Electronic Companion to "Empowering or Exploiting? The Implications of Direct Market Access for Improving Smallholder Farmers' Welfare"

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EC.1. Proofs

For simplicity, we denote the buyer's and the MCN's expected profit functions by $\mathbb{E}_Z[\bar{\Pi}_C(w,Z)]$ and $\mathbb{E}_Z[\bar{\Pi}_L(\theta,Z)]$.

Proof of Proposition 1. We first derive the equilibrium solutions to the two models by backward induction. In the contract farming model, it can be calculated that the farmers' optimal planting quantity in the second stage equals $y_C(w)=\max\{\frac{w-\mu}{2\sigma},0\}$ and the buyer's optimal price equals $p_C(w,z)=1-\frac{(w-\mu)\cdot z}{2\alpha\sigma}$ if $\frac{(w-\mu)\cdot z}{\alpha\sigma}\le 1$, and $p_C(w,z)=\frac{1}{2}$ otherwise. When $z=0$ (possible only if $\epsilon=1$) there is no output to sell and the objective does not depend on the retail price; we set $p_C(w,0)=1$, consistent with the expression above. This is immaterial, as Z is continuously distributed. Accordingly, we can calculate the buyer's expected profit function in the first stage.

$$\mathbb{E}_Z[\bar{\Pi}_C(w,Z)]=\begin{cases}0, & \text{if } w\in[0,\mu];\\ \Pi_{C_1}\doteq\frac{(w-\mu)\left(6\alpha\sigma(1-w)-\left(\left(\epsilon^2+3\right)(w-\mu)\right)\right)}{12\alpha\sigma^2}, & \text{if } w\in[\mu,\mu+\frac{\alpha\sigma}{1+\epsilon}];\\ \Pi_{C_2}\doteq-\frac{\alpha^3\sigma^3-3\alpha^2\sigma^2(\epsilon+1)(w-\mu)+3\alpha\sigma(w-\mu)^2\left(4w\epsilon+(\epsilon-1)^2\right)+(\epsilon-1)^3(w-\mu)^3}{24\alpha\sigma^2\epsilon(w-\mu)}, & \text{if } w\in[\mu+\frac{\alpha\sigma}{1+\epsilon},\mu+\frac{\alpha\sigma}{1-\epsilon}];\\ \Pi_{C_3}\doteq\frac{\alpha\sigma+2w(\mu-w)}{4\sigma}, & \text{if } w\ge\mu+\frac{\alpha\sigma}{1-\epsilon}.\end{cases} \tag{EC.1}$$

We can calculate that $\mathbb{E}_Z[\bar{\Pi}_C(w,Z)]$ is concave in w. Furthermore, $\frac{\partial\mathbb{E}_Z[\bar{\Pi}_C(w,Z)]}{\partial w}\Big|_{w=\mu}=\frac{1-\mu}{2\sigma}$. When $\mu\ge 1$, this derivative is no greater than 0; therefore, $\mu\ge1$ implies $w^*\le\mu$ and $y_C(w^*)\le 0$. We refer to this result as Statement †.

It can be calculated that when $\epsilon=0$, the optimal wholesale price that maximizes this function is $w^*=\frac{\alpha(\mu+1)\sigma+\mu}{2\alpha\sigma+1}$. Accordingly, the planting quantity in equilibrium is $y_C^*\doteq y_C(w^*)=\max\{\frac{\alpha(1-\mu)}{4\alpha\sigma+2},0\}$.

In the rural livestreaming model, we can derive the optimal price decision in the third stage, which equals $p_L(\theta,y,z)=1-\frac{y\cdot z}{\alpha}$ if $\frac{y\cdot z}{\alpha}\le\frac{1}{2}$, and $p_L(\theta,y,z)=\frac{1}{2}$ otherwise. Plugging $p_L(\theta,y,z)$ into Equation (3), we obtain

$$\Gamma_L(y|\theta)=\begin{cases}\Gamma_{L_1}\doteq\frac{y\left((1-\theta)\left(3\alpha-y\left(3+\epsilon^2\right)\right)-3y\cdot\alpha\sigma\right)}{3\alpha}-\mu y, & \text{if } y\in[0,\frac{\alpha}{2(1+\epsilon)}];\\ \Gamma_{L_2}\doteq\frac{(1-\theta)\left(8y^3(1-\epsilon)^3-\alpha^3-12\alpha y^2(1-\epsilon)^2+6\alpha^2y(1+\epsilon)\right)}{48\alpha y\epsilon}-\mu y-\sigma y^2, & \text{if } y\in[\frac{\alpha}{2(1+\epsilon)},\frac{\alpha}{2(1-\epsilon)}];\\ \Gamma_{L_3}\doteq\frac{1}{4}\alpha(1-\theta)-\mu y-\sigma y^2, & \text{if } y\ge\frac{\alpha}{2(1-\epsilon)}.\end{cases} \tag{EC.2}$$

We first note that $y_L(\theta)=0$ whenever $\mu\ge1$. To show this, we directly calculate that $\Gamma_L(y|\theta)$ is concave. Hence, a negative derivative at $y=\alpha/[2(1+\epsilon)]$ implies that the maximizer lies in the Γ_{L_1} region. When $\mu\ge1$, this derivative is bounded above by its value at $\mu=1$, namely, $\frac{\partial\Gamma_{L_1}}{\partial y}\Big|_{y=\frac{\alpha}{2(1+\epsilon)},\mu=1}=-\frac{3\alpha\sigma+\epsilon(3\theta+\epsilon(1-\theta))+3}{\epsilon+1}<0$. Hence, the maximizer indeed lies in the Γ_{L_1} region. Solving the first-order condition for Γ_{L_1} gives the unconstrained candidate $y_L'(\theta)=\frac{3\alpha(1-\theta-\mu)}{2(1-\theta)\left(\epsilon^2+3\right)+6\alpha\sigma}$. If $\mu\ge1$, $y_L'(\theta)$ is weakly negative; thus, $y_L(\theta)=0$. We refer to this result as Statement ‡.

It can be further calculated that when $\epsilon=0$, the optimal planting quantity is $y_L(\theta)=\max\{\frac{\alpha(1-\theta-\mu)}{2\alpha\sigma-2\theta+2},0\}$. Hence, the MCN's expected profit is $\mathbb{E}_Z[\bar{\Pi}_L(\theta,Z)]=\frac{\alpha\theta(1-\theta-\mu)(2\alpha\sigma-\theta+\mu+1)}{4(\alpha\sigma-\theta+1)^2}$, which is concave on $[0,1]$. In addition, since its derivative is positive at $\theta=0$ and is negative at $\theta=1$, its unique maximizer must be strictly interior. Hence, solving the first-order condition yields the unique optimum $\theta^*\in(0,1)$, which is given by $(\alpha\sigma+1)F_0(t)$, where $F_0(t)\doteq\left(-\frac{t^2}{\sqrt[3]{3}B}+\frac{B}{3^{2/3}}+1\right)$, $B\doteq\sqrt[3]{t^2\left(\sqrt{3}\sqrt{t^2+27}-9\right)}$, and $t\doteq\frac{\alpha\sigma+\mu}{\alpha\sigma+1}$. Plugging θ^* into $y_L(\theta)$ gives y_L^*.

According to Statements † and ‡, the optimal planting quantities under both models are zero when $\mu\ge1$. We next compare the equilibrium quantities under $\mu<1$. When $\mu<1$, we calculate that

$$y_C^*-y_L^*=\alpha\left(\frac{1-\mu}{4\alpha\sigma+2}-\frac{1-\theta^*-\mu}{2\alpha\sigma-2\theta^*+2}\right)=\alpha\cdot\frac{\sqrt[3]{3}\alpha B^2\sigma+Bt\left(6\alpha\sigma+\sqrt[3]{3}B(\alpha\sigma+1)+3\right)-3^{2/3}t^3(\alpha\sigma+1)-3^{2/3}\alpha\sigma t^2}{2(2\alpha\sigma+1)(1-\theta^*+\alpha\sigma)} \tag{EC.3}$$

Since the right-hand-side fraction depends on α and σ only through their product, for notational convenience, define $\beta\doteq\alpha\sigma$. The denominator of this fraction is positive because θ^* is smaller than 1. The numerator is a quadratic function in B with two zeros, which we denote by $x_1<x_2$. Since it is convex, we know that $x_2<B$ implies that the

numerator is positive, indicating $y_C^* > y_L^*$. We can show that $x_2 < B$ holds as follows.

$$x_2^3 - B^3 = \underbrace{\sqrt{3}t^2\left(2\beta\left(t^2+t+6\right)+t^2+\beta^2(t(t+2)+13)+3\right)\sqrt{R_0}}_{K_1} - \underbrace{9K_0(1-t)t^2+2\sqrt{3}t^2\sqrt{t^2+27}(\beta+\beta t+t)^3}_{K_2},$$

where $R_0 \doteq t^2\left(4\beta(4\beta+3)+4(\beta+1)^2t^2+8\beta(\beta+1)t+3\right)$ and $K_0 \doteq -2\beta^3+(\beta+1)^2t^2+(\beta+1)(\beta(2\beta+5)+1)t$. Because $t=\frac{\mu+\beta}{1+\beta}$ and $\mu<1$, we have $0<t<1$. In addition, plugging $t=\frac{\mu+\beta}{1+\beta}$ into K_0 yields $\mu^2+\mu\beta(2\beta+7)+\mu+6\beta^2+\beta>0$. Taken together, $K_1, K_2>0$, and it follows that $(x_2)^3-B^3<0 \Leftrightarrow (K_1)^2-(K_2)^2<0$. To prove that $(K_1)^2-(K_2)^2<0$, we can calculate that it equals $-36\cdot K_0(1-t)t^4\left(\sqrt{3}\sqrt{t^2+27}-9\right)(\beta+\beta t+t)^3<0$. Therefore, $(K_1)^2-(K_2)^2<0$, which implies $B<x_2$, hence the numerator in (EC.3) is positive and $y_C^*>y_L^*$. □

Proof of Proposition 2. Given any $\mu\in(0,1)$, define $\beta\doteq\alpha\cdot\sigma$ to simplify notation. Direct calculation gives $\Gamma_C^*-\Gamma_L^*=\alpha\cdot G_2(\beta,\mu)$; the G_2 function equals

$$G_2(\beta,\mu)\doteq\frac{\beta(1-\mu)^2}{4(2\beta+1)^2}-\frac{\left(3^{2/3}(\mu+\beta)^2-\sqrt[3]{3}(\beta+1)^2V^2+3(\beta+1)V(\mu+\beta)\right)^2}{12\ 3^{2/3}(\beta+1)V(\mu+\beta)^2-12\cdot\sqrt[3]{3}(\beta+1)^3V^3},$$

where $V=\sqrt[3]{(\mu+\beta)^2\left(\sqrt{3}\sqrt{\frac{(\mu+\beta)^2}{(\beta+1)^2}+27}-9\right)/(\beta+1)^2}$ and can be shown to be positive. To analyze $G_2(\beta,\mu)$, we first combine the two fractions in G_2. The denominator of the resulting fraction can be shown to be positive. The resulting numerator equals $f_2(V)$ where f_2 is a quartic function. Since it can be shown that the two smallest zeros of $f_2(x)$ are negative and the largest zero is greater than V for all β and μ, we only need to compare V with the second largest zero, which we denote by $x_1\doteq-\frac{S-\sqrt[6]{3}S_1}{4(\beta+1)(2\beta+1)^2}$, where $S_1=[6\beta^2(\mu+\beta)^4+24\beta(3\beta(\beta+1)+1)(\mu+\beta)^3 4(\beta(\beta+1)(73\beta(\beta+1)+44)+7)(\mu+\beta)^2+24\beta(\beta+1)^2(3\beta(\beta+1)+1)(\mu+\beta)6(1-\mu)K_5\left(\mu^2\beta+2\mu(\beta(4\beta+3)+1)+\beta(8\beta(\beta+1)+3)\right)]^{\frac{1}{2}}$, $S=3^{2/3}\left(\mu^2\beta-\mu(K_5-2\beta(4\beta+3)-2)+K_5+\beta(8\beta(\beta+1)+3)\right)$, $K_5=\sqrt{\beta\left(\mu^2\beta+2\mu(\beta(8\beta+7)+2)+\beta(16\beta(\beta+1)+5)\right)}$. Since the leading coefficient of f_2 is negative, we have $f_2(V)\le 0\Leftrightarrow V\le x_1$. We prove $V\le x_1$ next.

