E-Companion to "Fighting Together: Agency vs. Wholesale Distribution Contracts for Digital Content Supply Chains in the Presence of Piracy"

EC.1. Tables

Table EC.1 Equilibrium under HH: $\theta = 1$, $r = 1$

Variables	V (VI Firm)	W (Wholesale)	A (Agency)
w^{HH*}	—	$\frac{1}{2}$	—
p^{HH*}	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$
π_V^{HH*}	$\frac{1}{4}$	—	—
π_M^{HH*}	—	$\frac{1}{8}$	$\frac{1-\phi}{4}$
π_P^{HH*}	—	$\frac{1}{16}$	$\frac{\phi}{4}$
π_T^{HH*}	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{4}$
CS^{HH*}	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{8}$

Table EC.2 Equilibrium under HL: $\theta = 1$, $r \leq 1$

Variables	V (VI Firm)	W (Wholesale)	A (Agency)
r^{HL*}	$\frac{\beta}{4}$	$\frac{\beta}{16}$	$\frac{\beta\phi}{4}$
w^{HL*}	—	$\frac{16(1-\beta)+\beta^2}{32}$	—
p^{HL*}	$\frac{(2-\beta)^2}{8}$	$\frac{3(16(1-\beta)+\beta^2)}{64}$	$\frac{4(1-\beta)+\beta^2\phi}{8}$
π_V^{HL*}	$\frac{8(1-\beta)+\beta^2}{32}$	—	—
π_M^{HL*}	—	$\frac{16(1-\beta)+\beta^2}{128}$	$\frac{(1-\phi)(4(1-\beta)+\beta^2\phi)}{16}$
π_P^{HL*}	—	$\frac{32(1-\beta)+\beta^2}{512}$	$\frac{\phi(8(1-\beta)+\beta^2\phi)}{32}$
π_T^{HL*}	$\frac{8(1-\beta)+\beta^2}{32}$	$\frac{96(1-\beta)+5\beta^2}{512}$	$\frac{8(1-\beta)+\beta^2(2-\phi)\phi}{32}$
CS^{HL*}	$\frac{4(1+3\beta)-3\beta^2}{32}$	$\frac{16(1+15\beta)-15\beta^2}{512}$	$\frac{4(1+3\beta)-3\beta^2\phi}{32}$

Table EC.3 Thresholds on the Cost of Quality Improvement c

Threshold	Definition
c_V	$\frac{(2-\beta)^2}{16}$
c_W	$\frac{8(1-\beta)+\beta^2}{64}$
c_A	$\frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8}$
c_A^θ	$\frac{\beta^2(1-\phi)^2}{8}$
c_A^M	$\frac{\beta^2(1-\phi)}{16}$
c_A^{CS}	$\frac{1}{72} - \frac{4+3\beta(4+3\beta(1-\phi)(1-9\phi-9\beta(1-\phi)))}{288(1+3\beta)} + \frac{\beta^2(1-\phi)\sqrt{1+9\beta^2(1-\phi)(9-17\phi)-18\beta(1-\phi)^2-\phi(58-121\phi)}}{32(1+3\beta)}$
c_{AW}^θ	$\frac{\beta^2(1-\phi)(1-4\phi)}{32(1-2\phi)}$
c_{AW}^r	$\frac{2(1-\beta)+\phi\beta^2}{64\phi}$
$\overline{c}_\pi$	$\frac{\beta^2(1-\phi)(1-3\phi)}{48-64(2-\phi)\phi}$
$\underline{c}_\pi$	$\frac{16(1-\beta)+\beta^2(1+8(1-\phi)\phi)+\sqrt{(16(1-\beta)(2\phi-1)+\beta^2(1-8(1-\phi)\phi))(16(3-2\phi)(1-\beta)+\beta^2(1-8(1-\phi)\phi))}}{128}$
c_{AW}^{CS}	$\frac{8(1-\phi)}{5} + \frac{\beta^2(1-\phi)\phi}{8} - \frac{8(11+230\beta)(1-\phi)}{80+75(16-\beta)\beta} + \frac{(1-\beta)(1-\phi)\sqrt{3(15(\beta-16)\beta-16)\beta^2\phi+64(1+3\beta)^2}}{16+15(16-\beta)\beta}$

EC.2. Detailed Analysis

Here, we provide a detailed analysis of our model. We begin with the general model, that is, scenario LL, followed by the baseline scenario with zero piracy, that is, Scenario LH, which is a special case of the general model. Then we analyze the two other special cases, Scenario HH, and Scenario HL, which are the *highest-quality* special cases of Scenario LH and LL, respectively. The analyses of the three special cases follow a logic similar to the general model, so we provide the most detailed analysis for Scenario LL.

EC.2.1. Analysis of the Base Model : $\theta \leq 1$ and $r \leq 1$ (Scenario LL)

Decisions in stages 1 and 2 are given as $\theta \leq 1$ and $r \leq 1$, while $c > 0$ and $k > 0$. In stage 4, the consumer with valuation v faces quality $\theta \leq 1$ and enforcement level $r \leq 1$. By purchasing genuine content at price p, the consumer will receive the utility given by (1), $\mathcal{U}(v, \theta, p) = v\theta - p$. However, by obtaining pirated content at zero price, the consumer will receive the utility given by (2), $\mathcal{U}_P(v, \theta, r) = (1-r)v\beta\theta$.

ASSUMPTION EC.1. *Tie-breaking rules: A consumer is indifferent between: (a) purchasing the genuine product and consuming the pirated product (that is, $\mathcal{U} = \mathcal{U}_P$), purchases the genuine product; (b) consuming the pirated product and doing nothing (that is, $\mathcal{U}_P = 0$), does nothing.*

Thus, the demand for the genuine product, given by (3a), becomes $\mathcal{D}(\theta, r, p) = 1 - \frac{p}{\theta} \cdot \frac{1}{1-\beta(1-r)}$, and following (3b), the demand for the pirated product is $\mathcal{D}_P(\theta, r, p) = \frac{p}{\theta} \cdot \frac{1}{1-\beta(1-r)}$ as long as $r < 1$, and zero otherwise.

Table EC.4 Thresholds on the Commission Rate ϕ and the Relative Quality of the Pirated Content β

Threshold	Definition
$\underline{\phi}$	$\frac{(2+\beta)\beta-2-\sqrt{4-\beta(8-\beta(8-32c-(4-\beta)\beta))}}{2\beta^2}$
$\overline{\phi}$	$\frac{(2+\beta)\beta-2+\sqrt{4-\beta(8-\beta(8-32c-(4-\beta)\beta))}}{2\beta^2}$
$\widehat{\phi}_\theta$	$1-\frac{2\sqrt{2c}}{\beta}$
$\widehat{\phi}_{M_1}$	$\frac{1}{2}-\frac{2(1-\beta)}{\beta^2}$
$\widehat{\phi}_{M_2}$	$1-\frac{16c}{\beta^2}$
$\widehat{\phi}_{CS}$	is the (real) solution to $\frac{\partial CS_A^*(c>c_A)}{\partial\phi}=0$
$\widehat{\phi}_\theta^{AW}$	$\frac{\left(5\beta^2-64c\right)-\sqrt{9\beta^4+4096c^2-128\beta^2c}}{8\beta^2}$
$\widehat{\phi}_r^{AW}$	$\frac{2(1-\beta)}{64c-\beta^2}$
$\widehat{\phi}_{M_1}^{AW}$	$\frac{2\beta(4+\beta)-8-\sqrt{2\beta(\beta^3+32\beta-64)+64}}{4\beta^2}$
$\widehat{\phi}_{M_2}^{AW}$	$\frac{A-\sqrt{B}}{2\beta(\beta((\beta-16)\beta-64c-16)+64)-64}$, where $A=(128-(48+(16-\beta)\beta+64c)\beta)\beta-64$, and $B=\left(16-64c-16\beta+\beta^2\right)\left(1024c-2048c\beta+512c\beta^2+2048c^2\beta^2+512c\beta^3+16\beta^4-96c\beta^4-16\beta^5+\beta^6\right)$
$\widehat{\phi}_{M_3}^{AW}$	$\frac{2\left(32c-\beta^2\right)+\sqrt{1024c^2-48c\beta^2+\beta^4}}{64c-3\beta^2}$
β_c	$\frac{4(1-2\phi)+2\sqrt{2(1-2\phi)(3-4\phi(3-2\phi))}}{8\phi(1-\phi)-1}$
β_M	$4\left(\frac{2-4\phi+\sqrt{5-2\phi(13-4\phi(5-2\phi))}}{8(1-\phi)\phi-1}\right)$
β_π	$\frac{4(3-8\phi+4\phi^2)+2\sqrt{2(3-2\phi)(1-2\phi)(7-8\phi(3-2\phi))}}{8\phi(1-\phi)-1}$

EC.2.1.1. Wholesale Contract With $\theta \le 1$ and $r \le 1$ (W_{LL}) Given $\theta \le 1$ and $r \le 1$, while $c > 0$ and $k > 0$, the profit functions for M and P are:

$$\pi_{MW}(\theta,r,w,p) = w\left(1-\frac{p}{\theta}\cdot\frac{1}{1-\beta(1-r)}\right)-\frac{1}{2}c\theta^2, \quad \text{and} \tag{EC.1}$$

$$\pi_{PW}(\theta,r,w,p) = (p-w)\left(1-\frac{p}{\theta}\cdot\frac{1}{1-\beta(1-r)}\right)-\frac{1}{2}r^2. \tag{EC.2}$$

Recall the sequence of events discussed in Figure 1(a). In stage 3(b), in response to M's wholesale price w and content quality θ, and its own decision on the protection level r, P chooses $p(w,\theta,r) \ge 0$ to maximize $\pi_{PW}(\theta,r,w,p)$, given by (EC.2), which is concave in p as $\frac{\partial^2\pi_{PW}(\theta,r,w,p)}{\partial p^2} = -\frac{2}{\theta(1-\beta(1-r))} < 0$ for any $0<\beta<1$, $0\le r\le 1$, and $0\le\theta\le 1$.

Applying first-order conditions (FOC) to (EC.2) with respect to p yields P's selling price, given by $p_W(w,\theta,r) = \frac{\theta+w-\beta\theta(1-r)}{2}$. Since $\theta, w \ge 0$, and $0 \le \beta, r \le 1$, note that $p_W(w,\theta,r) \ge 0$. We obtain P's demand and profit functions at this selling price, given by, $\mathcal{D}_W(w,\theta,r) = \frac{\theta-\beta\theta(1-r)-w}{2\theta(1-\beta(1-r))}$, and $\pi_{PW}(w,\theta,r) = \frac{(\theta-w-\beta\theta(1-r))^2}{4\theta(1-\beta(1-r))} - \frac{1}{2}r^2$, respectively.

After substituting $p_W(w,\theta,r)$ in (EC.1), M's profit-maximization problem in stage 3(a) becomes:

$$\underset{w\ge 0}{\text{Maximize}}\ \pi_{MW}(\theta,r,w,p_W(\theta,r,w)) = w\mathcal{D}_W(\theta,r,w)-\frac{1}{2}c\theta^2 = w\left(\frac{\theta-\beta\theta(1-r)-w}{2\theta(1-\beta(1-r))}\right)-\frac{1}{2}c\theta^2, \tag{EC.3}$$

which is concave in w, as $\frac{\partial^2\pi_{MW}(\theta,r,w,p_W(\theta,r,w))}{\partial w^2} = -\frac{1}{\theta(1-\beta(1-r))} < 0$ for any $0<\beta<1$, $0\le r\le 1$, and $0\le\theta\le 1$.

Applying FOC to EC.3 yields M's wholesale price, and subsequently, its profit function, given by $w_W(\theta,r) = \frac{\theta-\beta\theta(1-r)}{2}$ and $\pi^*_{MW}(\theta,r) = \frac{\theta-\beta\theta(1-r)}{8} - \frac{1}{2}c\theta^2$, respectively, in response to P's protection level decision r and its own content quality decision θ. Since $\theta \ge 0$, and $0 \le \beta, r \le 1$, note that $w_W(\theta,r) \ge 0$. Substituting $w_W(\theta,r)$ in P's response functions, we obtain P's selling price and demand functions, given by $p_W(\theta,r) = \frac{3\theta(1-\beta(1-r))}{4}$ and $\mathcal{D}_W(\theta,r) = \frac{1}{4}$, respectively.

Substituting $w_W(\theta,r)$ and $p_W(\theta,r)$ in (EC.2), P's profit-maximization problem in stage 2 becomes:

$$\underset{0\le r\le 1}{\text{Maximize}}\ \pi_{PW}(\theta,r,w_W(\theta,r),p_W(\theta,r)) = \frac{\theta-\beta\theta(1-r)}{16}-\frac{r^2}{2}, \tag{EC.4}$$

which is concave in r, as $\frac{\partial^2\pi_{PW}(\theta,r,w_W(\theta,r),p_W(\theta,r))}{\partial r^2} = -1 < 0$.

Applying FOC on EC.4 yields P's protection level, which in turn leads us to the selling price and profit in response to M's choice of θ, given by $r_W(\theta) = \frac{\beta\theta}{16}$, $p_W(\theta) = \frac{48\theta(1-\beta)+3\beta^2\theta^2}{64}$, and $\pi_{PW}(\theta) = \frac{32\theta(1-\beta)+\beta^2\theta^2}{512}$. Since $\theta \ge 0$, and $0 \le \beta \le 1$, note that all of these values are non-negative.

Anticipating P's reaction, and substituting $r_W(\theta)$, $w_W(\theta)$, and $p_W(\theta)$ in (EC.1), M's profit-maximization problem in stage 1 becomes:

$$\underset{0\le\theta\le 1}{\text{Maximize}}\ \pi_{MW}(\theta,r_W(\theta),w_W(\theta),p_W(\theta)) = \frac{16\theta(1-\beta)+\beta^2\theta^2}{128}-\frac{1}{2}c\theta^2. \tag{EC.5}$$

Applying FOC to (EC.5) with respect to θ yields M's content quality decision in stage 1, given by $\theta_W^* = \frac{8(1-\beta)}{64c-\beta^2}$, which in turn leads us to M's wholesale price and profit in equilibrium, given by $w_W^* = \frac{2(1-\beta)^2(128c-\beta^2)}{(64c-\beta^2)^2}$, and $\pi_{MW}^* = \frac{(1-\beta)^2}{128c-2\beta^2}$, respectively.

Note that for this solution to be valid in equilibrium, we need $c > c_W = \frac{8(1-\beta)+\beta^2}{64}$. If this condition holds, we note that $\pi_{MW}(\theta, r_W(\theta), w_W(\theta), p_W(\theta))$ is concave in θ as $\frac{\partial^2 \pi_{MW}(\theta, r_W(\theta), w_W(\theta), p_W(\theta))}{\partial\theta^2} = -c + \frac{\beta^2}{64} < 0$ for any $c > \frac{8(1-\beta)+\beta^2}{64}$ and $0 < \beta < 1$.

Substituting w_W^* and θ_W^* in P's response functions, we obtain P's selling price, protection level, and profit in equilibrium, given by $p_W^* = \frac{3(1-\beta)^2(128c-\beta^2)}{(64c-\beta^2)^2}$, $r_W^* = \frac{\beta(1-\beta)}{2(64c-\beta^2)}$, and $\pi_{PW}^* = \frac{(1-\beta)^2(256c-3\beta^2)}{8(64c-\beta^2)^2}$, respectively. The channel profit is given by $\pi_{TW}^* = \pi_{MW}^* + \pi_{PW}^* = \frac{(1-\beta)^2(512c-7\beta^2)}{8(64c-\beta^2)^2}$. The consumer surplus is given by

$$CS_W^* = \int_0^{\tilde{v}_W^*} \mathcal{U}_P(v,\theta,p)dv + \int_{\tilde{v}_W^*}^1 \mathcal{U}(v,\theta,p_V^*)dv$$

$$= \int_0^{\frac{3}{4}} \left(1 - \frac{\beta(1-\beta)}{2(64c-\beta^2)}\right) v\beta\theta dv + \int_{\frac{3}{4}}^1 \left(v\frac{8(1-\beta)}{64c-\beta^2} - \frac{3(1-\beta)^2(128c-\beta^2)}{(64c-\beta^2)^2}\right) dv = \frac{(1-\beta)(128(1+15\beta)c-(17+15\beta)\beta^2}{8(64c-\beta^2)^2}.$$

When $c \le c_W = \frac{8(1-\beta)+\beta^2}{64}$, M's content quality is at the boundary, given by $\theta_W^* = 1$. Substituting $\theta_W^* = 1$, we obtain M's wholesale price and profit in equilibrium if , given by $w_W^* = \frac{16(1-\beta)+\beta^2}{32}$, and $\pi_{MW}^* = \frac{16(1-\beta)+\beta^2-64c}{128}$, respectively. Similarly, P's selling price, protection level, and firm profit in equilibrium are given by $p_W^* = \frac{48(1-\beta)+3\beta^2}{64}$, $r_W^* = \frac{\beta}{16}$, and $\pi_{PW}^* = \frac{32(1-\beta)+\beta^2}{512}$, respectively. The channel profit and the consumer surplus are given by $\pi_{TW}^* = \frac{96(1-\beta)+5\beta^2-256c}{512}$ and $CS_W^* = \frac{16(1+15\beta)-15\beta^2}{512}$, respectively. We summarize the results in Table 4 in column "W (Wholesale)" with the subscript W omitted.

EC.2.1.2. Agency Contract With $\theta \le 1$ and $r \le 1$ (A_{LL})

Given $\theta \le 1$ and $r \le 1$, while $c > 0$ and $k > 0$, the profit functions for M and P are:

$$\pi_{MA}(\theta, r, \phi, p) = (1-\phi)\, p\left(1 - \frac{p}{\theta}\cdot\frac{1}{1-\beta\,(1-r)}\right) - \frac{1}{2}c\theta^2, \quad \text{and} \tag{EC.6}$$

$$\pi_{PA}(\theta, r, \phi, p) = \phi p\left(1 - \frac{p}{\theta}\cdot\frac{1}{1-\beta\,(1-r)}\right) - \frac{1}{2}r^2, \tag{EC.7}$$

Recall the sequence of events discussed in Figure 1(b). In stage 3, in response to its content quality θ and P's protection level r, M chooses $p_A(\theta, r)$ that maximizes $\pi_{MA}(\theta, r, \phi, p)$ given by (EC.6), which is concave in p, as $\frac{\partial^2 \pi_{PA}(\theta,r,\phi,p)}{\partial p^2} = -\frac{2(1-\phi)}{\theta(1-\beta(1-r))} < 0$ for any $0 < \beta < 1$, $0 < \theta < 1$, $0 \le r \le 1$, and $0 < \phi \le \frac{1}{2}$.

