E-Companion for "Platform Compatibility in a Two-sided Market: Pricing Schemes and Coopetition Analysis"

This e-companion consists of two parts: Appendix A (Main Proofs) and Appendix B (Supplementary Analysis of Model Extensions).

Appendix A. Proof of Main Results

By explicitly solving the two platforms' problems under three scenarios, the equilibria exist under the following conditions: $t \le \frac{1}{2}$, $c \le \frac{1-3\alpha t^2}{\alpha}$, $\omega \le \min\{\frac{11t^2+5c-3}{5(4t^2-1)}, \frac{-376t^4+115(\lambda-1)t^3+(-200c+344)t^2+(\lambda-1)(53c-31)t+128c-64}{-128-800t^4+212(\lambda-1)t^3+712t^2-53(\lambda-1)t}\} = \frac{-376t^4+115(\lambda-1)t^3+(-200c+344)t^2+(\lambda-1)(53c-31)t+128c-64}{-128-800t^4+212(\lambda-1)t^3+712t^2-53(\lambda-1)t}$, which technically guarantee the existence of second-order conditions and the positive equilibrium solutions. Intuitively, these conditions are reasonable, as they collectively imply that t should not be too large; otherwise, network externalities would dominate price effects and may diminish the interior equilibrium conditions. Moreover, c and ω cannot be too large to ensure that customer valuation of choosing the service offered by the niche platform, i.e., v, does not exceed one in the compatible case. Economically, the condition on c (incompatible case) and the condition on ω (compatible case) ensure that the discounted base value of the niche platform is no greater than that of the dominant platform.

Proof of Proposition 1

In the incompatible case (scenario I), the profit functions of the two platforms are $\Pi_1^I(p_1) = \lambda p_1\left[\alpha v^I + (1-\alpha)\right]$ and $\Pi_2^I(p_2) = \lambda p_2 \alpha\left(1 - v^I\right)$, where $v^I = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) + c$ and $\alpha \in (0,1)$. The optimal pricing decision of each platform can be obtained by solving: $\max \Pi_i^I(p_i|p_{3-i}^*)$ subject to $p_i \geqslant 0$, $i \in \{1,2\}$. Additionally, $u_i^s = (1-\lambda)p_i + td_i - \sqrt{s_i}$. Given $u_i^s \ge 0$, we have $s_1^I = [(1-\lambda)p_1 + td_1^I]^2$ and $s_2^I = [(1-\lambda)p_2 + td_2^I]^2$. Substituting s_1^I and s_2^I into d_1^I and d_2^I, we have $d_1^I = \frac{\left(t^2+(p_1-p_2)(\lambda-1)t-c+p_1-p_2\right)\alpha-1}{2\alpha t^2-1}$, $d_2^I = \frac{\left(t^2-(p_1-p_2)(\lambda-1)t+c-p_1+p_2\right)\alpha}{2\alpha t^2-1}$. As such, we have the following profit functions:

$$\Pi_1^I(p_1|p_2) = \frac{\lambda p_1(\left(t^2 + (p_1-p_2)(\lambda-1)t - c + p_1 - p_2\right)\alpha - 1)}{2\alpha t^2 - 1}, \tag{A.1}$$

$$\Pi_2^I(p_2|p_1) = \frac{\lambda p_2(\left(t^2 - (p_1-p_2)(\lambda-1)t + c - p_1 + p_2\right))\alpha}{2\alpha t^2 - 1}. \tag{A.2}$$

Taking the first-order and second-order conditions of Π_1^I and Π_2^I with respect to p_1 and p_2, we have $\frac{\partial \Pi_1^I}{\partial p_1} = \frac{((-t^2-(\lambda-1)(2p_1-p_2)t+c-2p_1+p_2)\alpha+1)\lambda}{1-2\alpha t^2}$, $\frac{\partial^2 \Pi_1^I}{\partial p_1^2} = \frac{2\lambda\alpha(1+(\lambda-1)t)}{2\alpha t^2-1}$, $\frac{\partial \Pi_2^I}{\partial p_2} = \frac{\alpha\lambda(t^2-(p_1-2p_2)(\lambda-1)t+c-p_1+2p_2)}{2\alpha t^2-1}$ and $\frac{\partial^2 \Pi_2^I}{\partial p_2^2} = \frac{2\lambda\alpha(1+(\lambda-1)t)}{2\alpha t^2-1}$. Thus, $\frac{\partial^2 \Pi_1^I}{\partial p_1^2} \le 0$, and $\frac{\partial^2 \Pi_2^I}{\partial p_2^2} \le 0$ when $t \le \frac{1}{\sqrt{2\alpha}}$. Since $\frac{1}{2} < \frac{1}{\sqrt{2}}$ and our analysis is restricted to $t \le \frac{1}{2}$, the concavity condition $t \le \frac{1}{\sqrt{2\alpha}}$ is always satisfied. Therefore, $\Pi_1^I(p_1|p_2)$ and $\Pi_2^I(p_2|p_1)$ are concave in p_1 and p_2, respectively. Then, the best response functions $p_1(p_2)$ and $p_2(p_1)$ can be determined by solving equations $\frac{\partial \Pi_1^I}{\partial p_1} = 0$ and $\frac{\partial \Pi_2^I}{\partial p_2} = 0$. Next, simultaneously solving the response functions, we obtain $p_1^* = \frac{(-3t^2+c)\alpha+2}{3\alpha(1+(\lambda-1)t)}$ and $p_2^* = \frac{(-3t^2-c)\alpha+1}{3\alpha(1+(\lambda-1)t)}$. Substituting p_1^* and p_2^* into the demand-side and supply-side functions, we can obtain $d_1^* = \frac{2-(3t^2-c)\alpha}{3(1-2\alpha t^2)}$ and $d_2^* = \frac{1-(3t^2+c)\alpha}{3(1-2\alpha t^2)}$; the equilibrium supply-side providers are $s_1^* = [\frac{(2+(c-3t^2)\alpha)((3(\lambda-1)t^2+t)\alpha-\lambda+1)}{3\alpha(1-2\alpha t^2)(1+t(\lambda-1))}]^2$ and $s_2^* = [\frac{((c+3t^2)\alpha-1)((3(\lambda-1)t^2+t)\alpha-\lambda+1)}{3\alpha(1-2\alpha t^2)(1+t(\lambda-1))}]^2$. Substituting these equilibrium outcomes into $(p_1 - t\sqrt{s_1}) - (p_2 - t\sqrt{s_2} + c)$, we have $(p_1^* - t\sqrt{s_1^*}) - (p_2^* - t\sqrt{s_2^*} + c) = \frac{1-(3t^2+c)\alpha}{3\alpha(1-2\alpha t^2)}$. This result implies that the valuation of the platform 2's service satisfies $v \le 1$ when $c \le \frac{1-3\alpha t^2}{\alpha}$. Q.E.D.

Proof of Proposition 2

In the compatible case with the unified pricing scheme (scenario CU), the profit functions of the two platforms are $\Pi_1^{CU}(\xi, p_1) = \lambda p_1 d_1^{CU} + \xi\lambda p_2 d_{21}^{CU}$ and $\Pi_2^{CU}(p_2) = \lambda p_2 d_{22}^{CU} + (1-\xi)\lambda p_2 d_{21}^{CU}$, where $d_1^{CU} = 1 - p_1 + p_2 + t(\sqrt{s_1} - \sqrt{s_2}) + c - \omega$, $d_{21}^{CU} = \omega$, and $d_{22}^{CU} = p_1 - p_2 - t(\sqrt{s_1} - \sqrt{s_2}) - c$. Additionally, $u_i^s = (1-\lambda)p_i + td_i - \sqrt{s_i}$. Given $u_i^s \geq 0$, we have $s_1^{CU} = [(1-\lambda)p_1 + td_1^{CU}]^2$ and $s_2^{CU} = [(1-\lambda)p_2 + td_2^{CU}]^2$. Substituting s_1^{CU} and s_2^{CU} into d_1^{CU} and d_2^{CU}, we have $d_1^{CU} = \frac{t^2+(p_1-p_2)(\lambda-1)t-c+\omega+p_1-p_2-1}{2t^2-1}$, $d_2^{CU} = \frac{t^2-(p_1-p_2)(\lambda-1)t+c-\omega-p_1+p_2}{2t^2-1}$. As such, we have the following profit functions:

$$\Pi_1^{CU}(p_1, p_2|\xi) = \frac{\lambda p_1(t^2 + (p_1 - p_2)(\lambda - 1)t - c + \omega + p_1 - p_2 - 1)}{2t^2 - 1} + \xi\lambda p_2\omega, \tag{A.3}$$

$$\Pi_2^{CU}(p_1, p_2|\xi) = (1-\xi)\lambda p_2\omega + \lambda p_2(\frac{t^2 - (p_1 - p_2)(\lambda - 1)t + c - \omega - p_1 + p_2}{2t^2 - 1} - \omega). \tag{A.4}$$

By backward induction, we first investigate the best responses with respect to prices. Taking the first-order and second-order conditions of Π_1^{CU} and Π_2^{CU} with respect to p_1 and p_2, respectively, we can obtain $\frac{\partial\Pi_1^{CU}}{\partial p_1} = \frac{\lambda(1-2p_1+p_2-t^2-(\lambda-1)(2p_1-p_2)t+c-\omega)}{1-2t^2}$, $\frac{\partial^2\Pi_1^{CU}}{\partial p_1^2} = \frac{2\lambda(1+(\lambda-1)t)}{2t^2-1}$, $\frac{\partial\Pi_2^{CU}}{\partial p_2} = \frac{\lambda((1-2\xi\omega)t^2-(p_1-2p_2)(\lambda-1)t-p_1+2p_2+\omega(\xi-1)+c)}{2t^2-1}$, and $\frac{\partial^2\Pi_2^{CU}}{\partial p_2^2} = \frac{2\lambda(1+(\lambda-1)t)}{2t^2-1}$. Thus, $\frac{\partial^2\Pi_1^{CU}}{\partial p_1^2} \leq 0$, and $\frac{\partial^2\Pi_2^{CU}}{\partial p_2^2} \leq 0$ when $t \leq \frac{1}{\sqrt{2}}$. Since $\frac{1}{2} < \frac{1}{\sqrt{2}}$ and our analysis is restricted to $t \leq \frac{1}{2}$, the concavity condition $t \leq \frac{1}{\sqrt{2}}$ is always satisfied. Therefore, $\Pi_1^{CU}(p_1, p_2|\xi)$ and $\Pi_2^{CU}(p_1, p_2|\xi)$ are concave in p_1 and p_2, respectively. Solving $\frac{\partial\Pi_1^{CU}}{\partial p_1} = 0$ and $\frac{\partial\Pi_2^{CU}}{\partial p_2} = 0$, we obtain the best response functions $p_1(p_2)$ and $p_2(p_1)$. Next, simultaneously solving the response functions yields $p_1 = \frac{(-\xi-1)\,\omega+c+2+t^2\,(2\,\xi\,\omega-3)}{3t(1+(\lambda-1))}$ and $p_2 = \frac{4\,\xi\,t^2\,\omega-2\,\xi\,\omega-3\,t^2-c+\omega+1}{3t(1+(\lambda-1))}$. Next, substituting them into (A.3), we have $\Pi_1^{CU}(\xi) = [\lambda((20\xi^2t^4 + 10\xi(1-2\xi)t^2 + 5\xi^2 - 5\xi - 1)\omega^2) - (6\xi t^4 + (6 + (10c-1)\xi)t^2 + (5c+1)\xi + 2c + 4)\omega - (c + 2 - 3t^2)^2]/[9(2t^2-1)(1 + t(\lambda-1))]$.

