta} = \frac{1}{1+\sqrt{1+p}}$. ■

Proof of Proposition 4:

Solving $IC_H^{D,\alpha}$ from (23) and IR_L^D from (6) for T_L and T_H yields:

$$T_L^{D,\alpha}(q_H, q_L) = \pi_R(q_L; L) + \frac{\delta p}{1+\alpha\delta p}\left(\Delta(q_L) - \Delta(q_L^S)\right), \tag{32}$$

$$T_H^{D,e}(q_H, q_L) = \pi_R(q_H; H) - \Delta(q_L) + \frac{(1+\alpha)\delta p}{1+\alpha\delta p}\big(\Delta(q_L) - \Delta(q_L^S)\big).$$

Substituting $T_L^{D,\alpha}(q_H, q_L)$ and $T_H^{D,\alpha}(q_H, q_L)$ into the manufacturer's expected profit, $pT_H + (1-p)T_L$, yields:

$$\Pi_M^{D,\alpha}(q_H, q_L) = p\pi_R(q_H; H) + (1-p)\pi_R(q_L; L) - p\Delta(q_L) \tag{33}$$

$$+\left[\frac{1+\alpha p}{1+\alpha\delta p}\right]\delta p\big(\Delta(q_L) - \Delta(q_L^S)\big).$$

Notice that given q_L and q_H, $\Pi_M^{D,\alpha}(q_H, q_L) \to \Pi_M^{D,ne}(q_H, q_L)$ as $\alpha \to 0$, $\Pi_M^{D,\alpha}(q_H, q_L)$ is increasing with α, and $\Pi_M^{D,\alpha}(q_H, q_L) \to \Pi_M^{D,e}(q_H, q_L)$ as $\alpha \to 1$. This proves the first part of the proposition. Differentiating $\Pi_M^{D,\alpha}(q_H, q_L)$ with respect to q_L and q_H yields $q_H^{D,\alpha}$ and $q_L^{D,\alpha}$ as defined by (24).

Turing to the retailer's expected profit, substituting $T_L^{D,\alpha}(q_H, q_L)$ and $T_H^{D,\alpha}(q_H, q_L)$ to the retailer's expected profit $p(\pi_R(q_H; H) - T_H) + (1-p)(\pi_R(q_L; L) - T_L)$, and evaluating at $q_H^{D,\alpha}$ and $q_L^{D,\alpha}$ yields the retailer's expected profit as a function of α:

$$\Pi_R^{D,\alpha}(\alpha) = \Pi_R^S + \delta(1-\delta)\frac{p^2(1+\alpha p)(V_H - V_L)^2}{2(1-p)(1+\alpha\delta p)^2}. \tag{34}$$

The first term, Π_R^S, is the static expected profit and is independent of α. The second term is maximized at: $\alpha = \frac{1-2\delta}{\delta p}$ (it is straightforward to verify that the second-order condition holds). Because $\frac{1-2\delta}{\delta p} < 0$

when $\delta < 1/2$ and $\frac{1-2\delta}{\delta p} > 1$ when $\delta > \frac{1}{2+p}$, we have that the optimal α for the retailer when taking into account that $0 \leq \alpha \leq 1$ is given by $\hat{\alpha}(\delta)$ as defined in the proposition. ■

Proof of Proposition 5:

Under limited liability, the retailer's participation constraint is identical to the static setting, IR_L^S, while the form of the incentive compatibility constraint is identical to the case of past sales information sharing, $IC_H^{D,e}$. Solving IR_L^S (equation (1)) and IC_H^D (equation (7)) for T_H and T_L, the manufacturer's payoff under limited liability with past sales information sharing, denoted as $\Pi_M^{D,l}$, is:

$$\begin{aligned}\Pi_M^{D,l}(q_H, q_L) &= p\pi_R(q_H; H) + (1-p)\pi_L(q_L; L) - p\big[\pi_R(q_L; H) - \pi_R(q_L; L)\big] \\ &+ \left[\frac{p}{1-\delta(1-p)}\right] \delta p\big(\pi_R(q_L; H) - \pi_R(q_L; L) - (\pi_R(q_L^S; H) - \pi_R(q_L^S; L))\big)\end{aligned} \tag{35}$$