Applying FOC to (EC.6) with respect to p yields M's selling price, and subsequently the market demand and profit function, given by $p_A(\theta, r) = \frac{\theta-\beta\theta(1-r)}{2}$, $\mathcal{D}_A(\theta, r) = \frac{1}{2}$, and $\pi_{MA}(\theta, r, \phi) = \frac{\theta(1-\phi)(1-\beta(1-r))}{4} - \frac{1}{2}c\theta^2$, respectively. Since $\theta \ge 0$, $0 \le \beta, r \le 1$, and $0 < \phi \le \frac{1}{2}$, note that $p_A(\theta, r) \ge 0$. Substituting $p_A(\theta, r)$ in (EC.7), P's profit-maximization problem in stage 2 becomes:

$$\underset{0\le r\le 1}{\text{Maximize}}\ \pi_{PA}(\theta, r, \phi, p_A(\theta, r)) = \frac{\phi\theta(1-\beta(1-r))}{4} - \frac{r^2}{2}, \tag{EC.8}$$

which is concave in r as $\frac{\partial^2 \pi_{PA}(\theta,r,\phi,p_A(\theta,r))}{\partial r^2} = -1 < 0$.

Applying FOC to EC.8 yields P's protection level, and subsequently P's profit, as functions of θ decided by M, given by $r_A(\theta) = \frac{\beta\theta\phi}{4}$ and $\pi_{PA}(\theta) = \frac{\phi\theta(8(1-\beta)+\phi\theta\beta^2)}{32}$, respectively. Substituting $r_A(\theta)$ in $p_A(\theta, r)$, we obtain M's retail price as a function of θ, given by $p_A(\theta) = \frac{4\theta(1-\beta)+\phi\theta^2\beta^2}{8}$, $q_A(\theta) = \frac{1}{2}$.

Anticipating P's reaction, and substituting $r_A(\theta)$ and $p_A(\theta)$ in (EC.6), M's profit-maximization problem in stage 1 becomes:

$$\underset{0\le\theta\le 1}{\text{Maximize}}\ \pi_{MA}(\theta, r_A(\theta), \phi, p_A(\theta)) = \frac{\theta(1-\phi)(4(1-\beta)+\phi\theta\beta^2)}{16} - \frac{1}{2}c\theta^2. \tag{EC.9}$$

Applying FOC to (EC.9) with respect to θ yields M's content quality, and subsequently, its selling price and profit in equilibrium, given by $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-(1-\phi)\phi\beta^2}$, $p_A^* = \frac{(1-\beta)^2(1-\phi)(16c-(1-\phi)\phi\beta^2)}{2(8c-(1-\phi)\phi\beta^2)^2}$, and $\pi_{MA}^* = \frac{(1-\beta)^2(1-\phi)^2}{4(8c-(1-\phi)\phi\beta^2)}$, respectively.

Note that for this solution to be valid in equilibrium, we need $c > c_A = \frac{(1-\phi)(2-(2-\beta\phi)\beta)}{8}$. Then, we can show that this solution is optimal as $\pi_{MA}(\theta, r_A(\theta), \phi, p_A(\theta))$ is concave in θ, as $\frac{\partial^2 \pi_{MA}(\theta, r_A(\theta), \phi, p_A(\theta))}{\partial\theta^2} = -c + \frac{\beta^2\phi(1-\phi)}{8} < 0$ for any $c > \frac{(1-\phi)(2-(2-\beta\phi)\beta)}{8}$, $0 < \beta < 1$, and $0 < \phi \le \frac{1}{2}$.

Substituting θ_A^* in P's response functions, we obtain P's protection level and profit in equilibrium, given by $r_A^* = \frac{\beta\phi(1-\beta)(1-\phi)}{2(8c-(1-\phi)\phi\beta^2)}$, and $\pi_{PA}^* = \frac{(1-\beta)^2(1-\phi)\phi(32c-3(1-\phi)\phi\beta^2)}{8(8c-(1-\phi)\phi\beta^2)^2}$, respectively. The channel profit is given by $\pi_{TA}^* = \pi_{MA}^* + \pi_{PA}^* = \frac{(1-\beta)^2(1-\phi)(16c(1+\phi)-(2-\phi-\phi^2)\phi\beta^2)}{8(8c-(1-\phi)\phi\beta^2)^2}$. The consumer surplus is given by

$$CS_A^* = \int_0^{\tilde{v}_A^*} \mathcal{U}_P(v,\theta,p)dv + \int_{\tilde{v}_A^*}^1 \mathcal{U}(v,\theta,p_V^*)dv$$

$$= \int_0^{\frac{1}{2}} \left(1 - \frac{\beta\phi(1-\beta)(1-\phi)}{2(8c-(1-\phi)\phi\beta^2)}\right) v\beta\theta dv + \int_{\frac{1}{2}}^1 \left(v\frac{2(1-\beta)(1-\phi)}{8c-(1-\phi)\phi\beta^2} - \frac{(1-\beta)^2(1-\phi)(16c-(1-\phi)\phi\beta^2)}{2(8c-(1-\phi)\phi\beta^2)^2}\right) dv$$
$$= \frac{(1-\beta)(1-\phi)(16c(1+3\beta)-(5+3\beta)(1-\phi)\phi\beta^2)}{8(8c-(1-\phi)\phi\beta^2)^2}.$$

When $c \le c_A = \frac{(1-\phi)(2-(2-\beta\phi)\beta)}{8}$, M's content quality is at the boundary, given by $\theta_A^* = 1$. Substituting $\theta_A^* = 1$, we obtain M's selling price, selling quantity, and profit in equilibrium, given by $p_A^* = \frac{4(1-\beta)+\phi\beta^2}{8}$, $q_A^* = \frac{1}{2}$, and $\pi_{MA}^* = \frac{(1-\phi)(4(1-\beta)+\phi\beta^2)-8c}{16}$, respectively. Similarly, P's protection level and firm profit in equilibrium are given by $r_A^* = \frac{\beta\phi}{4}$, $\pi_{PA}^* = \frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$, respectively. The channel profit and the consumer surplus are given by $\pi_{TA}^* = \frac{8(1-\beta)+\phi(2-\phi)\beta^2-16c}{32}$ and $CS_A^* = \frac{4(1+3\beta)-3\beta^2\phi}{32}$, respectively. The results are summarized in Table 4 in column "A (Agency)" with the subscript A omitted.

EC.2.1.3. Vertically Integrated Supply Chain With $\theta \le 1$ and $r \le 1$ (V_{LL})

In this scenario, the demand for genuine content is as given by (3a). The profit-maximization problem for firm V then becomes:

$$\underset{p\ge0, r\ge0, \theta\ge0}{\text{Maximize}}\ \pi_V(\theta,r,p) = p\mathcal{D}(\theta,r,p) - \frac{1}{2}c\theta^2 - \frac{1}{2}r^2 = p\left(1 - \frac{p}{\theta}\cdot\frac{1}{1-\beta(1-r)}\right) - \frac{1}{2}c\theta^2 - \frac{1}{2}r^2, \qquad \text{(EC.10)}$$

Applying first-order conditions (FOC) on (EC.10), we obtain firm V's content quality, protection level, selling price, selling quantity, and profit in equilibrium, given by $\theta_V^* = \frac{4(1-\beta)}{16c-\beta^2}$, $r_V^* = \frac{\beta(1-\beta)}{16c-\beta^2}$, $p_V^* = \frac{32c(1-\beta)^2}{(16c-\beta^2)^2}$, and $\pi_V^* = \frac{(1-\beta)^2}{32c-2\beta^2}$, respectively. This equilibrium solution exists when $c > c_V = \frac{(2-\beta)^2}{16}$. Then, we can show that this solution is optimal as the Hessian $\begin{pmatrix} -\frac{2}{\theta(1-\beta(1-r))} & \frac{2p}{\theta^2(1-\beta(1-r))} & \frac{2\beta\theta p}{\theta^2(1-\beta(1-r))^2} \\ \frac{2p}{\theta^2(1-\beta(1-r))} & -c-\frac{2p^2}{\theta^3(1-\beta(1-r))} & -\frac{\beta p^2}{\theta^2(1-\beta(1-r))^2} \\ \frac{2\beta\theta p}{\theta^2(1-\beta(1-r))^2} & -\frac{\beta p^2}{\theta^2(1-\beta(1-r))^2} & -\frac{2\beta^2\theta^2p^2}{(\theta+\beta\theta(r-1))^3}-1 \end{pmatrix}$ is negative definite at $\theta_V^* = \frac{4(1-\beta)}{16c-\beta^2}$, $r_V^* = \frac{\beta(1-\beta)}{16c-\beta^2}$, $p_V^* = \frac{32c(1-\beta)^2}{(16c-\beta^2)^2}$. The value of determinant of the Hessian (or the third-order principal minor) is $-\frac{(16c-\beta^2)^3}{512c(1-\beta)^2} < 0$ for any $0<\beta<1$ and $c > \frac{(2-\beta)^2}{16}$. Similarly, we can easily show that the determinant of all second-order principal minors is positive and that the first-order principal minors are negative. The channel profit is the same as firm V's profit, given by $\pi_{TV}^* = \frac{(1-\beta)^2}{32c-2\beta^2}$, while the consumer surplus is given by

$$CS_V^* = \int_0^{\tilde{v}_V^*} \mathcal{U}_P(v,\theta,p)dv + \int_{\tilde{v}_V^*}^1 \mathcal{U}(v,\theta,p_V^*)dv$$
$$= \int_0^{\frac{1}{2}} \left(1 - \frac{\beta(1-\beta)}{16c-\beta^2}\right) v\beta\theta dv + \frac{4(1-\beta)}{16c-\beta^2}\int_{\frac{1}{2}}^1 \left(v - \frac{8c(1-\beta)}{16c-\beta^2}\right) dv = \frac{2(1-\beta)(4c(1+3\beta)-\beta^2)}{(16c-\beta^2)^2}.$$

When $c \le c_V = \frac{(2-\beta)^2}{16}$, firm V's content quality is set at the boundary, i.e., $\theta_V^{LH*} = 1$. Substituting $\theta_V^* = 1$, we obtain firm V's protection level, selling price, selling quantity, firm profit, channel profit, and consumer surplus in equilibrium, given by $r_V^* = \frac{\beta}{4}$, $p_V^* = \frac{(2-\beta)^2}{8}$, $\pi_V^{LH*} = \frac{8(1-\beta)+\beta^2}{32}$, $\pi_{TV}^* = \frac{8(1-\beta)+\beta^2}{32}$, and $CS_V^* = \frac{4(1+3\beta)-3\beta^2}{32}$, respectively. We summarize the results in Table 5, in column "V (VI Firm)," with the subscript V omitted.

EC.2.2. Analysis of $\theta \le 1$ and $r = 1$ (LH)

The decisions in stages 1 and 2 are given as $r=1$ and $\theta \le 1$, while $k=0$ and $c>0$. In stage 4, the consumer with valuation v faces enforcement level $r=1$ and quality $\theta \le 1$. By buying the genuine content at price p, the consumer will receive the utility given by $\mathcal{U}(v,\theta,p) = v\theta - p$ (see (1)). However, by obtaining the pirated content at zero price, the consumer will receive the utility given by substituting $r=1$ and $\theta \le 1$ in (2), which yields $\mathcal{U}_P(v,\theta,1) = 0$. Thus, the demand for the genuine product, given by (3a), becomes $\mathcal{D}(\theta,1,p) = 1 - \frac{p}{\theta}$. Since $\mathcal{U}_P(v,\theta,1) = 0$ for all v, following (3b), the demand for the pirated product is $\mathcal{D}_P(\theta,1,p) = 0$. The analysis is quite similar to that in Section EC.2.1, so we provide the key steps.

EC.2.2.1. Vertically Integrated Supply Chain With $\theta \le 1$ and $r = 1$ (V_{LH})

The profit-maximization problem in stage 3 for firm V is to maximize its profit given by (EC.11):

$$\underset{p\ge0, 0\le\theta\le1}{\text{Maximize}}\ \pi_V(\theta,1,p) = p\mathcal{D}(\theta,1,p) - \frac{1}{2}c\theta^2 = p\left(1-\frac{p}{\theta}\right) - \frac{1}{2}c\theta^2. \qquad \text{(EC.11)}$$

Note that $\pi_V(\theta,1,p)$ is jointly concave in p and θ as the Hessian $\begin{pmatrix} -\frac{2}{\theta} & \frac{2p}{\theta^2} \\ \frac{2p}{\theta^2} & -c-\frac{2p^2}{\theta^3} \end{pmatrix}$ is negative definite as the value of determinant is $\frac{2c}{\theta} > 0$ for any $c>0$ and $\theta>0$ (the first-order principal minors are negative too). Applying FOC on (EC.11) yields firm V's content quality and selling price, and hence the profit in equilibrium, given by $\theta_V^{LH*} = \frac{1}{4c}$, $p_V^{LH*} = \frac{1}{8c}$, and $\pi_V^{LH*} = \frac{1}{32c}$, respectively. This equilibrium solution is valid when $c > \frac{1}{4}$. The channel profit is the same as firm V's profit, given by $\pi_{CV}^{LH*} = \frac{1}{32c}$, while the consumer surplus is given by

$$CS_V^{LH*} = \int_0^{\frac{p_V^{LH*}}{\theta_V^{LH*}}} \mathcal{U}_P(v,\theta,1)dv + \int_{\frac{p_V^{LH*}}{\theta_V^{LH*}}}^1 \mathcal{U}(v,\theta,p_V^{LH*})dv = \int_0^{\frac{1}{2}} 0dv + \frac{1}{4c}\int_{\frac{1}{2}}^1 \left(v-\frac{1}{2}\right)dv = \frac{v(v-1)}{8c}\bigg|_{v=\frac{1}{2}}^{v=1} = \frac{1}{32c}.$$

When $c \le \frac{1}{4}$, firm V's content quality is set at the boundary, given by $\theta_V^{LH*} = 1$. Substituting $\theta_V^{LH*} = 1$, we obtain the firm V's selling price, firm profit, channel profit, and consumer surplus in equilibrium, given by $p_V^{LH*} = \frac{1}{2}$, $\pi_V^{LH*} = \frac{1-2c}{4}$, $\pi_T^{LH*} = \frac{1-2c}{4}$, and $CS_{CV}^{LH*} = \frac{1}{8}$, respectively. The results are summarized in Table 6, in column "V (VI Firm)," with the subscript V omitted.

EC.2.2.2. Wholesale Contract With $\theta \le 1$ and $r = 1$ (W_{LH}) Given $\theta \le 1$ and $r = 1$, while $c > 0$ and $k = 0$, the profit functions for M and P are:

$$\pi_{MW}(\theta, 1, w, p) = w\left(1 - \frac{p}{\theta}\right) - \frac{1}{2}c\theta^2, \quad \text{and} \tag{EC.12}$$

$$\pi_{PW}(\theta, 1, w, p) = (p - w)\left(1 - \frac{p}{\theta}\right). \tag{EC.13}$$

In response to M's wholesale price w and content quality θ, firm P chooses $p(w,\theta) \ge 0$ in stage 3(b) to maximize $\pi_{PW}(\theta,1,w,p)$, given by (EC.13), which is concave in p, as $\frac{\partial^2 \pi_{PW}(\theta,1,w,p)}{\partial p^2} = -\frac{2}{\theta} < 0$ for any $\theta > 0$. Applying FOC to (EC.13) with respect to p yields P's selling price, and hence demand and profit as functions of w and θ, given by $p_W^{LH}(w,\theta) = \frac{\theta+w}{2}$, $\mathcal{D}_W^{LH}(w,\theta) = \frac{\theta-w}{2\theta}$, and $\pi_{PW}^{LH}(w,\theta) = \frac{(\theta-w)^2}{4\theta}$, respectively. Because $r = 1$, there is no stage 2 decision. Therefore, anticipating P's reaction, M's stage 1 profit maximization problem becomes:

$$\underset{w \ge 0, 0 \le \theta \le 1}{\text{Maximize}}\ \pi_{MW}(\theta, 1, w, p_W^{LH}(w,\theta)) = w\mathcal{D}_W^{HH}(w,\theta) - \frac{1}{2}c\theta^2 = w\left(\frac{\theta - w}{2\theta}\right) - \frac{1}{2}c\theta^2, \tag{EC.14}$$

which is obtained by substituting $p_W^{LH}(w,\theta)$ in (EC.12). Now, $\pi_{MW}(\theta,1,w,p_W^{LH}(w,\theta)$ is jointly concave in w and θ as the Hessian $\begin{pmatrix} -\frac{1}{\theta} & \frac{w}{\theta^2} \\ \frac{w}{\theta^2} & -c - \frac{w^2}{\theta^3} \end{pmatrix}$ is negative definite as the value of determinant is $\frac{c}{\theta} > 0$ for any $c > 0$ and $\theta > 0$ (the first-order principal minors are negative too). Applying FOC to (EC.12) with respect to w and θ yields M's wholesale price and content quality, and thus profit, given by $w_W^{LH*} = \frac{1}{16c}$, $\theta_W^{LH*} = \frac{1}{8c}$, and $\pi_{MW}^{LH*} = \frac{1}{128c}$, respectively. This equilibrium solution exists when $c > \frac{1}{8}$. Substituting w_W^{LH*} and θ_W^{LH*} in P's response functions, we obtain P's selling price and profit in equilibrium, given by $p_W^{LH*} = \frac{3}{32c}$ and $\pi_{PW}^{LH*} = \frac{1}{128c}$, respectively. The channel profit is $\pi_{TW}^{LH*} = \pi_{MW}^{LH*} + \pi_{PW}^{LH*} = \frac{1}{64c}$, while the consumer surplus is:

$$CS_W^{LH*} = \int_0^{\frac{p_W^{LH*}}{\theta_W^{LH*}}} \mathcal{U}_P(v,\theta,1)dv + \int_{\frac{p_W^{LH*}}{\theta_W^{LH*}}}^1 \mathcal{U}(v,\theta,p_W^{LH*})dv = \int_0^{\frac{3}{4}} 0dv + \frac{1}{8c}\int_{\frac{3}{4}}^1 \left(v - \frac{3}{4}\right)dv = \frac{v(2v-3)}{32c}\bigg|_{v=\frac{3}{4}}^{v=1} = \frac{1}{256c}.$$

When $c \le \frac{1}{8}$, M's content quality $\theta_W^{LH*} = 1$. Substituting $\theta_W^{LH*} = 1$, we obtain M's wholesale price and profit in equilibrium, given by $w_W^{LH*} = \frac{1}{2}$ and $\pi_{MW}^{LH*} = \frac{1-4c}{8}$, respectively. Similarly, P's selling price and profit, and consequently the channel profit and consumer surplus in equilibrium are given by $p_W^{LH*} = \frac{3}{4}$, $\pi_{PW}^{LH*} = \frac{1}{16}$, $\pi_{TW}^{LH*} = \frac{3-8c}{16}$, and $CS_W^{LH*} = \frac{1}{32}$, respectively. The results are summarized in Table 6, in column "W (Wholesale)," with the subscript W omitted.

