Online Appendices for 'Managing Consumer Bracketing Behaviours for E-tailing Operations'

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Review of the notation:

Notation	Description
Parameter	
v	Consumers' willingness to pay for the perfect match product.
m_i	Probability that the product's attributes perfectly match the expectation of the consumer i, $i \in \{n, b\}$.
d_i	Disutility of the consumer i keeping the mismatched product, $i \in \{n, b\}$.
c	Reverse logistics cost of the e-tailer (the product's reverse logistics cost incurred for returns).
Decision variable	
p	Selling price of the e-tailer.
f	Unit return service fee.
Calculated Parameter	
U_i	Utility of the consumer i, $i \in \{n, b\}$.
S_i	Size of the consumers i, $i \in \{n, b\}$.
Π	Expected profit of the e-tailer.

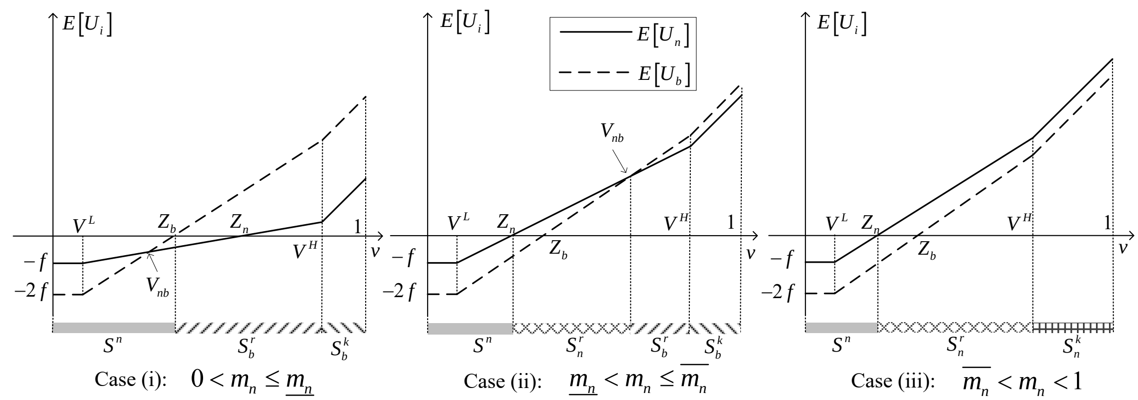
Appendix A: Consumer Segmentation

Appendix A1: Same Disutility

We use backward induction to prove the consumer best response as shown in Table 1. First, we assume that the consumer has purchased the product and assessed whether it matches their needs. The consumer makes decisions based on utility maximisation, i.e., if the payoff from keeping the product is higher than the fee of returning it, then they keep it; otherwise, they return the product and pay the return fee. Specifically, (i) the consumer with $0 < v \leq V^L$=$p - f$ returns the product regardless of match; (ii) the consumer with $V^L < v \leq V^H$=$p - f + d$ keeps the product if it matches and returns it otherwise; and (iii) the consumer with $V^H < v \leq 1$ keeps the product regardless of match.

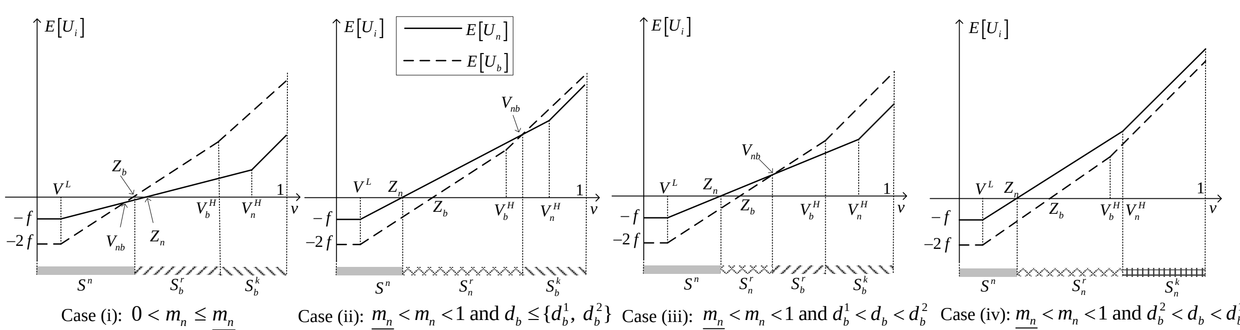
Now, we return to the second stage to assess whether the consumer is inclined to purchase the product. When $0 < v \leq V^L$, the consumer's expected payoff is $E[U_i] = -f(n_i + 1) < 0$, where $n_b = 1$ and $n_n = 0$. In this case, the consumer does not buy the product. When $V^L < v \leq V^H$, the consumer's expected payoff is $E[U_i]$ =$m_i(v - p - n_i f) - f(1 - m_i)(n_i + 1)$ and they purchases the product if $E[U_i] \geq 0$. This implies $v \geq Z_i = \frac{f(1-m_i+n_i)+m_i p}{m_i}$, where $Z_i > V^L$. To align with reality, we assume $Z_i < V^H$; otherwise, all the consumers buy the product and keep it.

Figure A.1 Consumer segmentation under the same disutility.



Let $V_{nb} = p + f\left(\frac{1}{m_b - m_n} - 1\right)$ denote the threshold where the consumer is indifferent between exhibiting the non-bracketing and bracketing behaviours. By comparing the slopes of the consumer's expected utilities $E[U_i]$, we derive three cases: (i) $Z_n \ge Z_b$, i.e., $0 < m_n \le \underline{m_n} = \frac{m_b}{2}$, (ii) $Z_n < Z_b$ and $V_{nb} \le V^H$, i.e., $\underline{m_n} < m_n \le \overline{m_n} = m_b - \frac{f}{d}$, and (iii) $Z_n < Z_b$ and $V_{nb} > V^H$, i.e., $\overline{m_n} < m_n < 1$. At one extreme, in case (i), consumers prefer bracketing over non-bracketing due to the low match probability m_n. At the other extreme, in case (iii), consumers prefer non-bracketing over bracketing due to the high m_n. Case (ii) represents a hybrid scenario where consumers have mixed preferences for bracketing and non-bracketing. Figure A.1 depicts consumer segmentation. Table 1 in the main text is derived from Figure A.1.

Figure A.2 Consumer segmentation under different disutilities.



Appendix A2: Different Disutilities

Figure A.2 depicts consumer segmentation, where $d_b^1 = \frac{f}{m_b - m_n}$, $d_b^2 = \frac{dn(1-m_n)-f}{1-m_b}$, $V_n^H{=}p - f + d_n$, and $V_b^H{=}p - f + d_b$. Figure A.2 demonstrates that the same consumer segmentation as in our main model occurs even when the disutility is different. Specifically, case i in Figure A.2 mirrors the consumer segmentation of case i in Figure A.1, case iii in Figure A.2 corresponds to case ii in Figure A.1, and case iv in Figure A.2 aligns with case iii in Figure A.1. Moreover, Figures A.1 and A.2 demonstrate that regardless of whether the disutility is the same or different, consumers' critical trade-off for the purchasing method remains unchanged. For instance, bracketing increases the likelihood that consumers will find a matching product, but it inevitably results in high return fees for them, a critical mechanism underlying our main findings. Therefore, when the disutility differs, our key insights remain unchanged.