As in the two previous cases of dynamic game without information sharing (equation (10)) and with information sharing (equation (19)), the manufacturer's profit is comprised from the static profit (the first line in equation (35)) and the dynamic information rents (the second line), which is positive if the information rents paid to the high-type retailer are higher than in the static setting. The difference between these three settings is captured in the last term of each profit. Recall from equation (10), that in the dynamic setting without information sharing, the equivalent to the term in the square brackets is 1, while with information sharing, the equivalent term is $\frac{1+p}{1+\delta p} > 1$. Now, under information sharing with limited liability, the equivalent term is $\frac{p}{1-\delta(1-p)}$. As $\frac{p}{1-\delta(1-p)} < 1 < \frac{1+p}{1+\delta p}$, limited liability with information sharing provides the manufacturer with lower expected profits compared with the previous two cases.

Proof of Proposition 6:

We first solve for the optimal contract under limited liability. Differentiating (35) with respect to q_L and q_H yields the following first-order conditions:

$$\begin{aligned}\frac{\partial \Pi^{D,l}(q_H, q_L)}{\partial q_H} &= p\pi'_R(q_H; H) = 0, \\ \frac{\partial \Pi_M^{D,l}(q_H, q_L)}{\partial q_L} &= -p\pi'_R(q_L; H) + \pi'_R(q_L; L) + \left[\frac{p}{1-\delta(1-p)}\right]\Delta'(q_L) = 0.\end{aligned} \tag{36}$$

Substituting $\pi'_R(q_{\widetilde{\theta}}; \theta) = V_\theta - 2q_{\widetilde{\theta}}$ (where $\theta, \widetilde{\theta} \in \{H, L\}$) and $\Delta'(q_L) = V_H - V_L$ into the first-order conditions and solving for q_L and q_H yields $q_H^{D,l} = q_H^S = q_H^*$ and $q_L^{D,l}$, where:

$$q_L^{D,l} = q_L^S + \frac{p}{1-\delta(1-p)} \frac{\delta p}{(1-p)} \frac{(V_H - V_L)}{2} < q_L^{D,ne} \tag{37}$$

Turning to $T_L^{D,l}$ and $T_H^{D,l}$, solving IR_L^S (in (1)) and IC_H^D (in (7)) for T_H and T_L given q_L and q_H yields:

$$T_L^{D,l} = \pi_R(q_L; L), \tag{38}$$

$$T_H^{D,l} = \pi_R(q_H; H) - \Delta(q_L) + \frac{\delta p}{1-\delta(1-p)}\left(\Delta(q_L) - \Delta(q_L^S)\right).$$

Because $q_L^{D,l} > q_L^S$ and $\frac{\delta p}{1-\delta(1-p)} < 1$, we have that $T_H^{D,l} < T_H^S$ even though the second line in $T_H^{D,l}$ is positive. Substituting $\pi_R(q_{\tilde{\theta}};\theta) = (V_\theta - q_{\tilde{\theta}})q_{\tilde{\theta}}$, $q_H = q_H^{D,l}$, $q_L = q_L^{D,l}$ and rearranging yields

$$T_H^{D,l} = T_H^S - \frac{1-\delta}{(1-\delta(1-p))^2}\frac{\delta p^2(V_H - V_L)^2}{2(1-p)}, \tag{39}$$

$$T_L^{D,l} = T_L^S + \frac{p\,(2-\delta(2-p))}{(1-\delta(1-p))^2}\frac{\delta p^2(V_H - V_L)^2}{4(1-p)^2}.$$

Next, we move to the retailer's expected profit in the case of ex-post information with limited liability. The retailer earns: $\Pi_R^{D,l} \equiv p(\pi_R(q_H^{D,l}; H) - T_H^{D,l}) + (1-p)(\pi_R(q_L^{D,l}; L) - T_L^{D,l})$, or:

$$\Pi_R^{D,l} = \Pi_R^{D,ne} - \frac{\delta(1-\delta)p^2(V_H - V_L)^2}{2(1-\delta(1-p))^2}\left[1 - \delta(2 - \delta(1-p))\right], \tag{40}$$

where $\Pi_R^{D,ne}$ is given by (15). Hence, the sign of the gap $\Pi_R^{D,l} - \Pi_R^{D,ne}$ is given by the sign of the term in squared brackets in (40). We have that $1 - \delta(2 - \delta(1-p)) > 0$ iff $\delta < \widehat{\delta} = \frac{1}{1+\sqrt{p}}$. ■

