Online Appendices to
"Platform Certification and Seller Disclosure in Online Selling"

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Appendix A: Proofs

Proof of Lemma 1. Given seller i's payoffs under disclosure $\widehat{\pi}_i^d = (1-\alpha)q_i^2/4 - h$ and under non-disclosure $\widehat{\pi}_i^{nd} = (1-\alpha)\widehat{q}_D^2/16$, it is evident that at $\widehat{q}_D^*$, we have $\widehat{\pi}_i^d = \widehat{\pi}_i^{nd}$, which leads to the equilibrium disclosure threshold $\widehat{q}_D^* = \min\{4\sqrt{\frac{h}{3(1-\alpha)}}, 1\}$.

Proof of Lemma 2. For certified seller i, the payoff under disclosure is given by $\widehat{\pi}_i^{c,d} = (1-\alpha)q_i^2/4 - h$ and under non-disclosure is given by $\widehat{\pi}_i^{c,nd} = (1-\alpha)(\widehat{q}_C + \widehat{q}_D^c)^2/16$. It is evident that at $\widehat{q}_D^c$ we have $\widehat{\pi}_i^{c,d} = \widehat{\pi}_i^{c,nd}$, which leads to the certified seller i's equilibrium disclosure threshold $\widehat{q}_D^c = \min(1, \frac{1}{3}\widehat{q}_C + \frac{2}{3}\sqrt{\widehat{q}_C^2 + \frac{12h}{1-\alpha}})$. Following a similar approach, we can derive uncertified seller i's equilibrium disclosure threshold $\widehat{q}_D^{nc} = \min(\widehat{q}_C, \widehat{q}_D^*)$.

Proof of Proposition 1. We first verify that the platform would never set a quality standard $\widehat{q}_C > \widehat{q}_D^*$. Let $\widehat{\Pi}_p^{nc}$ denote the platform's profit without certification. Then, in this scenario, $\widehat{\Pi}_p - \widehat{\Pi}_p^{nc} = -\frac{\alpha}{48}\left(\widehat{q}_D^c - \widehat{q}_C\right)^3 < 0$. This option always makes the platform obtain a lower profit than that without certification, so it cannot be the equilibrium strategy.

Next, we move to the scenario $\widehat{q}_C \leqslant \widehat{q}_D^*$. Note that $\widehat{q}_D^c < 1$ requires $\widehat{q}_C < -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$, which leads to the following two subcases:

Case 1: $\widehat{q}_D^* \leqslant -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$, which requires $h \leqslant \frac{3(7-\sqrt{13})(1-\alpha)}{128}$, $\widehat{\Pi}_p = \displaystyle\int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C + \widehat{q}_D^c)^2}{16}dq + \int_{\widehat{q}_D^c}^{1} \frac{\alpha q^2}{4}dq$. Solving the maximization problem, we can obtain $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C} = 288\frac{h}{1-\alpha}\widehat{q}_C - (96\frac{h}{1-\alpha} + 5(\widehat{q}_C)^2)\sqrt{(\widehat{q}_C)^2 + 12\frac{h}{1-\alpha}} + 32(\widehat{q}_C)^3$, and $\frac{\partial^2 \widehat{\Pi}_p}{\partial(\widehat{q}_C)^2} < 0$. This together with the fact that $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=0} > 0$ and $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=\widehat{q}_D^*} < 0$ suggests that there exists a unique $\widehat{q}_C^*$ satisfying

$$288\frac{h}{1-\alpha}\widehat{q}_C - (96\frac{h}{1-\alpha} + 5(\widehat{q}_C)^2)\sqrt{(\widehat{q}_C)^2 + 12\frac{h}{1-\alpha}} + 32(\widehat{q}_C)^3 = 0 \tag{A.1}$$

that maximizes the platform's profit.

Case 2: $\widehat{q}_D^* > -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$, which requires $h > \frac{3(7-\sqrt{13})(1-\alpha)}{128}$, under which

$$\widehat{\Pi}_p = \begin{cases} \displaystyle\int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C + \widehat{q}_D^c)^2}{16}dq + \int_{\widehat{q}_D^c}^{1} \frac{\alpha q^2}{4}dq, & \text{if } \widehat{q}_C \leqslant -1 + 2\sqrt{1 - \dfrac{4h}{1-\alpha}}; \\ \displaystyle\int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{1} \frac{\alpha(\widehat{q}_C + 1)^2}{16}dq, & \text{if } -1 + 2\sqrt{1 - \dfrac{4h}{1-\alpha}} < \widehat{q}_C < \widehat{q}_D^*. \end{cases}$$

If $-1 + 2\sqrt{1 - \frac{4h}{1-\alpha}} < \widehat{q}_C < \widehat{q}_D^*$, then $\widehat{\Pi}_p = \frac{\alpha}{16}(1 + \widehat{q}_C - (\widehat{q}_C)^2)$. We can show that $\widehat{q}_C^* = 1/2$, which exists only when $h > \frac{7(1-\alpha)}{64}$. If $\widehat{q}_C \leqslant -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$, $\widehat{q}_C^*$ is the solution of $288\frac{h}{1-\alpha}\widehat{q}_C - (96\frac{h}{1-\alpha} + 5(\widehat{q}_C)^2)\sqrt{(\widehat{q}_C)^2 + 12\frac{h}{1-\alpha}} + 32(\widehat{q}_C)^3 = 0$, which exists only when $h < \frac{1}{16}\sqrt[3]{-\alpha^3 + 3\alpha^2 + 78\sqrt{2}\sqrt{\alpha^6 - 6\alpha^5 + 15\alpha^4 - 20\alpha^3 + 15\alpha^2 - 6\alpha + 1} - 3\alpha + 1} - \frac{847872\alpha^2 - 1695744\alpha + 847872}{589824\sqrt[3]{-\alpha^3 + 3\alpha^2 + 78\sqrt{2}\sqrt{\alpha^6 - 6\alpha^5 + 15\alpha^4 - 20\alpha^3 + 15\alpha^2 - 6\alpha + 1} - 3\alpha + 1}} + \frac{1-\alpha}{8}$. Comparing the platform's profits under the two scenarios, we can obtain that there exists a h^0 such that when $h > h^0$, $\widehat{q}_C^* = 1/2$ and when $h < h^0$, $\widehat{q}_C^*$ is the solution of $288\frac{h}{1-\alpha}\widehat{q}_C - (96\frac{h}{1-\alpha} + 5(\widehat{q}_C)^2)\sqrt{(\widehat{q}_C)^2 + 12\frac{h}{1-\alpha}} + 32(\widehat{q}_C)^3 = 0$, where h^0 is the solution of

$$\frac{1}{4} - (\frac{1}{3}\widehat{q}_C^* + \frac{2}{3}\sqrt{(\widehat{q}_C^*)^2 + \frac{12h}{1-\alpha}})^3 + 3(\frac{1}{3}\widehat{q}_C^* + \frac{2}{3}\sqrt{(\widehat{q}_C^*)^2 + \frac{12h}{1-\alpha}})^2\widehat{q}_C^* - 3(\frac{1}{3}\widehat{q}_C^* + \frac{2}{3}\sqrt{(\widehat{q}_C^*)^2 + \frac{12h}{1-\alpha}})(\widehat{q}_C^*)^2 = 0. \tag{A.2}$$

Based on the implicit function theorem, we can further show $\partial\widehat{q}_C^*/\partial\alpha \geqslant 0$, $\partial\widehat{q}_C^*/\partial h \geqslant 0$.

Proof of Proposition 2. Without platform certification, the platform's and the seller i's expected profits are $\widehat{\Pi}_p^{nc} = \int_0^{\widehat{q}_D^*} \frac{\alpha(\widehat{q}_D^*)^2}{16} dq + \int_{\widehat{q}_D^*}^1 \frac{\alpha q^2}{4} dq = \frac{\alpha}{108}(9 - \frac{16\sqrt{3}h^{3/2}}{(1-\alpha)^{3/2}})$ and $\widehat{\Pi}_i^{nc} = \int_0^{\widehat{q}_D^*} \frac{(1-\alpha)(\widehat{q}_D^*)^2}{16} dq + \int_{\widehat{q}_D^*}^1 \frac{(1-\alpha)q^2}{4} - h\, dq = \frac{1-\alpha}{12} + \frac{32h^{3/2}}{9\sqrt{3-3\alpha}} - h$, respectively.

With platform certification, the platform's and the seller i's expected profits are, respectively,

$$\widehat{\Pi}_p = \begin{cases} \frac{5}{64}\alpha, & if\ h \geqslant h^0; \\ \int_0^{\widehat{q}_C^*} \frac{\alpha(\widehat{q}_C^*)^2}{16} dq + \int_{\widehat{q}_C^*}^{\widehat{q}_D^{c*}} \frac{\alpha(\widehat{q}_C^* + \widehat{q}_D^{c*})^2}{16} dq + \int_{\widehat{q}_D^{c*}}^1 \frac{\alpha q^2}{4} dq & otherwise. \end{cases}$$

$$\widehat{\Pi}_i = \begin{cases} \frac{5}{64}(1-\alpha), & if\ h \geqslant h^0; \\ \int_0^{\widehat{q}_C^*} \frac{(1-\alpha)(\widehat{q}_C^*)^2}{16} dq + \int_{\widehat{q}_C^*}^{\widehat{q}_D^{c*}} \frac{(1-\alpha)(\widehat{q}_C^* + \widehat{q}_D^{c*})^2}{16} dq + \int_{\widehat{q}_D^{c*}}^1 \frac{(1-\alpha)q^2}{4} - h\, dq & otherwise. \end{cases}$$

We then compare the platform's and sellers' expected profit with and without platform certification. If $h \geqslant h^0$, $\widehat{\Pi}_p - \widehat{\Pi}_p^{nc} = \frac{\alpha(-9+9\alpha+256\sqrt{\frac{3h}{1-\alpha}}h)}{1728(1-\alpha)}$. Since $\frac{\partial(\widehat{\Pi}_p - \widehat{\Pi}_p^{nc})}{\partial\alpha} = -\frac{1}{192} + \frac{2(2+\alpha)h}{9(1-\alpha)^2}\sqrt{\frac{h}{3-3\alpha}} > 0$ for $h \geqslant h^0$, $\widehat{\Pi}_p - \widehat{\Pi}_p^{nc} > \widehat{\Pi}_p - \widehat{\Pi}_p^{nc}|_{\alpha=0} = 0$. Moreover, $\frac{\partial(\widehat{\Pi}_p - \widehat{\Pi}_p^{nc})}{\partial h} = \frac{2\alpha}{3(1-\alpha)}\sqrt{\frac{h}{3-3\alpha}} > 0$. $\widehat{\Pi}_i - \widehat{\Pi}_i^{nc} = h - \frac{32h^{3/2}}{9\sqrt{3-3\alpha}} - \frac{1-\alpha}{192}$, $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial h} = 1 - \frac{16\sqrt{h}}{3\sqrt{3-3\alpha}} > 0$ for $h < \frac{27(1-\alpha)}{256}$ while $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial h} < 0$ otherwise. This together with the fact that $\widehat{\Pi}_i - \widehat{\Pi}_i^{nc}|_{h=h^0} > 0$ and $\widehat{\Pi}_i - \widehat{\Pi}_i^{nc}|_{h=3(1-\alpha)/16} > 0$ suggests $\widehat{\Pi}_i - \widehat{\Pi}_i^{nc} > 0$ always holds. Also, $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial\alpha} = \frac{1}{192} - \frac{16h^{3/2}}{3(3-3\alpha)^{3/2}} < 0$ always holds.

If $h < h^0$, then according to the envelope theorem, $\frac{\partial(\widehat{\Pi}_p - \widehat{\Pi}_p^{nc})}{\partial h} = \frac{2\alpha\left(\widehat{q}_C^*\left(\sqrt{(1-\alpha)[(1-\alpha)(\widehat{q}_C^*)^2+12h]}+(\alpha-1)\widehat{q}_C^*\right)-6h\right)}{9(1-\alpha)\sqrt{(1-\alpha)[(1-\alpha)(\widehat{q}_C^*)^2+12h]}} + \frac{2\alpha\sqrt{h}}{3\sqrt{3}(1-\alpha)^{3/2}}$. Through the mathematical derivation and algebraic calculation, we can obtain $\frac{\partial(\widehat{\Pi}_p - \widehat{\Pi}_p^{nc})}{\partial h} > 0$ and $\widehat{\Pi}_p - \widehat{\Pi}_p^{nc}|_{h=0} > 0$, suggesting $\widehat{\Pi}_p - \widehat{\Pi}_p^{nc} > 0$ always holds. Following the similar approach, we can show that $\frac{\partial(\widehat{\Pi}_p - \widehat{\Pi}_p^{nc})}{\partial\alpha} > 0$.

As for seller i's profit, $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial h} = \frac{\left(5(1-\alpha)(\widehat{q}_C^*)^2\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}+240h\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}-32(1-\alpha)^2(\widehat{q}_C^*)^3\right)\frac{\partial\widehat{q}_C^*}{\partial h}}{432\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}} + \frac{48\left(5\widehat{q}_C^*\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}-9\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}+96h-4\alpha(\widehat{q}_C^*)^2+4(\widehat{q}_C^*)^2\right)}{432\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}} - (\frac{16\sqrt{h}}{3\sqrt{3-3\alpha}} - 1)$ and $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial\alpha} = \frac{3(1-\alpha)\left(5(\alpha-1)(\widehat{q}_C^*)^2\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}-240h\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}+32(\alpha-1)^2(\widehat{q}_C^*)^3\right)\frac{\partial\widehat{q}_C^*}{\partial\alpha}}{1296(\alpha-1)\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}} + \frac{-4608h^2+192(\alpha-1)h(\widehat{q}_C^*)^2-(\alpha-1)\left(108\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}+5(\widehat{q}_C^*)^3\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}+32(\alpha-1)(\widehat{q}_C^*)^4\right)}{1296(\alpha-1)\sqrt{(1-\alpha)\left((1-\alpha)(\widehat{q}_C^*)^2+12h\right)}} - (\frac{16h^{3/2}}{3(3-3\alpha)^{3/2}} - \frac{1}{12})$. Through the mathematical derivation and algebraic calculation, we can show that $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial h} > 0$ and $\frac{\partial(\widehat{\Pi}_i - \widehat{\Pi}_i^{nc})}{\partial\alpha} > 0$.

