Online Appendix: Mergers of Consumer Cooperatives

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EC.1. Proofs

Proof of Proposition 1. *Unique Equilibrium Characterization.* Each cooperative c's action is its procurement budget $b_c \in \mathbb{R}_+$, which is constrained by the cooperative's total income $N_c I$. Thus, its action set is $[0, N_c I]$. Each cooperative c's objective function $U_c = \gamma + \frac{1}{1-\theta}[(\frac{1}{N_c}\frac{b_c}{b_c+\widehat{B}_{-c}}\widehat{O})^{1-\theta} - 1] + I - \frac{b_c}{N_c}$ is continuous in b_c. The necessary first-order optimality condition with respect to b_c is

$$(\frac{1}{N_c}\frac{b_c}{b_c+\widehat{B}_{-c}}\widehat{O})^{-\theta}\frac{\widehat{B}_{-c}}{(b_c+\widehat{B}_{-c})^2}\widehat{O} - 1 = 0, \tag{EC.1}$$

which is also sufficient since U_c is concave in b_c (given $\theta > 0$). Notice that as $b_c \to 0$, the left-hand side of (EC.1) converges to a strictly positive value and as $b_c \to N_c I$, the left-hand side converges to a strictly negative value for a sufficiently large I which we assume ($I > \overline{I}$). Hence, the optimal b_c admits an interior solution.

Symmetry of cooperative sizes and equilibrium procurement budgets. The detailed proof that each cooperative contains $1/C$ measure of consumers is provided in the proof of Theorem 1, steps 1 and 2, and is hence omitted here. Given the symmetry in cooperative sizes, we will prove that no asymmetric procurement budgets can exist in equilibrium. Suppose to the contrary that there exists an equilibrium with asymmetric procurement budgets. Then, there must be at least two cooperatives (1 and 2 without loss of generality) such that $\widehat{b}_1 > \widehat{b}_2$. Then, using (EC.1) and $N_c = 1/C$, we obtain $(\frac{C\widehat{b}_1}{\widehat{B}}\widehat{O})^{-\theta}\frac{\widehat{B}-\widehat{b}_1}{\widehat{B}^2}\widehat{O} - 1 < (\frac{C\widehat{b}_2}{\widehat{B}}\widehat{O})^{-\theta}\frac{\widehat{B}-\widehat{b}_2}{\widehat{B}^2}\widehat{O} - 1$, which implies that either $\widehat{b}_1$ or $\widehat{b}_2$ cannot satisfy (EC.1), and hence cannot be an equilibrium procurement budget; a contradiction.

Let $\widehat{b}$ denote each cooperative's equilibrium procurement budget. By evaluating (EC.1) we obtain

$$\widehat{B} = C\widehat{b} = \frac{C-1}{C}\widehat{O}^{1-\theta}. \tag{EC.2}$$

Moving backward to the first stage, the action of each supplier s is its production quantity $o_s \in \mathbb{R}_+$ to maximize its profit function $\pi_s = po_s - ao_s^r = \frac{o_s\widehat{B}}{o_s+\widehat{O}_{-s}} - ao_s^r = \frac{C-1}{C}o_s(o_s+\widehat{O}_{-s})^{-\theta} - ao_s^r$ which is continuous in o_s. Since $\theta > 1-r$, $\pi_s < 0$ as $o_s \to +\infty$. Thus, the action set of each supplier is restricted from above. The necessary first-order optimality condition with respect to o_s is

$$\frac{\partial \pi_s}{\partial o_s} = \frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta} - \frac{C-1}{C}\theta o_s(o_s+\widehat{O}_{-s})^{-\theta-1} - aro_s^{r-1} = 0. \tag{EC.3}$$

To verify that the second-order optimality condition is met, we examine the second derivative of the supplier's optimization problem as follows:

$$-\theta\frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta-2}\Big[2(o_s+\widehat{O}_{-s}) - (1+\theta)o_s\Big] - ar(r-1)o_s^{r-2}. \tag{EC.4}$$

We now claim that the above expression is strictly negative (implying that the sufficiency condition is met) for any candidate production quantity o_s that satisfies the necessary

condition in (EC.3) and generates a positive profit for supplier s. To see this, we can rewrite (EC.4) as follows: $-\theta\frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta-1}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}+1-\frac{o_s}{o_s+\widehat{O}_{-s}}\right]-ar(r-1)o_s^{r-2}\propto -\theta\frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta-1}o_s\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}+1-\frac{o_s}{o_s+\widehat{O}_{-s}}\right]-(r-1)\frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]\propto -\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}+1-\frac{o_s}{o_s+\widehat{O}_{-s}}\right]-(r-1)\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]=-\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]\left[r-1+\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\left[1-\frac{o_s}{o_s+\widehat{O}_{-s}}\right]<0$, where the second and third expressions are obtained by multiplying the first line by o_s and substituting aro_s^{r-1} from (EC.3), and the fourth term is obtained by dropping some common positive terms. To see why the last expression is strictly negative, it is enough to show that the first two brackets are positive. The first bracket must be positive to hold (EC.3) equal to zero, since $\frac{C-1}{C}(o_s+\widehat{O}_{-s})^{-\theta}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]-aro_s^{r-1}=0$. In addition, given that the supplier must earn a positive profit at any candidate o_s, we can write

$$\begin{aligned}
&\pi_s=\frac{C-1}{C}o_s(o_s+\widehat{O}_{-s})^{-\theta}-ao_s^r\geq 0\\
\iff\quad &\frac{C-1}{C}o_s(o_s+\widehat{O}_{-s})^{-\theta}-\frac{C-1}{Cr}o_s(o_s+\widehat{O}_{-s})^{-\theta}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]\geq 0\\
\iff\quad &r-1+\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\geq 0, \qquad \text{(EC.5)}
\end{aligned}$$

where the second line is obtained by substituting ao_s^r from (EC.3). Therefore, we prove our claim.

Next, we show that there is only one candidate production quantity o_s that satisfies the necessary condition in (EC.3) and generates a positive profit for supplier s. To see this, we can multiply (EC.3) by o_s^{1-r} and rearrange terms to obtain

$$\frac{C-1}{C}o_s^{1-r}(o_s+\widehat{O}_{-s})^{-\theta}\left[1-\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\right]=ar, \qquad \text{(EC.6)}$$

for any candidate production quantity that satisfies the necessary optimality condition. As shown above, the term in the brackets above cannot be negative for any candidate optimum, so we can restrict our attention to such production quantities. In this case, the term in the brackets strictly decreases with such o_s. In addition, $o_s^{1-r}(o_s+\widehat{O}_{-s})^{-\theta}$ also decreases in o_s if $r-1+\theta\frac{o_s}{o_s+\widehat{O}_{-s}}\geq 0$ which holds given that the supplier must earn a positive profit at any candidate o_s (as shown above). Therefore, the left-hand side of (EC.6) strictly decreases in o_s values of focus, whereas its right-hand side remains constant. As a result, (EC.3) admits a unique solution as long as the profit function is positive, and that solution is the global maximum.

Symmetry of equilibrium production quantities. Given the necessary condition above, we will prove that no asymmetric production quantities can exist in equilibrium. Suppose to the contrary that suppliers' equilibrium production quantities are asymmetric. Then, there must be at least two suppliers (1 and 2 without loss of generality) such that $\widehat{o}_1>\widehat{o}_2$. It is straightforward to see that the function $\frac{C-1}{C}\theta o\widehat{O}^{-\theta-1}+aro^{r-1}$ is strictly increasing in o for $r\geq 1$. To see why it is also strictly increasing in o for $r\in(0,1)$, note that $\frac{C-1}{C}\theta\widehat{O}^{-\theta-1}+ar(r-1)o^{r-2}\geq\frac{C-1}{C}\theta\widehat{O}^{-\theta-1}+\frac{C-1}{C}\widehat{O}^{-\theta}r(r-$

$1)o^{-1} = \frac{C-1}{C}\widehat{O}^{-\theta-1}\left[\theta - r(1-r)\frac{\widehat{O}}{o}\right] \geq \frac{C-1}{C}\widehat{O}^{-\theta-1}\theta(1-r) > 0$, where the first inequality results from $\pi_s \geq 0$, and the last inequality results from (EC.5). This implies that $\frac{C-1}{C}\widehat{O}^{-\theta} - \frac{C-1}{C}\theta\widehat{o}_1\widehat{O}^{-\theta-1} - ar\widehat{o}_1{}^{r-1} < \frac{C-1}{C}\widehat{O}^{-\theta} - \frac{C-1}{C}\theta\widehat{o}_2\widehat{O}^{-\theta-1} - ar\widehat{o}_2{}^{r-1}$. Thus, either $\widehat{o}_1$ or $\widehat{o}_2$ cannot satisfy (EC.3) which is a contradiction. Therefore, production quantities in equilibrium should be symmetric.

Let $\widehat{o}$ denote each supplier's equilibrium production quantity. By evaluating (EC.3) we obtain (7). We can verify that in equilibrium, each supplier earns a strictly positive profit given that $S(r-1)+\theta > 0$. Finally, the price can be obtained as $\widehat{p} = \frac{\widehat{B}}{\widehat{O}}$. This completes the proof. ■

Proof of Proposition 2. (i) The necessary (and sufficient) first-order optimality condition of the merged cooperative utility-maximization problem given in (8) is

$$(\frac{C}{2}\frac{b_{c_{12}}}{b_{c_{12}}+\widehat{B}_{-c_{12}}}\widehat{O}_m)^{-\theta}\frac{\widehat{B}_{-c_{12}}}{(b_{c_{12}}+\widehat{B}_{-c_{12}})^2}\widehat{O}_m - 1 = 0. \tag{EC.7}$$

As in the proof of Proposition 1, we can show that the optimal $b_{c_{12}}$ admits an interior solution. Each non-merged cooperative $c \in \{3,...,C\}$ also best-responds by solving

$$(C\frac{b_{c_m}}{b_{c_m}+\widehat{B}_{-c_m}}\widehat{O}_m)^{-\theta}\frac{\widehat{B}_{-c_m}}{(b_{c_m}+\widehat{B}_{-c_m})^2}\widehat{O}_m - 1 = 0, \tag{EC.8}$$

where subscript m represents post-merger outcomes. Using the exact same argument as in the proof of Proposition 1, one can verify that in any equilibrium, the procurement budgets of non-merged cooperatives are symmetric. Let $\widehat{b}_m$ denote each non-merged cooperative's equilibrium procurement budget. Considering (EC.7) and (EC.8) yields the following simultaneous equations for the equilibrium procurement budgets $\widehat{b}_{c_{12}}$ and $\widehat{b}_m$:

$$(\frac{C}{2}\frac{\widehat{b}_{c_{12}}}{\widehat{B}_m}\widehat{O}_m)^{-\theta}\frac{\widehat{B}_m-\widehat{b}_{c_{12}}}{\widehat{B}_m^2}\widehat{O}_m - 1 = 0, \tag{EC.9}$$

$$(C\frac{\widehat{b}_m}{\widehat{B}_m}\widehat{O}_m)^{-\theta}\frac{\widehat{B}_m-\widehat{b}_m}{\widehat{B}_m^2}\widehat{O}_m - 1 = 0. \tag{EC.10}$$

