Online Appendix to *Strategic Communications with Socializing Agents under Unknown Public Health Threats*

Ailing Xu, Zhenxiao Chen, Qiao-Chu He, Ying-Ju Chen

This online appendix consists of two parts. Part A presents the formal proofs for the analysis of the basic model. Part B discusses model extensions. Specifically, Section B.1 provides proofs for the three extensions analyzed in the main text, and Section B.2 examines a more realistic setting in which the government's choice of penalties is constrained relative to the basic model.

A Proofs for Basic Model

Proof of Lemma 1. By comparing Equation (3) with Equation (4), we observe two findings. First, $u_s(A=1) \geq u_t(A=1)$ always holds, indicating that socializing agents consistently derive greater utility from social interactions than traditional agents and are more likely to violate. Second, $u_s(A=1)$ increases with η, implying that socializing agents either fully comply or fully violate. Given any pair (s,p), four equilibrium outcomes may emerge:

- All agents violate, i.e., $u_t(A=1,\eta=1) \geq 0$;
- Only some traditional agents comply, i.e., $u_t(A=1,\eta=1) < 0$ and $u_t(A=1,\eta=\lambda) \geq 0$;
- All traditional agents comply while all socializing agents violate, i.e., $u_t(A=1,\eta=\lambda) < 0$ and $u_s(A=1,\eta=\lambda) \geq 0$;
- All agents comply, i.e., $u_s(A=1,\eta=\lambda) < 0$.

These four cases correspond to the outcomes described in Equation (6). By restricting the range of p, we further derive the results summarized in Table 1. □

Before proving Proposition 1, we introduce two lemmas to support the subsequent analysis. Because both messages and penalties can convey information about the pandemic state, their interaction makes it difficult to directly characterize the optimal policy. To address this, we introduce an intermediate step by considering how the public's belief is formed. Given any actual state $s \in S$, a public policy $\Gamma = \{\mathbb{F}, P, M\}$ determines both the penalty level and the public's perceived severity of the pandemic. This allows us to represent public policy in an alternative form.

Lemma A.1. *For any public policy $\Gamma = \{\mathbb{F}, P, M\}$, there exists a unique pair (p_Γ, s_Γ), where $p_\Gamma : S \to P$ denotes the penalty level applied at state s, and $s_\Gamma : S \to S$ represents the public's* *perceived pandemic severity when the true state is s.*

This equivalence follows naturally from the way public beliefs are updated following Bayes' rule. In what follows, we focus on identifying the optimal pair (p_Γ, s_Γ), and then derive the corresponding public policy Γ. A notable special case is full information disclosure, recorded as s^c_Γ. Within such an information structure, the public perception aligns with the true state, i.e., $s^c_\Gamma(s) = s$ for all $s \in S$. Under full information disclosure, the optimal penalty mapping that solves the government's optimization problem (5) is denoted by p^c_Γ, and is given by:

$$p^c_\Gamma(s) = \arg\max_p -s\eta^*(s,p) - ap, \quad \text{for any } s \in S. \tag{A.1}$$

The corresponding utility function of the government is:

$$u^c(s) = -s\eta^*(s, p^c_\Gamma(s)) - ap^c_\Gamma(s). \tag{A.2}$$

Lemma A.2. *If the utility function $u^c(s)$ is convex in s, then the optimal public policy is characterized by full information disclosure, i.e., (p^c_Γ, s^c_Γ).*

To prove this result, we show that any deviation involving pooling over a subset of states cannot improve the government's expected utility. Suppose the government pools information across a measurable subset $S^c \subseteq S$, while maintaining full information disclosure outside this subset. Given that s is uniformly distributed, the perceived severity within the pooling region is the conditional expectation:

$$\tilde{s} := \mathbb{E}[s \mid s \in S^c] = \frac{1}{\mu(S^c)} \int_{S^c} s\, ds, \tag{A.3}$$

where $\mu(S^c)$ denotes the Lebesgue measure of the set S^c. Within the pooling region, the government applies a unified penalty $p^c_\Gamma(\tilde{s})$ based on the conditional expectation $\tilde{s}$. The total utility over this region is:

$$\mu(S^c) \cdot u^c(\tilde{s}). \tag{A.4}$$

In contrast, under full information, the total utility over the same region is:

$$\int_{S^c} u^c(s)\, ds. \tag{A.5}$$

Since $u^c(s)$ is strictly convex, Jensen's inequality implies:

$$\frac{1}{\mu(S^c)}\int_{S^c} u^c(s)\,ds > u^c(\tilde{s}). \tag{A.6}$$

This establishes that pooling information within S^c strictly decreases the government's total utility over the region compared to full information disclosure.

Proof of Proposition 1. Our proof proceeds in two steps. First, we derive the optimal pair (p_Γ, s_Γ) using Lemma A.2. Then, following Lemma A.1, we identify the corresponding public policy Γ.

Step 1. Based on Equations (A.1) and (A.2), under the full information scenario, the optimal penalty $p^c_\Gamma(s)$ and the corresponding government utility $u^c(s)$ depend on the specific parameter conditions.