The comparison between x_1 and V involves two sub-steps. In particular, we derive $V\le x_1\Leftrightarrow V^3-(x_1)^3=D_5-D_6\cdot S_1\Leftrightarrow V\le x_1\Leftrightarrow (D_5)^2-(D_6)^2\cdot(S_1)^2\le 0$, where D_5 and D_6 can be shown to be positive. Simplifying $(D_5)^2-(D_6)^2\cdot(S_1)^2$ yields $D_7-D_8\cdot K_5$, where $D_7, D_8>0$. Therefore, $V\le x_1$ is further equivalent to $(D_7)^2-(D_8)^2\cdot(K_5)^2=4(\mu+\beta)^2(1+2\beta)^6\cdot f(\beta)\le 0$, where $f(\beta)=-4\mu^2-\beta\cdot 4\mu(\mu(\mu+7)+2)-\beta^2\cdot(\mu(\mu(\mu(\mu+18)+97)+44)+4)-\beta^3\cdot 2(\mu(\mu(\mu(\mu+19)+96)+53)+7)-\beta^4\cdot(\mu(\mu(2\mu(\mu+20)+237)+134)+3)+\beta^5\left(60-4\mu(6\mu^2+45\mu+28)\right)+\beta^6\cdot(108-4\mu(21\mu+22))+\beta^7\cdot 64(1-\mu)$. Let c_i denote the coefficient of β to the i^{th} power. We can observe that $c_i\le 0$ for $i\le 4$ and $c_7>0$. By Descartes' Rule of Signs, the number of positive real roots is at most the number of sign changes in the sequence of coefficients. In this case, the only way $f(\beta)$ could have more than one positive root is if $c_5>0$ and $c_6<0$, which is impossible given $\beta>0$ and $\mu\in(0,1)$. Hence, $f(\beta)$ has at most one positive root. Moreover, since $c_0<0$ and $c_7>0$, we have $f(0)<0$ and $f(\beta)\to+\infty$ as $\beta\to+\infty$. Therefore, $f(\beta)$ must cross zero at least once. As a result, $f(\beta)$ has exactly one positive root. Denote this root by t_1'. We have $\alpha\cdot\sigma\le t_1'\Leftrightarrow f(\beta)\le 0\Leftrightarrow V\le x_1\Leftrightarrow f_2(V)\le 0\Leftrightarrow\Gamma_C^*-\Gamma_L^*\le 0$. Thus, defining $t_1=t_1'$ completes the proof.

Moreover, it can be verified (by calculating $f(\beta)$'s derivative with respect to μ directly) that $f(\beta)$ strictly decreases in μ for all $\mu>0$. In addition, $f(\beta,\mu)$ crosses the β-axis from negative to positive at t_1. Consequently, the positive root t_1 strictly increases in μ. □

Proof of Lemma 1. In the contract farming model, supply depletion, i.e., $y_C^*\cdot z\le\alpha(1-p_C^*(z))\ \forall z\in[1-\epsilon,1+\epsilon]$, is equivalent to $y_C^*\le\frac{\alpha}{2(1+\epsilon)}\Leftrightarrow w^*\le\mu+\frac{\alpha\sigma}{1+\epsilon}$. Since Π_{C_3} (defined in Equation (EC.1)) decreases in w, $w\ge\mu+\frac{\alpha\sigma}{1-\epsilon}$ is always suboptimal; therefore, we only need to consider Π_{C_1} and Π_{C_2}. Π_{C_1} is concave because $\frac{\partial^2\Pi_{C_1}}{\partial w^2}=-\frac{6\alpha\sigma+\epsilon^2+3}{6\alpha\sigma^2}<0$. Π_{C_2} is also concave because: $\frac{\partial^3\Pi_{C_2}}{\partial w^3}=\frac{\alpha^2\sigma}{4(w-\mu)^4\epsilon}>0$, $\frac{\partial^2\Pi_{C_2}}{\partial w^2}\Big|_{w=\mu+\frac{\alpha\sigma}{1+\epsilon}}=-\frac{6\alpha\sigma+\epsilon^2+3}{6\alpha\sigma^2}<0$, and $\frac{\partial^2\Pi_{C_2}}{\partial w^2}\Big|_{w=\mu+\frac{\alpha\sigma}{1-\epsilon}}=-\frac{1}{\sigma}<0$. When $\epsilon=1$, the third branch is empty and $\frac{\partial^2\Pi_{C2}}{\partial w^2}=-\frac{\alpha^2\sigma}{12(w-\mu)^3}-\frac{1}{\sigma}<0$ for all $w>\mu$, so concavity continues to hold. Taken together, the buyer's expected profit function is concave in w. Therefore, $w^*\le\mu+\frac{\alpha\sigma}{1+\epsilon}$ is equivalent to $\frac{\partial\Pi_{C_1}}{\partial w}\Big|_{w=\mu+\frac{\alpha\sigma}{1+\epsilon}}=\frac{\partial\Pi_{C_2}}{\partial w}\Big|_{w=\mu+\frac{\alpha\sigma}{1+\epsilon}}=-\frac{3(2\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{6\sigma(\epsilon+1)}\le 0\Leftrightarrow\alpha\sigma\ge-\frac{3\mu(\epsilon+1)+(\epsilon-3)\epsilon}{6}$. Defining $t_2^C(\mu,\epsilon)$ by $-\frac{3\mu(\epsilon+1)+(\epsilon-3)\epsilon}{6}$ proves Lemma 1 for contract farming.

Similarly, in the rural livestreaming model, we can show that supply depletion, i.e., $y_L^* \cdot z \le \alpha(1-p_L^*(z))$ $\forall z \in [1-\epsilon, 1+\epsilon]$, is equivalent to $y_L^* \le \frac{\alpha}{2(1+\epsilon)}$. Based on similar arguments, we can derive that $y_L^* \le \frac{\alpha}{2(1+\epsilon)}$ is equivalent to $\frac{\partial \Gamma_{L_1}}{\partial y}\big|_{y=\frac{\alpha}{2(1+\epsilon)}} = \frac{\partial \Gamma_{L_2}}{\partial y}\big|_{y=\frac{\alpha}{2(1+\epsilon)}} = \frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon} \le 0 \Leftrightarrow \theta \ge \frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon}$ (where Γ_{L_1} and Γ_{L_2} are defined in Equation (EC.2)); therefore, what remains is to show that $\theta^* \ge \frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon}$ holds if and only if $\alpha\sigma$ is larger than a $t_2^L(\mu,\epsilon)$ threshold. To accomplish this, we first show that the MCN's expected profit function $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)]$ is concave in θ, and then derive $t_2^L(\mu,\epsilon)$ by analyzing the sign of $\frac{\partial \mathbb{E}_Z[\bar{\Pi}_L(\theta,Z)]}{\partial \theta}\big|_{\theta=\frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon}}$.

Since there is no closed-form characterization of $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)]$, we use the chain rule to analyze its sensitivity property. Let us write $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)]$ as $\theta \cdot R(y_L(\theta))$, where $R(y)$ represents the expected livestream sales revenue under a given y. Applying the chain rule, we have

$$\frac{\partial^2(\theta \cdot R(y_L(\theta)))}{\partial \theta^2} = 2 \cdot \frac{\partial R(y)}{\partial y}\bigg|_{y=y_L(\theta)} \cdot \frac{\partial y_L(\theta)}{\partial \theta} + \theta\left(\left(\frac{\partial y_L(\theta)}{\partial \theta}\right)^2 \cdot \frac{\partial^2 R(y)}{\partial y^2}\bigg|_{y=y_L(\theta)} + \frac{\partial R(y)}{\partial y}\bigg|_{y=y_L(\theta)} \cdot \frac{\partial^2 y_L(\theta)}{\partial \theta^2}\right). \tag{EC.4}$$

$\frac{\partial R(y)}{\partial y} > 0$ and $\frac{\partial^2 R(y)}{\partial y^2} < 0$ can be shown based on the same arguments used to prove concavity of Π_{C_1} and Π_{C_2} in the first paragraph of this proof. Next, we will further show that $\frac{\partial y_L(\theta)}{\partial \theta} < 0$ and $\frac{\partial^2 y_L(\theta)}{\partial \theta^2} < 0$. To do so, we use the chain rule to analyze the sensitivity property of $y_L(\theta)$. By definition, $y_L(\theta)$ is the maximizer of $\Gamma_L(y|\theta)$. Consider $\frac{\partial \Gamma_L(y,\theta)}{\partial y}$ as a function of y and θ, which we denote by $G_1(y,\theta)$. Following the chain rule, we have

$$\frac{\partial G_1(y_L(\theta),\theta)}{\partial \theta} = \frac{\partial G_1(y,\theta)}{\partial y}\bigg|_{y=y_L(\theta)} \cdot \frac{\partial y_L(\theta)}{\partial \theta} + \frac{\partial G_1(y,\theta)}{\partial \theta}\bigg|_{y=y_L(\theta)}. \tag{EC.5}$$

Since $y_L(\theta)$ is the maximizer of $\Gamma_L(y_L(\theta),\theta)$, we have $G_1(y_L(\theta),\theta) \equiv 0$, which implies $\frac{\partial G_1(y_L(\theta),\theta)}{\partial \theta} = 0$. Plugging this value into Equation (EC.5), we obtain $\frac{\partial G_1(y,\theta)}{\partial \theta}|_{y=y_L(\theta)} = -\frac{\partial G_1(y,\theta)}{\partial y}|_{y=y_L(\theta)} \cdot \frac{\partial y_L(\theta)}{\partial \theta}$. Additionally, it can be shown that $\Gamma_L(y,\theta)$ is concave in y, thus $\frac{\partial G_1(y,\theta)}{\partial y}|_{y=y_L(\theta)} < 0$. Taken together, the sign of $\frac{\partial y_L(\theta)}{\partial \theta}$ is the same as that of $\frac{\partial G_1(y,\theta)}{\partial \theta}|_{y=y_L(\theta)}$, which by definition equals $\frac{\partial^2 \Gamma_L(y,\theta)}{\partial y \partial \theta}|_{y=y_L(\theta)}$ and can be calculated as $\frac{2y_L(\theta)\left(\epsilon^2+3\right)}{3\alpha} - 1$ if $y_L(\theta) < \frac{\alpha}{2(1+\epsilon)}$, or $\frac{(\alpha+2y_L(\theta)(\epsilon-1))^2(4y_L(\theta)(\epsilon-1)-\alpha)}{48\alpha(y_L(\theta))^2\epsilon}$ if $y_L(\theta) \in [\frac{\alpha}{2(1+\epsilon)}, \frac{\alpha}{2(1-\epsilon)})$ ($y > \frac{\alpha}{2(1-\epsilon)}$ is always suboptimal, and this range is empty when $\epsilon = 1$). Both values are negative because $\frac{\partial^2 \Gamma_L(y,\theta)}{\partial y \partial \theta} < 0$ for all $y \in [0, \frac{\alpha}{2(1-\epsilon)})$. Hence, $\frac{\partial y_L(\theta)}{\partial \theta}$ is also negative.

We further show that $y_L(\theta)$ is concave in θ. Differentiating both sides of Equation (EC.5) with respect to θ yields

$$0 = 2 \cdot \frac{\partial^2 G_1(y,\theta)}{\partial y \partial \theta}\bigg|_{y=y_L(\theta)} \cdot \frac{\partial y_L(\theta)}{\partial \theta} + \frac{\partial^2 G_1(y,\theta)}{\partial y^2}\bigg|_{y=y_L(\theta)} \cdot \left(\frac{\partial y_L(\theta)}{\partial \theta}\right)^2 + \frac{\partial G_1(y,\theta)}{\partial y}\bigg|_{y=y_L(\theta)} \cdot \frac{\partial^2 y_L(\theta)}{\partial \theta^2} + 0. \tag{EC.6}$$

$\frac{\partial^2 G_1(y,\theta)}{\partial y \partial \theta}$ and $\frac{\partial^2 G_1(y,\theta)}{\partial y^2}$ can be calculated to be nonnegative for all y, which implies $\frac{\partial^2 y_L(\theta)}{\partial \theta^2} < 0$.

