Online Appendix to "The Role of Product Quality in Marketplaces"

Leela Nageswaran[a], Aditya Jain[b], Haresh Gurnani[c]
[a]Michael G. Foster School of Business, University of Washington, Seattle, WA 98195, lnages@uw.edu
[b]Zicklin School of Business, The City University of New York, New York, NY 10010, Aditya.Jain@baruch.cuny.edu
[c]College of Business, Stony Brook University, Stony Brook, NY 11794, haresh.gurnani@stonybrook.edu

Appendix A: Proofs of Main Results

Proof of Lemma 1. (i) From (1), the retailer's profit function is $(1-\frac{p}{q})(p-w)$. Then, the optimal price $p^w(q)=\frac{1}{2}(q+w)$ as long as $p^w(q)<q$. Substituting $p=p^w(q)$ in $p<q$, we get $\frac{1}{2}(q+w)<q$. This is equivalent to $w<q$. The supplier's profit is $(1-\frac{p}{q})(w-\kappa q^2)$ from (2) if $w<q$ and 0 otherwise, which is maximized at $w^*=\frac{q+\kappa q^2}{2}$. Enforcing the condition $w<q$ yields $\kappa q^2<q$ or $\kappa q<1$. Then the profits for the supplier and the retailer are $\frac{q}{8}(1-\kappa q)^2$ and $\frac{q}{16}(1-\kappa q)^2$, respectively.

(ii) The supplier's profit function in (2) is maximized at $p^m(q)=\frac{q}{2}+\frac{\kappa q^2}{2(1-\phi)}$. The condition $p<q$ is equivalent to $q>\frac{\kappa q^2}{1-\phi}$. Therefore, the supplier of quality q contracts if $q>\frac{\kappa q^2}{1-\phi}$ and does not contract otherwise. This condition is equivalent to $\phi<1-\kappa q$. Then we get profits $\pi^m_{(\cdot)}(q,\phi)$ as shown. $\square$

Proof of Proposition 1. This proof proceeds in two steps. In Step 1 we establish the commission rate under the pure marketplace contract. In Step 2 we derive the retailer's optimal contract choice.

Step 1: Establishing the commission rate under the pure marketplace contract.

The first and second derivatives of $\pi^m_r(q,\phi)=\frac{\phi}{4}q\left(1-\frac{(\kappa q)^2}{(1-\phi)^2}\right)$ with respect to (w.r.t.) ϕ are $\frac{1}{4}q\left(1-\frac{(1+\phi)\kappa^2q^2}{(1-\phi)^3}\right)$ and $-\frac{(2+\phi)(\kappa^2q^3)}{2(1-\phi)^4}$ respectively. Thus, the unconstrained maximizer of $\pi^m_r(q,\phi)$ is the unique solution to $(1-\phi)^3=\kappa^2q^2(1+\phi)$; let ϕ^b be this value. We have $\pi^m_r(q,0)=0$ and $\pi^m_r(q,1-\kappa q)=0$. Therefore, $\phi^b\in(0,1-\kappa q)$, and the optimal commission rate is ϕ^b, the value of $\phi\in(0,1)$ that solves $(1-\phi)^3=\kappa^2q^2(1+\phi)$.

Step 2: Deriving the retailer's optimal contract choice.

From Step 1, the unconstrained maximizer of $\pi^m_r(q,\phi)$ is ϕ^b. We need to compare $\pi^m_r(q,\phi^b)$ and $\pi^w_r(q)=\frac{q}{16}(1-\kappa q)^2$. The difference $\pi^m_r(q,\phi^b)-\pi^w_r(q)=\frac{q}{4}\phi^b\cdot\left(1-\frac{(\kappa q)^2}{(1-\phi^b)^2}\right)-\frac{q}{16}(1-\kappa q)^2=\frac{1}{16}q\left[4\phi^b\cdot\left(1-\frac{(\kappa q)^2}{(1-\phi^b)^2}\right)-(1-\kappa q)^2\right]$. By definition of ϕ^b we have $\frac{(1-\phi^b)^3}{(1+\phi^b)}=\kappa^2q^2$. Then substituting $(\kappa q)^2$ we get $\pi^m_r(q,\phi^b)-\pi^w_r(q)=\frac{1}{16}q\left[\frac{8(\phi^b)^2}{1+\phi^b}-(1-\kappa q)^2\right]$. We are left to show that $\frac{8(\phi^b)^2}{1+\phi^b}-\left(1-\sqrt{\frac{(1-\phi^b)^3}{(1+\phi^b)}}\right)^2>0$, which we do next. Equivalently, we need to show $8\phi^2-\left(\sqrt{1+\phi}-\sqrt{(1-\phi)^3}\right)^2>0$ for any ϕ. This simplifies to

$$2\sqrt{(1-\phi^2)}(1-\phi)>-\phi^3-5\phi^2-2\phi+2. \tag{6}$$

By Descartes' Rule of Signs, the right hand side (RHS) of (6) has one positive root. Since the value of the RHS of (6) is positive at $\phi=0$ and negative at $\phi=1/2$, we can conclude that its positive root occurs at some $\phi\in(0,1/2)$. Moreover, the RHS of (6) is negative for $\phi\geq1/2$. Since the left hand side (LHS) is non-negative, we can conclude that (6) holds true for $\phi\geq1/2$. To show (6) holds for $\phi<1/2$, we define $g(\phi)\equiv2\sqrt{(1-\phi^2)}(1-\phi)-(-\phi^3-5\phi^2-2\phi+2)$. Then $g(\phi)=2\sqrt{(1-\phi)(1+\phi)}(1-\phi)-(1+\phi)(2-\phi(4+\phi))=\sqrt{1+\phi}(2\sqrt{(1-\phi)}(1-\phi)-\sqrt{1+\phi}(2-\phi(4+\phi)))$. Since $\sqrt{1+\phi}$ is positive, we can show $g(\phi)\geq0$ if $2\sqrt{(1-\phi)}(1-\phi)\geq\sqrt{1+\phi}(2-\phi(4+\phi))$. Squaring both sides and simplifying, the inequality we need to show becomes $4(1-\phi)^3 \geq (1+\phi)(2-\phi(4+\phi))^2$, or $\phi^2(-\phi^3-9\phi^2-24\phi+16)>0$. Finally, $-\phi^3-9\phi^2-24\phi+16>0$: This is because the LHS is decreasing in ϕ and takes the value of 13/8 for $\phi=1/2$. □

Proof of Proposition 2. We prove (i). From (1), the retailer's profit function is $\pi_r^w(p_H,p_L) = d_H(p_H;q_H)(p_H-w_H)+d_L(p_L;q_L)(p_L-w_L)$. For this equilibrium to be feasible, we need $d_i(\cdot,\cdot)>0$. Substituting d_i into the retailer's profit function, and noting that the two profit terms can be optimized separately, we can conclude retail prices are as shown in the proposition statement. Since the optimal price for quality q_i product, p_i, only depends on wholesale price w_i, we can conclude that the wholesale prices are optimized separately. The demands are $d_i = \frac{1}{4}\gamma_i(1-\kappa q_i)>0$, since $\kappa q_H<1$.

It follows that it is optimal to contract with both suppliers. □

Proof of Proposition 3. We prove (i). From (2), we have that the suppliers' profit functions have negative second derivative w.r.t. p_i and the retail prices are as shown in the proposition. At the optimal prices, the demands are positive iff $\phi < 1-\kappa q_i$. Plugging in the optimal prices into the suppliers' and retailer's profit functions gives the required expressions.