Proof of Proposition 7:

No past - sales information. Given the first period menu, $\{(q_H, T_H), (q_L, T_L)\}$, if the state in all periods is L and the retailer chooses the contract that reveals this state, the retailer receives in the first period the contract (q_L, T_L) followed by the contract $(q_L^*, \pi_R(q_L^*; L))$ in all future periods which yields to the retailer 0 future profits. Hence, the retailer's individual rationality constraint in state L is identical to the static:

$$IR_L^S : \pi_R(q_L; L) - T_L + 0 \cdot \delta/(1-\delta) \geq 0.$$

If the state is H the retailer chooses the contract (q_H, T_H) in the first period based on the revelation principle. This contract is followed by the contract $(q_H^*, \pi_R(q_H; H))$ in all future periods which yields 0 future profits to the retailer. If the retailer misrepresents a high state as a low state, the retailer receives at the first period the contract (q_L, T_L) followed by the contract $(q_L^*, \pi_R(q_L^*; L))$ in all future periods, which yields positive profit because $\pi_R(q_L^*; H) - \pi_R(q_L^*; L) = \Delta(q_L^*) > 0$. Given that the retailer already misrepresented the state at the first period, the retailer will accept the state L centralized contract in all future periods because this is the only contract that will be offered. Hence, the retailer's dynamic incentive compatibility constraint in state H under perfect correlation is:

$$\pi_R(q_H;H) - T_H + \frac{\delta}{1-\delta}\cdot 0 \geq \pi_R(q_L;H) - T_L + \frac{\delta}{1-\delta}\Delta(q_L^*).$$

Solving the two constraints for T_H and T_L and substituting into the manufacturer's expected profit in the first period, $pT_H + (1-p)T_L$, yields (we use the notation $\widehat{\Pi}$ to capture the case of correlated states of demand):

$$\begin{aligned}\widehat{\Pi}_M^{D,ne}(q_H,q_L) = p\pi_R(q_H;H) + (1-p)\pi_R(q_L;L) - p\Delta(q_L) \\ -\frac{\delta p}{1-\delta}\Delta(q_L^*).\end{aligned}$$

Notice that the first line is identical to the static profits under asymmetric information. The second line is negative and represents the additional information rents that the manufacturer needs to leave the retailer due to the retailer's potential profits from misrepresenting the type in all future periods. This last line is independent of the quantities of the first period, hence, the manufacturer sets in the first period the static quantities under asymmetric information, $q_\theta = q_\theta^S$ as defined by (4), and earns $\widehat{\Pi}_M^{D,ne}(q_H^S, q_L^S)$ while the retailer earns the expected profit of:

$$\widehat{\Pi}_R^{D,ne}(q_H^S, q_L^S) = p\Delta(q_L^S) + \frac{\delta p}{1-\delta}\Delta(q_L^*).$$

Past - sales information. Assume that sales information is exchanged, and the manufacturer observes it at the end of the first period. In the second period onward the manufacturer offers the contract that maximizes the centralized profit based on the observable state, regardless of the retailer's chosen contract during the first period. Therefore, in the first period, the manufacturer offers the static menu, $\{(q_H^S, T_H^S), (q_L^S, T_L^S)\}$, because the retailer's chosen contract in the first period does not affect future periods. Hence, the manufacturer earns in the first period $\widehat{\Pi}_M^{D,e}(q_H^S, q_L^S) = \Pi_M^S(q_H^S, q_L^S)$ as given by (3), while the retailer earns $\widehat{\Pi}_R^{D,e}(q_H^S, q_L^S) = p\Delta(q_L^S)$.