Case 1: When $a \le 1$ and $\beta \ge \frac{1}{a\lambda}$, the functions are given by:

$$p^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{a}{1+a\lambda-\lambda}, \\ 1 - s\lambda, & \text{if } \frac{a}{1+a\lambda-\lambda} \le s < \frac{1}{\lambda}, \\ 0, & \text{if } \frac{1}{\lambda} \le s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \ge \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \quad u^c(s) = \begin{cases} -s, & \text{if } s < \frac{a}{1+a\lambda-\lambda}, \\ -s\lambda - a + as\lambda, & \text{if } \frac{a}{1+a\lambda-\lambda} \le s < \frac{1}{\lambda}, \\ -s\lambda, & \text{if } \frac{1}{\lambda} \le s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ -a + as\lambda - a\beta\lambda, & \text{if } s \ge \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \tag{A.7}$$

The third interval arises because the penalty $1 - s\lambda$ decreases with s and eventually reaches zero.

Case 2: When $a \le 1$ and $\frac{1}{1-\lambda+a\lambda} \le \beta < \frac{1}{a\lambda}$, the functions are given by:

$$p^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{a}{1+a\lambda-\lambda}, \\ 1 - s\lambda, & \text{if } \frac{a}{1+a\lambda-\lambda} \le s < a\beta, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \ge a\beta. \end{cases} \quad u^c(s) = \begin{cases} -s, & \text{if } s < \frac{a}{1+a\lambda-\lambda}, \\ -s\lambda - a + as\lambda, & \text{if } \frac{a}{1+a\lambda-\lambda} \le s < a\beta, \\ -a + as\lambda - a\beta\lambda, & \text{if } s \ge a\beta. \end{cases} \tag{A.8}$$

Case 3: When $a \le 1$ and $\beta < \frac{1}{1-\lambda+a\lambda}$, the functions are given by:

$$p^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{a(1+\beta\lambda)}{1+a\lambda}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \ge \frac{a(1+\beta\lambda)}{1+a\lambda}. \end{cases} \quad u^c(s) = \begin{cases} -s, & \text{if } s < \frac{a(1+\beta\lambda)}{1+a\lambda}, \\ -a + as\lambda - a\beta\lambda, & \text{if } s \ge \frac{a(1+\beta\lambda)}{1+a\lambda}. \end{cases} \tag{A.9}$$

Case 4: When $a > 1$, the functions are given by:

$$p_\Gamma^c(s) = \begin{cases} 0, & \text{if } s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \geq \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \quad u^c(s) = \begin{cases} -s, & \text{if } s < 1, \\ -1, & \text{if } 1 \leq s < \frac{1}{\lambda}, \\ -s\lambda, & \text{if } \frac{1}{\lambda} \leq s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ -a + as\lambda - a\beta\lambda, & \text{if } s \geq \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \tag{A.10}$$

For Case 2 and Case 3, it can be verified that $u^c(s)$ is convex in s and therefore the optimal public policy can be characterized by the full-information pair (p_Γ^c, s_Γ^c). The analysis for Case 4 follows the same procedure as in Case 1. To avoid redundancy, we present the detailed proof only for Case 1 below. For Case 1, we establish the following characteristics of the optimal pooling region S^c.

Lemma A.3. *The optimal pooling region $S^c \subseteq [b, c] \subseteq S$ in Case 1 satisfies the following properties:*

- *S^c is a connected interval.*
- *The conditional expectation within this region, $\tilde{s} = \frac{1}{\mu(S^c)} \int_{S^c} s\, ds$, equals the threshold $s = \frac{1}{\lambda}$.*

To prove the first part, we begin by fixing the measure of the pooling set, $\mu(S^c) = \zeta$, and suppose by contradiction that S^c is not connected. That is, S^c consists of multiple disjoint subintervals with total measure ζ. Let us consider two benchmark connected intervals at the boundary of the support $[b, c]$, each of measure ζ: one at the lower end, $[b, b+\zeta]$, and one at the upper end, $[c-\zeta, c]$. The conditional expectations for these intervals are:

$$\tilde{s}_L = b + \frac{\zeta}{2}, \quad \tilde{s}_H = c - \frac{\zeta}{2}. \tag{A.11}$$

Since S^c is not connected, its conditional expectation $\tilde{s} = \frac{1}{\mu(S^c)} \int_{S^c} s\, ds$ must satisfy:

$$b + \frac{\zeta}{2} \leq \tilde{s} \leq c - \frac{\zeta}{2}. \tag{A.12}$$

Now, observe that the utility function $u^c(s)$, as defined in Equation (A.7), first decreases with s and then increases beyond a certain threshold, i.e., $s = \frac{a(1+\beta\lambda)}{\lambda(1+a)}$. Because of this shape, the function $u^c(s)$ achieves its minimum around the middle and increases toward both ends of the interval. Consequently, we have:

$$u^c(\tilde{s}) \leq \max\left\{ u^c\left(b + \frac{\zeta}{2}\right), u^c\left(c - \frac{\zeta}{2}\right) \right\}. \tag{A.13}$$

This implies that there always exists a connected interval (either $[b, b+\zeta]$ or $[c-\zeta, c]$) with equal measure but weakly higher utility than any disconnected pooling set. Hence, the optimal pooling region must be connected.