The retailer's profit function in terms of ϕ has a negative second derivative. Thus, we can conclude that the retailer's optimal commission rate is the solution ϕ^m to the first order condition, and is given by: $(1-\phi)^3(\gamma_H q_H+\gamma_L q_L) = \kappa^2(1+\phi)(\gamma_H q_H^3+\gamma_L q_L^3)$, as long as it is less than $1-\kappa q_H$. Otherwise, the commission rate is $1-\kappa q_H$. Define $\gamma \equiv \gamma_H$ so that $\gamma_L = 1-\gamma$. Now, there is an interior solution as long as the slope of the profit function at $1-\kappa q_H$ is negative. This translates to the following condition: $\gamma\kappa^2(2q_H^3(-1+\kappa q_H)+2q_L^3-\kappa q_H q_L(q_H^2+q_L^2)) + \kappa^2(-2q_L^3+\kappa q_H q_L(q_H^2+q_L^2))<0$. The coefficient of $\gamma\kappa^2$ is $-2q_H^3+2q_L^3+\kappa q_H(q_H-q_L)(2q_H^2+q_Hq_L+q_L^2)$. Since $\kappa<1/q_H$, this quantity is less than $-2q_H^3+2q_L^3+(q_H-q_L)(2q_H^2+q_Hq_L+q_L^2) = -(q_H-q_L)q_L(q_H+q_L)<0$. Thus, the required condition is

$$\gamma > \frac{\kappa q_H q_L(q_H^2+q_L^2)-2q_L^3}{2(q_H^3-q_L^3)-\kappa q_H(q_H-q_L)(2q_H^2+q_Hq_L+q_L^2))}. \tag{7}$$

Next we find when the retailer finds it optimal to contract with both suppliers. It is easy to see that the retailer prefers contracting with both suppliers to contracting with the high quality supplier alone. Thus, we are left to compare their profit when contracting with both suppliers against that when contracting with the low quality supplier alone. Let us focus on γ larger than the threshold in (7); otherwise, an exclusive contract with the low quality supplier is more profitable.

Define $\underline{\phi}$ as the commission rate when contracting with both suppliers, that is, it solves $(1-\phi)^3(\gamma_H q_H+\gamma_L q_L) = \kappa^2(1+\phi)(\gamma_H q_H^3+\gamma_L q_L^3)$, and $\overline{\phi}$ as that when contracting with the low quality supplier, that is, it solves $(1-\phi)^3 = \kappa^2 q_L^2(1+\phi)$. Consider the difference between the optimal profits, the sign of which we need:

$$\xi(\gamma) \equiv \left(\gamma\pi_r^m(q_H,\underline{\phi})+(1-\gamma)\pi_r^m(q_L,\underline{\phi})\right)-(1-\gamma)\pi_r^m(q_L,\bar{\phi}). \tag{8}$$

The rest of the proof proceeds in three steps. We will show key properties of $\xi(\gamma)$ in Step 1; show that $\frac{d^2\xi(\gamma)}{d\gamma^2}>0$ in Step 2; and characterize $\xi(\gamma)$ in terms of γ in Step 3.

Step 1: Showing key properties of $\xi(\gamma)$.

When $\gamma = 0$ we have $\bar{\phi} = \underline{\phi}$ so that $\xi(0) = 0$. We will show that $\underline{\phi}$ is decreasing in γ. The ratio $\frac{\gamma q_H^3 + (1-\gamma) q_L^3}{\gamma q_H + (1-\gamma) q_L}$ is increasing in γ, since its first derivative in γ is $\frac{(q_H^2 - q_L^2) q_H q_L}{(\gamma q_H + (1-\gamma) q_L)^2} > 0$. $\underline{\phi}$ satisfies $\frac{(1-\phi)^3}{(1+\phi)} = \kappa^2 \frac{\gamma q_H^3 + (1-\gamma) q_L^3}{\gamma q_H + (1-\gamma) q_L}$, where the LHS is decreasing in ϕ. This combined with the RHS increasing in γ, implies that $\underline{\phi}$ is decreasing in γ.

Step 2: Showing that $\frac{d^2 \xi(\gamma)}{d\gamma^2} > 0$.

Since $\bar{\phi}$ does not depend on γ, we have $\frac{d^2 \xi(\gamma)}{d\gamma^2} = \frac{d^2}{d\gamma^2} \left(\gamma \pi_r^m (q_H, \underline{\phi}) + (1-\gamma) \pi_r^m (q_L, \underline{\phi}) \right)$. Evaluating this derivative by substituting the functional form of $\pi_r^m(q, \phi)$ and employing the envelope theorem, we get $\frac{d^2 \xi(\gamma)}{d\gamma^2} = (q_H - q_L) \frac{1}{4} \cdot \frac{d}{d\gamma} \left[\underline{\phi} \cdot \left(1 - \frac{\kappa^2}{(1-\underline{\phi})^2} (q_H^2 + q_H q_L + q_L^2) \right) \right]$. Since $q_H > q_L$, in order to show that $\frac{d^2 \xi(\gamma)}{d\gamma^2} > 0$ we effectively need to show that $\frac{d}{d\gamma} \left[\underline{\phi} \cdot \left[1 - \frac{\kappa^2}{(1-\underline{\phi})^2} (q_H^2 + q_H q_L + q_L^2) \right] \right] > 0$. By using the chain rule, we have $\frac{d}{d\gamma} \left[\underline{\phi} - \frac{\kappa^2 \underline{\phi}}{(1-\underline{\phi})^2} (q_H^2 + q_H q_L + q_L^2) \right] = \frac{d}{d\underline{\phi}} \left[\underline{\phi} - \frac{\kappa^2 \underline{\phi}}{(1-\underline{\phi})^2} (q_H^2 + q_H q_L + q_L^2) \right] \cdot \frac{d\underline{\phi}}{d\gamma}$. This simplifies to $\left[1 - (q_H^2 + q_H q_L + q_L^2) \kappa^2 \frac{1+\underline{\phi}}{(1-\underline{\phi})^3} \right] \cdot \frac{d\underline{\phi}}{d\gamma}$. Using its definition, $(1-\underline{\phi})^3 = \kappa^2 \frac{\gamma q_H^3 + (1-\gamma) q_L^3}{\gamma q_H + (1-\gamma) q_L} (1+\underline{\phi})$ we can further simplify this expression as $\left[1 - (q_H^2 + q_H q_L + q_L^2) \frac{\gamma q_H + (1-\gamma) q_L}{(\gamma q_H^3 + (1-\gamma) q_L^3)} \right] \cdot \frac{d\underline{\phi}}{d\gamma} = -\frac{q_H q_L (q_H + q_L)}{\gamma (q_H^3 - q_L^3) + q_L^3} \cdot \frac{d\underline{\phi}}{d\gamma}$. Since $\underline{\phi}$ is decreasing in γ (see Step 1), we can conclude that $\frac{d}{d\gamma} \left[\underline{\phi} \cdot \left[1 - \frac{\kappa^2}{(1-\underline{\phi})^2} (q_H^2 + q_H q_L + q_L^2) \right] \right] > 0$, and therefore, $\frac{d^2 \xi(\gamma)}{d\gamma^2} > 0$.

Step 3: Characterizing $\xi(\gamma)$ in terms of γ.