Comparison between no past sales and past sales information. Under both no-past sales and past sales information, the manufacturer earns in the second period onward the centralized profits while the retailer earns a payoff of 0. In the first period, under both cases, the offered quantities are identical to the static quantities. Moreover, the manufacturer always benefits from past-sales information while the retailer is always hurt by past sales information because:

$$\widehat{\Pi}_M^{D,e}(q_H^S, q_L^S) - \widehat{\Pi}_M^{D,ne}(q_H^S, q_L^S) = \widehat{\Pi}_R^{D,ne}(q_H^S, q_L^S) - \widehat{\Pi}_R^{D,e}(q_H^S, q_L^S) = \frac{\delta p}{1-\delta}\Delta(q_L^*),$$

where the last term is positive because $\Delta(q_L^*) > 0$.■

Appendix C: Proof that IR_H and IC_L are not binding

Below we show that in the dynamic setting, given that IR_L and IC_H are binding, IR_H and IC_L are satisfied.

Repeated game with no ex-post information

We prove that $IR_H^{D,ne}$ and $IC_L^{D,ne}$ are not binding given any arbitrary q_L and q_H, as long as $q_L < q_H$, which holds in equilibrium because $q_L^{D,ne} \leq q_L^* < q_H^* = q_H^{D,ne}$. We start with $IR_H^{D,ne}$. Rearranging IR_H^D from (6), IR_H^D holds when:

$$\pi_R(q_H; H) - T_H + \frac{\delta}{1-\delta}\left[p(\pi_R(q_H; H) - T_H) + (1-p)(\pi_R(q_L; L) - T_L)\right] - \frac{\delta}{1-\delta}\left[p\Delta(q_L^S)\right] > 0 \tag{41}$$

Substituting $T_L^{D,ne}$ and $T_H^{D,ne}$ from (27) into (41) and rearranging, condition $IR_H^{D,ne}$ becomes $\Delta(q_L) > 0$, which holds because of our assumption that $\pi_R(q; H) > \pi_R(q; L)$ for any q.

Turning to $IC_L^{D,ne}$, this condition is identical to the static constraint in (2). Substituting $T_L^{D,ne}$ and $T_H^{D,ne}$ from (27) into (2), we have that $\pi_R(q_L; L) - T_L - (\pi_R(q_H; L) - T_H) > 0$ if $\Delta(q_H) - \Delta(q_L) > 0$. This condition holds because of our assumption that the gap $\Delta(q) > 0$ is increasing in q and because $q_L^{D,ne} < q_H^{D,ne}$.

Repeated game with ex-post information

We prove that $IR_H^{D,e}$ and $IC_L^{D,e}$ are not binding given any arbitrary q_L and q_H, as long as $q_L < q_H$ and $q_L > q_L^S$, which holds in equilibrium because $q_L^S < q_L^{D,e} < q_H^{D,e}$. We start with $IR_H^{D,e}$. Given T_L and T_H, the IR_H^D is identical to the case of no ex-post information described above. Substituting $T_L^{D,n}$ and $T_H^{D,n}$ from (29) into (41) and rearranging, condition $IR_H^{D,ne}$ holds when:

$$\frac{\Delta(q_L) + \delta p \Delta(q_L^S)}{1 + \delta p} > 0,$$

which holds for any arbitrary q_L and q_H because by assumption, $\Delta(q_L) > 0$.

Next consider $IC_L^{D,e}$. Rearranging IC_L^D from (7), $IC_L^{D,e}$ holds when:

$$\pi_R(q_L; L) - T_L + \frac{\delta}{1-\delta}\left[p(\pi_R(q_H; H) - T_H) + (1-p)(\pi_R(q_L; L) - T_L)\right] \tag{42}$$

$$-\left(\pi_R(q_H; L) - T_H + \frac{\delta}{1-\delta}\left[p\Delta(q_L^S)\right]\right) > 0.$$

Substituting $T_L^{D,n}$ and $T_H^{D,n}$ from (29) into (42) and rearranging, condition $IC_L^{D,ne}$ holds when:

$$\frac{\Delta(q_H)-\Delta(q_L)}{1+\delta p}+\frac{\delta p\left(\Delta(q_H)+\Delta(q_L)-2\Delta(q_L^S)\right)}{1+\delta p}>0. \tag{43}$$

The first term is positive because $\Delta(q_H)-\Delta(q_L)>0$ when $q_H>q_L$. The second term is also positive because $\Delta(q_H)+\Delta(q_L)-2\Delta(q_L^S)>\Delta(q_H)+\Delta(q_L)-2\Delta(q_L)=\Delta(q_H)-\Delta(q_L)>0$, where the first inequality follows because $q_L>q_L^S$ and $\Delta(q)$ is increasing in q, and the second inequality follows because $\Delta(q_H)-\Delta(q_L)>0$ when $q_H>q_L$.