Appendix B: The E-tailer's Profit Maximisation Problem

We write the Lagrangian as $\mathcal{L}(p,f) = \Pi(p,f) + \eta_1 S_b^r(p,f) + \eta_2 S_b^k(p,f) + \eta_3 S_n^r(p,f) + \eta_4 S_n^k(p,f)$, where η_i for $i = 1, \ldots, 4$ are Lagrangian multipliers associated with the model constraints. The stationary conditions are $\frac{\partial \mathcal{L}}{\partial S_b^r} = 0$, $\frac{\partial \mathcal{L}}{\partial S_b^k} = 0$, $\frac{\partial \mathcal{L}}{\partial S_n^r} = 0$, and $\frac{\partial \mathcal{L}}{\partial S_n^k} = 0$. Additionally, the feasibility conditions $S_b^r \geq 0$, $S_b^k \geq 0$, $S_n^r \geq 0$, $S_n^k \geq 0$, and $\eta_i \geq 0$ for $i = 1, \ldots, 4$, and the complimentary slackness conditions $\frac{\partial \mathcal{L}}{\partial \eta_1} = 0$, $\frac{\partial \mathcal{L}}{\partial \eta_2} = 0$, $\frac{\partial \mathcal{L}}{\partial \eta_3} = 0$, and $\frac{\partial \mathcal{L}}{\partial \eta_4} = 0$ should hold. We verify that the Hessian in each case is negative definite. We then solve for the optimal solutions. All the effective solutions are summarised in Table B. We first focus on the case where the e-tailer charges the return fee, i.e., $f > 0$.

Case (i): $0 < m_n \leq \underline{m_n}$ ($S_n^r = 0$ and $S_n^k = 0$)

Solution 1. ($S_b^r > 0$ and $S_b^k > 0$) $p = \frac{1}{4}(2 - 2d + dm_b)$, $f = \frac{1}{4}(2c + dm_b)$, $\eta_1 = 0$, and $\eta_2 = 0$. Inputting in these values, we have: $S_b^r = \frac{d}{2} - \frac{c}{m_b}$ and $S_b^k = \frac{1}{2}(1 + c - d)$. Check feasibility: $S_b^r \geq 0$ as $c \leq \frac{dm_b}{2} = c_l$, and $S_b^k \geq 0$. Therefore, when $0 < m_n \leq \underline{m_n}$ and $c \leq c_l$, this solution is optimal.

Solution 2. ($S_b^r = 0$ and $S_b^k > 0$) $p = \frac{1}{2}(1 + c - d)$, $f = \frac{dm_b}{2}$, $\eta_1 = (1 - m_b)(2c - dm_b)$, $\eta_2 = 0$. Inputting in these values, we have: $S_b^r = 0$ and $S_b^k = \frac{1}{2}(1 - c - d + dm_b)$. Check feasibility: $S_b^k \geq 0$ as $c \leq 1 - d + dm_b = \bar{c}$, and $\eta_1 \geq 0$ as $c \geq c_l$. Therefore, when $0 < m_n \leq \underline{m_n}$ and $c_l \leq c \leq \bar{c}$, this solution is optimal.

Solution 3. ($S_b^r > 0$ and $S_b^k = 0$) $p = \frac{1}{4}(2(2 + c - 2d) - m_b(1 + c - 2d))$, $f = \frac{1}{4}(2c - (1 + c - 2d)m_b)$, $\eta_1 = 0$, $\eta_2 = (1 + c - d)(m_b - 1)$. Check feasibility: this solution is never optimal because η_2 is negative.

Solution 4. ($S_b^r = 0$ and $S_b^k = 0$) $p = \frac{1}{2}(2 - 2d + dm_b)$, $f = \frac{dm_b}{2}$, $\eta_1 = 2c - m_b - cm_b$, $\eta_2 = d - dm_b - 1 + c$. Inputting in these values, we have: $S_b^r = 0$, and $S_b^k = 0$. Check feasibility: $\eta_1 \geq 0$ and $\eta_2 \geq 0$ as $c \geq 1 - d + dm_b = \overline{c_l}$. This solution is eliminated to ensure that the e-tailer's profit is positive.

Case (ii): $\underline{m_n} < m_n \leq \overline{m_n}$, ($S_n^r \geq 0$, $S_b^r \geq 0$, $S_b^k \geq 0$ and $S_n^k = 0$)

Solution 5. ($S_b^r > 0$, $S_b^k > 0$, and $S_n^r > 0$) $p = \frac{1}{4}\left(2 - d + dm_b\left(1 - \frac{1 - m_b}{m_b(1 - 4m_n) + 4m_n^2}\right)\right)$, $f = \frac{1}{4}\left(2c - d + dm_b\left(1 + \frac{1 - m_b}{4m_n^2 + m_b(1 - 4m_n)}\right)\right)$, $\eta_1 = 0$, $\eta_2 = 0$, and $\eta_3 = 0$. Inputting in these values, we have: $S_b^r = d - \frac{c}{2(m_b - m_n)} - \frac{d(1 - m_b)m_n}{m_b - 4m_b m_n + 4m_n^2}$, $S_b^k = \frac{1}{2}\left(1 + c - 2d + \frac{d(1 - m_b)m_b}{4m_n^2 + m_b(1 - 4m_n)}\right)$, and $S_n^r = \frac{1}{4}\left(\frac{1}{m_b - m_n} - \frac{1}{m_n}\right)\left(2c - d + dm_b\left(1 + \frac{1 - m_b}{4m_n^2 + m_b(1 - 4m_n)}\right)\right)$. Check feasibility: $S_b^r \geq 0$ as $c \leq \frac{2d(m_b - m_n)\left(4m_n^2 - m_n + m_b(1 - 3m_n)\right)}{4m_n^2 + m_b(1 - 4m_n)} = c_{m1}$, $S_b^k \geq 0$ as $c \geq 2d - \frac{d(1 - m_b)m_b}{4m_n^2 + m_b(1 - 4m_n)} - 1 = c_{m2}$, and $S_n^r \geq 0$. Therefore, when $\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m2} \leq c \leq c_{m1}$, this solution is optimal.

Solution 6. ($S_b^r > 0$, $S_b^k = 0$, and $S_n^r > 0$) $p = \frac{1}{2}(2+c-2d) - \frac{m_n}{m_b}(1+c-2d)(m_b-m_n)$, $f = \frac{c}{2} - \frac{m_n}{m_b}(1+c-2d)(m_b-m_n)$, $\eta_1 = 0$, $\eta_2 = 4(1+c-2d)m_n - 1 - c + d + dm_b - \frac{4(1+c-2d)m_n^2}{m_b}$, and $\eta_3 = 0$. Inputting in these values, we have: $S_b^r = d + \frac{(1+c-2d)m_n}{m_b} - \frac{c}{2(m_b-m_n)}$, $S_b^k = 0$, and $S_n^r = \left(\frac{1}{m_b-m_n} - \frac{1}{m_n}\right)\left(\frac{c}{2} - \frac{m_n}{m_b}(1+c-2d)(m_b-m_n)\right)$. Check feasibility: $S_b^r \geq 0$ as $c \leq \frac{2(m_b-m_n)(dm_b+(1-2d)m_n)}{2m_n^2+m_b(1-2m_n)} = c_{m2}^{NB}$, $\eta_2 \geq 0$ as $c \leq c_{m2}$, and $S_n^r \geq 0$. Therefore, when $\underline{m_n} < m_n \leq \overline{m_n}$ and $c \leq \min\{c_{m2}, c_{m2}^{NB}\}$, this solution is optimal.