Let's consider the following two possibilities: (i) Suppose $\frac{d\xi(\gamma)}{d\gamma} \geq 0$ at $\gamma = 0$. Then due to Step 2, $\frac{d\xi(\gamma)}{d\gamma} > 0, \forall \gamma > 0$. Combined with Step 1, this implies that $\xi(\gamma) > 0, \forall \gamma > 0$, or $\phi^m = \underline{\phi}$. (ii) Suppose $\frac{d\xi(\gamma)}{d\gamma} < 0$ at $\gamma = 0$. Then due to Step 2, $\frac{d\xi(\gamma)}{d\gamma} < 0$ for low γ and $\frac{d\xi(\gamma)}{d\gamma} \geq 0$ otherwise. Combined with Step 1, this implies that there exists γ^m such that $\xi(\gamma) < 0$ for $\gamma \leq \gamma^m$, which implies $\phi^m = \bar{\phi}$. And $\xi(\gamma) \geq 0$ otherwise, which implies $\phi^m = \underline{\phi}$. By redefining γ^m as the maximum of the above γ^m and the threshold in (7), we have our result. □

Proof of Proposition 4. We start with (i). In this case, the prices are as in Lemma 1 under the wholesale contract for the high-quality product and under the marketplace contract for the low-quality product. Working backwards, we get the wholesale price for the high quality product as in Lemma 1 as well. The retailer's profit is then maximized at the ϕ that is tailored to maximize profits when partnering with the low-quality supplier alone. This concludes the proof of (i).

Next we find when the retailer finds it optimal to contract in WM vs. MW. Define $\gamma \equiv \gamma_H$ so that $\gamma_L = 1 - \gamma$. Define ϕ_L as the commission rate under WM, that is, it solves $(1-\phi)^3 = \kappa^2 q_L^2 (1+\phi)$, and ϕ_H as the commission rate under MW, that is, it solves $(1-\phi)^3 = \kappa^2 q_H^2 (1+\phi)$. Consider the difference between the optimal profits: $\xi_h(\gamma) \equiv (\gamma \pi_r^w (q_H) + (1-\gamma) \pi_r^m (q_L, \phi_L)) - (\gamma \pi_r^m (q_H, \phi_H) + (1-\gamma) \pi_r^w (q_L))$. Now, $\xi_h(\gamma)$ is decreasing in γ: The slope of $\xi_h(\gamma)$ w.r.t. γ is $\pi_r^w(q_H) - \pi_r^m(q_H, \phi_H) + \pi_r^w(q_L) - \pi_r^m(q_L, \phi_L) < 0$. Here the inequality follows from Proposition 1, which establishes that $\pi_r^w(q_H) - \pi_r^m(q_H, \phi_H) < 0$ and $\pi_r^w(q_L) - \pi_r^m(q_L, \phi_L) < 0$. This also implies that it takes positive and negative values at $\gamma = 0$ and $\gamma = 1$, respectively. Combined with $\xi_h(\gamma)$ being linear in γ, we can conclude that $\exists \gamma_h$ such that $\xi_h(\gamma) > 0$ iff $\gamma < \gamma_h$. □

Proof of Proposition 5. It is easy to see from Proposition 1 that the hybrid contracts are better than pure wholesale. Moreover, WM is better than pure marketplace with only the low quality supplier. We are left to compare the two hybrid contracts against the pure marketplace contract with both supplier types. Define $\gamma \equiv \gamma_H$ so that $\gamma_L = 1 - \gamma$. Depending on the value of γ relative to γ^h, the profit under hybrid could be that under WM or MW. Define the two functions: $\psi_I(\gamma) \equiv \gamma \pi_r^w (q_H) + (1-\gamma) \pi_r^m (q_L, \phi_L) -$

$\left(\gamma\pi_r^m(q_H,\underline{\phi})+(1-\gamma)\pi_r^m(q_L,\underline{\phi})\right), \psi_{II}(\gamma)\equiv\gamma\pi_r^m(q_H,\phi_H)+(1-\gamma)\pi_r^w(q_L)-\left(\gamma\pi_r^m(q_H,\underline{\phi})+(1-\gamma)\pi_r^m(q_L,\underline{\phi})\right).$ Here ϕ_i, independent of γ, is the commission rate for quality q_i supplier under the hybrid contract, and $\underline{\phi}$ is the commission rate under the pure marketplace contract when partnering with both suppliers and it is decreasing in γ (from the proof of Proposition 3).

When $\gamma=0$ we have $\underline{\phi}=\phi_L$ so that $\psi_I(0)=0$. When $\gamma=1$ we have $\underline{\phi}=\phi_H$ so that $\psi_{II}(1)=0$. Since ϕ_i does not depend on γ, we have $\frac{d^2\psi_{(\cdot)}(\gamma)}{d\gamma^2}=-\frac{d^2}{d\gamma^2}\left(\gamma\pi_r^m(q_H,\underline{\phi})+(1-\gamma)\pi_r^m(q_L,\underline{\phi})\right)$. From Step 2 in the proof of Proposition 3 we can conclude that $\frac{d^2\psi_{(\cdot)}(\gamma)}{d\gamma^2}<0$. The sign of the profit difference depends on the signs of $\psi_{(\cdot)}(\gamma^h)$, the slope of $\psi_I(\gamma)$ at $\gamma=0$, and the slope of $\psi_{II}(\gamma)$ at $\gamma=1$. We can conclude that the profit under pure marketplace exceeds that under WM and MW for intermediate values of γ. Further imposing $\gamma>\gamma^m$, so that the pure marketplace contract involves partnering with both suppliers, we get our result.

Furthermore, we can derive when (a) the slope of $\psi_I(\gamma)$ at $\gamma=0$ is negative and (b) the slope of $\psi_{II}(\gamma)$ at $\gamma=1$ is positive. For the former (a) we need $\pi_r^w(q_H)-\pi_r^m(q_H,\phi_L)-\frac{d\pi_r^m(q_L,\underline{\phi})}{d\gamma}|_{\gamma=0}<0$. Since $\frac{d\pi_r^m(q_L,\phi)}{d\phi}|_{\gamma=0}=0$, we need $\pi_r^w(q_H)-\pi_r^m(q_H,\phi_L)<0$. Now $[\pi_r^w(q_H)-\pi_r^m(q_H,\phi_L)]/q_H=\frac{1}{16}(1-\kappa q_H)^2-\frac{\phi_L}{4}\left(1-\frac{(\kappa q_H)^2}{(1-\phi_L)^2}\right)$ is a quadratic in q_H with a positive leading coefficient and it takes a negative value at $q_H=q_L$, so we need q_H low enough, say $q_H<\underline{q}$, for this condition. For the latter (b), proceeding similarly, we need $\pi_r^m(q_L,\phi_H)-\pi_r^w(q_L)>0$. For $q_H=q_L$ we have $\pi_r^m(q_L,\phi_H)-\pi_r^w(q_L)=\pi_r^m(q_L,\phi_L)-\pi_r^w(q_L)>0$ and $\pi_r^m(q_L,\phi)$ achieves its maximum at $\phi=\phi_L$. As q_H increases ϕ_H decreases so that $\pi_r^m(q_L,\phi)$, and therefore $\pi_r^m(q_L,\phi_H)-\pi_r^w(q_L)$, also decreases. Once again, we need q_H low enough, say $q_H<\overline{q}$, for this latter condition. Now $\overline{q}>\underline{q}$: This can be shown by plugging in the value $q_H=\overline{q}$ into $\pi_r^w(q_H)-\pi_r^m(q_H,\phi_L)$ which results in a positive value for all q_L. Given the shape of $\psi_{(\cdot)}$, i.e., $\frac{d^2\psi_{(\cdot)}(\gamma)}{d\gamma^2}<0$, $\psi_I(0)=0$, $\psi_{II}(1)=0$, and that $\psi_I(1)<0$ and $\psi_{II}(0)<0$, we can define another threshold $\tilde{q}$ as the value of q_H such that $\psi_I(\gamma^h)=\psi_{II}(\gamma^h)=0$.