EC.2.2.3. Agency Contract With $\theta \le 1$ and $r = 1$ (A_{LH}) Given $\theta \le 1$ and $r = 1$, while $c > 0$ and $k = 0$, the profit functions for M and P are:

$$\pi_{MA}(\theta, 1, \phi, p) = (1 - \phi)\,p\left(1 - \frac{p}{\theta}\right) - \frac{1}{2}c\theta^2, \quad \text{and} \tag{EC.15}$$

$$\pi_{PA}(\theta, 1, \phi, p) = \phi p\left(1 - \frac{p}{\theta}\right), \tag{EC.16}$$

Given the value of ϕ and knowing that $r = 1$, M chooses $p \ge 0$ and $0 < \theta \le 1$ to maximize $\pi_{MA}(\theta,1,\phi,p)$ given by (EC.15). Now, $\pi_{MA}(\theta,1,\phi,p)$ is jointly concave in p and θ as the Hessian $\begin{pmatrix} -\frac{2(1-\phi)}{\theta} & \frac{2(1-\phi)p}{\theta^2} \\ \frac{2(1-\phi)p}{\theta^2} & -\frac{2(1-\phi)p^2}{\theta^3} - c \end{pmatrix}$ is negative definite as the value of determinant is $\frac{2c(1-\phi)}{\theta} > 0$ for any $c, \theta > 0$, and $0 < \phi \le \frac{1}{2}$ (the first-order principal minors are negative too).

Applying FOC to (EC.15) with respect to p and θ yields M's selling price, content quality, and profit in equilibrium, given by $p_A^{LH*} = \frac{1-\phi}{8c}$, $\theta_A^{LH*} = \frac{1-\phi}{4c}$, and $\pi_{MA}^{LH*} = \frac{(1-\phi)^2}{32c}$, respectively. This equilibrium solution is valid when $c > \frac{1-\phi}{4}$. Substituting p_A^{LH*} and θ_A^{LH*} in (EC.16), we obtain P's profit in equilibrium, given by $\pi_{PA}^{LH*} = \frac{(1-\phi)\phi}{16c}$. The channel profit is $\pi_{TA}^{LH*} = \pi_{MA}^{LH*} + \pi_{PA}^{LH*} = \frac{1-\phi^2}{32c}$, while the consumer surplus is given by

$$CS_A^{LH*} = \int_0^{\frac{p_A^{LH*}}{\theta_A^{LH*}}} \mathcal{U}_P(v,\theta,1)dv + \int_{\frac{p_A^{LH*}}{\theta_A^{LH*}}}^1 \mathcal{U}(v,\theta,p_A^{LH*})dv = \int_0^{\frac{1}{2}} 0dv + \frac{1-\phi}{4c}\int_{\frac{1}{2}}^1 \left(v - \frac{1}{2}\right)dv = \frac{(1-\phi)v(v-1)}{8c}\bigg|_{v=\frac{1}{2}}^{v=1} = \frac{1-\phi}{32c}.$$

When $c \le \frac{1-\phi}{4}$, M's content quality $\theta_A^{LH*} = 1$. Substituting $\theta_A^{LH*} = 1$, we obtain M's selling price and profit in equilibrium, given by $p_A^{LH*} = \frac{1}{2}$ and $\pi_{MA}^{LH*} = \frac{1-\phi-2c}{4}$, respectively. Consequently, P's profit, channel profit, and consumer surplus in equilibrium are given by $\pi_{PA}^{LH*} = \frac{\phi}{4}$, $\pi_{TA}^{LH*} = \frac{1-2c}{4}$, and $CS_A^{LH*} = \frac{1}{8}$, respectively. The results are summarized in Table 6, under column "A (Agency)," with the subscript A omitted.

EC.2.3. Analysis of $\theta = 1$ and $r = 1$ (HH)

The decisions in stages 1 and 2 are given as $\theta = 1$ and $r = 1$, while $c = 0$ and $k = 0$. In stage 4, the consumer with valuation v faces quality $\theta = 1$ and enforcement level $r = 1$. By buying the genuine content at price p, the consumer will receive the utility given by substituting $\theta = 1$ in (1), which yields $\mathcal{U}(v,1,p) = v - p$. However, by obtaining the pirated content at zero price, the consumer will receive the utility given by substituting $\theta = 1$ and $r = 1$ in (2), which yields $\mathcal{U}_P(v,1,1) = 0$. Thus, the demand for the genuine product, given by (3a), becomes $\mathcal{D}(1,1,p) = 1 - p$. Since $\mathcal{U}_P(v,1,1) = 0$ for all v, following (3b), the demand for the pirated product is $\mathcal{D}_P(1,1,p) = 0$. The analysis is very similar to, and is a further special case of, Scenario LH with $\theta = 1$ (see Section EC.2.2). For the sake of brevity, we omit repeating the detailed steps. The results are summarized in Table EC.1.

EC.2.4. Analysis of $\theta = 1$ and $r \leq 1$ (HL)

The decisions in stages 1 and 2 are given as $\theta = 1$ and $r \leq 1$, while $c = 0$ and $k > 0$. In stage 4, the consumer with valuation v faces quality $\theta = 1$ and enforcement level $r \leq 1$. By buying the genuine content at price p, the consumer will receive the utility given by substituting $\theta = 1$ in (1), which yields $\mathcal{U}(v,1,p) = v - p$. However, by obtaining the pirated content at zero price, the consumer will receive the utility given by substituting $\theta = 1$ and $r \leq 1$ in (2), which yields $\mathcal{U}_P(v,1,r) = (1-r)v\beta$. Thus, the demand for the genuine product, given by (3a), becomes $\mathcal{D}(1,r,p) = 1 - \frac{p}{1-\beta(1-r)}$, and following (3b), the demand for the pirated product is $\mathcal{D}_P(1,r,p) = \frac{p}{1-\beta(1-r)}$ as long as $r < 1$, and zero otherwise. The analysis is very similar to, and is special case of, Scenario LL with $\theta = 1$ (see Section EC.2.1). For the sake of brevity, we omit repeating the detailed steps. The results are summarized in Table EC.2.

EC.3. Proofs of Key Results

We now provide the proofs of the lemmas and propositions in Section 4.

EC.3.1. Proof of Proposition 1

From Table 4, observe that for $c \leq c_A = \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8}$, the optimal quality, $\theta_A^* = 1$, and hence $\frac{\partial\theta_A^*}{\partial\phi} = 0$. However, when $c > c_A$, $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-\beta^2(1-\phi)\phi}$, and thus $\frac{\partial\theta_A^*}{\partial\phi} = -\frac{2(1-\beta)\left(8c-\beta^2(1-\phi)^2\right)}{(8c-\phi(1-\phi)\beta^2)^2}$. Since $1-\beta > 0$ and $8c - \phi(1-\phi)\beta^2 > 0$ for any $0 < \beta < 1$, $0 < \phi \leq \frac{1}{2}$, and $c > c_A$, therefore, $\frac{\partial\theta_A^*}{\partial\phi} > 0$, when $8c - \beta^2(1-\phi)^2 < 0$ and $c > c_A$, that is, when $c \in \left(c_A, c_A^\theta\right)$, where $c_A^\theta = \frac{\beta^2(1-\phi)^2}{8}$. Now, $c_A^\theta - c_A = \frac{\beta^2(1-\phi)^2}{8} - \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8} = \frac{(1-\phi)\left(\beta^2-2(1-\beta)-2\phi\beta^2\right)}{8} > 0$ when $\phi < \frac{1}{2} - \frac{1-\beta}{\beta^2}$ as $1 - \phi > 0$. Furthermore, $\frac{1}{2} - \frac{1-\beta}{\beta^2} < \frac{1}{2}$ for any $0 < \beta < 1$. However, $\frac{1}{2} - \frac{1-\beta}{\beta^2} > 0$ if $\beta > \sqrt{3} - 1$. This implies that $c_A < c_A^\theta$ only if $\beta > \sqrt{3} - 1$ and $\phi < \frac{1}{2} - \frac{1-\beta}{\beta^2}$. Hence, $\frac{\partial\theta_A^*}{\partial\phi} > 0$ if and only if $\beta > \sqrt{3} - 1$, $\phi < \frac{1}{2} - \frac{1-\beta}{\beta^2}$, and $c \in \left(c_A, c_A^\theta\right)$.

Graphical Representation in Figure 2: Analyzing c_A, we find that $c \leq c_A$ only if $\underline{\phi} \leq \phi \leq \overline{\phi}$. When $c > c_A$, then M sets $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-\beta^2(1-\phi)\phi}$, which is concave in ϕ and is maximized at $\phi = \widehat{\phi}_\theta = 1 - \frac{2\sqrt{2c}}{\beta}$. Analyzing $\widehat{\phi}_\theta$, we observe that when $c > c_A$, θ_A^* increases in ϕ only if $c < c_A^\theta$. Given that $\theta_A^* = 1$ when $c \leq c_A$ and θ_A^* is decreasing in ϕ when $c > c_A^\theta$, we deduce that θ_A^* increases in ϕ if and only if $c \in \left(c_A, c_A^\theta\right)$. For this condition to be feasible, we need $c_A < c_A^\theta$, which happens only if $\beta > \sqrt{3} - 1$ and $\phi < \frac{1}{2} - \frac{1-\beta}{\beta^2}$.

EC.3.2. Proof of Proposition 2

From Table 4, observe that for $c \leq c_A = \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8}$, the manufacturer's profit, $\pi_{MA}^* = \frac{(1-\phi)(4(1-\beta)+\beta^2\phi)-8c}{16}$. We differentiate π_{MA}^* with respect to ϕ, and note that $\frac{\partial\pi_{MA}^*}{\partial\phi} = -\frac{4-\beta(4+\beta-2\beta\phi)}{16} > 0$ when $0 < \phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$. Now, $\frac{1}{2} - \frac{2(1-\beta)}{\beta^2} < \frac{1}{2}$ for any $0 < \beta < 1$. However, $\frac{1}{2} - \frac{2(1-\beta)}{\beta^2} > 0$ if $2\left(\sqrt{2}-1\right) < \beta < 1$. This implies that $\frac{\partial\pi_{MA}^*}{\partial\phi} > 0$ only if $2\left(\sqrt{2}-1\right) < \beta < 1$, $0 < \phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$, and $0 < c < c_A$. However, for $c > c_A$, $\pi_{MA}^* = \frac{(1-\beta)^2(1-\phi)^2}{4(8c-\beta^2(1-\phi)\phi)}$. Differentiating π_{MA}^* with respect to ϕ, we obtain $\frac{\partial\pi_{MA}^*}{\partial\phi} = -\frac{(1-\beta)^2(1-\phi)\left(16c-\beta^2(1-\phi)\right)}{4(8c-\beta^2(1-\phi)\phi)^2}$. Since $1-\beta > 0$, $1-\phi > 0$, and $8c - \phi(1-\phi)\beta^2 > 0$ for any $0 < \beta < 1$, $0 < \phi \leq \frac{1}{2}$, and $c > c_A$, therefore, $\frac{\partial\pi_{MA}^*}{\partial\phi} > 0$, when $16c - \beta^2(1-\phi) < 0$, that is, $c < c_A^M = \frac{\beta^2(1-\phi)}{16}$ Now, $c_A^M - c_A = \frac{\beta^2(1-\phi)}{16} - \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8} = \frac{(1-\phi)\left(\beta^2-4(1-\beta)-2\phi\beta^2\right)}{16} > 0$ when $\phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$ as $1 - \phi > 0$. Furthermore, $\frac{1}{2} - \frac{2(1-\beta)}{\beta^2} < \frac{1}{2}$ for any $0 < \beta < 1$. However, $\frac{1}{2} - \frac{2(1-\beta)}{\beta^2} > 0$ if $\beta > 2\left(\sqrt{2}-1\right)$. This implies that $c_A < c_A^M$ when $\beta > 2\left(\sqrt{2}-1\right)$ and $\phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$. Hence, $\frac{\partial\pi_{MA}^*}{\partial\phi} > 0$ when $\beta > 2\left(\sqrt{2}-1\right)$, $\phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$, and $c \in \left(c_A, c_A^M\right)$. Taking a summary of these two scenarios, we conclude that $\frac{\partial\pi_{MA}^*}{\partial\phi} > 0$, when $2\left(\sqrt{2}-1\right) < \beta < 1$ and $0 < \phi < \frac{1}{2} - \frac{2(1-\beta)}{\beta^2}$, and $c < \max\{c_A, c_A^M\}$.

Graphical Representation in Figure 3: Given that $\phi \geq 0$, for $0 < \phi < \widehat{\phi}_{M_1}$, it is necessary that $\widehat{\phi}_{M_1} > 0$, or $\beta > 2\left(\sqrt{2}-1\right)$, which can be verified by analyzing $\widehat{\phi}_{M_1}$. Intuitively, M benefits from an increase in ϕ when the relative quality of the pirated content β is sufficiently high, which means that the pirated content is quite similar to the genuine content and, therefore, attractive to consumers. This is when the *AC effect* dominates the *PE effect*, but only as long as the commission rate is not too high. Similarly, for $\phi < \widehat{\phi}_{M_2}$ to hold, it can be easily verified that $c < c_A^M = \frac{\beta^2(1-\phi)}{16}$.

EC.3.3. Proof of Proposition 3

From Table 4, observe that for $c \leq c_A$, the consumer surplus, $CS_A^* = \frac{4(1+3\beta)-3\beta^2\phi}{32}$. Differentiating CS_A^* with respect to ϕ, we note that $\frac{\partial CS_A^*}{\partial \phi} = -\frac{3\beta^2}{32} < 0$ when $0 < \beta < 1$. However, when $c > c_A$, the consumer surplus, $CS_A^* = \frac{(1-\beta)(1-\phi)(16c(1+3\beta)-(1-\phi)\phi\beta^2(5+3\beta))}{8(8c-\beta^2(1-\phi)\phi)^2}$, therefore $\frac{\partial CS_A^*}{\partial \phi} = -\frac{(1-\beta)\left((3\beta+5)\beta^4(1-\phi)^3\phi+128(3\beta+1)c^2-8\beta^2c(\phi-1)(9\beta(\phi-1)-9\phi+1)\right)}{8(8c-\phi(1-\phi)\beta^2)^3}$. Since $1-\beta > 0$ and $8c - \phi(1-\phi)\beta^2 > 0$ for any $0 < \beta < 1$, $0 < \phi \leq \frac{1}{2}$, and $c > c_A$, therefore, $\frac{\partial CS_A^*}{\partial \phi} > 0$, when $128(3\beta+1)c^2 - 8\beta^2c(\phi-1)(9\beta(\phi-1)-9\phi+1) + (3\beta+5)\beta^4(1-\phi)^3\phi < 0$ and $c > c_A$.

Define $f_{ACS}(c) = 128(3\beta+1)c^2 - 8\beta^2c(\phi-1)(9\beta(\phi-1)-9\phi+1) + (3\beta+5)\beta^4(1-\phi)^3\phi$. Note that $\frac{\partial^2 f_{ACS}(c)}{\partial c^2} = 256(3\beta+1) > 0$ and thus $f_{ACS}(c)$ is a convex function in $c > c_A$. Setting $f_{ACS}(c) = 0$, we obtain two roots, $X - \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y}$ and $X + \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y}$, where $X = \frac{1}{72} - \frac{4+3\beta(4+3\beta(1-\phi)(1-9\phi-9\beta(1-\phi)))}{288(1+3\beta)}$ and $Y = 1 + 9\beta^2(1-\phi)(9-17\phi) - 18\beta(1-\phi)^2 - \phi(58-121\phi)$. As $f_{ACS}(c)$ is a convex function, $f_{ACS}(c) < 0, \Rightarrow \frac{\partial CS_A^*}{\partial \phi} > 0$ when $X - \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y} < c < X + \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y}$. However, the smaller root $X - \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y} < c_A$ for any $0 < \beta < 1$ and $0 < \phi < \frac{1}{2}$, but the larger root: $X + \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y} = c_A^{CS} > c_A$, when $0 < \phi < \frac{3\beta^2+21\beta-5-\sqrt{121-18\beta+147\beta^2-90\beta^3+9\beta^4}}{12\beta^2}$ (see Table EC.3 for the full expression of c_A^{CS}).

Given that we require $c > c_A$, we observe that the appropriate region for $\frac{\partial CS_A^*}{\partial \phi} > 0$ is $c_A < c < X - \frac{\beta^2(1-\phi)}{32(1+3\beta)}\sqrt{Y} = c_A^{CS}$. Hence, we conclude that $\frac{\partial CS_A^*}{\partial \phi} > 0$ for $c \in \left(c_A, c_A^{CS}\right)$.

EC.3.4. Proof of Proposition 4

We prove Proposition 4 in two parts.

EC.3.4.1. Proof of Proposition 4(a).

To prove Proposition 4(a), we state and prove three lemmas.