Because $\frac{\partial y_L(\theta)}{\partial \theta} < 0$ and $\frac{\partial^2 y_L(\theta)}{\partial \theta^2} < 0$, we derive that $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)]$ is concave based on (EC.4). Consequently, whether $y_L^* \le \frac{\alpha}{2(1+\epsilon)}$ holds or not depends on the sign of $\frac{\partial \mathbb{E}_Z[\bar{\Pi}_L(\theta,Z)]}{\partial \theta}\big|_{\theta=\frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon}}$. Define $t_2^L(\mu,\epsilon) \doteq -\frac{2(\epsilon-3)^3\epsilon^3+3\mu(\epsilon+1)(\epsilon(\epsilon((\epsilon-6)\epsilon+18)+18)+9)}{3(\epsilon(\epsilon(\epsilon(2\epsilon-15)+33)+27)+9)}$. It can be calculated that if and only if $\alpha\sigma \ge t_2^L(\mu,\epsilon)$, $\frac{\partial \mathbb{E}_Z[\bar{\Pi}_L(\theta,Z)]}{\partial \theta}\big|_{\theta=\frac{3(\alpha\sigma+\mu)+\epsilon(3\mu+\epsilon-3)}{(\epsilon-3)\epsilon}} \ge 0$, hence $y_L^* \le \frac{\alpha}{2(1+\epsilon)}$ and $y_L^* \cdot z \le \alpha(1-p_L^*(z))$ $\forall z \in [1-\epsilon, 1+\epsilon]$ hold. □

Proof of Lemma 2. <u>Contract farming:</u> As shown in the proof of Proposition 1, $y_C^* = \frac{w^*-\mu}{2\sigma}$. The farmers' optimal expected income for a given w, $\Gamma_C(y_C(w)|w)$, equals $\frac{(w-\mu)^2}{4\sigma}$, and the expected income in equilibrium $\Gamma_C^*(\epsilon) = \frac{(w^*(\epsilon)-\mu)^2}{4\sigma}$; hence, $\frac{\partial \Gamma_C^*(\epsilon)}{\partial \epsilon} = (w^*(\epsilon)-\mu)(\frac{\partial w^*(\epsilon)}{\partial \epsilon} \cdot \frac{1}{2\sigma}) = (w^*(\epsilon)-\mu)\frac{\partial y_C^*(\epsilon)}{\partial \epsilon}$. Therefore, $y_C^*(\epsilon)$, $\Gamma_C^*(\epsilon)$ and $w^*(\epsilon)$ share the same sensitivity properties with respect to ϵ. Furthermore, it can be shown that the buyer's expected profit function $\mathbb{E}_Z[\bar{\Pi}_C(w, Z)]$ in Equation (EC.1) is maximized at $w^* \doteq \frac{3\alpha(\mu+1)\sigma+\mu\left(\epsilon^2+3\right)}{6\alpha\sigma+\epsilon^2+3}$. Accordingly, we can calculate that $\frac{\partial w^*}{\partial \epsilon} < 0$.

<u>Rural Livestreaming:</u> We first prove that y_L^* decreases in ϵ. As indicated in Lemma 1, when $\alpha\sigma \ge t_2^L(\mu,\epsilon)$, $y_j^* \cdot z \le \alpha(1-p_j^*(z))$ $\forall z \in [1-\epsilon, 1+\epsilon]$. As a result, $\Gamma_L(y|\theta) = \Gamma_{L_1}$ as presented in Equation (EC.2). Applying the first order condition, we can derive $y_L(\theta) = \frac{3\alpha(\theta-1+\mu)}{2(\theta-1)\left(\epsilon^2+3\right)-6\alpha\sigma}$. In particular, the planting quantity in equilibrium is $y_L^*(\epsilon) = \frac{3\alpha(\theta^*(\epsilon)-1+\mu)}{2(\theta^*(\epsilon)-1)\left(\epsilon^2+3\right)-6\alpha\sigma}$. Its derivative with respect to ϵ, $\frac{\partial y_L^*(\epsilon)}{\partial \epsilon}$, can be calculated as $-\frac{6\alpha\epsilon(1-\theta^*(\epsilon))(1-\mu-\theta^*(\epsilon))+3\alpha((3+\epsilon^2)\mu+3\alpha\sigma)\partial\theta^*(\epsilon)/\partial\epsilon}{2(3+\epsilon^2+3\alpha\sigma-(3+\epsilon^2)\theta^*(\epsilon))^2}$. Since the denominator is positive, $\partial y_L^*(\epsilon)/\partial\epsilon < 0$ follows once the numerator is positive. It is therefore sufficient to establish $\theta^*(\epsilon) \le 1-\mu$ and $\partial\theta^*(\epsilon)/\partial\epsilon > 0$. To prove $\theta^*(\epsilon) \le 1-\mu$, we first derive that when $\theta = 1-\mu$, farmers' best response $y_L(\theta)$ corresponds to the $y_L'(\theta)$ function defined in the proof of

Proposition 1 by checking $\frac{\partial \Gamma_L(y|1-\mu)}{\partial y}\Big|_{y=\frac{\alpha}{2(1+\epsilon)}} < 0$. Therefore, $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)] = -\frac{3\alpha\theta(\theta+\mu-1)\left(6\alpha\sigma-\left(\epsilon^2+3\right)(\theta-\mu-1)\right)}{4\left((\theta-1)\left(\epsilon^2+3\right)-3\alpha\sigma\right)^2}$. Since its derivative equals $\frac{3\alpha(\mu-1)}{6\alpha\sigma+2\mu\left(\epsilon^2+3\right)} < 0$ at $\theta = 1-\mu$, by concavity of $\mathbb{E}_Z[\bar{\Pi}_L(\theta, Z)]$, $\theta^* < 1-\mu$.

To prove $\frac{\partial \theta^*(\epsilon)}{\partial \epsilon} > 0$, we first analyze $\frac{\partial \theta^*(t(\epsilon))}{\partial \epsilon}$, where $t = \frac{\alpha\sigma}{3+\epsilon^2}$, then apply the chain rule. We calculate that $\theta^*(t(\epsilon)) = \frac{-3^{2/3}(\mu+3t)^2 + \sqrt[3]{3}W^2 Y(\mu+3t) + 3(3t+1)WY^2}{3W(\mu+3t)^{2/3}}$, where $W = \sqrt[3]{\sqrt{3}\sqrt{(\mu+3t)^2+27(3t+1)^2} - 9(3t+1)}$, $Y = \sqrt[3]{\mu+3t}$. Taking its derivative with respect to t and plugging in $\frac{\partial W(t)}{\partial t} = \frac{3\left(\mu+3t-3W^3\right)}{W^2\left(27t+W^3+9\right)}$ yield $\frac{\partial \theta^*(t)}{\partial t} = \frac{H_1(W)}{3W^4(\mu+3t)^{2/3}\left(27t+W^3+9\right)}$. Since $(\sqrt{3}\sqrt{(\mu+3t)^2+27(3t+1)^2})^2 - (9(3t+1))^2 > 0$, W and accordingly, $27t + W^3 + 9$ are positive. Therefore, $\frac{\partial \theta^*(t)}{\partial t} < 0 \Leftrightarrow H_1(W) < 0$, where $H_1(W)$ is given by $-4\ 3^{2/3}(W^3)^2(\mu+3t) + 2\sqrt[3]{3}(W^3)^2\sqrt[3]{\mu+3t}W^2 + 9(W^3)^2(\mu+3t)^{2/3}W - 9\ 3^{2/3}(W^3)(\mu+15t+4)(\mu+3t) + 9\sqrt[3]{3}(W^3)(-\mu+3t+2)\sqrt[3]{\mu+3t}W^2 + 81(W^3)(3t+1)(\mu+3t)^{2/3}W + 3\ 3^{2/3}(\mu+3t)^3 + 3\sqrt[3]{3}(\mu+3t)^{7/3}W^2$. Plugging $W^3 = \sqrt{3}\sqrt{(\mu+3t)^2+27(3t+1)^2} - 9(3t+1)$ into $H_1(W)$ results in a convex quadratic function in W. Since it can be verified that the intercept of $H_1(x)$ is negative, its smaller zero is negative. Therefore, to prove $H_1(W) < 0$, it suffices to show that W is always smaller than the larger zero, denoted by x_2.

Next, we calculate that $W^3 - x_2^3 = \frac{F_2 - F_3\sqrt{R_3}}{F_1}$, where F_1, F_2, $F_3 > 0$ and R_3 is given by $(\mu+3t)^{4/3}\left(\mu(24-5\mu)+1764t^2+42(\mu+27)t+177\right)\left(\mu^2+495t^2+6t\left(\mu-3\sqrt{3}\sqrt{R_4}+54\right)-6\sqrt{3}\sqrt{R_4}+54\right)$, where $R_4 \doteq \mu^2+6\mu t+18t(14t+9)+27$. Therefore, $W < x_2 \Leftrightarrow L_1 \doteq F_2^2 - F_3^2 \cdot R_3 < 0$. L_1 can be re-arranged into $F_5 - F_6\sqrt{R_4}$, where $F_5,\ F_6 > 0$. As a result, $W < x_2$ is further equivalent to $L_2 \doteq (F_5)^2 - R_4 \cdot (F_6)^2 < 0$. Since $L_2 = -31104(\mu-1)^4(\mu+3t)^{18}(5\mu+63t+16)^2\left(-(\mu-3)\mu+126t^2+3(\mu+27)t+12\right)^3\left(\mu^2+252t^2+6(\mu+27)t+27\right) < 0$, $W < x_2$; thus, $H_1(W) < 0$ and $\frac{\partial \theta^*(t)}{\partial t} < 0$. Lastly, by the chain rule, $\frac{\partial \theta^*(t(\epsilon))}{\partial \epsilon} = \frac{\partial \theta^*(t(\epsilon))}{\partial t} \cdot \frac{\partial t(\epsilon)}{\partial \epsilon} = \frac{\partial \theta^*(t(\epsilon))}{\partial t} \cdot \frac{-2\alpha\sigma\epsilon}{\left(\epsilon^2+3\right)^2} > 0$.

Finally, we show that $\frac{\partial \Gamma_L^*(\epsilon)}{\partial \epsilon} < 0$. $\frac{\partial \Gamma_L^*(\epsilon)}{\partial \epsilon} = \frac{-3\alpha(1-\theta^*-\mu)\left(6\alpha\sigma+\left(\epsilon^2+3\right)(1-\theta^*+\mu)\right)\cdot(\partial\theta^*/\partial\epsilon)+6\alpha(\theta^*-1)\epsilon(\theta^*+\mu-1)^2}{4\left(3\alpha\sigma-\theta^*\left(\epsilon^2+3\right)+\epsilon^2+3\right)^2}$. We denote the numerator by F_7 and it suffices to show that $F_7 < 0$. Since $\theta^* < 1-\mu$ as established earlier in this proof, F_7 is linear and decreasing in $\frac{\partial \theta^*}{\partial \epsilon}$. Hence, $F_7 < 0$ if and only if $\frac{\partial \theta^*}{\partial \epsilon} > -\frac{2(1-\theta^*)\epsilon(1-\theta^*-\mu)}{\left(\epsilon^2+3\right)(1-\theta^*+\mu)+6\alpha\sigma}$, which holds because the fraction on the right-hand side is negative as $\theta^* < 1-\mu$, while $\frac{\partial \theta^*}{\partial \epsilon} > 0$. $\square$

Proof of Proposition 3. To compare the equilibrium income in the two models, we first calculate that $\Gamma_C^* - \Gamma_L^* = \alpha \cdot F(\alpha\cdot\sigma, \cdot)$. Letting $\beta \doteq \alpha\sigma$, the F function is formulated as $F(\beta, \cdot) \doteq \frac{u^2\left(\epsilon^2+3\right)^2/\left(9\tau^{4/3}\left(\sqrt{3}\sqrt{\tau^2+27}-9\right)^{2/3}\right)\cdot M_1(W_1)}{4(6t+1)^2\left(\epsilon^2+3\right)^2\left((1-\theta^*)\left(\epsilon^2+3\right)+3\beta\right)}$. The $M_1(W_1)$ function is defined by $-3\sqrt[3]{3}\tau^2(1-2v)^2 - 3^{2/3}\tau^{2/3}(1-2v)^2(W_1)^4 - 3\sqrt[3]{3}(W_1)^3(2\tau+(\tau(\tau+6)+1)(v-1)v) - 3\tau^{4/3}(1-2v)^2(W_1)^2 + 3\ 3^{2/3}\tau^{2/3}W_1(2\tau+(\tau(\tau+6)+1)(v-1)v)$ where $W_1 = \sqrt[3]{\sqrt{3}\sqrt{\tau^2+27}-9}$, $\tau = u/v$, $u = 3t+\mu$, $v = 3t+1$, $t = \beta/(3+\epsilon^2)$, and $\beta = \alpha\sigma$. We analyze $F(\beta, \cdot)$ in four steps:

- Because $\theta^* < 1$ by concavity and first order conditions, we have $F(\beta, \cdot) \le 0 \Leftrightarrow M_1(W_1) \le 0$.
- The function $M_1(x)$ is quartic and has four zeros. Since it can be shown that the two smallest zeros of $M_1(x)$ are negative, we only need to compare W_1 with the two larger zeros, which we denote by $x_0 < x_1$. Since the leading coefficient of $M_1(x)$ is negative, we have $M_1(W_1) \le 0$ if and only if $W_1 \le x_0$ or $W_1 \ge x_1$.
- We then compare W_1 with x_0. The comparison between x_0 and W_1 involves two sub-steps.