Last, we compare the sellers' ex post profits with and without platform certification, which are given by the following table:

	Without platform certification	With platform certification
$q_i \in [0, \widehat{q}_C^*]$,	$(\widehat{q}_D^*)^2/16$	$(\widehat{q}_C^*)^2/16$
$q_i \in (\widehat{q}_C^*, \widehat{q}_D^*]$,	$(\widehat{q}_D^*)^2/16$	$(\widehat{q}_C^* + \widehat{q}_D^{c*})^2/16$
$q_i \in (\widehat{q}_D^*, \widehat{q}_D^{c*}]$,	$q_i^2/4 - h$	$(\widehat{q}_C^* + \widehat{q}_D^{c*})^2/16$
$q_i \in (\widehat{q}_D^{c*}, 1]$,	$q_i^2/4 - h$	$q_i^2/4 - h$

We then can obtain the results stated in Proposition 2.

Proof of Lemma 3. Given the seller's payoffs under disclosure $\tilde{\pi}_i^d = q_i^2/4 - h - s$ and under non-disclosure $\tilde{\pi}_i^{nd} = \max(\tilde{q}_D^2/16 - s, 0)$, it is evident that at $\tilde{q}_D^*$, we have $\pi_i^d = \pi_i^{nd}$. We first assume that

$\tilde{q}_D^2/16 - s \geqslant 0$, which leads to the equilibrium $\tilde{q}_D^* = \min\left\{4\sqrt{\frac{h}{3}}, 1\right\}$. The existence condition for this equilibrium result is $h \geqslant 3s$. We then assume that $\tilde{q}_D^2/16 - s < 0$, which leads to the equilibrium $\tilde{q}_D^* = 2\sqrt{s+h}$, which exists only when $h < 3s$. Summarizing the above results, we can obtain Lemma 3.

Proof of Lemma 4. We first discuss the scenario when the disclosure cost $h \geqslant 3s$, wherein all the sellers sell through the platform in the absence of platform certification. If $\tilde{q}_C < \tilde{q}_D^*$, then all the uncertified sellers withhold quality information, and their profit $\tilde{\pi}_i^{nc,nd} = \tilde{q}_C^2/16 - s \geqslant 0$ only when $\tilde{q}_C \geqslant 4\sqrt{s}$. Otherwise, they quit platform selling due to the negative profit. If $\tilde{q}_C \geqslant \tilde{q}_D^*$, then according to Lemma 3, all sellers sell through the platform.

We now move to the scenario when the disclosure cost $h < 3s$. If $\tilde{q}_C < \tilde{q}_D^*$, then all the uncertified sellers withhold quality information and quit platform selling. As for a certified seller i, its payoff under disclosure is $\tilde{\pi}_i^{c,d} = q_i^2/4 - h - s$ and under non-disclosure is $\tilde{\pi}_i^{c,nd} = (\tilde{q}_C + \tilde{q}_D^c)^2/16 - s$. Thus, it is evident that at $\tilde{q}_D^c$, we have $\tilde{\pi}_i^{c,d} = \tilde{\pi}_i^{c,nd}$, which leads to the certified seller i's equilibrium disclosure threshold $\tilde{q}_D^c = \min(1, \frac{1}{3}\tilde{q}_C + \frac{2}{3}\sqrt{\tilde{q}_C^2 + 12h})$. The requirement for a certified non-disclosure seller's profit to be nonnegative is $\tilde{q}_C \geqslant 4\sqrt{s} - \tilde{q}_D^*$. If that is negative, they quit platform selling. The other certified sellers always sell through the platform and reveal their quality information. If $\tilde{q}_C > \tilde{q}_D^*$, then platform certification does not change the sellers' participation strategy.

Proof of Proposition 3. When $h \geqslant 3s$ and $\tilde{q}_C < 4\sqrt{s}$, $\tilde{\Pi}_p = s(1 - \tilde{q}_C)$, which decreases in $\tilde{q}_C$ and the equilibrium $\tilde{q}_C^* = 0$. When $h \geqslant 3s$ and $\tilde{q}_C \geqslant 4\sqrt{s}$, we have $\tilde{\Pi}_p = s$, which is independent of $\tilde{q}_C$. Therefore, when $h \geqslant 3s$, $\tilde{q}_C^* = 1$, and $\tilde{\Pi}_p = s$. When $h < 3s$, $\tilde{\Pi}_p = s(1 - \max\{4\sqrt{s} - \tilde{q}_D^*, \min\{\tilde{q}_C, \tilde{q}_D^*\}\})$, which decreases in $\tilde{q}_C$, and the equilibrium $\tilde{q}_C^* = 4\sqrt{s} - \tilde{q}_D^*$. Obviously, $\tilde{q}_C^*$ increases in s and decreases in h.

Proof of Proposition 4. When $h \geqslant 3s$, the platform does not certify the sellers, and both the platform's and the sellers' profits remain unchanged. When $h < 3s$, sellers whose quality $q_i \in [4\sqrt{s} - \tilde{q}_D^*, \tilde{q}_D^*)$ switch from quitting to selling through the platform, leading to an increase in the platform's profit $\Delta\tilde{\Pi}_p = s[\tilde{q}_D^* - (4\sqrt{s} - \tilde{q}_D^*)] = s(4\sqrt{s+h} - 4\sqrt{s})$ and $\frac{\partial \Delta\tilde{\Pi}_p}{\partial h} = \frac{2s}{\sqrt{h+s}} > 0, \frac{\partial \Delta\tilde{\Pi}_p}{\partial s} = \frac{4h+6s-6\sqrt{s}\sqrt{h+s}}{\sqrt{h+s}} > 0$. However, the profit of a seller with product quality $q_i \in [4\sqrt{s} - \tilde{q}_D^*, \tilde{q}_D^*)$ still equals zero since its sales revenue can just cover the listing fee. Besides, when $h < 3s$, platform certification does not change the participation nor the disclosure strategies of those sellers whose quality level $q_i \in [0, 4\sqrt{s} - \tilde{q}_D^*) \cup [\tilde{q}_D^*, 1)$, which implies that these sellers' profits remain unchanged. Thus, platform certification has no impact on the sellers' profits.

Proof of Proposition 5. For an uncertified seller i, its payoff under disclosure is given by $\pi_i^{nc,d} = (1-\alpha)q_i^2/4 - h - s$ and under non-disclosure is given by $\pi_i^{nc,nd} = (1-\alpha)(q_D^{nc})^2/16 - s$. It is evident that at q_D^{nc}, we have $\pi_i^{nc,d} = \pi_i^{nc,nd}$. We first assume that $(1-\alpha)(q_D^{nc})^2/16 - s \geqslant 0$, which leads to the equilibrium $q_D^{nc} = \min\{q_D^*, q_C\}$. The requirement for this equilibrium result is $h \geqslant 3s$. To ensure the existence of this solution, we must have $\alpha < \overline{\alpha} = \max\{1 - \frac{16}{3}h, 1 - 4(s+h)\}$, $h < \overline{h} = 3/16$, and $s < \overline{s} = 1/4 - h$. We then assume that $(1-\alpha)(q_D^{nc})^2/16 - s < 0$, so $q_D^{nc} = \sqrt{\frac{4(s+h)}{1-\alpha}}$, which exists only when $h < 3s$ and $q_C > \sqrt{\frac{4(s+h)}{1-\alpha}}$. Following a similar approach, we can obtain a certified seller i's equilibrium disclosure threshold $q_D^h = \min\left(1, \frac{1}{3}q_C + \frac{2}{3}\sqrt{q_C^2 + \frac{12h}{1-\alpha}}\right)$.

(i) When $h \geqslant 3s$, following the logic in the proof of Proposition 1, we can easily verify that the platform never sets a quality standard $q_C > q_D^*$. Next, we move to the scenario $q_C \leqslant q_D^*$. Note that $q_D^c < 1$ requires $q_C < -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$ and the profit of the uncertified sellers $\pi_i^{nc,nd} = (1-\alpha)(q_C)^2/16 - s \geqslant 0$ requires $q_C \geqslant 4\sqrt{\frac{s}{1-\alpha}}$, which leads to the following three subcases:

Case 1: $4\sqrt{\frac{s}{1-\alpha}} < q_D^* < -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}$, which requires $h < \frac{3(7-\sqrt{13})(1-\alpha)}{128}$, under which

$$\Pi_p = \begin{cases} \displaystyle\int_{q_C}^{q_D^c} \frac{\alpha(q_C + q_D^c)^2}{16} + s\,dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } q_C < 4\sqrt{\dfrac{s}{1-\alpha}}; \\ \displaystyle\int_{0}^{q_C} \frac{\alpha q_C^2}{16} dq + \int_{q_C}^{q_D^c} \frac{\alpha(q_C + q_D^c)^2}{16} dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} dq + s, & \text{if } 4\sqrt{\dfrac{s}{1-\alpha}} \leqslant q_C < q_D^*. \end{cases}$$

(1) If $q_C < 4\sqrt{\frac{s}{1-\alpha}}$, then $\frac{\partial \Pi_p}{\partial q_C} = \frac{\alpha\left(288h^2 - 12hq_C\left(6\sqrt{(\alpha-1)\left((\alpha-1)q_C^2 - 12h\right)} - 17(\alpha-1)q_C\right)\right)}{108(\alpha-1)\left((\alpha-1)q_C^2 - 12h\right)} +$

$\frac{\alpha(\alpha-1)q_C^3\left(8\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}-19(\alpha-1)q_C\right)}{108(\alpha-1)\left((\alpha-1)q_C^2-12h\right)} - s$ and $\frac{\partial^2\Pi_p}{\partial q_C^2} < 0$. We can show that $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=0} = \frac{2\alpha h}{9-9\alpha} - s > 0$ for $s < \frac{2\alpha h}{9-9\alpha}$ and $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=0} < 0$ otherwise. We can also obtain that $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} > 0$ for $s < \bar{s}$ and $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} < 0$ otherwise, where $\bar{s}$ is the solution of $4(49\alpha+27)s^2 + 3(41\alpha+27)sh + 64\alpha s\sqrt{\frac{s}{1-\alpha}}\sqrt{(1-\alpha)(4s+3h)} + 18\alpha h\left(2\sqrt{\frac{s}{1-\alpha}}\sqrt{(1-\alpha)(4s+3h)} - h\right) = 0$. We then have that $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$ when $s < \bar{s}$, $q_C^* = 0$ when $s > \frac{2\alpha h}{9-9\alpha}$, and q_C^* is the solution of $\frac{\alpha\left(288h^2-12hq_C\left(6\sqrt{(1-\alpha)\left((1-\alpha)q_C^2+12h\right)}-17(\alpha-1)q_C\right)+(\alpha-1)q_C^3\left(8\sqrt{(1-\alpha)\left((1-\alpha)q_C^2+12h\right)}+19(1-\alpha)q_C\right)\right)}{108(1-\alpha)\left((1-\alpha)q_C^2+12h\right)} - s = 0$ otherwise.

(2) If $4\sqrt{\frac{s}{1-\alpha}} \leqslant q_C < q_D^*$, $\frac{\partial\Pi_p}{\partial q_C} = \frac{\alpha\left((\alpha-1)q_C^2\left(5\sqrt{(1-\alpha)\left((1-\alpha)q_C^2+12h\right)}-32(1-\alpha)q_C\right)-96h\left(\sqrt{(1-\alpha)\left((1-\alpha)q_C^2+12h\right)}-3(1-\alpha)q_C\right)\right)}{432(\alpha-1)\sqrt{(1-\alpha)\left((1-\alpha)q_C^2+12h\right)}}$ and $\frac{\partial^2\Pi_p}{\partial q_C^2} < 0$. Therefore, we can show that $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} > 0$ for $s < \frac{1}{148}\left(\sqrt[3]{10064\sqrt{2}h^3+36123h^3} + \frac{1033h^2}{\sqrt[3]{10064\sqrt{2}h^3+36123h^3}} - 53h\right)$ and $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} < 0$ otherwise. Also, we can show that $\left.\frac{\partial\Pi_p}{\partial q_C}\right|_{q_C=q_D^*} < 0$. Then, we shall have $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$ for $s > \frac{1}{148}\left(\sqrt[3]{10064\sqrt{2}h^3+36123h^3} + \frac{1033h^2}{\sqrt[3]{10064\sqrt{2}h^3+36123h^3}} - 53h\right)$ and q_C^* is the solution of $\frac{\alpha\left((\alpha-1)q_C^2\left(5\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}+32(\alpha-1)q_C\right)-96h\left(\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}+3(\alpha-1)q_C\right)\right)}{432(\alpha-1)\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}} = 0$ otherwise.