LEMMA EC.1. ***Comparison of** c_V, c_W, **and** c_A:* *(a) $c_V > \max\{c_W, c_A\}$ for all $\beta \in (0,1)$ and $\phi \in (0, \frac{1}{2})$. (b) $c_W > c_A$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$. (c) $c_A > c_W$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $0 < \beta < \beta_c$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$.*

Proof of Lemma EC.1: Note: The values of c_V, c_W, and c_A are defined in Table EC.3, while the value of β_c is defined in Table EC.4. By comparing c_V, c_W, and c_A, we can verify the ordering provided in Lemma EC.1. We omit the detailed steps for the sake of brevity. □

LEMMA EC.2. ***Comparison of** c_{AW}^{θ}, c_W, **and** c_A:* *(a) $c_{AW}^{\theta} > c_W > c_A$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$. (b) $c_A > c_W > c_{AW}^{\theta}$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $0 < \beta < \beta_c$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$.*

Proof of Lemma EC.2 Note: The values of c_{AW}^{θ}, c_W, and c_A are defined in Table EC.3, while the value of β_c is defined in Table EC.4. Similar to the proof of Lemma EC.1, we can verify the ordering provided in Lemma EC.2 by comparing c_{AW}^{θ}, c_W, and c_A, and omit the detailed steps for the sake of brevity. □

LEMMA EC.3. ***Comparison of** θ_V^*, θ_W^*, **and** θ_A^*:* *(i) $\theta_W^* = \theta_A^* = \theta_V^*$ if $c \in (0, \min\{c_W, c_A\})$; (ii) $\theta_W^* < \theta_A^* = \theta_V^*$ if $c \in (c_W, c_A)$; (iii) $\theta_A^* < \theta_W^* = \theta_V^*$ if $c \in (c_A, c_W)$; (iv) $\theta_A^* < \theta_W^* < \theta_V^*$ if $c \in \left(c_W, c_{AW}^{\theta}\right)$ and $c_A < c_W$; and (v) $\theta_W^* < \theta_A^* < \theta_V^*$ otherwise. Note: $c_V > c_{AW}^{\theta}$.*

Proof of Lemma EC.3: From Tables 5 and 4, we get the value of optimal quality θ_j, $j \in \{V, W, A\}$. Moreover, c_V, c_W, and c_A are defined in Table EC.3. We consider the following four cases and prove our results:

(i) For $c \in (0, \min\{c_W, c_A\})$, from Table 4, we get $\theta_W^* = \theta_A^* = 1$. Now, from Lemma EC.1, $c_V > \max\{c_W, c_A\}$. Hence, for $c \in (0, \min\{c_W, c_A\})$, $\theta_V^* = 1$. Thus, in this case, $\theta_W^* = \theta_A^* = \theta_V^* = 1$.

(ii) When $c \in (c_W, c_A)$, we get the ordering $c_W < c_A < c_V$ from Lemma EC.1. From Tables 5 and 4, we obtain $\theta_A^* = \theta_V^* = 1$ but $\theta_W^* = \frac{8(1-\beta)}{64c-\beta^2} < 1$. Hence, $\theta_W^* < \theta_A^* = \theta_V^*$.

(iii) When $c \in (c_A, c_W)$, we get the order: $c_A < c_W < c_V$. From Tables 5 and 4, we obtain $\theta_W^* = \theta_V^* = 1$ but $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-\phi(1-\phi)\beta^2} < 1$. Hence, $\theta_A^* < \theta_W^* = \theta_V^*$.

(iv) For $\max\{c_W, c_A\} < c < c_V$, from Tables 5 and 4, we obtain $\theta_V^* = 1 > \max\{\theta_W^*, \theta_A^*\}$, where $\theta_W^* = \frac{8(1-\beta)}{64c-\beta^2}$ and $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-\phi(1-\phi)\beta^2}$. Solving $\theta_A^* = \theta_W^*$, $\Rightarrow c = \frac{\beta^2(1-\phi)(1-4\phi)}{32(1-2\phi)} = c_{AW}^{\theta}$. From Lemma EC.2, we obtain $c_{AW}^{\theta} > c_W > c_A$ when $\beta_c < \beta < 1$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Otherwise, $c_{AW}^{\theta} < c_W < c_A$. Thus, $\theta_W^* > \theta_A^*$ when $c \in \left(c_W, c_{AW}^{\theta}\right)$ and $c_A < c_W$. These relationships hold for $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta \in (\beta_c, 1)$. Otherwise, $\theta_A^* > \theta_W^*$. Furthermore, for $c > c_V$, from Tables 5 and 4, we obtain $\theta_V^* = \frac{4(1-\beta)}{16c-\beta^2}$, $\theta_W^* = \frac{8(1-\beta)}{64c-\beta^2}$, and $\theta_A^* = \frac{2(1-\beta)(1-\phi)}{8c-\phi(1-\phi)\beta^2}$. Now, $\theta_V^* - \theta_W^* = 4(1-\beta)\left(\frac{1}{16c-\beta^2} - \frac{2}{64c-\beta^2}\right) = 4(1-\beta)\left(\frac{32c+\beta^2}{(16c-\beta^2)(64c-\beta^2)}\right) > 0$ and $\theta_V^* - \theta_A^* = 2(1-\beta)\left(\frac{2}{16c-\beta^2} - \frac{1-\phi}{8c-\beta^2(1-\phi-1)\phi}\right) = 2(1-\beta)\left(\frac{16c\phi+\beta^2(1-\phi)(1-2\phi)}{(16c-\beta^2)(8c-\beta^2(1-\phi-1)\phi)}\right) > 0$ for any $0 < \beta < 1$ and $0 < \phi < \frac{1}{2}$. In summary, for $c > \max\{c_W, c_A\}$, the following relationships hold $\theta_V^* > \theta_W^* > \theta_A^*$ when $c \in \left(c_W, c_{AW}^{\theta}\right)$ and $c_A < c_W$. Otherwise, $\theta_V^* > \theta_A^* > \theta_W^*$.

□

From Lemma EC.2, we have $c^{\theta}_{AW} > c_W > c_A$, when $\beta_c < \beta < 1$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Furthermore, from Lemma EC.3, we find that $\theta^*_W > \theta^*_A$ under two different scenarios. First, when $c \in (c_A, c_W)$, then $\theta^*_W = \theta^*_V = 1$ but $\theta^*_A = \frac{2(1-\beta)(1-\phi)}{8c-\phi(1-\phi)\beta^2} < 1$. Furthermore, when $c > \max\{c_W, c_A\}$, from Lemma EC.3, we find that $\theta^*_V > \theta^*_W > \theta^*_A$ when $c \in \left(c_W, c^{\theta}_{AW}\right)$ and $c_A < c_W$. Both relationships hold true when $\beta_c < \beta < 1$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Thus, (a) $\theta^*_A < \theta^*_W = \theta^*_V$ if $c \in (c_A, c_W)$, and (b) $\theta^*_A < \theta^*_W < \theta^*_V$ if $c \in \left(c_W, c^{\theta}_{AW}\right)$.

EC.3.4.2. Proof of Proposition 4(b). We consider the following four scenarios:

(i) For $c \in (0, \min\{c_W, c_A\})$, from Table 4, we get $r^*_W = \frac{\beta}{16}$ and $r^*_A = \frac{\beta\phi}{4}$. Thus, $r^*_W - r^*_A = \frac{\beta}{16} - \frac{\beta\phi}{4} = \frac{\beta(1-4\phi)}{16} > 0$ when $\phi \in \left(0, \frac{1}{4}\right]$. Otherwise, $r^*_W - r^*_A < 0$. Therefore, $r^*_W > r^*_A$ when $\phi \in \left(0, \frac{1}{4}\right]$.

(ii) For $c \in (c_W, c_A)$, from Tables 4, we get $r^*_W = \frac{\beta(1-\beta)}{2(64c-\beta^2)}$ and $r^*_A = \frac{\beta\phi}{4}$. Given r^*_W is decreasing in c, we solve $r^*_A = r^*_W$ to obtain the threshold $c = \frac{2(1-\beta)+\phi\beta^2}{64\phi} = c^r_{AW}$, such that $r^*_A > r^*_W$ when $c > c^r_{AW}$, and $r^*_A < r^*_W$ when $c < c^r_{AW}$.

Note that $c^r_{AW} - c_W = \frac{2(1-\beta)+\phi\beta^2}{64\phi} - \frac{8(1-\beta)+\beta^2}{64} = \frac{(1-\beta)(1-4\phi)}{32\phi} > 0$ for any $\phi \in \left(0, \frac{1}{4}\right]$. Furthermore, since $c_A - c^r_{AW} = \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8} - \frac{2(1-\beta)+\phi\beta^2}{64\phi} = \frac{(8(1-\phi)\phi-1)(2(1-\beta)+\phi\beta^2)}{64\phi}$, $c_A > c^r_{AW}$ when $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$, and $c^r_{AW} > c_A$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Remember from Lemma EC.1 that $c_W < c_A$ implies either $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $0 < \beta < \beta_c$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$. Combining these relationships, we conclude that $c_W < c_A < c^r_{AW}$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$; $c_W < c^r_{AW} < c_A$ when $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$; and $c^r_{AW} < c_W < c_A$ when $\phi \in \left(\frac{1}{4}, \frac{1}{2}\right)$.

Therefore, $r^*_W > r^*_A$ when $c_W < c < c^r_{AW}$ and $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$, or $c_W < c < c_A$, $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and $\beta_c < \beta < 1$.

(iii) For $c \in (c_A, c_W)$, from Table 4, we get $r^*_W = \frac{\beta}{16}$ and $r^*_A = \frac{(1-\beta)\beta(1-\phi)\phi}{2(8c-(\phi 1-\phi)\beta^2)}$. Solving $r^*_W = r^*_A$, we obtain the threshold $c = \frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8}$, such that $r^*_W > r^*_A$ when $c > \frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8}$. Remember from Lemma EC.1 that $c_W > c_A$ implies $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$. Now, $c_A - \frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8} = \frac{(1-\phi)(2(1-\beta)+\phi\beta^2)}{8} - \frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8} = \frac{(1-\beta)(1-\phi)(1-4\phi)}{4} > 0$, for any $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Therefore, $r^*_W > r^*_A$ when $c \in (c_A, c_W)$.

(iv) For $\max\{c_W, c_A\} < c$, from Table 4, we get $r^*_W = \frac{\beta(1-\beta)}{2(64c-\beta^2)}$ and $r^*_A = \frac{(1-\beta)\beta(1-\phi)\phi}{2(8c-(\phi 1-\phi)\beta^2)}$. Therefore, $r^*_A - r^*_W = \frac{4(1-\beta)\beta c(8(1-\phi)\phi-1)}{(64c-\beta^2)(8c-\beta^2(1-\phi)\phi)}$. Now, $1-\beta > 0$, $64c - \beta^2 > 0$, and $8c - \beta^2(1-\phi)\phi > 0$ for $c > \max\{c_W, c_A\}$, $0 < \beta < 1$. Hence, $r^*_A > r^*_W$ when $8(1-\phi)\phi - 1 > 0$, that is, $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$. Otherwise, $r^*_A < r^*_W$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$.

Combining the four cases, we arrive at the result shown in Proposition 4(b).

EC.3.5. Proof of Proposition 5

We compare π^*_{MW} and π^*_{MA} in four different cases to arrive at our result.

EC.3.5.1. When $c < \min\{c_W, c_A\}$: From Table 4, we obtain the values of π^*_{MW} and π^*_{MA} when $c < \min\{c_W, c_A\}$. Let $g_{\pi MWA}(\beta) = \pi^*_{MW} - \pi^*_{MA} = \frac{16(1-\beta)+\beta^2-64c}{128} - \frac{(1-\phi)(4(1-\beta)+\beta^2\phi)-8c}{16} = \frac{16(2\phi-1)(1-\beta)+\beta^2(1-8(1-\phi)\phi)}{128}$. Note that, $\frac{\partial^2 g_{\pi MWA}(\beta)}{\partial\beta^2} = \frac{1-8(1-\phi)\phi}{64} > 0$ when $1 - 8(1-\phi)\phi > 0$, i.e., $\phi < \frac{2-\sqrt{2}}{4}$, from Lemma EC.2. Thus, $g_{\pi MWA}(\beta)$ is convex in β when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and concave in β when $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$.

If $g_{\pi MWA}(\beta)$ is convex in β, observe that $\frac{\partial g_{\pi MWA}(\beta)}{\partial\beta} = \frac{-8(2\phi-1)+\beta(1-8(1-\phi)\phi}{64}$ and the optimal $\beta^*_{\pi MWA} = -\frac{8(1-2\phi)}{1-8(1-\phi)\phi} < 0$ for $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Hence, in this region, $g_{\pi MWA}(\beta)$ is an increasing convex function of $\beta \in (0,1)$ and we obtain the maximum of $g_{\pi MWA}(\beta)$ at $\beta = 1$. Next, we determine $g_{\pi MWA}(\beta = 0) = -\frac{1-2\phi}{8} < 0$ and $g_{\pi MWA}(\beta = 1) = \frac{1-8(1-\phi)\phi}{128} > 0$ as $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Thus, $g_{\pi MWA}(\beta)$ changes sign within $\beta \in (0,1)$. Solving $g_{\pi MWA}(\beta) = 0$, we obtain two roots: $\frac{4\left(2-4\phi-\sqrt{5-2\phi(13-4\phi(5-2\phi))}\right)}{8(1-\phi)\phi-1}$ and $\frac{4\left(2-4\phi+\sqrt{5-2\phi(13-4\phi(5-2\phi))}\right)}{8(1-\phi)\phi-1}$. As $g_{\pi MWA}(\beta)$ is an increasing convex function of $\beta \in (0,1)$ and it changes its sign only once within this range ($\beta \in (0,1)$), we note that the smaller root $4\left(\frac{2-4\phi-\sqrt{5-2\phi(13-4\phi(5-2\phi))}}{8(1-\phi)\phi-1}\right) < 0$, and the larger root $0 < 4\left(\frac{2-4\phi+\sqrt{5-2\phi(13-4\phi(5-2\phi))}}{8(1-\phi)\phi-1}\right) = \beta_M < 1$ (see Table EC.4 for the value of β_M). Furthermore, from Lemma EC.1, we obtain that $c_W > c_A$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$. Otherwise, $c_A > c_W$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $0 < \beta < \beta_c$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$. Now, $\beta_M > \beta_c$ for any $\phi > 0$. Therefore, $g_{\pi MWA}(\beta) > 0 \Rightarrow \pi^*_{MW} > \pi^*_{MA}$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, $\beta_M < \beta < 1$, and $0 < c < c_A$.

If $g_{\pi MWA}(\beta)$ is concave in β, it is maximized at $\beta^*_{\pi MWA} = \frac{8(1-2\phi)}{8(1-\phi)\phi-1} > 0$. However, we find that $\beta^*_{\pi MWA} = \frac{8(1-2\phi)}{8(1-\phi)\phi-1} > 1$ for $\frac{2-\sqrt{2}}{4} < \phi < \frac{3(2-\sqrt{2})}{4}$. Thus, in this range of ϕ, $g_{\pi MWA}(\beta)$ is an increasing concave function of $\beta \in (0,1)$, and we obtain the maximum of $g_{\pi MWA}(\beta)$ at $\beta = 1$. But in this case, $g_{\pi MWA}(\beta = 1) = -\frac{8(1-\phi)\phi-1}{128} < 0$ (as $\phi > \frac{2-\sqrt{2}}{4}$) and $g_{\pi MWA}(\beta = 0) = -\frac{1-2\phi}{8} < 0$. Hence, $g_{\pi MWA}(\beta) < 0 \Rightarrow \pi^*_{MW} < \pi^*_{MA}$ for any $\frac{2-\sqrt{2}}{4} < \phi < \frac{3(2-\sqrt{2})}{4}$. Finally, for $\frac{3(2-\sqrt{2})}{4} < \phi < \frac{1}{2}$, $g_{\pi MWA}(\beta)$ is concave with the optimal $0 < \beta^*_{\pi MWA} = \frac{8(1-2\phi)}{8(1-\phi)\phi-1} < 1$. Substituting the value of $\beta^*_{\pi MWA}$, we get the maximum value of $g_{\pi MWA}(\beta = \beta^*_{\pi MWA}) = \frac{(1-2\phi)(5-8(2-\phi)\phi)}{8(1-8(1-\phi)\phi)} < 0$ when ϕ lies between $\frac{3(2-\sqrt{2})}{4}$ and $\frac{1}{2}$. Hence, $g_{\pi MWA} < 0 \Rightarrow \pi^*_{MW} < \pi^*_{MA}$ for any values of $\phi \in \left(\frac{3(2-\sqrt{2})}{4}, \frac{1}{2}\right)$.

Therefore, when $c < \min\{c_W, c_A\}$, $\pi^*_{MW} > \pi^*_{MA}$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, $\beta_M < \beta < 1$, and $0 < c < c_A$.

EC.3.5.2. When $c \in (c_W, c_A)$: From Table 4, we obtain the values of $\pi^*_{MW}(c) = \frac{(1-\beta)^2}{128c-2\beta^2}$ and $\pi^*_{MA}(c) = \frac{(1-\phi)(4(1-\beta)+\beta^2\phi)-8c}{16}$. Comparing the two profit functions, we note that $\pi^*_{MA} > \pi^*_{MW}$ for all $c \in (c_W, c_A)$, and thus this case is not part of Proposition 5.

