Online Supplement: Experience and Pivoting in Entrepreneurial Product Development

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Appendix

A.1 Summary of Pivot Survey Results

Table 2: Summary statistics

Number of responses	49	Average number of pivots per team	2.7
Number of teams	25	Average time between pivots (in weeks)	5.2
Average similarity of pivots†		Average riskiness of pivots††	
Early (weeks 5-9)	0.57	Early (weeks 5-9)	0.26
Late (weeks 10-14)	0.92	Late (weeks 10-14)	0.15

† 0 = major pivot (unrelated), 1 = incremental pivot (similar)

†† 0 = low risk, 1 = high risk

A.2 Notation

Table 3: List of parameters

Parameter	**Description**
t	Time stage
μ	Expected revenue of the initial product
M_p, μ_p	Random and realized expected revenue of the pivot opportunity
α, β	Lower and upper bounds for the distribution of the expect revenue of pivot opportunity
m, s	Mean and standard deviation of the expected revenue of the pivot opportunity
U_i, $i = p, c, a$	Utility of pivoting, continuing, and abandoning the product
d_i, $i = p, c, pp$	Development cost for pivot, continue, post-launch pivot
σ, σ_p	Market uncertainties for the initial product and pivot alternative
ρ	Risk aversion
η	Degree of overprecision
$\mathcal{P}$ and $\mathcal{P}_p$	Pivot and post-launch pivot probabilities
$\bar{\mathcal{P}}$ and $\bar{C}$	Probabilities of pivot and continuation being errors
$\mathcal{E}^E$ and $\mathcal{E}^I$	Level of entrepreneurial and industry experience

A.3 Proofs

Proof of Lemma 2.

The region where pivoting is the optimal decision is bounded by two inequalities. The first inequality is as follows

$$\begin{aligned}U_p(\mu_p) > U_c &\Leftrightarrow \\ \mu_p - d_p - \rho(1-\eta)\sigma_p^2 > \mu - d_c - \rho(1-\eta)\sigma^2 &\Leftrightarrow \\ \mu_p > \mu + d_p - d_c + \rho(1-\eta)(\sigma_p^2 - \sigma^2).\end{aligned}$$

The second inequality is

$$\begin{aligned}U_p(\mu_p) > U_a &\Leftrightarrow \\ \mu_p - d_p - \rho(1-\eta)\sigma_p^2 > 0 &\Leftrightarrow \\ \mu_p > d_p + \rho(1-\eta)\sigma_p^2.\end{aligned}$$

Continuing is the optimal decision when $U_c > U_p(\mu_p)$ and $U_c > U_a$. Similar approach as deriving the pivot inequalities results in

$$\mu > \mu_p + d_c - d_p + \rho(1-\eta)(\sigma^2 - \sigma_p^2) \text{ and } \mu > d_c + \rho(1-\eta)\sigma^2.$$

Abandoning is the optimal decision when $U_a > U_c$ and $U_a > U_p(\mu_p)$. Similar approach as deriving the pivot inequalities results in

$$\mu < d_c + \rho(1-\eta)\sigma^2 \text{ and } \mu_p < d_p + \rho(1-\eta)\sigma_p^2.$$

□

Proof of Proposition 3.
We evaluate the impact of η on the pivot-continue threshold, i.e., $U_p(\mu_p) - U_c = 0 \Leftrightarrow \mu_p - \mu + d_c - d_p - \rho(1-\eta)(\sigma_p^2 - \sigma^2) = 0$. In other words, we assess

$$\frac{\partial}{\partial \eta}\mu_p - \mu + d_c - d_p - \rho(1-\eta)(\sigma_p^2 - \sigma^2) = \rho(\sigma_p^2 - \sigma^2).$$

Term $\rho(\sigma_p^2 - \sigma^2)$ is positive if $\sigma_p > \sigma$ and negative if $\sigma_p < \sigma$. Therefore, the pivot-continue threshold moves in favor of increasing the pivot (continue) region when overprecision increases if $\sigma_p > \sigma$ ($\sigma_p < \sigma$).

Next, we assess the impact of η on the pivot-abandon, i.e., $U_p(\mu)_p - U_a = 0 \Leftrightarrow \mu_p - d_p - \rho(1-\eta)\sigma_p^2 = 0$, and continue-abandon, $U_c - U_a = 0 \Leftrightarrow \mu - d_c - \rho(1-\eta)\sigma^2 = 0$, thresholds. We obtain

$$\frac{\partial}{\partial \eta}U_p(\mu_p) - U_a = \rho\sigma_p^2 \text{ and } \frac{\partial}{\partial \eta}U_c - U_a = \rho\sigma^2.$$

Consequently, the abandon region decreases relative to the pivot and continue regions when overprecision increases. Furthermore, if $\sigma_p > \sigma$ ($\sigma_p < \sigma$), then a larger proportion of the lost abandonment region goes to the pivot (continue) region if overprecision increases. □

Proof of Proposition 5.