Solution 7. ($S_b^r = 0$, $S_b^k > 0$, and $S_n^r > 0$) $p = \frac{1}{2}(1+c-d-dm_b+2dm_n)$, $f = d(m_b-m_n)$, $\eta_1 = 2(2c-d-7dm_b)m_n + 8dm_n^2 - 2m_b(2c-2d-3dm_b) + \frac{m_b(c-2dm_b)}{m_n}$, $\eta_2 = 0$, and $\eta_3 = 0$. Inputting in these values, we have: $S_b^r = 0$, $S_b^k = \frac{1}{2}(1-c-d+3dm_b-4dm_n)$, and $S_n^r = d\left(2-\frac{m_b}{m_n}\right)$. Check feasibility: $S_b^k \geq 0$ as $c \leq 1-d+3dm_b-4dm_n = c_{m1}^{NB}$, $\eta_1 \geq 0$ as $c \geq c_{m1}$, and $S_n^r \geq 0$. Therefore, when $\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m1} \leq c \leq c_{m1}^{NB}$, this solution is optimal.

Solution 8. ($S_b^r = 0$, $S_b^k = 0$, and $S_n^r > 0$) $p = 1 - d + dm_b - dm_n$, $f = d(m_b-m_n)$, $\eta_1 = \frac{2(1+c-2d)m_n^2+m_b(c-2(1+c-3d)m_n)-2dm_b^2}{m_n}$, $\eta_2 = c+d-3dm_b+4dm_n-1$, and $\eta_3 = 0$. Inputting in these values, we have: $S_b^r = 0$, $S_b^k = 0$, and $S_n^r = d\left(2-\frac{m_b}{m_n}\right)$. Check feasibility: $\eta_1 \geq 0$ as $c \geq c_{m2}^{NB}$, $\eta_2 \geq 0$ as $c \geq c_{m1}^{NB}$, and $S_n^r \geq 0$. In this case, if $c > \frac{dm_b+m_n-2dm_n}{1-m_n} = \overline{c_m}$, the e-tailer's profit is negative. Therefore, when $\underline{m_n} < m_n \leq \overline{m_n}$ and $\max\{c_{m1}^{NB}, c_{m2}^{NB}\} \leq c \leq \overline{c_m}$, this solution is optimal.

Solution 9. ($S_b^r > 0$, $S_b^k > 0$, and $S_n^r = 0$) $p = \frac{1}{2}(1+c-d+dm_b)$, $f = 0$, $\eta_1 = 0$, $\eta_2 = 0$, and $\eta_3 = \frac{cm_b-2m_b(2c-d+dm_b)m_n+2(2c-d+dm_b)m_n^2}{m_b-2m_n}$. This solution was eliminated because $f = 0$.

Solution 10. ($S_b^r > 0$, $S_b^k = 0$, and $S_n^r = 0$) $p = 1-d$, $f = 0$, $\eta_1 = 0$, $\eta_2 = c+d+dm_b-1$, and $\eta_3 = \frac{2(1+c-2d)m_n^2+m_b(c-2(1+c-2d)m_n)}{m_b-2m_n}$. This solution was eliminated because $f = 0$.

Solution 11. ($S_b^r = 0$, $S_b^k > 0$, and $S_n^r = 0$) There is no viable solution.

Solution 12. ($S_b^r = 0$, $S_b^k = 0$, and $S_n^r = 0$) There is no viable solution.

Case (iii): $\overline{m_n} < m_n < 1$ ($S_b^r = 0$ and $S_b^k = 0$)

Table B Effective solutions and corresponding conditions.

Scenario	Solution	S_b^r	S_b^k	S_n^r	S_n^k	Conditions
$f > 0$	1	+	+	0	0	$0 < m_n \leq \underline{m_n}$ and $c \leq c_l$
	2	0	+	0	0	$0 < m_n \leq \underline{m_n}$ and $c_l \leq c \leq \overline{c}$
	5	+	+	+	0	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m_2} \leq c \leq c_{m_1}$
	6	+	0	+	0	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c \leq \min\{c_{m_2}, c_{m_2}^{NB}\}$
	7	0	+	+	0	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m_1} \leq c \leq c_{m_1}^{NB}$
	8	0	0	+	0	$\underline{m_n} < m_n \leq \overline{m_n}$ and $\max\{c_{m_1}^{NB}, c_{m_2}^{NB}\} \leq c \leq \overline{c}$
	13	0	0	+	+	$\overline{m_n} < m_n < 1$ and $c \leq c_h^R$
	14	0	0	0	+	$\overline{m_n} < m_n < 1$ and $c \geq c_h^R$
$f = 0$	17	+	+	0	0	$c \leq c_h$
	18	+	0	0	0	$c \geq c_h$

Solution 13. ($S_n^r > 0$ and $S_n^k > 0$) $p = \frac{1}{2}(1 - d + dm_n)$, $f = \frac{1}{2}(c + dm_n)$, $\eta_3 = 0$, and $\eta_4 = 0$. Inputting in these values, we have: $S_n^r = \frac{1}{2}\left(d - \frac{c}{m_n}\right)$ and $S_n^k = \frac{1}{2}(1 + c - d)$. Check feasibility: $S_n^r \geq 0$ as $c \leq dm_n = c_h^R$, and $S_b^k \geq 0$. Therefore, when $\overline{m_n} < m_n < 1$ and $c \leq c_h^R$, this solution is optimal.

Solution 14. (S_n^r=0 and $S_n^k > 0$) $p = \frac{1}{2}(1 - d + dm_n)$, $f = dm_n$, $\eta_3 = (1 - m_n)(c - dm_n)$, and $\eta_4 = 0$. Inputting in these values, we have: $S_n^r = 0$ and $S_n^k = \frac{1}{2}(1 - d + dm_n)$. Check feasibility: $S_n^k \geq 0$, and $\eta_3 \geq 0$ as $c \geq c_h^R$. Therefore, when $\overline{m_n} < m_n < 1$ and $c \geq c_h^R$, this solution is optimal.

Solution 15. ($S_n^r > 0$ and $S_n^k = 0$) $p = \frac{1}{2}(2 + c - 2d - (1 + c - 2d)m_n)$, $f = \frac{1}{2}(c - (1 + c - 2d)m_n)$, $\eta_3 = 0$, and $\eta_4 = (1 + c - d)(m_n - 1)$. Check feasibility: this solution is never optimal because η_4 is negative.

Solution 16. ($S_n^r = 0$ and $S_n^k = 0$) $p = 1 - d + dm_n$, $f = dm_n$, $\eta_3 = c - (1 + c)m_n$, and $\eta_4 = d - dm_n - 1$. Inputting in these values, we have: $S_n^r = 0$ and $S_n^k = 0$. Check feasibility: this solution is never optimal because η_4 is negative.

Next, we focus on the case where the e-tailer offers free returns, i.e., $f = 0$. In this case, consumers prefer bracketing than non-bracketing, i.e., $S_n^r = 0$ and $S_n^k = 0$.

Solution 17. ($S_b^r > 0$ and $S_b^k > 0$) $p = \frac{1}{2}(1 + c - d + dm_b)$, $\eta_1 = 0$, and $\eta_2 = 0$. Inputting in these values, we have: $S_b^r = d$ and $S_b^k = \frac{1}{2}(1 - c - d - dm_b)$. Check feasibility: $S_b^r \geq 0$ and $S_b^k \geq 0$ as $c \leq 1 - d - dm_b = c_h$. Therefore, when $f = 0$ and $c \leq c_h$, this solution is optimal.

Solution 18. ($S_b^r > 0$ and $S_b^k = 0$) $p = 1 - d$, $\eta_1 = 0$, and $\eta_2 = c + d + dm_b - 1$. Inputting in these values, we have: $S_b^r = d$ and $S_b^k = 0$. Check feasibility: $S_b^k \geq 0$, and $\eta_2 \geq 0$ as $c \geq c_h$. Therefore, when $f = 0$ and $c \geq c_h$, this solution is optimal.

Solution 19. ($S_b^r = 0$ and $S_b^k > 0$) There is no viable solution.

Solution 20. ($S_b^r = 0$ and $S_b^k = 0$) There is no viable solution.