Putting all the above statements together we have the optimal contract choice is as follows: (i) when $q_H\le\underline{q}$, pure marketplace for all γ; (ii) when $\underline{q}<q_H\le\overline{q}$, WM for $\gamma<\underline{\gamma}$ and pure marketplace otherwise; (iii) when $\overline{q}<q_H\le\tilde{q}$, WM for $\gamma<\underline{\gamma}$, pure marketplace for $\underline{\gamma}<\gamma<\overline{\gamma}$ and MW otherwise; and finally, (iv) when $q_H>\tilde{q}$, WM $\gamma<\gamma^h$ and MW otherwise. □

Proof of Proposition 6. This proof proceeds in three steps. In Step 1 we derive the best response functions for the supplier and the retailer. In Step 2 we show that the pure marketplace contract is a pure strategy equilibrium. In Step 3 we show that the pure wholesale contract is not a pure strategy equilibrium.

Step 1: Deriving the best response functions for the supplier and the retailer.

Suppose the supplier chooses quality q and the retailer chooses to offer marketplace or wholesale mode in the first stage. Based on Lemma 1, we can write the payoff matrix for both players. We can then derive the best response of each player. For the supplier, the profit maximizing quality choice when the retailer chooses wholesale mode is $1/(3\kappa)$ and that when the retailer chooses marketplace mode is $(1-\phi)/(3\kappa)$. For the retailer, the profit maximizing commission rate $\phi^m(q)$ under the marketplace mode is the solution to $\frac{(1-\phi)^3}{1+\phi}=(\kappa q)^2$ given the supplier chooses quality q (Proposition 1).

Step 2: Showing that the pure marketplace contract is a pure strategy equilibrium.

We need both parties to not deviate from this equilibrium. The supplier will not choose any other quality since $(1-\phi)/(3\kappa)$ is their best response. We are left to ensure that the retailer does not deviate to the other pure strategy (i.e., wholesale contract). From the retailer's perspective, the commission rate under marketplace (setting $q=(1-\phi)/(3\kappa)$) solves $\frac{1-\phi}{1+\phi}=\frac{1}{9}$, or $\phi^m=0.8$. Then the retailer's profit under marketplace is $0.0118519/\kappa$. This exceeds their profit under the wholesale contract, $0.00362963/\kappa$, obtained by substituting the supplier's chosen quality $(1-\phi^m)/(3\kappa)$ into the retailer's payoff function under the wholesale contract. The quality level chosen under the pure marketplace contract is $1/(15\kappa)$.

Step 3: Showing that the pure wholesale contract is not a pure strategy equilibrium.

We show that the retailer will deviate from this equilibrium. Recall that the supplier chooses quality $q=1/(3\kappa)$ under this strategy. From the retailer's perspective, the optimal commission rate under marketplace when considering the supplier's quality choice $q=1/(3\kappa)$ solves $\frac{(1-\phi)^3}{1+\phi}=1/9$, or $\phi^m\approx 0.455212$. Then the retailer's profit under marketplace is $0.0237328/\kappa$, which exceeds that under wholesale (i.e., $0.00925926/\kappa$). So the retailer will deviate from this equilibrium. □

Proof of Proposition 7. Under the WM outcome, the best responses are $q_H=1/(3\kappa), q_L=(1-\phi)/(3\kappa)$ and $\phi=\phi_L$, where ϕ_L is the solution to $\frac{(1-\phi)^3}{1+\phi}=(\kappa q_L)^2$ (from the proof of Proposition 6). Then, $q_H^*=1/(3\kappa), q_L^*=1/(15\kappa), \phi^*=0.8$ with the retailer's profit $\Pi^H(\gamma_H)=(32-7\gamma_H)/(2700\kappa)$. We need to consider all the possible deviations for the retailer. By deviating to the pure wholesale contract, the retailer's profit would be $(98+27\gamma_H)/(13500\kappa)$. This is smaller than $\Pi^H(\gamma_H), \forall\gamma_H$. By deviating to the other hybrid contract MW, the retailer can re-optimize the commission rate to solve $\frac{(1-\phi)^3}{1+\phi}=(\kappa q_H^*)^2$, that is, $\phi^{MW}=0.455212$ (from the proof of Proposition 6) with a profit of $(0.00362963+0.0201032\gamma_H)/\kappa$. For low enough $\gamma_H<0.36228$, this is smaller than $\Pi^H(\gamma_H)$. By deviating to an exclusive pure marketplace contract, the retailer is strictly worse off, since WM (MW, respectively) is better than an exclusive marketplace contract with the low type (high type, respectively). By deviating to the pure marketplace contract with both types, the retailer can re-optimize the commission rate to ϕ^M, the solution to $(1-\phi)^3=\kappa^2\frac{\sum_i \gamma_i (q_i^*)^3}{\sum_i \gamma_i q_i^*}(1+\phi)$ (Proposition 3); let the retailer's profit be $\Pi^m(\gamma_H)\equiv\frac{\phi\left(4\gamma_H\left(225\phi^2-450\phi+194\right)+225\phi^2-450\phi+224\right)}{13500\kappa(\phi-1)^2}$ with $\phi=\phi^M$. We want to see when $\Pi^m(\gamma_H)<\Pi^H(\gamma_H)$. $\Pi^m(0)=\Pi^H(0)$, and from the proof of Proposition 3, $\frac{d^2\Pi^m(\gamma_H)}{d\gamma_H^2}>0$. Since $\Pi^H(\gamma_H)$ is decreasing in γ_H, this implies that may exist a $\gamma_H\in(0,1)$ where the two curves cross each other. By plugging in the optimal ϕ^M in terms of γ_H and solving for the crossing point, we get its value as $\gamma_H\approx 0.2$. Since this is smaller than the previous threshold on γ_H to not deviate to MW, we can conclude that the hybrid contract with WM outcome is a pure strategy equilibrium for $\gamma_H<\overline{\gamma}_E\approx 0.2$.

Consider the pure marketplace contract. Proceeding as in the proof above, the quality levels chosen are $q_L^*=q_H^*=1/(15\kappa)$ and $\phi^*=0.8$. Deviating to the pure wholesale or the hybrid contracts will not make the retailer better off (Proposition 1). Thus, the pure marketplace contract is also a pure strategy equilibrium.

Lastly, we rule out the MW equilibrium under the hybrid contract and the pure wholesale contract. For MW, proceeding as in the proof above, there is no feasible equilibrium because the quality levels chosen violate the assumption that $q_H>q_L$. For the pure wholesale contract, identical quality levels are chosen, in which case, we know from Proposition 1 that the retailer will deviate to the pure marketplace contract. □

Proof of Proposition 8. We will use the following lemma in this proof.

LEMMA A1. *Define* $g(\phi,x)\equiv 4\phi\left(1-\frac{(x)^2}{(1-\phi)^2}\right)-(1-x)^2$. *Then* $g\left(\phi,\frac{\sqrt{2(1-\phi)}-1}{\sqrt{2/(1-\phi)}-1}\right)>0$ *for* $\phi\in(0,1/2)$.