From (EC.9) and (EC.10), we obtain $\frac{\widehat{B}_m-\widehat{b}_{c_{12}}}{\widehat{B}_m-\widehat{b}_m} = \frac{(C-2)\widehat{b}_m}{\widehat{b}_{c_{12}}+(C-3)\widehat{b}_m} = (\frac{\widehat{b}_{c_{12}}}{2\widehat{b}_m})^{\theta}$. Let $\eta = \frac{\widehat{b}_m}{\widehat{b}_{c_{12}}}$. Simplifying the above expression, η solves

$$1+(C-3)\eta-(C-2)\eta(2\eta)^{\theta} = 0. \tag{EC.11}$$

The derivative of the left-hand side of (EC.11) with respect to η is $(C-3)-(C-2)(1+\theta)(2\eta)^{\theta} = (C-3)-(C-2)(1+\theta)\frac{1+(C-3)\eta}{(C-2)\eta} = -\theta(C-3)-\frac{1+\theta}{\eta} < 0$. Observe that if $\eta = \frac{1}{2}$, the left-hand side of (EC.11) becomes $\frac{1}{2} > 0$ and if $\eta = 1$, the left-hand side of (EC.11) becomes $(C-2)(1-2^{\theta}) < 0$. Hence, $\eta \in (\frac{1}{2},1)$ uniquely solves (EC.11) for each C. Following this observation, we can write

$$\frac{\partial\eta}{\partial C} = -\frac{\eta[1-(2\eta)^{\theta}]}{(C-3)-(C-2)(1+\theta)(2\eta)^{\theta}} = \frac{\eta(\eta-1)}{(C-2)[\theta(C-3)\eta+1+\theta]} < 0, \tag{EC.12}$$

by the implicit function theorem and after substituting for $(2\eta)^{\theta}$ from (EC.11). Letting $C=3$ and $C\to\infty$ in (EC.11), we find that $\frac{1}{2} < \eta \leq (\frac{1}{2})^{\frac{\theta}{1+\theta}}$. In addition, $\widehat{B}_m = \widehat{b}_{c_{12}} + (C-2)\widehat{b}_m = [1+(C-$

$2)\eta]\widehat{b}_{c_{12}} \Rightarrow \frac{\widehat{b}_{c_{12}}}{\widehat{B}_m} = \frac{1}{1+(C-2)\eta}$ and $\frac{\widehat{b}_m}{\widehat{B}_m} = \frac{\eta}{1+(C-2)\eta}$. Using (EC.9) and (EC.10), we obtain

$$\widehat{B}_m = (\frac{C}{2}\frac{1}{1+(C-2)\eta})^{-\theta}\frac{(C-2)\eta}{1+(C-2)\eta}\widehat{O}_m^{1-\theta} = (\frac{C\eta}{1+(C-2)\eta})^{-\theta}\frac{1+(C-3)\eta}{1+(C-2)\eta}\widehat{O}_m^{1-\theta}. \tag{EC.13}$$

Anticipating this behavior by cooperatives, the action of each supplier s in the first stage is its production quantity $o_s \in \mathbb{R}_+$ to maximize its profit function $\pi_{s_m} = p_m o_{s_m} - a o_{s_m}^r = \frac{o_{s_m}\widehat{B}_m}{o_{s_m}+\widehat{O}_{-s_m}} - a o_{s_m}^r = (\frac{C}{2}\frac{1}{1+(C-2)\eta})^{-\theta}\frac{(C-2)\eta}{1+(C-2)\eta}o_{s_m}(o_{s_m}+\widehat{O}_{-s_m})^{-\theta} - a o_{s_m}^r$ which is continuous in o_{s_m}. The necessary first-order optimality condition with respect to o_{s_m} is $\frac{\partial \pi_{s_m}}{\partial o_{s_m}} = (\frac{C}{2}\frac{1}{1+(C-2)\eta})^{-\theta}\frac{(C-2)\eta}{1+(C-2)\eta}[(o_{s_m}+\widehat{O}_{-s_m})^{-\theta} - \theta o_{s_m}(o_{s_m}+\widehat{O}_{-s_m})^{-\theta-1}] - aro_{s_m}^{r-1} = 0$. Using the same arguments as in the proof of Proposition 1, we can verify that production quantities are symmetric in any equilibrium and that the above condition is sufficient for optimality. Thus, the equilibrium production quantity is given by

$$\begin{aligned}\widehat{o}_m &= [(\frac{C}{2}\frac{1}{1+(C-2)\eta})^{-\theta}\frac{(C-2)\eta}{1+(C-2)\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}} \\ &= [(\frac{C\eta}{1+(C-2)\eta})^{-\theta}\frac{1+(C-3)\eta}{1+(C-2)\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}.\end{aligned} \tag{EC.14}$$

Finally, the price can be obtained as $\widehat{p}_m = \frac{\widehat{B}_m}{\widehat{O}_m}$.

(ii) To compare the utility that a consumer receives in the merged cooperative with the utility in a non-merged cooperative, we need to evaluate the following expression $\widehat{U}_{c_{12}} - \widehat{U}_{c_m} = \frac{1}{1-\theta}(\frac{C}{2}\frac{\widehat{b}_{c_{12}}\widehat{O}_m}{\widehat{B}_m})^{1-\theta} - \frac{C}{2}\widehat{b}_{c_{12}} - \frac{1}{1-\theta}(C\frac{\widehat{b}_{c_m}\widehat{O}_m}{\widehat{B}_m})^{1-\theta} + C\widehat{b}_{c_m} = (\frac{C}{2}\frac{1}{1+(C-2)\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}] - (\frac{C\eta}{1+(C-2)\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1+(C-3)\eta}{1+(C-2)\eta}] = (\frac{C}{2}\frac{1}{1+(C-2)\eta})(\frac{C\eta}{1+(C-2)\eta})^{-\theta}\frac{1+(C-3)\eta}{(C-2)\eta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}] - (\frac{C\eta}{1+(C-2)\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1+(C-3)\eta}{1+(C-2)\eta}] \propto (\frac{C}{2}\frac{1}{1+(C-2)\eta})\frac{1+(C-3)\eta}{(C-2)\eta}[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}] - (\frac{C\eta}{1+(C-2)\eta})[\frac{1}{1-\theta} - \frac{1+(C-3)\eta}{1+(C-2)\eta}] < 0$, where the third equality results from (EC.13). To see the inequality, it is enough to prove that $\frac{1+(C-3)\eta}{(C-2)\eta}[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}] < 2\eta[\frac{1}{1-\theta} - \frac{1+(C-3)\eta}{1+(C-2)\eta}] \iff (2\eta)^\theta[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}] < 2\eta[\frac{1}{1-\theta} - \frac{(C-2)\eta}{1+(C-2)\eta}(2\eta)^\theta] \iff \frac{(C-2)\eta}{1+(C-2)\eta}(2\eta-1) < \frac{1}{1-\theta}[(2\eta)^{1-\theta}-1]$, where the second term results from the expression for η in (EC.11) and the third term results from rearranging terms and dropping the term $(2\eta)^\theta$ from both sides. To see why the third inequality holds, consider the function $G(C) = \frac{(C-2)\eta}{1+(C-2)\eta}(2\eta-1) - \frac{1}{1-\theta}[(2\eta)^{1-\theta}-1]$, and note that we want to show that $G(C) < 0$ for all $C \geq 3$. To see this, we take the total derivative of $G(C)$ with respect to C: $\frac{dG(C)}{dC} = \frac{\partial G}{\partial C} + \frac{\partial G}{\partial \eta} \times \frac{\partial \eta}{\partial C} = \frac{\eta(2\eta-1)}{[1+(C-2)\eta]^2} + [\frac{2\eta-1}{[1+(C-2)\eta]^2} + \frac{2\eta}{1+(C-2)\eta} - \frac{2\eta}{1+(C-3)\eta}]\frac{\eta(\eta-1)}{\theta(C-3)\eta+1+\theta} = \frac{\eta(2\eta-1)}{[1+(C-2)\eta]^2}[\frac{\theta(C-3)\eta+\eta+\theta}{\theta(C-3)\eta+1+\theta}] + [\frac{2\eta}{1+(C-2)\eta} - \frac{2\eta}{1+(C-3)\eta}]\frac{\eta(\eta-1)}{\theta(C-3)\eta+1+\theta} > 0$ using (EC.11) and (EC.12). As a result, $G(C)$ attains its maximum when $C \to \infty$ which in that case $\eta \to \frac{1}{2}$ by (EC.11). Observe that $\lim_{C\to\infty} G(C) = 0$ and hence $G(C) < 0$ for all $C \geq 3$. This means $\widehat{U}_{c_{12}} < \widehat{U}_{c_m}$ for all $C \geq 3$. ■

Proof of Theorem 1. In two steps, we shall prove a stronger result that shows both Theorem 1 and the symmetry of cooperative sizes in Proposition 1.

Step 1. We prove that there is no (pre-merger or post-merger) equilibrium in which $m \geq 1$ cooperatives have size $N' > \frac{1}{\widetilde{C}}$ and the remaining $(\widetilde{C}-m)$ cooperatives have size $\frac{1-mN'}{\widetilde{C}-m} < \frac{1}{\widetilde{C}}$.

Suppose to the contrary that such an equilibrium exists. We already show in the proof of Proposition 1 that cooperatives of the same size spend symmetric procurement budgets in equilibrium.