Appendix C: Proofs of the Theorems

Proof of Theorem 1

By comparing the e-tailer's profit from charging a return fee ($\prod_{17}^*$ and $\prod_{18}^*$) with offering free returns ($\prod_i^*$ for $i = 1, 2, 5, 6, 7, 8, 13, 14$), we have $\prod_{17}^* > \prod_i^*$ for $i = 1, 2, 5, 6, 7, 8, 13, 14$ iff $\overline{m_n} < m_n < 1$ and $c \leq \min\{c_{h1}^{NB}, c_h\}$, and $\prod_{18}^* \geq \prod_i^*$ for $i = 1, 2, 5, 6, 7, 8, 13, 14$ iff $\overline{m_n} < m_n < 1$ and $c_h \leq c \leq c_{h2}^{NB}$, where $c_{h1}^{NB}, c_{h2}^{NB} < c_h^R$.

Based on the conditions corresponding to the equilibrium solutions under the optimal return charge model, we provide the optimal selling and return strategies for the e-tailer in Figure 2 in the main text. Table C.1 summarises the regions shown in Figure 2. To prove that Figure 2 is mathematically valid, we analyse the mathematical properties of the conditions corresponding to all the equilibrium solutions. Taking Region II as an example:

(i) The first derivative of c_{m1} with respect to d is $\frac{\partial c_{m1}}{\partial d} = \frac{2(m_b-m_n)\left(m_n-4m_n^2-m_b(1-3m_n)\right)}{-4m_n^2-m_b(1-4m_n)}$. Moreover, $c_{m1}|_{d=0} = 0$. $\frac{\partial c_{m1}}{\partial d} > 0$ because $\underline{m_n} < m_n \leq \overline{m_n}$.

(ii) The first derivative of c_{m1}^{NB} with respect to d is $\frac{\partial c_{m1}^{NB}}{\partial d} = 3m_b - 4m_n - 1$. Moreover, $c_{m1}^{NB}\big|_{d=0} = 1$. $\frac{\partial c_{m1}^{NB}}{\partial d} < 0$ because $\underline{m_n} < m_n \leq \overline{m_n}$.

(iii) The first derivative of c_{m2} with respect to d is $\frac{\partial c_{m2}}{\partial d} = 2 - \frac{(1-m_b)m_b}{m_b(1-4m_n)+4m_n^2}$. Moreover, $c_{m2}|_{d=d_m} = 0$, where $d_m = \frac{1}{2} + \frac{(1-m_b)m_b}{2\left(m_b^2+m_b(1-8m_n)+8m_n^2\right)}$. $\frac{\partial c_{m2}}{\partial d} > 0$ and $d_m > 0$ because $\underline{m_n} < m_n \leq \overline{m_n}$.

(iv) The first derivative of c_{m2}^{NB} with respect to d is $\frac{\partial c_{m2}^{NB}}{\partial d} = \frac{2(m_b-2m_n)(m_b-m_n)}{2m_n^2+m_b(1-2m_n)}$. Moreover, $c_{m2}^{NB}\big|_{d=1} = \frac{2(m_b-m_n)(m_b+(1-2)m_n)}{2m_n^2+m_b(1-2m_n)}$. $\frac{\partial c_{m1}^{NB}}{\partial d} < \frac{\partial c_{m2}^{NB}}{\partial d} < 0$ and $c_{m2}^{NB}\big|_{d=1} > 0$ because $\underline{m_n} < m_n \leq \overline{m_n}$.

The mathematical properties ensure that the conditions for Region II correspond correctly to the visual representation. Proofs for other regions are similar to those of Region II. Table C.2 summarises the mathematical properties of the conditions for the equilibrium solutions.

Table C.1 Summary of the regions depicted in Figure 2 and the corresponding solutions.

Region in Figure 2	Solution	Market composition	Return strategy
$\mathrm{I_a}$	2	Target bracketers	Partial-refund, allow returns
$\mathrm{I_b}$	1		
$\mathrm{II_a}$	7	Target bracketers and non-bracketers	Partial-refund, allow returns
$\mathrm{II_b}$	5		
$\mathrm{II_c}$	6		
$\mathrm{III_a}$	8	Target non-bracketers	Partial-refund, allow returns
$\mathrm{III_b}$	13		
$\mathrm{IV_a}$	17	Target bracketers	Full-refund, allow returns
$\mathrm{IV_b}$	18		
V	14	Target non-bracketers	Partial-refund, disallow returns

Table C.2 Mathematical properties of the conditions corresponding to equilibrium solutions.

Region in Figure 2	Solution	Condition	Mathematical property
$\mathrm{I_a}$	2	$0 < m_n \leq \underline{m_n}$ and $c_l \leq c \leq \overline{c_l}$	$\frac{\partial \overline{c_l}}{\partial d} < 0$, $\overline{c_l}|_{d=0} = 1$, and $\overline{c_l}|_{d=1} = m_b$
$\mathrm{I_b}$	1	$0 < m_n \leq \underline{m_n}$ and $c \leq c_l$	$\frac{\partial c_l}{\partial d} > 0$, $c_l|_{d=0} = 0, c_l|_{d=1} = \frac{m_b}{2}$, and $c_l|_{d=1} < \overline{c_l}|_{d=1}$
$\mathrm{II_a}$	7	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m1} \leq c \leq c_{m1}^{NB}$	$\frac{\partial c_{m1}}{\partial d} > 0$, $\frac{\partial c_{m1}^{NB}}{\partial d} < 0$, $c_{m1}|_{d=0} = 0$, and $c_{m1}^{NB}\big|_{d=0} = 1$
$\mathrm{II_b}$	5	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c_{m2} \leq c \leq c_{m1}$	$\frac{\partial c_{m2}}{\partial d} > 0$, $c_{m2}|_{d=d_m} = 0$, and $d_m > 0$
$\mathrm{II_c}$	6	$\underline{m_n} < m_n \leq \overline{m_n}$ and $c \leq \min\{c_{m2}, c_{m2}^{NB}\}$	$\frac{\partial c_{m1}^{NB}}{\partial d} < \frac{\partial c_{m2}^{NB}}{\partial d} < 0$ and $c_{m2}^{NB}\big|_{d=1} > 0$
$\mathrm{III_a}$	8	$\underline{m_n} < m_n \leq \overline{m_n}$ and $\max\{c_{m1}^{NB}, c_{m2}^{NB}\} \leq c \leq \overline{c_m}$	$\frac{\partial \overline{c_m}}{\partial d} < 0$
$\mathrm{III_b}$	13	$\overline{m_n} < m_n < 1$ and $c \leq c_h^R$	$\frac{\partial c_h^R}{\partial d} > 0$, $c_h^R\big|_{d=0} = 0$, and $c_h^R\big|_{d=1} = m_n$
$\mathrm{IV_a}$	17	$c < \min\{c_{h1}^{NB}, c_h\}$	$\frac{\partial c_h}{\partial d} < 0$, $c_h|_{d=d_h} = 0$, and $d_h > 0$
$\mathrm{IV_b}$	18	$c_h \leq c \leq c_{h2}^{NB}$	c_{h1}^{NB} and c_{h2}^{NB} are non-monotonic
V	14	$\overline{m_n} < m_n < 1$ and $c \geq c_h^R$	$\frac{\partial c_h^R}{\partial d} > 0$, $c_h^R\big|_{d=0} = 0$, and $c_h^R\big|_{d=1} = m_n$

Proof of Theorem 2

The conclusion of Theorem 2 follows from a systematic integration of all candidate optima derived in Appendix B. Specifically, Appendix B characterises the retailer's analytical optimal solutions under different feasibility conditions. We then map each candidate optimum, together with its corresponding conditions, into the parameter-space partition shown in Figure 2, thereby obtaining the global optimal strategy as a function of (m_n, c, d).