Next, we examine the optimal fraction of sharing revenue charged by the dominant platform. Taking the first-order and second-order conditions of Π_1^{CU} with respect to ξ, we have $\frac{\partial\Pi_1^{CU}}{\partial\xi} = \frac{\lambda\omega(5\omega(2\xi-4\xi t^2-1)+3t^2+5c+1)}{-9(1+(\lambda-1)t)}$ and $\frac{\partial^2\Pi_1^{CU}}{\partial\xi^2} = \frac{10\lambda\omega^2(2t^2-1)}{9(1+(\lambda-1)t)}$. In this case, we can show that $\frac{\partial^2\Pi_1^{CU}}{\partial\xi^2} \leq 0$ when $t \leq \frac{1}{\sqrt{2}}$. Given we have stated earlier that we discuss the results within the range of $t \leq \frac{1}{2} < \frac{1}{\sqrt{2}}$, we can obtain that $\Pi_1^{CU}(\xi)$ is concave in ξ when $t \leq \frac{1}{2}$. Solving $\frac{\partial\Pi_1^{CU}}{\partial\xi} = 0$ yields the optimal fraction of shared revenue charged by the dominant platform, i.e., $\xi^\dagger = \frac{5\omega-3t^2-5c-1}{10\omega(1-2t^2)}$. Substituting $\xi^\dagger$ into the response functions $p_1(p_2)$ and $p_2(p_1)$, we have $p_1^\dagger = \frac{7-9t^2-5\omega+5c}{10(1+(\lambda-1)t)}$ and $p_2^\dagger = \frac{2(1-2t^2)}{5(1+(\lambda-1)t)}$. Accordingly, substituting $p_1^\dagger$, $p_2^\dagger$, and $\xi^\dagger$ into both the demand-side and supply-side functions, we can then obtain $d_1^\dagger = \frac{7-9t^2-5\omega+5c}{10(1-2t^2)}$, $d_2^\dagger = \frac{3-11t^2+5\omega-5c}{10(1-2t^2)}$, $s_1^\dagger = [\frac{(3t^2(\lambda-1)-\lambda+t+1)(9t^2+5\omega-5c-7)}{10(2t^2-1)(t(\lambda-1)+1)}]^2$, and $s_2^\dagger = [\frac{27(\lambda-1)t^4+11t^3+(\lambda-1)(5c-5\omega-19)t^2+(5c-5\omega-3)t+4(\lambda-1)}{10(2t^2-1)(t(\lambda-1)+1)}]^2$. Leveraging the obtained equilibrium outcomes, we can show that $(p_1^\dagger - t\sqrt{s_1^\dagger}) - (p_2^\dagger - t\sqrt{s_2^\dagger} + c) = \frac{(11-20\omega)t^2+5c+5\omega-3}{10(2t^2-1)}$. This result indicates that the valuation of the platform 2's service satisfies $v \leq 1$ when $\omega \leq \frac{11t^2+5c-3}{5(4t^2-1)}$. Q.E.D.

Proof of Proposition 3

In the compatible case with the differential pricing scheme (scenario CD), the two platforms' profit functions are: $\Pi_1^{CD}(\xi, p_1) = \lambda p_1 d_1^{CD} + \xi\lambda p_c d_{21}^{CD}$ and $\Pi_2^{CD}(\mu, p_2) = \lambda p_2 d_{22}^{CD} + (1-\xi)\lambda p_c d_{21}^{CD}$, where $p_c = p_2 - \mu$, $d_1^{CD} = (\sqrt{s_1} - \sqrt{s_2})\,t + c - \mu - \omega - p_1 + p_2 + 1$, $d_{21}^{CD} = \omega + \mu$, and $d_{22}^{CD} = (\sqrt{s_2} - \sqrt{s_1})\,t - c + p_1 - p_2$. Additionally, we have $u_1^s = (1-\lambda)p_1 + td_1 - \sqrt{s_1}$ and $u_2^s = (1-\lambda)(p_2 + (p_2 - \mu))/2 + td_2 - \sqrt{s_2}$. Given $u_i^s \geq 0$, we have $s_1^{CD} = [(1-\lambda)p_1 + td_1^{CD}]^2$ and $s_2^{CD} = [(1-\lambda)(p_2 + (p_2-\mu))/2 + td_2^{CD}]^2$. Substituting s_1^{CD} and s_2^{CD} into d_1^{CD} and d_2^{CD}, we have $d_1^{CD} = \frac{2t^2+(\mu+2p_1-2p_2)(\lambda-1)\,t-2c+2\mu+2\omega+2p_1-2p_2-2}{2(2t^2-1)}$ and $d_2^{CD} = \frac{2t^2-(\mu+2p_1-2p_2)(\lambda-1)\,t+2c-2\mu-2\omega-2p_1+2p_2}{2(2t^2-1)}$. As such, we have the following profit functions:

$$\pi_1^{CD}(p_1,p_2|\xi)=\frac{\lambda p_1\left(-2t^2-(\mu+2p_1-2p_2)(\lambda-1)t+2(c+p_2-\mu-\omega-p_1+1)\right)}{-4t^2+2}+\xi\lambda(p_2-\mu)(\mu+\omega), \quad (A.5)$$

$$\pi_2^{CD}(p_1,p_2|\xi)=\lambda p_2\left(\frac{-2t^2+(\mu+2p_1-2p_2)(\lambda-1)t-2(c+p_2-\mu-\omega-p_1)}{-4t^2+2}-\mu-\omega\right)+(1-\xi)\lambda(p_2-\mu)(\mu+\omega). \quad (A.6)$$

By backward induction, we first examine $p_i^\ddagger=\arg\max_{p_i\geqslant 0}\ \Pi_i^{CD}(p_i|p_{3-i}^\ddagger,\xi,\mu)$, where $i\in\{1,2\}$. Given ξ and μ, taking the first-order and second-order conditions of Π_1^{CD} and Π_2^{CD} with respect to p_1 and p_2, respectively, we can obtain the following results: $\frac{\partial\Pi_1^{CD}}{\partial p_1}=\frac{\lambda(2t^2+(\lambda-1)(4p_1-2p_2+\mu)t-2c+2\mu+2\omega+4p_1-2p_2-2)}{2(2t^2-1)}$, $\frac{\partial^2\Pi_1^{CD}}{\partial p_1^2}=\frac{2\lambda(1+(\lambda-1)t)}{2t^2-1}$, $\frac{\partial\Pi_2^{CD}}{\partial p_2}=\frac{\lambda(2(1-2\xi\mu-2\xi\omega)t^2-(\mu+2p_1-4p_2)(\lambda-1)t+2(\xi-1)\mu+2\xi\omega+2c-2\omega-2p_1+4p_2)}{2(2t^2-1)}$, and $\frac{\partial^2\Pi_2^{CD}}{\partial p_2^2}=\frac{2\lambda(1+(\lambda-1)t)}{2t^2-1}$. Thus, $\frac{\partial^2\Pi_1^{CD}}{\partial p_1^2}\leq 0$, and $\frac{\partial^2\Pi_2^{CD}}{\partial p_2^2}\leq 0$ when $t\leq\frac{1}{\sqrt{2}}$. Since we have stated earlier that we discuss the results within the range of $t\leq\frac{1}{2}<\frac{1}{\sqrt{2}}$, we can obtain that Π_1^{CD} is concave in p_1 and Π_2^{CD} is concave in p_2 when $t\leq\frac{1}{2}$. Then, solving $\frac{\partial\Pi_1^{CD}}{\partial p_1}=0$ and $\frac{\partial\Pi_2^{CD}}{\partial p_2}=0$, we have the best response functions: $p_1=\frac{(4\xi\mu+4\xi\omega-6)\,t^2-\mu(\lambda-1)\,t-2(\xi+1)\,\mu-2\xi\omega+2c-2\omega+4}{6(1+(\lambda-1)\,t)}$ and $p_2=\frac{(8\,\xi\,\mu+8\,\xi\,\omega-6)\,t^2+\mu(\lambda-1)\,t+(2-4\,\xi)\,\mu-4\,\xi\,\omega-2\,c+2\,\omega+2}{6(1+(\lambda-1)\,t)}$. Substituting them into Π_1^{CD} and then taking the first-order and second-order conditions of Π_1^{CD} with respect to ξ, we can show Π_1^{CD} is concave in ξ when $t\leq 1/\sqrt{2}$. Solving $\frac{\partial\Pi_1^{CD}}{\partial\xi}=0$ yields $\xi(\mu)=\frac{6t^2+13\mu(\lambda-1)t+10c+8\mu-10\omega+2}{20(\mu+\omega)(2t^2-1)}$. Substituting $\xi(\mu)$ into the response functions, we have $p_1(\mu)=\frac{-18\,t^2+\mu(\lambda-1)\,t+10\,c-4\,\mu-10\,\omega+14}{20(1+(\lambda-1)t)}$ and $p_2(\mu)=\frac{-4t^2+3\mu(\lambda-1)t+3\mu+2}{5(1+(\lambda-1)t)}$. Plugging $\xi(\mu)$, $p_1(\mu)$, and $p_2(\mu)$ into $\Pi_2^{CD}(\mu)$, and taking the first-order and second-order conditions of $\Pi_2^{CD}(\mu)$ with respect to μ, we have

$$\frac{\partial\Pi_2^{CD}(\mu)}{\partial\mu}=\frac{\lambda((-200\mu-100\omega+63)t^2+29\mu(\lambda-1)t+25c+104\mu+25\omega-19)}{50(2t^2-1)}, \quad (A.7)$$

$$\frac{\partial^2\Pi_2^{CD}(\mu)}{\partial\mu^2}=\frac{\lambda(29\lambda t-200t^2-29t+104)}{50(2t^2-1)}. \quad (A.8)$$

We can show that $2t^2-1<0$ and $29\lambda t-200t^2-29t+104>0$ always hold when $t\leq\frac{1}{2}$. Thus, Π_2^{CD} is concave in μ. Solving $\frac{\partial\Pi_2^{CD}}{\partial\mu}=0$ yields $\mu^\ddagger=\frac{(100\,\omega-63)\,t^2-25\,c-25\,\omega+19}{-200\,t^2+(29\,\lambda-29)\,t+104}$. Then, the equilibrium outcomes can be obtained by plugging $\mu^\ddagger$ into p_1, p_2, ξ and the demand functions. Leveraging the equilibrium outcomes, we can show that $(p_1^\ddagger-t\sqrt{s_1^\ddagger})-(p_2^\ddagger-t\sqrt{s_2^\ddagger}+c)\geq 0$ holds when $\omega\leq\frac{-376t^4+115(\lambda-1)t^3+(-200c+344)t^2+(\lambda-1)(53c-31)t+128c-64}{-128-800t^4+212(\lambda-1)t^3+712t^2-53(\lambda-1)t}$. In this case, the platform 2's valuation $v<1$ can be ensured. Q.E.D.

Proof of Proposition 4

To investigate the impact of compatibility on the pricing decisions, we compare the equilibrium prices of each channel in different scenarios. Specifically, to compare the prices of various channels, we define pricing differentials across scenarios.

We first examine the equilibrium prices between scenarios I and CU; by comparing p_1^* and $p_1^\dagger$, we can obtain $\Delta p_1^\dagger(\omega)=p_1^\dagger-p_1^*=-\frac{1}{2\left((\lambda-1)t+1\right)}\omega+\frac{-20+(3t^2+5c+21)\,\alpha}{30\,\alpha\left((\lambda-1)t+1\right)}$, where $-\frac{1}{2\left((\lambda-1)t+1\right)}<0$ always holds. If $\alpha\geq\alpha_{p1}^\dagger=\frac{20}{3(t^2+7)+5c}$, then $\frac{-20+(3t^2+5c+21)\,\alpha}{30\,\alpha\left((\lambda-1)t+1\right)}\geq 0$. In this case, $\Delta p_1^\dagger(\omega)$ is greater than zero when ω is no more than the threshold value. Thus, solving $\Delta p_1^\dagger(\omega)=0$ yields the threshold $\omega_{p1}^\dagger=\frac{\alpha(3t^2+5c+21)-20}{15\alpha}$. In summary, we have the following results:

$$\begin{cases}\Delta p_1^\dagger(\omega)=p_1^\dagger-p_1^*\geq 0, & \text{if } (\alpha,\omega)\in\Lambda_0,\\ \Delta p_1^\dagger(\omega)=p_1^\dagger-p_1^*<0, & \text{otherwise,}\end{cases} \quad (A.9)$$

where $\Lambda_0=\{(\alpha,\omega):\alpha\geq\alpha^{\dagger}_{p1},\omega\leq\omega^{\dagger}_{p1}\}$. Similarly, $\Delta p^{\dagger}_2{=}p^{\dagger}_2-p^{*}_2{=}\frac{-5+(3t^2+5c+6)\,\alpha}{15\alpha((\lambda-1)t+1)}$, where $15\alpha((\lambda-1)t+1)\geq 0$ always holds. That is, $\Delta p^{\dagger}_2\geq 0$ when $\alpha\geq\alpha^{\dagger}_{p2}=\frac{5}{3t^2+5c+6}$.