EC.3.5.3. When $c \in (c_A, c_W)$: From Table 4, we obtain the values of $\pi^*_{MW}(c) = \frac{16(1-\beta)+\beta^2-64c}{128}$ and $\pi^*_{MA}(c) = \frac{(1-\beta)^2(1-\phi)^2}{4(8c-\beta^2(1-\phi)\phi)}$. Now, the difference $\pi^*_{MW}(c) - \pi^*_{MA}(c)$ is a decreasing concave function in $c \in (c_A, c_W)$, as $\frac{\partial^2(\pi^*_{MW}(c)-\pi^*_{MA}(c))}{\partial c^2} = -\frac{32(1-\beta)^2(1-\phi)^2}{(8c-\beta^2(1-\phi)\phi)^3} < 0$ and $\frac{\partial(\pi^*_{MW}(c)-\pi^*_{MA}(c))}{\partial c} = -\frac{1}{2} + \frac{(1-\beta)^2(1-\phi)^2}{(8c-\beta^2(1-\phi)\phi)^2} < 0$, for $0 < \beta < 1$, $0 < \phi < \frac{1}{2}$, and $c \in (c_A, c_W)$. Solving $\pi^*_{MW}(c) - \pi^*_{MA}(c) = 0$, we obtain two roots: $\frac{X^M_{AW} - \sqrt{Y^M_{AW}}}{128}$ and $\frac{X^M_{AW} + \sqrt{Y^M_{AW}}}{128}$, where $X^M_{AW} = 16(1-\beta) + \beta^2(1+8(1-\phi)\phi)$, and $Y^M_{AW} = (16(1-\beta)(2\phi-1) + \beta^2(1-8(1-\phi)\phi))(16(3-2\phi)(1-\beta) + \beta^2(1-8(1-\phi)\phi))$, respectively. As $\pi^*_{MW}(c) - \pi^*_{MA}(c)$ is a decreasing concave function in c, the smaller root satisfies $\frac{X^M_{AW} - \sqrt{Y^M_{AW}}}{128} < c_A$, while the larger root $\frac{X^M_{AW} + \sqrt{Y^M_{AW}}}{128} = \underline{c}_\pi \in (c_A, c_W)$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_M < \beta < \beta_\pi$ (see Table EC.4 for values of β_M and β_π). It can be verified that $\underline{c}_\pi \not< c_A$ for any $\phi \in \left(0, \frac{1}{2}\right)$ and $0 < \beta < 1$, and $c_A < c_W$. In addition, $\underline{c}_\pi > c_W > c_A$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_\pi < \beta < 1$. Thus, when $c \in (c_A, c_W)$, $\pi^*_{MW} > \pi^*_{MA}$ when $c \in \{c_A, \underline{c}_\pi\}$, $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and $\beta_M < \beta < \beta_\pi$, or $c \in (c_A, c_W)$, $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and $\beta_\pi < \beta < 1$.

EC.3.5.4. When $c > \max\{c_W, c_A\}$: From Table 4, we obtain the values of $\pi^*_{MW} = \frac{(1-\beta)^2}{128c-2\beta^2}$ and $\pi^*_{MA} = \frac{(1-\beta)^2(1-\phi)^2}{4(8c-\beta^2(1-\phi)\phi)}$. We observe that $\pi^*_{MW} - \pi^*_{MA} = \frac{(1-\beta)^2}{4}\left(\frac{2}{64c-\beta^2} - \frac{(1-\phi)^2}{4(8c-\beta^2(1-\phi)\phi)}\right) > 0$, when $c < \frac{\beta^2(1-\phi)(1-3\phi)}{48-64(2-\phi)\phi} = \overline{c}_\pi$ (see Table EC.3 for the value of $\overline{c}_\pi$). Next, we compare $\overline{c}_\pi$ with c_W.

Let $g_{\pi M\overline{c}W}(\beta) = \overline{c}_\pi - c_W = \frac{1}{64}\left(\frac{\beta^2(1-8(1-\phi)\phi)}{3-4(2-\phi)\phi} - 8(1-\beta)\right)$. Because $\frac{\partial^2 g_{\pi M\overline{c}W}(\beta)}{\partial\beta^2} = \frac{1-8(1-\phi)\phi}{32(3-4(2-\phi)\phi)}$, $g_{\pi M\overline{c}W}(\beta)$ is convex (concave) in β if $\frac{1-8(1-\phi)\phi}{32(3-4(2-\phi)\phi)} > 0$, i.e., $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ (if $\frac{1-8(1-\phi)\phi}{32(3-4(2-\phi)\phi)} < 0$, i.e., $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$). Differentiating $g_{\pi M\overline{c}W}(\beta)$ with respect to β, we obtain the optimal $\beta^*_{\overline{c}W} = -\frac{4(3-4(2-\phi)\phi)}{1-8(1-\phi)\phi}$.

When $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, i.e., $g_{\pi M\overline{c}W}(\beta)$ is convex, $\beta^*_{\overline{c}W} = -\frac{4(3-4(2-\phi)\phi)}{1-8(1-\phi)\phi} < 0$. Hence, $g_{\pi M\overline{c}W}(\beta)$ is an increasing convex function of $\beta \in (0,1)$ with $g_{\pi M\overline{c}W}(\beta = 0) = -\frac{1}{8} < 0$ and $g_{\pi M\overline{c}W}(\beta = 1) = \frac{1-8(1-\phi)\phi}{64(3-4(2-\phi)\phi)} > 0$ as $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Thus, $g_{\pi M\overline{c}W}(\beta)$ changes its sign (from negative to positive) within $\beta \in (0,1)$. Solving $g_{\pi M\overline{c}W}(\beta) = 0$, we get two roots: $\frac{4(3-8\phi+4\phi^2)-2\sqrt{2(3-2\phi)(1-2\phi)(7-8\phi(3-2\phi))}}{8\phi(1-\phi)-1}$ and $\frac{4(3-8\phi+4\phi^2)+2\sqrt{2(3-2\phi)(1-2\phi)(7-8\phi(3-2\phi))}}{8\phi(1-\phi)-1}$. As $g_{\pi M\overline{c}W}(\beta)$ is an increasing convex function in $\beta \in (0,1)$, the smaller root $\frac{4(3-8\phi+4\phi^2)-2\sqrt{2(3-2\phi)(1-2\phi)(7-8\phi(3-2\phi))}}{8\phi(1-\phi)-1} < 0$ and the larger root $0 < \frac{4(3-8\phi+4\phi^2)+2\sqrt{2(3-2\phi)(1-2\phi)(7-8\phi(3-2\phi))}}{8\phi(1-\phi)-1} = \beta_\pi < 1$ (see Table EC.4 for the value of β_π). Thus, $g_{\pi M\overline{c}W}(\beta) > 0 \Rightarrow \overline{c}_\pi > c_W$, when $\beta > \beta_\pi$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Otherwise, $c_W > \overline{c}_\pi$, when $\beta < \beta_\pi$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$.

Furthermore, when $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$, i.e., $g_{\pi M\overline{c}W}(\beta)$ is a concave function, the optimal $\beta^*_{\overline{c}W} = \frac{4(3-4(2-\phi)\phi)}{8(1-\phi)\phi-1} > 1$, when $\frac{2-\sqrt{2}}{4} < \phi < \frac{10-\sqrt{22}}{12}$ and thus, $g_{\pi M\overline{c}W}(\beta)$ is an increasing concave function in $\beta \in (0,1)$. We determine $g_{\pi M\overline{c}W}(\beta = 0) = -\frac{1}{8} < 0$ and $g_{\pi M\overline{c}W}(\beta = 1) = -\frac{8(1-\phi)\phi-1}{64(3-4(2-\phi)\phi)} < 0$ when $\frac{2-\sqrt{2}}{4} < \phi < \frac{10-\sqrt{22}}{12}$. Thus, in this region, $g_{\pi M\overline{c}W}(\beta) < 0 \Rightarrow \overline{c}_\pi < c_W$. Finally, when $\frac{10-\sqrt{22}}{12} < \phi < \frac{1}{2}$, $g_{\pi M\overline{c}W}(\beta)$ is concave with $0 < \beta^*_{\overline{c}W} = \frac{4(3-4(2-\phi)\phi)}{8(1-\phi)\phi-1} < 1$. Again, $g_{\pi M\overline{c}W}(\beta = \beta^*_{\overline{c}W}) = -\frac{7-8(3-2\phi)\phi}{8(8(1-\phi)\phi-1)} < 0$, for $\frac{10-\sqrt{22}}{12} < \phi < \frac{1}{2}$. Thus, in this region, $g_{\pi M\overline{c}W}(\beta) < 0, \Rightarrow \overline{c}_\pi < c_W$. In summary, $\overline{c}_\pi > c_W$, when $\beta_\pi < \beta < 1$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Otherwise, $c_W > \overline{c}_\pi$, when $0 < \beta < \beta_\pi$ and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$.

From Lemma EC.1, we get $c_W > c_A$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_c < \beta < 1$. Now, $\beta_\pi > \beta_c$ for any $0 < \phi < \frac{1}{2}$. Hence, $\bar{c}_\pi > c_W > c_A$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $\beta_\pi < \beta < 1$. For any other parametric ranges of β and ϕ, $\bar{c}_\pi < \max\{c_A, c_W\}$.

Hence, we can conclude that when $c > \max\{c_W, c_A\}$), $\pi^*_{MW} > \pi^*_{MA}$ if $c_W < c < \bar{c}_\pi$, $\beta_\pi < \beta < 1$, and $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$.

EC.3.5.5. Summary considering all c: Considering all of the aforementioned parameter spaces together, we summarize that $\pi^*_{MW} > \pi^*_{MA}$ iff $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and either (i) $c \in (0, \underline{c}_\pi)$ and $\beta \in (\beta_M, \beta_\pi)$, or (ii) $c \in (0, \bar{c}_\pi)$ and $\beta \in (\beta_\pi, 1)$.

EC.3.6. Proof of Lemma 1

To compare P's profit, we consider four distinct cases, as follows.

(i) For $c \in (0, \min\{c_W, c_A\})$, from Table 4, we obtain values of $\pi^*_{PW} = \frac{32(1-\beta)+\beta^2}{512}$ and $\pi^*_{PA} = \frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$. Now, $\pi^*_{PW} - \pi^*_{PA} = \frac{(1-4\phi)(32(1-\beta)+\beta^2(1+4\phi))}{512} > 0$ when $\phi \in \left(0, \frac{1}{4}\right]$ and $0 < \beta < 1$.

(ii) For $c \in (c_W, c_A)$, from Table 4, we obtain values of $\pi^*_{PW} = \frac{(1-\beta)^2(256c-3\beta^2)}{8(64c-\beta^2)^2}$ and $\pi^*_{PA} = \frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$. Solving $\pi^*_{PW} - \pi^*_{PA} = 0$, we get two roots: $\frac{\beta^2(6(1-\beta)+\phi\beta^2)}{64(8(1-\beta)+\phi\beta^2)}$ and $\frac{2(1-\beta)+\phi\beta^2}{64\phi}$. However, the smaller root $\frac{\beta^2(6(1-\beta)+\phi\beta^2)}{64(8(1-\beta)+\phi\beta^2)} < c_W$ for any $0 < \beta < 1$ and $0 < \phi < \frac{1}{2}$, while the larger root $c_W < \frac{2(1-\beta)+\phi\beta^2}{64\phi} = c^r_{AW} < c_A$, for any $0 < \beta < 1$ and $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$ (see EC.3 for the value of c^r_{AW}). In this region, $\pi^*_{PW} > \pi^*_{PA}$ when $c_W < c < c^r_{AW}$. Furthermore, $c_A > c_W > c^r_{AW}$ when $\phi \in \left(\frac{1}{4}, \frac{1}{2}\right)$ and $0 < \beta < 1$. Conversely, $c^r_{AW} > c_A > c_W$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ and $0 < \beta < \beta_c$. Thus, we conclude that $\pi^*_{PW} > \pi^*_{PA}$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, $c \in (c_W, c_A)$, and $\beta \in (0, \beta_c)$, or $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$, $c \in (c_W, c^r_{AW})$ and $c^r_{AW} < c_A$.

(iii) For $c \in (c_A, c_W)$, from Table 4, we obtain values of $\pi^*_{PW} = \frac{32(1-\beta)+\beta^2}{512}$ and $\pi^*_{PA} = \frac{(1-\beta)^2(1-\phi)\phi(32c-3\beta^2(1-\phi)\phi)}{8(8c-\beta^2(1-\phi)\phi)^2}$. Solving $\pi^*_{PW} - \pi^*_{PA} = 0$, we get two roots: $\frac{\beta^2(24(1-\beta)+\beta^2)(1-\phi)\phi}{8(32(1-\beta)+\beta^2)}$ and $\frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8}$. Now, $\pi^*_{PW} > \pi^*_{PA}$ when $c < \frac{\beta^2(24(1-\beta)+\beta^2)(1-\phi)\phi}{8(32(1-\beta)+\beta^2)}$ or $c > \frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8}$. The smaller root $\frac{\beta^2(24(1-\beta)+\beta^2)(1-\phi)\phi}{8(32(1-\beta)+\beta^2)} < c_A$ for any $0 < \beta < 1$ and $0 < \phi < \frac{1}{2}$. However, the larger root $\frac{(8(1-\beta)+\beta^2)(1-\phi)\phi}{8} < c_A$, whenever $c_A < c_W$. Hence, we conclude that $\pi^*_{PW} > \pi^*_{PA}$ for all $c \in (c_A, c_W)$.

(iv) Finally, for $c > \max\{c_W, c_A\}$, from Table 4, we obtain values of $\pi^*_{PW}(c) = \frac{(1-\beta)^2(256c-3\beta^2)}{8(64c-\beta^2)^2}$ and $\pi^*_{PA}(c) = \frac{(1-\beta)^2(1-\phi)\phi(32c-3\beta^2(1-\phi)\phi)}{8(8c-\beta^2(1-\phi)\phi)^2}$. Solving $\pi^*_{PW}(c) - \pi^*_{PA}(c) = 0$, we get two roots of c: $X_1 - \sqrt{Y_1}$ and $X_1 + \sqrt{Y_1}$, where $X_1 = \frac{3(1-\beta)^2\beta^2(1-64(1-\phi)^2\phi^2)}{512(1-\beta)^2(1-8(1-\phi)\phi)}$ and $Y_1 = \frac{(1-\beta)^4\beta^4(1-8(1-\phi)\phi)^2(9-16(1-\phi)\phi(7-36(1-\phi)\phi))}{(512(1-\beta)^2(1-8(1-\phi)\phi))^2}$. Now, both roots $X_1 \pm Y_1 < \max\{c_W, c_A\}$. Hence, the order of the function, $\pi^*_{PW}(c) - \pi^*_{PA}(c)$, depends on the sign of the second-order condition. We find that $\frac{\partial^2(\pi^*_{MW}(c)-\pi^*_{MA}(c))}{\partial c^2} > 0$, when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$. Similarly, we can find that $\frac{\partial^2(\pi^*_{MW}(c)-\pi^*_{MA}(c))}{\partial c^2} < 0$, when $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$. This implies that $\pi^*_{MW}(c) - \pi^*_{MA}(c)$ is increasing convex (decreasing concave) in $c \in (c_A, c_W)$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$ ($\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{2}\right)$). Hence, we conclude that $\pi^*_{PW} > \pi^*_{PA}$ when $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$.

Hence, $\pi^*_{PW} > \pi^*_{PA}$ iff either (i) $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, or (ii) $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$, and $c < c^r_{AW}$, where $c^r_{AW} \in (c_W, c_A)$.

EC.3.7. Proof of Proposition 6

From Proposition 5, we have $\pi^*_{MW} > \pi^*_{MA}$ iff $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and either (i) $c \in (0, \underline{c}_\pi)$ and $\beta \in (\beta_M, \beta_\pi)$, or (ii) $c \in (0, \bar{c}_\pi)$ and $\beta \in (\beta_\pi, 1)$. Furthermore, from Lemma 1, we find that $\pi^*_{PW} > \pi^*_{PA}$ iff either (i) $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, or (ii) $\phi \in \left(\frac{2-\sqrt{2}}{4}, \frac{1}{4}\right]$, and $c < c^r_{AW}$, where $c^r_{AW} \in (c_W, c_A)$. Combining these relationships, we conclude that both firms M and P prefer the wholesale contract over the agency contract, i.e., $\pi^*_{MW} > \pi^*_{MA}$ and $\pi^*_{PW} > \pi^*_{PA}$ iff $\phi \in \left(0, \frac{2-\sqrt{2}}{4}\right]$, and either (i) $c \in (0, \underline{c}_\pi)$ and $\beta \in (\beta_M, \beta_\pi)$, or (ii) $c \in (0, \bar{c}_\pi)$ and $\beta \in (\beta_\pi, 1)$.

EC.3.8. Proof of Corollary 1

We compare the consumer surplus under the two different contracts, but limit our focus to the win-win region given by Proposition 6. We omit the computational details for the sake of brevity.

EC.4. Analysis of Extensions

We now provide the detailed analysis and results for each of the extensions discussed in Section 5.

EC.4.1. Analysis of Section 5.1: Equilibrium under LL for Wholesale (W) and Agency (A) Contracts in Alternate Sequence

In this extension, we consider the decision-making process of M and P when their antipiracy actions take place in a sequence different from that shown in Section 3.3. In this alternative sequence, in stage 1, P determines the protection level P. In stage 2, M sets the quality of the digital product, θ. Note that our analysis in scenarios HH, LH, and HL, where at most one firm makes a decision, remains unchanged. The only scenario where both firms make decisions is LL. The profit functions of the two firms and the decisions of firm V remain the same as in scenario LL. Due to the change in the decisions to be made in stages 1 and 2, the backward induction is different in those two stages. We provide closed-form solutions in the equilibrium in Table EC.5. It can be verified that our qualitative insights remain unchanged when the sequence of the antipiracy actions of M and P are altered. For brevity, we omit derivations for the last two stages of contract W and the final stage of contract A.