Based on the results from the first part, we focus on the interval $S^c = [d, d+\zeta] \subseteq S$, where the conditional expectation within this interval is $\tilde{s} = d + \frac{\zeta}{2}$. In the second part, according to Lemma A.2, the pooling interval must include the non-convex region of $u^c(s)$. The function $u^c(s)$ can be separated into two parts: it is strictly convex in s over the intervals $s \in \left[0, \frac{1}{\lambda}\right]$ and $s \in \left[\frac{1}{\lambda}, \overline{s}\right]$. This implies that the condition $d \leq \frac{1}{\lambda} \leq d + \zeta$ must hold, ensuring that the pooling interval contains the threshold. Next, we consider how the public responds to the policy after pooling within this interval. From Equation A.7, $u^c(s)$ is divided by four regions, and the corresponding public response is:

$$\eta^* = \begin{cases} 1, & \text{if } s < \frac{a}{1+a\lambda-\lambda}, \\ \lambda, & \text{if } \frac{a}{1+a\lambda-\lambda} \leq s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ 0, & \text{if } s \geq \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \tag{A.14}$$

Suppose the pooling interval leads to $\eta^* = 1$. This requires the lower bound of the pooling interval to be below the first threshold, i.e., $d < \frac{a}{1+a\lambda-\lambda}$. In this case, the utility for each state $s \in S^c$ becomes $u_1^c(s) = -s$. It is easy to verify that $u_1^c(s)$ coincides with the full-information utility $u^c(s)$ when $s \in [d, \frac{a}{1+a\lambda-\lambda}]$. However, for $s \in [\frac{a}{1+a\lambda-\lambda}, d+\zeta]$, the revised utility $u_1^c(s)$ is strictly lower than $u^c(s)$. This indicates that the pooling policy strictly reduces the government's utility in this region. A similar argument shows that the pooling interval cannot lead to $\eta^* = 0$. Therefore, the only feasible case is that the pooling interval results in $\eta^* = \lambda$. We now derive the optimal value of the conditional expectation within such an interval. Let the total utility generated over the pooling interval of length ζ, starting at d, be denoted as $U(d)$. This total utility can be written as:

$$U(d) = \zeta \cdot u^c\left(d + \frac{\zeta}{2}\right). \tag{A.15}$$

Since $u^c(s)$ is piecewise-defined and differentiable within each region, $U(d)$ is continuously differentiable (i.e., smooth) as long as $d + \frac{\zeta}{2}$ lies strictly within a single piece. In such cases, any interior extremum of $U(d)$ must satisfy the first-order condition $U'(d) = 0$. However, because $u^{c\prime}(s)$ is monotonic within each piece, the derivative $U'(d)$ does not change sign in the interior of a single piece. This implies that the function $U(d)$ does not attain a maximum inside any piece unless it is constant. Therefore, the maximum must occur at a boundary between two pieces. That is, the pooled expectation $d + \frac{\zeta}{2}$ equals the threshold $\frac{1}{\lambda}$.

Lemma A.3 provides two key insights that guide the identification of the optimal pooling interval. Building on this result, the final step is to determine the boundaries of the pooling interval. Given that the midpoint of the interval satisfies $d + \frac{\zeta}{2} = \frac{1}{\lambda}$, we can rewrite the pooling interval as $S^c = [\frac{1}{\lambda} - \sigma, \frac{1}{\lambda} + \sigma]$, where σ represents half the length of the pooling interval. With this formulation, the optimization problem becomes:

$$\max_{\sigma} \int_{\frac{1}{\lambda}-\sigma}^{\frac{1}{\lambda}+\sigma} -s\lambda \, ds - \int_{\frac{1}{\lambda}-\sigma}^{\frac{1}{\lambda}+\sigma} u^c(s) \, ds, \tag{A.16}$$

where the first term represents the government's total utility after pooling, and the second term represents the total utility under full information. Solving this problem yields the following result for Case 1:

$$\sigma^* = \begin{cases} a\beta - \frac{1}{\lambda}, & \text{if } \frac{1}{a\lambda} \leq \beta < \frac{a\lambda - 2\lambda + 2}{a^2\lambda^2 - a\lambda^2 + a\lambda}, \\ \frac{a\beta\lambda^2 - 2\lambda + 1}{\lambda(a\lambda + 1)}, & \text{if } \frac{a\lambda - 2\lambda + 2}{a^2\lambda^2 - a\lambda^2 + a\lambda} \leq \beta < \frac{a+2}{a\lambda}. \end{cases} \tag{A.17}$$

When $\beta \geq \frac{a+2}{a\lambda}$, the lower bound of the pooling interval reaches zero, making this a special case. In this case, the optimal pooling interval is given by $S^c = \left[0, \frac{a(1+\beta\lambda)}{\lambda(1+a)}\right]$.

Step 2. Based on the (p_Γ, s_Γ) for different situations, we can derive the corresponding public policy.

Case 1-1: When $a \leq 1$ and $\frac{1}{a\lambda} \leq \beta < \frac{a\lambda - 2\lambda + 2}{a^2\lambda^2 - a\lambda^2 + a\lambda}$, the policy functions are defined as follows:

$$p_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{a}{1 + a\lambda - \lambda}, \\ 1 - s\lambda, & \text{if } \frac{a}{1 + a\lambda - \lambda} \leq s < \frac{2}{\lambda} - a\beta, \\ 0, & \text{if } \frac{2}{\lambda} - a\beta \leq s < a\beta, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \geq a\beta. \end{cases} \qquad s_\Gamma(s) = \begin{cases} s, & \text{if } s < \frac{2}{\lambda} - a\beta, \\ \frac{1}{\lambda}, & \text{if } \frac{2}{\lambda} - a\beta \leq s < a\beta, \\ s, & \text{if } s \geq a\beta. \end{cases} \tag{A.18}$$