Comparing the platform's profit under the two scenarios, we can obtain that when $s < s_1$, $q_C^* = q_0$, which is the solution of

$$\frac{\alpha\left((\alpha-1)q_C^2\left(5\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}+32(\alpha-1)q_C\right)-96h\left(\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}+3(\alpha-1)q_C\right)\right)}{432(\alpha-1)\sqrt{(\alpha-1)\left((\alpha-1)q_C^2-12h\right)}} = 0; \tag{A.3}$$

when $s_1 < s < s_2$, $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$; and $q_C^* = 1$ otherwise, where

$$s_1 = \frac{1}{148}\left(\sqrt[3]{10064\sqrt{2}h^3+36123h^3} + \frac{1033h^2}{\sqrt[3]{10064\sqrt{2}h^3+36123h^3}} - 53h\right) \tag{A.4}$$

and s_2 is the solution of

$$s\left(5s - 16\sqrt{-\frac{s}{\alpha-1}}\sqrt{(\alpha-1)(-(3h+4s))}\right) + 3h\sqrt{-\frac{s}{\alpha-1}}\left(\sqrt{3}\sqrt{h-\alpha h} - \sqrt{(\alpha-1)(-(3h+4s))}\right) + 18hs = 0. \tag{A.5}$$

What's more,

$$q_D^{c*} = \frac{1}{3}q_C^* + \frac{2}{3}\sqrt{(q_C^*)^2 + \frac{12h}{1-\alpha}}. \tag{A.6}$$

Case 2: $4\sqrt{\frac{s}{1-\alpha}} < -1 + 2\sqrt{1-\frac{4h}{1-\alpha}} < q_D^*$, which requires $\frac{3(7-\sqrt{13})(1-\alpha)}{128} < h < \frac{1-\alpha}{16}(3 - 8\sqrt{\frac{s}{1-\alpha}}) - s$,

$$\Pi_p = \begin{cases} \int_{q_C}^{q_D^c} \frac{\alpha(q_C+q_D^c)^2}{16} + s\,dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } q_C < 4\sqrt{\frac{s}{1-\alpha}}; \\ \int_0^{q_C} \frac{\alpha q_C^2}{16}dq + \int_{q_C}^{q_D^c} \frac{\alpha(q_C+q_D^c)^2}{16}dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4}dq + s, & \text{if } 4\sqrt{\frac{s}{1-\alpha}} \leqslant q_C < -1 + 2\sqrt{1-\frac{4h}{1-\alpha}}; \\ \int_0^{q_C} \frac{\alpha q_C^2}{16}dq + \int_{q_C}^{1} \frac{\alpha(q_C+1)^2}{16}dq + s, & \text{if } -1 + 2\sqrt{1-\frac{4h}{1-\alpha}} \leqslant q_C < q_D^*. \end{cases}$$

(1) If $q_C < 4\sqrt{\frac{s}{1-\alpha}}$, or $4\sqrt{\frac{s}{1-\alpha}} \leqslant q_C < -1 + 2\sqrt{1-\frac{4h}{1-\alpha}}$, the equilibrium strategy is the same as that in Case 1. (2) If $-1 + 2\sqrt{1-\frac{4h}{1-\alpha}} \leqslant q_C < q_D^*$, then $\Pi_p = s + \frac{\alpha}{16}(1 + q_C - q_C^2)$. We can show that $q_C^* = 1/2$, which exists only when $h > \frac{7(1-\alpha)}{64}$.

Comparing the platform's profit under the above three scenarios, we can obtain that when $h > h^1$, $q_C^* = 1/2$; When $h < h^1$, if $s < s_1$, then $q_C^* = q_0$; when $s_1 < s < s_2$, we have $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$; otherwise, $q_C^* = 1$, wherein

$$h^1 = \min\{h^0, h^2\} \tag{A.7}$$

and h^2 is the solution of

$$\frac{768h\left(\sqrt{(1-\alpha)(4s+3h)} - \frac{6s}{\sqrt{\frac{s}{1-\alpha}}}\right)}{(1-\alpha)^2} + \frac{256s\left(16\sqrt{(1-\alpha)(4s+3h)} - \frac{5s}{\sqrt{\frac{s}{1-\alpha}}}\right)}{(1-\alpha)^2} - 27 = 0 \tag{A.8}$$

Case 3: $-1+2\sqrt{1-\frac{4h}{1-\alpha}} < 4\sqrt{\frac{s}{1-\alpha}} < q_D^*$, which requires $h > \frac{1-\alpha}{16}(3 - 8\sqrt{\frac{s}{1-\alpha}}) - s$, under which

$$\Pi_p = \begin{cases} \int_{q_C}^{q_D^c} \frac{\alpha(q_C+q_D^c)^2}{16} + s\,dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } q_C < -1+2\sqrt{1-\frac{4h}{1-\alpha}}; \\ \int_{q_C}^{1} \frac{\alpha(q_C+1)^2}{16} + s\,dq, & \text{if } -1+2\sqrt{1-\frac{4h}{1-\alpha}} \leqslant q_C < 4\sqrt{\frac{s}{1-\alpha}}; \\ \int_{0}^{q_C} \frac{\alpha q_C^2}{16}dq + \int_{q_C}^{1} \frac{\alpha(q_C+1)^2}{16}dq + s, & \text{if } 4\sqrt{\frac{s}{1-\alpha}} \leqslant q_C < q_D^*. \end{cases}$$

(1) If $q_C < -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, then $\frac{\partial \Pi_p}{\partial q_C} < 0$. Thus, $q_C^* = 0$.

(2) If $-1+2\sqrt{1-\frac{4h}{1-\alpha}} \leqslant q_C < 4\sqrt{\frac{s}{1-\alpha}}$, then $\Pi_p = (1-q_C)[s + \frac{\alpha}{16}(1+q_C)^2]$, $\frac{\partial \Pi_p}{\partial q_C} = \frac{-16s+\alpha-2\alpha q_C-3\alpha q_C^2}{16}$, and $\frac{\partial^2 \Pi_p}{\partial q_C^2} < 0$. Note that $\frac{\partial \Pi_p}{\partial q_C}\Big|_{q_C=-1+2\sqrt{1-\frac{4h}{1-\alpha}}} > 0$ for $s < \frac{1}{2}\sqrt{\frac{\alpha^3-\alpha^2+4\alpha^2 h}{\alpha-1}} - \frac{3(\alpha^2-\alpha+4\alpha h)}{4(\alpha-1)}$ and $\frac{\partial \Pi_p}{\partial q_C}\Big|_{q_C=-1+2\sqrt{1-\frac{4h}{1-\alpha}}} < 0$ otherwise. We can also show that $\frac{\partial \Pi_p}{\partial q_C}\Big|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} > 0$ for $s < \frac{-4\alpha^3+3\alpha^2+\alpha}{16(2\alpha+1)^2} - \frac{1}{8}\sqrt{\frac{3\alpha^6-5\alpha^5+\alpha^4+\alpha^3}{(2\alpha+1)^4}}$ and $\frac{\partial \Pi_p}{\partial q_C}\Big|_{q_C=4\sqrt{\frac{s}{1-\alpha}}} < 0$ otherwise. Therefore, the equilibrium solution shall be one of the following three: $q_C^* = -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$ or $q_C^* = \frac{-\alpha+2\sqrt{-12s\alpha+a^2}}{3\alpha}$.

(3) If $4\sqrt{\frac{s}{1-\alpha}} \leqslant q_C < q_D^*$, then $\Pi_p = s + \frac{\alpha}{16}(1+q_C-q_C^2)$. Thus, we have $q_C^* = 1/2$, which exists only when $s < \frac{1-\alpha}{64}$. Otherwise, Π_p decreases in q_C, and thus $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$.

Comparing the platform's profit under the above three scenarios, we can obtain that when $s < \frac{1-\alpha}{64}$, we have $q_C^* = 1/2$; when $\frac{1-\alpha}{64} < s < s_2$, we have $q_C^* = 4\sqrt{\frac{s}{1-\alpha}}$; otherwise, $q_C^* = 1$.

(ii) When $h < 3s$, $q_D^c < 1$ requires $q_C < -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, which leads to the following two subcases:

Case 1: $q_D^* = \sqrt{\frac{4(s+h)}{1-\alpha}} < -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, which requires $h < \frac{17-20s-17\alpha-8\sqrt{1+5s-2\alpha-5\alpha s+a^2}}{100}$, under which

$$\Pi_p = \begin{cases} \int_{q_D^*}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } q_C < 4\sqrt{\frac{s}{1-\alpha}} - q_D^c; \\ \int_{q_C}^{q_D^c} \frac{\alpha(q_C+q_D^*)^2}{16} + s\,dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } 4\sqrt{\frac{s}{1-\alpha}} - q_D^c \leqslant q_C < q_D^*. \end{cases}$$

Case 2: $q_D^* = \sqrt{\frac{4(s+h)}{1-\alpha}} > -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, which requires $h > \frac{17-20s-17\alpha-8\sqrt{1+5s-2\alpha-5\alpha s+a^2}}{100}$, under which

$$\Pi_p = \begin{cases} \int_{q_D^*}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } q_C < 4\sqrt{\frac{s}{1-\alpha}} - q_D^c; \\ \int_{q_C}^{q_D^c} \frac{\alpha(q_C+q_D^*)^2}{16} + s\,dq + \int_{q_D^c}^{1} \frac{\alpha q^2}{4} + s\,dq, & \text{if } 4\sqrt{\frac{s}{1-\alpha}} - q_D^c \leqslant q_C < -1+2\sqrt{1-\frac{4h}{1-\alpha}}; \\ \int_{q_C}^{1} \frac{\alpha(q_C+1)^2}{16} + s\,dq & \text{if } -1+2\sqrt{1-\frac{4h}{1-\alpha}} \leqslant q_C < q_D^*. \end{cases}$$

Solving the maximization problem for the above two cases, we can easily obtain $q_C^* = 4\sqrt{\frac{s}{1-\alpha}} - 2\sqrt{\frac{s+h}{1-\alpha}}$ and $q_D^{c*} = 2\sqrt{\frac{s+h}{1-\alpha}}$. Summarizing the above results, we obtain Proposition 5.

Proof of Proposition 6. The platform decides the commission rate and the listing fee simultaneously, but we can solve the platform's optimal strategy sequentially. Given the commission rate α, we first obtain the optimal listing fee. (1) When $h < 3s$, we have $\Pi_p = \frac{\alpha\left(\alpha+8h\sqrt{\frac{s+h}{1-\alpha}}-1\right)+4s\left(12\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s+h}{1-\alpha}}+\alpha\left(3-4\sqrt{\frac{s+h}{1-\alpha}}\right)-3\right)}{12(\alpha-1)}$, $\frac{\partial\Pi_p}{\partial s} = \frac{s\left(2\alpha\sqrt{\frac{s}{1-\alpha}}+3\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s+h}{1-\alpha}}\right)+\sqrt{\frac{s}{1-\alpha}}\left((1-\alpha)^2\sqrt{\frac{s+h}{1-\alpha}}+(\alpha+2)h\right)}{(1-\alpha)\sqrt{s(s+h)}}$ and $\frac{\partial\Pi_p}{\partial s}|_{s=h/3} = 1 - \frac{3-5\alpha}{2(1-\alpha)}\sqrt{\frac{h}{3-3\alpha}} > 0$, $\frac{\partial\Pi_p}{\partial s}|_{s=(1-\alpha)/4-h} < 0$ for $\alpha < \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$ while $\frac{\partial\Pi_p}{\partial s}|_{s=(1-\alpha)/4-h} > 0$ otherwise. Therefore, when $\alpha < \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$, the optimal s^* is the solution of $\frac{s\left(2\alpha\sqrt{\frac{s}{1-\alpha}}+3\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s+h}{1-\alpha}}\right)+\sqrt{\frac{s}{1-\alpha}}\left((1-\alpha)^2\sqrt{\frac{s+h}{1-\alpha}}+(\alpha+2)h\right)}{(1-\alpha)\sqrt{s(s+h)}} = 0$ and otherwise $s^* = (1-\alpha)/4-h$. (2) When $s < s_1$ and $h \geqslant h^1$, we have $\Pi_p = \frac{5\alpha}{64}+s$, which increases in s, and the optimal $s^* = s_1$. (3) When $s < s_1$ and $3s < h < h^1$, $\Pi_p = s - \frac{2\alpha h\left(\sqrt{(\alpha-1)\left((\alpha-1)(q_C^*)^2-12h\right)}+3(\alpha-1)q_C^*\right)}{27(\alpha-1)^2} + \frac{\alpha\left(\frac{32(q_C^*)^2\sqrt{(\alpha-1)\left((\alpha-1)(q_C^*)^2-12h\right)}}{\alpha-1}+5(q_C^*)^3+108\right)}{1296}$, which increases in s, and the optimal $s^* = s_1$. (4) When $h > 3s$ and $s_1 < s < s_2$, we have $\Pi_p = \frac{3\alpha\left(9(1-\alpha)^2-16h\left(6\alpha\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s}{1-\alpha}}+\sqrt{(1-\alpha)(4s+3h)}\right)\right)-4s\left(\alpha^2\left(20\sqrt{\frac{s}{1-\alpha}}-81\right)+\alpha\left(-20\sqrt{\frac{s}{1-\alpha}}+64\sqrt{(1-\alpha)(4s+3h)}+162\right)-81\right)}{324(1-\alpha)^2}$ and $\frac{\partial\Pi_p}{\partial s} = \frac{27(1-\alpha)^2\sqrt{s(4s+3h)}+12\alpha h\left(6\alpha\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s}{1-\alpha}}+\sqrt{(1-\alpha)(4s+3h)}\right)+2\alpha h\left(64\alpha\sqrt{\frac{s}{1-\alpha}}-64\sqrt{\frac{s}{1-\alpha}}+5\sqrt{(1-\alpha)(4s+3h)}\right)}{27(1-\alpha)^2\sqrt{s(4s+3h)}} > 0$, which leads to the optimal $s^* = s_2$. (5) When $h > 3s$ and $s > s_2$, we have $\Pi_p = s + \frac{\alpha(9-9\alpha-16h\sqrt{\frac{3h}{1-\alpha}})}{108(1-\alpha)}$, which increases in s, and the optimal $s^* = h/3$. Summarizing the above results, we can obtain that given the commission rate α, when $\alpha < \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$, the optimal s^* is the solution of

$$\frac{s\left(2\alpha\sqrt{\frac{s}{1-\alpha}}+3\sqrt{\frac{s}{1-\alpha}}-6\sqrt{\frac{s+h}{1-\alpha}}\right)+\sqrt{\frac{s}{1-\alpha}}\left((1-\alpha)^2\sqrt{\frac{s+h}{1-\alpha}}+(\alpha+2)h\right)}{(1-\alpha)\sqrt{s(s+h)}} = 0 \tag{A.9}$$

and when $\alpha > \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$, $s^* = (1-\alpha)/4-h$.