Proof. By definition, $g(\phi,x)=-x^2\frac{(1+\phi)^2}{(1-\phi)^2}+2x+(4\phi-1)$, which is a quadratic in x with a negative leading coefficient. Since its discriminant is $\frac{16\phi^2(\phi+2)}{(1-\phi)^2}>0$, $g(\phi,x)>0$ for those values of x between the two roots of this quadratic. The smaller and larger roots of the quadratic are $x_1(\phi)\equiv\frac{1-\phi}{(1+\phi)^2}(1-\phi-2\phi\sqrt{2+\phi})$ and $x_2(\phi)\equiv\frac{1-\phi}{(1+\phi)^2}(1-\phi+2\phi\sqrt{2+\phi})$ respectively. Given $x_0(\phi)\equiv\frac{\sqrt{2(1-\phi)}-1}{\sqrt{2/(1-\phi)}-1}$, we then need to show that $x_1(\phi)<x_0(\phi)<x_2(\phi)$ for $\phi\in(0,1/2)$. We will show this result in four steps:

In Step 1, we will show that $x_1(\phi)<\frac{1-2(1-\phi)}{1-2/(1-\phi)}$: Since $2\sqrt{2+\phi}>1$, we have $x_1(\phi)<\frac{1-\phi}{(1+\phi)^2}(1-2\phi)$. Now, $\frac{2(1-\phi)-1}{2/(1-\phi)-1}=\frac{1-2\phi}{1+\phi}(1-\phi)$. This implies that $x_1(\phi)<\frac{1-\phi}{(1+\phi)^2}(1-2\phi)<\frac{1-\phi}{1+\phi}(1-2\phi)=\frac{2(1-\phi)-1}{2/(1-\phi)-1}$, where the inequality uses $1<1+\phi$ and $\phi<1/2$.

In Step 2, we will show that $\frac{1-2(1-\phi)}{1-2/(1-\phi)}<x_0(\phi)$: This can be equivalently expressed by cross multiplying as $(2(1-\phi)-1)(\sqrt{2/(1-\phi)}-1)-(2/(1-\phi)-1)(\sqrt{2(1-\phi)}-1)<0$. The LHS simplifies to $3\sqrt{2(1-\phi)}-2(1-\phi)-3\sqrt{2/(1-\phi)}+2/(1-\phi)$ or $3\sqrt{2}(\sqrt{1-\phi}-\frac{1}{\sqrt{1-\phi}})+2(\frac{1}{1-\phi}-(1-\phi))=-3\sqrt{2}\frac{\phi}{\sqrt{1-\phi}}+\frac{2\phi}{1-\phi}(2-\phi)$. To show $-3\sqrt{2}\frac{\phi}{\sqrt{1-\phi}}+\frac{2\phi}{1-\phi}(2-\phi)<0$ we effectively need to show that $\sqrt{2}(2-\phi)<3\sqrt{1-\phi}$, or $2(2-\phi)^2-9(1-\phi)<0$. The LHS of this inequality is $2\phi^2+\phi-1$, which is less than 0 for $\phi<1/2$.

In Step 3, we will show that $x_0(\phi)<\frac{1-\phi}{1+\phi}$: This can be equivalently expressed by cross multiplying as $(\sqrt{2(1-\phi)}-1)(1+\phi)-(1-\phi)(\sqrt{2/(1-\phi)}-1)<0$. The LHS simplifies to $\sqrt{2(1-\phi)}(1+\phi)-2\phi-\sqrt{2(1-\phi)}=(\sqrt{2(1-\phi)}-2)\phi<0$.

In Step 4, we will show that $\frac{1-\phi}{1+\phi}<x_2(\phi)$: Since $\sqrt{2+\phi}>1$, we have $x_2(\phi)>\frac{1-\phi}{(1+\phi)^2}(1+\phi)=\frac{1-\phi}{1+\phi}$. □

We now prove Proposition 8. The maximum commission rate for the quality q_i supplier to opt for marketplace mode is the solution to $\sqrt{\frac{1-\phi}{2}}=1-\frac{\phi}{1-\kappa q_i}$, obtained by rearranging the supplier's profit terms from Lemma 1. Moreover, the low-quality supplier is willing to tolerate a higher commission rate than the high-quality supplier, so the MW outcome is infeasible; define $\tilde{\phi}$ as this threshold for the low-quality supplier.

We next show that the retailer's optimal commission rate under hybrid is $\tilde{\phi}$: The retailer's profit under WM is $\pi_H(\phi)\equiv\gamma_H\pi_r^w(q_H)+\gamma_L\pi_r^m(q_L,\phi)$, whose derivative w.r.t. ϕ is $\gamma_L q_L\frac{(1-\phi)^3-(\kappa q_L)^2(1+\phi)}{4(1-\phi)^3}$. Define $\bar{\phi}$ as the commission rate when contracting with the low quality supplier alone, that is, it solves $(1-\phi)^3=\kappa^2 q_L^2(1+\phi)$. Then, $\pi_H(\phi)$ is increasing for $\phi\le\tilde{\phi}$ as long as $\tilde{\phi}<\bar{\phi}$ (based on the proof of Proposition 3). $\tilde{\phi}<\bar{\phi}$ is true as follows: By the definition of $\tilde{\phi}$, we have that for any $\phi>\tilde{\phi}$, we must have $\sqrt{\frac{1-\phi}{2}}>1-\frac{\phi}{1-\kappa q_L}$. Thus we need to show that $\sqrt{\frac{1-\bar{\phi}}{2}}+\frac{\bar{\phi}}{1-\kappa q_L}>1$. Plugging in $\bar{\phi}$ and noting that $(\kappa q_L)^2=(1-\bar{\phi})^3/(1+\bar{\phi})$, we need to show that $\sqrt{\frac{1-\bar{\phi}}{2}}+\frac{\bar{\phi}}{1-\sqrt{(1-\bar{\phi})^3/(1+\bar{\phi})}}>1$. We will show that $\sqrt{\frac{1-\phi}{2}}+\frac{\phi}{1-\sqrt{(1-\phi)^3/(1+\phi)}}>1$ for any $\phi\in(0,1)$, as follows. Cross-multiplying and simplifying, we need to show $\sqrt{(1-\phi)/2}(\sqrt{1+\phi}-(1-\phi)\sqrt{1-\phi})+\phi\sqrt{1+\phi}>\sqrt{1+\phi}-(1-\phi)\sqrt{1-\phi}$. This simplifies to $\sqrt{(1-\phi)/2}\sqrt{1+\phi}-(1-\phi)^2/\sqrt{2}>\sqrt{1+\phi}(1-\phi)-(1-\phi)\sqrt{1-\phi}$, that is, $\sqrt{1+\phi}(1-\sqrt{2(1-\phi)})>(1-\phi)(\sqrt{1-\phi}-\sqrt{2})$. Since $\sqrt{1+\phi}>1-\phi$ for $\phi\in(0,1)$ we are left to show that $1-\sqrt{2(1-\phi)}>\sqrt{1-\phi}-\sqrt{2}$. This is the same as $1+\sqrt{2}>\sqrt{1-\phi}(1+\sqrt{2})$, which is true for $\phi\in(0,1)$.