The pivot probability, i.e., the probability that $U_p(M_p) \geq \max(U_c, U_a)$, is

$$\begin{aligned} \mathbb{P}\left[U_p(M_p) \geq \max\left(U_c, U_a\right)\right] &\Leftrightarrow \\ \mathbb{P}\left[M_p - d_p - \rho(1-\eta)\sigma_p^2 \geq \max\left(\mu - d_c - \rho(1-\eta)\sigma^2, 0\right)\right] &\Leftrightarrow \\ \mathbb{P}\left[M_p \geq d_p + \rho(1-\eta)\sigma_p^2 + \max\left(\mu - d_c - \rho(1-\eta)\sigma^2, 0\right)\right]&. \end{aligned}$$

We define $x = d_p + \rho(1-\eta)\sigma_p^2 + \max(\mu - d_c - \rho(1-\eta)\sigma^2, 0)$. Since, $U_p(M_p) - max(U_c, U_a)$ is positive for some μ_p and negative for other, this implies $\alpha \leq x \leq \beta$. Also, since $M_p \sim \mathcal{U}[\alpha,\beta]$, the pivot probability is

$$\mathbb{P}\left[M_p \geq x\right] = 1 - \frac{x-\alpha}{\beta-\alpha}. \tag{3}$$

We have two cases from the max function of x to investigate. First case $U_c \geq U_a$ holds if $\mu - d_c - \rho(1-\eta)\sigma^2 \geq 0 \Leftrightarrow \eta \geq 1 - \frac{\mu-d_c}{\rho\sigma^2}$. We obtain

$$\begin{aligned} \frac{\partial}{\partial\eta}\left(1 - \frac{x-\alpha}{\beta-\alpha}\right) &\Leftrightarrow \\ \frac{\partial}{\partial\eta}\left(1 - \frac{\mu - d_c + d_p + \rho(1-\eta)(\sigma_p^2 - \sigma^2) - \alpha}{\beta-\alpha}\right) &\Leftrightarrow \\ \frac{\rho(\sigma_p^2 - \sigma^2)}{\beta-\alpha}&. \end{aligned}$$

Since terms ρ and $\beta - \alpha$ are always positive, pivoting probability increases with η if $\sigma_p > \sigma$ and decreases if $\sigma_p < \sigma$.

Second case $U_c < U_a$ holds if $\eta < 1 - \frac{\mu-d_c}{\rho\sigma^2}$. We can solve that

$$\begin{aligned} \frac{\partial}{\partial\eta}\left(1 - \frac{d_p + \rho(1-\eta)\sigma_p^2 - \alpha}{\beta-\alpha}\right) &\Leftrightarrow \\ \frac{\rho\sigma_p^2}{\beta-\alpha}&. \end{aligned}$$

This is always positive resulting in that pivoting probability increases with η. □

Proof of Proposition 7.

The probabilities for pivot errors, $\bar{\mathcal{P}}$, can be solved from the following three possible cases:

1. $U_c \geq U_a = 0|(\eta > 0)$ and $U_c < 0|(\eta' = 0)$, in which case $\bar{\mathcal{P}} = \mathbb{P}\left[U_p(M_p) \geq 0|(\eta > 0) \ \wedge \ U_p(M_p) < 0|(\eta' = 0)\right]$.
2. $U_c \geq 0|(\eta > 0)$ and $U_c \geq 0|(\eta' = 0)$, in which case $\bar{\mathcal{P}} = \mathbb{P}\left[U_p(M_p) \geq U_c|(\eta > 0) \wedge U_p(M_p) < U_c|(\eta' = 0)\right]$.
3. $U_c < 0|(\eta > 0)$ and $U_c < 0|(\eta' = 0)$, in which case $\bar{\mathcal{P}} = \mathbb{P}\left[U_p(M_p) \geq U_c|(\eta > 0) \ \wedge \ U_p(M_p) < 0|(\eta' = 0)\right]$.