Next, we solve for the optimal commission rate. (1) When $\alpha > \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$, $\Pi_p = \frac{(\alpha+4h-1)\left(\sqrt{\frac{1-\alpha-4h}{1-\alpha}}-1\right)}{2(1-\alpha)}$, and then $\frac{\partial\Pi_p}{\partial\alpha} = \frac{h\left(12h+(1-\alpha)\left(2\sqrt{\frac{1-\alpha-4h}{1-\alpha}}-3\right)\right)}{(\alpha-1)^3\sqrt{\frac{1-\alpha-4h}{1-\alpha}}}$ and $\frac{\partial^2\Pi_p}{\partial\alpha^2} < 0$. Note that $\frac{\partial\Pi_p}{\partial\alpha}|_{\alpha=1-16/3h} < 0$ and $\frac{\partial\Pi_p}{\partial\alpha}|_{\alpha=\max\{0,\frac{-11+72h-16c^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}} > 0$ for $\frac{1}{36} < h < \frac{5}{36}$ while $\frac{\partial\Pi_p}{\partial\alpha}|_{\alpha=\max\{0,\frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}} < 0$ otherwise, which implies that when $\frac{1}{36} < h < \frac{5}{36}$, $\alpha^* = \frac{5-36h}{5}$ and $\alpha^* = 0$ otherwise. (2) When $\alpha < \max\{0, \frac{-11+72h-16h^2+24h\sqrt{3-8h}}{-11+40h+16h^2}\}$, $\Pi_p = \frac{\alpha\left(\alpha+8h\sqrt{\frac{s^*+h}{1-\alpha}}-1\right)+4s^*\left(12\sqrt{\frac{s^*}{1-\alpha}}-6\sqrt{\frac{s^*+h}{1-\alpha}}+\alpha\left(3-4\sqrt{\frac{s^*+h}{1-\alpha}}\right)-3\right)}{12(\alpha-1)}$ and we can show that the optimal $\alpha^* = \alpha_1$ is the solution of

$$\begin{aligned}&\frac{(s^*+h)\left(s^*\left(2\alpha\sqrt{\frac{s^*}{1-\alpha}}+3\sqrt{\frac{s^*}{1-\alpha}}-6\sqrt{\frac{s^*+h}{1-\alpha}}\right)+\sqrt{\frac{s^*}{1-\alpha}}\left((\alpha-1)^2\sqrt{\frac{s^*+h}{1-\alpha}}+(\alpha+2)h\right)\right)\left(s^*\left(2\alpha\sqrt{\frac{s^*}{1-\alpha}}+13\sqrt{\frac{s^*}{1-\alpha}}-18\sqrt{\frac{s^*+h}{1-\alpha}}\right)+(\alpha+8)h\sqrt{\frac{s^*}{1-\alpha}}\right)}{\left(s^*\left(2\alpha\sqrt{\frac{s^*}{1-\alpha}}+3\sqrt{\frac{s^*}{1-\alpha}}-6\sqrt{\frac{s^*+h}{1-\alpha}}\right)+h\left(3\alpha\sqrt{\frac{s^*}{1-\alpha}}+4\sqrt{\frac{s^*}{1-\alpha}}-6\sqrt{\frac{s^*+h}{1-\alpha}}\right)\right)}\\&+\frac{(s^*)^2\left(-8\alpha\sqrt{\frac{s^*}{1-\alpha}}-52\sqrt{\frac{s^*}{1-\alpha}}+72\sqrt{\frac{s^*+h}{1-\alpha}}\right)+\sqrt{\frac{s^*}{1-\alpha}}\left((\alpha-1)^3\sqrt{\frac{s^*+h}{1-\alpha}}+4(\alpha+2)h^2\right)-4(\alpha+11)s^*h\sqrt{\frac{s^*}{1-\alpha}})}{12} = 0.\end{aligned} \tag{A.10}$$

Comparing the platform's profit in above two scenarios, we can complete the proof of Proposition 6.

Proof of Proposition 7. Given the consumers' quality expectation $\overline{q}_i$, with backward induction, the platform determines the optimal retail price to maximize its profit $\Pi_p = \sum_i (\overline{q}_i - p_i)(p_i - w_i)$, which leads to the optimal retail price $p_i = \frac{\overline{q}_i + w_i}{2}$. Substituting the optimal retail price into the seller i's profit function $\pi_i = (\overline{q}_i - p_i)w_i - \mathbb{1} \cdot h$ and solving the first order condition of π_i with respect to w_i, we can obtain $w_i = \frac{\overline{q}_i}{2}$. In equilibrium, the platform's profit is $\Pi_p = \sum_i \frac{\overline{q}_i^2}{16}$ and the seller i's profit is $\pi_i = \frac{\overline{q}_i^2}{8} - \mathbb{1} \cdot h$.

Next, we derive the seller i's optimal quality disclosure strategy. For a certified seller i, the payoff is $\pi_i^{w,c,d} = q_i^2/8 - h$ under disclosure and $\pi_i^{w,c,nd} = (q_C^w + q_D^{w,c})^2/32$ under non-disclosure. Solving $\pi_i^{w,c,d} = \pi_i^{w,c,nd}$ leads to the equilibrium disclosure threshold $q_D^{w,c} = \min(1, \frac{1}{3}q_C^w + \frac{2}{3}\sqrt{(q_C^w)^2 + 24h})$. Similarly, we can obtain that the uncertified seller i's equilibrium disclosure threshold $q_D^{w,nc} = \min(q_C^w, q_D^{w*})$, where $q_D^{w*} = 4\sqrt{\frac{2h}{3}}$.

Following the similar reasoning for the proof of Proposition 1, we can show that the platform would never set a quality standard $q_C^w > q_D^{w*}$. Therefore, we focus on the scenario $q_C^w \leqslant q_D^{w*}$. Note that $q_D^{w,c} < 1$ requires $q_C^w < -1 + 2\sqrt{1-8h}$, which leads to the following two cases:

Case 1: $q_D^{w*} \leqslant -1 + 2\sqrt{1-8h}$, which requires $h \leqslant \frac{3(7-\sqrt{13})}{256}$, under which $\Pi_p^w = \int_0^{q_C^w} \frac{(q_C^w)^2}{64} dq + \int_{q_C^w}^{q_D^{w,c}} \frac{(q_C^w + q_D^{w,c})^2}{64} dq + \int_{q_D^{w,c}}^1 \frac{q^2}{16} dq$. Solving the maximization problem, we can obtain $\frac{\partial \Pi_p^w}{\partial q_C^w} = \frac{h}{9}(1 + \frac{q_C^w}{\sqrt{24h + (q_C^w)^2}}) + \frac{q_C^w(5q_C^w - 32\sqrt{24h + (q_C^w)^2})}{1728}$, and $\frac{\partial^2 \Pi_p^w}{\partial (q_C^w)^2} < 0$. This together with the fact that $\frac{\partial \Pi_p^w}{\partial q_C^w}|_{q_C^w = 0} > 0$ and $\frac{\partial \Pi_p^w}{\partial q_C^w}|_{q_C^w = q_D^{w*}} < 0$ suggests that there exists a unique $q_C^{w*} = q_w$ satisfying

$$\frac{h}{9}(1 + \frac{q_C^w}{\sqrt{24h + (q_C^w)^2}}) + \frac{q_C^w(5q_C^w - 32\sqrt{24h + (q_C^w)^2})}{1728} = 0 \tag{A.11}$$

that maximizes the platform's profit.

Case 2: $q_D^{w*} > -1 + 2\sqrt{1-8h}$, which requires $h > \frac{3(7-\sqrt{13})}{256}$, under which

$$\Pi_p^w = \begin{cases} \int_0^{q_C^w} \frac{(q_C^w)^2}{64} dq + \int_{q_C^w}^{q_D^{w,c}} \frac{(q_C^w + q_D^{w,c})^2}{64} dq + \int_{q_D^{w,c}}^1 \frac{q^2}{16} dq, & \text{if } q_C^w \leqslant -1 + 2\sqrt{1-8h}; \\ \int_0^{q_C^w} \frac{(q_C^w)^2}{64} dq + \int_{q_C^w}^1 \frac{(q_C^w + 1)^2}{64} dq, & \text{if } -1 + 2\sqrt{1-8h} < q_C^w < q_D^{w*}. \end{cases}$$

If $-1 + 2\sqrt{1-8h} < q_C^w < q_D^{w*}$, then $\Pi_p^w = \frac{1}{64}(1 + q_C^w - (q_C^w)^2)$. We can show that $q_C^{w*} = 1/2$, which exists only when $h > \frac{7}{128}$. If $q_C^w \leqslant -1 + 2\sqrt{1-8h}$, q_C^{w*} is the solution of $\frac{h}{9}(1 + \frac{q_C^w}{\sqrt{24h + (q_C^w)^2}}) + \frac{q_C^w(5q_C^w - 32\sqrt{24h + (q_C^w)^2})}{1728} = 0$, which exists only when $h < \frac{1}{32}(2 - \frac{23}{(1+78\sqrt{2})^{1/3}} + (1 + 78\sqrt{2})^{1/3})$. Comparing the platform's profits under the two scenarios, we can obtain that there exists a h^w such that when $h > h^w$, $q_C^{w*} = 1/2$ and when $h < h^w$, q_C^{w*} is the solution of $\frac{h}{9}(1 + \frac{q_C^w}{\sqrt{24h + (q_C^w)^2}}) + \frac{q_C^w(5q_C^w - 32\sqrt{24h + (q_C^w)^2})}{1728} = 0$, where h^w is the solution of

$$27 - 128(q_C^{w*})^2\sqrt{24h + (q_C^{w*})^2} - 768h\left(\sqrt{24h + (q_C^{w*})^2} - 3q_C^{w*}\right) + 20(q_C^{w*})^3 = 0. \tag{A.12}$$

Proof of Proposition 8. Under the commission fee scheme, we define a threshold $\widehat{q}_D^c$ at which the seller i is indifferent between opting for quality disclosure and obtaining platform certification. If a seller i chooses to disclose its quality information, its payoff is $\widehat{\pi}_i^d = (1-\alpha)q_i^2/4 - h$; if it chooses to obtain platform certification, its payoff is $\widehat{\pi}_i^c = (1-\alpha)(\widehat{q}_C + \widehat{q}_D^c)^2/16 - t$. Solving $\widehat{\pi}_i^d = \widehat{\pi}_i^c$ leads to the seller i's equilibrium disclosure threshold $\widehat{q}_D^c = \min\left\{1, \frac{1}{3}\widehat{q}_C + \frac{2}{3(1-\alpha)}\sqrt{(\widehat{q}_C^2(1-\alpha) + 12h - 12t)(1-\alpha)}\right\}$. To ensure that the seller i is willing to obtain platform certification, it is required $\widehat{\pi}_i^c \geqslant \frac{1}{16}(\widehat{q}_D^c)^2$ if $\widehat{q}_C \leqslant \widehat{q}_D^*$; and $\widehat{\pi}_i^c \geqslant \frac{1}{16}(\widehat{q}_D^*)^2$ if $\widehat{q}_C > \widehat{q}_D^*$. We can easily obtain that the seller i would obtain platform certification only if $\frac{4(\sqrt{(h+3t)(1-\alpha)} - \sqrt{h(1-\alpha)})}{\sqrt{3}(1-\alpha)} \leqslant \widehat{q}_C \leqslant \widehat{q}_D^*$; otherwise the seller i would never obtain platform certification and the platform's profit remains the same as that of the benchmark scenario without platform certification.

We now mainly focus on the scenario where $\frac{4(\sqrt{(h+3t)(1-\alpha)}-\sqrt{h(1-\alpha)})}{\sqrt{3}(1-\alpha)} \leqslant \widehat{q}_C \leqslant \widehat{q}_D^*$. Note that $\widehat{q}_D^c < 1$ requires $\widehat{q}_C < -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}}$, which leads to the following two cases:

Case 1: $\widehat{q}_D^* \leqslant -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}}$, which requires $h \leqslant \frac{3(7(1-\alpha)+32t-\sqrt{(1-\alpha)(13-13\alpha+64t)}}{128}$, under which $\widehat{\Pi}_p = \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C+\widehat{q}_D^c)^2}{16} + tdq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4}dq$. Solving the maximization problem, we can obtain $\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C} = \frac{(\alpha-1)\alpha\widehat{q}_C^2\left(5\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+32(\alpha-1)\widehat{q}_C\right)-96\alpha h\left(\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+3(\alpha-1)\widehat{q}_C\right)-96(2\alpha-3)t\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+288(\alpha-1)t\widehat{q}_C}{432(\alpha-1)\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}}$, and $\frac{\partial^2\widehat{\Pi}_p}{\partial(\widehat{q}_C)^2} < 0$. This together with the fact that $\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C}\Big|_{\widehat{q}_C=\frac{4(\sqrt{(h+3t)(1-\alpha)}-\sqrt{h(1-\alpha)})}{\sqrt{3}(1-\alpha)}} > 0$ only for $h > h_1^p$ and $\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C}\Big|_{\widehat{q}_C=\widehat{q}_D^*} < 0$ suggests that when $h > h_1^p$, there exists a unique $\widehat{q}_C^* = q_0^p$ satisfying

$$\begin{array}{c}(\alpha-1)\alpha\widehat{q}_C^2\left(5\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+32(\alpha-1)\widehat{q}_C\right)\\ -96\alpha h\left(\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+3(\alpha-1)\widehat{q}_C\right)\\ -96(2\alpha-3)t\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)\widehat{q}_C^2\right)}+288(\alpha-1)t\widehat{q}_C\end{array} = 0. \tag{A.13}$$

that maximizes the platform's profit; when $h < h_1^p$, $\widehat{q}_C^* = \frac{4(\sqrt{(h+3t)(1-\alpha)}-\sqrt{h(1-\alpha)})}{\sqrt{3}(1-\alpha)}$.