We next show the pure wholesale contract is dominated by hybrid with WM outcome. We need to show that $\pi_r^m(q_L,\tilde{\phi})>\pi_r^w(q_L)$, or $4\tilde{\phi}\left(1-\frac{(\kappa q_L)^2}{(1-\tilde{\phi})^2}\right)>(1-\kappa q_L)^2$. By its definition, $\kappa q_L=\frac{\sqrt{2(1-\tilde{\phi})}-1}{\sqrt{2/(1-\tilde{\phi})}-1}$. That is, we need to show that $g(\tilde{\phi}, \kappa q_L) = g\left(\tilde{\phi}, \frac{\sqrt{2(1-\tilde{\phi})}-1}{\sqrt{2/(1-\tilde{\phi})}-1}\right) > 0$, which is true from Lemma A1, provided that $\tilde{\phi} \in (0, 1/2)$. This implies that we are done if we can show that $\tilde{\phi} \in (0, 1/2)$, which we do next. By its definition, $\tilde{\phi}$ is the value of ϕ that solves $\kappa q_L = \frac{1-\sqrt{2(1-\phi)}}{1-\sqrt{2/(1-\phi)}}$. Since $\kappa q_L \in (0, 1)$, the potential values of $\tilde{\phi}$ are such that $\frac{1-\sqrt{2(1-\phi)}}{1-\sqrt{2/(1-\phi)}} \in (0, 1)$. Now $\frac{1-\sqrt{2(1-\phi)}}{1-\sqrt{2/(1-\phi)}}$ is positive for $\phi < 1/2$, 0 at $\phi = 1/2$, and negative for $1/2 < \phi < 1$: The denominator $1 - \sqrt{2/(1-\phi)} < 0$ for $\phi \in [0, 1)$; and the numerator $1 - \sqrt{2(1-\phi)}$ is positive for $\phi > 1/2$, negative for $\phi < 1/2$, and zero for $\phi = 1/2$. This implies $\tilde{\phi} \in (0, \frac{1}{2})$ and the pure wholesale contract is suboptimal.

Consider $\gamma < \gamma^m$. In this case, we have ϕ^m solves $\frac{(1-\phi)^3}{1+\phi} = (\kappa q_L)^2$. This implies that the retailer's profit under the marketplace contract is $\Pi_r^m = (1-\gamma)\frac{\phi^m}{4} q_L \left(1 - \frac{(\kappa q_L)^2}{(1-\phi^m)^2}\right) = (1-\gamma)\frac{\phi^m}{4} q_L \left(1 - \frac{1-\phi^m}{1+\phi^m}\right) = (1-\gamma)\frac{q_L}{2}\frac{(\phi^m)^2}{1+\phi^m}$. Now the profit under the hybrid contract is $\pi_H(\tilde{\phi}) = \gamma \pi_r^w(q_H) + (1-\gamma)\pi_r^m(q_L, \tilde{\phi})$ from above, which can be either increasing, decreasing or constant in γ (depending on the relative magnitudes of $\pi_r^w(q_H)$ and $\pi_r^m(q_L, \tilde{\phi})$, which are independent of γ). Combined with $\Pi_r^m < \pi_H(\tilde{\phi})$ at $\gamma = 1$ and the decreasing nature of Π_r^m with γ, we can conclude that the hybrid contract is optimal at sufficiently large γ, that is $\gamma > \underline{\gamma}$ for some $\underline{\gamma}$. Consider $\gamma > \gamma^m$. Then for $\gamma = 1$, the pure marketplace contract with both types dominates the hybrid WM outcome (Proposition 1). This proves that the optimal contract is either the pure marketplace contract or the hybrid WM contract in general. □

Proof of Proposition 9. (i) The pricing stage decision follows from Lemma 1, i.e., for quality level q_i and wholesale price w_i, the price is $p_i^*(w_i) = \frac{1}{2}(q_i + w_i)$. Then the retailer's profit is $\Pi_r^w(p_i^*(w_i); q_i) = \frac{(q_i - w_i)^2}{4q_i}$ and the supplier's profit is $\Pi_s^w(p_i^*(w_i); q_i) = \frac{(q_i - w_i)(w_i - \kappa q_i^2)}{2q_i}$. Then $\Pi_r^w(p_i^*(w_i); q_i)^\beta \Pi_s^w(p_i^*(w_i); q_i)^{1-\beta}$ is maximized at w_i as shown in the proposition. The retailer's profit is $\sum_i \gamma_i \frac{1}{16}(1+\beta)^2 q_i (1-\kappa q_i)^2$ and the supplier of quality i's profit is $\gamma_i \frac{1}{8}(1-\beta)(1+\beta) q_i (1-\kappa q_i)^2$.

(ii) This follows from Proposition 1. The retailer's profit is $\sum_i \gamma_i \pi_r^m(q_i, \phi_i)$ and the supplier of quality i's profit is $\gamma_i \pi_s^m(q_i, \phi_i)$, where $\pi_{(\cdot)}^m(q_i, \phi_i)$ are defined in Lemma 1(ii).

(iii) There are two potential types of hybrid equilibria, WM or MW. The equilibrium outcome under each type follows from the proof of Proposition 9(i)-(ii). Under WM, the retailer's profit is $\gamma \frac{1}{16}(1+\beta)^2 q_H (1-\kappa q_H)^2 + (1-\gamma)\pi_r^m(q_L, \phi_L)$, the high-quality supplier's profit is $\gamma \frac{1}{8}(1-\beta)(1+\beta) q_H (1-\kappa q_H)^2$, and the low quality supplier's profit is $(1-\gamma)\pi_s^m(q_L, \phi_L)$. □

Proof of Proposition 10. The retailer's profit from partnering with supplier of quality q, that is, $\frac{1}{16}(1+\beta)^2 q(1-\kappa q)^2$ is increasing in β. We will next show that there exists a threshold q_0 such that $\pi_r^m(q, \phi^b) > \frac{1}{16}(1+\beta)^2 q(1-\kappa q)^2$ iff $q < q_0$. Then we have the result stated in (a). This would also imply that all three types of contract can be optimal depending on the relative values of q_L and q_H w.r.t. q_0 as stated in (b).

The difference $\pi_r^m(q, \phi^b) - \frac{1}{16}(1+\beta)^2 q(1-\kappa q)^2$ has the same sign as $4\phi^b\left(1 - \frac{\kappa^2 q^2}{(1-\phi^b)^2}\right) - (\beta+1)^2(1-\kappa q)^2$. By using the definition of ϕ^b, we can substitute the value of κq in this term to simplify it to: $\frac{32(\phi^b)^2 - (\beta+1)^2\left(2\sqrt{\phi^b+1} - (1-\phi^b)^3\right)^2}{4(\phi+1)}$. Now there exists a threshold on ϕ, say $\phi_0 \in (0, 1)$, such that $\sqrt{32}\phi^b - (\beta+1)(2\sqrt{\phi^b+1} - (1-\phi^b)^3)$ is negative for low $\phi^b < \phi_0$ and positive for $\phi > \phi_0$. Since ϕ^b is decreasing with κq, ranging from 1 at $\kappa q = 0$ to 0 at $\kappa q = 1$, this in turn implies that there threshold q_0 such that $\pi_r^m(q, \phi^b) > \frac{1}{16}(1+\beta)^2 q(1-\kappa q)^2$ iff $q < q_0$. $\square$

Proof of Proposition 11. (i) Since $\mu + \Delta < 1/\kappa$ all suppliers find it profitable to partner with the retailer. The retailer gets profit $\pi_r^w(q)$ from the supplier of quality q. Given the probability density function of quality levels and full market coverage, we have the retailer's expected profit is

$$\int_{\mu-\Delta}^{\mu+\Delta} \pi_r^w(q) \cdot \frac{1}{2\Delta} dq.$$

This simplifies to $\frac{1}{48}\Delta^2\kappa(3\kappa\mu - 2) + \frac{1}{16}\mu(1-\kappa\mu)^2$.