Repeated game with limited liability

Consider first $IR_H^{D,l}$. Substituting $T_L^{D,l}$ and $T_H^{D,l}$ from (38) into IR_H^D as defined by (41) and rearranging, IR_H^D is: $\pi_R(q_L;H)-\pi_R(q_L;L)>0$, which holds by assumption. As the retailer is financially constraint in both states, we need to verify that $\pi_R(q_H;H)-T_H^{D,l}>0$. Substituting $T_H^{D,l}$ from (38) into $\pi_R(q_H;H)-T_H^{D,l}$, we have:

$$\pi_R(q_H;H)-T_H^{D,l}=\frac{(1-\delta)\Delta(q_L)+\delta p\Delta(q_L^S)}{1-\delta(1-p)},$$

which is always positive.

Next, consider $IC_L^{D,l}$. To prove that the financially constraint retailer will not deviate in state L to the contract of state H, it is sufficient to show that $\pi_R(q_H;L)-T_H^{D,l}<0$. Substituting $T_H^{D,l}$ from (38) into $\pi_R(q_H;L)-T_H^{D,l}$, we have:

$$\pi_R(q_H;L)-T_H^{D,l}=\frac{\delta p\Delta(q_L^S)-(1-\delta(1-p))\Delta(q_H)+(1-\delta)\Delta(q_L)}{1-\delta(1-p)}<$$

$$\frac{\delta p\Delta(q_L)-(1-\delta(1-p))\Delta(q_H)+(1-\delta)\Delta(q_L)}{1-\delta(1-p)}=$$

$$-\left(\Delta(q_H)-\Delta(q_L)\right)<0,$$

where the first inequality follows because $q_L=q_L^{D,l}>q_L^S$ and $\Delta(q)$ is increasing in q, and the last inequality follows whenever $q_H<q_L$.

Appendix D: An alternative mechanism under past-sales information

Consider the scenario in which past-sales information is shared. Suppose that the manufacturer offers the following mechanism. In each period, as long as the retailer reported the true state during the previous period:

Stage 1: The manufacturer pays F

Stage 2: The manufacturer offers a menu of contracts $\{(q_H^*, T_H^*), (q_L^*, T_L^*)\}$ where recall that q_θ^* is the centralized selling quantity in state $\theta \in \{H, L\}$ and $T_\theta^* = \pi_R(q_\theta^*; \theta)$.

Stage 3: The retailer observes the true realization of $\theta \in \{H, L\}$ and chooses a contract from the menu.

Stage 4: At the end of the period, the manufacturer observes the realization of demand.

The manufacturer continues to offer the above menu as long as the retailer chose the contract according to the actual demand. If the retailer deviated, then the manufacturer stops to pay F and instead offers the static menu in all future periods (which is also the punishment in our paper).

Below we show the following results:

1. This alternative mechanism does not result in positive profits for the manufacturer for all values of δ. In particular, it result negative profit for the manufacturer when δ is small, and positive profit if δ is high.
2. This alternative mechanism always (for all $\delta < 1$) provides lower profits to the manufacturer than the profit of the mechanism in our paper. Hence, the manufacturer would never find it optimal to offer it.
3. If the manufacturer wishes to implement a mechanism with the quantities that maximize total industry profits, the manufacturer would rather do so using the mechanism in our model (and setting the quantities q_H^* and q_L^* instead of $q_H^{D,e}$ and $q_L^{D,e}$).