More precisely, for any candidate solution identified in Appendix B, we can specify the necessary and sufficient conditions under which it is valid. These conditions can be expressed as a set of inequalities in (m_n, c, d), which in turn determine the corresponding region in the parameter space. For example, when $0 < m_n < \underline{m_n}$ and $c < c_l$, the optimum associated with this region in Figure 2 is given by p_1^* and f_1^* (see the Appendix B). Applying the same procedure to all candidate optima in Appendix B, we assign each of them to a mutually exclusive region defined over (m_n, c, d), which yields the complete partition in Figure 2 and the region–optimum correspondence summarised in the table.Based on this partition, we further compare the outcomes across regions and find that the optimal pair (p^*, f^*) varies not only in magnitude but also in the induced consumer segmentation, that is, the equilibrium patterns of bracketing versus no bracketing and returning versus not returning. These differences are driven by the relative magnitudes of (m_n, c, d). Consequently, the optimal management of bracketing is not simply a matter of encouraging or discouraging bracketing uniformly. Instead, the e-tailer must tailor pricing and return fee combinations to product attributes (m_n, c) and consumer attribute d across different regions, thereby influencing bracketing and return decisions in a targeted manner. This establishes the result stated in Theorem 2.

Proof of Theorem 3

Consider Solution 1 in Appendix B, which serves as a representative case where, even with $c = 0$, the optimal return fee $f^* = \frac{1}{4}dm_b$ remains strictly positive. This implies that, at $c = 0$, the retailer's optimal policy within this regime is to charge a strictly positive return fee. The remaining solutions in Appendix B can be verified analogously by setting $c = 0$ under their respective feasibility constraints. We establish that a free-return policy ($f^*|_{c=0} = 0$) is optimal when $m_n > \overline{m_n}$ and $d < \hat{d}$ (corresponding to Regions IV_a and IV_b). Here, $\hat{d} = \frac{1+2m_b-m_n \pm \sqrt{4m_b^2-m_n-4m_b m_n+m_n^2}}{1+4m_b-m_n}$.

Proof of Theorem 4

Consider Solution 2 in Appendix B. Under this solution, the size of the bracketing segment is $S_b = \frac{1}{2}(1-c-d+dm_b)$. The feasibility conditions for this solution are $0 < m_n < \underline{m_n}$ and $c_l \leq c \leq \bar{c}$. Under these conditions, since $d > 0$ and $m_b > 0$, it follows that $S_b > 0$. Differentiating the threshold $\bar{c}$ with respect to d yields $\frac{\partial \bar{c}}{\partial d} = -(1-m_b) < 0$, indicating that a lower mismatch disutility allows the e-tailer to tolerate higher reverse logistics costs. Moreover, taking the limit as $d \to 0$, we find $\lim_{d\to 0} \bar{c} = 1$. Similarly, for the case where $\underline{m_n} < m_n \le \overline{m_n}$, the upper bound c_{m1}^{NB} exhibits identical properties. Hence, even when the reverse-logistics cost is relatively high, as long as the feasibility conditions are satisfied, the equilibrium can still feature a strictly positive bracketing segment, which allows the e-tailer to benefit from bracketing.

Proof of Theorem 5

We conduct a sensitivity analysis to examine how the impact of the reverse logistics cost on the e-tailer's price and return service fee. We exclude Region IV from our analysis as it involves free returns. Using Region II$_c$ as an example, the first derivatives of the optimal price and the return service fee with respect to c are $\frac{\partial p^*}{\partial c} = \frac{1}{2} + m_n\left(\frac{m_n}{m_b} - 1\right)$ and $\frac{\partial f^*}{\partial c} = \frac{1}{2} + m_n\left(\frac{m_n}{m_b} - 1\right)$. $\frac{\partial p}{\partial c} > 0$ and $\frac{\partial f}{\partial c} > 0$ because $\underline{m_n} < m_n \le \overline{m_n}$. The sensitivity analysis for other regions follows the same logic as in Region II$_c$. Table C.3 summarises the impact of the reverse logistics cost on the equilibrium solutions. From Table C.3, we find that when the return rate from bracketing is low, i.e., $R_i^b < \underline{R^b} = \begin{cases} R_1^b, & 0<m_n \le \underline{m_n} \\ \min\{R_5^b, R_6^b\}, & \underline{m_n}<m_n \le \overline{m_n} \end{cases}$, and the e-tailer benefits from bracketing (Regions I$_a$ and II$_a$), then $\frac{\partial p}{\partial c} > \frac{\partial f}{\partial c} = 0$. This result indicates that when the reverse logistics cost increases, the e-tailer should transfer this cost to all the consumers by raising the price, rather than recouping them from consumers who make returns as the return service fee. In addition, we also find that when the return rate from bracketing is high, i.e., $R_i^b > \overline{R^b} = \begin{cases} R_2^b, & 0<m_n \le \underline{m_n} \\ \max\{R_5^b, R_7^b\}, & \underline{m_n}<m_n \le \overline{m_n} \end{cases}$, and the e-tailer can benefit from bracketing (Region II$_c$), then $\frac{\partial p}{\partial c} > 0$ and $\frac{\partial f}{\partial c} > 0$.

Table C.3 **Impact of the reverse logistics cost on the equilibrium solutions.**

Region in Figure 2	**Solution**	**Impact on price $\frac{\partial p}{\partial c}$**	**Impact on return service fee $\frac{\partial f}{\partial c}$**	**Return rate from bracketing R_i^b**
I$_a$	2	>0	$=0$	$R_2^b = \frac{1}{2}$
I$_b$	1	$=0$	>0	$R_1^b = \frac{m_b(1+3c+d-dm_b)-4c}{2(1+c)m_b-4c} > R_2^b$
II$_a$	7	>0	$=0$	$R_7^b = \frac{1}{2} < R_5^b$
II$_b$	5	$=0$	>0	$R_5^b = \frac{s_b^k+(2-m_b)s_b^r}{2(s_b^k+s_b^r)} < R_6^b$
II$_c$	6	>0	>0	$R_6^b = 1 - \frac{m_b}{2}$
III$_a$	8	$=0$	$=0$	$R_8^b = 0$
III$_b$	13	$=0$	>0	$R_{13}^b = 0$
V	14	$=0$	$=0$	$R_{14}^b = 0$

Proof of Theorem 6

Using Region II$_c$ as an example, the first derivative of the optimal profit with respect to m_n is $\frac{\partial \pi}{\partial m_n} = 1 + 2c - cd(3+\gamma) - (1-d)d(3+\gamma) + c^2\left(1 + \frac{1}{4\gamma(1-m_n)^2}\right) - \frac{c^2}{4m_n^2} + \frac{(1+c-2d)^2(1-\gamma)m_n^2}{(\gamma+(1-\gamma)m_n)^2} - \frac{2(1+c-2d)^2 m_n}{\gamma+(1-\gamma)m_n}$. We can prove that iff $m_6 < m_n < \overline{m_n}$ and $0 < c < \min\{c_6, c_{m2}, c_{m2}^{NB}\}$, then $\frac{\partial \pi}{\partial m_n} < 0$; otherwise, $\frac{\partial \pi}{\partial m_n} \ge 0$. Table C.4 summarises the results.

Table C.4 Impact of the match probability m_n on the equilibrium profits.