Let $\widehat{b}'$ and $\widehat{b}$ be the symmetric procurement budgets of cooperatives with size N' and $\frac{1-mN'}{\widetilde{C}-m}$, respectively. Then, similar to (EC.9) and (EC.10), we can write

$$(\frac{\widehat{b}'}{N'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}'}{\widehat{B}_m^2}\widehat{O}_m-1=0, \tag{EC.15}$$

$$(\frac{\widehat{b}(\widetilde{C}-m)}{1-mN'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}}{\widehat{B}_m^2}\widehat{O}_m-1=0. \tag{EC.16}$$

From (EC.15) and (EC.16), we obtain $\frac{\widehat{B}_m-\widehat{b}'}{\widehat{B}_m-\widehat{b}}=\frac{(m-1)\widehat{b}'+(\widetilde{C}-m)\widehat{b}}{m\widehat{b}'+(\widetilde{C}-1-m)\widehat{b}}=(\frac{\widehat{b}'(1-mN')}{N'(\widetilde{C}-m)\widehat{b}})^{\theta}$. Let $\eta'=\frac{\widehat{b}}{\widehat{b}'}$. Simplifying the above expression, η' solves

$$\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-1-m)\eta'}=(\frac{1-mN'}{N'(\widetilde{C}-m)\eta'})^{\theta}. \tag{EC.17}$$

We can rearrange the above expression and rewrite it as

$$\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-1-m)\eta'}[N'(\widetilde{C}-m)\eta']^{\theta}-(1-mN')^{\theta}=0.$$

The derivative of the left-hand side of the above expression with respect to η' is

$$\frac{\widetilde{C}-1}{[m+(\widetilde{C}-1-m)\eta']^2}[N'(\widetilde{C}-m)\eta']^{\theta}+\theta N'(\widetilde{C}-m)[N'(\widetilde{C}-m)\eta']^{\theta-1}\frac{m-1+\theta(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-1-m)\eta'}>0.$$

Observe that if $\eta'\to 0$, the left-hand side converges to $-(1-mN')^{\theta}<0$ and if $\eta'\to 1$, the left-hand side converges to $[N'(\widetilde{C}-m)]^{\theta}-(1-mN')^{\theta}>0$ for $N'>\frac{1}{\widetilde{C}}$. Hence, $\eta'\in(0,1)$ uniquely solves (EC.17) for each $\widetilde{C},m$ and N'. Following this observation, we can write

$$\frac{\partial\eta'}{\partial\widetilde{C}}=-\frac{\frac{\eta'(1-\eta')}{[m+(\widetilde{C}-1-m)\eta']^2}+\theta(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{-\theta-1}\frac{N'\eta'}{1-mN'}}{\frac{\widetilde{C}-1}{[m+(\widetilde{C}-1-m)\eta']^2}+\theta(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{-\theta-1}\frac{N'(\widetilde{C}-m)}{1-mN'}}<0, \tag{EC.18}$$

by the implicit function theorem. In addition, $\widehat{B}_m=m\widehat{b}'+(\widetilde{C}-m)\widehat{b}\Rightarrow\frac{\widehat{b}'}{\widehat{B}_m}=\frac{1}{m+(\widetilde{C}-m)\eta'}$ and $\frac{\widehat{b}}{\widehat{B}_m}=\frac{\eta'}{m+(\widetilde{C}-m)\eta'}$. Using (EC.15) and (EC.16), we obtain

$$\begin{aligned}\widehat{B}_m=&(\frac{1}{N'}\frac{1}{m+(\widetilde{C}-m)\eta'})^{-\theta}\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}\widehat{O}_m^{1-\theta}\\=&(\frac{\widetilde{C}-m}{1-mN'}\frac{\eta'}{m+(\widetilde{C}-m)\eta'})^{-\theta}\frac{m+(\widetilde{C}-1-m)\eta'}{m+(\widetilde{C}-m)\eta'}\widehat{O}_m^{1-\theta}.\end{aligned} \tag{EC.19}$$

We now compare the utility that a consumer receives in these two types of cooperatives to show that it cannot form an equilibrium. Let $\widehat{U}'$ and $\widehat{U}$ represent the utilities that a consumer receives in cooperatives with size N' and $\frac{1-mN'}{C-1-m}$, respectively. Then, we have $\widehat{U}'-\widehat{U}=\frac{1}{1-\theta}(\frac{1}{N'}\frac{1}{m+(\widetilde{C}-m)\eta'})^{1-\theta}\widehat{O}_m^{1-\theta}-\frac{1}{N'}\frac{1}{m+(\widetilde{C}-m)\eta'}\widehat{B}_m-\frac{1}{1-\theta}(\frac{\widetilde{C}-m}{1-mN'}\frac{\eta'}{m+(\widetilde{C}-m)\eta'})^{1-\theta}\widehat{O}_m^{1-\theta}+\frac{\widetilde{C}-m}{1-mN'}\frac{\eta'}{m+(\widetilde{C}-m)\eta'}\widehat{B}_m=(\frac{1}{N'}\frac{1}{m+(\widetilde{C}-m)\eta'})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}]-(\frac{\widetilde{C}-m}{1-mN'}\frac{\eta'}{m+(\widetilde{C}-m)\eta'})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{m+(\widetilde{C}-1-m)\eta'}{m+(\widetilde{C}-m)\eta'}]\propto(\frac{1}{N'}\frac{1}{m+(\widetilde{C}-m)\eta'})^{\frac{m+(\widetilde{C}-1-m)\eta'}{m-1+(\widetilde{C}-m)\eta'}}[\frac{1}{1-\theta}-\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}]-(\frac{\widetilde{C}-m}{1-mN'}\frac{\eta'}{m+(\widetilde{C}-m)\eta'})[\frac{1}{1-\theta}-\frac{m+(\widetilde{C}-1-m)\eta'}{m+(\widetilde{C}-m)\eta'}]<0$, where the left-hand side of the inequality is obtained by using (EC.19) and after factoring out some positive common terms. To see the inequality, it is enough to prove that $\frac{m+(\widetilde{C}-1-m)\eta'}{m-1+(\widetilde{C}-m)\eta'}[\frac{1}{1-\theta}-\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}]<\frac{N'(\widetilde{C}-m)\eta'}{1-mN'}[\frac{1}{1-\theta}-\frac{m+(\widetilde{C}-1-m)\eta'}{m+(\widetilde{C}-m)\eta'}]\iff(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{\theta}[\frac{1}{1-\theta}-\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}]<\frac{N'(\widetilde{C}-m)\eta'}{1-mN'}[\frac{1}{1-\theta}-$

$\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{\theta}] \iff \frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}[\frac{N'(\widetilde{C}-m)\eta'}{1-mN'}-1] < \frac{1}{1-\theta}[(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{1-\theta}-1]$, where the second inequality results from the expression for η' in (EC.17) and the third inequality results from rearranging terms and dropping the term $(\frac{N'(\widetilde{C}-m)\eta'}{1-mN'})^{\theta}$ from both sides. To see why the third inequality holds, consider the function $H(\zeta)=\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}(\zeta-1)-\frac{1}{1-\theta}(\zeta^{1-\theta}-1)$, where $\zeta \equiv \frac{N'(\widetilde{C}-m)\eta'}{1-mN'}$ and note that we want to show that $H(\zeta)<0$. To see this, we take the partial derivative of $H(\zeta)$ with respect to ζ as follows $\frac{\partial H(\zeta)}{\partial \zeta}=\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-m)\eta'}-\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-1-m)\eta'}<0$, where we use the fact that $\zeta^{-\theta}=\frac{m-1+(\widetilde{C}-m)\eta'}{m+(\widetilde{C}-1-m)\eta'}$ by (EC.17). Since $\zeta>1$, we have $H(\zeta)<H(1)=0$. This means $\widehat{U}'<\widehat{U}$ and hence we cannot have an equilibrium with two cooperative sizes.

Step 2. We prove that there is no (pre-merger or post-merger) equilibrium in which $m \geq 3$ cooperatives have asymmetric sizes.

Suppose to the contrary that such an equilibrium exists, that is there are at least three different equilibrium sizes in the post-merger equilibrium. Let $N_1'>N_2'>N_3'$ be the cooperatives sizes and $\widehat{b}_1$, $\widehat{b}_2$, and $\widehat{b}_3$ be the corresponding equilibrium procurement budgets for any of these cooperatives. Then, similar to (EC.15) and (EC.16) we can write

$$(\frac{\widehat{b}_1}{N_1'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}(\widehat{B}_m-\widehat{b}_1)=\frac{\widehat{B}_m^2}{\widehat{O}_m}, \tag{EC.20}$$

$$(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}(\widehat{B}_m-\widehat{b}_2)=\frac{\widehat{B}_m^2}{\widehat{O}_m}, \tag{EC.21}$$

$$(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}(\widehat{B}_m-\widehat{b}_3)=\frac{\widehat{B}_m^2}{\widehat{O}_m}. \tag{EC.22}$$

Since the left-hand sides of (EC.20), (EC.21), and (EC.22) are equal and $N_1'>N_2'>N_3'$, we must have $\widehat{b}_1>\widehat{b}_2>\widehat{b}_3$. Also, since the left-hand sides of (EC.20), (EC.21), and (EC.22) are equal and $\widehat{B}_m-\widehat{b}_1<\widehat{B}_m-\widehat{b}_2<\widehat{B}_m-\widehat{b}_3$, we conclude that $\frac{\widehat{b}_1}{N_1'}<\frac{\widehat{b}_2}{N_2'}<\frac{\widehat{b}_3}{N_3'}$. Next, in equilibrium, we must have $\widehat{U}_1=\widehat{U}_2=\widehat{U}_3$ and hence

$$\begin{aligned}
&\frac{1}{1-\theta}(\frac{\widehat{b}_1}{N_1'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_1}{N_1'}=\frac{1}{1-\theta}(\frac{\widehat{b}_1}{N_1'}\frac{\widehat{O}_m}{\widehat{B}_m})(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_1}+I-\frac{\widehat{b}_1}{N_1'}\\
=\;&\frac{1}{1-\theta}(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_2}{N_2'}\\
=\;&\frac{1}{1-\theta}(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_3}{N_3'}=\frac{1}{1-\theta}(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_3}+I-\frac{\widehat{b}_3}{N_3'},\\
\Rightarrow\;&\frac{1}{1-\theta}\frac{\widehat{O}_m}{\widehat{B}_m}(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}[\frac{\widehat{b}_2}{N_2'}-(\frac{\widehat{b}_1}{N_1'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_1}]=\frac{\widehat{b}_2}{N_2'}-\frac{\widehat{b}_1}{N_1'},
\end{aligned} \tag{EC.23}$$

by using (EC.20), (EC.21), and (EC.22) in the first three lines and then rearranging the terms in the first and second lines. Similarly, by rearranging the terms in the second and third lines, we obtain

$$\frac{1}{1-\theta}\frac{\widehat{O}_m}{\widehat{B}_m}(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}[(\frac{\widehat{b}_3}{N_3'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_3}-\frac{\widehat{b}_2}{N_2'}]=\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_2}{N_2'}. \tag{EC.24}$$