Case 2: $\widehat{q}_D^* > -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}}$, which requires $h \leqslant \frac{3(7(1-\alpha)+32t-\sqrt{(1-\alpha)(13-13\alpha+64t)}}{128}$, under which

$$\widehat{\Pi}_p = \begin{cases} \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C+\widehat{q}_D^c)^2}{16} + tdq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4}dq, & \text{if } \widehat{q}_C \leqslant -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}}; \\ \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^1 \frac{\alpha(\widehat{q}_C+1)^2}{16} + tdq, & \text{if } -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}} < \widehat{q}_C < \widehat{q}_D^*. \end{cases}$$

If $-1+2\sqrt{1-\frac{4h-4t}{1-\alpha}} < \widehat{q}_C < \widehat{q}_D^*$, $\widehat{\Pi}_p = \frac{\alpha}{16}(1+\widehat{q}_C-(\widehat{q}_C)^2)+t(1-\widehat{q}_C)$, then $\widehat{q}_C^* = 1/2-8t/\alpha$, which exists only when $h > \frac{7(1-\alpha)}{64} - \frac{4(1-\alpha)t^2}{\alpha^2} + \frac{t(3-\alpha)}{2\alpha}$. If $\widehat{q}_C \leqslant -1+2\sqrt{1-\frac{4h-4t}{1-\alpha}}$, $\widehat{q}_C^* = q_0^p$, which exists only when $h < h_2^p$, which is the solution of

$$\begin{aligned}(\alpha-1)\alpha&\left(32(\alpha-1)\left(2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}-1\right)+5\sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}\right)^2-12h+12t\right)}\right)\left(1-2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}\right)^2\\ &+288(\alpha-1)t\left(2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}-1\right)-96(2\alpha-3)t\sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}\right)^2-12h+12t\right)}\\ &-96\alpha h\left(3(\alpha-1)\left(2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}-1\right)+\sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-4t-1}{\alpha-1}}\right)^2-12h+12t\right)}\right)=0\end{aligned}$$

Comparing the platform's profits under the two cases, we can obtain that there exists a h^p such that when $h > h^p$, $\widehat{q}_C^* = 1/2-8t/\alpha$ and when $h < h^p$, $\widehat{q}_C^* = \max\left\{q_0^p, \frac{4(\sqrt{(h+3t)(1-\alpha)}-\sqrt{h(1-\alpha)})}{\sqrt{3}(1-\alpha)}\right\}$, where h^p is the solution of

$$\frac{\begin{array}{c}\alpha\left((\alpha-1)\left(108(\alpha-1)+32(\widehat{q}_C^*)^2\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)(\widehat{q}_C^*)^2\right)}+5(\alpha-1)(\widehat{q}_C^*)^3\right)\right)\\ -96\alpha h\left(\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)(\widehat{q}_C^*)^2\right)}+3(\alpha-1)\widehat{q}_C^*\right)\\ -96t\left((8\alpha-9)\sqrt{(\alpha-1)\left(-12h+12t+(\alpha-1)(\widehat{q}_C^*)^2\right)}+3\left(2\alpha^2-5\alpha+3\right)\widehat{q}_C^*\right)\end{array}}{1296(1-\alpha)^2} t = \frac{5\alpha}{64}+\frac{4t^2}{\alpha}+\frac{t}{2}. \tag{A.14}$$

Proof of Proposition 9. Under commission fee scheme, according to the proof of Lemma 2 and Proposition 1, when the platform suffers a reputation loss, the platform would set a quality standard $\widehat{q}_C \leqslant \widehat{q}_D^*$. We now discuss the following two subcases:

Case 1: $\widehat{q}_D^* \leqslant -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, which requires $h \leqslant \frac{3(7-\sqrt{13})(1-\alpha)}{128}$, under which $\widehat{\Pi}_p = \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C+\widehat{q}_D^c)^2}{16}dq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4}dq + \beta(\int_0^{\widehat{q}_C/2} \frac{q-\widehat{q}_C/2}{\widehat{q}_C}dq +$

$\int_{\widehat{q}_C}^{(\widehat{q}_C+\widehat{q}^c_D)/2} \frac{q-(\widehat{q}_C+\widehat{q}^c_D)/2}{\widehat{q}^c_D-\widehat{q}_C}dq)$. Solving the maximization problem, we can obtain $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C} = \frac{18\alpha\beta-18\beta+96\alpha h-5\alpha^2\widehat{q}_C^2+5\alpha\widehat{q}_C^2}{432(1-\alpha)} + \frac{\widehat{q}_C\sqrt{(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)}\left(9\alpha\beta-9\beta-72\alpha h+8\alpha^2\widehat{q}_C^2-8\alpha\widehat{q}_C^2\right)}{108(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)}$, and $\frac{\partial^2\widehat{\Pi}_p}{\partial(\widehat{q}_C)^2} < 0$. We can further show that $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=0} > 0$ for $\beta < \frac{16\alpha h}{3(1-\alpha)}$ and $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=0} < 0$ otherwise. This together with fact that $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=\widehat{q}^*_D} < 0$ suggests that when $\beta < \frac{16\alpha h}{3(1-\alpha)}$, there exists a unique $\widehat{q}^*_C = q_0^\beta$ satisfying

$$\frac{18\alpha\beta-18\beta+96\alpha h-5\alpha^2\widehat{q}_C^2+5\alpha\widehat{q}_C^2}{432(1-\alpha)} + \frac{\widehat{q}_C\sqrt{(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)}\left(9\alpha\beta-9\beta-72\alpha h+8\alpha^2\widehat{q}_C^2-8\alpha\widehat{q}_C^2\right)}{108(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)} = 0 \tag{A.15}$$

that maximizes the platform's profit; When $\beta > \frac{16\alpha h}{3(1-\alpha)}$, the optimal $\widehat{q}^*_C = 1$ (when the platform is indifferent between certifying and not certifying, we assume that it opts not to certify sellers).

Case 2: $\widehat{q}^*_D > -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, which requires $h > \frac{3(7-\sqrt{13})(1-\alpha)}{128}$, under which

$$\widehat{\Pi}_p = \begin{cases} \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}^c_D} \frac{\alpha(\widehat{q}_C+\widehat{q}^c_D)^2}{16}dq + \int_{\widehat{q}^c_D}^1 \frac{\alpha q^2}{4}dq & \text{if } \widehat{q}_C \leqslant -1+2\sqrt{1-\dfrac{4h}{1-\alpha}}; \\ +\beta(\int_0^{\widehat{q}_C/2} \frac{q-\widehat{q}_C/2}{\widehat{q}_C}dq + \int_{\widehat{q}_C}^{(\widehat{q}_C+\widehat{q}^c_D)/2} \frac{q-(\widehat{q}_C+\widehat{q}^c_D)/2}{\widehat{q}^c_D-\widehat{q}_C}dq), & \\ \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^1 \frac{\alpha(\widehat{q}_C+1)^2}{16}dq & \text{if } -1+2\sqrt{1-\dfrac{4h}{1-\alpha}} < \widehat{q}_C < \widehat{q}^*_D. \\ +\beta(\int_0^{\widehat{q}_C/2} \frac{q-\widehat{q}_C/2}{\widehat{q}_C}dq + \int_{\widehat{q}_C}^{(\widehat{q}_C+1)/2} \frac{q-(\widehat{q}_C+1)/2}{1-\widehat{q}_C}dq), & \end{cases}$$

If $-1+2\sqrt{1-\frac{4h}{1-\alpha}} < \widehat{q}_C < \widehat{q}^*_D$, then $\widehat{\Pi}_p = \frac{\alpha}{16}(1+\widehat{q}_C-(\widehat{q}_C)^2) - \frac{\beta}{8}$. We can show that $\widehat{q}^*_C = 1/2$, which exists only when $h > \frac{7(1-\alpha)}{64}$. If $\widehat{q}_C \leqslant -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, $\widehat{q}^*_C = q_0^\beta$, which exists only when $\beta < \frac{16\alpha h}{3(1-\alpha)}$ and $h < h_0^\beta$, where h_0^β is the solution of

$$\begin{gathered} (\alpha-1)\left(36(\alpha-1)\beta\left(2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}-1\right) - 18\beta\sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}\right)^2-12h\right)} + 32(\alpha-1)\alpha\left(2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}-1\right)^3\right) \\ +5(\alpha-1)\alpha\left(1-2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}\right)^2\sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}\right)^2-12h\right)} \qquad = 0. \\ -96\alpha h\left(3(\alpha-1)\left(2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}-1\right) + \sqrt{(\alpha-1)\left((\alpha-1)\left(1-2\sqrt{\tfrac{\alpha+4h-1}{\alpha-1}}\right)^2-12h\right)}\right) \end{gathered}$$

Comparing the platform's profits and summarizing the above results, we can show that under the commission fee scheme, in equilibrium, (1) when $\beta > \frac{16\alpha h}{3(1-\alpha)}$and $h < \frac{7(1-\alpha)}{64}$, $\widehat{q}^*_C = 0$; (2) otherwise when $h < h^\beta$, $\widehat{q}^*_C$ is the solution of $\frac{18\alpha\beta-18\beta+96\alpha h-5\alpha^2\widehat{q}_C^2+5\alpha\widehat{q}_C^2}{432(1-\alpha)} + \frac{\widehat{q}_C\sqrt{(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)}\left(9\alpha\beta-9\beta-72\alpha h+8\alpha^2\widehat{q}_C^2-8\alpha\widehat{q}_C^2\right)}{108(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)} = 0$ and when $h > h^\beta$, $\widehat{q}^*_C = 1/2$, where h^β is the solution of

$$\begin{gathered} \tfrac{1}{64}(8\beta-5\alpha) + \frac{\beta\left(2\sqrt{(\alpha-1)\left((\alpha-1)(q_0^\beta)^2-12h\right)}-\alpha q_0^\beta+q_0^\beta\right)}{24(\alpha-1)} \\ + \frac{\alpha\left((\alpha-1)\left(108(\alpha-1)+32(q_0^\beta)^2\sqrt{(\alpha-1)\left((\alpha-1)(q_0^\beta)^2-12h\right)}+5(\alpha-1)(q_0^\beta)^3\right)-96h\left(\sqrt{(\alpha-1)\left((\alpha-1)(q_0^\beta)^2-12h\right)}+3(\alpha-1)q_0^\beta\right)\right)}{1296(\alpha-1)^2} \end{gathered} = 0. \tag{A.16}$$

and

$$\widehat{q}^c_D(q_0^\beta) = \frac{1}{3}q_0^\beta + \frac{2}{3}\sqrt{(q_0^\beta)^2 + \frac{12h}{1-\alpha}}. \tag{A.17}$$

Proof of Proposition 10. Under the listing fee scheme, when the disclosure cost $h < 3s$, it can be easily verified that the platform would never set the quality standard higher than $\widetilde{q}^*_D$, as this option only increases the platform's reputation cost. Then, we focus on the scenario when $\widetilde{q}_C \leqslant \widetilde{q}^*_D$. According to the proof of Lemma 4, a certified seller i would disclose its quality information only when $q_i \geqslant \widetilde{q}^c_D = \min(1, \frac{1}{3}\widetilde{q}_C + \frac{2}{3}\sqrt{\widetilde{q}_C^2+12h})$ and the requirement for a certified non-disclosure seller's profit to be nonnegative is $\widetilde{q}_C \geqslant 4\sqrt{s} - \widetilde{q}^*_D$. Therefore, when $h < 3s$, the platform's profit function can be written as

$$\widetilde{\Pi}_p = \begin{cases} \int_{\widetilde{q}^*_D}^1 s\,dq, & \text{if } \widetilde{q}_C < 4\sqrt{s}-\widetilde{q}^*_D; \\ \int_{\widetilde{q}_C}^1 s\,dq + \beta(\int_{\widehat{q}_C}^{(\widehat{q}_C+\widetilde{q}^c_D)/2} \frac{q-(\widehat{q}_C+\widetilde{q}^c_D)/2}{\widetilde{q}^c_D-\widehat{q}_C}dq), & \text{if } 4\sqrt{s}-\widetilde{q}^*_D \leqslant \widetilde{q}_C \leqslant \widetilde{q}^*_D. \end{cases}$$

If $\widetilde{q}_C < 4\sqrt{s} - \widetilde{q}_D^*$, $\widetilde{\Pi}_p = s(1 - 2\sqrt{s+h})$, which is independent of $\widetilde{q}_C$. If $4\sqrt{s} - \widetilde{q}_D^* \leqslant \widetilde{q}_C \leqslant \widetilde{q}_D^*$, solving the maximization problem, we can obtain $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C} = -s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h+\widetilde{q}_C}})$ and $\frac{\partial^2 \widetilde{\Pi}_p}{\partial (\widetilde{q}_C)^2} < 0$. This together with the fact that $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = 0} = -s + \frac{\beta}{12} > 0$ only if $\beta > 12s$ and $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = \widetilde{q}_D^*} = -s + \frac{\beta}{12}(1 - \frac{\sqrt{s+h}}{\sqrt{4s+h}}) > 0$ only if $\beta > \frac{12s\sqrt{4s+h}}{\sqrt{4s+h} - \sqrt{s+h}}$ suggests that when $\beta < 12s$, $\widetilde{q}_C^* = 4\sqrt{s} - \widetilde{q}_D^*$; when $12s < \beta < \frac{12s\sqrt{4s+h}}{\sqrt{4s+h} - \sqrt{s+h}}$, $\widetilde{q}_C^* = q_1^\beta$ is the solution of $-s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h+\widetilde{q}_C}}) = 0$; otherwise $\widetilde{q}_C^* = \widetilde{q}_D^*$. Comparing the platform's profit under the above two scenarios, we can obtain that if $h < 3s$, in equilibrium, when $\beta < 12s$, $\widetilde{q}_C^* = 4\sqrt{s} - \widetilde{q}_D^*$; when $12s < \beta < \beta_1$, $\widetilde{q}_C^* = q_1^\beta$; otherwise $\widetilde{q}_C^* = 1$ (if the platform's profit remains the same whether or not it provides certification, we assume that the platform would choose not to certify sellers), where β_1 is the solution of $s(1 - q_1^\beta) + \frac{\beta}{12}(q_1^\beta - \sqrt{12h + (q_1^\beta)^2}) = s(1 - 2\sqrt{s+h})$.