We next examine the equilibrium prices between scenarios I and CD. By comparing p^{*}_1 and $p^{\ddagger}_1$, we have $\Delta p^{\ddagger}_1(\omega){=}p^{\ddagger}_1-p^{*}_1{=}\frac{\left(20\,t^3\lambda-20\,t^3-63\,t\,\lambda+320\,t^2+63\,t-188\right)}{4(29\,t\,\lambda-200\,t^2-29\,t+104)(t\,\lambda-t+1)}\omega+(-240\,\alpha\,t^4-3\,\alpha\,(\lambda-1)\,t^3+(1600+(-400\,c-1404)\,\alpha)t^2+(\lambda-1)t(-232+(c+255)\alpha)t-832+(268\,c+828)\,\alpha)/(12\,\alpha\,(29\,t\,\lambda-200\,t^2-29\,t+104)\,(t\,\lambda-t+1))$, where $\frac{\left(20\,t^3\lambda-20\,t^3-63\,t\,\lambda+320\,t^2+63\,t-188\right)}{4(29\,t\,\lambda-200\,t^2-29\,t+104)(t\,\lambda-t+1)}<0$ always holds. If $\alpha\geq\alpha^{\ddagger}_{p1}=\frac{832-1600t^2+232(\lambda-1)t}{-240t^4+3(1-\lambda)t^3(400c+1404)t^2+(43c+255)(\lambda-1)t+268c+828}$, then $(-240\,\alpha\,t^4-3\,\alpha\,(\lambda-1)\,t^3+(1600+(-400\,c-1404)\,\alpha)t^2+(\lambda-1)t(-232+(c+255)\alpha)t-832+(268\,c+828)\,\alpha)/(12\,\alpha\,(29\,t\,\lambda-200\,t^2-29\,t+104)\,(t\,\lambda-t+1))\geq 0$. In this case, $\Delta p^{\ddagger}_1(\omega)$ is greater than zero when ω is no more than the threshold $\omega^{\ddagger}_{p1}=\frac{240\alpha t^4+3\alpha(\lambda-1)t^3+(-1600+(400c+1404)\alpha)t^2-(\lambda-1)(-232+(43c+255)\alpha)t+832+(-268c-828)\alpha}{60\alpha(\lambda-1)t^3+960\alpha t^2-189(\lambda-1)\alpha t-564\alpha}$. Thus, we get the following results:

$$\begin{cases}\Delta p^{\ddagger}_1(\omega)=p^{\ddagger}_1-p^{*}_1\geq 0, & \text{if } (\alpha,\omega)\in\Lambda_1,\\ \Delta p^{\ddagger}_1(\omega)=p^{\ddagger}_1-p^{*}_1<0, & \text{otherwise,}\end{cases}\tag{A.10}$$

where $\Lambda_1=\{(\alpha,\omega):\alpha\geq\alpha^{\ddagger}_{p1},\omega\leq\omega^{\ddagger}_{p1}\}$.

Similarly, $\Delta p^{\ddagger}_2(\omega){=}p^{\ddagger}_2-p^{*}_2{=}\frac{15(4t^2-1)}{104-200t^2+29(\lambda-1)t}\omega+(-120\,\alpha t^4-96\,\alpha(\lambda-1)t^3+(200-(200c+291)\alpha)t^2-(\lambda-1)(29+(16c-69)\alpha)t-104+(59c+159)\alpha)/(3\,\alpha\,(29t(\lambda-1)-200t^2+104)\,(\lambda t-t+1))$, where $\frac{15(4t^2-1)}{104-200t^2+29(\lambda-1)t}\leq 0$ always holds. That is, when $\alpha\geq\alpha^{\ddagger}_{p2}=\frac{-104+200t^2+29(1-\lambda)t}{120t^4+(96\lambda-96)t^3+(200c+291)t^2+(\lambda-1)(16c-69)t-59c-159}$, then $(-120\,\alpha t^4-96\,\alpha(\lambda-1)t^3+(200-(200c+291)\alpha)t^2-(\lambda-1)(29+(16c-69)\alpha)t-104+(59c+159)\alpha)/(3\,\alpha\,(29t(\lambda-1)-200t^2+104)\,(\lambda t-t+1))\geq 0$. In this case, $\Delta p^{\ddagger}_2(\omega)$ is greater than zero when ω is no more than the threshold $\omega^{\ddagger}_{p2}=\frac{120\,\alpha t^4+96\,\alpha(\lambda-1)t^3+\left(-200+(200c+291)\alpha\right)t^2+(\lambda-1)(29+(16c-69)\alpha)t+104+(-59c-159)\alpha}{45\alpha(\lambda t-t+1)(4t^2-1)}$. Thus, we have the following results:

$$\begin{cases}\Delta p^{\ddagger}_2(\omega)=p^{\ddagger}_2-p^{*}_2\geq 0, & \text{if } (\alpha,\omega)\in\Lambda_2,\\ \Delta p^{\ddagger}_2(\omega)=p^{\ddagger}_2-p^{*}_2<0, & \text{otherwise,}\end{cases}\tag{A.11}$$

where $\Lambda_2=\{(\alpha,\omega):\alpha\geq\alpha^{\ddagger}_{p2},\omega\leq\omega^{\ddagger}_{p2}\}$.

Finally, by comparing the equilibrium prices of the compatible channel and the platform 2' exclusive channel, we have $\Delta p^{\ddagger}_c{=}p^{\ddagger}_c-p^{*}_2{=}\frac{10(1-4t^2)}{104-200\,t^2+29(\lambda-1)\,t}\omega+(-120\,\alpha t^4+93\,\alpha(\lambda-1)t^3+(200+(-200c-102)\alpha)t^2+(-29+(59c+12)\alpha)(\lambda-1)t-104+(134c+102)\alpha)/(3\,(29t(\lambda-1)-200t^2+104)\,(\lambda t-t+1)\,\alpha)$, where $\frac{10(1-4t^2)}{104-200\,t^2+29(\lambda-1)\,t}\geq 0$ always holds. We can show that if $\alpha\leq\alpha^{\ddagger}_{pc}=\frac{104-200t^2+(29\lambda-29)t}{-120t^4+(93\lambda-93)t^3+(-200c-102)t^2+(59c+12)(\lambda-1)t+134c+102}$, $(-120\,\alpha t^4+93\,\alpha(\lambda-1)t^3+(200+(-200c-102)\alpha)t^2+(-29+(59c+12)\alpha)(\lambda-1)t-104+(134c+102)\alpha)/(3\,(29t(\lambda-1)-200t^2+104)\,(\lambda t-t+1)\,\alpha)\leq 0$. In this case, $\Delta p^{\ddagger}_c(\omega)$ is less than zero when ω is no more than the threshold $\omega^{\ddagger}_{pc}=\frac{-120\,\alpha\,t^4+93\,\alpha\,(\lambda-1)\,t^3+(200+(-200c-102)\alpha)t^2+(-29+(59c+12)\alpha)(\lambda-1)t-104+(134c+102)\alpha}{30\,\alpha\,(\lambda t-t+1)(4t^2-1)}$. In summary, we can obtain the following results:

$$\begin{cases}\Delta p^{\ddagger}_c(\omega)=p^{\ddagger}_c-p^{*}_2\leq 0, & \text{if } (\alpha,\omega)\in\Lambda_3,\\ \Delta p^{\ddagger}_c(\omega)=p^{\ddagger}_c-p^{*}_2>0, & \text{otherwise,}\end{cases}\tag{A.12}$$

where $\Lambda_3=\{(\alpha,\omega):\alpha\leq\alpha^{\ddagger}_{pc},\omega\leq\omega^{\ddagger}_{pc}\}$. Q.E.D.

Proof of Proposition 5

To investigate the impact of compatibility on demands, we compare the equilibrium demands of each channel across scenarios. We first examine the equilibrium demands between scenarios I and CU. By comparing d_1^* and $d_1^\dagger$, we have $\Delta d_1^\dagger(\omega)=d_1^\dagger - d_1^*=\frac{1}{2(2t^2-1)}\omega + \frac{-6\alpha t^4+(13+(-10c-12)\alpha)t^2-10\alpha c+15c+1}{60\alpha t^2(2t^2-1)+30(1-2t^2)}$, where $\frac{1}{2(2t^2-1)} < 0$ and $\frac{-6\alpha t^4+(13+(-10c-12)\alpha)t^2-10\alpha c+15c+1}{60\alpha t^2(2t^2-1)+30(1-2t^2)} > 0$ holds; in this case, $\Delta d_1^\dagger(\omega)$ is greater than zero when ω is less than the threshold $\hat{\omega} = \frac{(-6t^4-(10c+12)t^2-10c)\alpha+13t^2+15c+1}{15(1-2\alpha t^2)}$. Thus, we can show $\Delta d_1^\dagger(\omega) \geq 0$ when $\omega \leq \hat{\omega}$. Similarly, by comparing d_2^* and $d_2^\dagger$, we have $\Delta d_2^\dagger(\omega)=d_2^\dagger - d_2^*=\frac{1}{2(1-2t^2)}\omega - \frac{-6\alpha t^4+(13+(-10c-12)\alpha)t^2-10\alpha c+15c+1}{60\alpha t^2(2t^2-1)+30(1-2t^2)}$, where $\frac{1}{2(1-2t^2)} > 0$ and $-\frac{-6\alpha t^4+(13+(-10c-12)\alpha)t^2-10\alpha c+15c+1}{60\alpha t^2(2t^2-1)+30(1-2t^2)} < 0$ holds; thus, $\Delta d_2^\dagger(\omega)$ is smaller than zero when ω is less than the threshold $\hat{\omega} = \frac{(-6t^4-(10c+12)t^2-10c)\alpha+13t^2+15c+1}{15(1-2\alpha t^2)}$.

We next examine the equilibrium demands between scenarios I and CD. By comparing d_1^* and $d_1^\ddagger$, we have $\Delta d_1^\ddagger(\omega)=d_1^\ddagger-d_1^* = -\frac{188+20(1-\lambda)t^3-320t^2+63(\lambda-1)t}{4(1-2t^2)(29t(\lambda-1)-200t^2+104)}\omega+\psi_{d1}(\alpha)$, where $-\frac{188+20(1-\lambda)t^3-320t^2+63(\lambda-1)t}{4(1-2t^2)(29t(\lambda-1)-200t^2+104)} \leq 0$ and $\psi_{d1}(\alpha)$ is decreasing with α. Thus, we have $\psi_{d1}(\alpha) \geq 0$, when $\alpha \leq \tilde{\alpha} = (-1040t^4 + 113(\lambda-1)t^3 + (-1200c+612)t^2 + (159c+23)(\lambda-1)t+684c-4)/(-480t^6+6(1-\lambda)t^5+(-800c-408)t^4+(86c+162)(\lambda-1)t^3+(-264c+408)t^2+116c(\lambda-1)t+416c)$. In this case, $\Delta d_1^\ddagger(\omega)$ is greater than zero when ω is less than the threshold $\tilde{\omega} = (480\alpha t^6 + 6(\lambda-1)\alpha t^5+(-1040+(800c+408)\alpha)t^4+(113-(86c+162)\alpha)(\lambda-1)t^3+((264c-408)\alpha-1200c+612)t^2+((159c+23)-116\alpha c)(\lambda-1)t-416\alpha c+684c-4)/(3(2\alpha t^2-1)(20(\lambda-1)t^3-63(\lambda-1)t+320t^2-188))$. Therefore,$\Delta d_1^\ddagger(\omega) \geq 0$ holds when $\alpha \leq \min\{\tilde{\alpha}, 1\}$ and $\omega \leq \tilde{\omega}$. Similarly, by comparing $d_2^\ddagger$ and d_2^*, we can show $\Delta d_2^\ddagger(\omega)=d_2^\ddagger - d_2^* = \frac{188+20(1-\lambda)t^3-320t^2+63(\lambda-1)t}{4(1-2t^2)(29t(\lambda-1)-200t^2+104)}\omega - \psi_{d1}(\alpha)$, where $\frac{188+20(1-\lambda)t^3-320t^2+63(\lambda-1)t}{4(1-2t^2)(29t(\lambda-1)-200t^2+104)} > 0$ and $\psi_{d1}(\alpha)$ is increasing with α. Therefore, we have $-\psi_{d1}(\alpha) \leq 0$, when $\alpha \leq \tilde{\alpha} = (-1040t^4+113(\lambda-1)t^3+(-1200c+612)t^2+(159c+23)(\lambda-1)t+684c-4)/(-480t^6+6(1-\lambda)t^5+(-800c-408)t^4+(86c+162)(\lambda-1)t^3+(-264c+408)t^2+116c(\lambda-1)t+416c)$. In this case, $\Delta d_2^\ddagger(\omega)$ is smaller than zero when ω is less than the threshold value. Letting $\Delta d_2^\ddagger(\omega) = 0$, we have the threshold $\tilde{\omega} = (480\alpha t^6+6(\lambda-1)\alpha t^5+(-1040+(800c+408)\alpha)t^4+(113-(86c+162)\alpha)(\lambda-1)t^3+((264c-408)\alpha-1200c+612)t^2+((159c+23)-116\alpha c)(\lambda-1)t-416\alpha c+684c-4)/(3(2\alpha t^2-1)(20(\lambda-1)t^3-63(\lambda-1)t+320t^2-188))$. Thus, $\Delta d_2^\ddagger(\omega) \leq 0$ holds when $\alpha \leq \min\{\tilde{\alpha}, 1\}$ and $\omega \leq \tilde{\omega}$. Q.E.D.