The above mechanism satisfies the retailer's individual rationality in state L because $T_L^* = \pi_R(q_L^*; L)$, and on top of that the retailer receives a fixed sum of F. We solve for the lowest F that motivates the retailer to choose the contract according to the realized demand. To this end, the incentive compatibility constraint in state H is the following:

$$IC_H^D: \quad 0 + \frac{\delta}{1-\delta} F \geq \pi_R(q_L^*; H) - \pi_R(q_L^*; L) + \frac{\delta}{1-\delta}\left[p\Delta(q_L^S)\right]. \tag{44}$$

To see why, suppose that in a certain period the demand is H. If, at stage 3 of this period, the retailer truthfully chooses the contract that was designed for state H, then the retailer earns 0 at the current period, because $T_H^* = \pi_R(q_H^*; H)$ (note that because the retailer already received the fixed sum of F at the beginning of the period, this payment is viewed as sunk). At stage 4, based on the sales information, the manufacturer observes that the retailer chose the correct contract, and hence the manufacturer will continue to offer this contract during all future periods. Thus, the LHS of IC_H^D captures the payoff of the retailer from choosing the correct contract during periods of high-demand.

Next, consider the possibility that the retailer mis-represents the type (i.e., chooses the contract not according to the realized demand). When the retailer chooses the contract that was designed for state

L, he earns at the current period $\pi_R(q_L^*; H) - T_L^* = \pi_R(q_L^*; H) - \pi_R(q_L^*; L) = \Delta(q_L^*) > 0$. When sales information is shared, the manufacturer infers that the retailer has deviated from the prescribed contract, and the manufacturer offers during all future periods the static contract (see Section 3). Again, notice that we omit the fixed payment of F from the current period payoff as it is sunk at the point the retailer decides whether to choose the contract according to the realized demand or the alternative one (furthermore, note that adding the fixed payment of F of the current period to the two sides of IC_H^D has no effect on the results). This payoff constitutes the right-hand-side of IC_H^D.

Solving IC_H^D for F in equality (which is the best situation for the manufacturer), we have that the lowest F that motivates the retailer to choose the appropriate contract according to the demand realization is:

$$F = p\Delta(q_L^S) + \frac{1-\delta}{\delta}\Delta(q_L^*). \tag{45}$$

The manufacturer's expected per-period profit from this mechanism is

$$\widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*) = -F + pT_H^* + (1-p)T_L^*.$$

Substituting $T_\theta^* = \pi_R(q_\theta^*; \theta)$ and F, we have:

$$\widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*) = \pi_R(q_L^*; H) - p(\Delta(q_L^S) - \Delta(q_L^*)) \tag{46}$$
$$-\frac{\Delta(q_L^*)}{\delta}.$$

Notice that the manufacturer's profit from this alternative mechanism is negative for low values of δ. To see why, substituting $\delta \to 0$ at the second line, this second line is: $-\infty$, because $\Delta(q_L^*) > 0$. Moreover, it is possible to confirm that $\widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*)$ is increasing in δ.

Comparing $\widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*)$ with the manufacturer's profit from the mechanism in our base model, $\Pi_M^{D,e}(q_H^{D,e}; q_L^{D,e})$, as given by equation (15) in our base model, we have that for all $\delta < 1$:

$$\Pi_M^{D,e}(q_H^{D,e}; q_L^{D,e}) > \Pi_M^{D,e}(q_H^*; q_L^*) > \widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*),$$

where the first inequality is obtained by revealed preferences: the quantities $q_H^{D,e}$ and $q_L^{D,e}$ maximize $\Pi_M^{D,e}(q_H^{D,e}; q_L^{D,e})$. The second inequality follows because:

$$\Pi_M^{D,e}(q_H^*; q_L^*) - \widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*) = \frac{(1-\delta)\left(\delta p\Delta(q_L^S) + \Delta(q_L^*)\right)}{\delta(1+\delta p)} > 0, \tag{47}$$

where this inequality follows because $\pi_R(q; H) > \pi_R(q; L)$ for all q. Hence, the manufacturer prefers to offer the mechanism in our base model over this alternative mechanism. The two mechanisms provide identical profit only when $\delta = 1$ because then $q_L^{D,e} = q_L^*$ and thus $\Pi_M^{D,e}(q_H^*; q_L^*) = \widetilde{\Pi}_M^{D,e}(q_H^*; q_L^*)$.

Finally, notice that the analysis above (see Equation (43)) implies that if the manufacturer would like to implement the quantities that maximize industry profits, q_H^* and q_L^*, the manufacturer would rather do so using the mechanism in our base model and set $q_\theta^{D,e} = q_\theta^*$, $\theta \in \{H, L\}$, and not using a fixed fee that is independent of the realized demand.