In the first case, $U_c \geq 0|(\eta > 0) \longrightarrow \sigma \leq \sqrt{\frac{\mu-d_c}{\rho(1-\eta)}}$ and $U_c < 0|(\eta' = 0) \longrightarrow \sigma > \sqrt{\frac{\mu-d_c}{\rho}}$. Together, this implies $\sqrt{\frac{\mu-d_c}{\rho}} < \sigma \leq \sqrt{\frac{\mu-d_c}{\rho(1-\eta)}}$. The probability for a pivot to be a mistake $\bar{\mathcal{P}}$ is

$$\begin{aligned} \mathbb{P}\left[U_p(M_p) \geq 0|(\eta > 0) \ \wedge \ U_p(M_p) < 0|(\eta' = 0)\right] &= \\ \mathbb{P}\left[\mu - d_c + d_p - \rho(1-\eta)(\sigma^2 - \sigma_p^2) \leq M_p < d_p + \rho\sigma_p^2\right]&. \end{aligned}$$

Therefore, it is required that $\mu - d_c + d_p - \rho(1-\eta)(\sigma^2 - \sigma_p^2) < d_p + \rho\sigma_p^2$, from which follows that $\sigma > \sqrt{\frac{\mu - d_c - \rho\eta\sigma_p^2}{\rho(1-\eta)}}$. Given this condition is satisfied, we have

$$\begin{aligned}
\mathbb{P}\left[\mu - d_c + d_p - \rho(1-\eta)(\sigma^2 - \sigma_p^2) < M_p < d_p + \rho\sigma_p^2\right] &= \\
\frac{d_p + \rho\sigma_p^2 - \left[\mu - d_c + d_p - \rho(1-\eta)(\sigma^2 - \sigma_p^2)\right]}{\beta - \alpha} &= \\
\frac{\rho\eta(\sigma_p^2 - \sigma^2) + \rho\sigma^2 - \mu - d_c}{\beta - \alpha}&.
\end{aligned}$$

In summary, under the first case $\bar{\mathcal{P}} = \frac{\rho\eta(\sigma_p^2 - \sigma^2) + \rho\sigma^2 - \mu - d_c}{\beta - \alpha}$, which holds when $\max\left\{\sqrt{\frac{\mu - d_c - \rho\eta\sigma_p^2}{\rho(1-\eta)}}, \sqrt{\frac{\mu - d_c}{\rho}}\right\} < \sigma \leq \sqrt{\frac{\mu - d_c}{\rho(1-\eta)}}$. In the second case, similar analysis provides $\bar{\mathcal{P}} = \frac{\rho\eta(\sigma_p^2 - \sigma^2)}{\beta - \alpha}$, if $\sigma \leq \min\left\{\sqrt{\frac{\mu - d_c}{\rho}}, \sigma_p\right\}$. In the third case, $\bar{\mathcal{P}} = \frac{\rho\eta\sigma_p^2}{\beta - \alpha}$ when $\sigma > \sqrt{\frac{\mu - d_c}{\rho(1-\eta)}}$.

The probabilities for persistency mistakes, $\bar{C}$, can be solved from the following two possible cases:

1. $U_c \geq 0|(\eta > 0)$ and $U_c < 0|(\eta' = 0)$, in which case $\bar{C} = \mathbb{P}\left[U_c \geq U_p(M_p)|(\eta > 0)\right]$.
2. $U_c > 0|(\eta > 0)$ and $U_c \geq 0|(\eta' = 0)$, in which case $\bar{C} = \mathbb{P}\left[U_c \geq U_p(M_p)|(\eta > 0) \wedge U_c < U_p(M_p)|(\eta' = 0)\right]$.

The first case results in $\bar{C} = \frac{\mu - d_c + d_p - \rho(1-\eta)(\sigma^2 - \sigma_p^2) - \alpha)}{\beta - \alpha}$ given that $\sqrt{\frac{\mu - d_c}{\rho}} < \sigma \leq \sqrt{\frac{\mu - d_c}{\rho(1-\eta)}}$. In the second case, $\bar{C} = \frac{\rho\eta(\sigma^2 - \sigma_p^2)}{\beta - \alpha}$ if $\sigma_p < \sigma \leq \sqrt{\frac{\mu - d_c}{\rho}}$.

The abandonment decision is not an error due to overconfidence. This is because if $U_a = 0 > \max\{U_c, U_p(\mu_p)\}|(\eta > 0)$ for a realized μ_p then also $0 > \max\{U_c, U_p(\mu_p)\}|(\eta' = 0)$ due to $U_c|(\eta > 0) > U_c|(\eta' = 0)$ and $U_p(\mu_p)|(\eta > 0) > U_p(\mu_p)|(\eta' = 0)$. □

Proof of Proposition 8.