Proof of Proposition 6

To investigate the effect of platform 2's awareness advantage and the increment in consumer utilities for choosing the compatible channel, we analyze the changes of such parameters on the spillover profit for both platforms. We first examine the equilibrium profits between scenarios I and CU. Specifically, by comparing $\Pi_1^\dagger$ and Π_1^*, we have $\Delta\Pi_1^\dagger(\omega)=\Pi_1^\dagger - \Pi_1^*=a_1^\dagger\omega^2 + b_1^\dagger\omega + c_1\omega$, where $a_1^\dagger = \frac{\lambda}{4(\lambda t-t+1)(1-2t^2)} \geq 0$, $b_1^\dagger = \frac{\lambda(1-t^2)}{2(\lambda t-t+1)(2t^2-1)} \leq 0$, and $c_1^\dagger = -\frac{\lambda(18\alpha^2t^6-288\alpha^2t^4+291\alpha t^4+162\alpha^2t^2-6\alpha t^2-160t^2-81\alpha+80)}{180\alpha(\lambda t-t+1)(1-2t^2)(1-2\alpha t^2)}$. This result indicates that if $\alpha \geq \bar{\alpha}$, then $c_1^\dagger \geq 0$, where $\bar{\alpha} = \frac{27-97t^4+2t^2-\sqrt{10689t^8-21508t^6+16526t^4-5652t^2+729}}{12t^2(t^4-16t^2+9)}$. In this case, $\Delta\Pi_1^\dagger(\omega)$ is greater than zero when ω is less than the threshold value. Solving $\Delta\Pi_1^\dagger(\omega) = 0$ yields the threshold $\bar{\omega} = (15\alpha(-2\alpha t^4+2\alpha t^2+t^2-1)-2(180\alpha^4t^8+270\alpha^4t^6-930\alpha^3t^6-180\alpha^4t^4+105\alpha^3t^4+820\alpha^2t^4+180\alpha^3t^2-320\alpha^2t^2-200\alpha t^2-45\alpha^2+100\alpha)^{\frac{1}{2}})/(15\alpha(2\alpha t^2-1))$. In summary, we can obtain the following results:

$$\begin{cases} \Delta\Pi_1^\dagger(\omega) = \Pi_1^\dagger - \Pi_1^* \geq 0, & \text{if } (\alpha, \omega) \in \Gamma, \\ \Delta\Pi_1^\dagger(\omega) = \Pi_1^\dagger - \Pi_1^* < 0, & \text{otherwise}, \end{cases} \tag{A.13}$$

where $\Gamma=\{(\alpha,\omega):\alpha\geq\overline{\alpha},\omega\leq\overline{\omega}\}$. By comparing $\Pi_2^{\dagger}$ and Π_2^{*}, we have $\Delta\Pi_2^{\dagger}(\omega)$=$\Pi_2^{\dagger}-\Pi_2^{*}$=$\frac{\lambda(-(81t^2+72)\alpha^2t^2+(78t^2+36)\alpha-25)}{225\alpha(1+(\lambda-1)t(1-2\alpha t^2))}$. We have $\Delta\Pi_2^{\dagger}(\omega)\geq 0$ holds when $\alpha\geq\alpha_{\Pi_2}^{\dagger}$ regardless of ω, where $\alpha_{\Pi_2}^{\dagger}=\frac{13t^2+6-2\sqrt{-14t^4-11t^2+9}}{3t^2(9t^2+8)}$. Additionally, by comparing $\overline{\alpha}$ and $\alpha_{\Pi_2}^{\dagger}$, we find that $\overline{\alpha}>\alpha_{\Pi_2}^{\dagger}$ always holds. Thus, compatibility can be sustained as an equilibrium in the CU scenario when $(\alpha,\omega)\in\Gamma$, where $\Gamma=\{(\alpha,\omega):\alpha\geq\min(\overline{\alpha},1),\omega\leq\overline{\omega}\}$.

We next analyze the equilibrium profits between scenarios I and CD. By comparing $\Pi_1^{\ddagger}$ and Π_1^{*}, we have $\Delta\Pi_1^{\ddagger}(\omega)$=$\Pi_1^{\ddagger}-\Pi_1^{*}$=$a_1^{\ddagger}\omega^2+b_1^{\ddagger}\omega+c_1^{\ddagger}$, where $a_1^{\ddagger}=-5250(\alpha t^2-\frac{1}{2})(\frac{943}{875}+(\lambda-1)^2t^6+\frac{24}{7}(\lambda-1)t^5+(-\frac{163}{210}\lambda^2+\frac{163}{105}\lambda+\frac{797}{210})t^4-\frac{10}{3}(\lambda-1)t^3+(\frac{2963}{14000}\lambda^2-\frac{2963}{7000}\lambda-\frac{57837}{14000})t^2+\frac{344}{375}(\lambda-1)t)\alpha\geq 0$, $b_1^{\ddagger}=((2\alpha t^2-1)(13700\lambda^2t^6+88000(\lambda-1)t^7-27400\lambda t^6-14943\lambda^2t^4+1600(\lambda-1)t^5-18300t^6+29886\lambda t^4+5591\lambda^2t^2-69632(\lambda-1)t^3-11182\lambda t^2+25680(\lambda-1)t+169057t^4-149273t^2+36464)\alpha)/8\geq 0$, and $c_1^{\ddagger}\leq 0$. Solving $\Delta\Pi_1^{\ddagger}(\omega)=0$ yields the threshold $\omega_{\Pi_1}^{\ddagger}=\frac{-b_1^{\ddagger}+\sqrt{\left(b_1^{\ddagger}\right)^2-4a_1^{\ddagger}c_1^{\ddagger}}}{2a_1^{\ddagger}}$. Additionally, we find that $\omega_{\Pi_1}^{\ddagger}$ is decreasing with α, and $\omega_{\Pi_1}^{\ddagger}(\alpha)\geq\omega_{\Pi_1}^{\ddagger}(\alpha=1)\geq 1$. Therefore, $\Delta\Pi_1^{\ddagger}\leq 0$ always holds. Similarly, we denote $\Delta\Pi_2^{\ddagger}(\omega)$=$\Pi_2^{\ddagger}-\Pi_2^{*}$=$a_2^{\ddagger}\omega^2+b_2^{\ddagger}\omega+c_2^{\ddagger}$, where $a_2^{\ddagger}=\frac{25\lambda(16t^4-8t^2+1)}{4\alpha(1-2t^2)(29t(\lambda-1)-200t^2+104)}\geq 0$ and $b_2^{\ddagger}=-\frac{\lambda(252t^4-139t^2+19)}{2\alpha(1-2t^2)(29t(\lambda-1)-200t^2+104)}\leq 0$. If $\alpha\geq\alpha_{\Pi_2}^{\ddagger}$, then $c_2^{\ddagger}\geq 0$, where $\alpha_{\Pi_2}^{\ddagger}=[-((1792t^6+1169t^5(\lambda-1)+1905t^4-1066t^3(\lambda-1)-2746t^2+297t(\lambda-1)+729)(-512t^6+233t^5(\lambda-1)+937t^4-170t^3(\lambda-1)-490t^2+33t(\lambda-1)+81))^{1/2}+243+1664t^6+235(\lambda-1)t^5-453t^4-278(\lambda-1)t^3-638t^2+99(\lambda-1)t]/[3456t^8+1404(\lambda-1)t^7+1452t^6-1344(\lambda-1)t^5-3384t^4+396(\lambda-1)t^3+972t^2]$. In this case, $\Delta\Pi_2^{\ddagger}(\omega)$ is greater than zero when ω is no more than the threshold $\omega_{\Pi_2}^{\ddagger}=\frac{-b_2^{\ddagger}-\sqrt{(b_2^{\ddagger})^2-4a_2^{\ddagger}c_2^{\ddagger}}}{2a_2^{\ddagger}}$. Thus, $\Delta\Pi_2^{\ddagger}\geq 0$ holds when $(\alpha,\omega)\in\Gamma'$, where $\Gamma'=\{(\alpha,\omega):\alpha\geq\min(\alpha,1),\omega\leq\omega_{\Pi_2}^{\ddagger}\}$. Note that platform 1 cannot benefit from compatibility; therefore, compatibility cannot become an equilibrium strategy in the CD scenario. Q.E.D.

Proof of Proposition 7

Leveraging the equilibrium outcomes, we analyze the effect of unified and differential pricing schemes in the compatible case on both the prices and demands. We first examine the prices of platform 1 in scenarios CU and CD. By comparing $p_1^{\dagger}$ and $p_1^{\ddagger}$, we have Δp_1=$p_1^{\ddagger}-p_1^{\dagger}=\frac{5\,(t\lambda-t-4)\,(4t^2-1)}{4\,(29t(\lambda-1)-200t^2+104)\,(t\lambda-t+1)}\omega-\frac{(t\lambda-t-4)\,(63\,t^2+25\,c-19)}{20\,(29t(\lambda-1)200\,t^2+104)\,(t\lambda-t+1)}$, where $\frac{5\,(t\lambda-t-4)\,(4t^2-1)}{4\,(29t(\lambda-1)-200t^2+104)\,(t\lambda-t+1)}\geq 0$ and $-\frac{(t\lambda-t-4)\,(63\,t^2+25\,c-19)}{20\,(29t(\lambda-1)-200\,t^2+104)\,(t\lambda-t+1)}\leq 0$. Thus, $\Delta p_1\leq 0$ holds when $\omega\leq\omega_{p1}=\frac{19-63t^2-25c}{25(1-4t^2)}$. Moreover, by comparing ω_{p1} with the upper bound of ω for which the equilibrium outcomes exist, we have $\frac{19-63t^2-25c}{25(1-4t^2)}-\frac{11t^2+5c-3}{5(4t^2-1)}=\frac{4(1-2t^2)}{25(1-4t^2)}>0$. Therefore, $\Delta p_1\leq 0$ always holds.

We next examine the prices of platform 2's exclusive channel and the compatible channel in scenarios CU and CD. By comparing $p_2^{\dagger}$ and $p_2^{\ddagger}$, we have Δp_2=$p_2^{\ddagger}-p_2^{\dagger}=\frac{-75(1-4t^2)\omega-189t^2+57-75c}{145t(\lambda-1)-1000t^2+520}$, where $-\frac{75(1-4t^2)}{145t(\lambda-1)-1000t^2+520}\leq 0$. Letting $\Delta p_2=0$, we have the threshold ω_{p2}=$\frac{19-63t^2-25c}{25(1-4t^2)}$. Therefore, $\Delta p_2\geq 0$ holds when $\omega\leq\omega_{p2}$. Furthermore, by comparing ω_{p2} with the upper bound of ω for which the equilibrium outcomes exist, we find that $\frac{19-63t^2-25c}{25(1-4t^2)}-\frac{11t^2+5c-3}{5(4t^2-1)}=\frac{4(1-2t^2)}{25(1-4t^2)}>0$. In this case, $\Delta p_2\geq 0$ always holds.

Additionally, by comparing $p_c^{\ddagger}$ and $p_2^{\dagger}$, we have Δp_c=$p_c^{\ddagger}$−$p_2^{\dagger}$ = $\frac{50(1-4t^2)\omega+126t^2+50c-38}{145(\lambda-1)-1000t^2+520}$, where $\frac{50(1-4t^2)}{145(\lambda-1)-1000t^2+520}\geq 0$. Letting $\Delta p_c=0$, we can obtain the threshold ω_{pc}=$\frac{19-63t^2-25c}{25(1-4t^2)}$. Therefore, $\Delta p_c\leq 0$ holds when $\omega\leq\omega_{pc}$. Furthermore, by comparing ω_{pc} with the upper bound of ω for which the equilibrium outcomes exist, we find that $\frac{19-63t^2-25c}{25(1-4t^2)}-\frac{11t^2+5c-3}{5(4t^2-1)}=\frac{4(1-2t^2)}{25(1-4t^2)}>0$. In this case, $\Delta p_c\leq 0$ always holds true.

Finally, we compare the equilibrium demand of each platform in scenarios CU and CD. By comparing $d_1^{\ddagger}$ and $d_1^{\dagger}$, we have Δd_1=$d_1^{\ddagger}-d_1^{\dagger}=\frac{5(1-4t^2)(t\lambda-t-4)}{4(2t^2-1)(29t(\lambda-1)-200t^2+104)}\omega+\frac{(63\,t^2+25\,c-19)\,(t\,\lambda-t-4)}{20\,(2\,t^2-1)\,(29t(\lambda-1)-200t^2+104)}$, where $\frac{5(1-4t^2)(t\lambda-t-4)}{4(2t^2-1)(29t(\lambda-1)-200t^2+104)}\geq 0$ and $\frac{(63\,t^2+25\,c-19)\,(t\,\lambda-t-4)}{20\,(2\,t^2-1)\,(29t(\lambda-1)-200t^2+104)}\leq 0$. Solving $\Delta d_1=0$ yields the threshold ω_{d1}=$\frac{19-63t^2-25c}{25(1-4t^2)}$. Thus, $\Delta d_1\leq 0$ holds when $\omega\leq\omega_{d1}$. Furthermore, by comparing ω_{d1} with the upper bound of ω for which the equilibrium outcomes exist, we find that $\frac{19-63t^2-25c}{25(1-4t^2)}-\frac{11t^2+5c-3}{5(4t^2-1)}=\frac{4(1-2t^2)}{25(1-4t^2)}>0$. In this case, $\Delta d_1 \leq 0$ always holds. Similarly, we can obtain $\Delta d_2 = d_2^{\ddagger} - d_2^{\dagger} \geq 0$. Q.E.D.

Proof of Proposition 8

We investigate the effect of unified and differential pricing schemes in the compatible case on platform profits. For simplicity, c is normalized to zero when comparing profits.