Region in Figure 2	Solution	Impact on profit $\frac{\partial \Pi}{\partial m_n}$
$\mathrm{I_{a,b}}$, $\mathrm{III_b}$, $\mathrm{IV_{a,b}}$, V	1, 2, 13, 14, 17, 18	>0
$\mathrm{II_a}$	7	$\begin{cases} <0 & \text{iff } \underline{m_n}<m_n<m_7 \text{ and } c_7<c<c_{m1}^{NB} \\ \geq 0 & \text{otherwise} \end{cases}$
$\mathrm{II_b}$	5	$\begin{cases} <0 & \text{iff } m_5<m_n<\overline{m_n} \text{ and } \max\{0,c_{m2}\}<c<c_5 \\ \geq 0 & \text{otherwise} \end{cases}$
$\mathrm{II_c}$	6	$\begin{cases} <0 & \text{iff } m_6<m_n<\overline{m_n} \text{ and } 0<c<\min\{c_6,c_{m2},c_{m2}^{NB}\} \\ \geq 0 & \text{otherwise} \end{cases}$
$\mathrm{III_a}$	8	$\begin{cases} <0 & \text{iff } m_8<m_n<\overline{m_n},\ c_{m2}^{NB}<c<c_8,\ d_8<d<1 \\ \geq 0 & \text{otherwise} \end{cases}$

Appendix D: Summary of the Thresholds

Thres.	Expression	Thres.	Expression
$\underline{m_n}$	$\frac{m_b}{2}$	c_{m2}^{NB}	$\frac{2(m_b-m_n)\left(dm_b+(1-2d)m_n\right)}{2m_n^2+m_b(1-2m_n)}$
$\overline{m_n}$	$m_b-\frac{f}{d}$	c_{h1}^{NB}	$\frac{m_n}{2m_n-1}\left(1+2d-3dm_b+dm_n-\frac{1}{2}\sqrt{(6dm_b-2(1+2d+dm_n))^2-\frac{4d(2m_n-1)(m_b(2-2d+dm_b)+(d-2)m_n)}{m_n}}\right)$
m_5, m_6	$1-\frac{1}{1+\sqrt{\gamma}}$	c_{h2}^{NB}	$\frac{m_n}{m_n-1}\left(3d-2dm_b+dm_n-\frac{1}{2}\sqrt{4d^2(3-2m_b+m_n)^2+\frac{4(m_n-1)((d-1)(d-1+4dm_b)-(d-2)dm_n)}{m_n}}\right)$
m_7	$\sqrt{2}\sqrt{\frac{\gamma}{3+5\gamma}}$	c_h^R	dm_n
m_8	$\sqrt{\frac{\gamma}{1+\gamma}}$	$\bar{c}$	$\begin{cases} 1-d+dm_b & m_n\leq\underline{m_n} \\ \frac{dm_b+m_n^2-2dm_n}{1-m_n} & \text{others} \end{cases}$
c_l	$\frac{dm_b}{2}$	c_5	$\frac{\gamma(m_n-1)^2m_n^2\left(d(\gamma-1)-\sqrt{\text{(complex expression)}}\right)}{2\gamma m_n-(\gamma-1)m_n^2-\gamma}$
c_{m1}	$\frac{2d(m_b-m_n)\left(4m_n^2-m_n+m_b(1-3m_n)\right)}{4m_n^2+m_b(1-4m_n)}$	c_6	$\frac{2\left(d(3+\gamma)-2-\frac{2(2d-1)(\gamma-1)m_n^2}{(\gamma-(\gamma-1)m_n)^2}+\frac{4(2d-1)m_n}{(\gamma-1)m_n-\gamma}+\sqrt{\text{(complex expression)}}\right)}{4\gamma/(\gamma-1)+1/\gamma(m_n-1)^2-1/m_n^2-4\gamma^2/(\gamma-1)(\gamma+m_n-\gamma m_n)^2}$
c_{m2}	$2d-1-\frac{d(1-m_b)m_b}{4m_n^2+m_b(1-4m_n)}$	c_7	$\frac{m_n^2\left(d-1+\gamma+d\gamma(4+11\gamma)-d(1+3\gamma)^2m_n\right)-2d\gamma^2}{(3+5\gamma)m_n^2-2\gamma}$
c_h	$1-d-dm_b$	c_8	$d-1+d\gamma+\frac{(d-1)\gamma}{(1+\gamma)m_n^2-\gamma}$
c_{m1}^{NB}	$1-d+3dm_b-4dm_n$	d_8	$\frac{m_n(2\gamma+(1-\gamma)m_n)}{2\gamma m_n+(1+\gamma^2)m_n^2-\gamma^2}$

Appendix E: Return Strategy

See Figures E.1–E.3 for details.

Appendix F: Proofs for Section 6

Under a fixed return fee policy, the expected-utility expressions for single-product purchases, i.e., purchasing only product 1 or only product 2, coincide with those in the main model, regardless of whether the consumer adopts bracketing. Consequently, this section focusses on analysing consumer decision-making regarding the joint purchase of both products. Given that the return fee is levied per return instance rather than per item, once a consumer opts to bracket one product type, the return fee becomes a sunk cost, as a return is inevitable. In this scenario, the marginal cost of extending the bracketing strategy to the second product is zero; however, doing so enhances the consumer's expected

Figure E.1 Free returns (Zappos.com).

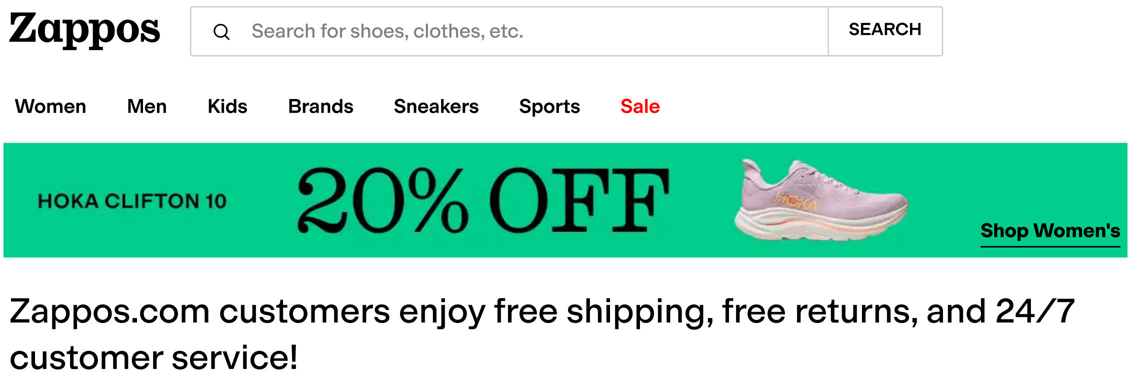


Figure E.2 Quantity fee (Amazon.co.uk).

In many cases, we'll provide you with a pre-paid returns label. If the item you purchased is not eligible for Free Return, and you're using a pre-paid return label to return an item which was sold or dispatched by Amazon, we'll deduct the cost of the return from your refund (unless the item is defective, damaged or incorrect, or was delivered late, see below for more details). For returns from within the UK, costs start at £1.99. International return costs start at £8.09. Unless an exact return cost was stated at the time of purchase, please note that returns that contain multiple items, or large items, may cost more. We offer various returns methods which will differ depending on the item(s) you're returning. We'll always display the most accurate return costs and methods available for your item(s) in our Online Returns Centre when you create your label and the exact return cost may also be displayed to you before you place your order.

Figure E.3 Fixed fee (Zara.com).

How To Return | Help | ZARA United Kingdom

You can return your items by dropping off your package at a drop point. SEARCH

The **cost of the return request** is 2.95 GBP, which will be deducted from your refund.

If you have **purchased as a registered user**, you can request the return from the Returns section of your account.