The corresponding values of η^* in the four intervals of $p_\Gamma(s)$ are 1, λ, λ, and 0, respectively. Put simply, the government should withhold information only for states $s \in [\frac{2}{\lambda} - a\beta, a\beta)$, and should commit to full disclosure for all other states. In the second and fourth intervals of $p_\Gamma(s)$, the penalty is state-dependent, which implies that no message is sufficient to influence the agent's action. In contrast, in the first interval, the optimal penalty is state-independent, which would normally require the government to send separating (state-dependent) messages for each state. However, since in this region the outcome is $\eta^* = 1$ for all $s < \frac{a}{1+a\lambda-\lambda}$ (as all such s are less than 1; see Equation (6)), sending a unified message can achieve the same result. By reporting a single message, the perceived severity becomes $\frac{a}{2(1+a\lambda-\lambda)}$ for all states in this interval, which still leads to $\eta^* = 1$. To reduce the size of the message space, we focus on the setting where the government sends a unified message m_1. Furthermore, to distinguish between the two intervals where the penalty is zero, the optimal policy introduces an additional message m_2 specifically for states in $[\frac{2}{\lambda} - a\beta, a\beta)$.

Case 1-2: When $a \le 1$ and $\frac{1}{a\lambda} \le \beta < \frac{a\lambda - 2\lambda + 2}{a^2\lambda^2 - a\lambda^2 + a\lambda}$, the policy functions are defined as follows:

$$p_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{-a\beta\lambda + a + 2}{a\lambda + 1}, \\ 0, & \text{if } \frac{-a\beta\lambda + a + 2}{a\lambda + 1} \le s < \frac{\lambda(a\beta\lambda + a - 2) + 2}{\lambda(a\lambda + 1)}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \ge \frac{\lambda(a\beta\lambda + a - 2) + 2}{\lambda(a\lambda + 1)}. \end{cases} \tag{A.19}$$

$$s_\Gamma(s) = \begin{cases} s, & \text{if } s < \frac{-a\beta\lambda + a + 2}{a\lambda + 1}, \\ \frac{1}{\lambda}, & \text{if } \frac{-a\beta\lambda + a + 2}{a\lambda + 1} \le s < \frac{\lambda(a\beta\lambda + a - 2) + 2}{\lambda(a\lambda + 1)}, \\ s, & \text{if } s \ge \frac{\lambda(a\beta\lambda + a - 2) + 2}{\lambda(a\lambda + 1)}. \end{cases} \tag{A.20}$$

The corresponding values of η^* in the three intervals are 1, λ, and 0, respectively. In summary, the government should withhold information only within the intermediate interval and commit to full disclosure in the other two regions. Compared to the optimal policy described in Case 1-1, the only difference here is that the interval associated with a separating (state-dependent) penalty disappears.

Case 1-3: When $a \le 1$ and $\beta \ge \frac{a+2}{a\lambda}$, the policy functions are defined as follows:

$$p_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \ge \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \quad s_\Gamma(s) = \begin{cases} \frac{a(1+\beta\lambda)}{2\lambda(1+a)}, & \text{if } s < \frac{a(1+\beta\lambda)}{\lambda(1+a)}, \\ s, & \text{if } s \ge \frac{a(1+\beta\lambda)}{\lambda(1+a)}. \end{cases} \tag{A.21}$$

The corresponding values of η^* in the two intervals are λ and 0, respectively. In summary, compared to the optimal policy described in Case 1-2, the only change is that the initial interval with $\eta^* = 1$ disappears. For *Case 2* and *Case 3*, the optimal policies are given by $p_\Gamma(s) = p^c_\Gamma(s)$ and $s_\Gamma(s) = s^c_\Gamma(s)$, as full information disclosure is optimal in these cases. The corresponding public policies can be viewed as degenerate cases of Case 1-1. All the results discussed above are summarized in the following Table A.1. As previously mentioned, we omit the detailed analysis for Case 4 and directly present the results in Table A.2. Unless otherwise specified, we focus on the most general case in the main text and in the appendix analysis, corresponding to the shaded row in Table A.1. This case reflects the most complex scenario in designing public policy, and the analysis of other degenerate cases can follow a similar approach. □

Proof Corollary 1. These results can be derived by calculating the government's utility as a function of s under various regimes:

Table A.1. Threshold Values (s_1, s_2, s_3) under Different Condition $(a \leq 1)$.

Condition	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{s_3}$
$\beta < \frac{1}{1-\lambda+a\lambda}$	$\frac{a(1+\beta\lambda)}{1+a\lambda}$		
$\frac{1}{1-\lambda+a\lambda} \leq \beta < \frac{1}{a\lambda}$	$\frac{a}{1+a\lambda-\lambda}$	$a\beta$	
$\frac{1}{a\lambda} \leq \beta < \frac{a\lambda-2\lambda+2}{a^2\lambda^2-a\lambda^2+a\lambda}$	$\frac{a}{1+a\lambda-\lambda}$	$\frac{2}{\lambda} - a\beta$	$a\beta$
$\frac{a\lambda-2\lambda+2}{a^2\lambda^2-a\lambda^2+a\lambda} \leq \beta < \frac{a+2}{a\lambda}$	$\frac{-a\beta\lambda+a+2}{a\lambda+1}$		$\frac{\lambda(a\beta\lambda+a-2)+2}{\lambda(a\lambda+1)}$
$\beta \geq \frac{a+2}{a\lambda}$	0		$\frac{a(1+\beta\lambda)}{\lambda(1+a)}$

Table A.2. Threshold Values (s_1, s_2, s_3) under Different Condition $(a > 1)$.