Using (EC.23) and (EC.24), and after rearranging the terms, we obtain

$$[\frac{\widehat{b}_2}{N_2'}-\frac{\widehat{b}_1}{N_1'}][(\frac{\widehat{b}_3}{N_3'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_3}-\frac{\widehat{b}_2}{N_2'}]=[\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_2}{N_2'}][\frac{\widehat{b}_2}{N_2'}-(\frac{\widehat{b}_1}{N_1'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_1}]$$

$$\Rightarrow \quad \frac{\widehat{b}_2}{N_2'}=\frac{\frac{\widehat{b}_1}{N_1'}(\frac{\widehat{b}_3}{N_3'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_3}-\frac{\widehat{b}_1}{N_1'}(\frac{\widehat{b}_3}{N_3'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_1}}{(\frac{\widehat{b}_3}{N_3'})\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_3}-\frac{\widehat{b}_3}{N_3'}+\frac{\widehat{b}_1}{N_1'}-\frac{\widehat{b}_1}{N_1'}\frac{\widehat{B}_m-\widehat{b}_2}{\widehat{B}_m-\widehat{b}_1}}. \tag{EC.25}$$

Next, note that we can rewrite the condition $\widehat{U}_1=\widehat{U}_2=\widehat{U}_3$ as follows

$$\begin{aligned}
&\frac{1}{1-\theta}(\frac{\widehat{b}_1}{N_1'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_1}{N_1'}=\frac{1}{1-\theta}(\frac{\widehat{b}_1}{N_1'}\frac{\widehat{O}_m}{\widehat{B}_m})(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_1}+I-\frac{\widehat{b}_1}{N_1'}\\
=\ &\frac{1}{1-\theta}(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_2}{N_2'}=\frac{1}{1-\theta}(\frac{\widehat{b}_2}{N_2'}\frac{\widehat{O}_m}{\widehat{B}_m})(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_2}+I-\frac{\widehat{b}_2}{N_2'},\\
=\ &\frac{1}{1-\theta}(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{1-\theta}+I-\frac{\widehat{b}_3}{N_3'}\\
\Rightarrow\ &\frac{1}{1-\theta}\frac{\widehat{O}_m}{\widehat{B}_m}(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}[\frac{\widehat{b}_3}{N_3'}-(\frac{\widehat{b}_1}{N_1'})\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_1}]=\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_1}{N_1'},
\end{aligned} \tag{EC.26}$$

by using (EC.20), (EC.21), and (EC.22) in the first three lines and then rearranging the terms in the first and third lines. Similarly, rearranging the terms in the second and third lines, we obtain

$$\frac{1}{1-\theta}\frac{\widehat{O}_m}{\widehat{B}_m}(\frac{\widehat{b}_3}{N_3'}\frac{\widehat{O}_m}{\widehat{B}_m})^{-\theta}[\frac{\widehat{b}_3}{N_3'}-(\frac{\widehat{b}_2}{N_2'})\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_2}]=\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_2}{N_2'}. \tag{EC.27}$$

Using (EC.26) and (EC.27), and after rearranging the terms, we obtain

$$[\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_1}{N_1'}][\frac{\widehat{b}_3}{N_3'}-(\frac{\widehat{b}_2}{N_2'})\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_2}]=[\frac{\widehat{b}_3}{N_3'}-\frac{\widehat{b}_2}{N_2'}][\frac{\widehat{b}_3}{N_3'}-(\frac{\widehat{b}_1}{N_1'})\frac{\widehat{B}_m-\widehat{b}_3}{\widehat{B}_m-\widehat{b}_1}].$$

Substituting $\frac{\widehat{b}_2}{N_2'}$ from (EC.25) into the above expression, and simplifying the expression using straightforward algebra, we obtain $\widehat{b}_1=\widehat{b}_2$ which is a contradiction. Thus, we cannot have three or more cooperative sizes in equilibrium. This completes the proof of step 2.

Therefore, the unique equilibrium comprises equal-sized cooperatives, and the equilibrium outcomes are obtained using the expressions in Proposition 1 by setting $C=\widetilde{C}$. ■

Proof of Proposition 3. The proof directly follows from Proposition 1 and Theorem 1. ■

Proof of Theorem 2. In equilibrium, consumer utility $\widehat{U}_c=\gamma+\frac{1}{1-\theta}[(S\widehat{o})^{1-\theta}-1]+I-C\widehat{b}$. Plugging in the values for $\widehat{o}$ and $\widehat{b}$ and ignoring the irrelevant terms, the derivative of $\widehat{U}_c$ with respect to C can be determined by evaluating the derivative of $[\frac{C-1}{C}]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta}-\frac{C-1}{C}]$ with respect to C. This is proportional to $\frac{1}{r+\theta-1}\left[1-r\left(\frac{C-1}{C}\right)\right]$. We now argue that $\widehat{U}_c$ is unimodal in C, that is there exists C^* such that $\frac{\partial\widehat{U}_c}{\partial C}>0$ for all $C<C^*$ and $\frac{\partial\widehat{U}_c}{\partial C}<0$ for all $C>C^*$. Observe that when $r\geq 2$ (which implies $r+\theta-1>0$), $1-r\left(\frac{C-1}{C}\right)\leq 0$ for all $C\geq 2$. Hence, $C^*=2$. In addition, when $r\in(1,2)$, $C^*=\frac{r}{r-1}$ is the unique value above which $\frac{\partial\widehat{U}_c}{\partial C}<0$ and below which $\frac{\partial\widehat{U}_c}{\partial C}>0$. Finally, when $r\leq 1$, we have $r+\theta-1>0$ and $1-r\left(\frac{C-1}{C}\right)>0$. Thus, $C^*\to\infty$. ■

Proof of Proposition 4. The proof directly follows from Theorem 2. ■

Proof of Proposition 5. (i) This result follows from the fact that in equilibrium, consumer utility is the same across all cooperatives. Therefore, under endogenous mergers, post-merger utility

increases for all consumers. (ii) In the pre-merger equilibrium with C cooperatives, each supplier earns $\pi(\widehat{o}) = \frac{C-1}{C} S^{-\theta} \widehat{o}^{1-\theta} - a\widehat{o}^{r}$ where $\widehat{o}$ is given by (7). Using (EC.3) and substituting for $a\widehat{o}^{r}$, we can write $\pi(\widehat{o}) = \frac{C-1}{C} \widehat{o}(S\widehat{o})^{-\theta} \left[1 - \frac{1}{r}\left(1 - \frac{\theta}{S}\right)\right]$ where the term in the brackets is positive by the assumption $S(r-1)+\theta > 0$. Ignoring irrelevant terms, $\frac{\partial \pi(\widehat{o})}{\partial C} \propto \frac{\partial \pi(\frac{C-1}{C})^{\frac{r}{r+\theta-1}}}{\partial C} \geq 0$. Thus, each supplier's profit decreases post-merger since $C^* < C$. (iii) Notice that each cooperative aims to maximize the average utility of its members, which is given by $\widehat{U}_c = \gamma + \frac{1}{1-\theta}[(S\widehat{o})^{1-\theta} - 1] + I - \widehat{B}$ in the pre-merger equilibrium. Thus, total consumer utility is $C \times \frac{1}{C}\widehat{U}_c = \widehat{U}_c$. The sum of suppliers' profits is $S\pi(\widehat{o}) = \widehat{B} - Sa\widehat{o}^{r}$. Therefore, social welfare (W) is $W = \gamma + \frac{1}{1-\theta}[(S\widehat{o})^{1-\theta} - 1] + I - Sa\widehat{o}^{r} \propto \frac{1}{1-\theta}(\widehat{o})^{1-\theta} - \frac{C-1}{Cr}(\widehat{o})^{1-\theta}\left(1 - \frac{\theta}{S}\right)$. Therefore, $\frac{\partial W}{\partial C} \propto \frac{1}{C^2}\left[\frac{C-1}{C}\right]^{\frac{1-\theta}{r+\theta-1}-1}\left[1 - \frac{C-1}{C}\left(1 - \frac{\theta}{S}\right)\right] > 0$. This implies that social welfare decreases post-merger since $C^* < C$. ■

Proof of Theorem 3. Part (i). The existence of equilibrium and the derivation of the procurement budgets for the merged and non-merged cooperatives, as well as the suppliers' production quantities, follow from the proof of Theorem 4 and are hence omitted. Without loss of generality, assume that consumers decide to switch post-merger one at a time. Notice that immediately after the merger of cooperatives 1 and 2, the merged cooperative contains $N_{c_{12}} = \frac{2}{3}$ measure of consumers and the non-merged cooperative c_3, the remaning $\frac{1}{3}$ measure. Therefore, the utilities that consumers receive in the merged and non-merged cooperatives upon merger are, respectively, $U_{c_{12}} = \gamma + \frac{1}{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{B}_m + v_M$, and $U_{c_3} = \gamma + \frac{1}{1-\theta}[(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{B}_m + v$, with $N_{c_{12}} = \frac{2}{3}$, and $\eta = (\frac{1-N_{c_{12}}}{N_{c_{12}}})^{\frac{\theta}{1+\theta}}$. Define $\Delta U = U_{c_3} - U_{c_{12}} > 0$ in the absence of vertical differentiation, where the inequality follows from Proposition 2(ii). Then, if $\Delta v \equiv v_M - v < \Delta U$, consumers want to switch from the merged cooperative to the non-merged one c_3. Here, if $\Delta v < \Delta U \leq \Delta v + \alpha$, no switching occurs; otherwise, if $\Delta v + \alpha < \Delta U$, consumers switch. Thus, we can derive the value of $N_{c_{12}} \in (\frac{1}{2}, \frac{2}{3})$ in equilibrium by setting $\widehat{U}_{c_{12}} = \widehat{U}_{c_3} - \alpha$ as follows $\gamma + \frac{1}{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{B}_m + v_M = \gamma + \frac{1}{1-\theta}[(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{B}_m + v - \alpha$

$$\begin{aligned} \iff & (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{\eta}{1+\eta}] - (\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] \\ & = v - v_M - \alpha = -\Delta v - \alpha < 0. \end{aligned} \tag{EC.28}$$

Next, if $\Delta v \geq \Delta U$, consumers want to switch from the non-merged cooperative to c_{12}. Here, if $\Delta v \geq \Delta U \geq \Delta v - \alpha$, no switching occurs. Otherwise, if $\Delta v - \alpha > \Delta U$, consumers switch. Thus, we can derive the value of $N_{c_{12}} \in (\frac{2}{3}, 1)$ in equilibrium by setting $\widehat{U}_{c_{12}} - \alpha = \widehat{U}_{c_3}$ as follows $\gamma + \frac{1}{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{B}_m + v_M - \alpha = \gamma + \frac{1}{1-\theta}[(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{B}_m + v$

$$\begin{aligned} \iff & (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{\eta}{1+\eta}] - (\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] \\ & = v - v_M + \alpha = -\Delta v + \alpha < 0. \end{aligned} \tag{EC.29}$$

Because $\Delta v < \overline{v}$, $N_{c_{12}} < 1$. The uniqueness of $N_{c_{12}}$ in the above cases follows from the same arguments used in the proof of Theorem 4. Next, if we bring all the terms in (EC.28) or (EC.29) to the left-hand side, it can be seen that the partial derivative with respect to Δv is 1. The partial derivative with respect to $N_{c_{12}}$ is strictly negative, as shown in the proof of Theorem 4. Therefore, by applying the implicit function theorem to (EC.28) or (EC.29), it can be verified that $\frac{\partial N_{c_{12}}}{\partial \Delta v} > 0$. Using the same argument, it can be verified that $\frac{\partial N_{c_{12}}}{\partial \alpha} > 0$ in (EC.28) and $\frac{\partial N_{c_{12}}}{\partial \alpha} < 0$ in (EC.29).