To prove the impact of the mean-preserving increase in the standard deviation, we replace α and β by $\alpha - \Delta$ and $\beta + \Delta$ in (5) and obtain

$$1 - \frac{x - (\alpha - \Delta)}{\beta - \alpha + 2\Delta}.$$

We solve

$$\begin{aligned}
\frac{\partial}{\partial \Delta}\left(1 - \frac{x - (\alpha - \Delta)}{\beta - \alpha + 2\Delta}\right) = 0 &\Leftrightarrow \\
x = \frac{\alpha + \beta}{2} = m&.
\end{aligned} \tag{4}$$

We consider first the case $U_c \geq U_a$, implying that $x = \mu - d_c + d_p + \rho(1-\eta)(\sigma_p^2 - \sigma^2)$. In order for the pivot probability to be increasing in Δ, we solve for $\mu - d_c + d_p + \rho(1-\eta)(\sigma_p^2 - \sigma^2) \geq m$. We obtain

$$\begin{cases} \eta \leq 1 - \frac{m - d_p - (\mu - d_c)}{\rho(\sigma_p^2 - \sigma^2)} & \text{if } \sigma_p \geq \sigma \text{ and} \\ \eta \geq 1 - \frac{m - d_p - (\mu - d_c)}{\rho(\sigma_p^2 - \sigma^2)} & \text{if } \sigma_p < \sigma. \end{cases}$$

Similarly, we can solve the case $U_c < U_a$ when $x = d_p + \rho(1-\eta)\sigma_p^2$. This results in pivot probability increasing with Δ when $\eta \leq 1 - \frac{m-d_p}{\rho\sigma_p^2}$. □

Proof of Corollary 9.

Since the utility of the post-launch pivot opportunity is $U_p(\mu_p) = \mu_p - d_{pp} - \rho(1-\eta)\sigma_p^2$, it is optimal to make a post-launch pivot if $U_p(\mu_p)$ exceeds the revenue of the original product launched on market v, i.e., $\mu_p - d_{pp} - \rho(1-\eta)\sigma_p^2 \geq v$. The post-launch pivot probability can be solved from

$$\begin{aligned}\mathcal{P}_p = \mathbb{P}\left[U_p(M) \geq v\right] \Leftrightarrow \\ \mathbb{P}\left[M \geq d_{pp} + \rho(1-\eta)\sigma_p^2 + v\right] = \\ 1 - \frac{d_{pp} + \rho(1-\eta)\sigma_p^2 + v - \alpha}{\beta - \alpha}.\end{aligned}$$

We solve

$$\frac{\partial}{\partial\eta}\left(1 - \frac{d_{pp} + \rho(1-\eta)\sigma_p^2 + v - \alpha}{\beta - \alpha}\right) = \frac{\rho\sigma_p^2}{\beta - \alpha}.$$

Since, $\frac{\rho\sigma_p^2}{\beta-\alpha} > 0$, post launch pivot probability increases with η. □

Proof of Proposition 11.

In the first part of proof, we show the impact of $\mathcal{E}^E$ on the pivot probability. The pivot probability, i.e., the probability that $U_p(M_p) \geq \max(U_c, U_a)$, is

$$\begin{aligned}\mathbb{P}\left[U_p(M_p) \geq \max(U_c, U_a)\right] \Leftrightarrow \\ \mathbb{P}\left[M_p - d_p - \rho\mathcal{E}^E\sigma_p^2 \geq \max\left(\mu - d_c - \rho\mathcal{E}^E\sigma^2, 0\right)\right] \Leftrightarrow \\ \mathbb{P}\left[M_p \geq d_p + \rho\mathcal{E}^E\sigma_p^2 + \max\left(\mu - d_c - \rho\mathcal{E}^E\sigma^2, 0\right)\right].\end{aligned}$$

We define $x = d_p + \rho\mathcal{E}^E\sigma_p^2 + \max(\mu - d_c - \rho\mathcal{E}^E\sigma^2, 0)$. Since, $U_p(M_p) - max(U_c, U_a)$ is positive for some μ_p and negative for other, this implies $\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha) \leq x \leq \beta$. Also, since $M_p \sim \mathcal{U}[\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha), \beta]$, the pivot probability is

$$\mathbb{P}\left[M_p \geq x\right] = 1 - \frac{x - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}. \tag{5}$$