Table EC.5 Equilibrium under LL for Wholesale (W) and Agency (A) Contracts in Alternate Sequence

Variables	W (Wholesale)		A (Agency)	
	$c \leq c_W$	$c > c_W$	$c \leq c_A$	$c > c_A$
θ^*	1	$\frac{8(1-\beta)}{64c-\beta^2}$	1	$\frac{2(1-\beta)(1-\phi)}{8c-\beta^2(1-\phi)\phi}$
r^*	$\frac{\beta}{16}$	$\frac{\beta(1-\beta)}{64c-\beta^2}$	$\frac{\beta\phi}{4}$	$\frac{\beta(1-\beta)\phi(1-\phi)}{8c-\beta^2(1-\phi)\phi}$
w^*	$\frac{16(1-\beta)+\beta^2}{32}$	$\frac{256c(1-\beta)^2}{(64c-\beta^2)^2}$	-	-
p^*	$\frac{3(16(1-\beta)+\beta^2)}{64}$	$\frac{384c(1-\beta)^2}{(64c-\beta^2)^2}$	$\frac{4(1-\beta)+\phi\beta^2}{8}$	$\frac{8(1-\beta)^2c(1-\phi)}{\left(8c-\beta^2(1-\phi)\phi\right)^2}$
π_M^*	$\frac{16(1-\beta)+\beta^2-64c}{128}$	$\frac{32c(1-\beta)^2}{(64c-\beta^2)^2}$	$\frac{(1-\phi)(4(1-\beta)+\phi\beta^2)-8c}{16}$	$\frac{2c(1-\beta)^2(1-\phi)^2}{\left(8c-\beta^2(1-\phi)\phi\right)^2}$
π_P^*	$\frac{32(1-\beta)+\beta^2}{512}$	$\frac{(1-\beta)^2}{2(64c-\beta^2)}$	$\frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$	$\frac{(1-\beta)^2(1-\phi)\phi}{2\left(8c-\beta^2(1-\phi)\phi\right)}$
π_T^*	$\frac{96(1-\beta)+5\beta^2-256c}{512}$	$\frac{(1-\beta)^2(128c-\beta^2)}{2(64c-\beta^2)^2}$	$\frac{8(1-\beta)+\phi(2-\phi)\beta^2-16c}{32}$	$\frac{(1-\beta)^2(1-\phi)\left(4c(\phi+1)-\beta^2(1-\phi)\phi^2\right)}{2\left(8c-\beta^2(1-\phi)\phi\right)^2}$
CS^*	$\frac{16(1+15\beta)-15\beta^2}{512}$	$\frac{4(1-\beta)(4c(1+15\beta)-\beta^2)}{(64c-\beta^2)^2}$	$\frac{4(1+3\beta)-3\phi\beta^2}{32}$	$\frac{(1-\beta)(1-\phi)\left((6\beta+2)c-\beta^2(1-\phi)\phi\right)}{\left(8c-\beta^2(1-\phi)\phi\right)^2}$

EC.4.1.1. Wholesale Contract

From the optimization problem EC.3, we obtain M's optimal wholesale price, $w_W(\theta,r)=\frac{\theta-\beta\theta(1-r)}{2}$ and P's selling price, $p_W(\theta,r)=\frac{3\theta(1-\beta(1-r))}{4}$. Substituting $w_W(\theta,r)$ and $p_W(\theta,r)$ in (EC.1), M's profit-maximization problem becomes:

$$\underset{0\leq\theta\leq 1}{\text{Maximize}}\, \pi_{MW}(r,\theta,w_W(\theta,r),p_W(\theta,r))=\frac{\theta-\beta\theta(1-r)}{8}-\frac{1}{2}c\theta^2, \tag{EC.17}$$

which yields M's content quality, wholesale price, and profit in equilibrium, given by $\theta_W(r)=\frac{1-\beta(1-r)}{8c}$, $w_W(r)=\frac{(1-\beta(1-r))^2}{16c}$, and $\pi^*_{MW}(r)=\frac{(1-\beta(1-r))^2}{128c}$. It may be noted that $\pi_{MW}(r,\theta,w_W(\theta,r),p_W(\theta,r))$ is concave in θ as $\frac{\partial^2\pi_{MW}(r,\theta,w_W(\theta,r),p_W(\theta,r))}{\partial\theta^2}=-c<0$. Therefore, P's selling price is given by $p_W(r)=\frac{3(1-\beta(1-r))^2}{32c}$. Anticipating M's reaction, and substituting $\theta_W(r)$, $w_W(r)$, and $p_W(r)$ in (EC.2), P chooses $0<r\leq 1$ to maximize:

$$\pi_{PW}(r,\theta_W(r),w_W(r),p_W(r))=\frac{(1-\beta(1-r))^2}{128c}-\frac{1}{2}r^2. \tag{EC.18}$$

$\pi_{PW}(r,\theta_W(r),w_W(r),p_W(r))$ is concave in θ as $\frac{\partial^2\pi_{PW}(r,\theta_W(r),w_W(r),p_W(r))}{\partial r^2}=-1+\frac{\beta^2}{64c}<0$. Applying FOC to (EC.18) with respect to r yields P's protection level, and consequently, the selling price and profit in equilibrium, given by $r^*_W=\frac{\beta(1-\beta)}{64c-\beta^2}$, $p^*_W=\frac{384c(1-\beta)^2}{(64c-\beta^2)^2}$, and $\pi^*_{PW}=\frac{(1-\beta)^2}{2(64c-\beta^2)}$, respectively. Substituting r^*_W and p^*_W in M's response functions, we obtain M's wholesale price, content quality, and profit in equilibrium, given by $w^*_W=\frac{256c(1-\beta)^2}{(64c-\beta^2)^2}$, $\theta^*_W=\frac{8(1-\beta)}{64c-\beta^2}$, and $\pi^*_{MW}=\frac{32c(1-\beta)^2}{(64c-\beta^2)^2}$, respectively. This equilibrium solution exists when $c>c_W=\frac{8(1-\beta)+\beta^2}{64}$. The channel profit and the consumer surplus are given by $\pi^*_{TW}=\frac{(1-\beta)^2(128c-\beta^2)}{2(64c-\beta^2)^2}$ and $CS^*_W=\frac{4(1-\beta)(4c(1+15\beta)-\beta^2)}{(64c-\beta^2)^2}$, respectively.

When $c\leq c_W=\frac{8(1-\beta)+\beta^2}{64}$, it is easy to show that M's wholesale price, content quality, and profit in equilibrium are given by $w^*_W=\frac{16(1-\beta)+\beta^2}{32}$, $\theta^*_W=1$, and $\pi^*_{MW}=\frac{16(1-\beta)+\beta^2-64c}{128}$, respectively. Similarly, P's selling price, protection level, and firm profit in equilibrium are given by $p^*_W=\frac{48(1-\beta)+3\beta^2}{64}$, $r^*_W=\frac{\beta}{16}$, and $\pi^*_{PW}=\frac{32(1-\beta)+\beta^2}{512}$, respectively. The channel profit and the consumer surplus are given by $\pi^*_{TW}=\frac{96(1-\beta)+5\beta^2-256c}{512}$ and $CS^*_W=\frac{16(1+15\beta)-15\beta^2}{512}$, respectively. The results are summarized in Table EC.5, under column "W (Wholesale)," with the subscript W omitted.

EC.4.1.2. Agency Contract

Applying FOC to (EC.6) with respect to p yields M's selling price and market demand, given by $p_A(\theta,r)=\frac{\theta-\beta\theta(1-r)}{2}$ and $\mathcal{D}_A(\theta,r)=\frac{1}{2}$, respectively. Substituting $p_A(\theta,r)$ in (EC.6), manufacturer M's profit-maximization problem becomes:

$$\underset{0\leq\theta\leq 1}{\text{Maximize}}\, \pi_{MA}(r,\theta,\phi,p_A(\theta,r))=\frac{\theta(1-\phi)(1-\beta(1-r))}{4}-\frac{1}{2}c\theta^2, \tag{EC.19}$$

which yields M's content quality and profit in equilibrium, given by $\theta_A(r)=\frac{(1-\phi)(1-\beta(1-r))}{4c}$ and $\pi^*_{MA}(r)=\frac{(1-\phi)^2(1-\beta(1-r))^2}{32c}$, respectively. It is easy to show that $\pi_{MA}(r,\theta,\phi,p_A(\theta,r))$ is concave in θ as $\frac{\partial^2\pi_{MA}(r,\theta,\phi,p_A(\theta,r))}{\partial\theta^2}=$

$-c<0$. Substituting $r_A(\theta)$ in $\pi_{PA}(r,\theta_A(r),\phi)$, we obtain M's selling price and market demand given by $p_A(r)=\frac{(1-\phi)(1-\beta(1-r))^2}{8c}$, $q_A(r)=\frac{1}{2}$. Anticipating M's reaction, P chooses $0<r\le 1$ to maximize:

$$\pi_{PA}(r,\theta_A(r),\phi,p_A(r))=\frac{\phi(1-\phi)(1-\beta(1-r))^2}{16c}-\frac{1}{2}r^2, \tag{EC.20}$$

which is obtained by substituting $\theta_A(r)$ and $p_A(r)$ in (EC.6). $\pi_{PA}(r,\theta_A(r),\phi,p_A(r))$ is concave in θ as $\frac{\partial^2\pi_{PA}(r,\theta_A(r),\phi,p_A(r))}{\partial r^2}=-1+\frac{\phi(1-\phi)\beta^2}{8c}<0$. Applying FOC to (EC.20) with respect to r yields P's protection level and profit in equilibrium, given by $r_A^*=\frac{\beta(1-\beta)\phi(1-\phi)}{8c-\beta^2(1-\phi)\phi}$ and $\pi_{PA}^*=\frac{(1-\beta)^2(1-\phi)\phi}{2(8c-\beta^2(1-\phi)\phi)}$, respectively. Substituting r_A^* in M's response functions, we obtain M's content quality, selling price, and profit in equilibrium, given by $\theta_A^*=\frac{2(1-\beta)(1-\phi)}{8c-\beta^2(1-\phi)\phi}$, $p_A^*=\frac{8(1-\beta)^2c(1-\phi)}{(8c-\beta^2(1-\phi)\phi)^2}$, and $\pi_{MA}^*=\frac{2c(1-\beta)^2(1-\phi)^2}{(8c-\beta^2(1-\phi)\phi)^2}$, respectively. This equilibrium solution exists when $c>c_A=\frac{(1-\phi)(2-(2-\beta\phi)\beta)}{8}$. The channel profit and consumer surplus are given by $\pi_{TA}^*=\pi_{MA}^*+\pi_{PA}^*=\frac{(1-\beta)^2(1-\phi)\left(4c(\phi+1)-\beta^2(1-\phi)\phi^2\right)}{2(8c-\beta^2(1-\phi)\phi)^2}$ and $CS_A^*=\frac{(1-\beta)(1-\phi)\left((6\beta+2)c-\beta^2(1-\phi)\phi\right)}{(8c-\beta^2(1-\phi)\phi)^2}$, respectively.

When $c\le c_A=\frac{(1-\phi)(2-(2-\beta\phi)\beta)}{8}$, it is easy to show that M's selling price, content quality, and profit in equilibrium are given by $p_A^*=\frac{4(1-\beta)+\phi\beta^2}{8}$, $\theta_A^*=1$, and $\pi_{MA}^*=\frac{(1-\phi)(4(1-\beta)+\phi\beta^2)-8c}{16}$, respectively. Similarly, P's protection level and firm profit in equilibrium, are given by $r_A^*=\frac{\beta\phi}{4}$, $\pi_{PA}^*=\frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$, respectively. The channel profit and the consumer surplus are given by $\pi_{TA}^*=\frac{8(1-\beta)+\phi(2-\phi)\beta^2-16c}{32}$ and $CS_A^*=\frac{4(1+3\beta)-3\beta^2\phi}{32}$, respectively. The results are summarized in Table EC.5, under column "A (Agency)," with the subscript A omitted.

Comparing the equilibria in Table EC.5, we find that our key results remain unchanged even with the alternate sequence. First, as the commission rate ϕ increases, the quality of genuine content θ_A^* and the content manufacturer's profit π_{MA}^* increase when $c\in\left(c_A,\frac{\beta^2(1-\phi)^2}{8}\right)$, $0<\beta<1$, and $0<\phi\le\frac{1}{2}$. Second, both M and P prefer the wholesale contract (W_{LL}) over the agency contract (A_{LL}) iff $0<\phi<\frac{2-\sqrt{2}}{4}$, $0<\beta<1$, and $c\in\left(\frac{\beta^2}{64},\frac{\beta^2(1-(5-4\phi)\phi)}{32(1-2\phi)}\right)$.

EC.4.2. Analysis of Section 5.2: Equilibrium under HL for Wholesale (W) and Agency (A) Contracts in Additive Utility Model

Similar to Lahiri and Dey (2013) and Kim et al. (2018), in this section, we consider an additive utility model for pirated content, i.e., $\mathcal{U}_P(v,\theta,r)=v\beta\theta-r$. The utility of consuming genuine content $\mathcal{U}(v,\theta,p)=v\theta-p$. Due to analytical complexities, we consider the HL scenario, i.e., the model in which $\theta=1$ and $r\le 1$, while $c=0$ and $k>0$ (remember that HL scenario is a special case of LL scenario). We show that even with HL scenario, our key results remain intact.

By substituting $\theta=1$, we obtain $\mathcal{U}(v,1,p)=v-p$ and $\mathcal{U}_P(v,1,r)=v\beta-r$. Equating $\mathcal{U}(v,1,p)$ to $\mathcal{U}_P(v,1,r)$ provides us the indifference valuation $\widetilde{v}=\frac{p-r}{1-\beta}$. Thus, the demand for the genuine product is given by $\mathcal{D}(1,r,p)=\begin{cases}1-\frac{p-r}{1-\beta}, & \text{if } p>\frac{r}{\beta}\\ 1-p, & \text{if } p\le\frac{r}{\beta}\end{cases}$, and the demand for the pirated product is $\mathcal{D}_P(1,r,p)=\begin{cases}\frac{p-r}{1-\beta}-\frac{r}{\beta}, & \text{if } p>\frac{r}{\beta}\\ 0, & \text{if } p\le\frac{r}{\beta}\end{cases}$.

EC.4.2.1. Wholesale Contract With $\theta=1$ and $r<1$ (W_{HL})

Under contract W, the profit functions for M and P become:

$$\pi_{MW}(1,r,w,p)=w\mathcal{D}(1,r,p)=\begin{cases}w\left(1-\frac{p-r}{1-\beta}\right), & \text{if } p>\frac{r}{\beta}\\ w(1-p), & \text{if } p\le\frac{r}{\beta}\end{cases}, \text{ and} \tag{EC.21}$$

$$\pi_{PW}(1,r,w,p)=(p-w)\,\mathcal{D}(1,r,p)-\frac{r^2}{2}=\begin{cases}(p-w)\left(1-\frac{p-r}{1-\beta}\right)-\frac{r^2}{2}, & \text{if } p>\frac{r}{\beta}\\ (p-w)(1-p)-\frac{r^2}{2}, & \text{if } p\le\frac{r}{\beta}\end{cases}, \text{ respectively.} \tag{EC.22}$$

In response to M's wholesale price offer w and its own protection level decision r, P chooses $p_W^{HL}(r,w)$ that maximizes $\pi_{PW}(1,r,w,p)$ given by (EC.22). When $p>\frac{r}{\beta}$, we show that $\pi_{PW}(1,r,w,p)$ is concave in p as $\frac{\partial^2\pi_{PW}(1,r,w,p)}{\partial p^2}=-\frac{2}{1-\beta}<0$, for all $0<\beta\le 1$. Applying FOC to (EC.22) with respect to p yields P's selling price, demand, and profit function, given by $p_W^{HL}(w,r)=\frac{1+r+w-\beta}{2}$, $\mathcal{D}_W^{HL}(w,r)=\frac{1+r-w-\beta}{2(1-\beta)}$, and $\pi_{PW}^{HL}(w,r)=\frac{(1+r-w-\beta)^2}{4(1-\beta)}-\frac{r^2}{2}$, respectively. Substituting $p_W^{HL}(w,r)$ in $p>\frac{r}{\beta}$, we obtain $w>\frac{\beta^2-\beta-\beta r+2r}{\beta}$. On the other hand, when $p\le\frac{r}{\beta}$, we show that $\pi_{PW}(1,r,w,p)$ is concave in p as $\frac{\partial^2\pi_{PW}(1,r,w,p)}{\partial p^2}=-2<0$. Applying FOC to (EC.22) with respect to p yields P's selling price, demand, and profit function, given by $p_W^{HL}(w)=\frac{1+w}{2}$, $\mathcal{D}_W^{HL}(w)=\frac{1-w}{2}$, and $\pi_{PW}^{HL}(w)=\frac{(1-w)^2}{4}-\frac{r^2}{2}$, respectively. Substituting $p_W^{HL}(w,r)$ in $p\le\frac{r}{\beta}$, we obtain $w\le\frac{2r-\beta}{\beta}$. Finally, when $\frac{2r-\beta}{\beta}<w\le\frac{\beta^2-\beta-\beta r+2r}{\beta}$, it can be easily shown that $p_W^{HL}(w,r)=\frac{r}{\beta}$, $\mathcal{D}_W^{HL}(w,r)=1-\frac{r}{\beta}$, and $\pi_{PW}^{HL}(w,r)=\frac{(\beta-r)(r-\beta w)}{\beta^2}-\frac{r^2}{2}$, respectively. Therefore, after substituting $p_W^{HL}(w,r)$ in (EC.21), M's profit-

maximization problem becomes:

$$\underset{w\geq 0}{\text{Maximize}}\ \pi_{MW}(1,r,w,p_W^{HL}(r,w)) = \begin{cases} w\left(\frac{1+r-w-\beta}{2(1-\beta)}\right), & \text{if } w > \frac{\beta^2-\beta-\beta r+2r}{\beta} \\ w\left(1-\frac{r}{\beta}\right), & \text{if } \frac{2r-\beta}{\beta} < w \leq \frac{\beta^2-\beta-\beta r+2r}{\beta} \\ w\left(\frac{1-w}{2}\right), & \text{if } w \leq \frac{2r-\beta}{\beta}. \end{cases} \quad \text{(EC.23)}$$

For $w > \frac{\beta^2-\beta-\beta r+2r}{\beta}$, $\pi_{MW}(1,r,w,p_W^{HL}(r,w))$ is concave in w as $\frac{\partial^2 \pi_{MW}(1,r,w,p_W^{HL}(r,w))}{\partial w^2} = -\frac{1}{1-\beta} < 0$, for all $0<\beta\leq 1$. The FOC to $\pi_{MW}(1,r,w,p_W^{HL}(r,w))$ yields M's wholesale price and profit function, given by $w_W^{HL}(r) = \frac{1+r-\beta}{2}$ and $\pi_{MW}^{HL}(r) = \frac{(1+r-\beta)^2}{8}$, respectively, in response to P's protection level decision r. Substituting $w_W^{HL}(r)$ in $w > \frac{\beta^2-\beta-\beta r+2r}{\beta}$, we obtain $r < \frac{3\beta(1-\beta)}{4-3\beta}$. On the other hand, for $w \leq \frac{2r-\beta}{\beta}$, we find that $\pi_{MW}(1,r,w,p_W^{HL}(r,w))$ is concave in w as $\frac{\partial^2 \pi_{MW}(1,r,w,p_W^{HL}(r,w))}{\partial w^2} = -1 < 0$. The FOC to $\pi_{MW}(1,r,w,p_W^{HL}(r,w))$ yields M's wholesale price and profit function, given by $w_W^{HL}(r) = \frac{1}{2}$ and $\pi_{MW}^{HL}(r) = \frac{1}{8}$, respectively. Substituting $w_W^{HL}(r)$ in $w \leq \frac{2r-\beta}{\beta}$, we obtain $r \geq \frac{3\beta}{4}$. Finally, for $\frac{2r-\beta}{\beta} < w \leq \frac{\beta^2-\beta-\beta r+2r}{\beta}$, $\pi_{MW}(1,r,w,p_W^{HL}(r,w))$ is increasing in w as $\frac{\partial \pi_{MW}(1,r,w,p_W^{HL}(r,w))}{\partial w} = 1-\frac{r}{\beta} > 0$, as $0<r<\beta<1$. It can be easily shown that $w_W^{HL}(r) = \frac{\beta^2-\beta-\beta r+2r}{\beta}$ and $\pi_{MW}^{HL}(r) = \frac{(\beta-r)\left((2-\beta)r-\beta+\beta^2\right)}{\beta^2}$.