We first examine the profits of platform 1 in scenarios CU and CD. By comparing $\Pi_1^{\dagger}$ and $\Pi_1^{\ddagger}$, we have $\Delta\Pi_1{=}\Pi_1^{\ddagger} - \Pi_1^{\dagger}{=}a_1\omega^2 + b_1\omega + c_1$, where $b_1 = -11000(\lambda-1)\lambda t^7 - \frac{3425}{2}(\lambda^2 - 2\lambda + \frac{1417}{137})\lambda t^6 + 5600(\lambda-1)\lambda t^5 + \frac{11579}{8}\lambda(\lambda^2 - 2\lambda + \frac{153979}{11579})t^4 - 112(\lambda-1)\lambda t^3 - \frac{2227}{8}(\lambda^2 - 2\lambda + \frac{57027}{2227})\lambda t^2 - 194(\lambda-1)\lambda t + 850\lambda \geq 0$, and $c_1 = [(47\lambda^2t^4 + 8800\lambda t^5 - 94\lambda t^4 - 8800t^5 - 51\lambda^2t^2 - 6816\lambda t^3 + 12847t^4 + 102\lambda t^2 + 6816t^3 + 1568\lambda t - 12099t^2 - 1568t + 3024)\lambda(63t^2 - 19)]/80 \leq 0$. Solving $\Delta\Pi_1 = 0$, we have two threshold values: $\omega_1^{(1)} = \frac{19-63t^2}{25(1-4t^2)}$ and $\omega_1^{(2)} = \frac{3024+8800(\lambda-1)t^5+(47\lambda^2-94\lambda+12847)t^4-6816(\lambda-1)t^3+(-51\lambda^2+102\lambda-12099)t^2+1568t(\lambda-1)}{-400+2100(\lambda-1)^2t^4+7200(\lambda-1)t^3+(-1105\lambda^2+2210\lambda+495)t^2-2880(\lambda-1)t}$. If $a_1 > 0$, then we have $\omega_1^{(1)} > 0$ and $\omega_1^{(2)} < 0$; if $a_1 \leq 0$, then we have $\omega_1^{(2)} > 1 > \omega_1^{(1)} > 0$. Thus, $\Delta\Pi_1 \leq 0$ holds when $\omega \leq \omega_1^{(1)}$. Additionally, by comparing $\omega_1^{(1)}$ with the upper bound of ω for which the equilibrium outcomes exist(c=0), we find that $\frac{19-63t^2}{25(1-4t^2)} - \frac{11t^2-3}{5(4t^2-1)} = \frac{4(1-2t^2)}{25(1-4t^2)} > 0$. In this case, $\Delta\Pi_1{=}\Pi_1^{\ddagger} - \Pi_1^{\dagger} \leq 0$ always holds.

We then examine the profits of platform 2's exclusive channel between scenarios CU and CD. By comparing $\Pi_{22}^{\dagger}$ and $\Pi_{22}^{\ddagger}$, we can obtain $\Delta\Pi_{22}{=}\Pi_{22}^{\ddagger}{-}\Pi_{22}^{\dagger}{=}a_{22}\omega^2{+}b_{22}\omega{+}c_{22}$, where $a_{22} = \frac{15\,(2t-1)^2\lambda(2t+1)^2\left(53t(\lambda-1)-200t^2+128\right)}{4\left(29t(\lambda-1)-200t^2+104\right)^2(1-2t^2)} \geq 0$, $b_{22} = 18062(t + \frac{1}{2})\lambda(\frac{2272}{9031} - \frac{80000}{9031}t^6 + \frac{19400}{9031}(1-\lambda)t^5 + (\lambda^2 - 2\lambda + \frac{89631}{9031})t^4 + \frac{27537}{9031}(\lambda-1)t^3 + (-\frac{2528}{9031}\lambda^2 + \frac{5056}{9031}\lambda - \frac{25272}{9031})t^2 + \frac{5881}{9031}(1-\lambda)t)(t - \frac{1}{2})/(5((\lambda-1)t+1)(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)) \geq 0$, and $c_{22} = -(63t^2{-}19)(1609\lambda^2t^4{-}200\lambda t^5{-}32000t^6{-}3218\lambda t^4{+}200t^5{-}407\lambda^2t^2{+}1893\lambda t^3{+}41409t^4{+}814\lambda t^2{-}1893t^3{-}289\lambda t{-}16623t^2 + 289t + 2368)\lambda/(100((\lambda-1)t+1)(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)) \leq 0$. Letting $\Delta\Pi_{22} = 0$, we have the threshold $\omega_{22} = \frac{19-63t^2}{25(1-4t^2)}$. Therefore, $\Delta\Pi_{22} \leq 0$ holds when $\omega \leq \omega_{22}$. Furthermore, by comparing ω_{22} with the upper bound of ω for which the equilibrium solutions exist, we find that $\frac{19-63t^2}{25(1-4t^2)} - \frac{11t^2-3}{5(4t^2-1)} = \frac{4(1-2t^2)}{25(1-4t^2)} > 0$. In this case, $\Delta\Pi_{22} = \Pi_{22}^{\ddagger} - \Pi_{22}^{\dagger} \leq 0$ always holds.

Finally, by comparing $\Pi_c^{\ddagger}$ and $\Pi_c^{\dagger}$, we have $\Delta\Pi_c{=}\Pi_c^{\ddagger}{-}\Pi_c^{\dagger}{=}a_c\omega^2{+}b_c\omega{+}c_c$, where $b_c = 18062\lambda(\frac{2272}{9031} - \frac{80000}{9031}t^6 + \frac{19400}{9031}(1-\lambda)t^5 + (\lambda^2 - 2\lambda + \frac{89631}{9031})t^4 + \frac{27537}{9031}(\lambda-1)t^3 + (-\frac{2528}{9031}\lambda^2 + \frac{5056}{9031}\lambda - \frac{25272}{9031})t^2 + \frac{5881}{9031}(1-\lambda)t)(t^2 - \frac{1}{4})/(5(2t^2-1)((\lambda-1)t+1)(29\lambda t - 200t^2 - 29t + 104)^2) \geq 0$, $c_c = -\lambda(1609\lambda^2t^4 - 200\lambda t^5 - 32000t^6 - 3218\lambda t^4 + 200t^5 - 407\lambda^2t^2 + 1893\lambda t^3 + 41409t^4 + 814\lambda t^2 - 1893t^3 - 289\lambda t - 16623t^2 + 289t + 2368)(63t^2-19)/(100(2t^2-1)((\lambda-1)t+1)(29\lambda t - 200t^2 - 29t + 104)^2) \leq 0$, and $a_c = \frac{15\,(2t-1)^2\,\lambda\left(53t(\lambda-1)-200\,t^2+128\right)(2t+1)^2}{4\left(29t(\lambda-1)-200\,t^2+104\right)^2(1-2t^2)} \geq 0$. Solving $\Delta\Pi_c = 0$, we have the threshold $\omega_c = \frac{19-63t^2}{25(1-4t^2)}$. Thus, $\Delta\Pi_c \geq 0$ holds when $\omega \leq \omega_c$. In addition, by comparing ω_c with the upper bound of ω for which the equilibrium outcomes exist, we find that $\frac{19-63t^2}{25(1-4t^2)} - \frac{11t^2-3}{5(4t^2-1)} = \frac{4(1-2t^2)}{25(1-4t^2)} > 0$. In this case, $\Delta\Pi_c = \Pi_c^{\ddagger} - \Pi_c^{\dagger} \geq 0$ always holds in the feasible range. Q.E.D.

Proof of Proposition 9

To analyze the impact of compatibility on the consumer surplus, we first define the surplus in each scenario and then systematically compare them respectively. In scenario I, the demand-side surplus is $S_d^I = \alpha\int_0^{v^I}[1 - p_1 + t\sqrt{s_1}]\,\mathrm{d}v + (1 - p_1 + t\sqrt{s_1})(1-\alpha) + \alpha\int_{v^I}^1[v - p_2 + t\sqrt{s_2}]\,\mathrm{d}v$, where $v^I = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) + c$. In scenario CU, the demand-side surplus is $S_d^{CU} = \int_0^{\underline{v}^{CU}}[1 - p_1 + t\sqrt{s_1}]\,\mathrm{d}v + \int_{\underline{v}^{CU}}^{\overline{v}^{CU}}[v - p_2 + t\sqrt{s_2} - (c-\omega)]\,\mathrm{d}v + \int_{\overline{v}^{CU}}^1[v - p_2 + t\sqrt{s_2} - c]\,\mathrm{d}v$, where $\underline{v}^{CU} = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) + c - \omega$, and $\overline{v}^{CU} = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) + c$. In scenario CD, the demand-side surplus is $S_d^{\ddagger} = \int_0^{\underline{v}^{CD}}[1 - p_1 + t\sqrt{s_1}]\,\mathrm{d}v + \int_{\underline{v}^{CD}}^{\overline{v}^{CD}}[v - (p_2 - \mu) + t\sqrt{s_2} - (c-\omega)]\,\mathrm{d}v + \int_{\overline{v}^{CD}}^1[v - p_2 + ts_2 - c]\,\mathrm{d}v$, where $\underline{v}^{CD} = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) - \omega - \mu + c$, and $\overline{v}^{CD} = 1 - (p_1 - p_2) + t(\sqrt{s_1} - \sqrt{s_2}) + c$.

Case 1: Scenario CU v.s. Scenario I. By comparing S_d^{CU} and S_d^I, we have $\Delta S_d^{CU} = S_d^{CU} - S_d^I = a_d^\dagger\omega^2 + b_d^\dagger\omega + c_d^\dagger$, where $a_d^\dagger = \frac{32t^4-24t^2+5}{8(2t^2-1)^2} \geq 0$, and $b_d^\dagger = \frac{16t^4-27t^2+7}{20(2t^2-1)^2} \geq 0$. Then, solving $\Delta S_d^{CU} = 0$ yields the threshold $\omega_d^{CU} = \frac{-48t^4+81t^2-21+2\sqrt{8640t^{10}-26136t^8+23172t^6-8429t^4+1218t^2-41}}{480t^4-360t^2+75}$. Thus, $\Delta S_d^{CU} \geq 0$ holds if $\omega \geq \omega_d^{CU}$.

Case 2: Scenario CD v.s. Scenario I. By comparing S_d^{CD} and S_d^I, we can show $\Delta S_d^{CD} = S_d^{CD} - S_d^I = a_d^\ddagger\omega^2 + b_d^\ddagger\omega + c_d^\ddagger$, where $a_d^\ddagger = [116240 + 1280000t^8 + -710400(\lambda-1)t^7 + (98768\lambda^2 - 197536\lambda - 2691632)t^6 + 1051136(\lambda-1)t^5 + (-76296\lambda^2 + 152592\lambda + 2206072)t^4 + -515792(\lambda-1)t^3 + (16265\lambda^2 - 32530\lambda - 818231)t^2 + 86880(\lambda-1)t]/[32\,(29t(\lambda-1) - 200t^2 + 104)^2(2t^2-1)^2] \geq 0$, $b_d^\ddagger = [94032 + 96000(\lambda-1)t^9 + (-13920\lambda^2 + 27840\lambda + 1624480)t^8 - 600256(\lambda-1)t^7 + (31068\lambda^2 - 62136\lambda - 3250468)t^6 + 735952(\lambda-1)t^5 + (-26193\lambda^2 + 52386\lambda + 2409391)t^4 - 331824(\lambda-1)t^3 + (5665\lambda^2 - 11330\lambda - 785391)t^2 + 49472(\lambda-1)t]/[16(29t(\lambda-1) - 200t^2 + 104)^2(2t^2-1)^2] \geq 0$, and $c_d^\ddagger = (271888 - 6912000t^{10} + 915840(\lambda-1)t^9 + (12528\lambda^2 - 25056\lambda + 24330096)t^8 - 3323088(\lambda-1)t^7 + (133641\lambda^2 - 267282\lambda - 27591735)t^6 + 2821632(\lambda-1)t^5 + (-62778\lambda^2 + 125556\lambda + 13926742)t^4 - 834512(\lambda-1)t^3 + (6097\lambda^2 - 12194\lambda - 3205135)t^2 + 77600(\lambda-1)t)/(288(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)^2) \leq 0$. Then, solving $\Delta S_d^{CD} = 0$ yields the threshold $\omega_d^{CD} = \frac{-b_d^\ddagger + \sqrt{(b_d^\ddagger)^2 - 4a_d^\ddagger c_d^\ddagger}}{2a_d^\ddagger}$. In this case, $\Delta S_d^{CD} \geq 0$ holds when $\omega \geq \omega_d^{CD}$.