When the disclosure cost $h \geqslant 3s$, to ensure the uncertified seller i's profit $\frac{\widetilde{q}_C^2}{16} - s \geqslant 0$, $\widetilde{q}_C < 4\sqrt{s}$ is required. We can easily verify that in equilibrium, the platform would never set the quality standard $\widetilde{q}_C \geqslant 4\sqrt{s}$, as this only increases the platform's reputation loss. Note that $\widetilde{q}_D^c < 1$ requires $\widetilde{q}_C < -1 + 2\sqrt{1-4h}$, which leads to the following two subcases:

Case 1: $4\sqrt{s} < -1 + 2\sqrt{1-4h}$, which requires $3s < h < \frac{3 - 8\sqrt{h} - 16h}{16}$, under which $\widetilde{\Pi}_p = \int_{\widetilde{q}_C}^1 s dq + \beta \int_{\widetilde{q}_C}^{(\widetilde{q}_C + \widetilde{q}_D^c)/2} \frac{q - (\widetilde{q}_C + \widetilde{q}_D^c)/2}{\widetilde{q}_D^c - \widetilde{q}_C} dq$. Solving the maximization problem, we can obtain $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C} = -s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h+\widetilde{q}_C}})$ and $\frac{\partial^2 \widetilde{\Pi}_p}{\partial (\widetilde{q}_C)^2} < 0$. This together with fact that $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = 0} = -s + \frac{\beta}{12} > 0$ only if $\beta > 12s$ and $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = 4\sqrt{s}} = -s + \frac{\beta}{12}(1 - \frac{4\sqrt{s}}{\sqrt{16s+12h}}) > 0$ only if $\beta > \frac{12s\sqrt{4s+3h}}{\sqrt{4s+3h} - 2\sqrt{s}}$ suggests that when $\beta < 12s$, $\widetilde{q}_C^* = 1$ (when the platform is indifferent between certifying and not certifying, we assume that it opts not to certify sellers); when $12s < \beta < \frac{12s\sqrt{4s+3h}}{\sqrt{4s+3h} - 2\sqrt{s}}$, $\widetilde{q}_C^* = q_1^\beta$, which is the solution of $-s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h+\widetilde{q}_C}}) = 0$; otherwise $\widetilde{q}_C^* = 4\sqrt{s} - \epsilon$, where ϵ is an infinitesimal number.

Case 2: $4\sqrt{s} > -1 + 2\sqrt{1-4h}$, which requires $h > \frac{3 - 8\sqrt{h} - 16h}{16}$, under which

$$\widetilde{\Pi}_p = \begin{cases} \displaystyle\int_{\widetilde{q}_C}^1 sdq + \beta \int_{\widetilde{q}_C}^{(\widetilde{q}_C + \widetilde{q}_D^c)/2} \frac{q - (\widetilde{q}_C + \widetilde{q}_D^c)/2}{\widetilde{q}_D^c - \widetilde{q}_C} dq, & \text{if } \widetilde{q}_C < -1 + 2\sqrt{1-4h}; \\ \displaystyle\int_{\widetilde{q}_C}^1 s\, dq + \beta (\int_{\widehat{q}_C}^{(\widehat{q}_C + 1)/2} \frac{q - (\widehat{q}_C + 1)/2}{1 - \widehat{q}_C} dq), & \text{if } -1 + 2\sqrt{1-4h} \leqslant \widetilde{q}_C \leqslant 4\sqrt{s}. \end{cases}$$

If $\widetilde{q}_C < -1 + 2\sqrt{1-4h}$, $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C} = -s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h+\widetilde{q}_C}})$ and $\frac{\partial^2 \widetilde{\Pi}_p}{\partial (\widetilde{q}_C)^2} < 0$. This together with fact that $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = 0} = -s + \frac{\beta}{12} > 0$ only for $\beta > 12s$ and $\frac{\partial \widetilde{\Pi}_p}{\partial \widetilde{q}_C}|_{\widetilde{q}_C = -1 + 2\sqrt{1-4h}} = -s + \frac{\beta}{12}(1 - \frac{4\sqrt{s}}{\sqrt{16s+12h}}) < 0$ suggests that when $\beta < 12s$, $\widetilde{q}_C^* = 1$ (when the platform is indifferent between certifying and not certifying, we assume that it opts not to certify sellers); when $\beta > 12s$, $\widetilde{q}_C^* = q_1^\beta$, which is the solution of

$$-s + \frac{\beta}{12}(1 - \frac{\widetilde{q}_C}{\sqrt{12h + \widetilde{q}_C}}) = 0 \tag{A.18}$$

If $-1 + 2\sqrt{1-4h} \leqslant \widetilde{q}_C \leqslant 4\sqrt{s}$, $\widetilde{\Pi}_p = \frac{(1 - \widetilde{q}_C)}{8}(8s - \beta)$, it can be easily verified that this can never be the equilibrium outcome.

Summarizing the above results, we can obtain the results in Proposition 10. It is worth noting that to ensure the impact of the reputation loss is not substantially large, we focus on the case

$$\beta < \overline{\beta} = \min\left\{\beta_1, \frac{12s\sqrt{4s+3h}}{\sqrt{4s+3h} - 2\sqrt{s}}\right\}. \tag{A.19}$$

Proof of Proposition 11. Under the commission fee scheme, according to Lemma 2, when the quality standard serves as an entry barrier, the platform's profit can be written as

$$\widehat{\Pi}_p = \begin{cases} \displaystyle\int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C + \widehat{q}_D^c)^2}{16} dq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4} dq, & \text{if } \widehat{q}_C \leqslant -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}; \\ \displaystyle\int_{\widehat{q}_C}^1 \frac{\alpha(\widehat{q}_C + 1)^2}{16} dq, & \text{if } \widehat{q}_C > -1 + 2\sqrt{1 - \frac{4h}{1-\alpha}}. \end{cases}$$

If $\widehat{q}_C \leqslant -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, solving the maximization problem, we can obtain $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C} = \frac{\alpha\left(24h+19\alpha\widehat{q}_C^2-19\widehat{q}_C^2\right)}{108(1-\alpha)} + \frac{2\alpha\widehat{q}_C\sqrt{-\left((\alpha-1)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)\right)\left(-9h+\alpha\widehat{q}_C^2-\widehat{q}_C^2\right)}}{27(1-\alpha)\left(12h-\alpha\widehat{q}_C^2+\widehat{q}_C^2\right)}$, and $\frac{\partial^2 \widehat{\Pi}_p}{\partial (\widehat{q}_C)^2} < 0$. This together with the fact that $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=0} > 0$ and $\frac{\partial \widehat{\Pi}_p}{\partial \widehat{q}_C}|_{\widehat{q}_C=-1+2\sqrt{1-\frac{4h}{1-\alpha}}} < 0$ suggests that there exists a unique $\widehat{q}_C^* = 2\sqrt{\frac{2(5\sqrt{3}-8)h}{11(1-\alpha)}}$ that maximizes the platform's profit.

If $\widehat{q}_C > -1+2\sqrt{1-\frac{4h}{1-\alpha}}$, then $\widehat{\Pi}_p = \frac{\alpha}{16}(1-\widehat{q}_C)(1+\widehat{q}_C)^2$. We can show that the optimal $\widehat{q}_C^* = 1/3$, which exists only when $h > \frac{5(1-\alpha)}{36}$.

Comparing the platform's profits under the two scenarios, we can obtain that there exists a h^b such that when $h > h^b$, $\widehat{q}_C^* = 1/3$ and when $h < h^b$, $\widehat{q}_C^* = 2\sqrt{\frac{2(5\sqrt{3}-8)h}{11(1-\alpha)}}$, where h^b is the solution of

$$\frac{\alpha}{108} + \frac{4\alpha h\left(\sqrt{2\left(5\sqrt{3}-8\right)}\left(95\sqrt{3}-251\right)\sqrt{(1-\alpha)h}-\sqrt{15450\sqrt{3}+23537}\sqrt{h-\alpha h}\right)}{891\sqrt{11}(1-\alpha)^2} = 0. \tag{A.20}$$

and

$$\widehat{q}_D^c(\widehat{q}_C^*) = \frac{2}{3}\left(\sqrt{2\left(5\sqrt{3}-8\right)}+2\sqrt{10\sqrt{3}+17}\right)\sqrt{\frac{\mathrm{cd}}{11-11\alpha}}. \tag{A.21}$$

Appendix B: Discussion and Analysis of Extensions

B.1 Price Signaling

To focus on the implications of information revelation strategies—platform certification and seller disclosure—on quality transparency, our study abstracts away from the potential signaling role of the seller's retail price. However, we can further show that even if price signaling is allowed, retail prices fail to convey any quality information to consumers in our baseline setting where the seller's production cost is normalized to zero. We demonstrate this result under the D1 refinement criterion (Ramey, 1996; Cho and Kreps, 1987), whose logic also applies to other common refinements such as the intuitive criterion and the lexicographically maximal sequential equilibrium (LMSE). The underlying reason is that our model does not satisfy the single-crossing condition, which requires that $-\frac{Q}{p}$ be strictly decreasing in product quality q. As a result, any separating equilibrium where sellers credibly signal quality through price is unsustainable, and only pooling equilibria can arise. In such pooling equilibria, retail prices fail to differentiate seller quality, rendering price uninformative. Consequently, the equilibrium outcomes degenerate to those of our baseline model, where quality information is revealed solely through platform certification and seller disclosure.

We note that when sellers incur nonzero production costs, the retail price may serve as a signaling device to convey product quality to consumers. Below, we briefly discuss the interplay between price signaling and quality disclosure, assuming that a seller's per-unit production cost is positively related to its product quality. Specifically, a seller i with quality q_i incurs a unit production cost of kq_i, where $k > 0$. To our knowledge, Daughety and Reinganum (2008) is the first to examine this issue in a setting with continuously distributed product quality. We begin with a benchmark scenario in which consumers can directly observe seller i's product quality. In this case, seller i sets its retail price to maximize profit $\pi_i = (q_i - p_i)(p_i - kq_i)$. The optimal price and resulting profit can be readily derived as $p_i^f = \frac{1+k}{2}q_i$ and $\pi_i^f = \frac{(1-k)^2}{4}q_i^2$, respectively.

Next, following Daughety and Reinganum (2008), we consider the price signaling game in the absence of quality disclosure. Seller i sets the retail price p_i to maximize $\pi_i = (\bar{q}_i - p_i)(p_i - kq_i)$, where $\bar{q}_i$ denotes consumers' belief about seller i's product quality. Analogous to the results in Daughety and Reinganum (2008), a unique separating equilibrium exists and the optimal pricing strategy $p_i(q_i)$ satisfies the following differential equation with boundary condition $p_i(0) = 0$: $\frac{dp_i}{dq_i} = \frac{p_i - kq_i}{2p_i - q_i - kq_i}$. Solving this equation and substituting $p_i(q_i)$ into the profit function yields seller i's equilibrium profit π_i^s under pure price signaling. A key insight is that high-quality sellers must increase their price more significantly to distinguish themselves from lower-quality sellers. This rising deviation imposes an increasing signaling cost, causing the seller's profit to decrease monotonically in its quality level.

When both quality disclosure and price signaling are feasible, sellers naturally compare the associated costs and choose the more advantageous channel. If seller i discloses its quality information, its optimal price and profit are $p_i^d = \frac{1+k}{2}q_i$ and $\pi_i^d = \frac{(1-k)^2}{4}q_i^2 - h$, where h is the disclosure cost. By comparing π_i^d with π_i^s, we can identify a threshold quality level q_s such that if $q_i > q_s$, seller i prefers to disclose its quality information; if $q_i \leqslant q_s$, seller i opts to withhold quality information and rely on price signaling instead. As shown by Daughety and Reinganum (2008), the seller's equilibrium profit exhibits a U-shaped relationship with product quality, as illustrated in Figure B.1.

Given the equilibrium outcomes, once the platform's pricing scheme is taken into account, we observe that under the listing fee scheme, sellers with intermediate-quality products may choose to quit platform selling, while those with either high or low quality opt to participate in platform selling. Incorporating price signaling into this setting—where seller participation is endogenously determined—introduces substantial analytical challenges. In particular, constructing a consistent equilibrium that combines price signaling and quality disclosure would require additional structural assumptions. We leave this extension for future research and provide a brief discussion in the concluding section.

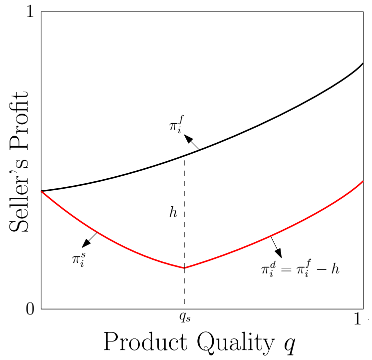


Figure B.1: Sellers' Profit in the Presence of Production Cost

B.2 When Sellers Pay for Platform Certification: Listing Fee Scheme

Proposition B.1. *When sellers pay for platform certification under the listing fee scheme, the platform sets a quality standard $\widetilde{q}_C^*$, and the following results hold in equilibrium:*

(1) *If the disclosure cost $h \geqslant 3s$ and the certification cost $t < t^p$, then $\widetilde{q}_C^* = \frac{4(\sqrt{h+3t}-\sqrt{h})}{\sqrt{3}}$. Sellers participate in platform selling only if $q_i \geqslant \widetilde{q}_C^*$. Moreover, they choose to obtain platform certification if $\widetilde{q}_C^* < q_i < \widetilde{q}_D^c(\widetilde{q}_C^*)$, and disclose their quality information only if $q_i \geqslant \widetilde{q}_D^c(\widetilde{q}_C^*)$.*

(2) *If $h \geqslant 3s$ and $t > t^p$, then $\widetilde{q}_C^* = \max\left\{\frac{4(\sqrt{h+3t}-\sqrt{h})}{\sqrt{3}}, 4\sqrt{s}\right\}$. In this case, all sellers participate in platform selling. Sellers obtain platform certification if $\widetilde{q}_C^* < q_i < \widetilde{q}_D^c(\widetilde{q}_C^*)$, and disclose their quality information only if $q_i \geqslant \widetilde{q}_D^c(\widetilde{q}_C^*)$.*

(3) *If $h < 3s$, then $\widetilde{q}_C^* = 4\sqrt{s+t-2\sqrt{s+h}}$. Sellers participate in platform selling only if $q_i \geqslant \widetilde{q}_C^*$. They obtain platform certification if $\widetilde{q}_C^* < q_i < 2\sqrt{s+h}$, and disclose their quality information only if $q_i \geqslant 2\sqrt{s+h}$.*

Proof of Proposition B.1. Similar to that under the commission fee scheme, the seller i would obtain quality certification only if $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \widetilde{q}_C \leqslant \widetilde{q}_D^*$ and the threshold $\widetilde{q}_D^c = \min\left\{1, \frac{1}{3}\widetilde{q}_C + \frac{2}{3}\sqrt{\widetilde{q}_C^2 + 12h - 12t}\right\}$, at which the seller i is indifferent between opting for quality disclosure and obtaining quality certification.