  —First, we calculate that $(W_1)^3 - (x_0)^3 = \frac{D_0 - D_1\cdot\sqrt{R_1}}{512\tau^2(1-2v)^6}$, where $R_1 = \frac{24\tau^4(v-1)^2v^2}{(1-2v)^4} + \frac{24(v-1)^2v^2}{(1-2v)^4} + \frac{24\tau^{2/3}(v-1)v\sqrt{R_2}}{(1-2v)^2} + \frac{48\tau^{5/3}(3(v-1)v+1)\sqrt{R_2}}{(1-2v)^2} + \frac{24\tau^{8/3}(v-1)v\sqrt{R_2}}{(1-2v)^2} + \frac{96\tau^3(v-1)v(3(v-1)v+1)}{(1-2v)^4} + \tau^2\left(\frac{9}{(1-2v)^4} + \frac{30}{(1-2v)^2} + 73\right) + \frac{96\tau(v-1)v(3(v-1)v+1)}{(1-2v)^4} \cdot \frac{1}{\tau^{4/3}}$, and $R_2 = \frac{(\tau-1)^2(v-1)v(4\tau+(\tau(\tau+14)+1)(v-1)v)}{\tau^{4/3}(1-2v)^4}$. Both R_1 and R_2 are positive because $v = 3t+1 = 3\beta/(3+\epsilon^2)+1 > 1$. Since D_0 and D_1 can be shown to be positive, $W_1 \le x_0 \Leftrightarrow (D_0)^2 - (D_1)^2 \cdot R_1 \le 0$. $(D_0)^2 - (D_1)^2 \cdot R_1$ can be simplified into $D_2 \cdot \sqrt{R_2} - D_3$, where D_2 and D_3 can be shown to be positive. Hence, $W_1 \le x_0 \Leftrightarrow L \doteq (D_2)^2 \cdot R_2 - (D_3)^2 \le 0$.

  —Second, we show that L can be rearranged as $D_4 \cdot (\tau-1)^2\tau^6(1-2v)^{18}\left(\sqrt{3}\sqrt{\tau^2+27}-9\right)^2 \cdot D_5$, where D_4 is a positive constant. Hence, $L \le 0 \Leftrightarrow D_5 \le 0$. Next, we substitute the values of τ, u, v, and t into D_5, simplify, and obtain $D_5 = \frac{M_2(\beta)}{(3+\epsilon^2)^8(1+3\beta/(3+\epsilon^2))^2}$. Comparing coefficients, $M_2(\beta) = (3+\epsilon^2)^8 f\left(\frac{3\beta}{3+\epsilon^2}\right)$, where f is the polynomial defined in the proof of Proposition 2. Since $3+\epsilon^2 > 0$, M_2 and f share the same sign structure, and by the argument in the proof of Proposition 2, f has exactly one positive root t_1. Hence $M_2(\beta)$ has exactly one positive root, $t_4' \doteq \frac{(3+\epsilon^2)t_1}{3}$, and $\beta \le t_4' \Leftrightarrow D_5 \le 0 \Leftrightarrow L \le 0 \Leftrightarrow W_1 \le x_0$.

Combining the results from these two sub-steps, we conclude that $W_1 \le x_0 \Leftrightarrow 0 < \beta \le t_4'$. Moreover, since t_1 does not depend on ϵ, t_4' increases in ϵ.

- To ensure t_4' is the only threshold that defines whether $M_1(W_1)$ is positive or not, what remains is to check that $W_1 < x_1$ for all β. The comparison between W_1 and x_1 is similar to that between W_1 and x_0; the key difference is that $W_1 - x_1$ shares the same sign as a polynomial $M_4(\beta)$, which is always negative. Therefore, $W_1 \ge x_1$ cannot occur.

Combining the above results, we conclude that $F(\beta, \cdot) \le 0 \Leftrightarrow 0 < \beta \le t_4'$. This indicates that $\Gamma_C^* \le \Gamma_L^* \Leftrightarrow 0 < \beta \le t_4'$, which proves Proposition 3 with the threshold $\bar{t}_1(\mu, \epsilon)$ defined by t_4', which increases in ϵ. $\square$

Proof of Lemma 3. Define $\Lambda \doteq \lambda - \mu$. It can be shown that holding μ fixed gives $\partial\Gamma_j^{S*}/\partial\lambda = \partial\Gamma_j^{S*}/\partial\Lambda \ \forall j \in \{C, L\}$. Hence, it suffices to examine the sensitivity of the equilibrium income with respect to Λ.

We derive the equilibrium solution using backward induction, following the same procedure as in the proof of Proposition 1, with the linear planting cost term $-\mu y$ replaced by Λy. This yields $y_C^S(w) = \frac{\Lambda + w}{2\sigma}$. Accordingly, we have $w^{S*}(\Lambda) = \frac{\alpha\sigma - \Lambda(1+\alpha\sigma)}{1+2\alpha\sigma}$ when $\epsilon = 0$. By plugging in $w^{S*}(\Lambda)$ and $y_C^{S*}(\Lambda)$, we can obtain $\Gamma_C^{S*}(\Lambda) = \frac{\alpha^2\sigma(1+\Lambda)^2}{4(1+2\alpha\sigma)^2}$. Taking the derivative of $\Gamma_C^{S*}(\Lambda)$ with respect to Λ, we have $\frac{\partial\Gamma_C^{S*}(\Lambda)}{\partial\Lambda} = \frac{\alpha^2\sigma(1+\Lambda)}{2(1+2\alpha\sigma)^2} > 0$, thus $\frac{\partial\Gamma_C^{S*}(\lambda)}{\partial\lambda} = \frac{\partial\Gamma_C^{S*}(\Lambda)}{\partial\Lambda} > 0$.

To prove the second statement, we calculate that $y_C^L(\theta) = \frac{\alpha(1-\theta+\Lambda)}{2\alpha\sigma - 2\theta + 2}$, and accordingly, $\theta^{S*}(\Lambda) = \alpha\sigma - \frac{(\Lambda - \alpha\sigma)^2}{\sqrt[3]{3}K_5} + \frac{K_5}{3^{2/3}} + 1$ when $\epsilon = 0$, where $K_5 \doteq \sqrt[3]{(\Lambda - \alpha\sigma)^2\left(\sqrt{3}\sqrt{(\Lambda - \alpha\sigma)^2 + 27(\alpha\sigma+1)^2} - 9\alpha\sigma - 9\right)}$. Next, we can further calculate $\Gamma_L^{S*}(\Lambda) = -\alpha(1+\alpha\sigma)f_7(\tau)$, where $f_7(\tau) \doteq \frac{\left(3^{2/3}\tau^2 - \sqrt[3]{\tau^2\left(\sqrt{3}\sqrt{\tau^2+27}-9\right)}\left(\sqrt[3]{3}\sqrt[3]{\tau^2\left(\sqrt{3}\sqrt{\tau^2+27}-9\right)}+3\tau\right)\right)^2}{12\sqrt[3]{3}\tau^2\left(-\sqrt[3]{3}\sqrt[3]{\tau^2\left(\sqrt{3}\sqrt{\tau^2+27}-9\right)}+\sqrt{3}\sqrt{\tau^2+27}-9\right)}$, and $\tau \doteq \frac{\alpha\sigma - \Lambda}{\alpha\sigma + 1}$ (a function of Λ). To analyze the sign of $\frac{\partial\Gamma_L^{S*}(\Lambda)}{\partial\Lambda}$, we can analyze $\frac{\partial f_7(\tau)}{\partial\tau}$ instead. This is because

$$\frac{\partial\Gamma_L^{S*}(\Lambda)}{\partial\Lambda} > 0 \Leftrightarrow -\alpha(1+\alpha\sigma)\frac{\partial f_7(\tau(\Lambda))}{\partial\Lambda} > 0 \Leftrightarrow -\frac{\partial f_7(\tau)}{\partial\tau}\cdot\frac{\partial\tau(\Lambda)}{\partial\Lambda} = -\frac{\partial f_7(\tau)}{\partial\tau}\cdot(-\frac{1}{\alpha\sigma+1}) > 0 \Leftrightarrow \frac{\partial f_7(\tau)}{\partial\tau} > 0.$$

We can further calculate that $\frac{\partial f_7(\tau)}{\partial\tau} < 0 \Leftrightarrow \tau < \tau_0$, where τ_0 is a constant approximately equal 0.1546. Let us define $\beta \doteq \alpha\sigma$ and $\bar{\Lambda} \doteq \bar{\lambda} - \mu$, which is the upper bound on Λ resulting from Assumption 1. We then define t_3' by the only real solution of β to the function $\frac{\beta - \bar{\Lambda}}{\beta + 1} = \tau_0$. Since τ increases in β, for any $\beta < t_3'$, we have $\tau(\beta, \bar{\Lambda}) < \tau_0$. Since τ decreases in Λ, for any $\beta < t_3'$, there must exist a $\underline{\Lambda} < \bar{\Lambda}$ such that $\tau(\beta, \Lambda) < \tau_0$ holds for all $\Lambda > \underline{\Lambda}$. On the other hand, for any $\beta \ge t_3'$, $\tau(\beta, \bar{\Lambda}) \ge \tau(t_3', \bar{\Lambda}) = \tau_0$. Since τ decreases in Λ, we have $\tau(\beta, \Lambda) > \tau(\beta, \bar{\Lambda}) \ge \tau_0$, $\forall \Lambda < \bar{\Lambda}$ (which is equivalent to $\lambda < \bar{\lambda}$ as previously defined). Defining $\underline{\lambda} \doteq \underline{\Lambda} + \mu$ and $t_3 \doteq t_3'$ completes the proof. $\square$

Proof of Proposition 4. We first prove Part 2(a), and then prove Part 1 and Part 2(b) together. Let $\beta \doteq \alpha\sigma$.

Part 2(a): When $\lambda < \mu$, the subsidy does not fully offset the linear planting cost, so the farmer faces a net linear planting cost of $\tilde{\mu} \doteq \mu - \lambda$. Since $\lambda > 0$, $0 < \mu < 1$, and $\lambda < \mu$, $\tilde{\mu} \in (0, 1)$. Because Proposition 2 holds for any μ in $(0, 1)$, it applies here for $\mu = \tilde{\mu}$. The result follows directly with $t_5(\mu, \lambda) = t_1(\tilde{\mu}) \equiv t_1(\mu - \lambda)$. Furthermore, it can be verified that Proposition 2 extends continuously to $\mu = 0$, the boundary case $\lambda = \mu$ follows by setting $\tilde{\mu} = \mu - \lambda = 0$. Thus, $t_5(\mu, \mu) = t_1(0)$. In addition, since Proposition 2 shows that $t_1(\mu)$ is strictly increasing in μ, it follows that $t_5(\mu, \lambda) = t_1(\mu - \lambda)$ is strictly decreasing in λ. This also implies that $t_5(\mu, \lambda) < t_5(\mu, 0) = t_1(\mu)$. Since it can be shown that $\underline{t}(\mu, \lambda) = 0$ when $\lambda \le \mu$, we have $t_5(\mu, \lambda) \in (\underline{t}(\mu, \lambda), t_1(\mu))$.

Part 1 and Part 2(b): Similar to the proof of Lemma 3, we analyze the comparison through the transformed parameter $\Lambda \doteq \lambda - \mu$. Following the derivation in the proof of Proposition 2, we can derive that $\Gamma_C^{S*} > \Gamma_L^{S*}$ is equivalent to $g(\beta, \Lambda) > 0$, where $g(\beta, \Lambda) \doteq -\beta^2(2\beta(\beta+1)+1)\Lambda^4 + 2\beta(\beta(\beta(4\beta(3\beta+5)+19)+9)+2)\Lambda^3 - (\beta(\beta(3\beta(\beta(4\beta(7\beta+15)+79)+64)+97)+28)+4)\Lambda^2 + 2\beta(\beta(\beta(\beta(4\beta(\beta(8\beta+11)+14)+67)+53)+22)+4)\Lambda - \beta^2\left(\beta\left(-4(\beta(16\beta+27)+15)\beta^2+3\beta+14\right)+4\right) > 0$. (This polynomial is exactly $f(\beta, -\Lambda)$, where $f(\cdot)$ is defined in the proof of Proposition 2.) We therefore characterize the sign of $g(\beta, \Lambda)$ by analyzing its zeros in β for a given Λ. We conduct a three-step analysis:

Step 1: By Assumption 1, we focus on the case $\Lambda < \bar{\Lambda} \doteq \bar{\lambda} - \mu$. We first reparametrize $g(\beta;\Lambda)$ as follows: Since $\bar{\Lambda}$ depends on $\alpha \cdot \sigma \equiv \beta$, we shall consider $\bar{\Lambda}$ as a function of β. Since $\bar{\Lambda}(0^+) = 0$ and $\lim_{\beta\to\infty} \bar{\Lambda}(\beta) = 1$, the admissibility condition $0 < \Lambda < \bar{\Lambda}(\beta)$ implies that the relevant range of Λ is $(0,1)$, which can be reparameterized through $\Lambda(\nu) = 2\nu^2/(1+\nu)$. Since $\Lambda(0) = 0$, $\Lambda(1) = 1$, and $\Lambda'(\nu) = 2\nu(2+\nu)/(1+\nu)^2 > 0$ for $\nu \in (0,1)$, every $\Lambda \in (0,1)$ corresponds to a unique $\nu \in (0,1)$. Then it remains to reparameterize β: For any admissible $\beta > \underline{t}(\mu,\lambda)$ (see Proposition EC.4), there exists a unique $\omega > 0$ such that $\beta = \underline{t}(\mu,\lambda) + \omega$. Under the reparametrization $\Lambda = 2\nu^2/(1+\nu)$, the lower bound on β becomes $\underline{t}(\mu,\lambda) = 2\nu^2/(1-\nu^2)$. Therefore, each admissible β corresponds one-to-one to $\beta = \frac{2\nu^2}{1-\nu^2} + \omega,\ \omega > 0$. Therefore, the analysis of $g(\beta;\lambda)$ can be equivalently conducted through the transformed polynomial $T_1(\omega;\nu)$, where $\omega > 0$ is the new polynomial variable and $\nu \in (0,1)$ is the reparameterized coefficient variable.