Case 3: Scenario CD v.s. Scenario CU. For the demand side, we have $\Delta S_d = S_d^{CD} - S_d^{CU} = a_d\omega^2 + b_d\omega + c_d$, where $a_d = (167312 + 480000(\lambda-1)t^9 + (-69600\lambda^2 + 139200\lambda + 5562400)t^8 - 2258880(\lambda-1)t^7 + (101516\lambda^2 - 203032\lambda - 9323764)t^6 + 2040912(\lambda-1)t^5 + (-40137\lambda^2 + 80274\lambda + 5832759)t^4 - 682864(\lambda-1)t^3 + (4777\lambda^2 - 9554\lambda - 1617575)t^2 + 78464(\lambda-1)t)/(80(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)^2) \leq 0$, $b_d = [167312 + (480000\lambda - 480000)t^9 + (-69600\lambda^2 + 139200\lambda + 5562400)t^8 + (-2258880\lambda + 2258880)t^7 + (101516\lambda^2 - 203032\lambda - 9323764)t^6 + (2040912\lambda - 2040912)t^5 + (-40137\lambda^2 + 80274\lambda + 5832759)t^4 + (-682864\lambda + 682864)t^3 + (4777\lambda^2 - 9554\lambda - 1617575)t^2 + (78464\lambda - 78464)t]/[80(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)^2] \geq 0$, and $c_d = (63t^2-19)(6960\lambda^2t^6 + 48000(1-\lambda)t^7 - 13920\lambda t^6 - 7013\lambda^2t^4 + 91920(\lambda-1)t^5 + 48560t^6 + 14026\lambda t^4 + 1489\lambda^2t^2 - 43376(\lambda-1)t^3 - 2978\lambda t^2 + 5728(\lambda-1)t - 89253t^4 + 51201t^2 - 9136)/(800(29\lambda t - 200t^2 - 29t + 104)^2(2t^2-1)^2) \geq 0$. Then, solving $\Delta S_d = 0$ yields the threshold $\omega_d = \frac{19-63t^2}{25(1-4t^2)}$. Thus, $S_d^{CD} - S_d^{CU} \geq 0$ holds when $\omega \leq \omega_d$. In addition, by comparing ω_d with the upper bound of ω for which the equilibrium outcomes exist, we find that $\frac{19-63t^2}{25(1-4t^2)} - \frac{11t^2-3}{5(4t^2-1)} = \frac{4(1-2t^2)}{25(1-4t^2)} > 0$. In this case, $\Delta S_d = S_d^{CD} - S_d^{CU} \geq 0$ always holds true in the feasible range. Q.E.D.

Appendix B. Supplementary Analysis of Model Extensions

Extension 1: Variations in the Level of Awareness

In this extension, we relax the exogenous awareness assumption of the baseline model and allow the awareness level of the niche platform to depend on consumer valuation. Specifically, we assume that consumers who derive higher utility from the niche platform are more likely to be aware of it. Formally, the awareness level is given by $\alpha = \kappa v$, where $\kappa > 0$. For expository simplicity, κ is normalized to one. Under this setting, the awareness level becomes endogenous and increases in the relative attractiveness of the niche platform. In the incompatible case, this leads to a non-linear demand system, whereas in the compatible case, all consumers are aware of both platforms and the demand system coincides with the baseline model. Because endogenous awareness level introduces additional non-linearity, analytical comparisons become intractable, and we complement our analysis with numerical experiments. Next, we illustrate the details of model setting and solution under the CU scenario.

To simplify the analysis, we treat s_1 and s_2 as exogenous and solve the game model via backward induction. Under the CU scenario, the demand functions are $d_1^{CU} = t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 - \omega + 1$ and $d_2^{CU} = -t\sqrt{s_1} + t\sqrt{s_2} - c + p_1 - p_2 + \omega$. The profit functions of the two platforms are given by $\Pi_1^{CU}(p_1, p_2|\xi) = \lambda p_1(t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 - \omega + 1) + \xi\lambda p_2\omega$ and $\Pi_2^{CU}(p_1, p_2|\xi) = \lambda p_2(-t\sqrt{s_1} + t\sqrt{s_2} - c + p_1 - p_2) + (1-\xi)\lambda p_2\omega$. We first derive the equilibrium prices. The first-order condition of platform 1 with respect to p_1 yields $\partial\Pi_1^{CU}/\partial p_1 = \lambda(t\sqrt{s_1} - t\sqrt{s_2} + c - 2p_1 + p_2 - \omega + 1)$, with second-order derivative $-2\lambda < 0$, ensuring concavity. Setting the first-order condition to zero gives $p_1^{CU}(p_2) = t\sqrt{s_1}/2 - t\sqrt{s_2}/2 + c/2 + p_2/2 - \omega/2 + 1/2$. Similarly, for platform 2, the first-order condition is $\partial\Pi_2^{CU}/\partial p_2 = -\lambda(t\sqrt{s_1} - t\sqrt{s_2} + (-1+\xi)\omega + c - p_1 + 2p_2)$, with second-order derivative $-2\lambda < 0$. Setting it to zero yields $p_2^{CU}(p_1) = -t\sqrt{s_1}/2 + t\sqrt{s_2}/2 + (1-\xi)\omega/2 - c/2 + p_1/2$. Solving the two equations jointly, we obtain $p_1^{CU}(\xi) = t\sqrt{s_1}/3 - t\sqrt{s_2}/3 + (-\xi - 1)\omega/3 + c/3 + 2/3$ and $p_2^{CU}(\xi) = -t\sqrt{s_1}/3 + t\sqrt{s_2}/3 + (-2\xi + 1)\omega/3 - c/3 + 1/3$. Substituting these expressions into the profit functions and maximizing with respect to ξ, we obtain $\xi^{CU} = (-5t\sqrt{s_1} + 5t\sqrt{s_2} - 5c + 5\omega - 1)/10\omega$. Substituting back, we obtain equilibrium prices $p_1^{CU} = t\sqrt{s_1}/2 - t\sqrt{s_2}/2 + c/2 - \omega/2 + 7/10$ and $p_2^{CU} = 2/5$. Substituting all equilibrium decisions into the profit functions yields $\Pi_1^{CU} = \lambda(2t(-t\sqrt{s_2} + c - \omega + 1)\sqrt{s_1} - 2t(c - \omega + 1)\sqrt{s_2} + (s_1 + s_2)t^2 + c^2 + (-2\omega + 2)c + \omega^2 - 2\omega + 9/5)/4$ and $\Pi_2^{CU} = 4\lambda/25$.

Under incompatibility, the demand functions become $d_1^I = 1/2 + (t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 + 1)^2/2$ and $d_2^I = 1/2 - (t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 + 1)^2/2$. Substituting these expressions into the profit functions yields $\Pi_1^I(p_1, p_2) = \lambda p_1\big(1/2 + (t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 + 1)^2/2\big)$ and $\Pi_2^I(p_1, p_2) = \lambda p_2\big(1/2 - (t\sqrt{s_1} - t\sqrt{s_2} + c - p_1 + p_2 + 1)^2/2\big)$. Following the same differentiation procedure as in the compatible case, we obtain the equilibrium prices as $p_1^I = -(-5 - 2c - 2t\sqrt{s_1} - c^2 + 2t\sqrt{s_2} - s_1t^2 - s_2t^2 + 2\sqrt{s_1}\sqrt{s_2}t^2 - 2\sqrt{s_1}ct + 2\sqrt{s_2}ct)/(4(t\sqrt{s_1} - t\sqrt{s_2} + c + 1))$ and $p_2^I = ((t\sqrt{s_1} - t\sqrt{s_2} + c + 1)\sqrt{9}\sqrt{(2t(-t\sqrt{s_2} + c + 1)\sqrt{s_1} - 2t(c+1)\sqrt{s_2} + (s_1 + s_2)t^2 + c^2 + 2c + 7/3)^2/(t\sqrt{s_1} - t\sqrt{s_2} + c + 1)^2} - 12t(-t\sqrt{s_2} + c + 1)\sqrt{s_1} + 12t(c+1)\sqrt{s_2} + (-6s_1 - 6s_2)t^2 - 6c^2 - 12c + 2)/(12t\sqrt{s_1} - 12t\sqrt{s_2} + 12c + 12)$.

Comparing profits under CU and incompatibility yields several insights. First, when awareness is endogenously driven by consumer valuation, platform 1 already captures a relatively large market share even prior to compatibility, as higher valuation simultaneously increases both utility and awareness. This creates an inherent advantage for platform 1 even in the absence of compatibility. Second, although compatibility intensifies competition and may reduce platform 1's demand, it relaxes price competition through the revenue-sharing mechanism. As a result, both platforms can sustain higher prices, which potentially increases total industry profits. Third, allowing awareness to depend on valuation does not qualitatively alter the main insights of the baseline model. In particular, compatibility can still emerge as an equilibrium under certain parameter configurations, and the trade-off between demand expansion and intensified competition remains central.

Extension 2: The Effect of Travel Costs

In this extension, we replace the vertical differentiation of the baseline model with horizontal differentiation captured by travel costs. Specifically, consumers are uniformly distributed on a Hotelling line with location $x \sim \mathcal{U}[0,1]$. Platform 1 (the dominant platform) is located at 0, while platform 2 (the niche platform) is located at 1. Consumers incur travel costs when accessing a platform; for a consumer located at x, the travel cost is $\tau_1 x$ for platform 1 and $\tau_2(1-x)$ for platform 2, where τ_i denotes the marginal travel cost.

Let θ denote the base value. In the CU scenario, the utilities are given as follows. A consumer located at x obtains $u_{d1}^{CU} = \theta - \tau_1 x - p_1 + t\sqrt{s_1}$ from choosing platform 1, $u_{dc}^{CU} = \theta - \tau_2(1-x) - p_2 + t\sqrt{s_2} - c + \omega$ from choosing the compatible channel, and $u_{d2}^{CU} = \theta - \tau_2(1-x) - p_2 + t\sqrt{s_2} - c$ from choosing platform 2's exclusive channel. Compared with the baseline model, this formulation introduces horizontal differentiation through travel costs, while compatibility reduces the effective switching cost. We solve this extension setting by backward induction. On the demand side, equating utilities yields the demand for platform 1, the compatible channel, and platform 2's exclusive channel as $d_1^{CU} = (\tau_2 + (\sqrt{s_1} - \sqrt{s_2})t + c - \omega - p_1 + p_2)/(\tau_2 + \tau_1)$, $d_{21}^{CU} = \omega/(\tau_2 + \tau_1)$, and $d_{22}^{CU} = (\tau_1 + (-\sqrt{s_1} + \sqrt{s_2})t - c + p_1 - p_2)/(\tau_2 + \tau_1)$, respectively, with total demand for platform 2 given by $d_2 = d_{21} + d_{22}$. On the supply side, providers' utilities are $u_{s1}^{CU} = (1-\lambda)p_1 + td_1 - \sqrt{s_1}$ and $u_{s2}^{CU} = (1-\lambda)p_2 + td_2 - \sqrt{s_2}$. Solving $u_{s1} = 0$ and $u_{s2} = 0$ yields $s_1^{CU} = [td_1 - (\lambda-1)p_1]^2$ and $s_2^{CU} = [td_2 - (\lambda-1)p_2]^2$. Substituting these into the profit functions gives $\Pi_1(p_1, p_2|\xi) = \lambda p_1(t^2 + (p_1 - p_2)(-1+\lambda)t - c + \omega + p_1 - p_2 - \tau_2)/(2t^2 - \tau_1 - \tau_2) + \xi\lambda p_2\omega/(\tau_2 + \tau_1)$ and $\Pi_2(p_1, p_2|\xi) = (1-\xi)\lambda p_2\omega/(\tau_2+\tau_1) + \lambda p_2((t^2 - (p_1 - p_2)(-1+\lambda)t + c - \omega - p_1 + p_2 - \tau_1)/(2t^2 - \tau_1 - \tau_2) - \omega/(\tau_2 + \tau_1))$.

We next derive equilibrium prices. Solving the first-order condition for platform 1 yields the reaction function $p_1(p_2) = (-t^2 + (-1+\lambda)p_2 t + c - \omega + p_2 + \tau_2)/(2 + (-2+2\lambda)t)$, and similarly, platform 2's first-order condition yields $p_2(p_1) = (\tau_1^2 + (-t^2 + \tau_2 + (-1+\lambda)p_1 t + p_1 + (1-\xi)\omega - c)\tau_1 + (-t^2 + (-1+\lambda)p_1 t + p_1 + (1-\xi)\omega - c)\tau_2 + 2\xi t^2\omega)/(2(\tau_2+\tau_1)((-1+\lambda)t+1))$. Solving the two reaction functions jointly generates equilibrium prices as functions of ξ. Substituting these expressions into the profit functions and maximizing with respect to ξ, we obtain $\xi^{CU} = ((3t^2 + 5c - 5\omega - 4\tau_1 + \tau_2)(\tau_2 + \tau_1))/(10\omega(2t^2 - \tau_1 - \tau_2))$. Substituting back yields equilibrium prices $p_1^{CU} = (-9t^2 + 5c - 5\omega + 2\tau_1 + 7\tau_2)/(10 + (10\lambda - 10)t)$ and $p_2^{CU} = (-4t^2 + 2\tau_1 + 2\tau_2)/(5 + (5\lambda - 5)t)$. The corresponding equilibrium profits are $\Pi_1 = -\lambda((21t^4)/5 + (-2c + 2\omega - (16\tau_1)/5 - (26\tau_2)/5)t^2 + (9\tau_2^2)/5 + (2c - 2\omega + (8\tau_1)/5)\tau_2 + (4\tau_1^2)/5 + (c-\omega)^2)/(4(2t^2 - \tau_1 - \tau_2)(\lambda t - t + 1))$ and $\Pi_2 = -4\lambda(2t^2 - \tau_1 - \tau_2)/(25 + (25\lambda - 25)t)$.