When $h < 3s$, $\widetilde{\Pi}_p = s(1 - \max\{4\sqrt{s} - \widetilde{q}_D^*, \min\{\widetilde{q}_C, \widetilde{q}_D^*\}\}) + t(\widetilde{q}_D^c - \widetilde{q}_C)$, which decreases in $\widetilde{q}_C$, and the equilibrium $\widetilde{q}_C^* = 4\sqrt{s} - \widetilde{q}_D^*$.

When $h > 3s$, note that the uncertified seller i is willing to participates in platform selling only if $\frac{\widetilde{q}_C^2}{16} - s \geqslant 0$, which requires $\widetilde{q}_C \geqslant 4\sqrt{s}$. If $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} < 4\sqrt{s}$, we have the following three cases:

Case 1: $4\sqrt{s} < 4\sqrt{\frac{h}{3}} < -1 + 2\sqrt{1-4h+4t}$, which requires $h \leqslant \frac{3(7+32t-\sqrt{13+64t})}{128}$, under which

$$\widehat{\Pi}_p = \begin{cases} \int_{\widetilde{q}_C}^{\widetilde{q}_D^c} t dq + \int_{\widetilde{q}_C}^{1} s dq, & \text{if } \frac{4(\sqrt{(h+3t)} - \sqrt{h})}{\sqrt{3}} \leqslant \widetilde{q}_C < 4\sqrt{s}; \\ \int_{\widetilde{q}_C}^{\widetilde{q}_D^c} t dq + \int_{0}^{1} s dq, & \text{if } 4\sqrt{s} \leqslant \widetilde{q}_C < 4\sqrt{\frac{h}{3}}. \end{cases}$$

If $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \tilde{q}_C < 4\sqrt{s}$, $\widehat{\Pi}_p = s(1-\tilde{q}_C) + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$. Therefore, the optimal $\tilde{q}^*_C = \frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}}$. If $4\sqrt{s} \leqslant \tilde{q}_C < 4\sqrt{\frac{h}{3}}$, $\widehat{\Pi}_p = s + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$. Therefore, the optimal $\tilde{q}^*_C = 4\sqrt{s}$. Comparing the platform's profit under two scenarios, we can obtain that there exists a t_1 such that when $t > \max\{t_1, 0\}$, $\tilde{q}^*_C = 4\sqrt{s}$ and when $t < \max\{t_1, 0\}$, $\tilde{q}^*_C = \frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}}$, where t_1 is the solution of $\frac{1}{9}\left(4\sqrt{3}\sqrt{h}(2t+3s) + 3s\left(3-4\sqrt{3}\sqrt{h+3t}\right) + 4\sqrt{3}t\left(\sqrt{-8\sqrt{h}\sqrt{h+3t}+17h+3t} - 2\sqrt{h+3t}\right)\right) - \left(t\left(\frac{1}{3}\left(2\sqrt{12h-12t+16s}+4\sqrt{s}\right)-4\sqrt{s}\right)+s\right) = 0$.

Case 2: $4\sqrt{s} < -1+2\sqrt{1-4h+4t} < 4\sqrt{\frac{h}{3}}$, which requires $\frac{3(7+32t-\sqrt{13+64t})}{128} < h < \frac{3+16t-8\sqrt{s-16s}}{16}$, under which

$$\widehat{\Pi}_p = \begin{cases} \displaystyle\int_{\tilde{q}_C}^{\tilde{q}^c_D} t\,dq + \int_{\tilde{q}_C}^{1} s\,dq, & \text{if } \dfrac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \tilde{q}_C < 4\sqrt{s}; \\ \displaystyle\int_{\tilde{q}_C}^{\tilde{q}^c_D} t\,dq + \int_{0}^{1} s\,dq, & \text{if } 4\sqrt{s} \leqslant \tilde{q}_C < -1+2\sqrt{1-4h+4t}; \\ \displaystyle\int_{\tilde{q}_C}^{1} t\,dq + \int_{0}^{1} s\,dq, & \text{if } -1+2\sqrt{1-4h+4t} \leqslant \tilde{q}_C < 4\sqrt{\dfrac{h}{3}}. \end{cases}$$

If $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \tilde{q}_C < 4\sqrt{s}$, $\widehat{\Pi}_p = s(1-\tilde{q}_C) + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$. Therefore, the optimal $\tilde{q}^*_C = \frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}}$. If $4\sqrt{s} \leqslant \tilde{q}_C < -1+2\sqrt{1-4h+4t}$, $\widehat{\Pi}_p = s + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$, and the optimal $\tilde{q}^*_C = 4\sqrt{s}$. If $-1+2\sqrt{1-4h+4t} \leqslant \tilde{q}_C < 4\sqrt{\frac{h}{3}}$, $\widehat{\Pi}_p = s + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$, and the optimal $\tilde{q}^*_C = -1+2\sqrt{1-4h+4t}$. Comparing the platform's profit under the above three scenarios, the results are the same as Case 1.

Case 3: $-1+2\sqrt{1-4h+4t} < 4\sqrt{s} < 4\sqrt{\frac{h}{3}}$, which requires $h > \frac{3+16t-8\sqrt{s-16s}}{16}$, under which

$$\widehat{\Pi}_p = \begin{cases} \displaystyle\int_{\tilde{q}_C}^{\tilde{q}^c_D} t\,dq + \int_{\tilde{q}_C}^{1} s\,dq, & \text{if } \dfrac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \tilde{q}_C < -1+2\sqrt{1-4h+4t}; \\ \displaystyle\int_{\tilde{q}_C}^{1} t\,dq + \int_{\tilde{q}_C}^{1} s\,dq, & \text{if } -1+2\sqrt{1-4h+4t} \leqslant \tilde{q}_C < 4\sqrt{s}; \\ \displaystyle\int_{\tilde{q}_C}^{1} t\,dq + \int_{0}^{1} s\,dq, & \text{if } 4\sqrt{s} \leqslant \tilde{q}_C < 4\sqrt{\dfrac{h}{3}}. \end{cases}$$

If $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} \leqslant \tilde{q}_C < -1+2\sqrt{1-4h+4t}$, $\widehat{\Pi}_p = s(1-\tilde{q}_C) + t(\tilde{q}^c_D - \tilde{q}_C)$, which decreases in $\tilde{q}_C$. Therefore, the optimal $\tilde{q}^*_C = \frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}}$. If $-1+2\sqrt{1-4h+4t} \leqslant \tilde{q}_C < 4\sqrt{s}$, $\widehat{\Pi}_p = s(1-\tilde{q}_C) + t(1-\tilde{q}_C)$, which decreases in $\tilde{q}_C$, and the optimal $\tilde{q}^*_C = -1+2\sqrt{1-4h+4t}$. If $4\sqrt{s} \leqslant \tilde{q}_C < 4\sqrt{\frac{h}{3}}$, $\widehat{\Pi}_p = s + t(1-\tilde{q}_C)$, which decreases in $\tilde{q}_C$, and the optimal $\tilde{q}^*_C = 4\sqrt{s}$. Comparing the platform's profit under the above three scenarios, we can obtain that there exists a t_2 such that when $t < t_2$, $\tilde{q}^*_C = \frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}}$ and when $t > t_2$, $\tilde{q}^*_C = 4\sqrt{s}$, where t_2 is the solution of $\frac{1}{9}\left(4\sqrt{3}\sqrt{h}(2t+3s) + 3s\left(3-4\sqrt{3}\sqrt{h+3t}\right) + 4\sqrt{3}t\left(\sqrt{-8\sqrt{h}\sqrt{h+3t}+17h+3t} - 2\sqrt{h+3t}\right)\right) - \left(t\left(1-4\sqrt{s}\right)+s\right) = 0$.

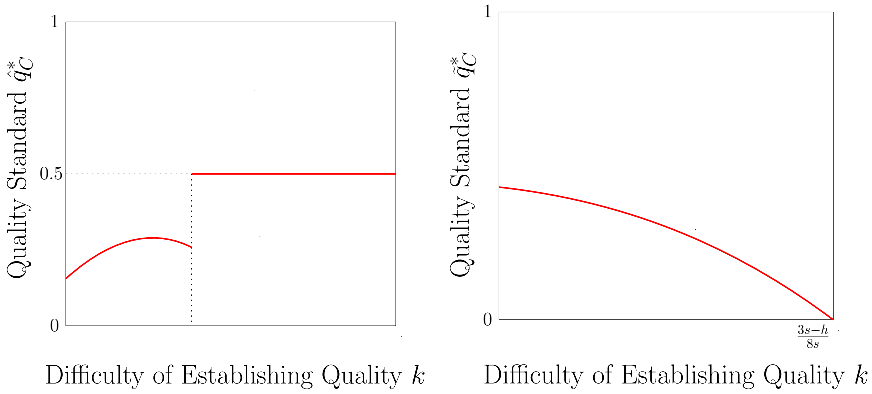
Following the similar approach, we can obtain the equilibrium outcomes for the scenario $\frac{4(\sqrt{(h+3t)}-\sqrt{h})}{\sqrt{3}} > 4\sqrt{s}$. Since the process is routine and tedious, we omit the details here. Summarizing the above results, we can obtain the results stated in the proposition, where $t^p = \max\{0, t_1, t_2\}$.

B.3 When the Disclosure Cost Increases in Product Quality

So far, we have assumed that the disclosure cost is a constant—an assumption commonly adopted in the literature on voluntary disclosure (see, e.g., Guo (2009b), Dranove and Jin (2010), and Zhang and Li (2021)). However, it is also reasonable to consider that a seller may incur a higher disclosure cost when revealing a higher level of product quality, as doing so may require more substantial evidence to credibly convince consumers. To capture this, we now assume that the disclosure cost, denoted by h', takes the following quadratic form: $h' = h + \frac{k}{2}q^2$, where $k > 0$ represents the difficulty of establishing product quality. To ensure that higher-quality sellers have (weakly) stronger incentives to disclose than lower-quality sellers, we require $k < \frac{3}{8}(1-\alpha)$. All other aspects of the model remain the same as in the baseline setting. We examine the implications of this increasing disclosure cost under both the commission fee and listing fee pricing schemes.

Proposition B.2. *When the disclosure cost increases with product quality, the following results hold:*

- *(i) Under the commission fee scheme: sellers always participate in platform selling.*
  - *(1) If $h \geqslant h^k$, the platform sets the optimal quality standard $\widehat{q}_C^* = \frac{1}{2}$, and no seller discloses its quality information. The threshold h^k decreases in k.*
  - *(2) If $h < h^k$, the platform sets $\widehat{q}_C^* = q_0^k < \frac{1}{2}$, and sellers disclose their quality information only when $q_i \geqslant \widehat{q}_D^{c*}$. Moreover, $\widehat{q}_C^*$ first increases and then decreases in k.*
- *(ii) Under the listing fee scheme:*
  - *(1) If $h \geqslant s(3-8k)$, the platform does not certify any sellers. Sellers always participate in platform selling and disclose their quality information only when $q_i \geqslant \frac{4\sqrt{h}}{\sqrt{3-8k}}$.*
  - *(2) If $h < s(3-8k)$, the platform sets $\widetilde{q}_C^* = 4\sqrt{s} - 2\sqrt{\frac{s+h}{1-2k}}$. Sellers participate in platform selling only if $q_i \geqslant \widetilde{q}_C^*$ and disclose their quality only when $q_i \geqslant 2\sqrt{\frac{s+h}{1-2k}}$.*



(a) Commission Fee Scheme (b) Listing Fee Scheme

Figure B.2: Impact of k on the Optimal Quality Standard

A careful comparison of the results in Proposition B.2 with those in Propositions 1 and 3 reveals that sellers' equilibrium information strategies remain qualitatively similar whether the disclosure cost is constant or increasing in product quality. In this subsection, we focus on how k, which captures the difficulty sellers face in substantiating their quality claims, influences the platform's certification decision. Figure B.2(a) illustrates that under the commission fee scheme, the platform's optimal quality standard $\widehat{q}_C^*$ exhibits an inverted U-shaped relationship with k. On one hand, as k increases, the cost of disclosure rises, discouraging sellers from voluntarily revealing their quality. This reduction in voluntary disclosure may incentivize the platform to raise the quality standard to better leverage the **information improvement effect**, thereby helping consumers better infer product quality. On the other hand, a higher disclosure cost also weakens the incentives of certified sellers to further disclose their quality, thus intensifying the **information reduction effect**. To mitigate this downside, the platform may respond by lowering the certification standard to maintain sellers' disclosure incentives. These opposing forces explain the non-monotonic behavior of $\widehat{q}_C^*$ as k increases. When k becomes sufficiently large such that no sellers find it worthwhile to disclose their quality, the platform sets the quality standard at $\widehat{q}_C^* = \frac{1}{2}$ to primarily exploit the information improvement effect. In contrast, under the listing fee scheme (Figure B.2(b)), the platform's optimal quality standard $\widetilde{q}_C^*$ decreases monotonically with k, whenever the platform chooses to certify sellers. In this case, a higher disclosure cost reduces sellers' willingness to participate, prompting the platform to lower the quality standard to encourage broader participation and maintain platform profitability.