(ii) From Lemma 1, for a given commission rate ϕ, the retailer offers the contract to suppliers whose product quality satisfies $\phi < 1 - \kappa q$, or effectively, those with quality levels $q < \frac{1-\phi}{\kappa}$ will participate and the rest will not. This implies that the equilibrium outcome falls into one of two types: (1) where all suppliers partner i.e., $\mu + \Delta \le \frac{1-\phi}{\kappa}$, or (2) where suppliers with high quality levels in $[\frac{1-\phi}{\kappa}, \mu + \Delta]$ are excluded from the partnership. Given ϕ, the retailer's profit in case (1) and (2) are, respectively,

$$\int_{\mu-\Delta}^{\mu+\Delta} \pi_r^m(q,\phi) \cdot \frac{1}{2\Delta} dq, \text{ and } \int_{\mu-\Delta}^{\frac{1-\phi}{\kappa}} \pi_r^m(q,\phi) \cdot \frac{1}{2\Delta} dq.$$

In case (1), the retailer's profit simplifies to $\frac{\mu\phi\left((1-\phi)^2 - (\Delta^2+\mu^2)\kappa^2\right)}{4(1-\phi)^2}$. In this case, the profit is maximized at ϕ such that

$$\frac{(1-\phi)^3}{1+\phi} = (\Delta^2 + \mu^2)\kappa^2. \tag{9}$$

It is easy to see that the optimal commission rate is decreasing in Δ. Define this commission rate as ϕ_1.

In case (2), the retailer's profit simplifies to $x^2(1+\phi) - 2x(1-\phi)^3 + (1-\phi)^4(1-3\phi)$ where $x = \kappa^2(\mu-\Delta)^2$. This quadratic in x has two solutions $x = (1-\phi)^2$ and $x = \frac{(1-\phi)^2(1-3\phi)}{1+\phi}$. The former cannot be a solution since it violates the feasibility condition for participation, i.e., $(1-\phi)/\kappa > \mu - \Delta$. Thus, the optimal commission rate in this case is given by the solution to

$$\frac{(1-\phi)^2(1-3\phi)}{1+\phi} = \kappa^2(\mu-\Delta)^2. \tag{10}$$

Moreover, the optimal commission rate in case (2) satisfies $\phi < 1/3$ and since the LHS of (10) is decreasing in ϕ for $\phi < 1/3$, the optimal commission rate is increasing in Δ. Define this commission rate as ϕ_2. Note that for $\Delta = 0$, the RHS of (9)-(10) are identical. The LHS of (9) is larger than that of (10), so that $\phi_1 > \phi_2$ for $\Delta = 0$. Now the threshold separating the two cases on ϕ is $1 - \kappa(\mu+\Delta)$: Case (1) is feasible for $\phi \le 1 - \kappa(\mu+\Delta)$ and case (2) is feasible for $\phi > 1 - \kappa(\mu+\Delta)$. Therefore, ϕ_2 is not feasible for $\Delta = 0$. We will show that there exists $\overline{\Delta}$ such that ϕ_1 is feasible for $\Delta \le \overline{\Delta}$ and ϕ_2 is feasible otherwise. Plugging in $\phi = 1 - \kappa(\mu+\Delta)$ into (9) or into (10) we get $\kappa = \frac{\Delta^2+\mu^2}{\Delta^3+2\Delta^2\mu+2\Delta\mu^2+\mu^3}$. Define $\overline{\Delta}$ as the solution to this equation. Then, all three curves, ϕ_1, ϕ_2 and $1-\kappa(\mu+\Delta)$ meet at $\Delta = \overline{\Delta}$. Due to the increasing nature of ϕ_2 in Δ and the decreasing nature of ϕ_1 and $1-\kappa(\mu+\Delta)$ in Δ, we have the required result. In other words, the outcome is such that there is a partnership with all suppliers for $\Delta < \overline{\Delta}$ and some suppliers are excluded otherwise. Furthermore, since $\overline{\Delta}$ is decreasing in μ and takes the value of $1/\kappa$ at $\mu = 0$, we must have that $\overline{\Delta} < \mu + \Delta$ for $\mu > \overline{\mu}$ for some $\overline{\mu}$; otherwise, the equilibrium involves all suppliers for $\mu < \overline{\mu}$.

(iii) Comparing $\pi_r^m(q,\phi)$ against $\pi_r^w(q)$, there are two thresholds q_1 and q_2 ($> q_1$) such that the retailer prefers contracting in the marketplace mode for $q \in [q_1, q_2]$ and wholesale mode otherwise. Here $q_1 = \frac{(1-\phi)^2 - 2(1-\phi)\phi\sqrt{\phi+2}}{\kappa(\phi+1)^2}$ and $q_2 = \frac{(1-\phi)^2 + 2\phi(1-\phi)\sqrt{\phi+2}}{\kappa(\phi+1)^2}$. It is, however, plausible that depending on the parameter values, the values of q_1 and q_2 may lie outside the quality range of potential suppliers. In general, we can then have three possible types of hybrid equilibria: (1) WM where the lower quality suppliers are offered marketplace mode and the rest are offered wholesale mode; (2) MW where the higher quality suppliers are offered wholesale mode and the rest are offered marketplace mode (provided their quality levels satisfy $q < (1-\phi)/\kappa$); or (3) WMW where the intermediate quality suppliers are offered marketplace mode and the rest are offered wholesale mode.

We will show that case (3) is not optimal. To show this, suppose the thresholds q_1, q_2 are valid, that is, they satisfy $\mu - \Delta < q_1 < q_2 < \mu + \Delta$. Then the retailer's profit in this case (3) is given by $\frac{1}{2\Delta} \cdot \left[\int_{\mu-\Delta}^{q_1} \pi_r^w(q)dq + \int_{q_1}^{q_2} \pi_r^m(q,\phi)dq + \int_{q_2}^{\mu+\Delta} \pi_r^w(q)dq\right] = \frac{1}{48}\left(\Delta^2\kappa(3\kappa\mu - 2) + \frac{16(1-\phi)^3\phi^3(\phi+2)^{3/2}}{\Delta\kappa^2(1+\phi)^6} + 3\mu(1-\kappa\mu)^2\right)$. In the range of feasible commission rates, i.e., $\phi \in (0,1)$, the first order condition leads to $\phi = 0.355301$, at which point the second derivative is negative. Thus, the above profit term is maximized at $\phi = 0.355301$. If the thresholds q_1 and q_2 are feasible at this point, then we can conclude that it can be optimal to induce equilibria in case (3): At this maximizer, q_1 is negative, so it is not feasible.