We have two cases from the max function of x to investigate. First case $U_c \geq U_a$ holds if $\mu - d_c - \rho\mathcal{E}^E\sigma^2 \geq 0 \Leftrightarrow \mathcal{E}^E \leq \frac{\mu-d_c}{\rho\sigma^2} \equiv \mathcal{E}_0^E$. We obtain

$$\begin{aligned}\frac{\partial}{\partial\mathcal{E}^E}\left(1 - \frac{x - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}\right) \Leftrightarrow \\ \frac{\partial}{\partial\mathcal{E}^E}\left(1 - \frac{\mu - d_c + d_p + \rho\mathcal{E}^E(\sigma_p^2 - \sigma^2) - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}\right) \Leftrightarrow \\ \frac{\mathcal{E}^I(\beta - \mu + d_c - d_p) + \rho(\sigma^2 - \sigma_p^2)}{(\beta-\alpha)(\mathcal{E}^E\mathcal{E}^I - 1)^2}.\end{aligned}$$

We can solve when this derivative is positive yielding that pivot probability increases with $\mathcal{E}^E$ if $\sigma_p < \sqrt{\sigma^2 + \frac{\mathcal{E}^I(\beta-\mu+d_c-d_p)}{\rho}} \equiv \sigma_1(\mathcal{E}^I)$ and decreases if $\sigma_p > \sigma_1(\mathcal{E}^I)$.

Second case $U_c < U_a$ holds if $\mathcal{E}^E > \mathcal{E}_0^E$. We can solve that

$$\frac{\partial}{\partial \mathcal{E}^E}\left(1 - \frac{d_p + \rho\mathcal{E}^E\sigma_p^2 - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}\right) \Leftrightarrow \frac{\mathcal{E}^I(\beta - d_p) - \rho\sigma_p^2}{(\beta-\alpha)(\mathcal{E}^E\mathcal{E}^I - 1)^2}.$$

This is positive when $\sigma_p < \sqrt{\frac{\mathcal{E}^I(\beta-d_p)}{\rho}} \equiv \sigma_2(\mathcal{E}^I)$, and therefore pivot probability increases with $\mathcal{E}^E$ if $\sigma_p < \sigma_2(\mathcal{E}^I)$ and decreases with $\mathcal{E}^E$ if $\sigma_p > \sigma_2(\mathcal{E}^I)$.

We have $\sigma_1(\mathcal{E}^I) \geq \sigma_2(\mathcal{E}^I)$ if and only if $\sigma \geq \sqrt{\frac{\mathcal{E}^I(\mu-d_c)}{\rho}}$. If $\sigma_1(\mathcal{E}^I) < \sigma_2(\mathcal{E}^I)$, i.e., $\sigma < \sqrt{\frac{\mathcal{E}^I(\mu-d_c)}{\rho}}$, then there exists no σ that also satisfies $\mathcal{E}^E < \mathcal{E}_0^E \Leftrightarrow \sigma > \sqrt{\frac{\mu-d_c}{\rho\mathcal{E}^E}}$. Therefore, when $\sigma_1(\mathcal{E}^I) < \sigma_2(\mathcal{E}^I)$ we always have $\mathcal{E}^E \geq \mathcal{E}_0^E$. Finally, we define equivalent conditions for $\sigma_p > \sigma_1(\mathcal{E}^I) = \sqrt{\sigma^2 + \frac{\mathcal{E}^I(\beta-\mu+d_c-d_p)}{\rho}}$ and $\sigma_p > \sigma_2(\mathcal{E}^I) = \sqrt{\frac{\mathcal{E}^I(\beta-d_p)}{\rho}}$, in terms of $\mathcal{E}^I$, as follows $\mathcal{E}^I < \frac{\rho(\sigma_p^2-\sigma^2)}{\beta-\mu+d_c-d_p} \equiv \mathcal{E}_1^I$ and $\mathcal{E}^I < \frac{\rho\sigma^2}{\beta-d_p} \equiv \mathcal{E}_2^I$, respectively.

In the second part of proof, we show the impact of $\mathcal{E}^I$ on the pivot probability. Given pivot probability in (5) as follows

$$\mathbb{P}\left[M_p \geq x\right] = 1 - \frac{x - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}$$

and $x = d_p + \rho\mathcal{E}^E\sigma_p^2 + \max(\mu - d_c - \rho\mathcal{E}^E\sigma^2, 0)$ with $\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha) \leq x \leq \beta$, the numerator $x - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))$ is always less or equal to the denominator $\beta - (\alpha + \mathcal{E}^E\mathcal{E}^I(\beta-\alpha))$. Therefore, an increase in $\mathcal{E}^I$ results in a monotonic decrease in $\frac{x-(\alpha+\mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}{\beta-(\alpha+\mathcal{E}^E\mathcal{E}^I(\beta-\alpha))}$, implying that the pivot probability increases monotonically. $\square$

A.4 Extension: Overestimation

We provide first a formal definition for overestimation that is aligned with Van den Steen (2011).