Under incompatibility, logic analogous to the compatible case, we can obtain the equilibrium prices: $p_1^I = ((-3t^2 + c - \tau_1)\alpha + 2\tau_1 + 2\tau_2)/(3\alpha((-1+\lambda)t+1))$ and $p_2^I = ((-3t^2 - c + \tau_1)\alpha + \tau_1 + \tau_2)/(3\alpha((-1+\lambda)t+1))$, with the corresponding equilibrium profits given by $\Pi_1^I = -\lambda(((-3t^2 + c - \tau_1)\alpha + 2\tau_1 + 2\tau_2)^2)/(9\alpha(2\alpha t^2 - \tau_1 - \tau_2)(\lambda t - t + 1))$ and $\Pi_2^I = -\lambda(((3t^2 + c - \tau_1)\alpha - \tau_1 - \tau_2)^2)/(9\alpha(2\alpha t^2 - \tau_1 - \tau_2)(\lambda t - t + 1))$.

Comparing profits under compatibility and incompatibility yields several insights. First, introducing travel costs preserves the main mechanism of the baseline model: compatibility can emerge as an equilibrium when consumer awareness is sufficiently high and the compatibility parameter is sufficiently small. Second, when the difference in travel costs between the two platforms is small, consumers are less strongly attached to platform 1 even without compatibility. In this case, compatibility reallocates demand from platform 1 to platform 2, making it more difficult for platform 1 to offset demand losses through price increases. Third, from platform 2's perspective, compatibility entails a trade-off between improved access and increased revenue-sharing payments. As the two platforms become more similar in terms of travel costs, this trade-off becomes less favorable, thereby shrinking the parameter region in which compatibility emerges as an equilibrium.

Extension 3: Platform Merger

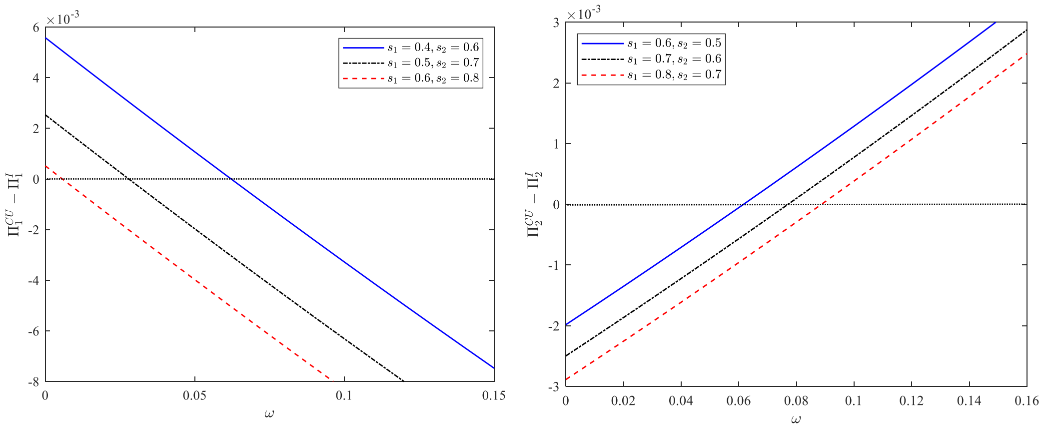
When platform mergers occur, the utilities of both the demand- and supply-side can be formulated as $u_d^m = \gamma + \omega - p_m + t\sqrt{s_m}$ and $u_s^m = (1-\lambda)p_m + td_m - l\sqrt{s_m}$. As such, we have $d_m = t\sqrt{s_m} + \omega - p_m + 1$ and $s_m = \frac{(td_m - \lambda p_m + p_m)^2}{l^2}$. Substituting them into the profit function of the merged platform yields $\Pi_m = \frac{\lambda p_m (l(\omega - p_m + 1) - p_m t(\lambda - 1))}{l - t^2}$. Taking the first-order and second-order conditions of Π_m with respect to p_m, we have $\frac{\partial \Pi_m}{\partial p_m} = \frac{\lambda((\omega - 2p_m + 1)l - 2p_m t(\lambda - 1))}{l - t^2}$, and $\frac{\partial^2 \Pi_m}{\partial p_m^2} = -\frac{2(t(\lambda-1)+l)\lambda}{l-t^2} \leq 0$. Therefore, Π_m is concave in p_m when $l - t^2 \leq 0$. Then, solving $\frac{\partial \Pi_m}{\partial p_m} = 0$, we can obtain the optimal price of the merged platform, i.e., $p_m^* = \frac{(\omega+1)l}{2(l+(\lambda-1)t)}$.

We first examine the equilibrium prices between the case of merger and scenario CU; specifically, by comparing p_m^* and p_1^{CU}, we can show that $p_m^* - p_1^{CU} = \frac{9t^2+5(\omega+1)(\lambda-1)t+10\omega-2}{10\,(1+(\lambda-1)t)\,((\lambda-1)t+l)} l + \frac{t\,(9t^2+5\omega-7)\,(\lambda-1)}{10\left(1+(\lambda-1)t\right)\left((\lambda-1)t+l\right)}$. We assumed that l is not is not so small that $(\lambda-1)t + l \geq 0$ can be guaranteed. In this case, we can obtain $\frac{t\,(9t^2+5\omega-7)\,(\lambda-1)}{10\left(1+(\lambda-1)t\right)\left((\lambda-1)t+l\right)} \geq 0$. In addition, if $\omega \leq \frac{2+5t-5\lambda t-9t^2}{5\,(\lambda t - t + 2)}$, then we have $\frac{9t^2+5(\omega+1)(\lambda-1)t+10\omega-2}{10\,(1+(\lambda-1)t)\,((\lambda-1)t+l)} \leq 0$. In this case, solving $p_m^* - p_1^{CU} = 0$ yields the threshold $\bar{l} = -\frac{t\left(9t^2+5\omega-7\right)(\lambda-1)}{9t^2+5(\omega+1)(\lambda-1)\,t+10\omega-2}$. Thus, $p_m^* - p_1^{CU} \leq 0$ when $l \geq \bar{l}$ and $\omega \leq \frac{2+5t-5\lambda t-9t^2}{5\,(\lambda t-t+2)}$. Similarly, by comparing p_m^* and p_2^{CU}, we have $p_m^* - p_2^{CU} = \frac{8t^2+5(\omega+1)(\lambda-1)t+5\omega+1}{10(1+(\lambda-1)t)((\lambda-1)t+l)} l + \frac{4(1-\lambda)(1-2t^2)}{10(1+(\lambda-1)t)((\lambda-1)t+l)}$, where $\frac{8t^2+5(\omega+1)(\lambda-1)t+5\omega+1}{10(1+(\lambda-1)t)((\lambda-1)t+l)} > 0$ and $\frac{4(1-\lambda)(1-2t^2)}{10(1+(\lambda-1)t)((\lambda-1)t+l)} > 0$. Thus, $p_m^* - p_2^{CU} > 0$ always holds.

Extension 4: Geographic Proximity Benefits

In the baseline setting, compatibility improves the matching efficiency through a reduction in switching costs. Here, we capture this effect by introducing waiting costs that depend on the size of the available driver pool. Specifically, we assume that when requesting a ride, consumers incur a waiting cost $d(s)$, where s denotes the number of available drivers. The waiting cost is decreasing and concave in s and approximated by $-1/\sqrt{s}$, capturing diminishing returns in matching efficiency.

Under incompatibility, each platform operates with its own driver pool. The utilities that a consumer obtains from choosing platform 1 and platform 2 are $u_{d1}^I = 1 - p_1 - r/\sqrt{s_1} + ts_1$ and $u_{d2}^I = v - p_2 - r/\sqrt{s_2} + ts_2 - c$. Under the CU scenario, platform 1 can access the combined driver pool, so the effective waiting cost becomes $-1/\sqrt{s_1+s_2}$, while platform 2's exclusive channel remains unchanged. Consumers can choose among three options, and utilities are adjusted accordingly. We first illustrate the equilibrium derivation under incompatibility. By equating utilities, we obtain demand functions $d_1^I = (\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + ((1 + (c - p_1 + p_2)\alpha)\sqrt{s_2} + \alpha r)\sqrt{s_1} - \sqrt{s_2}\alpha r)/(\sqrt{s_2}\sqrt{s_1})$ and $d_2^I = -\alpha(ts_1^{3/2}\sqrt{s_2} - ts_2^{3/2}\sqrt{s_1} + ((c - p_1 + p_2)\sqrt{s_2} + r)\sqrt{s_1} - r\sqrt{s_2})/(\sqrt{s_2}\sqrt{s_1})$. Substituting into profit functions yields $\Pi_1^I(p_1, p_2) = \lambda p_1(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + ((1 + (c - p_1 + p_2)\alpha)\sqrt{s_2} + \alpha r)\sqrt{s_1} - \sqrt{s_2}\alpha r)/(\sqrt{s_2}\sqrt{s_1})$ and $\Pi_2^I(p_1, p_2) = -\lambda p_2 \alpha(ts_1^{3/2}\sqrt{s_2} - ts_2^{3/2}\sqrt{s_1} + ((c - p_1 + p_2)\sqrt{s_2} + r)\sqrt{s_1} - r\sqrt{s_2})/(\sqrt{s_2}\sqrt{s_1})$. Solving the first-order condition for platform 1's problem yields $p_1(p_2) = (\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + ((1+(c+p_2)\alpha)\sqrt{s_2} + \alpha r)\sqrt{s_1} - \sqrt{s_2}\alpha r)/(2\sqrt{s_1}\sqrt{s_2}\alpha)$, and similarly for platform 2, we can obtain $p_2(p_1) = -(ts_1^{3/2}\sqrt{s_2} - ts_2^{3/2}\sqrt{s_1} + ((c-p_1)\sqrt{s_2} + r)\sqrt{s_1} - r\sqrt{s_2})/(2\sqrt{s_1}\sqrt{s_2})$. Solving jointly, we obtain equilibrium prices $p_1^I = (\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha r + 2\sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha r)/(3\sqrt{s_1}\sqrt{s_2}\alpha)$ and $p_2^I = -(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha r - \sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha r)/(3\sqrt{s_1}\sqrt{s_2}\alpha)$. Substituting back yields equilibrium profits $\Pi_1^I = \lambda\left(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha r + 2\sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha r\right)^2/(9\alpha s_1 s_2)$ and $\Pi_2^I = \lambda\left(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha r - \sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha r\right)^2/(9\alpha s_1 s_2)$. Following analogous steps, we obtain equilibrium prices under CU scenario as $p_1^{CU} = ((((-s_1 t - \omega)\xi + c - \omega + 2)\sqrt{s_2} - r\xi)\sqrt{s_1+s_2} + \sqrt{s_2}r\xi)/(3\sqrt{s_2}\sqrt{s_1+s_2})$ and $p_2^{CU} = -(((((2s_1t+2\omega)\xi+c-\omega-1)\sqrt{s_2}+2r\xi)\sqrt{s_1+s_2}-2\sqrt{s_2}r\xi)/(3\sqrt{s_2}\sqrt{s_1+s_2})))$.



(a) Illustration of Platform 1 (b) Illustration of Platform 2

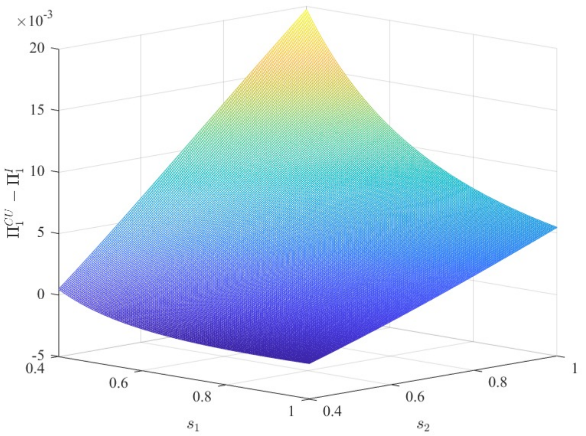
Figure 1: Profit Variance of Each Platform by Adopting Different Pricing Schemes (Parameter Value: $c = 0.05,\ \alpha = 0.9,\ \xi = 0.1,\ t = 0.2,\ \kappa = 0.5,\ \lambda = 0.2$)

Based on the setting above, numerical experiments are conducted to illustrate our analytical results. As we can see from Figure 1, we find that platform 2 is more likely to benefit from compatibility than platform 1. This result is driven by two factors. First, compatibility directly expands platform 2's dispatchable driver pool. Second, the establishment of the compatible channel does not substantially cannibalize demand from its exclusive channel. Thus, platform 2 has an incentive to participate in compatibility. In contrast, platform 1's incentive to participate in compatibility depends on the scale of platform 2's supply side. When platform 2 operates with a small provider base, compatibility leads to only a limited expansion of total supply; thus, this generates insufficient improvements in waiting time for platform 1's consumers. In this case, the compatible channel primarily reallocates consumers away from platform 1's exclusive channel, which adversely affects its profit. When platform 2's provider base is sufficiently large, however, compatibility substantially expands the total dispatchable supply and significantly reduces waiting times. This improvement increases consumers' utility from accessing the compatible channel. As a result, the demand expansion induced by shorter waiting times can partially or fully offset the cannibalization effect on platform 1's exclusive channel. Thus, when the compatibility-induced convenience gain, ω, is below a threshold, platform 1 can achieve a positive profit increment from compatibility and would chooses to participate in compatibility.