Part (ii). To compare post-merger utility with pre-merger utility in equilibrium, we consider the utility received by members under the non-merged cooperative c_3 (given that $\widehat{U}_{c_3} = \widehat{U}_{c_{12}} \pm \alpha$ post-merger). The difference between post-merger and pre-merger utility is $(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] - O^{1-\theta}[\frac{1}{1-\theta} - \frac{2}{3}] \propto (\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] - [\frac{2}{3}]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - \frac{2}{3}]$ where we use the fact that consumers derive vertical value v from cooperative c_3 before and after the merger. Now, let v^* be the unique value such that if $\Delta v - \alpha = v^*$, the value of $N_{c_{12}}$ determined by (EC.29) makes $\eta = \frac{1}{2}$ (according to (EC.32)). First, we prove that the above expression is strictly negative when $r \in (0,1)$. To see this note that the expression is negative if and only if $[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}]^{\frac{1-\theta}{r+\theta-1}}/[\frac{2}{3}]^{\frac{1-\theta}{r+\theta-1}} < (\geq)[\frac{1}{1-\theta} - \frac{2}{3}]/[(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}]]$ when $\theta < 1 (\geq 1)$. Given that $(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta} < \frac{2}{3}$, this condition holds for $r \in (0,1)$ if it holds at $r = 1$. Then all the steps in the proof of Theorem 4, part (ii), can be applied to this case to prove the results. ■

Proof of Proposition 6. Recall from Theorem 3 that when $\Delta v \leq \Delta U$, consumers either switch from the merged cooperative to the non-merged one or no switching occurs. In both cases, in equilibrium we have $\widehat{U}_{c_{12}} \leq \widehat{U}_{c_3}$. As a result, if cooperatives 1 and 2 find it optimal to merge endogenously, the merger increases the utility of all members. However, when $\Delta v > \Delta U$, in equilibrium we have $\widehat{U}_{c_{12}} > \widehat{U}_{c_3}$. Now suppose r^* (and r_1^*) are the unique thresholds above which post-merger utility is higher than pre-merger utility for the members of the merged (non-merged) cooperative. Since $\widehat{U}_{c_{12}} > \widehat{U}_{c_3}$, there exists a region $[r^*, r_1^*)$ where a merger benefits its members but harms other consumers. In addition, when $r \geq r_1^*$, a merger benefits all consumers. (ii) In the pre-merger equilibrium with $C = 3$ cooperatives, each supplier earns $\pi(\widehat{o}) = \frac{2}{3}S^{-\theta}\widehat{o}^{1-\theta} - a\widehat{o}^r$ where $\widehat{o}$ is given by (7). Using (EC.5), we can write $\pi(\widehat{o}) = \frac{2}{3}\widehat{o}(S\widehat{o})^{-\theta}\left[1 - \frac{1}{r}\left(1 - \frac{\theta}{S}\right)\right]$ where the term in the brackets is positive by the assumption $S(r-1) + \theta > 0$. Similarly, in the post-merger equilibrium, each supplier earns $\pi_m(\widehat{o}_m) = (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}S^{-\theta}\widehat{o}_m^{1-\theta} - a\widehat{o}_m^r = (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}S^{-\theta}\widehat{o}_m^{1-\theta}\left[1 - \frac{1}{r}\left(1 - \frac{\theta}{S}\right)\right]$. Thus, $\pi_m(\widehat{o}_m) - \pi(\widehat{o}) \propto (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}\widehat{o}_m^{1-\theta} - \frac{2}{3}\widehat{o}^{1-\theta} = \left(\frac{2}{3}\right)^{\frac{\theta-1}{r+\theta-1}}\widehat{o}^{1-\theta}[[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}]^{\frac{r}{r+\theta-1}} - (\frac{2}{3})^{\frac{r}{r+\theta-1}}] < 0$, given that $(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta} < \frac{2}{3}$. (iii) In the post-merger equilibrium, total consumer utility is $N_{c_{12}}\widehat{U}_{c_{12}} + (1 - N_{c_{12}})\widehat{U}_{c_3}$, and total suppliers' profit is $S\pi_m(\widehat{o}_m) = \widehat{B}_m - Sa\widehat{o}_m^r$. Therefore, the difference between social welfare post-merger and pre-merger $(W_m - W)$ is $W_m - W \propto \frac{N_{c_{12}}}{1-\theta}(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta} + N_{c_{12}}\Delta v + \frac{1-N_{c_{12}}}{1-\theta}(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta} - Sa\widehat{o}_m^r - \frac{1}{1-\theta}(S\widehat{o})^{1-\theta} + Sa\widehat{o}^r =$

$\frac{N_{c_{12}}}{1-\theta}(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta} + N_{c_{12}}\Delta v + \frac{1-N_{c_{12}}}{1-\theta}(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta} - N_{c_{12}}(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}\widehat{O}_m^{1-\theta}\frac{1}{r}(1 - \frac{\theta}{S}) - (1 - N_{c_{12}})(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}\widehat{O}_m^{1-\theta}\frac{1}{r}(1 - \frac{\theta}{S}) - \frac{1}{1-\theta}(\widehat{O})^{1-\theta} + \frac{2}{3r}(\widehat{O})^{1-\theta}(1 - \frac{\theta}{S}) = N_{c_{12}}(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}\widehat{O}_m^{1-\theta}[\frac{1}{(1-\theta)N_{c_{12}}\eta} - \frac{1}{r}\left(1 - \frac{\theta}{S}\right)] + (1 - N_{c_{12}})(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}\widehat{O}_m^{1-\theta}[\frac{\eta}{(1-\theta)(1-N_{c_{12}})} - \frac{1}{r}\left(1 - \frac{\theta}{S}\right)] + N_{c_{12}}\Delta v - \widehat{O}^{1-\theta}[\frac{1}{1-\theta} - \frac{2}{3r}\left(1 - \frac{\theta}{S}\right)] = [(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}]^{1+\frac{1-\theta}{r+\theta-1}}\frac{1}{(\frac{2}{3})^{\frac{1-\theta}{r+\theta-1}}}\widehat{O}^{1-\theta}[\frac{1}{(1-\theta)\eta} - N_{c_{12}}\frac{1}{r}(1 - \frac{\theta}{S})] + [(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}]^{1+\frac{1-\theta}{r+\theta-1}}\frac{1}{(\frac{2}{3})^{\frac{1-\theta}{r+\theta-1}}}\widehat{O}^{1-\theta}[\frac{\eta}{(1-\theta)} - (1 - N_{c_{12}})\frac{1}{r}\left(1 - \frac{\theta}{S}\right)] + N_{c_{12}}\Delta v - \widehat{O}^{1-\theta}[\frac{1}{1-\theta} - \frac{2}{3r}\left(1 - \frac{\theta}{S}\right)]$. Now, suppose $\Delta v \to 0$, which implies $N_{c_{12}} \in [\frac{1}{2}, \frac{2}{3}]$. We evaluate the sign of the above expression for two cases, $\theta < 1$ and $\theta \geq 1$. Suppose $\theta < 1$. Observe that $A := (\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta} = (\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta} < \frac{2}{3}$. In addition, $[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}]\frac{1}{\eta} + [(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}]\eta < 1$. Let $k := \frac{1-\theta}{r+\theta-1}$ and $F := A^{1+k}/(2/3)^k$. Then, the expression is proportional to $\frac{F(\eta+1/\eta)-1}{1-\theta} + \frac{1}{r}(1 - \frac{\theta}{S})(2/3 - F)$. Given that $\eta + \frac{1}{\eta} < \frac{1}{A}$, $F(\eta + 1/\eta) < \frac{F}{A} = (\frac{3A}{2})^k$. Define $z := \frac{3A}{2} \in (0, 1)$, then $\frac{F(\eta+1/\eta)-1}{1-\theta} < -\frac{1-z^k}{1-\theta}$. Similarly, $\frac{2}{3} - F = \frac{2}{3}(1 - z^{k+1})$. Observe that $1 - z^{k+1} \leq \frac{k+1}{k}(1 - z^k)$. To see this, note that $\frac{1-z^{k+1}}{1-z^k} \leq \frac{k+1}{k} \iff z^k((k+1) - kz) \leq 1$, and the function $z^k((k+1) - kz)$ is increasing on $(0, 1)$ with limit 1 as $z \to 1$. Next, from the definition of k, $r = (1 - \theta)\frac{k+1}{k}$, and therefore $\frac{1}{r}(1 - \frac{\theta}{S}) = (\frac{S-\theta}{S})\frac{k}{(1-\theta)(k+1)}$, which implies $\frac{1}{r}(1 - \frac{\theta}{S}) < \frac{k}{(1-\theta)(k+1)}$. As a result, $\frac{1}{r}(1 - \frac{\theta}{S})(2/3 - F) < (\frac{2}{3})\frac{1-z^k}{1-\theta}$. Thus, $\frac{F(\eta+1/\eta)-1}{1-\theta} + \frac{1}{r}(1 - \frac{\theta}{S})(2/3 - F) < -\frac{1-z^k}{1-\theta} + (\frac{2}{3})\frac{1-z^k}{1-\theta} < 0$. A similar argument can be provided to prove that the expression is negative when $\theta \geq 1$. Therefore, as $\Delta v \to 0$, $W_m - W \leq 0$. Next, suppose $\Delta v > 0$ and let r_2^* be the first $r \geq r^*$ at which $W_m - W = 0$. We claim that for any $r > r_2^*$, $W_m - W > 0$. To see this, note that $\frac{d(W_m-W)}{d(r)} = \frac{\partial(W_m-W)}{\partial r} + \frac{\partial(W_m-W)}{\partial N_{c_{12}}} \times \frac{\partial N_{c_{12}}}{\partial r}$. First, it is straightforward to verify that $\frac{\partial(W_m-W)}{\partial r} > 0$. Second, $\frac{\partial(W_m-W)}{\partial N_{c_{12}}} > 0$ since both $(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{1}{1+\eta}$ and $(\frac{1}{1-N_{c_{12}}}\frac{\eta}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}$ are increasing in $N_{c_{12}}$ for $N_{c_{12}} \in [\frac{1}{2}, \frac{2}{3}]$ and $\theta > 0$. Third, $\frac{\partial N_{c_{12}}}{\partial r} > 0$ as provided in the proof of Theorem 4. Putting these together, we prove our claim. In addition, as r increases, $N_{c_{12}}$ increases, and hence the positive term $N_{c_{12}}\Delta v$ adds to social welfare which ensures that $W_m - W > 0$ for $r > r_2^*$. Finally, by the implicit function theorem, as Δv decreases, r_2^* increases. ■