Comparing the manufacturer's profit under the three scenarios provided in (EC.23), we find that there are two thresholds on r, given by $\underline{r}_W = \frac{3\beta(1-\beta)}{4-3\beta}$ and $\overline{r}_W = \frac{\beta}{2(2-\beta)}\left(\sqrt{\frac{\beta}{2}}+(3-2\beta)\right)$, such that the manufacturer's optimal response to r is to set $w_{MW}^{HL}(r) = \frac{1+r-\beta}{2}$ if $r < \underline{r}_W$, $w_{MW}^{HL}(r) = \frac{\beta^2-\beta-\beta r+2r}{\beta}$ if $\underline{r}_W \leq r < \overline{r}_W$, and $w_{MW}^{HL}(r) = \frac{1}{2}$ if $r \geq \overline{r}_W$. Consequently, $\pi_{MW}^{HL}(r) = \frac{(1+r-\beta)^2}{8}$ if $r<\underline{r}_W$, $\frac{(\beta-r)\left((2-\beta)r-\beta+\beta^2\right)}{\beta^2}$ if $\underline{r}_W \leq r < \overline{r}_W$, and $\frac{1}{8}$ if $r \geq \overline{r}_W$. Substituting $w_W^{HL}(r)$ in P's response functions, we obtain P's selling price and demand functions, given by $p_W^{HL}(r) = \frac{3(1+r-\beta)}{4}$ if $r<\underline{r}_W$, $\frac{r}{\beta}$ if $\underline{r}_W \leq r < \overline{r}_W$, and $\frac{3}{4}$ if $r \geq \overline{r}_W$, and $\mathcal{D}_W^{HL}(r) = \frac{1+r-\beta}{4(1-\beta)}$ if $r<\underline{r}_W$, $1-\frac{r}{\beta}$ if $\underline{r}_W \leq r < \overline{r}_W$, and $\frac{1}{4}$ if $r \geq \overline{r}_W$, respectively.

Furthermore, substituting $w_W^{HL}(r)$ and $p_W^{HL}(r)$ in (EC.22), P's profit-maximization problem becomes:

$$\underset{0\leq r\leq 1}{\text{Maximize}}\ \pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r)) = \begin{cases} \frac{(1+r-\beta)^2}{16}-\frac{r^2}{2}, & \text{if } r<\underline{r}_W \\ \frac{(1-\beta)(\beta-r)^2}{\beta^2}-\frac{r^2}{2}, & \text{if } \underline{r}_W \leq r < \overline{r}_W \\ \frac{1}{16}-\frac{r^2}{2}, & \text{if } r \geq \overline{r}_W \end{cases}. \quad \text{(EC.24)}$$

For $r<\underline{r}_W$, we find that $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ is concave in r as $\frac{\partial^2 \pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))}{\partial r^2} = -1+\frac{1}{8(1-\beta)} < 0$, for $0<\beta\leq\frac{7}{8}$. The FOC to $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ yields P's protection level, selling price, quantity, and profit in equilibrium, given by $r_W^{HL*} = \frac{1-\beta}{7-8\beta}$, $p_W^{HL*} = \frac{6(1-\beta)^2}{7-8\beta}$, $q_W^{HL*} = \frac{2(1-\beta)}{7-8\beta}$, and $\pi_{PW}^{HL*} = \frac{(1-\beta)^2}{2(7-8\beta)}$. Substituting r_W^{HL*} in $r<\underline{r}_W$, we obtain $\frac{3-\sqrt{3}}{6} < \beta < \frac{3+\sqrt{3}}{6}$. In can be easily shown that for $0<\beta<\frac{3-\sqrt{3}}{6}$ or $\frac{3+\sqrt{3}}{6}<\beta<1$, $r_W^{HL*} = \underline{r}_W$, $p_W^{HL*} = 1-\frac{1}{4-3\beta}$, $q_W^{HL*} = \frac{1}{4-3\beta}$, and $\pi_{PW}^{HL*} = \frac{(1-\beta)\left(2-9(1-\beta)\beta^2\right)}{2(4-3\beta)^2}$.

For $\underline{r}_W \leq r < \overline{r}_W$, we find that $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ is concave in r as $\frac{\partial^2 \pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))}{\partial r^2} = \frac{2-\beta(2+\beta)}{\beta^2} < 0$, for $\sqrt{3}-1<\beta<1$. The FOC to $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ yields an interior solution $\frac{2\beta(1-\beta)}{2-\beta(2+\beta)}$. However, for $\sqrt{3}-1<\beta<1$, we find that $\frac{2\beta(1-\beta)}{2-\beta(2+\beta)} < \underline{r}_W$. Hence, for $\sqrt{3}-1<\beta<1$, $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ is decreasing in r. Hence, the equilibrium protection level, $r_W^{HL*} = \underline{r}_W$. On the other hand, for $0<\beta<\sqrt{3}-1$, it can be easily shown that equilibrium protection level, $r_W^{HL*} = \underline{r}_W$. Hence, in this case, for any $0<\beta<1$, $r_W^{HL*} = \underline{r}_W$, $p_W^{HL*} = 1-\frac{1}{4-3\beta}$, $q_W^{HL*} = \frac{1}{4-3\beta}$, and $\pi_{PW}^{HL*} = \frac{(1-\beta)\left(2-9(1-\beta)\beta^2\right)}{2(4-3\beta)^2}$.

For $r \geq \overline{r}_W$, we find that, $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r)) = \frac{1}{16}-\frac{r^2}{2}$, is decreasing in r. It can be easily shown that P's profit $\pi_{PW}(1,r,w_W^{HL}(r),p_W^{HL}(r))$ at $r_W^{HL*} = \overline{r}_W$ is dominated by previous scenarios.

Therefore, for $\frac{3-\sqrt{3}}{6} < \beta < \frac{3+\sqrt{3}}{6}$, the equilibrium results are given by $r_W^{HL*} = \frac{1-\beta}{7-8\beta}$, $w_W^{HL*} = \frac{4(1-\beta)^2}{7-8\beta}$, $p_W^{HL*} = \frac{6(1-\beta)^2}{7-8\beta}$, $\pi_{MW}^{HL*} = \frac{8(1-\beta)^3}{(7-8\beta)^2}$, and $\pi_{PW}^{HL*} = \frac{(1-\beta)^2}{2(7-8\beta)}$. The consumer surplus and the total supply chain profit are given by $CS_W^{HL*} = \frac{1-4(1-\beta)\beta(1-3\beta)(3-5\beta)+1}{2(7-8\beta)^2\beta}$ and $\pi_{TW}^{HL*} = \pi_{MW}^{HL*}+\pi_{PW}^{HL*} = \frac{(1-\beta)^2(23-24\beta)}{2(7-8\beta)^2}$, respectively. For $0<\beta<\frac{3-\sqrt{3}}{6}$ or $\frac{3+\sqrt{3}}{6}<\beta<1$, the equilibrium outcomes are given by $r_W^{HL*} = \frac{3\beta(1-\beta)}{4-3\beta}$, $w_W^{HL*} = \frac{2(1-\beta)}{4-3\beta}$, $p_W^{HL*} = 1-\frac{1}{4-3\beta}$, $q_W^{HL*} = \frac{1}{4-3\beta}$, $\pi_{MW}^{HL*} = \frac{2(1-\beta)}{(4-3\beta)^2}$ and $\pi_{PW}^{HL*} = \frac{(1-\beta)\left(2-9(1-\beta)\beta^2\right)}{2(4-3\beta)^2}$. The consumer surplus and the total supply chain profit are given by $CS_W^{HL*} = \frac{1}{2(4-3\beta)^2}$ and $\pi_{TW}^{HL*} = \frac{3(1-\beta)\left(2-3(1-\beta)\beta^2\right)}{2(4-3\beta)^2}$, respectively.

EC.4.2.2. Agency Contract With $\theta = 1$ and $r < 1$ (A_{HL})

Under contract A, the profit functions for M and P become:

$$\pi_{MA}(1,r,\phi,p) = (1-\phi)p\mathcal{D}(1,r,p) = \begin{cases} (1-\phi)\,p\left(1-\frac{p-r}{1-\beta}\right), & \text{if } p > \frac{r}{\beta} \\ (1-\phi)p(1-p), & \text{if } p \le \frac{r}{\beta} \end{cases}, \text{ and} \tag{EC.25}$$

$$\pi_{PA}(1,r,\phi,p) = \phi p\mathcal{D}(1,r,p) - \frac{r^2}{2} = \begin{cases} \phi p\left(1-\frac{p-r}{1-\beta}\right) - \frac{1}{2}r^2, & \text{if } p > \frac{r}{\beta} \\ \phi p(1-p) - \frac{r^2}{2}, & \text{if } p \le \frac{r}{\beta} \end{cases}, \text{ respectively.} \tag{EC.26}$$

In response to P's protection level decision r, M chooses $p_A^{HL}(r)$ that maximizes $\pi_{MA}(1,r,\phi,p)$ given by (EC.25). When $p > \frac{r}{\beta}$, we show that $\pi_{MA}(1,r,\phi,p)$ is concave in p as $\frac{\partial^2 \pi_{MA}(1,r,\phi,p)}{\partial p^2} = -\frac{2(1-\phi)}{1-\beta} < 0$, for all $0 < \beta \le 1$ and $0 < \phi \le \frac{1}{2}$. Applying FOC to (EC.25) with respect to p yields M's selling price, demand, and profit function, given by $p_A^{HL}(r) = \frac{1+r-\beta}{2}$, $\mathcal{D}_A^{HL}(r) = \frac{1+r-\beta}{2(1-\beta)}$, and $\pi_{MA}^{HL}(r) = \frac{(1-\phi)(1+r-\beta)^2}{4(1-\beta)}$, respectively. Substituting $p_A^{HL}(r)$ in $p > \frac{r}{\beta}$, we obtain $r < \underline{r}_A = \frac{\beta(1-\beta)}{2-\beta}$. On the other hand, when $p \le \frac{r}{\beta}$, we show that $\pi_{MA}(1,r,\phi,p)$ is concave in p as $\frac{\partial^2 \pi_{MA}(1,r,\phi,p)}{\partial p^2} = -2(1-\phi) < 0$ for any $0 < \phi \le \frac{1}{2}$. Applying FOC to (EC.25) with respect to p yields M's selling price, demand, and profit function, given by $p_A^{HL}(r) = \frac{1}{2}$, $\mathcal{D}_A^{HL}(r) = \frac{1}{2}$, and $\pi_{MA}^{HL}(r) = \frac{1-\phi}{4}$, respectively. Substituting $p_A^{HL}(r)$ in $p \le \frac{r}{\beta}$, we obtain $r \ge \overline{r}_A = \frac{\beta}{2}$. Finally, when $\underline{r}_A < r \le \overline{r}_A$, it can be easily shown that $p_A^{HL}(r) = \frac{r}{\beta}$, $\mathcal{D}_A^{HL}(r) = 1 - \frac{r}{\beta}$, and $\pi_{MA}^{HL}(r) = \frac{r(1-\phi)(\beta-r)}{\beta^2}$, respectively.

Substituting $p_A^{HL}(r)$ in (EC.26), P's profit-maximization problem becomes:

$$\underset{0\le r\le 1}{\text{Maximize}}\ \pi_{PA}(1,r,\phi,p_A^{HL}(r)) = \begin{cases} \frac{\phi(1+r-\beta)^2}{4(1-\beta)} - \frac{r^2}{2}, & \text{if } r < \underline{r}_A \\ \frac{\phi r(\beta-r)}{\beta^2} - \frac{r^2}{2}, & \text{if } \underline{r}_A < r \le \overline{r}_A \\ \frac{\phi}{4} - \frac{r^2}{2}, & \text{if } r \ge \overline{r}_A \end{cases}, \tag{EC.27}$$

For $r < \underline{r}_A$, we find that $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ is concave in r as $\frac{\partial^2 \pi_{PA}(1,r,\phi,p_A^{HL}(r))}{\partial r^2} = -1 + \frac{\phi}{2(1-\beta)} < 0$, for $0 < \phi \le \frac{1}{2}$ and $0 < \beta < \frac{2-\phi}{2}$. The FOC to $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ yields P's protection level, selling price, and profit in equilibrium, given by $r_A^{HL*} = \frac{\phi(1-\beta)}{2(1-\beta)-\phi}$, $p_A^{HL*} = \frac{(1-\beta)^2}{2(1-\beta)-\phi}$, and $\pi_{PA}^{HL*} = \frac{\phi(1-\beta)^2}{2(2(1-\beta)-\phi)}$. Substituting r_A^{HL*} in $r < \underline{r}_A$, we obtain $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < \frac{1+\sqrt{1-4\phi}}{2}$. In can be easily shown that for $0 < \phi < \frac{1}{4}$ and $0 < \beta < \frac{1-\sqrt{1-4\phi}}{2}$, or $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < 1$, or $\frac{1}{4} < \phi < \frac{1}{2}$ and $0 < \beta < 1$, the equilibrium outcomes are $r_A^{HL*} = \underline{r}_A$, $p_A^{HL*} = 1 - \frac{1}{2-\beta}$ and $\pi_{PA}^{HL*} = \frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$.

For $\underline{r}_A < r \le \overline{r}_A$, we find that $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ is concave in r as $\frac{\partial^2 \pi_{PA}(1,r,\phi,p_A^{HL}(r))}{\partial r^2} = -1 - \frac{2\phi}{\beta^2} < 0$, for $\phi > 0$ and $\beta > 0$. The FOC to $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ yields P's protection level, selling price, and profit in equilibrium, given by $r_A^{HL*} = \frac{\beta\phi}{2\phi+\beta^2}$, $p_A^{HL*} = \frac{\phi}{2\phi+\beta^2}$, and $\pi_{PA}^{HL*} = \frac{\phi^2}{4\phi+2\beta^2}$. Substituting r_A^{HL*} in $\underline{r}_A < r \le \overline{r}_A$, we obtain conditions: $0 < \phi < \frac{1}{4}$ and $0 < \beta < \frac{1-\sqrt{1-4\phi}}{2}$, or $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < 1$, or $\frac{1}{4} < \phi < \frac{1}{2}$ and $0 < \beta < 1$. In can be easily shown that for $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < \frac{1+\sqrt{1-4\phi}}{2}$, the equilibrium outcomes are $r_A^{HL*} = \underline{r}_A$, $p_A^{HL*} = 1 - \frac{1}{2-\beta}$, $q_A^{HL*} = \frac{1}{2-\beta}$, and $\pi_{PA}^{HL*} = \frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$.

For $r \ge \overline{r}_A$, we find that, $\pi_{PA}(1,r,\phi,p_A^{HL}(r)) = \frac{\phi}{4} - \frac{r^2}{2}$, is decreasing in r. It can be easily shown that P's profit $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ at $r_A^{HL*} = \overline{r}_A$ is dominated by previous scenarios.

Now, when $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < \frac{1+\sqrt{1-4\phi}}{2}$, P's profit function $\frac{\phi(1-\beta)^2}{2(2(1-\beta)-\phi)}$ dominates $\frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$, i.e., $\frac{\phi(1-\beta)^2}{2(2(1-\beta)-\phi)} > \frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$. On the other hand, when $0 < \phi < \frac{1}{4}$ and $0 < \beta < \frac{1-\sqrt{1-4\phi}}{2}$, or $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < 1$, or $\frac{1}{4} < \phi < \frac{1}{2}$ and $0 < \beta < 1$, P's profit function $\frac{\phi^2}{4\phi+2\beta^2}$ dominates $\frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$, i.e., $\frac{\phi^2}{4\phi+2\beta^2} > \frac{(1-\beta)\left(2\phi-(1-\beta)\beta^2\right)}{2(2-\beta)^2}$.

Therefore, for $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < \frac{1+\sqrt{1-4\phi}}{2}$, the equilibrium results are given by $r_A^{HL*} = \frac{\phi(1-\beta)}{2(1-\beta)-\phi}$, $p_A^{HL*} = \frac{(1-\beta)^2}{2(1-\beta)-\phi}$, $\pi_{MA}^{HL*} = \frac{(1-\beta)^3(1-\phi)}{(2(1-\beta)-\phi)^2}$, and $\pi_{PA}^{HL*} = \frac{\phi(1-\beta)^2}{2(2(1-\beta)-\phi)}$. The consumer surplus and the total supply chain profit are given by $CS_A^{HL*} = \frac{\beta(1+3\beta)(1-\beta)^2-4\phi\beta(1-\beta)+\phi^2}{2\beta(2(1-\beta)-\phi)^2}$ and $\pi_{TA}^{HL*} = \pi_{MA}^{HL*} + \pi_{PA}^{HL*} = \frac{(1-\beta)^2\left(2(1-\beta)-\phi^2\right)}{2(2(1-\beta)-\phi)^2}$, respectively. Otherwise, for $0 < \phi < \frac{1}{4}$ and $0 < \beta < \frac{1-\sqrt{1-4\phi}}{2}$, or $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < 1$, or $\frac{1}{4} < \phi < \frac{1}{2}$ and $0 < \beta < 1$, the equilibrium outcomes are given by $r_A^{HL*} = \frac{\beta\phi}{2\phi+\beta^2}$, $p_A^{HL*} = \frac{\phi}{2\phi+\beta^2}$, and $\pi_{PA}^{HL*} = \frac{\phi^2}{4\phi+2\beta^2}$. The consumer surplus and the total supply chain profit are given by $CS_A^{HL*} = \frac{1}{2(2-\beta)^2}$ and $\pi_{TA}^{HL*} = \frac{(1-\beta)(1+\beta)(2-(2-\beta)\beta)}{2(2-\beta)^2}$, respectively.