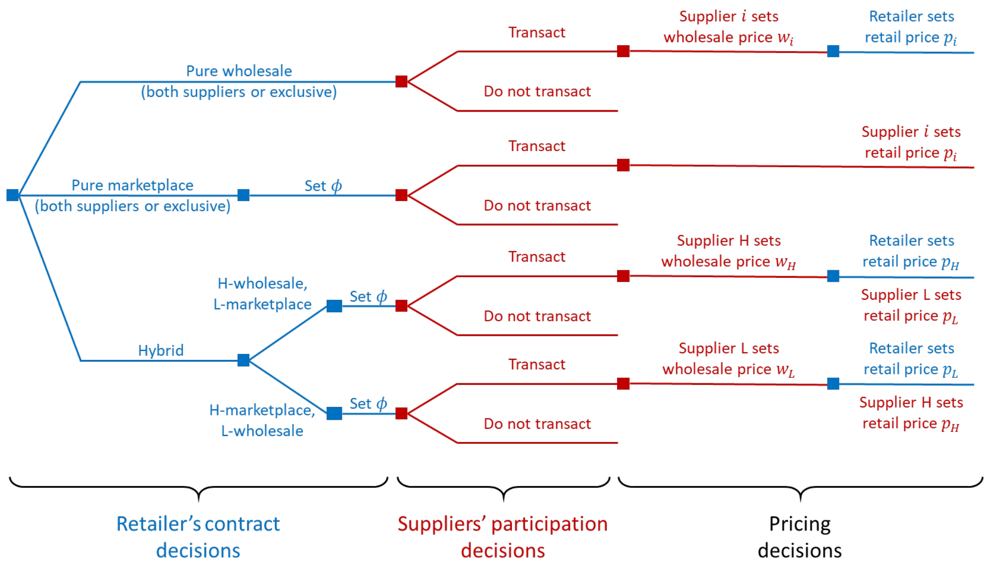
Next we simplify case (2). The retailer's profit in this case (2) is $\frac{1}{2\Delta} \cdot \left[\int_{\mu-\Delta}^{q_1} \pi_r^w(q)dq + \int_{q_1}^{(1-\phi)/\kappa} \pi_r^m(q,\phi)dq\right]$. By definition, we have $q_1 > 0$ iff $\phi < 1/4$. If the optimal commission rate turns out to be such that $\phi \geq 1/4$, then this case can never be feasible for any μ, Δ. The first order condition on the retailer's profit function yields the solution $\phi = 0.320551$. Since this exceeds the feasibility threshold, i.e., 1/4, we can conclude that the case (2) cannot be optimal.

Next we simplify case (1). The retailer's profit in this case (1) is $\frac{1}{2\Delta} \cdot \left[\int_{\mu-\Delta}^{q_2} \pi_r^m(q,\phi)dq + \int_{q_2}^{\mu+\Delta} \pi_r^w(q)dq\right]$. For feasibility, we must then have that the optimal contract satisfies $q_2 \in \{\mu-\Delta, \mu+\Delta\}$ and $q_1 < \mu - \Delta$. Taking the first order condition and verifying that the second derivative is negative yields the solution:

$$\frac{(1-\phi)^3}{1+\phi}\left(1 - \frac{\phi}{(1-\phi)(1+\phi)^3}\sqrt{g(\phi)}\right) = \kappa^2(\mu-\Delta)^2. \tag{11}$$

Here $g(\phi) \equiv \phi^6 + 12\phi^5 + \left(54 - 8\sqrt{\phi+2}\right)\phi^4 + \left(116 - 40\sqrt{\phi+2}\right)\phi^3 + \left(33 - 24\sqrt{\phi+2}\right)\phi^2 + 8\left(13\sqrt{\phi+2} - 15\right)\phi - 32\sqrt{\phi+2} + 48$. Now define the LHS of (11) as $\xi(\phi)$. Then $\xi(\phi)$ is decreasing in ϕ and positive for $\phi \in [0, 0.386919)$. Thus, the optimal commission rate, ϕ^h, satisfies $\phi^h \in [0, 0.386919)$. Next we need to check the feasibility conditions, i.e., $q_2 \in \{\mu-\Delta, \mu+\Delta\}$ and $q_1 < \mu - \Delta$, at the optimal ϕ^h. By their definitions $q_1 \cdot \kappa$ is decreasing in ϕ and we have $q_1 \cdot \kappa < \sqrt{\xi(\phi)}$. Then, since the optimal

Figure B1 Sequence of decisions: detailed decision tree version.



value ϕ^h satisfies $\sqrt{\xi(\phi)} = \kappa(\mu - \Delta)$, we can conclude that $q_1 < \mu - \Delta$. Similarly, $q_2 \cdot \kappa$ is decreasing in ϕ and we have $q_2 \cdot \kappa > \sqrt{\xi(\phi)}$. Then, since the optimal value ϕ^h satisfies $\sqrt{\xi(\phi)} = \kappa(\mu - \Delta)$, we can conclude that $q_2 > \mu - \Delta$. To ensure that $q_2 < \mu + \Delta$, we must have $\phi^h > \phi^c$ where ϕ^c is the value at which $q_2 = \mu + \Delta$. Since the RHS of (11) is decreasing in Δ, the value of ϕ^h is increasing in Δ. Since $\mu + \Delta$ is increasing in Δ, the value of ϕ^c is decreasing in Δ. Furthermore, at $\Delta = 0$, $\phi^c > \phi^h$ due to $q_2 \cdot \kappa > \sqrt{\xi(\phi)}$. Similarly, at $\Delta = \mu$, $\phi^h = 0.386919$. Furthermore, at $\Delta = \mu$, $q_2 < \mu + \Delta$ only if μ is large enough; specifically, we need $\mu > \frac{(1-\phi)\left(\phi\left(2\sqrt{\phi+2}-1\right)+1\right)}{2\kappa(\phi+1)^2}$. Now $\frac{(1-\phi)\left(\phi\left(2\sqrt{\phi+2}-1\right)+1\right)}{2\kappa(\phi+1)^2}$ is decreasing in ϕ, so for $q_2 < \mu + \mu$ to be satisfied for $\phi^h = 0.386919$ we must have $\mu > 0.288228/\kappa$. In other words, as long as $\mu > 0.288228/\kappa$ we have $\phi^c < \phi^h$ for large enough Δ. And the threshold on Δ in this case would be the unique feasible solution to $\phi^c = \phi^h$. Thus, the equilibrium is feasible for large enough μ and Δ. □

Appendix B: Additional Results

Figure B1 illustrates the detailed sequence of decisions, where supplier i has product quality $q_i, i \in \{L, H\}$.

We now present the analyses for our result that the pure marketplace contract is the unique equilibrium in the sequential move game variant where the retailer sets the contract terms first followed by the suppliers' quality decisions. Based on Proposition 6, the suppliers' decision is $1/(3\kappa)$ when assigned the wholesale mode and $(1-\phi)/(3\kappa)$ when assigned the marketplace mode. Then the retailer takes these decisions into account while choosing their contract. We discuss the retailer's profit from offering either the wholesale or marketplace mode to a single supplier next:

1. Wholesale mode: In this case, the retailer's profit is $\pi_r^w(q)$ with $q = 1/(3\kappa)$ from Lemma 1, which becomes $1/(108\kappa)$.

2. Marketplace mode: In this case, the retailer's profit is $\pi_r^m(q, \phi)$ with $q = (1-\phi)/(3\kappa)$ from Lemma 1, which becomes $\frac{2(1-\phi)\phi}{27\kappa}$. This profit is maximized at $\phi = 1/2$, resulting in the retailer's profit $1/(54\kappa)$.

Since the retailer's profit when offering the marketplace mode to a supplier exceeds that when offering the wholesale mode, the retailer will be better off assigning the marketplace mode to both suppliers, i.e., the pure marketplace contract forms the equilibrium. Note also that the resulting profit exceeds that in the simultaneous game counterpart, i.e., $0.0118519/\kappa$ as seen in Step 2 of the proof of Proposition 6.