Proof of Proposition 7. Notice that in choosing their procurement budgets, cooperatives take their size as given and try to optimize the average utility of their members. Hence, the existence of mismatch costs in the model only influences consumers' decisions in the first stage of the game and not the optimization problem of the cooperatives. This observation implies that the exact argument in the proof of Theorem 1 can be used to prove that the pre-merger market game has a unique equilibrium in which each cooperative contains $1/C$ measure of consumers. ■

Proof of Theorem 4. Part (i). The existence of equilibrium and symmetry of production quantities follow from previous proofs and are hence omitted. Suppose that in equilibrium, $N_{c_{12}}$ consumers join cooperative c_{12} and $(1 - N_{c_{12}})$ consumers join cooperative c_3. Then, the merged cooperative c_{12} solves the following optimization problem: $\max_{b_{c_{12}}} \gamma + \frac{1}{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{b_{c_{12}}}{b_{c_{12}}+\hat{b}_{c_3}}\widehat{O}_m)^{1-\theta} - 1] + I - \frac{b_{c_{12}}}{N_{c_{12}}}$. Notice that each cooperative chooses its procurement budget given its size, where the size of each

cooperative is determined by backward induction given the mismatch cost and anticipating the procurement strategy of cooperatives in equilibrium. Thus, the mismatch cost can be omitted from the cooperative's problem. The first-order optimality condition with respect to $b_{c_{12}}$ is given by

$$(\frac{1}{N_{c_{12}}}\frac{b_{c_{12}}}{b_{c_{12}}+\widehat{b}_{c_3}}\widehat{O}_m)^{-\theta}\frac{\widehat{b}_{c_3}}{(b_{c_{12}}+\widehat{b}_{c_3})^2}\widehat{O}_m-1=0, \tag{EC.30}$$

which is both necessary and sufficient. Similarly, considering the optimization problem of the non-merged cooperative and deriving the first-order optimality condition with respect to b_{c_3}, we obtain

$$(\frac{1}{[1-N_{c_{12}}]}\frac{b_{c_3}}{b_{c_3}+\widehat{b}_{c_{12}}}\widehat{O}_m)^{-\theta}\frac{\widehat{b}_{c_{12}}}{(b_{c_3}+\widehat{b}_{c_{12}})^2}\widehat{O}_m-1=0. \tag{EC.31}$$

The above two conditions imply that

$$\eta=\frac{\widehat{b}_{c_3}}{\widehat{b}_{c_{12}}}=(\frac{1-N_{c_{12}}}{N_{c_{12}}})^{\frac{\theta}{1+\theta}}. \tag{EC.32}$$

Since $\widehat{B}_m=\widehat{b}_{c_{12}}+\widehat{b}_{c_3}=(1+\eta)\widehat{b}_{c_{12}}$, we have $\frac{\widehat{b}_{c_{12}}}{\widehat{B}_m}=\frac{1}{1+\eta}$ and $\frac{\widehat{b}_{c_3}}{\widehat{B}_m}=\frac{\eta}{1+\eta}$. Using (EC.30) and (EC.31),

$$\widehat{B}_m=(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}\widehat{O}_m^{1-\theta}=(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}\widehat{O}_m^{1-\theta}. \tag{EC.33}$$

In addition, each supplier s chooses its production quantity o_{s_m} to maximize its profit $\pi_{s_m}=p_m o_{s_m}-ao_{s_m}^r=\frac{o_{s_m}\widehat{B}_m}{o_{s_m}+\widehat{O}_{-s_m}}-ao_{s_m}^r=(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}o_{s_m}(o_{s_m}+\widehat{O}_{-s_m})^{-\theta}-ao_{s_m}^r$. Taking the necessary and sufficient (using the exact same arguments as in the proof of Proposition 1) first-order optimality condition with respect to o_{s_m}, the production quantity by each supplier solves:

$$\widehat{o}_m=[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{-\theta}\frac{\eta}{1+\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}=[(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{-\theta}\frac{1}{1+\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}. \tag{EC.34}$$

Now, let us derive the value of $N_{c_{12}}$ in equilibrium. Note that in Figure 4(b), a consumer at the margin between c_3 and c_{12} from left or right should be indifferent between joining the merged cooperative c_{12} or the non-merged one c_3 in equilibrium. To satisfy this condition, we must have $\gamma+\frac{1}{1-\theta}[(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta}-1]+I-\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{B}_m-t(\frac{N_{c_{12}}}{2}-\frac{1}{6})=\gamma+\frac{1}{1-\theta}[(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta}-1]+I-\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{B}_m-t(\frac{1-N_{c_{12}}}{2})\iff$

$$(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{\eta}{1+\eta}]-(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{1}{1+\eta}]=t(N_{c_{12}}-\frac{2}{3}), \tag{EC.35}$$

where in deriving the mismatch costs in the first equality, we use two facts: 1) in any equilibrium, the distance of the marginal consumers on both sides of c_3 from c_3, must be equal; otherwise, these two consumers would not be receiving equal utilities (given different mismatch costs); and 2) the marginal consumer in any equilibrium cannot locate between c_1 and c_2, as otherwise, any consumer between the marginal consumer and c_1 or c_2 would be better off joining the merged cooperative and paying lesser mismatch costs. Figure 4(b) depicts the market configuration.

Now, suppose $t=0$. Then recall from the proof of Theorem 1, step 1, that if $N_{c_{12}}\leq\frac{1}{2}$, the utility of a consumer in the merged cooperative c_{12} would be higher than their utility in the

other cooperative c_3. This makes the left-hand side of (EC.35) positive. However, under $t>0$ and if $N_{c_{12}} \leq \frac{1}{2}$, the right-hand side of (EC.35) would be strictly negative, which is a contradiction. Therefore, in order to satisfy (EC.35), we must have that $N_{c_{12}} > \frac{1}{2}$.

Following this observation, the left-hand side of (EC.35) is strictly negative, implying that $N_{c_{12}} < \frac{2}{3}$ in order to make the right-hand side also strictly negative. Therefore, $\frac{1}{2} < N_{c_{12}} < \frac{2}{3}$.

Next, we prove the uniqueness of $N_{c_{12}}$ that satisfies (EC.35). To see this, notice that the left-hand side of (EC.35) can be written as $F(N_{c_{12}}) - F(1-N_{c_{12}})$ where $F(1-N_{c_{12}}) = (\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] \propto [\frac{(1-N_{c_{12}})^{-\frac{1}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}]^{\frac{(1-\theta)(r-1)}{r+\theta-1}}[\frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - \frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}]$, after substituting for η from (EC.32) and $\widehat{O}_m$ from (EC.34), and ignoring irrelevant terms. We claim that $F(1-N_{c_{12}})$ is strictly increasing in $N_{c_{12}}$. To prove this, we take its derivative with respect to $N_{c_{12}}$. After ignoring some positive common terms, the derivative simplifies to $\frac{(1-\theta)(r-1)}{r+\theta-1}N_{c_{12}}^{\frac{\theta}{1+\theta}}((1-N_{c_{12}})^{-\frac{2}{1+\theta}} - \frac{\theta}{1+\theta}N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{1}{1+\theta}} + \frac{1}{1+\theta}N_{c_{12}}^{\frac{\theta}{1+\theta}}(1-N_{c_{12}})^{-\frac{2+\theta}{1+\theta}}) \times (\frac{1}{1-\theta} - \frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}) + \frac{(1-\theta)}{r+\theta-1}(\frac{\theta}{1+\theta})N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{2}{1+\theta}}(\frac{1}{1-\theta} - \frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}) - \frac{\theta}{1+\theta}N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{2}{1+\theta}}\frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}} > (\frac{1-\theta}{1+\theta})N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{2}{1+\theta}}(\frac{1}{1-\theta} - \frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}) - \frac{\theta}{1+\theta}N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{2}{1+\theta}}\frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}} = \frac{1}{1+\theta}N_{c_{12}}^{-\frac{1}{1+\theta}}(1-N_{c_{12}})^{-\frac{2}{1+\theta}}(1 - \frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}}) > 0$, where the first inequality is obtained by setting $r=1$. Now, since $F(1-N_{c_{12}})$ is strictly increasing in $N_{c_{12}}$, we conclude that $F(N_{c_{12}})$ is strictly decreasing in $N_{c_{12}}$ and hence $F(N_{c_{12}}) - F(1-N_{c_{12}})$ is strictly decreasing in $N_{c_{12}}$, whereas the right-hand side of (EC.35) is strictly increasing in $N_{c_{12}}$. Therefore, there exists a unique $N_{c_{12}}$ that satisfies (EC.35). Next, if we bring all the terms in (EC.35) to the left-hand side, it can be seen that the partial derivative with respect to t is $-(N_{c_{12}} - \frac{2}{3}) > 0$ given that $N_{c_{12}} < \frac{2}{3}$. The partial derivative with respect to $N_{c_{12}}$ is strictly negative, as argued above. Thus, by applying the implicit function theorem to (EC.35), it can be verified that $\frac{\partial N_{c_{12}}}{\partial t} > 0$.