Step 2: To analyze $T_1(\omega;\nu)$, note that it can be written as $T_2(\omega;\nu)/((1-\nu)^7(1+\nu)^8)$. Since the denominator is strictly positive for $\nu \in (0,1)$, $T_1(\omega;\nu)$ and $T_2(\omega;\nu)$ have the same sign and, in particular, the same zeros in ω. We can write $T_2(\omega;\nu)$ as $\sum_{k=0}^{7} c_k(\nu)\omega^k$. We calculate that $c_0(\nu), c_5(\nu), c_6(\nu)$, and $c_7(\nu)$ are strictly positive for all $\nu \in (0,1)$. The remaining coefficients $c_1(\nu), c_2(\nu), c_3(\nu)$, and $c_4(\nu)$ each change sign exactly once on $(0,1)$. Let ν_i denote the corresponding sign-change cutoff for $c_i(\nu)$, $i = 1,2,3,4$. Then $c_i(\nu) < 0$ for $\nu < \nu_i$ and $c_i(\nu) > 0$ for $\nu > \nu_i$, with $\nu_4 < \nu_1 < \nu_3 < \nu_2$. As ν increases and crosses the ordered cutoffs, the signs of c_4, c_1, c_3, c_2 change from negative to positive in that order. Together with $c_0, c_5, c_6, c_7 > 0$, this implies that the coefficient sequence $(c_0,\ldots,c_7)$ has exactly two sign changes for $\nu \in (0,\nu_2)$, and no sign change for $\nu \in [\nu_2, 1)$. Hence, by Descartes' rule of signs, $T_2(\omega;\nu)$ has either no positive zero or two positive zeros for $\nu \in (0,\nu_2)$, and no positive zero for $\nu \in [\nu_2,1)$.

To further characterize the number of positive zeros, we first identify the possible double zero of $T_2(\omega;\nu)$. Such a double zero must satisfy $T_2(\omega;\nu) = 0$ and $\partial T_2(\omega;\nu)/\partial\omega = 0$. Solving this system over $\omega > 0$ and $\nu \in (0,1)$ yields a unique admissible solution, denoted by (ω^*,ν^*). Hence, $T_2(\omega;\nu^*)$ has a double positive zero at $\omega = \omega^*$, and no other double positive zero exists. We next show that no positive zero exists for $\nu > \nu^*$. At $\nu = \nu^*$, the two positive zeros merge into one double zero. For $\nu \in [\nu_2, 1)$, we have shown that no positive zero exists by Descartes' rule. Now consider the intermediate region (ν^*, ν_2). If there were a positive zero for some ν in this region, then Descartes' rule would imply that there must be two positive zeros. However, these two zeros would have to disappear before reaching ν_2, where no positive zero exists. Since $T_2(0;\nu) > 0$ and the leading coefficient of T_2 is positive throughout $(0,1)$, the zeros cannot disappear through the boundary $\omega = 0$ or through infinity. Hence, they would have to merge into another double zero at some $\nu \in (\nu^*,\nu_2)$, contradicting the uniqueness of (ω^*,ν^*). Therefore, no positive zero exists for any $\nu > \nu^*$.

We then show that there are exactly two positive zeros for $\nu \in (0,\nu^*)$. It can be verified that $T_2(\omega^*;\nu) < 0$ for all $\nu \in (0,\nu^*)$. In addition, $T_2(0;\nu) = c_0(\nu) > 0$ and $T_2(\omega;\nu) \to \infty$ as $\omega \to \infty$ because the leading coefficient $c_7(\nu)$ is positive. Hence, by continuity, $T_2(\omega;\nu)$ must have one positive zero in $(0,\omega^*)$ and another positive zero in (ω^*,∞). Since Descartes' rule has already shown that $T_2(\omega;\nu)$ has at most two positive zeros for $\nu \in (0,\nu_2)$, it follows that $T_2(\omega;\nu)$ has exactly two positive zeros, which we shall denote by $\omega_4(\nu) < \omega_5(\nu)$ for every $\nu \in (0,\nu^*)$.

Step 3: Finally, we recover the corresponding thresholds in the original variables. Specifically, we obtain $t_4(\mu,\lambda)$ and $t_5(\mu,\lambda)$ from the two zeros ω_4 and ω_5, respectively, and $\underline{\lambda}'$ from the cutoff ν^* given μ.

The sensitivity properties of the thresholds with respect to λ: We first introduce the following lemma.

LEMMA EC.1. *When $\alpha\sigma \le t_1(\mu)$, there exists a threshold $\underline{\lambda}'' < \bar{\lambda}$ such that $\Gamma_L^{S*}(\lambda) \ge \Gamma_C^{S*}(\lambda)$ if and only if $\lambda \in (0, \underline{\lambda}'']$.*

The proofs of all lemmas and propositions in this e-companion are presented in the supplementary appendix (link). Based on this lemma, we prove that the threshold $t_5(\mu,\lambda)$ decreases in λ by contradiction: assume there exists $\lambda_1 < \lambda_2$ such that $t_5(\mu,\lambda_1) \le t_5(\mu,\lambda_2)$. Then by the definition of $t_5(\mu,\lambda)$ established previously in this proof, we know that when $\alpha\sigma = t_5(\mu,\lambda_2)$, $\Gamma_L^{S*}(\lambda_2) = \Gamma_C^{S*}(\lambda_2)$, whereas $\Gamma_L^{S*}(\lambda_1) \le \Gamma_C^{S*}(\lambda_1)$. However, the fact that $\Gamma_L^{S*}(\lambda_2) = \Gamma_C^{S*}(\lambda_2)$ implies that the threshold in Lemma EC.1 satisfies $\underline{\lambda}'' \ge \lambda_2$. Hence, according to Lemma EC.1, $\Gamma_L^{S*}(\lambda_1) > \Gamma_C^{S*}(\lambda_1)$ should also hold since $\lambda_1 < \lambda_2 \le \underline{\lambda}''$, which contradicts $\Gamma_L^{S*}(\lambda_1) \le \Gamma_C^{S*}(\lambda_1)$ derived previously. Therefore, $t_5(\mu,\lambda)$ must strictly decrease in λ. We use similar arguments to prove that $t_4(\mu,\lambda)$ strictly increases in λ. □

EC.2. Supplementary Results

EC.2.1. Supplementary Analytical Results

We first provide an analytical characterization of the phenomenon illustrated in Figure 1 in §5.2 under contract farming. The result is presented for the limiting case where $\mu \to 0$.

PROPOSITION EC.1. *In the contract farming model, assume $\alpha\sigma < t_2^C(\epsilon)$ and $\mu \to 0$. There exists a threshold $t_3(\epsilon) \leq t_2^C(\epsilon)$ $\forall \epsilon \in (0,1]$ such that if $\alpha\sigma < t_3(\epsilon)$, y_C^* and Γ_C^* strictly increase in ϵ.*

We further present two analytical results to support the discussion of the impact of MCN's promotion effort in §8. Before doing so, we introduce the detailed model setup of the two promotion extensions of the rural livestreaming model: let $r \geq 1$ denote the MCN's promotion effort, under which consumer valuation of the product increases to rv, and the MCN incurs a promotion cost $\delta(r-1)^2$ where $\delta > 0$. Accordingly, the MCN's profit function becomes $\Pi_L - \delta(r-1)^2$, where Π_L is defined as in Equation (4) with p replaced by $\frac{p}{r}$ to capture the increase in demand when consumer valuation becomes higher. In the process-based promotion model, the MCN maximizes its profit by jointly deciding r with the commission rate θ in the first stage. In the product-based promotion model, the MCN decides r while the farmers decide the retail price p simultaneously in the third stage. The rest of the model remains unchanged.

PROPOSITION EC.2. *Assume $\epsilon = 0$ and $\mu \to 0$. In the process-based promotion model, the MCN's promotion effort and the farmers' income in equilibrium decrease in the cost coefficient δ.*

Proposition EC.2 indicates that a lower promotion cost motivates the MCN to exert higher effort in process-based promotion and thereby improves the farmers' income. Interestingly, this intuitive result does not extend to product-based promotion: a smaller promotion cost may result in higher product-based promotion effort from the MCN yet lower income for the farmers. Although an analytical characterization of this phenomenon cannot be derived due to the intractability of the product-based promotion model, the following proposition enables insights into its causes.

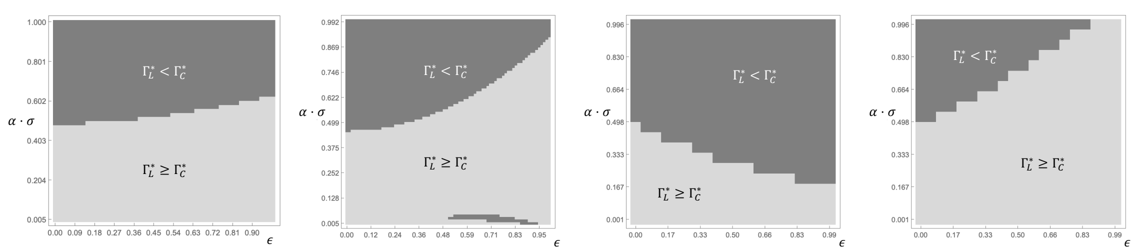
PROPOSITION EC.3. *Assume $\epsilon = 0$. In the product-based promotion model, the MCN's optimal promotion effort in the third stage increases in the farmers' planting quantity y.*

Proposition EC.3 shows the farmers can stimulate the MCN's product-based promotion effort by planting more.

EC.2.2. Supplementary Figures

All figures presented in this e-companion are plotted based on $\mu = 0.02$, except for Figure EC.1(b) where $\mu = 0.005$.

Alternative yield distributions: Comparing Figure EC.1 with Figure 2 in §5.2 illustrates the results in §5.3.



(a) Truncated normal (b) Two-point (c) Uniform on $[1-\epsilon, 1]$ (d) Uniform on $[1, 1+\epsilon]$

Figure EC.1 Income comparison between the two strategies under alternative yield distributions.

Comparison between the product-based promotion model and the base model: Figure EC.2 shows that the farmers' income may be lower in the product-based promotion model than in the base model. Figure EC.3 illustrates the comparative results between the process-based promotion model (superscript M) and product-based promotion model (superscript W) discussed in §8.

EC.2.3. Alternative Planting Cost Functions

We re-examine our comparative study under labor cost functions of the form $\sigma \cdot y^m$, where $m = 3, 4$. When the cost incurred is $\sigma \cdot y^3$, we first solve for farmers' optimal planting quantity function by backward induction as $\alpha \cdot G_3(\alpha^2\sigma, \theta)$, where $G_3(\alpha^2\sigma,\theta) \doteq \frac{\sqrt{(1-\theta)^2+3\alpha^2\sigma(1-\theta-\mu)}-(1-\theta)}{3\alpha^2\sigma}$. Next, we plug it into the MCN's profit function and obtain $\alpha\cdot\theta\cdot G_3(\alpha^2\sigma,\theta)\cdot(1-G_3(\alpha^2\sigma,\theta))$. As a result, the equilibrium commission rate is a function of $\alpha^2\sigma$, which we denote by $\theta_3(\alpha^2\sigma)$. Lastly, let us consider the farmers' income in equilibrium in terms of G_3 and θ_3, which equals $\alpha[(1-\theta_3)(1-G_3(\theta_3))G_3(\theta_3)-(G_3(\theta_3))^3\cdot\alpha^2\sigma]$. As for contract farming, we find that the farmers' income in equilibrium equals $\alpha\cdot\frac{2(w_3-\mu)^{3/2}}{3\sqrt{3}(\alpha^2\sigma)^{1/2}}$, where w_3 is the equilibrium wholesale price and is a function of $\alpha^2\sigma$. Hence, the comparison between the farmers' income is determined by functions of $\alpha^2\sigma$, and Proposition 2 continues to apply by replacing $\alpha\sigma$ in the condition by $\alpha^2\sigma$. Based on a similar approach, we can show that Proposition 2 also holds when the cost function is $\sigma\cdot y^4$; in that case, we need to replace $\alpha\sigma$ in the condition by $\alpha^3\sigma$.