Condition	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{s_3}$
$\beta < \frac{a-a\lambda+1}{a\lambda}$	1	$\frac{a-a\beta\lambda+1}{a\lambda}$	$\frac{a+a\beta\lambda-1}{a\lambda}$
$\frac{a-a\lambda+1}{a\lambda} \leq \beta < \frac{a+2}{a\lambda}$	$\frac{-a\beta\lambda+a+2}{a\lambda+1}$		$\frac{\lambda(a\beta\lambda+a-2)+2}{\lambda(a\lambda+1)}$
$\beta \geq \frac{a+2}{a\lambda}$	0		$\frac{a(1+\beta\lambda)}{\lambda(1+a)}$

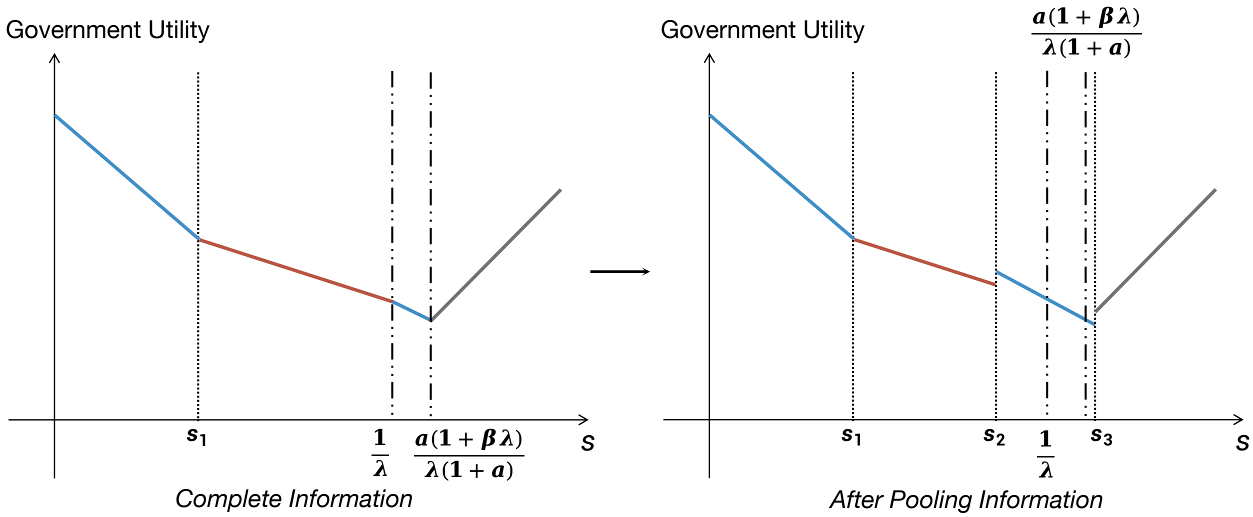
- Under the laissez-faire regime, the government's utility is $u_g^{\text{lai}}(s) = -s$ or $-s\lambda$.
- Under the intermediate-intervention regime, the government's utility is $u_g^{\text{int}}(s) = -s\lambda - a(1-s\lambda)$.
- Under the zero-out regime, the government's utility is $u_g^{\text{zero}}(s) = -a(1 - s\lambda + \beta\lambda)$. □

As noted in the main text, the discontinuous jump in utility is caused by the pooling interval. This effect can be better understood by analyzing the government's utility under the complete-information benchmark, which is visualized below. To further explain the impact of the pooling interval, we present the following corollary:

Corollary A.1. *Compared to the complete-information benchmark, the pooling strategy over the interval $s \in (s_2, s_3]$ increases total utility in the subinterval $s \in \left(s_2, \frac{1}{\lambda}\right]$, but decreases total utility in $s \in \left(\frac{a(1+\beta\lambda)}{1+a\lambda}, s_3\right]$. The increase in the former subinterval outweighs the decline in the latter, resulting in a net gain in overall utility across the entire pooling interval.*

Under the complete-information benchmark, the government's utility is continuous across severity levels. The introduction of a pooling strategy over the interval $s \in \left(s_2, \frac{1}{\lambda}\right]$ brings two key changes. First, for $s \in \left(s_2, \frac{1}{\lambda}\right]$, the penalty drops from $p^* = 1 - s\lambda$ to zero, while compliance from traditional agents remains unchanged at $\eta^* = \lambda$. As a result, the infection cost remains constant, but the enforcement cost is eliminated, leading to an increase in government utility within this subinterval. Second, for $s \in \left(\frac{a(1+\beta\lambda)}{1+a\lambda}, s_3\right)$, the penalty decreases from $p^* = 1 - s\lambda + \beta\lambda$ to

Figure A.1. Government Utility under Complete-Information Benchmark and After Pooling Information.



zero, but compliance from socializing agents is no longer achieved, increasing the infection cost as η^* rises from 0 to λ. The resulting utility in this subinterval declines due to the higher infection burden. Taken together, the overall increase in utility from pooling is primarily driven by the gains in lower severity states, where enforcement costs are reduced without additional infection costs. This highlights a fundamental trade-off in selecting the pooling interval. If the government lowers the lower bound of the interval to $s_2 - \Delta$, it must also raise the upper bound to $s_3 + \Delta$ to maintain the same perceived severity. However, it can be shown that such an extended pooling interval strictly reduces government utility, due to increased losses in higher severity states. □

Proof of Proposition 2. We compare the government's performance under public policies with and without a message scheme. When a message scheme is used, the public policy takes the form $\Gamma = \{\{P, \mathbb{F}_P\}, \{M, \mathbb{F}_M\}\}$, where $\mathbb{F}_P : S \to P$ specifies the penalty policy and $\mathbb{F}_M : S \to M$ specifies the message rule. Without a message scheme, the policy simplifies to $\tilde{\Gamma} = \left\{\tilde{P}, \tilde{\mathbb{F}}_P\right\}$, where the government can only assign penalties based on s.