If you have made a **guest purchase**, you can request it by accessing the link that appears in any of the emails we sent you about your order.

utility by increasing the match probability. Accordingly, in the context of joint purchasing, the strategy of bracketing only a single product is strictly dominated by the strategy of bracketing both products. We therefore exclude this partial bracketing scenario from subsequent analysis. Given the above, a rational consumer faces three feasible options:

(i) Non bracketing: Purchase both product 1 and product 2 without bracketing, yielding the expected utility:

$$U_{NB} = \underbrace{m_{n1}m_{n2}(v_1 - p_1 + v_2 - p_2)}_{\text{Keep 1 and 2}} + \underbrace{m_{n1}(1-m_{n2})(v_1 - p_1 - f)}_{\text{Keep 1 and return 2}} + \underbrace{m_{n2}(1-m_{n1})(v_2 - p_2 - f)}_{\text{Keep 2 and return 1}} - \underbrace{f(1-m_{n1})(1-m_{n2})}_{\text{Return 1 and 2}}.$$

(ii) Full bracketing: Purchase both product 1 and product 2 with bracketing, yielding the expected utility:

$$U_{FB} = \underbrace{m_b m_b (v_1 - p_1 + v_2 - p_2)}_{\text{Keep 1 and 2 and return the other}} + \underbrace{m_b(1-m_b)(v_1 - p_1 - f)}_{\text{Keep 1 and return the other}} + \underbrace{m_b(1-m_b)(v_2 - p_2 - f)}_{\text{Keep 2 and return the other}} - \underbrace{f(1-m_b)(1-m_b)}_{\text{Return all}}.$$

(iii) No purchase: Exit the market, yielding the expected utility 0.

Table F.1 The consumer's best response to the e-tailer's price and return fee.

Conditions	S_n^{12}	S_b^{12}
Case 1: $m_{n1} > m_{n2}$		
$\bar{v} > 0, \bar{w} > 0$	$\frac{f^2(m_{n1}+m_b\phi)^2}{2wm_b(m_b-m_{n1})m_{n1}(m_{n1}-m_{n2})}$	$\frac{1}{w}\left(\Omega - \frac{f^2}{2m_b^2} - \frac{f^2(m_{n1}+m_b\phi)^2}{2m_b^2(m_b-m_{n1})(m_{n1}-m_{n2})}\right)$
$\bar{v} \le 0, \bar{w} > 0$	$\frac{f^2}{2w}\left(2 - \psi + \frac{(m_{n1}m_{n2})^2}{(m_b-m_{n1})(m_b-m_{n2})}\right)$	$\frac{1}{w}\left(\Omega - \frac{f^2(m_{n1}m_{n2})^2}{2(m_b-m_{n1})(m_b-m_{n2})}\right)$
$\bar{v} > 0, \bar{w} \le 0$	0	$\frac{1}{w}\left(\Omega - \frac{f^2}{2m_b^2}\right)$
Case 2: $m_{n1} < m_{n2}$		
$\bar{v} > 0, \bar{w} > 0$	$\frac{f^2(m_{n2}+m_b\phi)^2}{2wm_b(m_b-m_{n2})m_{n2}(m_{n2}-m_{n1})}$	$\frac{1}{w}\left(\Omega - \frac{f^2[m_b(m_{n1}+m_{n2}-2m_{n1}m_{n2})-m_{n1}m_{n2}-m_b^2\phi^2]}{2m_b^2(m_b-m_{n2})(m_{n2}-m_{n1})}\right)$
$\bar{v} \le 0, \bar{w} > 0$	0	$\frac{1}{w}\left(\Omega - \frac{f^2}{2m_b^2}\right)$
$\bar{v} > 0, \bar{w} \le 0$	$\frac{f^2}{2w}\left(2 - \psi + \frac{(m_{n1}m_{n2})^2}{(m_b-m_{n1})(m_b-m_{n2})}\right)$	$\frac{1}{w}\left(\Omega - \frac{f^2(m_{n1}m_{n2})^2}{2(m_b-m_{n1})(m_b-m_{n2})}\right)$

Note: $S_n^1 = f(1 - \frac{1}{m_{n1}} + \frac{m_{n1}}{m_b-m_{n1}})$; $S_b^1 = 1 - (\frac{fm_{n1}}{m_b-m_{n1}} + p_1)$; $S_n^2 = \frac{f}{w}(1 - \frac{1}{m_{n1}} + \frac{m_{n1}}{m_b-m_{n1}})$; $S_b^2 = 1 - \frac{1}{w}(\frac{fm_{n2}}{m_b-m_{n2}} + p_2)$.
Variables: $\Omega = (1-p_1)(w-p_2)$; $\phi = m_{n1}m_{n2} - 1$; $\psi = m_{n1}m_{n2} + \frac{1}{m_{n1}m_{n2}}$. Thresholds: $\bar{v} = -\frac{m_{n2}+m_b\phi}{m_b(m_{n1}-m_{n2})}$ and $\bar{w} = \frac{m_{n1}+m_b\phi}{m_b(m_{n1}-m_{n2})}$.

Based on the assumption that products are neither complements nor substitutes, consumers consider purchasing both products only when $v_1 > p_1$ and $v_2 > p_2$ (Bhargava 2013). The product is kept if it is matched; otherwise, it is returned, consistent with Gao and Su (2017). In this case, for any given combination of price p_j and return fee f, consumers self-select among the no bracketing, full bracketing, and no purchase strategies by maximising the expected utility. Table F.1 reports the details of the market segments. We can derive the expected profit under a fixed return fee:

$$\begin{aligned}
\pi_{df} = {} & \rho_1\Big(S_n^1\big(m_{n1}p_1 + (1-m_{n1})(f-c)\big) + S_n^2\big(m_{n2}p_2 + (1-m_{n2})(f-c)\big)\Big) \\
& + \rho_2\Big(S_b^1\big(m_b(p_1+f-c) + (1-m_b)(f-2c)\big) + S_b^2\big(m_b(p_2+f-c) + (1-m_b)(f-2c)\big)\Big) \\
& + \rho_{12}\Bigg(S_n^{12}\big(m_{n1}m_{n2}(p_1+p_2) + m_{n1}(1-m_{n2})(p_1+f-c) + m_{n2}(1-m_{n1})(p_2+f-c) \\
& \qquad + (1-m_{n1})(1-m_{n2})(f-2c)\big) + S_b^{12}\big(m_b(p_1+p_2+f-2c) + (1-m_b)(f-4c)\big)\Bigg).
\end{aligned}$$

The e-tailer offers either a full refund policy $f = 0$ or partial refund policy $f > 0$. Based on the parameter settings reported in Online Appendix G, we jointly solve for the e-tailer's optimal pricing decisions (p_1, p_2) and fee f under different scenarios, and use these results to construct Figure 2 and Table F.2. Table F.2 confirms the robustness of our findings and shows that fixed return fees induce bracketing spillover. The results are as follows.

Table F.2 Optimal decisions and profit under different cases.