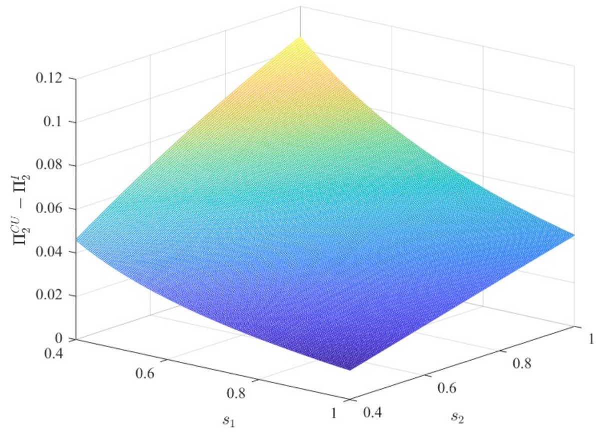
Extension 5: Probabilistic Routing

In this extension, we consider a probabilistic routing mechanism in which matching outcomes depend on the relative supply of drivers across platforms. Specifically, under compatibility, a consumer requesting service through platform 1 is matched to drivers from the two platforms with probabilities proportional to their supply shares. Let s_1 and s_2 denote the driver supply of platform 1 and platform 2, respectively. Under CU scenario, the total demand generated on platform 1 is denoted by d_{1c}. This demand is probabilistically allocated across the exclusive and compatible channels according to supply shares, such that $d_1 = \frac{s_1}{s_1+s_2}d$ and $d_c = \frac{s_2}{s_1+s_2}d$. Under the

CU scenario, consumers choosing platform 1 (including platform 1's exclusive and compatible channels) obtain utility $u_{d1c}^{CU} = 1 - p_1 + t\sqrt{s_1}$, while those choosing platform 2's exclusive channel obtain $u_{d22}^{CU} = \theta - p_2 + t\sqrt{s_2} - c$. This yields the demand of platform 1's exclusive and compatible channels $d_{1c}^{CU} = -t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1$. Accordingly, demand allocations are $d_1^{CU} = s_1(-t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1)/(s_2 + s_1)$, $d_c^{CU} = s_2(-t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1)/(s_2 + s_1)$, and platform 2's exclusive demand is $d_{22}^{CU} = t\sqrt{s_2} - t\sqrt{s_1} - c + p_1 - p_2$. Substituting into the profit functions yields $\Pi_1^{CU}(p_1, p_2) = \lambda p_1 s_1(-t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1)/(s_2 + s_1) + \xi\lambda p_2 s_2(-t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1)/(s_2 + s_1)$, and $\Pi_2^{CU}(p_1, p_2) = \lambda p_2(t\sqrt{s_2} - t\sqrt{s_1} - c + p_1 - p_2) + (1 - \xi)\lambda p_2 s_2(-t\sqrt{s_2} + t\sqrt{s_1} + c - p_1 + p_2 + 1)/(s_2 + s_1)$. Solving the first-order condition for platform 1's problem yields the reaction function $p_1(p_2) = (s_1^{3/2}t - \sqrt{s_2}s_1 t + (c + p_2 + 1)s_1 - \xi p_2 s_2)/(2s_1)$, with second-order condition $-2\lambda s_1/(s_2 + s_1) < 0$ ensuring concavity. Similarly, platform 2's reaction function is given by $p_2(p_1) = (-s_1^{3/2}t + s_2^{3/2}\xi t - \sqrt{s_1}\xi s_2 t + \sqrt{s_2}s_1 t + (1 + (-c + p_1 - 1)\xi)s_2 - s_1(c - p_1))/(2\xi s_2 + 2s_1)$. Solving the two functions jointly yields equilibrium prices $p_1^{CU} = (2s_1^{3/2}\xi s_2 t + s_1^{5/2}t - 2s_2^{3/2}\xi t s_1 - \xi^2 s_2^{5/2}t + \xi^2 s_2^2 t\sqrt{s_1} - \sqrt{s_2}s_1^2 t + s_2^2(c + 1)\xi^2 + 2(-s_2/2 + s_1(c + 1/2))s_2\xi + s_1(s_2 + s_1(c + 2)))/((\xi s_2 + s_1)(\xi s_2 + 3s_1))$ and $p_2^{CU} = -(s_1^{3/2}t - s_2^{3/2}\xi t + \sqrt{s_1}\xi s_2 t - \sqrt{s_2}s_1 t + (c - 1)s_1 + (-2 + (c + 1)\xi)s_2)s_1/((\xi s_2 + s_1)(\xi s_2 + 3s_1))$. Substituting back, the equilibrium profits are $\Pi_1^{CU} = \lambda(s_1^{3/2}t - \sqrt{s_2}s_1 t + s_1(c + 2) + s_2)^2 s_1/((s_2 + s_1)(\xi s_2 + 3s_1)^2)$ and $\Pi_2^{CU} = \lambda(s_1^{3/2}t - s_2^{3/2}\xi t + \sqrt{s_1}\xi s_2 t - \sqrt{s_2}s_1 t + (c - 1)s_1 + (-2 + (c + 1)\xi)s_2)^2 s_1^2/((s_2 + s_1)(\xi s_2 + 3s_1)^2(\xi s_2 + s_1))$. Following analogous steps, under incompatibility we obtain equilibrium prices $p_1^I = (\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha\kappa + 2\sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha\kappa)/(3\sqrt{s_1}\sqrt{s_2}\alpha)$ and $p_2^I = -(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha\kappa - \sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha\kappa)/(3\sqrt{s_1}\sqrt{s_2}\alpha)$, with profits $\Pi_1^I = \lambda(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha\kappa + 2\sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha\kappa)^2/(9\alpha s_1 s_2)$ and $\Pi_2^I = \lambda(\sqrt{s_2}s_1^{3/2}\alpha t - s_2^{3/2}\sqrt{s_1}\alpha t + (\sqrt{s_2}\alpha c + \alpha\kappa - \sqrt{s_2})\sqrt{s_1} - \sqrt{s_2}\alpha\kappa)^2/(9\alpha s_1 s_2)$.



(a) Illustration of Platform 1



(b) Illustration of Platform 2

Figure 2: Profit Variance of Each Platform in the $(s_1,\ s_2)$ Space (Parameter Value: $c = 0.1,\ \alpha = 0.9,\ \xi = 0.1,\ t = 0.2,\ \kappa = 0.5,\ \lambda = 0.2$)

Figure 2 depicts the variance in profit induced by platform compatibility under the probabilistic routing scheme. We find that when platform 2 has a sufficiently large driver base (s_2), compatibility allows consumers to access a larger pool of external service capacity in a probabilistic sense. Under the unified pricing scheme, this capacity-bolstering effect drives consumer demand. Thus, the demand expansion effect outweighs the potential channel diversion caused by compatibility, thereby increasing platform 1's total profit. In contrast, when s_2 is small, the additional capacity provided by compatibility is limited and fails to significantly improve matching outcomes; thus, platform 1 derives fewer benefits from compatibility. For platform 2, a larger driver pool can not only secure a higher matching weight within the probabilistic routing framework but also allow it to capitalize on its capacity advantage. This improvement in matching probabilities expands transaction volumes within the compatible channel. Therefore, platform 2 is also more inclined to adopt compatibility when its driver base is sufficiently large.

Extension 6: Multihoming of Supply Side

In this extension, we relax the single-homing assumption of the baseline model and allow service providers to multihome across platforms. The total mass of service providers is normalized to one. Providers can either single-home on platform 1 or platform 2, or multihome across platforms. Let s_1 and s_2 denote the mass of providers who exclusively serve platform 1 and platform 2, respectively, and the remaining providers choose multihoming. On the supply side, providers incur a cost ξ when participating in multiple platforms. The parameter h captures horizontal differentiation among providers. On the demand side, consumer utilities depend on platform prices and effective service capacity, which is determined by both exclusive and multihoming providers.

We first illustrate the equilibrium derivation under incompatibility. From the supply-side indifference conditions, letting providers be indifferent between single-homing and multihoming yields $s_1(d_1) = (2d_1t - p_1\lambda + p_2\lambda + 2h + p_1 - p_2 - 2t)/(2h)$ and $s_2(d_2) = -(2d_2t + p_1\lambda - p_2\lambda - p_1 + p_2 - 2t)/(2h)$. On the demand side, the consumer indifference condition implies $d_1(s_1, s_2) = \alpha((s_1 - s_2)t - p_1 + p_2 + 1) + 1 - \alpha$ and $d_2(s_1, s_2) = \alpha(-(s_1 - s_2)t + p_1 - p_2)$. Solving these equations jointly yields $d_1(p_1, p_2) = ((1 + (t - p_1 + p_2)\alpha)h - \alpha t^2)/h$ and $d_2(p_1, p_2) = -((t - p_1 + p_2)h - t^2)\alpha/h$. Substituting into the profit functions gives $\Pi_1^I(p_1, p_2) = \lambda p_1((1 + (t - p_1 + p_2)\alpha)h - \alpha t^2)/h$ and $\Pi_2^I(p_1, p_2) = -\lambda p_2((t - p_1 + p_2)h - t^2)\alpha/h$. Solving the first-order condition for platform 1's problem, we obtain equilibrium prices $p_1^I = (t(h - t)\alpha + 2h)/(3h\alpha)$ and $p_2^I = (-t(h - t)\alpha + h)/(3h\alpha)$. The corresponding equilibrium profits are $\Pi_1^I = \lambda(t(h - t)\alpha + 2h)^2/(9h^2\alpha)$ and $\Pi_2^I = \lambda(t(h - t)\alpha - h)^2/(9h^2\alpha)$. Following analogous steps, under CU scenario we obtain equilibrium prices $p_1^{CU} = ((((-\xi - 1)\omega + t + 2)h - t^2)/(3h))$ and $p_2^{CU} = ((((-2\xi + 1)\omega - t + 1)h + t^2)/(3h))$. The corresponding equilibrium profits are $\Pi_1^{CU} = -(5\lambda((((\xi^2 - \xi - 1/5)\omega^2 + ((\xi + 2/5)t + \xi/5 + 4/5)\omega - (t + 2)^2/5)h^2 - ((\xi + 2/5)\omega - (2t)/5 - 4/5)t^2h - t^4/5)))/(9h^2)$ and $\Pi_2^{CU} = (4\lambda((((\xi - 1/2)\omega + t/2 - 1/2)h - t^2/2)^2))/(9h^2)$.

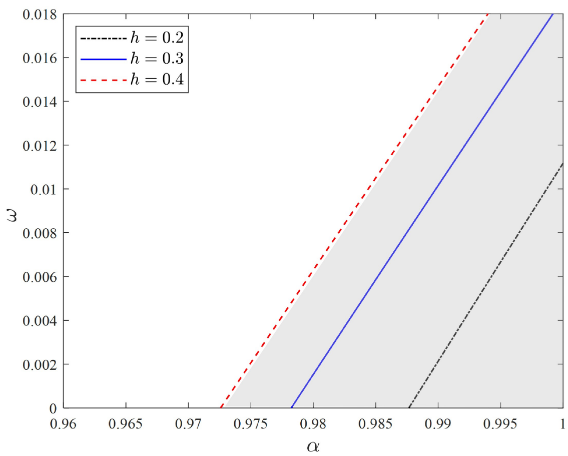


Figure 3: Effect of Multihoming on Equilibrium Strategy (Parameter Value: $c = 0$, $t = 0.2, \lambda = 0.2$, $\alpha \in [0.96, 1]$)

We numerically examine the effect of multihoming on platforms' compatibility decisions, as analytical comparisons are intractable (see Figure 3 for an illustration). Our analysis yields several robust insights. First, in the multihoming setting, both platforms benefit from adopting the unified pricing scheme. Specifically, platform 1 can improve its profitability under unified pricing, while platform 2 consistently gains from the same pricing scheme. This finding is consistent with the results of the baseline model. Second, the cost of multihoming plays a critical role. As h increases, the operating costs for supply-side providers rise, which in turn affects platforms' pricing incentives. In particular, under compatibility, platform 1 increases its price with h, thereby passing part of the increased operating cost onto consumers. As a result, compatibility becomes more attractive, and the parameter region in which the CU scenario emerges as an equilibrium expands with h.