Definition 12 *The degree of overestimation $\xi \in [0, \infty)$ specifies how much entrepreneurs overestimate the expected revenues of the initial and the pivot opportunity. We operationalize this by multiplying μ, μ_p by $(1+\xi)$.*

Relying on the definition, we show the impact of the overestimation on the pivot, continuation, and abandon regions in Proposition 13, which is followed by its proof.

Proposition 13 *The pivot region increases relative to the continuation region due to the overestimation ξ if $\mu_p > \mu$ and decreases if $\mu_p < \mu$. The abandoning region decreases due to overestimation.*

Proof of Proposition 13.
We evaluate the impact of ξ on the pivot-continue threshold, i.e., $U_p((1+\xi)\mu_p) - U_c = 0 \Leftrightarrow (1+\xi)(\mu_p - \mu) + d_c - d_p - \rho(\sigma_p^2 - \sigma^2) = 0$. In other words, we assess

$$\frac{\partial}{\partial \xi}(1+\xi)(\mu_p - \mu) + d_c - d_p - \rho(\sigma_p^2 - \sigma^2) = \mu_p - \mu.$$

Term $\mu_p - \mu$ is positive if $\mu_p > \mu$ and negative if $\mu_p < \mu$. Therefore, the pivot-continue threshold moves in favor of increasing the pivot (continue) region when overestimation increases if $\mu_p > \mu$ ($\mu_p < \mu$).

Similar analysis for the pivot-abandon and continue-abandon thresholds results that the abandon region decreases relative to the pivot and continue regions when overestimation increases. □

The result in Proposition 13 is analogous to that of Proposition 3, except that the threshold changes depend on the relative expected revenues instead of the relative revenue standard deviations. Next, we formalize the impact of the overestimation on the pivot probability in Proposition 14.

Proposition 14 *If $\sigma_p > \sqrt{\sigma^2 + \frac{d_c - d_p}{\rho}}$, then pivot probability, $\mathcal{P}$ increases monotonically with ξ. If $\sigma_p < \sqrt{\sigma^2 + \frac{d_c - d_p}{\rho}}$, then $\mathcal{P}$ increases monotonically until a critical point $\xi_0 = \frac{\rho\sigma^2 + d_c}{\mu} - 1$, after which $\mathcal{P}$ decreases monotonically with ξ.*

Proof of Proposition 14.

The pivot probability, i.e., the probability that $U_p((1+\xi)M_p) \geq \max(U_c, U_a)$, is

$$\mathbb{P}\left[U_p((1+\xi)M_p) \geq \max\left(U_c, U_a\right)\right] \Leftrightarrow$$
$$\mathbb{P}\left[(1+\xi)M_p - d_p - \rho\sigma_p^2 \geq \max\left((1+\xi)\mu - d_c - \rho\sigma^2, 0\right)\right] \Leftrightarrow$$
$$\mathbb{P}\left[M_p \geq \frac{d_p + \rho\sigma_p^2 + \max\left((1+\xi)\mu - d_c - \rho\sigma^2, 0\right)}{1+\xi}\right].$$

We solve this similar to what we did in the proof of Proposition 5. When $U_c \geq U_a \Leftrightarrow \xi \geq \frac{\rho\sigma^2 + d_c}{\mu} - 1$, we obtain that the pivot probability increases monotonically in overestimation when $\sigma_p > \sqrt{\sigma^2 + \frac{d_c - d_p}{\rho}}$ and otherwise it decreases in overestimation. When $U_c < U_a \Leftrightarrow \xi < \frac{\rho\sigma^2 + d_c}{\mu} - 1$, we can solve that the pivot probability increases if $d_p + \rho\sigma_p^2 > 0$, which is always true. □

The pivot probability in Proposition 14 behaves qualitatively the same way as if the entrepreneurs are plagued by overprecision given in Proposition 5. In particular, the pivot probability increases for a risky pivot with overconfidence (i.e., either η or ξ), whereas the pivot probability for a safe pivot increases until a critical point, which after it decreases with overconfidence. However, we also notice that risk aversion has a stronger role when overconfidence is caused by overestimation. Namely, more conservative, risk-averse, founders are more likely to take costly risky pivots (i.e., the condition $\sigma_p > \sqrt{\sigma^2 + \frac{d_c - d_p}{\rho}}$, given $d_p > d_c$, is more likely to hold).