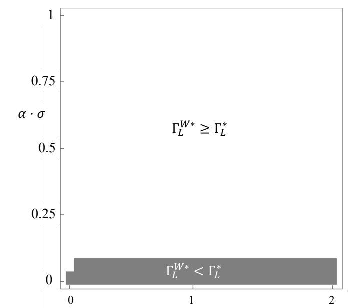


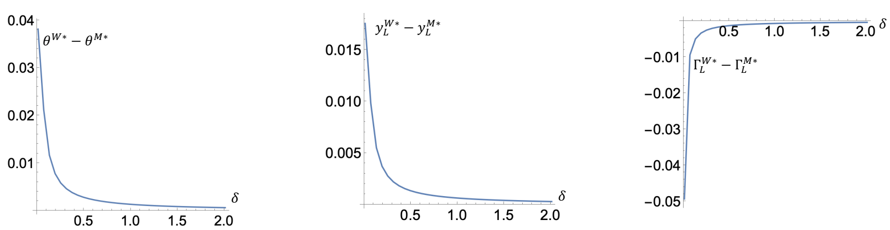
Figure EC.2: Impact of product-based promotion

EC.2.4. Characterization of $\bar{\lambda}$ in Assumption 1

Proposition EC.4. *Assume* $\epsilon = 0$.

1. *The upper bound* $\bar{\lambda} \doteq \min\{\lambda : w^{S*}(\lambda)=0 \text{ or } \theta^{S*}(\lambda)=1\} = \mu+\alpha\sigma-\frac{\alpha^2\sigma^2}{\sqrt{\alpha\sigma(2+\alpha\sigma)}}$.
2. *The condition* $\lambda < \bar{\lambda}$ *is equivalent to* $\alpha\sigma > \underline{t}(\mu,\lambda)$, *where* $\underline{t}(\mu,\lambda)=0$ *if* $\lambda\leq\mu$, *and* $\underline{t}(\mu,\lambda) = \frac{(\lambda-\mu)\left((4-\lambda+\mu)+\sqrt{(\lambda-\mu)(\lambda-\mu+8)}\right)}{4-4(\lambda-\mu)}$ *if* $\lambda>\mu$.

When $\epsilon>0$, $\bar{\lambda}$ cannot be characterized in closed form and is numerically determined in all of our experiments.

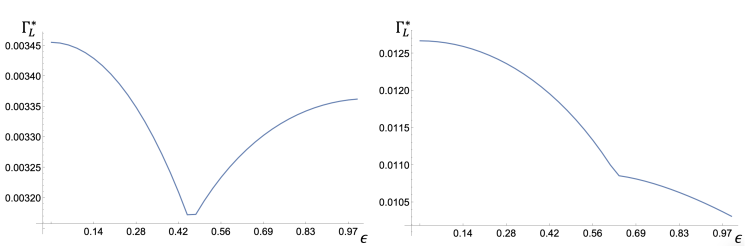


(a) $\theta^{W*}-\theta^{M*}$ (b) $y_L^{W*}-y_L^{M*}$ (c) $\Gamma_L^{W*}-\Gamma_L^{M*}$

Figure EC.3 Equilibrium comparisons between the two promotion models ($\alpha\sigma=0.6$, $\epsilon=0$).

EC.2.5. The Impact of Demand Uncertainty

We examine the impact of increased demand uncertainty by comparing the nominal contract farming model (without demand uncertainty) with a modified rural livestreaming model with demand uncertainty. To formulate the latter model, we assume that the MCN is able to reach a potential market of size $\alpha\cdot X$, where X is uniformly distributed in $[1-\tau,1+\tau]$ and $\tau\in[0,1]$. We further assume that the demand is realized in the retailing stage, and denote the realized demand by x. We modify the farmers' income function and the MCN's profit function, formulated in Equations (3)-(4) in the paper, by replacing α with $\alpha\cdot x$. To isolate the impact of demand uncertainty, we focus on the case with no yield uncertainty. Hence, the farmers' expected income and the MCN's expected profit are calculated based on the expectations over x, based on which the planting quantity and commission rate are determined. Due to analytical intractability, we analyze this extension based on a numerical study. We find that the impact of demand uncertainty is similar to that of yield uncertainty. When $\alpha\sigma$ is large, demand uncertainty reduces



(a) $\alpha\sigma=0.122$ (b) $\alpha\sigma=0.415$

Figure EC.4: Impact of demand uncertainty under rural livestreaming.

the farmers' income, paralleling Proposition 3. However, when $\alpha\sigma$ is small, demand uncertainty could improve the farmers' income, paralleling the observation made in §5.2. Figure EC.4 illustrates this set of findings. Nevertheless, the former impact is generally predominant, as is the case for yield uncertainty. Consequently, we observe that Proposition 2 continues to hold, yet increased demand uncertainty under rural livestreaming is likely to leave it a less preferable strategy for income generation for the farmers compared to contract farming.

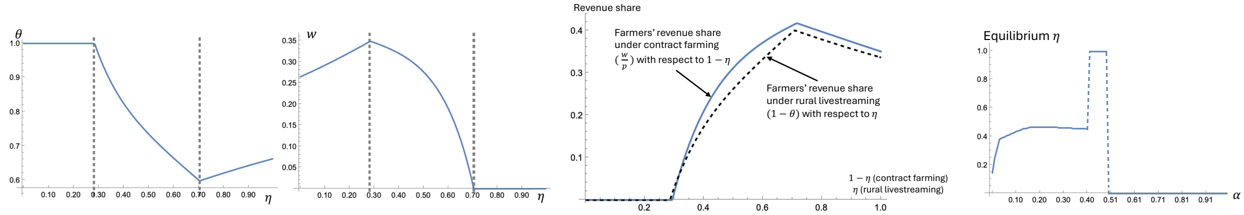
EC.3. Extensions

EC.3.1. Dual-Channel Market Access Strategy

In this extension, we examine the implications of introducing dual channels of market access for improving the farmers' income. We analyze a setting where the local government introduces both contract farming and rural livestreaming programs to the farmers. The government organizes planting among the farmers—as it is assumed in the main model—as well as facilitates the allocation of the yield among the two channels. This allocation decision can be made at different times, particularly, before and after the contracts are offered. We examine each scenario in this extension through an extensive numerical study, focusing on the case without yield uncertainty to isolate the effects.

EC.3.1.1. Pre-contract Allocation. This scenario occurs when intermediaries request information regarding the scope of participation in their corresponding programs before offering contracts. This is likely to be observed when the farmers lack power and the leverage to influence contracts. We model a sequence of events as follows: In the first stage, the farmers decide the proportion $\eta \in [0,1]$ of the planting quantity—and correspondingly, of the realized yield—to allocate to rural livestreaming. The remaining $1-\eta$ proportion will be supplied to the buyer under contract farming. A practical interpretation of this decision is that the government determines the proportion of farmers participating in each program, which, under the assumption of farmer homogeneity, also determines the share of the aggregate planting quantity allocated to each program. In the second stage, the buyer and the MCN simultaneously determine the wholesale price w and the commission rate θ applied to the yield from their allocated portion of the planting quantity, respectively. In the third stage, the farmers decide the total planting quantity y to maximize the sum of their expected income from the two channels. In the fourth stage, the buyer and the farmers decide the retail price for the portion of the yield to be sold through contract farming and rural livestreaming, respectively. For simplicity, we assume that the markets that the buyer and the MCN have access to do not overlap. Essentially, we examine how the farmers may benefit assuming the highest market access potential of dual channels.

We apply backward induction to solve the dual-channel model. One complexity is that because the farmers' optimal planting quantity depends on their income from both channels, it is a function of the wholesale price w and the commission rate θ. Plugging this optimal planting quantity into the buyer and the MCN's profit functions in the second stage leads to a non-cooperative game, where their profits depend on each other's decisions. We address this complexity by analyzing the Nash equilibrium of the (w,θ) outcome in the second stage. In case multiple equilibria exist, we focus on the equilibrium that leads to the lowest income for the farmers to provide a conservative estimate of how much the farmers can benefit without power to influence contracts. Additional analyses show that the insights are structurally robust if an alternative equilibrium (e.g., one with the highest income for the farmers) is chosen.



(a) Commission rate (b) Wholesale price (c) Farmers' revenue share comparison (d) Yield allocation

Figure EC.5 Equilibrium solution under pre-contract yield allocation

We first analyze how the intermediaries respond to the proportion of the yield allocated to them. Figure EC.5(a) and (b) illustrate the equilibrium commission rate and wholesale price with respect to η. The two figures exhibit similar patterns: The intermediaries first retain the entire sales revenue (via $\theta = 1$ or $w = 0$) when allocated a very low proportion of the yield. They then either reduce θ or increase w to share more revenue with the farmers as their yield proportions increase. However, when their yield proportion is sufficiently large, the intermediaries would reverse the course of action and share less revenue with the farmers. This equilibrium outcome illustrates the tradeoff between two strategic considerations that influence the intermediaries' decision. On the one hand, they have an incentive to share more to encourage planting. On the other hand, because the total planting quantity depends on both contracts, the intermediaries also have an incentive to free ride on each other's revenue-sharing efforts and share less themselves. It can be observed that the free riding incentive prevails for the intermediary that receives a small yield proportion. Meanwhile, free riding can cause the other intermediary to scale back its revenue share with the farmers in equilibrium, which explains the decrease in w or increase in θ when the buyer or the MCN receives a sufficiently large yield proportion. Nevertheless, when the yield proportion is intermediate, the incentive to encourage planting takes precedence, resulting in increased revenue shares from both intermediaries with the farmers.

Despite the similarities in the intermediaries' contract decisions, we observe that the buyer always shares more with the farmers than the MCN, given the same yield proportion. Figure EC.5(c) illustrates this finding by comparing the percentage of the retail price that the farmers obtain under contract farming (i.e., w/p) and under rural livestreaming (i.e., $1-\theta$) in equilibrium as functions of the yield proportion received by the buyer and the MCN, respectively. The root cause of this result is that the MCN tends to have more incentives to free ride, because the wholesale price contract is a more effective tool to motivate the farmers to plant (as demonstrated by Proposition 1 in the paper). Because of this, we find that the farmers tend to favor contract farming and allocate a larger proportion of the yield to the buyer in the majority of the cases (see Figure EC.5(d)). In the extreme case where $\alpha\sigma$ becomes sufficiently high, it may become optimal for the farmers to adopt only contract farming to eliminate free riding incentives between the intermediaries. The only exception is when $\alpha\sigma$ is intermediate; in that case, the farmers may adopt rural livestreaming only. Additional analyses show that such cases are more likely to arise as the unit input cost μ increases; when $\mu \to 0$, that region disappears. This is because a larger μ amplifies rural livestreaming's income advantage as demonstrated by Proposition 2. Nevertheless, the fact that the farmers may prefer to adopt one form of market access suggests that a single channel may perform better than dual channels, even assuming the highest market access potential of dual channels (i.e., no overlap between the consumer markets accessible by the single channels).

EC.3.1.2. Post-contract allocation. We analyze another scenario where the farmers can request contracts from the intermediaries and then decide how to allocate the yield among them. This scenario may arise when the farmers' power improves, because basing the yield allocation on contracts introduces competition between the intermediaries and enhances the farmers' market position. We change the sequence of events by letting the farmers decide the allocation η alongside the planting quantity y based on the (w, θ) decisions of the intermediaries. Equilibrium analysis shows that injecting competition for the yield is likely to increase the intermediaries' incentives to share revenue with the farmers and accordingly, improve the farmers' income. Furthermore, the competition effects outweigh the free riding effect previously discussed under pre-contract allocation. Consequently, it is optimal for the farmers to utilize both channels. Nevertheless, the impact of free riding persists, and the farmers may continue to prefer contract farming to rural livestreaming, especially as the market size increases. We refer readers to Figure EC.11 in the supplementary appendix (link) for an illustration of these results.

EC.3.2. Quantity Cap under Contract Farming

In this extension, we examine an alternative contract farming model where in the first stage, the buyer determines both a wholesale price w and a cap k, which represents the maximum amount of total yield it procures after harvest.

The key observation is that under the uniform yield distribution considered in the main model, it is optimal for the buyer not to utilize a quantity cap in the contract. This is because a quantity cap may only benefit the buyer when (i) undersupply predominates, as the cap reduces quantity mildly and can be effectively managed to adjust the farmers' planting quantity, or (ii) oversupply predominates, as it benefits the buyer to slash the planting quantity. Under a uniform distribution where the probability of undersupply equals oversupply, neither benefit manifests, and thus it is optimal to forgo the option to introduce a quantity cap. We note that this result shares a similar spirit with empirical evidence that contract farming implementations with additional provisions do not significantly outperform the plain-vanilla ones where only the wholesale price is involved (e.g., Arouna et al. 2021). Accordingly, we conclude that the option of incorporating a quantity cap under contract farming does not affect our results in the paper under the uniform distribution on $[1-\epsilon, 1+\epsilon]$. Nevertheless, under distributions where a quantity cap does appear in the optimal contract, introducing a quantity cap can harm the farmers' income, and consequently, rural livestreaming may become a more preferred approach for income generation for the farmers. However, extensive numerical analyses suggest that the comparison between rural livestreaming and contract farming remains structurally robust in such cases.