Comparing the two contracts, we find that our key results remain unchanged even with the additive utility model. First, as the commission rate ϕ increases, M's profit π_{MA}^{HL*} increases when $0 < \phi < \frac{1}{4}$ and $\frac{1-\sqrt{1-4\phi}}{2} < \beta < \frac{1+\sqrt{1-4\phi}}{2}$. Second, both M and P prefer the wholesale contract (W_{HL}) over the agency contract (A_{HL}) iff $0.74 < \beta < 0.756$ and

$\frac{17-8\beta(2+(5-\beta)(1-\beta)\beta)}{(7-8\beta)^2} < \phi < \frac{2(1-\beta)^2(1+2\beta^2)}{7-8\beta}$, or $\frac{6-\sqrt{2}}{8} < \beta < 0.756$ and $0 < \phi < \beta(5-4\beta) - \frac{17+(7-8\beta)\sqrt{17-16\beta(3-4\beta)}}{16}$, or $0.756 < \beta < \frac{3+\sqrt{3}}{6}$ and $0 < \phi < \frac{2(1-\beta)^2(1+2\beta^2)}{7-8\beta}$.

EC.4.3. Analysis of Section 5.3: Equilibrium under HL for Wholesale (W) and Agency (A) Contracts in Bundling Model

In this case, we consider a scenario in which the P bundles contents from two manufacturers, M_1 and M_2. Again, we analyze the HL scenario ($\theta = 1$ and $r \leq 1$, while $c = 0$ and $k > 0$) and show that our key results are robust. Thus, the demand for the genuine product, given by (3a), becomes $\mathcal{D}(1,r,p) = 1 - \frac{p}{1-\beta(1-r)}$, and following (3b), the demand for the pirated product is $\mathcal{D}_P(1,r,p) = \frac{p}{1-\beta(1-r)}$ as long as $r < 1$, and zero otherwise.

EC.4.3.1. Wholesale Contract With $\theta = 1$ and $r < 1$ (W_{HL})

Similar to Kim et al. (2018), under contract W, the profit functions for manufacturers (M_i, $i = 1,2$) and P become:

$$\pi_{M_iW}(1,r,w_i,p) = w_1\mathcal{D}(1,r,p) = w_i\left(1 - \frac{p}{1-\beta(1-r)}\right)\ \forall i \in \{1,2\} \quad \text{and} \tag{EC.28}$$

$$\pi_{PW}(1,r,w_1,w_2,p) = (p - w_1 - w_2)\,\mathcal{D}(1,r,p) = (p - w_1 - w_2)\left(1 - \frac{p}{1-\beta(1-r)}\right) - \frac{r^2}{2}, \tag{EC.29}$$

respectively. In response to w_1 and w_2 by M_1 and M_2, and its own protection level decision r, P chooses $p_W^{HL}(w_1,w_2,r)$ that maximizes $\pi_{PW}(1,r,w_1,w_2,p)$ given by (EC.29). $\pi_{PW}(1,r,w_1,w_2,p)$ is concave in p as $\frac{\partial^2\pi_{PW}(1,r,w_1,w_2,p)}{\partial p^2} = -\frac{2}{1-\beta(1-r)} < 0$. Applying FOC to (EC.29) with respect to p yields P's selling price, demand, and profit functions, given by $p_W^{HL}(w_1,w_2,r) = \frac{1+w_1+w_2-\beta(1-r)}{2}$, $\mathcal{D}_W^{HL}(w_1,w_2,r) = \frac{1-\beta(1-r)-w_1-w_2}{2(1-\beta(1-r))}$, and $\pi_{PW}^{HL}(w_1,w_2,r) = \frac{(1-w_1-w_2-\beta(1-r))^2}{4(1-\beta(1-r))} - \frac{r^2}{2}$.

In the previous stage, M_1 and M_2 compete and decide their wholesale prices. Substituting $p_W^{HL}(w_1,w_2,r)$ in (EC.28), the manufacturers' profit-maximization problems become:

$$\underset{w_i \geq 0}{\text{Maximize}}\ \pi_{M_iW}(1,r,w_i,p_W^{HL}(w_1,w_2,r)) = w_i\mathcal{D}_W^{HL}(w_1,w_2,r) = w_i\left(\frac{1-\beta(1-r)-w_i}{2(1-\beta(1-r))}\right)\ \forall i \in \{1,2\}. \tag{EC.30}$$

$\pi_{M_iW}(1,r,w_i,p_W^{HL}(w_1,w_2,r))$ is concave in w_i as $\frac{\partial^2\pi_{M_iW}(1,r,w_i,p_W^{HL}(w_1,w_2,r))}{\partial w_i^2} = -\frac{1}{1-\beta(1-r)} < 0$, for $0 < \beta < 1$, $0 < r < 1$, and $i \in \{1,2\}$. The FOCs from EC.30 yield the manufacturers' wholesale prices and profit functions, given by $w_{iW}^{HL}(r) = \frac{1-\beta(1-r)}{3}$ and $\pi_{M_iW}^{HL}(r) = \frac{1-\beta(1-r)}{18}$, for all $i \in \{1,2\}$, respectively, in response to P's protection level r. Substituting $w_{iW}^{HL}(r)$, $i \in \{1,2\}$, in P's response functions, we obtain P's selling price and demand functions, given by $p_W^{HL}(r) = \frac{5(1-\beta(1-r))}{6}$ and $\mathcal{D}_W^{HL}(r) = \frac{1}{6}$, respectively. Furthermore, substituting $w_{iW}^{HL}(r)$, $i = 1,2$, and $p_W^{HL}(r)$ in (EC.29), P's profit-maximization problem becomes:

$$\underset{0 \leq r \leq 1}{\text{Maximize}}\ \pi_{PW}(1,r,w_{1W}^{HL}(r),w_{2W}^{HL}(r),p_W^{HL}(r)) = \frac{1-\beta(1-r)}{36} - \frac{r^2}{2}. \tag{EC.31}$$

$\pi_{PW}(1,r,w_{1W}^{HL}(r),w_{2W}^{HL}(r),p_W^{HL}(r))$ is concave in r as $\frac{\partial^2\pi_{PW}(1,r,w_{1W}^{HL}(r),w_{2W}^{HL}(r),p_W^{HL}(r))}{\partial r^2} = -1 < 0$. Applying FOC to EC.31 yields P's protection level, and subsequently the selling price and profit in equilibrium, given by $r_W^{HL*} = \frac{\beta}{36}$, $p_W^{HL*} = \frac{5(36(1-\beta)+\beta^2)}{216}$, and $\pi_{PW}^{HL*} = \frac{72(1-\beta)+\beta^2}{2592}$. Substituting r_W^{HL*} in $w_{iW}^{HL}(r_W^{HL*})$ and $\pi_{M_iW}^{HL}(r_W^{HL*})$, $i = 1,2$, we obtain the manufacturers' wholesale prices and profits in equilibrium, given by $w_{iW}^{HL*} = \frac{36(1-\beta)+\beta^2}{108}$ and $\pi_{M_iW}^{HL*} = \frac{36(1-\beta)+\beta^2}{648}$ for all $i \in \{1,2\}$. The consumer surplus is given by $CS_W^{HL*} = \frac{36(1+35\beta)-35\beta^2}{2592}$. The channel profit is therefore $\pi_{TW}^{HL*} = \pi_{M_1W}^{HL*} + \pi_{M_2W}^{HL*} + \pi_{PW}^{HL*} = \frac{40(1-\beta)+\beta^2}{288}$.

EC.4.3.2. Agency Contract With $\theta = 1$ and $r < 1$ (A_{HL})

Similar to Bhargava (2021), under contract A, the profit functions for manufacturers (M_i, $i = 1,2$) and P become:

$$\pi_{M_iA}(1,r,\phi,p) = \frac{(1-\phi)\,p\mathcal{D}(1,r,p)}{2} = \frac{(1-\phi)\,p}{2}\left(1 - \frac{p}{1-\beta(1-r)}\right)\ \forall i \in \{1,2\} \quad \text{and} \tag{EC.32}$$

$$\pi_{PA}(1,r,\phi,p) = \phi p\left(1 - \frac{p}{1-\beta(1-r)}\right) - \frac{1}{2}r^2, \quad \text{respectively.} \tag{EC.33}$$

It may be noted that, as the manufacturers are symmetric, their profit functions are same. In response to the protection level r, the retailer P chooses $p_A^{HL}(r)$ that maximizes $\pi_{PA}(1,r,\phi,p)$ given by (EC.33). $\pi_{PA}(1,r,\phi,p)$ is concave in p as $\frac{\partial\pi_{PA}(1,r,\phi,p)}{\partial p} = -\frac{2\phi}{1-\beta(1-r)} < 0$, for $0 < \beta < 1$ and $0 < r < 1$. Applying FOC to (EC.33) with respect to p yields P's selling price, market demand, and profit function, given by $p_A^{HL}(r) = \frac{1-\beta(1-r)}{2}$, $\mathcal{D}_A^{HL}(r) = \frac{1}{2}$, and $\pi_{MA}^{HL}(r) = \frac{\phi(1-\beta(1-r))}{4}$, respectively. The manufacturers' profits are $\pi_{M_1A}^{HL}(r) = \frac{(1-\phi)(1-\beta(1-r))}{8}$, $i = 1,2$, respectively. Substituting $p_A^{HL}(r)$ in (EC.33), P's profit-maximization problem becomes:

$$\underset{0 \leq r \leq 1}{\text{Maximize}}\ \pi_{PA}(1,r,\phi,p_A^{HL}(r)) = \frac{\phi(1-\beta(1-r))}{4} - \frac{r^2}{2}, \tag{EC.34}$$

which yields P's protection level, selling price, and profit in equilibrium, given by $r_A^{HL*} = \frac{\beta\phi}{4}$, $p_A^{HL*} = \frac{4(1-\beta)+\phi\beta^2}{8}$, and $\pi_{PA}^{HL*} = \frac{\phi(8(1-\beta)+\phi\beta^2)}{32}$, respectively. $\pi_{PA}(1,r,\phi,p_A^{HL}(r))$ is concave in r as $\frac{\partial \pi_{PA}(1,r,\phi,p_A^{HL}(r))}{\partial r} = -1 < 0$. Substituting r_A^{HL*} in $\pi_{A_iW}^{HL}(r_W^{HL*}), i \in \{1,2\}$, we obtain $\pi_{M_1A}^{HL*} = \pi_{M_2A}^{HL*} = \frac{(1-\phi)(4(1-\beta)+\phi\beta^2)}{32}$. The total supply chain profit and consumer surplus are given by $\pi_{TA}^{HL*} = \frac{8(1-\beta)+\phi(2-\phi)\beta^2}{32}$ and $CS_A^{HL*} = \frac{4(1+3\beta)-3\beta^2\phi}{32}$, respectively.

Observe that our key results remain unchanged even with the content bundling. First, as the commission rate ϕ increases, the content manufacturer's profit $\pi_{M_iA}^{HL*}$ increase when $2\left(\sqrt{2}-1\right) < \beta < 1$ and $0 < \phi \leq \frac{\beta(4+\beta)-4}{2\beta^2}$. In addition, M_1, M_2, and P prefer the wholesale contract (W_{HL}) over the agency contract (A_{HL}) iff $\frac{3(7\sqrt{5}-15)}{2} < \beta < 1$ and $\phi < \frac{(4+\beta)\beta-4}{2\beta^2} - \frac{\sqrt{1296-2592\beta+1368\beta^2-72\beta^3+65\beta^4}}{18\beta^2}$. In contrast, M_1, M_2, and P prefer contract A_{HL} over contract W_{HL} for all β of $\frac{1}{9} < \phi < \frac{1}{2}$.

EC.4.4. Analysis of Section 5.4: Equilibrium under HL for Two-Part Tariff (TT) Contract

In the two-part tariff contract (TT), P pays the content manufacturer a fixed fee F and offers a unit price w for the content (Jiang et al. 2023). We assume that the reservation profit of M is Π_M. Again, for this extension, we consider the HL scenario ($\theta = 1$ and $r \leq 1$, while $c = 0$ and $k > 0$). Similar to our base case analyses, the demand for the genuine product and pirated product are given by $\mathcal{D}(1,r,p) = 1 - \frac{p}{1-\beta(1-r)}$ and $\mathcal{D}_P(1,r,p) = \frac{p}{1-\beta(1-r)}$, respectively.

Under contract TT, the profit functions for M and P become:

$$\pi_{MTT}(r,w,F,p) = w\mathcal{D}(1,r,p) + F = w\left(1 - \frac{p}{1-\beta(1-r)}\right) + F \quad \text{and} \tag{EC.35}$$

$$\pi_{PTT}(r,w,F,p) = (p-w)\,\mathcal{D}(1,r,p) - F = (p-w)\left(1 - \frac{p}{1-\beta(1-r)}\right) - \frac{r^2}{2} - F, \tag{EC.36}$$

respectively. In response to its own unit price offer w and protection level decision r, P chooses $p_{TT}^{HL}(r,w)$ that maximizes $\pi_{PTT}(r,w,F,p)$ given by (EC.36). $\pi_{PTT}(r,w,F,p)$ is concave in p as $\frac{\partial^2 \pi_{PTT}(r,w,F,p)}{\partial p^2} = -\frac{2}{1-\beta(1-r)} < 0$ for $0 < \beta < 1$ and $0 < r < 1$. Applying FOC to (EC.36) with respect to p yields P's selling price, demand, and profit functions, given by $p_{TT}^{HL}(w,r) = \frac{1+w-\beta(1-r)}{2}$, $\mathcal{D}_{TT}^{HL}(w,r) = \frac{1}{2} - \frac{w}{2(1-\beta(1-r))}$, and $\pi_{PTT}^{HL}(w,F,r) = \frac{(1-w-\beta(1-r))^2}{4(1-\beta(1-r))} - \frac{r^2}{2} - F$, respectively. Corresponding M's profit is given by $\pi_{MTT}^{HL}(w,F,r) = w\left(\frac{1-\beta(1-r)-w}{2(1-\beta(1-r))}\right) + F$. In the previous stage, P solves the following problem:

$$\underset{w\geq 0,F}{\text{Maximize}} \quad \pi_{PTT}^{HL}(w,F,r,p_{TT}^{HL}(r,w)) = \frac{(1-w-\beta(1-r))^2}{4(1-\beta(1-r))} - F \tag{EC.37}$$

$$\text{s.t.} \quad \pi_{MTT}^{HL}(w,F,r) = w\left(\frac{1-\beta(1-r)-w}{2(1-\beta(1-r))}\right) + F \geq \Pi_M. \tag{EC.38}$$

It may be noted that $\pi_{PTT}^{HL}(w,F,r)$ is decreasing in F. Hence, the constraint (EC.38) binds. Thus, $F_{TT}^{HL}(w,r) = \Pi_M - w\left(\frac{1-\beta(1-r)-w}{2(1-\beta(1-r))}\right)$ and $\pi_{MTT}^{HL} = \Pi_M$. After substituting $p_{TT}^{HL}(w,r)$ and $F_{TT}^{HL}(w,r)$ in (EC.36), P's profit-maximization problem becomes:

$$\underset{w\geq 0}{\text{Maximize}}\, \pi_{PTT}(r,w,F_{TT}^{HL}(r,w),p_{TT}^{HL}(r,w)) = \frac{1}{4}\left(1+\beta(r-1) - \frac{w^2}{1-\beta(1-r)}\right) - \frac{r^2}{2} - \Pi_M, \tag{EC.39}$$

$\pi_{PTT}(1,r,w,F_{TT}^{HL}(r,w),p_{TT}^{HL}(r,w))$ is concave in w as $\frac{\partial^2 \pi_{PTT}(1,r,w,F_{TT}^{HL}(r,w),p_{TT}^{HL}(r,w))}{\partial w^2} = -\frac{1}{2(1-\beta(1-r))} < 0$, for $0 < \beta < 1$ and $0 < r < 1$. Applying FOC on (EC.39) yields P's unit price and profit functions, given by $w_{TT}^{HL}(r) = 0$ and $\pi_{PTT}^{HL}(r) = \frac{1+\beta(1-r)}{4} - \frac{r^2}{2} - \Pi_M$, respectively, in response to P's protection level decision r. Substituting $w_{TT}^{HL}(r)$ in P's response functions, we obtain P's selling price and demand functions, given by $p_W^{HL}(r) = \frac{1-\beta(1-r)}{2}$ and $\mathcal{D}_W^{HL}(r) = \frac{1}{2}$, respectively. In the first stage, P solves the following profit-maximization problem:

$$\underset{0\leq r\leq 1}{\text{Maximize}}\, \pi_{PTT}(r,w_{TT}^{HL}(r),p_{TT}^{HL}(r)) = \frac{1+\beta(1-r)}{4} - \frac{r^2}{2} - \Pi_M, \tag{EC.40}$$

$\pi_{PTT}(r,w_{TT}^{HL}(r),p_{TT}^{HL}(r))$ is concave in r as $\frac{\partial^2 \pi_{PTT}(r,w_{TT}^{HL}(r),p_{TT}^{HL}(r))}{\partial r^2} = -1 < 0$. Applying FOC on (EC.40) yields P's protection level, and subsequently the selling price and profit in equilibrium, given by $r_{TT}^{HL*} = \frac{\beta}{4}, p_{TT}^{HL*} = \frac{(2-\beta)^2}{8}$, Substituting r_W^{HL*} in $w_W^{HL}(r_W^{HL*})$ and $\pi_{MW}^{HL}(r_W^{HL*})$, we obtain the consumer surplus, given by $CS_{TT}^{HL*} = \frac{4(1+3\beta)-3\beta^2}{32}$. The channel profit is therefore $\pi_{TTT}^{HL*} = \pi_{MTT}^{HL*} + \pi_{PTT}^{HL*} = \frac{8(1-\beta)+\beta^2}{32}$. Comparing the channel profit with the corresponding vertically integrated profit from Table EC.2, we have $\pi_{TV}^{HL*} = \pi_{TTT}^{HL*}$.