A.5 Extension: Correlated Entrepreneurs' Experience Level (Overconfidence) and Risk Aversion

The conceptual paper of Salamouris (2013) suggests that overconfidence leads to increased risk-taking. While our current model captures this effect indirectly, since overprecision reduces perceived uncertainties and thereby encourages entrepreneurs to pursue riskier products, we now consider an extension in which overconfidence is directly correlated with risk aversion. Although such a direct correlation is considered unlikely, as noted by Michailova et al. (2017), we include an analysis to explore this possibility.

We model the negative correlation between overprecision, η, and risk aversion, ρ, by defining $\rho' = \rho(1-\eta)$. Under this formulation, expert entrepreneurs without overconfidence bias, $\eta = 0$, retain their orignal risk aversion, $\rho' = \rho$, while novice entrepreneurs exhibiting high overconfidence, $\eta = 1$, become risk neutral, $\rho' = 0$. We formalize the impacts of the direct correlation between η and ρ on pivot probability as follows:

Proposition 15 *Given $\rho' = \rho(1-\eta)$, the optimal pivot probability, $\mathcal{P}$ behaves as in Proposition 5 except when $\sigma_p < \sigma$ then the critical point is $\eta_0 = 1 - \sqrt{\frac{\mu - d_c}{\rho\sigma^2}}$.*

Proof of Proposition 15.
The pivot probability when the risk aversion is $\rho' = \rho(1-\eta)$ can be solved as follows

$$\mathbb{P}\left[U_p(M_p) \geq \max\left(U_c, U_a\right)\right] \Leftrightarrow$$
$$\mathbb{P}\left[M_p \geq d_p + \rho(1-\eta)^2\sigma_p^2 + \max\left(\mu - d_c - \rho(1-\eta)^2\sigma^2, 0\right)\right].$$

The max function results in two possible cases. In the first case, $\eta \geq 1 - \sqrt{\frac{\mu - d_c}{\rho\sigma^2}}$, we solve

$$\frac{\partial}{\partial\eta}\left(1 - \frac{\mu - d_c + d_p + \rho(1-\eta)^2(\sigma_p^2 - \sigma^2) - \alpha}{\beta - \alpha}\right) = \frac{2(1-\eta)\rho(\sigma_p^2 - \sigma^2)}{\beta - \alpha}.$$

The expression is positive if $\sigma_p > \sigma$ and negative if $\sigma_p < \sigma$. Therefore, the pivot probability increases (decreases) if $\sigma_p > \sigma$ ($\sigma_p < \sigma$) with η.

In the second case, $\eta < 1 - \sqrt{\frac{\mu - d_c}{\rho\sigma^2}}$, we solve

$$\frac{\partial}{\partial\eta}\left(1 - \frac{d_p + \rho(1-\eta)^2\sigma_p^2 - \alpha}{\beta - \alpha}\right) = \frac{2(1-\eta)\rho\sigma_p^2}{\beta - \alpha},$$

which is always positive. Hence, the pivot probability increases with η. □

Proposition 15 implies that pivot probability increases for high-risk pivot opportunities under overconfidence, whereas for low-risk pivot opportunities, it increases up to a critical point and then declines as overconfidence grows. These results are qualitative consistent with those obtained without a direct correlation (see Proposition 5), thereby confirming the robustness of our findings. Similarly, we find our results to be robust when considering overestimation instead of overprecision, as demonstrated in Proposition 16.

Proposition 16 *Given $\rho' = \rho(1 - \epsilon\xi)$, $0 \leq \epsilon \leq 1/\xi$, if $\sigma_p > \sqrt{\sigma^2 + \frac{d_c - d_p}{(1+\epsilon)\rho}}$, then pivot probability, $\mathcal{P}$ increases monotonically with ξ. If $\sigma_p < \sqrt{\sigma^2 + \frac{d_c - d_p}{(1+\epsilon)\rho}}$, then $\mathcal{P}$ increases monotonically until a critical point $\xi_0' = \frac{(1+\epsilon)\rho\sigma^2 + d_c}{\mu + \rho\epsilon\sigma^2} - 1$, after which $\mathcal{P}$ decreases monotonically with ξ.*

The proof of Proposition 16 is similar to that of Proposition 14. Proposition 16 shows that the behavior of the pivot probability remains qualitatively the same as if the direct correlation were not included as given in Proposition 14.