To establish the result, consider a special case where the message space contains only one element, i.e., $M = \{m_0\}$. In this case, the message provides no additional information, and the public's posterior belief under Γ coincides with that under $\tilde{\Gamma}$. Therefore, the policy Γ is equivalent to $\tilde{\Gamma}$ in terms of influencing public beliefs and outcomes. More generally, introducing a non-trivial message scheme allows the government to influence the public's posterior belief in ways that are not possible under $\tilde{\Gamma}$. We have explained an example in Figure 4. Since the government's performance depends on the induced belief distribution and the corresponding penalties, expanding the feasible set of belief distributions can only weakly improve the outcome. Hence, the government's performance under Γ is weakly better than under $\tilde{\Gamma}$. In terms of equality situations, this happens when the thresholds satisfy $s_2 = s_3$ in Proposition 1. Under this condition, the government already achieves the optimal belief distribution using only the penalty scheme, and the addition of a message scheme does not further improve performance. □

Proof of Corollary 2 and Corollary 3. These results can be derived by checking how λ and β affect $u_g^{\text{lai}}(s)$, $u_g^{\text{int}}(s)$, and $u_g^{\text{zero}}(s)$. The influence of β on the derived thresholds and on the simplification of the general public policy is summarized in Table A.1. This table also presents five conditions defined in terms of β, which can be equivalently expressed using λ. The corresponding effect of λ can then be verified through these equivalent conditions. Overall, the trend shows that as either λ or β increases, the optimal public policy moves from the cases listed in the upper rows of the table to those in the lower rows. □

B Extensions and Additional Discussions

B.1 Proof of Extensions

Proof of Proposition 3. The introduction of the additional PBE constraint reduces the feasible set of public policies. Consequently, the no-credibility case is dominated by the full-credibility case. Furthermore, the deviation described in Equation (8) occurs in all cases presented in Proposition 1, which establishes strict dominance. □

Proof of Proposition 4. We follow a similar process as in the derivation of Proposition 1. With the revised utility function reflecting the government's objective that includes economic considerations, the optimal penalty mapping under full information disclosure, denoted by $\dot{p}_\Gamma^c$, solves the government's problem in Equation (9) and is given by:

$$\dot{p}_\Gamma^c(s) = \arg\max_p \, (e - s)\eta^*(s, p) - ap, \quad \text{for any } s \in S. \tag{B.1}$$

The corresponding utility function of the government is:

$$\dot{u}^c(s) = (e - s)\eta^*(s, \dot{p}_\Gamma^c(s)) - a\dot{p}_\Gamma^c(s). \tag{B.2}$$

Lemma B.1. *When the weight on economic growth is small, i.e., $e \leq 1$, the structure of the optimal public policy remains the same as in the basic model.*

This lemma follows directly from the functional form of $\dot{u}^c(s)$, which, under $e \leq 1$, retains the same structure as in Proposition 1. Accordingly, the optimal public policy remains unchanged in this regime. In the remainder of the proof, we focus on the more interesting case where $e > 1$, such that the economic objective becomes more prominent and leads to a distinct policy structure.

Furthermore, since our focus is on how the newly introduced parameter e affects policy formation, we concentrate on the most generalized case, which includes all possible thresholds and intervals (similar to *Case 1* in Proposition 1). This choice allows us to fully characterize the influence of the economic weight e on policy design through a concise and minimal proof. Deriving the complete optimal public policy, including degenerate cases, simply requires repeating the structure of this proof under parameter values that eliminate one or more intervals. Under this generalized structure, the optimal penalty and corresponding government utility take the form:

$$\dot{p}^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{e}{1-a}, \\ 1 - s\lambda, & \text{if } \frac{e}{1-a} \leq s < \frac{1}{\lambda}, \\ 0, & \text{if } \frac{1}{\lambda} \leq s < \frac{a(1+\beta\lambda)+e\lambda}{\lambda(1+a)}, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \geq \frac{a(1+\beta\lambda)+e\lambda}{\lambda(1+a)}. \end{cases} \tag{B.3}$$

$$\dot{u}^c(s) = \begin{cases} e - s, & \text{if } s < 1, \\ \frac{e-s}{s}, & \text{if } 1 \leq s < \frac{e}{1-a}, \\ (e-s)\lambda - a + as\lambda, & \text{if } \frac{e}{1-a} \leq s < \frac{1}{\lambda}, \\ (e-s)\lambda, & \text{if } \frac{1}{\lambda} \leq s < \frac{a(1+\beta\lambda)+e\lambda}{\lambda(1+a)}, \\ -a + as\lambda - a\beta\lambda, & \text{if } s \geq \frac{a(1+\beta\lambda)+e\lambda}{\lambda(1+a)}. \end{cases} \tag{B.4}$$