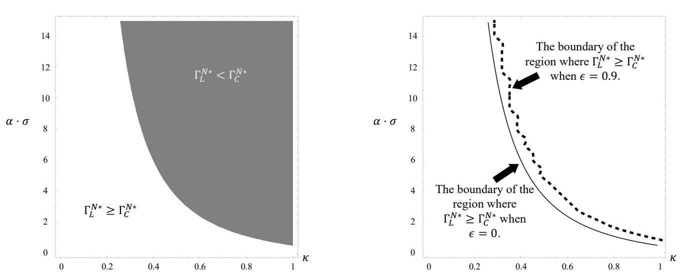
EC.3.3. Contract Negotiation

In this extension, we examine how our results may change if the farmers could influence the contracts through negotiation. We introduce a Nash bargaining model in the first stage where the contract decisions are negotiated.

$$\textit{Contract farming:}\quad \max_{w\geq 0} \left(\hat{\Pi}_C(w)\right)^{\kappa} \cdot \left(\hat{\Gamma}_C(w)\right)^{1-\kappa} \; s.t.\; \hat{\Pi}_C(w)\geq 0, \hat{\Gamma}_C(w)\geq 0. \tag{EC.7}$$

$$\textit{Rural livestreaming:}\quad \max_{0\leq\theta\leq 1} \left(\hat{\Pi}_L(\theta)\right)^{\kappa} \cdot \left(\hat{\Gamma}_L(\theta)\right)^{1-\kappa} \; s.t.\; \hat{\Pi}_L(\theta)\geq 0, \hat{\Gamma}_L(\theta)\geq 0. \tag{EC.8}$$

In (EC.7), which pertains to the contract farming model, the function $\hat{\Pi}_C(w)$ denotes the buyer's expected profit with its optimal retail price decision in the third stage of the game taken into account. Similarly, the function $\hat{\Gamma}_C(w)$ denotes the farmers' expected income under its optimal planting quantity decision. The functions $\hat{\Pi}_L$ and $\hat{\Gamma}_L$ in (EC.8) are defined in the same way based on the rural livestreaming model. The parameters κ (which is between 0 and 1) and $1-\kappa$ capture the bargaining power of the intermediaries and the farmers, respectively. We denote the farmers' expected income in equilibrium in this extension by Γ_C^{N*} and Γ_L^{N*} under contract farming and rural livestreaming, respectively.

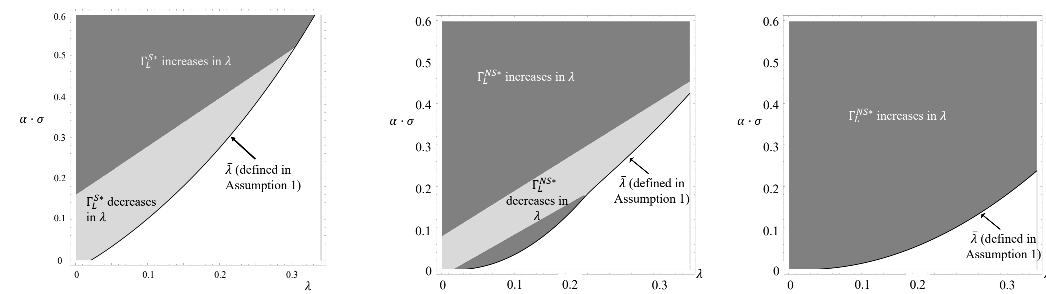


(a) $\epsilon = 0$ (b) $\epsilon = 0$ versus $\epsilon = 0.9$.

Figure EC.6: Farmers' income comparison under negotiation.

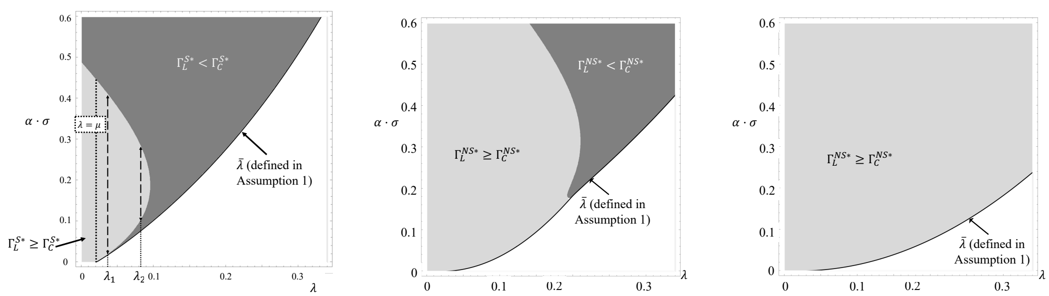
Extensive numerical analysis indicates that all else being equal, the parametric region where the farmers' income is higher under rural livestreaming than contract farming becomes larger as the farmers' bargaining power increases (see Figure EC.6(a)). To understand this observation, note that the farmers' income depends on the total revenue from the market sales of their products as well as their share of that revenue. When the farmers are oblivious to market sales, such as under contract farming, they may blindly negotiate for a higher share (via a higher wholesale price) without realizing that it may hurt the total revenue (as a higher wholesale price leads to a higher quantity which drives down the retail price). By contrast, farmers who are more informed about such market dynamics due to direct market access, such as under rural livestreaming, can better balance between the total revenue and their share in negotiation and therefore more effectively increase their income. This advantage is strengthened as the farmers' bargaining power increases. In other words, direct market access and enhanced power for the farmers are complementary in improving their income. We also find that such complementarity appears stronger when the farmers are subject to yield uncertainty, as illustrated by Figure EC.6(b).

We further analyze the impact of subsidies. We denote the farmers' expected income in equilibrium in this extension under subsidies by Γ_C^{NS*} and Γ_L^{NS*}. It can be observed that the region where Γ_L^{NS*} decreases in λ becomes smaller as κ decreases (see Figure EC.7). This suggests that increased bargaining power of the farmers can also mitigate or even rectify the negative joint impact of direct market access and subsidies as shown in §6. This is because that negative impact is derived from the MCN's strong incentive to increase the commission rate under subsidies. The farmers' ability to negotiate limits the MCN's power to do so. As such, rural livestreaming is more likely to lead to higher income for the farmers in the presence of subsidy (see Figure EC.8).



(a) $\kappa = 1$ (main model) (b) $\kappa = 0.85$ (c) $\kappa = 0.7$

Figure EC.7 Sensitivity of Γ_L^{NS*} with respect to λ assuming $\epsilon = 0$.



(a) $\kappa = 1$ (main model) (b) $\kappa = 0.85$ (c) $\kappa = 0.7$

Figure EC.8 Farmers' income comparison under subsidy in the negotiation model assuming $\epsilon = 0$.

EC.3.4. Farmer Competition and Heterogeneity

In this extension, we consider heterogeneous farmers who make their individual planting decisions. We use the superscript "H" to denote key notation of this model. We consider a unit mass of infinitesimally small farmers with varying planting capability u that is assumed to be uniformly distributed in $[1-h, 1+h]$. Farmers with capability u achieve a unit yield of $u \cdot Z$ from their crops. For a focused study, we ignore yield uncertainty in this extension.

Contract farming: Given the wholesale price w, the expected income of a farmer with planting capability u in the second stage can be formulated by $\Gamma_C^H(y|w,u) \doteq \mathbb{E}_Z[w \cdot (y \cdot Z \cdot u) - \mu \cdot y - \sigma \cdot y^2]$. Accordingly, their optimal planting quantity decision is $y_C^H(w,u) \doteq \arg\max_{y \geq 0} \Gamma_C^H(y|w,u)$. The total number of units planted by the farmers equals $Y_C(w) \doteq \int_{1-h}^{1+h} y_C^H(w,u) \cdot u \cdot \frac{1}{2h}\, du$. In the third stage, the buyer decides the retail price based on the total quantity, hence, its profit function can be formulated by replacing $y_C(w)$ with $Y_C(w)$ in Equation (2) in §3.1.

Rural livestreaming: Given the commission rate θ, each farmer obtains a $(1-\theta)$ share of the retail price for every unit of his products sold. We make an additional assumption that when the demand is smaller than supply in the retailing stage, that demand is proportionally distributed among the farmers based on the percentages of the total supply that they contribute, i.e., $\frac{y \cdot u \cdot z}{Y_L(\theta) \cdot z} = \frac{y \cdot u}{Y_L(\theta)}$, where $Y_L(\theta)$ denotes the total number of units planted by the farmers under rural livestreaming. Therefore, in the second stage, the expected income of a farmer with planting capability u equals $\Gamma_L^H(y|\theta, u, Y_L(\theta)) \doteq \mathbb{E}_Z\left[(1-\theta) \cdot p_L(\theta, Y_L(\theta), Z) \cdot \min\{y \cdot u \cdot Z, \frac{y \cdot u}{Y_L(\theta)} \cdot \alpha \cdot (1 - p_L(\theta, Y_L(\theta), Z))\}\right] - \mu \cdot y - \sigma \cdot y^2$.

Note that an individual farmer's income now depends on the total output. Following Tang et al. (2024), we adopt the rational expectations equilibrium (REE) framework. Specifically, we assume that the farmers have formed an expectation (for example, from historical data) about $Y_L(\theta)$ and each farmer would decide their planting quantity based on such an expectation. Hence, their optimal planting quantity can be formulated as $y_L^H(\theta, u, Y_L(\theta)) \doteq \arg\max_{y \geq 0} \Gamma_L^H(y|\theta, u, Y_L(\theta))$. An REE is achieved when $\int_{1-h}^{1+h} y_L^H(\theta, u, Y_L(\theta)) \cdot u \cdot \frac{1}{2h}\, du = Y_L(\theta)$. In the third stage, each farmer decides the retail price of their own yield. Because the demand is assumed to be proportioned among the farmers based on their yield, individual farmers' income functions in the third stage are proportional to each other, and thus their optimal prices are the same as the price that maximizes their total income given the total output, reflecting quantity competition. We also modify the MCN's profit function in Equation (4) of the paper by replacing y with $Y_L(\theta)$.

We derive two analytical results that illustrate the impact of farmer competition and heterogeneity.

Proposition EC.5. *In the farmer competition and heterogeneity extension, the planting quantity and expected income of each farmer in equilibrium are identical in the livestreaming and contract farming models.*

Proposition EC.5 shows that the equilibrium solutions under contract farming and rural livestreaming become identical in the extended models. This is because under quantity competition, the farmers become price takers and can no longer strategically manage retail price even with direct market access. However, this may not necessarily hurt the farmers because as previously shown, indirect market access can benefit the farmers due to lessened exploitation. This extension shows that letting the farmers compete under direct market access can have similar effects.

Proposition EC.6. *The total planting quantity in equilibrium under either strategy increases in h.*

Proposition EC.6 shows that a higher degree of heterogeneity in farmers' production capabilities increases the total output in equilibrium. This results in similar incentives for the buyer or the MCN as those explained in §4: the intermediaries have less/more incentives to share revenue with the farmers to motivate planting, which decreases/increases the farmers' overall income when the market size or planting cost is small/large. Overall, the insight is that increased heterogeneity among the farmers may improve or harm their total income under either strategy. Figure EC.9 illustrates this finding, where Γ_C^H and Γ_L^H denote the farmers' total income under the two strategies in the extended model.

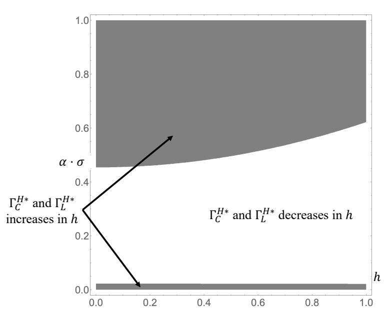


Figure EC.9: Income comparison.

Our results in this extension provide two practical insights. First, while competition between smallholder farmers that grow the same product is generally believed to reduce their income, we show that the opposite may occur. Hence, coordination of farmers' planting efforts that aims to improve their welfare may not always achieve the intended goals. Second, programs aiming to improve smallholder farmers' planting capabilities are widely observed. Some focus on introducing advanced technology—often to a small number of farmers to begin with—and may increase farmer heterogeneity, while others target farmers who lag behind in the local region to help them catch up and may reduce heterogeneity. Our analysis suggests that which type of focus is more beneficial to the farmers depends on the market and operating characteristics of their crops.