A.6 Extension: Experience Impacts both Lower and Upper Bounds of Pivot Distribution

We consider that entrepreneurial experience, in combination with industry experience improves the expected quality of the pivot opportunity by increasing both the lower and upper bounds of the pivot opportunity, i.e., we replace α, by $\mathcal{E}^E\mathcal{E}^I\kappa_1$ and β, by $\mathcal{E}^E\mathcal{E}^I\kappa_2$ when $\kappa_2 < \kappa_1$ and $\kappa_1 - \kappa_2 < \beta - \alpha$. With this adjustment, pivot probability depends on the founders' entrepreneurial experience and industry experience as follows.

Corollary 17 *There exists thresholds* $\mathcal{E}_1^I = \frac{\rho(\sigma_p^2-\sigma^2)(\beta-\alpha)}{\kappa_1(\beta-\mu+d_c-d_p)-\kappa_2(\alpha-\mu+d_c-d_p)}$ *and* $\mathcal{E}_2^I = \frac{\rho\sigma^2(\beta-\alpha)}{\kappa_1(\beta-d_p)-\kappa_2(\alpha-d_p)}$ *such that:*

- *If* $\mathcal{E}^I < \mathcal{E}_2^I$*, then* $\mathcal{P}$ *decreases monotonically with* $\mathcal{E}^E$.
- *If* $\mathcal{E}_2^I < \mathcal{E}^I < \mathcal{E}_1^I$*, then* $\mathcal{P}$ *decreases monotonically until* $\mathcal{E}_0^E = \frac{\mu-d_c}{\rho\sigma^2}$*, after which* $\mathcal{P}$ *increases monotonically in* $\mathcal{E}^E$*. This region only exists if* $\sigma \geq \sqrt{\frac{\mathcal{E}^I(\mu-d_c)(\kappa_1-\kappa_2)}{\rho(\beta-\alpha)}}$.
- *If* $\mathcal{E}^I > \max\{\mathcal{E}_1^I, \mathcal{E}_2^I\}$*, then* $\mathcal{P}$ *increases monotonically with* $\mathcal{E}^E$.

Furthermore, pivot probability, $\mathcal{P}$*, increases monotonically with industry experience* $\mathcal{E}^I$.

The proof of Corollary 17 is similar to that of Proposition 11. Corollary 17 shows that the behavior of the pivot probability remains qualitatively the same as if experience impacts only the lower bound α of the pivot distribution. Indeed, only the thresholds $\mathcal{E}_1^I$, $\mathcal{E}_2^I$, and $\sigma \geq \sqrt{\frac{\mathcal{E}^I(\mu-d_c)(\kappa_1-\kappa_2)}{\rho(\beta-\alpha)}}$ are altered.

A.7 The Impact of Overprecision on Pivot Propensity When Accounting for Industry Experience

The impact of overprecision on the pivot probability in the presence of industry experience is as follows:

Corollary 18 *There exists thresholds* $\sigma_1 = \sqrt{\sigma^2 + \frac{\mathcal{E}^I(\beta-\mu+d_c-d_p)}{\rho}}$ *and* $\sigma_2 = \sqrt{\frac{\mathcal{E}^I(\beta-d_p)}{\rho}}$ *such that:*

- *If* $\sigma_p > \sigma_1$*, then pivot probability,* $\mathcal{P}$ *increases monotonically with* η.
- *If* $\sigma_2 < \sigma_p < \sigma_1$*, then* $\mathcal{P}$ *increases monotonically until* $\eta_0 = 1 - \frac{\mu-d_c}{\rho\sigma^2}$*, after which* $\mathcal{P}$ *decreases monotonically in* η*. This region only exists if* $\sigma \geq \sqrt{\frac{\mathcal{E}^I(\mu-d_c)}{\rho(\beta-\alpha)}}$.
- *If* $\sigma_p < \min\{\sigma_1, \sigma_2\}$*, then* $\mathcal{P}$ *decreases monotonically with* η.

The proof is similar to that of Proposition 11. Corollary 18 allows direct comparison with Proposition 5. As can be observed, the main implication of Proposition 5 remains unchanged in terms of the types of pivots the founders are likely to pursue. In other words, founders with little entrepreneurial experience (high overconfidence η) are more inclined to pursue riskier pivots ($\sigma_p > \sigma_1$), whereas founders with more entrepreneurial experience tend to favor safer pivots ($\sigma_p < \sigma_1$).