Proof of Proposition B.2. Under the commission fee scheme, the payoff for a certified seller i is $\widehat{\pi}_i^{c,d} = (1-\alpha)q_i^2/4 - h - \frac{k}{2}q_i^2$ under disclosure and $\widehat{\pi}_i^{c,nd} = (1-\alpha)(\widehat{q}_C + \widehat{q}_D^c)^2/16$ under non-disclosure. Solving

$\widehat{\pi}_i^{c,d} = \widehat{\pi}_i^{c,nd}$ leads to the certified seller i's equilibrium disclosure threshold $\widehat{q}_D^c = \min(1, \frac{1-\alpha}{3-3\alpha-8k}\widehat{q}_C + \frac{2}{3-3\alpha-8k}\sqrt{\widehat{q}_C^2(1-\alpha)(1-\alpha-2k)+4h(3-3\alpha-8k)})$, which requires $k < \frac{3(1-\alpha)}{8}$. For an uncertified seller i, the payoff is $\widehat{\pi}_i^{nc,d} = (1-\alpha)q_i^2/4 - h - \frac{k}{2}q_i^2$ under disclosure and $\widehat{\pi}_i^{nc,nd} = (1-\alpha)\widehat{q}_C^2/16$ under non-disclosure. Solving $\widehat{\pi}_i^{nc,d} = \widehat{\pi}_i^{nc,nd}$ leads to the uncertified seller i's equilibrium disclosure threshold $\widehat{q}_D^{nc} = \min(\widehat{q}_C, 4\sqrt{\frac{h}{3-3\alpha-8k}})$, which also requires $k < \frac{3(1-\alpha)}{8}$. Similar to that in Proposition 1, we can verify that the platform never sets a quality standard $\widehat{q}_C > \widehat{q}_D^* = 4\sqrt{\frac{h}{3-3\alpha-8k}}$. Next, we move to the scenario $\widehat{q}_C \leqslant \widehat{q}_D^*$. Note that $\widehat{q}_D^c < 1$ requires $\widehat{q}_C < -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}$, which leads to the following two cases:

Case 1: $\widehat{q}_D^* = 4\sqrt{\frac{h}{3-3\alpha-8k}} < -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}$, which requires $h < \frac{(3-3\alpha-8k)\left((\alpha-1)\left(7\alpha+\sqrt{13(\alpha-1)^2+64k^2+56(\alpha-1)k}-7\right)+32k^2+28(\alpha-1)k\right)}{128(\alpha+2k-1)^2}$, $\widehat{\Pi}_p = \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C+\widehat{q}_D^c)^2}{16}dq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4}dq$. We can show that $\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C} = \frac{\alpha\left((\alpha-1)^2\widehat{q}_C^2(5\alpha+24k-5)-32h\left(3(\alpha-1)^2+32k^2+20(\alpha-1)k\right)\right)}{16(3\alpha+8k-3)^3} + \frac{\alpha\widehat{q}_C\left((\alpha-1)\widehat{q}_C^2\left(2(\alpha-1)^3+48k^3+50(\alpha-1)k^2+17(\alpha-1)^2k\right)-2h\left(9(\alpha-1)^3+256k^3+240(\alpha-1)k^2+78(\alpha-1)^2k\right)\right)}{(3\alpha+8k-3)^3\sqrt{(\alpha-1)\widehat{q}_C^2(\alpha+2k-1)-4h(3\alpha+8k-3)}}$, and $\frac{\partial^2\widehat{\Pi}_p}{\partial(\widehat{q}_C)^2} < 0$. This together with the fact that $\left.\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C}\right|_{\widehat{q}_C=0} > 0$ and $\left.\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C}\right|_{\widehat{q}_C=\widehat{q}_D^*} < 0$ indicate that there exists a unique $\widehat{q}_C^*$ satisfying $\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C} = 0$ that maximizes the platform's profit, where $\widehat{q}_C^* = q_0^k$ is the solution of

$$\frac{\alpha\left((\alpha-1)^2\widehat{q}_C^2(5\alpha+24k-5)-32h\left(3(\alpha-1)^2+32k^2+20(\alpha-1)k\right)\right)}{16(3\alpha+8k-3)^3} + \frac{\alpha\widehat{q}_C\left((\alpha-1)\widehat{q}_C^2\left(2(\alpha-1)^3+48k^3+50(\alpha-1)k^2+17(\alpha-1)^2k\right)\right)}{(3\alpha+8k-3)^3\sqrt{(\alpha-1)\widehat{q}_C^2(\alpha+2k-1)-4h(3\alpha+8k-3)}} - \frac{\alpha\widehat{q}_C\left(2h\left(9(\alpha-1)^3+256k^3+240(\alpha-1)k^2+78(\alpha-1)^2k\right)\right)}{(3\alpha+8k-3)^3\sqrt{(\alpha-1)\widehat{q}_C^2(\alpha+2k-1)-4h(3\alpha+8k-3)}} = 0. \tag{B.22}$$

Case 2: $\widehat{q}_D^* = 4\sqrt{\frac{h}{3-3\alpha-8k}} > -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}$, which requires $h > \frac{(3-3\alpha-8k)\left((\alpha-1)\left(7\alpha+\sqrt{13(\alpha-1)^2+64k^2+56(\alpha-1)k}-7\right)+32k^2+28(\alpha-1)k\right)}{128(\alpha+2k-1)^2}$, under which

$$\widehat{\Pi}_p = \begin{cases} \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^{\widehat{q}_D^c} \frac{\alpha(\widehat{q}_C+\widehat{q}_D^c)^2}{16}dq + \int_{\widehat{q}_D^c}^1 \frac{\alpha q^2}{4}dq, & \text{if } \widehat{q}_C < -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}; \\ \int_0^{\widehat{q}_C} \frac{\alpha(\widehat{q}_C)^2}{16}dq + \int_{\widehat{q}_C}^1 \frac{\alpha(\widehat{q}_C+1)^2}{16}dq, & \text{if } -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}} \leqslant \widehat{q}_C < \widehat{q}_D^*. \end{cases}$$

(1). If $-1+2\sqrt{1-\frac{4h+2k}{1-\alpha}} \leqslant q_C < 4\sqrt{\frac{h}{3-3\alpha-8k}}$, $\widehat{\Pi}_p = \frac{\alpha}{16}(1+\widehat{q}_C-(\widehat{q}_C)^2)$. In this case, we can easily show that the optimal solution $\widehat{q}_C^* = 1/2$, which exists only when $h > \frac{7(1-\alpha)-32k}{64}$. (2). If $\widehat{q}_C < -1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}$, the optimal solution $\widehat{q}_C^*$ is the root of $\frac{\alpha\left((\alpha-1)^2\widehat{q}_C^2(5\alpha+24k-5)-32h\left(3(\alpha-1)^2+32k^2+20(\alpha-1)k\right)\right)}{16(3\alpha+8k-3)^3} + \frac{\alpha\widehat{q}_C\left((\alpha-1)\widehat{q}_C^2\left(2(\alpha-1)^3+48k^3+50(\alpha-1)k^2+17(\alpha-1)^2k\right)-2h\left(9(\alpha-1)^3+256k^3+240(\alpha-1)k^2+78(\alpha-1)^2k\right)\right)}{(3\alpha+8k-3)^3\sqrt{(\alpha-1)\widehat{q}_C^2(\alpha+2k-1)-4h(3\alpha+8k-3)}} = 0$, which exists only when $\left.\frac{\partial\widehat{\Pi}_p}{\partial\widehat{q}_C}\right|_{\widehat{q}_C=-1+2\sqrt{1-\frac{4h+2k}{1-\alpha}}} < 0$.

Comparing the platform's profits under the above two scenarios, we can obtain that there exists a h^k such that when $h > h^k$, $\widehat{q}_C^* = 1/2$ and when $h < h^k$, the optimal solution $\widehat{q}_C^*$ is the root of $\frac{\alpha\left((\alpha-1)^2\widehat{q}_C^2(5\alpha+24k-5)-32h\left(3(\alpha-1)^2+32k^2+20(\alpha-1)k\right)\right)}{16(3\alpha+8k-3)^3} + \frac{\alpha\widehat{q}_C\left((\alpha-1)\widehat{q}_C^2\left(2(\alpha-1)^3+48k^3+50(\alpha-1)k^2+17(\alpha-1)^2k\right)-2h\left(9(\alpha-1)^3+256k^3+240(\alpha-1)k^2+78(\alpha-1)^2k\right)\right)}{(3\alpha+8k-3)^3\sqrt{(\alpha-1)\widehat{q}_C^2(\alpha+2k-1)-4h(3\alpha+8k-3)}} = 0$, where h^k is the solution of

$$\begin{aligned} &1/4 - \Big(\frac{1-\alpha}{3-3\alpha-8k}q_0^k + \frac{2}{3-3\alpha-8k}\sqrt{(q_0^k)^2(1-\alpha)(1-\alpha-2k)+4h(3-3\alpha-8k)}\Big)^3 \\ &+ 3\Big(\frac{1-\alpha}{3-3\alpha-8k}q_0^k + \frac{2}{3-3\alpha-8k}\sqrt{(q_0^k)^2(1-\alpha)(1-\alpha-2k)+4h(3-3\alpha-8k)}\Big)^2 q_0^k \quad = 0, \\ &- 3\Big(\frac{1-\alpha}{3-3\alpha-8k}q_0^k + \frac{2}{3-3\alpha-8k}\sqrt{(q_0^k)^2(1-\alpha)(1-\alpha-2k)+4h(3-3\alpha-8k)}\Big)(q_0^k)^2 \end{aligned} \tag{B.23}$$

and

$$\widehat{q}_D^{c*} = \frac{1-\alpha}{3-3\alpha-8k}\widehat{q}_C^* + \frac{2}{3-3\alpha-8k}\sqrt{(\widehat{q}_C^*)^2(1-\alpha)(1-\alpha-2k)+4h(3-3\alpha-8k)}. \tag{B.24}$$

From the implicit function theorem, we can further show that $\widehat{q}_C^*$ first increases and then decreases in k and h^k decreases in k.

Under the listing fee scheme, in the absence of platform certification, the seller i's payoff is $\widetilde{\pi}_i^d = q_i^2/4 - s - h - \frac{k}{2}q_i^2$ under disclosure and $\widetilde{\pi}_i^{nd} = \max(\widetilde{q}_D^2/16 - s, 0)$ under non-disclosure. When $\widetilde{q}_D^2/16 - s \geqslant 0$, solving $\widetilde{\pi}_i^d = \widetilde{\pi}_i^{nd}$ leads to the equilibrium $\widetilde{q}_D^* = \min\left\{4\sqrt{\frac{h}{3-8k}}, 1\right\}$, which requires $h \geqslant s(3-8k)$. When $\widetilde{q}_D^2/16 - s < 0$, solving $\widetilde{\pi}_i^d = \widetilde{\pi}_i^{nd}$ leads to the equilibrium $\widetilde{q}_D^* = 2\sqrt{\frac{s+h}{1-2k}}$, which exists only when $h < s(3-8k)$.

We now discuss the sellers' participation strategy in the presence of platform certification. When the disclosure cost $h \geqslant s(3-8k)$, if $\widetilde{q}_C < \widetilde{q}_D^*$, then all the uncertified sellers withhold their quality information, and their profit $\pi_i^{nc,nd} = \widetilde{q}_C^2/16 - s \geqslant 0$ only when $\widetilde{q}_C \geqslant 4\sqrt{s}$; otherwise, they quit platform selling due to the negative profit incurred. If $\widetilde{q}_C \geqslant \widetilde{q}_D^*$, then all the sellers sell through the platform. When the disclosure cost $h < s(3-8k)$, a certified seller i's payoff under disclosure is $\widetilde{\pi}_i^{c,d} = q_i^2/4 - h - \frac{k}{2}q_i^2 - s$ and under non-disclosure is $\widetilde{\pi}_i^{c,nd} = (\widetilde{q}_C + \widetilde{q}_D^c)^2/16 - s$. Solving $\widetilde{\pi}_i^{c,d} = \widetilde{\pi}_i^{c,nd}$, leads to the certified seller i's equilibrium disclosure threshold $\widetilde{q}_D^c = \min\left(1, \frac{\widetilde{q}_C + 2\sqrt{12h - 32kh + (\widetilde{q}_C)^2 - 2k(\widetilde{q}_C)^2}}{3-8k}\right)$. The requirement for a certified non-disclosure seller's profit to be nonnegative is $\widetilde{q}_C \geqslant 4\sqrt{s} - \widetilde{q}_D^*$. If $\widetilde{q}_C > \widetilde{q}_D^*$, then platform certification does not affect the sellers' participation strategy.

Based on the above discussion, the platform's expected profit can be written as

$$\widetilde{\Pi}_p = \begin{cases} \displaystyle\int_{\widetilde{q}_C}^1 s\,dq, & \text{if } h \geqslant s(3-8k) \text{ and } \widetilde{q}_C < 4\sqrt{s}; \\[2ex] \displaystyle\int_0^1 s\,dq, & \text{if } h \geqslant s(3-8k) \text{ and } \widetilde{q}_C \geqslant 4\sqrt{s}; \\[2ex] \displaystyle\int_{\max\{4\sqrt{s}-\widetilde{q}_D^*,\ \min\{\widetilde{q}_C, \widetilde{q}_D^*\}\}}^1 s\,dq, & \text{if } h < s(3-8k). \end{cases}$$

Solving the maximization problem of $\widetilde{\Pi}_p$, we can obtain that when $h \geqslant s(3-8k)$, $\widetilde{q}_C^* = 1$, and a seller i reveals quality information only when $q_i \geqslant 4\sqrt{\frac{h}{3-8k}}$. When $h < s(3-8k)$, $\widetilde{q}_C^* = 4\sqrt{s} - 2\sqrt{\frac{s+h}{1-2k}}$, and a seller i reveals quality information only when $q_i \geqslant 2\sqrt{\frac{s+h}{1-2k}}$.

B.4 When the Quality Standard Serves as Entry Barrier: Listing Fee Scheme

Proposition B.3. *When the platform enforces a quality standard as a prerequisite for entry, under the listing fee scheme, the equilibrium outcomes remain the same as in the baseline model.*

Proof of Proposition B.3. Under the listing fee scheme, according to Lemma 4, the platform's profit can be written as

$$\widetilde{\Pi}_p = \begin{cases} \displaystyle\int_{\widetilde{q}_C}^1 s\,dq, & \text{if } h \geqslant 3s \text{ and } \widetilde{q}_C < 4\sqrt{s}; \\[2ex] \displaystyle\int_0^1 s\,dq, & \text{if } h \geqslant 3s \text{ and } \widetilde{q}_C \geqslant 4\sqrt{s}; \\[2ex] \displaystyle\int_{\max\{4\sqrt{s}-\widetilde{q}_D^*,\ \min\{\widetilde{q}_C, \widetilde{q}_D^*\}\}}^1 s\,dq, & \text{otherwise.} \end{cases}$$

It is the same as Equation (6). Therefore, the equilibrium outcomes remain unchanged.

Reference 2

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