The function $\dot{u}^c(s)$ is non-convex in two regions: one around $s = 1$ and another around $s = \frac{1}{\lambda}$. As in *Case 1* of Proposition 1, we identify two pooling intervals. Applying the result in Lemma A.3, the conditional expectations in these intervals are $\tilde{s}_1 = 1$ and $\tilde{s}_2 = \frac{1}{\lambda}$, respectively. Thus, the pooling intervals are: $S^c_1 = [1 - \sigma_1, 1 + \sigma_1]$ and $S^c_2 = \left[\frac{1}{\lambda} - \sigma_2, \frac{1}{\lambda} + \sigma_2\right]$. The pooling around $s = \frac{1}{\lambda}$ follows the same rationale as before, reducing social interaction by lowering η^* to λ. In contrast, the new pooling interval around $s = 1$ emerges because the government now has an incentive to increase social interaction (i.e., raise η^* to 1) when the pandemic is relatively mild. We now formulate the optimization problems that determine the sizes of these intervals:

$$\max_{\sigma_1} \int_{1-\sigma_1}^{1+\sigma_1} -s\,ds - \int_{1-\sigma_1}^{1+\sigma_1} \dot{u}^c(s)\,ds, \tag{B.5}$$

$$\max_{\sigma_2} \int_{\frac{1}{\lambda}-\sigma_2}^{\frac{1}{\lambda}+\sigma_2} -s\lambda\,ds - \int_{\frac{1}{\lambda}-\sigma_2}^{\frac{1}{\lambda}+\sigma_2} \dot{u}^c(s)\,ds, \tag{B.6}$$

Each objective captures the difference in government utility before and after pooling information. Solving the above yields:

$$\sigma_1^* = e-1, \quad \sigma_2^* = e + a\beta - \frac{1}{\lambda}. \tag{B.7}$$

Based on these results, the optimal combination $(\dot{p}_\Gamma, \dot{s}_\Gamma)$ is:

$$\dot{p}_\Gamma(s) = \begin{cases} 0, & \text{if } s < \frac{e}{1-a}, \\ 1 - s\lambda, & \text{if } \frac{e}{1-a} \leq s < \frac{2}{\lambda} - a\beta - e, \\ 0, & \text{if } \frac{2}{\lambda} - a\beta - e \leq s < e + a\beta, \\ 1 - s\lambda + \beta\lambda, & \text{if } s \geq e + a\beta. \end{cases} \qquad \dot{s}_\Gamma(s) = \begin{cases} s, & \text{if } s < 2-e, \\ 1, & \text{if } 2-e \leq s < e, \\ s, & \text{if } e \leq s < \frac{e}{1-a}, \\ s, & \text{if } \frac{e}{1-a} \leq s < \frac{2}{\lambda} - a\beta - e, \\ \frac{1}{\lambda}, & \text{if } \frac{2}{\lambda} - a\beta - e \leq s < e + a\beta, \\ s, & \text{if } s \geq e + a\beta. \end{cases} \tag{B.8}$$

The resulting public responses η^* in the six intervals of $\dot{s}_\Gamma(s)$ are 1, 1, $1/s$, λ, λ, and 0, respectively. Focusing on the first three intervals, which arise due to the newly introduced economic weight e, we observe that the government pools information over $[2-e, e)$ to raise social interaction levels. That is, it seeks to induce $\eta^* = 1$ when the pandemic is relatively mild ($s < e$). This strategy can be implemented in two equivalent ways.

- If we strictly follow the above results in characterizing the optimal public policy, the government would provide separate messages with no penalty for $s < 2-e$, and a pooled message with no penalty for $2 - e \leq s < e$.
- An alternative method to achieve the same public response is that the government commits to no penalty and a pooling message directly for $s < e$.

In both cases, the public response remains $\eta^* = 1$, and the simplified structure enables the government to reduce the size of the message space. Therefore, we adopt the latter case to describe the results and Figure 7. To summarize, the values of thresholds are: $\dot{s}_0 = e$, $\dot{s}_1 = \frac{e}{1-a}$, $\dot{s}_2 = \frac{2}{\lambda} - a\beta - e$ and $\dot{s}_3 = e + a\beta$. To complete the proof of Corollary 4, we can examine how each threshold varies with respect to e. □

Proof of Proposition 5. We follow a similar approach as in the derivation of Proposition 1. Under full information disclosure and the revised equilibrium response, the government's optimal penalty mapping, denoted by $\ddot{p}^c_\Gamma$, solves the following optimization problem for each $s \in S$:

$$\ddot{p}^c_\Gamma(s) = \arg\max_p -s\ddot{\eta}^*(s,p) - ap, \quad \text{for any } s \in S. \tag{B.9}$$

The corresponding utility function of the government is given by:

$$\ddot{u}^c(s) = -s\ddot{\eta}^*(s, \ddot{p}^c_\Gamma(s)) - a\ddot{p}^c_\Gamma(s). \tag{B.10}$$

Depending on the value of the marginal cost a, the government's optimal strategy and utility under complete information take different forms. When $a < 1$, the functions are given by:

$$\ddot{p}^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < a, \\ 1, & \text{if } s \geq a. \end{cases} \quad \ddot{u}^c(s) = \begin{cases} -s, & \text{if } s < a, \\ -a, & \text{if } s \geq a. \end{cases} \tag{B.11}$$