EC.4. Case Study

For a relevant study, we focus on Western China, which is a less developed area of the country and where smallholder farmers' income heavily depends on local specialty crops. In 2020, the Chinese government published a list of specialty crops from 52 underdeveloped areas, most of which are located in the west of the country (source link). Based on data availability and reliability, we chose eight specialty crops with different features from the list, and added one more (Longsheng Sticky Rice) to enrich our sample. Note that these specialty crops are often grown in a geographically extensive region. For a more focused study, for each specialty crop, we picked a representative village where the farmers reportedly cultivate that crop differently (and typically in ways that better preserve the natural appeal of the crops) from farmers in the rest of the region where the crop is grown. For example, Longsheng Sticky Rice is cultivated with traditional farming methods on terraced fields in Diling, Guangxi (source link). In other words, these are villages where premium versions of the specialty crops considered are grown, which provides a fitting context for our study. Figure EC.10 shows the geolocations of the villages. We collected the following data for each crop considered.

Figure EC.10: Geolocations of the sampled villages.

Unit yield: The unit yield is estimated to be 1.6 ton/mu for Yuexi Apple grown in Datun (source link), 0.65 ton/mu for Semaiti Apricot grown in Yuqarqi Tesek (source link), 2 ton/mu for Yutian Grape grown in Yingkulebaishi (source link), 0.4 ton/mu for Nayong Agate Cherry grown in Taoying (source link), 0.5 ton/mu for Pingbian Lychee grown in Liyuan,[9] 0.5 ton/mu for Piyaman Pomegranate grown in Langankule (source link), 0.5 ton/mu for Yuanyang Red Rice grown in Guoli (source link), 1 ton/mu for Bapan Pear grown in Shangping (source link), and 0.3 ton/mu for Longsheng Sticky Rice grown in Diling (source link).

Consumer valuation: We use the highest reported market price as a proxy for the maximum consumer valuation (i.e., willingness to pay) for each specialty crop. Most of the specialty crops are priced at twice the price of their commodity counterparts; for some, the market price reaches three to four times that of the commodity counterparts. Specifically, Yuexi Apple is priced at 15 $yuan/kg$ (source link) versus 5 $yuan/kg$ of regular apple (source link); Semaiti Apricot is priced at 30 $yuan/kg$ (source link) versus 15 $yuan/kg$ of regular apricot (source link); Yutian Grape is priced at 20 $yuan/kg$ (source link) versus 10 $yuan/kg$ of regular grape (source link); Nayong Agate Cherry is priced at 60 $yuan/kg$ versus 20 $yuan/kg$ of regular cherry (source link); Pingbian Lychee is priced at 60 $yuan/kg$ (source link) versus 30 $yuan/kg$ of regular lychee (source link); Piyaman Pomegranate is priced at 30 $yuan/kg$ (source link) versus 20 $yuan/kg$ of regular pomegranate (source link); Bapan Pear is priced at 20 $yuan/kg$ (source link) versus 5 $yuan/kg$ of regular pear (source link); Longsheng Sticky Rice and Yuanyang Red Rice are priced at 20 $yuan/kg$ (source link) and 25 $yuan/kg$ (source link), respectively, whereas regular rice is priced at 5 $yuan/kg$ (source link).

Planting cost: For each specialty crop, we estimate the unit input cost μ and the labor cost coefficient σ in two steps. First, we estimate the total unit planting cost as the average unit planting cost of the corresponding commodity product, multiplied by the price ratio between the specialty crop and its commodity counterpart. The rationale is that consumers' willingness to pay a premium for specialty crops reflects both the higher quality of inputs and the additional effort invested in the planting process (e.g., manual weeding), which together constitute the principal source of the value added relative to the commodity counterpart; the price ratio therefore serves as a proxy for the proportional increase in planting cost. Second, we allocate the total unit planting cost between the two cost components in a $1:2$ ratio, reflecting empirical evidence that labor accounts for roughly two-thirds of the unit planting cost for labor-intensive specialty crops in smallholder settings.[10] Accordingly, μ is set so that linear input expenditures equal one-third of the total unit planting cost, while σ is set so that the quadratic labor cost, evaluated at the current village-level planting quantity, equals the remaining two-thirds; that is, σ is obtained by dividing two-thirds of the total unit planting cost by the current village-level planting quantity. A similar approach is adopted in Alizamir et al. (2019) for evaluating the quadratic planting cost coefficient.

Implementing the above calibration procedure requires two sets of data for each specialty crop:

- **Average planting cost:** 1500 $yuan/mu$ for apple (source link), 3000 $yuan/mu$ for grapes (source link), 250 $yuan/mu$ for cherry (source link), 1000 $yuan/mu$ for lychee (source link), and 500 $yuan/mu$ for rice (source link).

[9] We take the average between two yield estimates (source link 1 and source link 2) to accommodate growth variation of lychee.

[10] Oduor B (2025). `https://eagmark.net/news/how-to-calculate-farm-cost-of-production.`

We use the planting cost for apple to approximate that for pear, and the planting cost for peach, which is about 500 *yuan/mu* (source link), to approximate that for pomegranate and apricot, given their similar planting requirements.

- **Current planting quantities:** 6,000 *mu* of Yuexi Apple grown in Datun (source link), 1,500 *mu* of Semaiti Apricot grown in Yuqarqi Tesek (source link), 1,800 *mu* of Yutian Grape grown in Yingkulebaishi (source link), 4,800 *mu* of Nayong Agate Cherry grown in Taoying (source link), 4,000 *mu* of Pingbian Lychee grown in Liyuan (source link), 4,250 *mu* of Piyaman Pomegranate grown in Langankule (source link), 750 *mu* of Yuanyang Red Rice grown in Guoli (source link), 450 *mu* of Bapan Pear grown in Shangping (source link), and 400 *mu* of Longsheng Sticky Rice grown in Diling (source link).

Market size: The detailed demand statistics for the sampled specialty crops are not readily available. Hence, we resort to the total output data of the specialty crops (i.e., the total amount grown not only in the village selected but also in other areas) as a proxy for their market sizes. Accordingly, we estimated that the market size is 300 *kilotonne* for Yuexi Apple (source link), 200 *kilotonne* for Semaiti Apricot (source link), 120 *kilotonne* for Yutian Grape (source link), 12 *kilotonne* for Nayong Agate Cherry (source link), 12 *kilotonne* for Pingbian Lychee (source link), 10.57 *kilotonne* for Piyaman Pomegranate (source link), 10 *kilotonne* for Yuanyang Red Rice (source link), 4.5 *kilotonne* for Bapan Pear (source link), and 1.8 *kilotonne* for Longsheng Sticky Rice (source link).

Calibration: To ensure the validity of our estimation approach, we crosschecked the equilibrium planting quantity calculated from our model based on the estimated numbers (see Columns 2 and 4 in Table EC.3 in the supplementary appendix (link)) against the reported planting quantities (see the "Current planting quantities" paragraph). Overall, the two sets of values are well aligned for five of the nine specialty crops. For the remaining four crops, the discrepancies observed for Yuexi Apple, Semaiti Apricot, and Yutian Grape are intuitive: these products correspond to specialty crops with the largest market sizes, for which the equilibrium quantity predicted by our model reflects the production scale that would be reached once supply has fully responded to demand. The reported quantities, in contrast, capture a snapshot of current production, which, for high-demand specialty crops, typically lags behind their long-run equilibrium. As for Nayong Agate Cherry, a substantial increase in the planting quantity has been reported since the list of specialty crops was published (source link).

Overview of existing market access programs: We surveyed the existing government-led market access programs for the sample products. Most of these programs are organized at the county level. Established programs rely on indirect approaches—we observe a clear presence of contract farming in a number of counties, such as Yengisar County (where Semaiti Apricot is cultivated, source link), Yutian County (source link), Yuexi County (source link), Pingbian County (source link), and Longsheng County (source link). More recent market expansion efforts are largely based on rural livestreaming initiatives, and we observe varying levels of adoption of livestreaming across the nine sample products. In some counties (e.g., Pishan County, where Piyaman Pomegranate is cultivated, and Yuanyang County), rural livestreaming initiatives have received strong government support and are rapidly becoming a dominant market access strategy for their specialty crops (source link 1 and source link 2). Some counties are experimenting with small-scale pilot projects of rural livestreaming. This is particularly the case for counties that have mature contract farming programs in place, including Yengisar County, Yutian County, Yuexi County, Pingbian County and Longsheng County (see source link 1, source link 2, source link 3, source link 4) for rural livestreaming activities in these counties). For Nayong County and Xihe County (where Bapan Pear is cultivated), we observe a rising adoption of rural livestreaming, alongside an expansion of contract farming programs (source link 1 and source link 2).

EC.4.1. Additional Results

We provide detailed results regarding the impact of yield uncertainty and subsidy to complement those presented in §7. First, we examine the impact of yield uncertainty based on the uniform distribution on $[1-\epsilon, 1+\epsilon]$. We find that the performance comparison remains unchanged for $\epsilon \in [0.1, 0.9]$, although the income difference increases (see Tables EC.3–EC.7 in the supplementary appendix (link)).

However, when factoring in the potential variation in planting costs, nuanced results can be observed for Pingbian Lychee. Specifically, when the labor cost increases by 90%, the farmers' income is higher under contract farming without yield uncertainty; however, with a yield uncertainty of 30%,[11] the result would reverse, i.e., rural livestreaming would outperform contract farming in improving the farmers' income (see Columns 3–5 of Table EC.1 for the changes). We also consider yield distributions that skew toward yield increase or decrease. For example, when the yield uncertainty follows a uniform distribution on $[0.1, 1]$, contract farming would outperform rural livestreaming in improving the income for farmers cultivating Pingbian Lychee (see Column 7 of Table EC.1).

Table EC.1 Performance comparison under different yield uncertainty (ϵ) and labor costs (σ) setups.

Product	Strategy with the higher income					
	$[1-\epsilon, 1+\epsilon]$				$[1-\epsilon, 1]$	
	$\epsilon = 0.3$	$\sigma \uparrow 90\%, \epsilon = 0$	$\sigma \uparrow 90\%, \epsilon = 0.2$	$\sigma \uparrow 90\%, \epsilon = 0.3$	$\epsilon = 0.3$	$\epsilon = 0.9$
Yuexi Apple	CF	CF	CF	CF	CF	CF
Semaiti Apricot	CF	CF	CF	CF	CF	CF
Yutian Grape	CF	CF	CF	CF	CF	CF
Nayong Agate Cherry	RL	RL	RL	RL	RL	RL
Pingbian Lychee	RL	CF	CF	RL	RL	CF
Piyaman Pomegranate	RL	RL	RL	RL	RL	RL
Yuanyang Red Rice	CF	CF	CF	CF	CF	CF
Bapan Pear	CF	CF	CF	CF	CF	CF
Longsheng Sticky Rice	CF	CF	CF	CF	CF	CF

Finally, we examine how government subsidies affect the performance of the two models. We find that under a regular input subsidy (e.g., 200 *yuan/mu*), the performance comparison between the two models remains unchanged unless the planting cost, particularly the labor cost component, increases by 90% or more relative to the baseline (see Column 3 of Table EC.2). However, when incorporating the GAP subsidy (e.g., 2,000 *yuan/mu*), contract farming may outperform rural livestreaming under a broader range of parametric settings, with the specific patterns varying across crops. For Piyaman Pomegranate, contract farming yields higher farmer income than rural livestreaming even at the nominal planting cost. For Pingbian Lychee, contract farming becomes the more favorable model once the labor cost rises by 30% or more. For Nayong Agate Cherry, the comparison is non-monotonic: under the nominal planting cost, contract farming outperforms rural livestreaming; when the labor cost increases by 20%, rural livestreaming yields higher farmer income than contract farming; once the labor cost increases by more than 90%, contract farming outperforms livestreaming again (see Columns 4-7 of Table EC.2). The detailed performance data related to Tables EC.1 and EC.2 (as well as Table 1 in §7 of the paper) are presented in the supplementary appendix (link).

Table EC.2 Performance comparison with subsidies under different planting costs (σ).

Product	Strategy with the higher income					
	200 *yuan/mu*		2,000 *yuan/mu*			
	σ	$\sigma \uparrow 90\%$	σ	$\sigma \uparrow 20\%$	$\sigma \uparrow 30\%$	$\sigma \uparrow 90\%$
Yuexi Apple	CF	CF	CF	CF	CF	CF
Semaiti Apricot	CF	CF	CF	CF	CF	CF
Yutian Grape	CF	CF	CF	CF	CF	CF
Nayong Agate Cherry	RL	RL	CF	RL	RL	CF
Pingbian Lychee	RL	CF	RL	RL	CF	CF
Piyaman Pomegranate	RL	RL	CF	CF	CF	CF
Yuanyang Red Rice	CF	CF	CF	CF	CF	CF
Bapan Pear	CF	CF	CF	CF	CF	CF
Longsheng Sticky Rice	CF	CF	CF	CF	CF	CF

[11] The yield uncertainty for lychee can be as high as 50% in practice (source link).