Part (ii). Considering the average utility received by members under the non-merged cooperative c_3, the difference between post-merger and pre-merger utility can be expressed as (a similar analysis can be applied to compare the utilities of members in c_{12}):

$$(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{1}{1+\eta}] - O^{1-\theta}[\frac{1}{1-\theta} - \frac{2}{3}] - \frac{1}{2}t[\frac{1}{3} - \frac{N_{c_{12}}}{2}], \tag{EC.36}$$

where the term for the mismatch cost captures the difference between the expected mismatch costs incurred by members of cooperative c_3 under these two settings. Notice that since $N_{c_{12}} \in (\frac{1}{2}, \frac{2}{3})$, consumers of c_3 pay a higher mismatch cost on average post-merger (see, Figure 4(b)). Now, suppose $r \to 1$. Then, (EC.36) simplifies to $S^{1-\theta}(\frac{1}{a}S^{-\theta-1}(S-\theta))^{\frac{1-\theta}{\theta}}[(\frac{N_{c_{12}}^{\frac{\theta}{1+\theta}}}{N_{c_{12}}^{\frac{\theta}{1+\theta}}+(1-N_{c_{12}})^{\frac{\theta}{1+\theta}}})^{\frac{1-\theta}{\theta}}(\frac{1}{1-\theta} -$

$\frac{N_{c12}^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}) - (\frac{2}{3})^{\frac{1-\theta}{\theta}}(\frac{1}{1-\theta}-\frac{2}{3})] - \frac{1}{2}t[\frac{1}{3}-\frac{N_{c12}}{2}] < 0$. To see the inequality, note that the function $x^{\frac{1-\theta}{\theta}}(\frac{1}{1-\theta}-x)$ is strictly increasing in x for $x<1$, and $N_{c12}^{\frac{\theta}{1+\theta}}/[N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}] = \frac{1}{1+\eta} < \frac{2}{3} \iff \eta > \frac{1}{2}$, which holds. Therefore, the first bracket in the above expression is strictly negative. The second bracket is also strictly positive, and hence the entire expression is strictly negative. This implies that as $r \to 1$, the pre-merger utility is strictly higher than the post-merger utility. Moreover, (EC.36) is strictly negative when $r \in (0,1)$ as shown in the proof of Theorem 3. Next, suppose $r \to \infty$, then (EC.36) simplifies to $(\frac{(1-N_{c12})^{-\frac{1}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}})^{1-\theta}(\frac{1}{1-\theta}-\frac{N_{c12}^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}) - (\frac{1}{1-\theta}-\frac{2}{3}) - \frac{1}{2}t[\frac{1}{3}-\frac{N_{c12}}{2}]$. Applying the implicit function theorem to (EC.35), it can be verified that $\frac{\partial N_{c12}}{\partial r} > 0$ given the facts that $\widehat{O}_m$ is strictly decreasing in r and the left-hand side of (EC.35) is a negative value. This implies that as $r \to \infty$, $N_{c_{12}} \to \frac{2}{3}$ (hence, no difference in mismatch cost compared to pre-merger). Therefore, the above expression simplifies to $(\frac{(1-N_{c12})^{-\frac{1}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}})^{1-\theta}(\frac{1}{1-\theta} - \frac{N_{c12}^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}) - (\frac{1}{1-\theta}-\frac{2}{3}) > (\frac{1}{1-\theta} - \frac{N_{c12}^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}) - (\frac{1}{1-\theta}-\frac{2}{3}) = \frac{2}{3} - \frac{1}{1+\eta} > 0$, where we use the fact that $(1-N_{c12})^{-\frac{1}{1+\theta}} > N_{c12}^{\frac{\theta}{1+\theta}} + (1-N_{c12})^{\frac{\theta}{1+\theta}}$, and for any sign of $1-\theta$, the first inequality holds. Now, given the continuity of (EC.36) in r, there exists a value $r^* \in (1,\infty)$ where the pre-merger utility is equal to the post-merger utility. We now claim that this r^* is unique where for all $r < r^*$, pre-merger utility is higher and for all $r \geq r^*$, post-merger utility is higher. To see this, recall that when $r \to 1$, pre-merger utility is higher. Let r^* be the smallest r at which pre-merger utility is equal to the post-merger utility. Then, we prove that if (EC.36) is positive at some $r' \geq r^*$, it remains strictly positive at $r'' > r'$. To see this, first notice that as $N_{c_{12}}$ increases, the mismatch cost term in (EC.36) decreases. In addition, the first expression in (EC.36) which is $F(1-N_{c_{12}})$ also increases in $N_{c_{12}}$. Therefore, (EC.36) increases in $N_{c_{12}}$. Owing to the fact that $\frac{\partial N_{c12}}{\partial r} > 0$, we conclude that the indirect derivative of (EC.36) with respect to r (through $N_{c_{12}}$) is positive. Next, we investigate the direct derivative of (EC.36) with respect to r. (EC.36) can be re-written as $\frac{3}{4}(\frac{1}{[1-N_{c12}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{1}{1+\eta}] - O^{1-\theta}[\frac{1}{1-\theta}-\frac{2}{3}] + \frac{1}{4}(\frac{1}{N_{c12}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta} - \frac{\eta}{1+\eta}]$, where we substitute for the mismatch cost term using (EC.35). After factoring out the common term $S^{1-\theta}(\frac{1}{ar}S^{-\theta-1}(S-\theta))^{\frac{1-\theta}{r+\theta-1}}$, the direct derivative of the above expression with respect to r is $\frac{3(1-\theta)\theta}{4(r+\theta-1)^2}[j_1]^{\frac{(1-\theta)(r-1)}{r+\theta-1}}[j_2]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta}-j_2]\log[j_1] - \frac{3(1-\theta)}{4(r+\theta-1)^2}[j_1]^{\frac{(1-\theta)(r-1)}{r+\theta-1}}[j_2]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - j_2]\log[j_2] + \frac{(1-\theta)\theta}{4(r+\theta-1)^2}[j_3]^{\frac{(1-\theta)(r-1)}{r+\theta-1}}[j_4]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - j_4]\log[j_3] - \frac{(1-\theta)}{4(r+\theta-1)^2}[j_3]^{\frac{(1-\theta)(r-1)}{r+\theta-1}}[j_4]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta} - j_4]\log[j_4] + \frac{(1-\theta)}{(r+\theta-1)^2}[\frac{2}{3}]^{\frac{1-\theta}{r+\theta-1}}[\frac{1}{1-\theta}-\frac{2}{3}]\log[\frac{2}{3}] > 0$, where $j_1 \equiv \frac{(1-N_{c12})^{-\frac{1}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}$, $j_2 \equiv \frac{N_{c12}^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}$, $j_3 \equiv \frac{(N_{c12})^{-\frac{1}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}$, and $j_4 \equiv \frac{(1-N_{c12})^{\frac{\theta}{1+\theta}}}{N_{c12}^{\frac{\theta}{1+\theta}}+(1-N_{c12})^{\frac{\theta}{1+\theta}}}$, given the facts that (EC.36) is positive at r and

$3\log[\frac{1}{1+\eta}]+\log[\frac{\eta}{1+\eta}]<4\log[\frac{2}{3}]$. Putting these observations together, if (EC.36) is positive at some $r'\geq r^*$, it remains strictly positive at $r''>r'$.

Finally, to see why r^* is decreasing in t, consider an arbitrary $t'>0$ and let $r^*_{t'}$ be the value of r at which post-merger utility is equal to pre-merger utility given t'. Now, holding $r=r^*_{t'}$ fixed, let t' increases to t''. Then, $N''_{c_{12}}$ that solves (EC.35) increases. Accordingly, since (EC.36) is increasing in $N_{c_{12}}$, (EC.36) is strictly positive. This means post-merger utility is higher than pre-merger utility under t'' at $r=r^*_{t'}$. Therefore, $r^*_{t''}$ must be smaller than $r^*_{t'}$. This completes the proof of part (ii). ■

Proof of Proposition 8. (i) Let $t_{c_{12}}$ and t_{c_3} represent the average mismatch cost incurred by the members of the merged and non-merged cooperatives, respectively, in the post-merger equilibrium. We prove that the average utility received by members of the non-merged cooperative is strictly higher than that received by members of the merged cooperative. To see this note that $(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{\eta}{1+\eta}]-t_{c_{12}}-(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{1}{1+\eta}]+t_{c_{12}}<(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{\eta}{1+\eta}]-\frac{t}{2}(N_{c_{12}}-\frac{1}{3})-(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta})^{1-\theta}\widehat{O}_m^{1-\theta}[\frac{1}{1-\theta}-\frac{1}{1+\eta}]+\frac{t}{2}(1-N_{c_{12}})=0$, where the first inequality follows from the fact that the marginal consumer, who pins down the value of $N_{c_{12}}$ in equilibrium, incurs the maximum difference in mismatch cost when joining the non-merged cooperative, and the second inequality follows from (EC.35). This implies that if two cooperatives merge endogenously, the merger increases the utility of all consumers. (ii) The proof is identical to the one provided in Proposition 6. (iii) In the post-merger equilibrium, total consumer utility is $N_{c_{12}}\widehat{U}_{c_{12}}+(1-N_{c_{12}})\widehat{U}_{c_3}$, and total suppliers' profit is $S\pi_m(\widehat{o}_m)=\widehat{B}_m-Sa\widehat{o}^r_m$. Therefore, the difference between social welfare post-merger and pre-merger (W_m-W) is $W_m-W<\frac{N_{c_{12}}}{1-\theta}(\frac{1}{N_{c_{12}}}\frac{1}{1+\eta}\widehat{O}_m)^{1-\theta}+\frac{1-N_{c_{12}}}{1-\theta}(\frac{1}{[1-N_{c_{12}}]}\frac{\eta}{1+\eta}\widehat{O}_m)^{1-\theta}-Sa\widehat{o}^r_m-\frac{1}{1-\theta}(S\widehat{o})^{1-\theta}+Sa\widehat{o}^r<0$, where the first inequality follows from the fact that consumers incur a higher mismatch cost post-merger when switching from the merged cooperative to the non-merged cooperative, and the second inequality follows from the proof of Proposition 6 (when $\Delta v=0$). Thus, we prove our result. ■

Proofs of Propositions 9, 10, 11, and Theorem 5. These proofs are omitted due to page limits and are available from the authors upon request. ■

EC.1.1. Downstream Competition with a For-Profit Retailer

Pre-merger equilibrium. The existence of equilibrium and symmetry of suppliers' production quantities follow from previous proofs and are hence omitted. As in our base model, strategic consumers first choose whether to join one of the two cooperatives or buy from the retailer. To keep the focus on the retailer's presence and its impact on consumers' gain or loss from the mergers of cooperatives, we focus on equilibria where $N_c>0$ (to be determined endogenously) measure of consumers joins each cooperative and the remaining $1-2N_c>0$ measure opts to buy from the retailer. Specifically, the marginal consumer who is indifferent between joining a cooperative or

buying from the retailer will be determined endogenously by anticipating the corresponding utility under each strategy in equilibrium. Given N_c, cooperative c solves $\max_{b_c} \gamma + \frac{1}{1-\theta}[(\frac{1}{N_c}\frac{b_c}{b_c+\widehat{B}_{-c}}\widehat{O})^{1-\theta} - 1] + I - \frac{b_c}{N_c}$. The necessary (and sufficient) optimality condition solves

$$(\frac{1}{N_c}\frac{b_c}{b_c+\widehat{B}_{-c}}\widehat{O})^{-\theta}\frac{\widehat{B}_{-c}}{(b_c+\widehat{B}_{-c})^2}\widehat{O} - 1 = 0. \tag{EC.37}$$