When $a \geq 1$, the functions are given by:

$$\ddot{p}^c_\Gamma(s) = \begin{cases} 0, & \text{if } s < 1, \\ 0, & \text{if } 1 \leq s < \frac{a\overline{\beta} - \sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}, \\ 1, & \text{if } s \geq \frac{a\overline{\beta} - \sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}. \end{cases} \tag{B.12}$$

$$\ddot{u}^c(s) = \begin{cases} -s, & \text{if } s < 1, \\ \frac{s\left(s-\beta-\sqrt{4\beta+(s-\beta)^2}\right)}{2\beta}, & \text{if } 1 \leq s < \frac{a\overline{\beta} - \sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}, \\ -a, & \text{if } s \geq \frac{a\overline{\beta} - \sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}. \end{cases} \tag{B.13}$$

Note that the case $a < 1$ can be viewed as a special instance of $a \geq 1$ in which the intermediate interval vanishes. Thus, we focus on the more general case with $a \geq 1$ in the discussion below. By analyzing the derivative of $\ddot{u}^c(s)$ with respect to s, we find that the utility function is convex throughout its domain. This implies that full information disclosure is optimal: the government benefits from sending fully separating messages across all pandemic states and applying penalties according to the mapping $\ddot{p}^c_\Gamma(s)$ defined in Equation (B.12). However, to reduce the complexity of the message space, a simplified communication scheme can be adopted. Specifically:

- Send a pooling message for all states $s < 1$, which maintains the equilibrium outcome $\eta^* = 1$ in that region.

- Send a pooling message for all states $s > \frac{a\overline{\beta}-\sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}$, preserving the equilibrium outcome $\eta^* = 0$ in that region.

This simplification reduces the size of the message space. Accordingly, the simplified case is presented in Figure 9. To summarize, the relevant thresholds under full information are $\ddot{s}_1 = 1$ and $\ddot{s}_2 = \frac{a\overline{\beta}-\sqrt{a^2\overline{\beta}(-4a+\overline{\beta}+4)}}{2(a-1)}$. □

B.2 When the Penalty Set Is Discrete

In practice, governments often face constraints in selecting and adjusting penalty levels. In other words, the set P in the public policy formulation $\Gamma = \{\mathbb{F}, P, M\}$ is limited in terms of its maximum cardinality. Let N_p denote the upper bound on the number of distinct penalties the government can implement. Under this constraint, we obtain the following result:

Proposition B.1. *When the maximum cardinality N_p decreases, the set of feasible policies that the government can credibly commit to becomes more restricted. As a result, the government's expected utility weakly decreases.*

This result is direct: reducing the size of the penalty set offers no strategic advantage to the government and only limits its flexibility. Beyond the current model, a realistic consideration is that increasing the number of penalty levels or adjusting them dynamically may incur administrative or political costs. Let $|P|$ denote the actual number of penalties used. If expanding $|P|$ entails additional costs, the government's optimal policy will adjust accordingly, potentially involving fewer penalty levels in the commitment phase.

A further question of interest is the role of messages when the penalty set is constrained. In what follows, we analyze a special case where $N_p = 1$ and characterize the corresponding optimal public policy. To prevent some repeated discussions, we let the upper bound of state be $\overline{s} = 1$ for ease of exposition. In this special case, the optimal public policy is as follows:

Proposition B.2. *When the maximum cardinality of the penalty set is constrained to one, i.e., $N_p = 1$, the government selects among three candidate public policies by comparing their associated expected utilities:*

- *No Penalty: The government sets the penalty $p_1^* = 0$ and issues no message, leading to full violation across all pandemic states ($\eta^* = 1$).*
- *Intermediate Penalty: The government sets $p_2^* = \frac{a\lambda^2-2\lambda^2+6\lambda-4}{4(\lambda-1)}$. The messaging strategy is as follows:*

- *For mild states* $s < \frac{2(1-p_2^*)-\lambda}{\lambda}$*, it sends separating messages, resulting in full violation* $(\eta^* = 1)$*.*
- *For severe states* $s \geq \frac{2(1-p_2^*)-\lambda}{\lambda}$*, it sends a pooling message, resulting in compliance of traditional agents* $(\eta^* = \lambda)$*.*

- *Heavy Penalty: The government sets* $p_3^* = \frac{-a\lambda+4\beta\lambda-2\lambda+4}{4}$*. The messaging strategy is as follows:*
  - *For mild states* $s < \frac{2(1-p_3^*+\beta\lambda)-\lambda}{\lambda}$*, it sends separating messages, resulting in compliance of traditional agents* $(\eta^* = \lambda)$*.*
  - *For severe states* $s \geq \frac{2(1-p_3^*+\beta\lambda)-\lambda}{\lambda}$*, it sends a pooling message, achieving full compliance* $(\eta^* = 0)$*.*

Compared with the benchmark model in which the penalty set is unconstrained, two key differences emerge. First, restricting the penalty set introduces inefficiencies in managing public behavior across varying states. In the benchmark case, the government implements a policy that induces a gradual shift in public response—from full violation ($\eta^* = 1$) to partial compliance ($\eta^* = \lambda$) and eventually to full compliance ($\eta^* = 0$). Under the constraint, however, the government can induce at most two distinct compliance responses. This reduction in flexibility leads to potential inefficiencies in policy implementation and strictly lowers the government's expected utility.

Second, when the government is constrained in its choice of penalties, it relies more heavily on message-based persuasion. In the constrained setting, separating messages are employed more frequently, reflecting a strategic shift toward communication rather than enforcement. This underscores how limitations in penalty design increase the importance of information management in shaping public behavior.