In addition, each consumer who buys from the retailer solves (not considering the mismatch cost as it won't change the consumption): $\max_x \gamma + \frac{1}{1-\theta}[x^{1-\theta} - 1] + I - p_f x$. The necessary and sufficient optimality condition with respect to x is $x^{-\theta} - p_f = 0$. Notice that as $x \to 0$, the left-hand side of this condition converges to a strictly positive value and as $x \to (I/p_f)$, the left-hand side converges to a strictly negative value for a sufficiently large I which we assume as in the base model ($I > \overline{I}$). Hence, the optimal x admits an interior solution in the form of $\widehat{x} = p_f^{-\frac{1}{\theta}}$. Therefore, the aggregate demand of the retailer $q_f = (1-2N_c)p_f^{-\frac{1}{\theta}}$. This implies that $q_f^{-\theta} = (1-2N_c)^{-\theta}p_f$. Moreover, $q_f = \frac{b_f}{p} = \frac{b_f}{B}O = \frac{b_f}{b_f+B_{-f}}O$. Then, given N_c, the retailer solves $\max_{b_f}(p_f - p)q_f \equiv \max_{b_f}(p_f q_f - b_f)$, where $p_f q_f = q_f^{1-\theta}(1-2N_c)^{\theta} = (\frac{b_f}{b_f+B_{-f}}O)^{1-\theta}(1-2N_c)^{\theta}$. The first-order optimality condition with respect to b_f must satisfy

$$(1-\theta)(\frac{1}{[1-2N_c]}\frac{b_f}{b_f+\widehat{B}_{-f}}\widehat{O})^{-\theta}\frac{\widehat{B}_{-f}}{(b_f+\widehat{B}_{-f})^2}\widehat{O} - 1 \le 0. \tag{EC.38}$$

Observe that if $1-\theta \le 0$, the above condition holds with strict inequality and hence $b_f = 0$. This is because when $\theta \ge 1$, consumers are extremely risk averse and prefer to join a non-profit cooperative rather than buy from a for-profit retailer. In this case, the retailer would be better off not spending any procurement budget and serving no consumers, as the net profit would be negative. This implies that when $\theta \ge 1$, the results are similar to our base model where the retailer does not exist in the market. Therefore, suppose, $1-\theta > 0$ and hence $b_f > 0$ satisfies the above necessary and sufficient condition with equality. (EC.37) and (EC.38) together imply that $\eta = \frac{\widehat{b}_c}{\widehat{b}_f}$ solves $\frac{2(1-\theta)\eta^{1+\theta}}{1+\eta} = (\frac{N_c}{1-2N_c})^{\theta}$. Rearranging the terms, we obtain $N_c = \frac{\omega}{1+2\omega}$ where $\omega = (\frac{2(1-\theta)\eta^{1+\theta}}{1+\eta})^{\frac{1}{\theta}}$. Since $\widehat{B} = 2\widehat{b}_c + \widehat{b}_f = (1+2\eta)\widehat{b}_f$, we have $\frac{\widehat{b}_f}{\widehat{B}} = \frac{1}{1+2\eta}$ and $\frac{\widehat{b}_c}{\widehat{B}} = \frac{\eta}{1+2\eta}$. Using (EC.37) and (EC.38), we obtain $\widehat{B} = (\frac{1}{N_c}\frac{\eta}{1+2\eta})^{-\theta}\frac{1+\eta}{1+2\eta}\widehat{O}^{1-\theta} = (1-\theta)(\frac{1}{[1-2N_c]}\frac{1}{1+2\eta})^{-\theta}\frac{2\eta}{1+2\eta}\widehat{O}^{1-\theta}$. In addition, each supplier s chooses its production quantity o_s to maximize its profit $\pi_s = po_s - ao_s^r = \frac{o_s\widehat{B}}{o_s+\widehat{O}_{-s}} - ao_s^r = (\frac{1}{N_c}\frac{\eta}{1+2\eta})^{-\theta}\frac{1+\eta}{1+2\eta}o_s(o_s + \widehat{O}_{-s})^{-\theta} - ao_s^r$. Taking the necessary (and sufficient) first-order optimality condition with respect to o_s, the production quantity by each supplier solves: $\widehat{o} = [(\frac{1}{N_c}\frac{\eta}{1+2\eta})^{-\theta}\frac{1+\eta}{1+2\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}} = [(1-\theta)(\frac{1}{[1-2N_c]}\frac{1}{1+2\eta})^{-\theta}\frac{2\eta}{1+2\eta}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}$. Now, we can derive the value of N_c in equilibrium. For a consumer who is indifferent between joining a cooperative or buying from the retailer, we must have $\widehat{U}_c - t(N_c - \frac{1}{6}) = \widehat{U}_f - t(\frac{1}{2} - N_c) \iff \widehat{U}_c - \widehat{U}_f = 2t(N_c - \frac{1}{3}) \iff \frac{1}{1-\theta}[(\frac{1}{N_c}\frac{\eta}{1+2\eta}\widehat{O})^{1-\theta} - 1] - \frac{1}{N_c}\frac{\eta}{1+2\eta}\widehat{B} - \frac{1}{1-\theta}[\widehat{x}^{1-\theta} - 1] + p_f\widehat{x} = 2t(N_c - \frac{1}{3})$, where $\widehat{U}_c$ and $\widehat{U}_f$ indicate the utility a consumer receives in equilibrium from joining a cooperative or visiting the retailer, respectively.

Post-merger equilibrium. As in §6, consider a post-merger setting where cooperatives 1 and 2 merge to form cooperative c_{12} and compete with the retailer downstream in the supply chain. Let N_c' be the size of merged cooperative c_{12} in equilibrium. Then, cooperative c_{12} solves $\max_{b_c'} \gamma + \frac{1}{1-\theta}[(\frac{1}{N_c'}\frac{b_c'}{b_c'+\widehat{B'}_{-c}}\widehat{O'})^{1-\theta}-1]+I-\frac{b_c'}{N_c'}$. The necessary (and sufficient) optimality condition solves

$$(\frac{1}{N_c'}\frac{b_c'}{b_c'+\widehat{B}'_{-c}}\widehat{O'})^{-\theta}\frac{\widehat{B'}_{-c}}{(b_c'+\widehat{B}'_{-c})^2}\widehat{O'}-1=0. \tag{EC.39}$$

In addition, those consumers who buy from the retailer solve: $\max_{x'} \gamma + \frac{1}{1-\theta}[x'^{1-\theta}-1]+I-p_f'x'$. The necessary first-order optimality condition with respect to x' is $x'^{-\theta}-p_f'=0$ which is also sufficient. Thus, $\widehat{x'}=p_f'^{-\frac{1}{\theta}}$. Thus the aggregate demand of the retailer $q_f'=(1-N_c')p_f'^{-\frac{1}{\theta}}$. This implies that $q_f'^{-\theta}=(1-N_c')^{-\theta}p_f'$. Moreover, $q_f'=\frac{b_f'}{p'}=\frac{b_f'}{\widehat{B'}}O'=\frac{b_f'}{b_f'+B'_{-f}}O'$. Then, the retailer solves $\max_{b_f'}(p_f'-p')q_f' \equiv \max_{b_f'}(p_f'q_f'-b_f')$, where $p_f'q_f'=q_f'^{1-\theta}(1-N_c')^{\theta}=(\frac{b_f'}{b_f'+B'_{-f}}O')^{1-\theta}(1-N_c')^{\theta}$. The first-order optimality condition with respect to b_f' must satisfy

$$(1-\theta)(\frac{1}{[1-N_c']}\frac{b_f'}{b_f'+\widehat{B}'_{-f}}\widehat{O'})^{-\theta}\frac{\widehat{B}'_{-f}}{(b_f'+\widehat{B}'_{-f})^2}\widehat{O'}-1\le 0. \tag{EC.40}$$

As in the pre-merger case, we assume that $1-\theta>0$ and hence $b_f'>0$ satisfies the above condition (which is also sufficient) with equality. (EC.39) and (EC.40) together imply that $\eta'=\frac{\widehat{b}_c'}{\widehat{b}_f'}$ solves $(1-\theta)\eta'^{1+\theta}=\left(\frac{N_c'}{1-N_c'}\right)^{\theta}\Rightarrow N_c'=\frac{\omega'}{1+\omega'}$, where $\omega'=((1-\theta)\eta'^{1+\theta})^{\frac{1}{\theta}}$. Since $\widehat{B}'=\widehat{b}_c'+\widehat{b}_f'=(1+\eta')\widehat{b}_f'$, we have $\frac{\widehat{b}_f'}{\widehat{B}'}=\frac{1}{1+\eta'}$ and $\frac{\widehat{b}_c'}{\widehat{B}'}=\frac{\eta'}{1+\eta'}$. Using (EC.39) and (EC.40), we obtain $\widehat{B}'=(\frac{1}{N_c'}\frac{\eta'}{1+\eta'})^{-\theta}\frac{1}{1+\eta'}\widehat{O}'^{1-\theta}=(1-\theta)(\frac{1}{[1-N_c']}\frac{1}{1+\eta'})^{-\theta}\frac{\eta'}{1+\eta'}\widehat{O}'^{1-\theta}$. In addition, solving each supplier's maximization problem as before, the production quantity by each supplier solves: $\widehat{o}'=[(\frac{1}{N_c'}\frac{\eta'}{1+\eta'})^{-\theta}\frac{1}{1+\eta'}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}=[(1-\theta)(\frac{1}{[1-N_c']}\frac{1}{1+\eta'})^{-\theta}\frac{\eta'}{1+\eta'}\frac{1}{ar}S^{-\theta-1}(S-\theta)]^{\frac{1}{r+\theta-1}}$. Now, we can derive the value of N_c' in equilibrium. To satisfy the indifference condition for the marginal consumer, N_c' must solve $\widehat{U}_c'-t(\frac{N_c'}{2}-\frac{1}{6})=\widehat{U}_f'-t(\frac{1}{2}-\frac{N_c'}{2}) \iff \widehat{U}_c'-\widehat{U}_f'=t(N_c'-\frac{2}{3})$. Finally, we are interested in comparing the difference between post-merger and pre-merger member-consumer utility which is given by $\widehat{U}_c'-\widehat{U}_c-\frac{1}{4}t(N_c'-2N_c)$. Here, the term for the mismatch cost captures the difference between the expected mismatch costs incurred by the members of cooperatives under these two settings. To answer this question, we conduct a rigorous numerical analysis with 10,000 random instances of (θ,t,S,a). In particular, at each instance, we randomly select the number of suppliers S from $U[2,10]$, the supplier cost function coefficient a from $U[0.5,1]$, consumer utility coefficient θ from $U(0,1)$, and the mismatch cost t from $U[0,10]$. In all of these 10,000 random instances, we verify that mergers of cooperatives harm member consumers if and only if the production scale-up cost r is lower than a threshold. Figure 8 in the main body illustrates the results for one such instance and shows that our results in Theorem 4 are robust in this case. ■