$c=0,\ \rho_1=0.2,\ \rho_2=0.2$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.94	0.442	0.158	0.005	0.001	0.110	0.147	0	0.001	0.246	0.210
0.65	0.75	0.371	0.117	0.117	0.038	0.075	0.133	0.015	0.023	0.277	0.235
0.75	0.65	0.364	0.124	0.124	0.085	0.034	0.067	0.069	0.026	0.272	0.238
0.94	0.55	0.441	0.157	0.006	0.112	0	0.001	0.145	0.001	0.246	0.211

$c=0,\ \rho_1=0.4,\ \rho_2=0.4$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.94	0.471	0.250	0.001	0	0.211	0.078	0.156	0	0.062	0.192
0.65	0.75	0.432	0.210	0.045	0.030	0.188	0.103	0.147	0.001	0.073	0.196
0.75	0.65	0.421	0.198	0.058	0.079	0.145	0.063	0.184	0.002	0.075	0.197
0.94	0.55	0.470	0.249	0.002	0.078	0.134	0.001	0.232	0	0.062	0.193

$c=0.06,\ \rho_1=0.2,\ \rho_2=0.2$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.375	0.164	0.164	0.018	0.080	0.089	0.027	0.001	0.257	0.202
0.55	0.85	0.463	0.159	0.052	0.006	0.093	0.144	0	0.007	0.230	0.182
0.55	0.94	0.488	0.190	0.004	0	0.101	0.137	0	0.001	0.209	0.175
0.65	0.55	0.371	0.167	0.167	0.054	0.054	0.031	0.068	0.001	0.256	0.204
0.65	0.75	0.416	0.126	0.126	0.041	0.062	0.144	0	0.027	0.245	0.194
0.75	0.65	0.396	0.145	0.145	0.099	0.012	0.079	0.047	0.036	0.233	0.197
0.75	0.85	0.460	0.160	0.052	0.035	0.069	0.143	0	0.027	0.210	0.182
0.85	0.55	0.432	0.166	0.067	0.111	0	0.012	0.114	0.011	0.234	0.185
0.85	0.75	0.437	0.164	0.066	0.110	0	0.075	0.063	0.044	0.201	0.185
0.94	0.55	0.472	0.205	0.006	0.106	0	0.001	0.129	0.001	0.208	0.173

$c=0.06,\ \rho_1=0.4,\ \rho_2=0.4$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.388	0.162	0.141	0.031	0.167	0.153	0.088	0	0.085	0.170
0.55	0.85	0.473	0.249	0.041	0.009	0.188	0.229	0	0.001	0.060	0.167
0.55	0.94	0.508	0.263	0.004	0.001	0.195	0.224	0	0	0.055	0.171
0.65	0.55	0.376	0.155	0.155	0.101	0.115	0.058	0.155	0	0.088	0.171
0.65	0.75	0.449	0.227	0.069	0.045	0.161	0.156	0.077	0.003	0.065	0.166
0.75	0.65	0.440	0.217	0.079	0.108	0.105	0.086	0.141	0.004	0.067	0.167
0.75	0.85	0.473	0.250	0.041	0.056	0.149	0.228	0	0.006	0.056	0.168
0.85	0.55	0.463	0.240	0.054	0.179	0.032	0.020	0.191	0.002	0.062	0.166
0.85	0.75	0.468	0.246	0.047	0.156	0.053	0.107	0.119	0.007	0.055	0.167
0.94	0.55	0.483	0.285	0.005	0.206	0	0.002	0.205	0	0.054	0.170

$c=0.08,\ \rho_1=0.2,\ \rho_2=0.2$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.387	0.169	0.169	0.019	0.076	0.092	0.021	0.001	0.248	0.189
0.55	0.85	0.478	0.171	0.050	0.006	0.090	0.140	0	0.006	0.217	0.170
0.55	0.94	0.504	0.200	0.004	0	0.098	0.133	0	0.001	0.198	0.164
0.65	0.55	0.383	0.172	0.172	0.056	0.049	0.032	0.064	0.001	0.247	0.190
0.65	0.75	0.435	0.126	0.126	0.041	0.058	0.144	0	0.027	0.236	0.181
0.75	0.65	0.406	0.152	0.152	0.104	0.005	0.082	0.040	0.039	0.220	0.185
0.75	0.85	0.477	0.169	0.051	0.035	0.067	0.141	0	0.026	0.199	0.171
0.85	0.55	0.443	0.183	0.065	0.109	0	0.012	0.109	0.010	0.221	0.172
0.85	0.75	0.442	0.181	0.066	0.109	0	0.075	0.058	0.043	0.190	0.174
0.94	0.55	0.482	0.220	0.006	0.104	0	0.001	0.124	0.001	0.196	0.162

$c=0.08,\ \rho_1=0.4,\ \rho_2=0.4$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.398	0.172	0.144	0.032	0.162	0.156	0.078	0	0.082	0.159
0.55	0.85	0.487	0.255	0.041	0.009	0.183	0.225	0	0.001	0.058	0.160
0.55	0.94	0.521	0.268	0.004	0.001	0.189	0.222	0	0	0.053	0.165
0.65	0.55	0.384	0.161	0.161	0.105	0.107	0.060	0.145	0	0.085	0.161
0.65	0.75	0.455	0.232	0.077	0.050	0.152	0.175	0.054	0.003	0.063	0.158
0.75	0.65	0.446	0.223	0.087	0.119	0.091	0.094	0.126	0.004	0.065	0.158
0.75	0.85	0.486	0.254	0.041	0.056	0.144	0.226	0	0.006	0.054	0.162
0.85	0.55	0.467	0.244	0.063	0.209	0	0.023	0.180	0.003	0.060	0.159
0.85	0.75	0.471	0.249	0.057	0.191	0.016	0.131	0.091	0.011	0.051	0.161
0.94	0.55	0.488	0.298	0.005	0.205	0	0.002	0.196	0	0.051	0.163

$c=0.10,\ \rho_1=0.2,\ \rho_2=0.2$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.398	0.175	0.175	0.019	0.072	0.095	0.016	0.001	0.238	0.176
0.55	0.85	0.494	0.183	0.049	0.005	0.088	0.136	0	0.006	0.205	0.159
0.55	0.94	0.519	0.210	0.004	0	0.095	0.130	0	0.001	0.187	0.154
0.65	0.55	0.395	0.178	0.178	0.058	0.044	0.033	0.059	0.001	0.237	0.177
0.65	0.75	0.453	0.134	0.124	0.041	0.056	0.142	0	0.026	0.225	0.169
0.75	0.65	0.414	0.156	0.156	0.107	0	0.085	0.035	0.042	0.212	0.173
0.75	0.85	0.493	0.178	0.050	0.034	0.064	0.138	0	0.025	0.189	0.161
0.85	0.55	0.454	0.199	0.064	0.107	0	0.012	0.104	0.010	0.208	0.161
0.85	0.75	0.448	0.198	0.065	0.108	0	0.074	0.053	0.042	0.179	0.164
0.94	0.55	0.491	0.236	0.005	0.102	0	0.001	0.119	0.001	0.184	0.151

$c=0.10,\ \rho_1=0.4,\ \rho_2=0.4$

m_{n1}	m_{n2}	p_1^*	p_2^*	f^*	S_n^1	S_b^1	S_n^2	S_b^2	S_n^{12}	S_b^{12}	Profit
0.55	0.65	0.408	0.183	0.146	0.033	0.156	0.159	0.067	0	0.078	0.150
0.55	0.85	0.500	0.261	0.040	0.009	0.178	0.221	0	0.001	0.055	0.153
0.55	0.94	0.535	0.272	0.003	0.001	0.184	0.219	0	0	0.051	0.158
0.65	0.55	0.391	0.167	0.167	0.109	0.099	0.062	0.136	0	0.083	0.151
0.65	0.75	0.460	0.237	0.085	0.055	0.143	0.194	0.029	0.004	0.061	0.150
0.75	0.65	0.452	0.228	0.095	0.129	0.077	0.103	0.111	0.005	0.062	0.149
0.75	0.85	0.500	0.258	0.040	0.055	0.140	0.223	0	0.005	0.052	0.155
0.85	0.55	0.473	0.258	0.062	0.206	0	0.023	0.171	0.003	0.057	0.152
0.85	0.75	0.473	0.257	0.062	0.206	0	0.141	0.073	0.013	0.047	0.156
0.94	0.55	0.492	0.311	0.005	0.203	0	0.002	0.187	0	0.049	0.157

Observation F.1. The e-tailer can induce a segment of consumers to forgo bracketing by choosing a price and return fee. This finding mirrors Theorem 1. For instance, Table F.2 demonstrates that when both m_{ni} and c are high, the e-tailer induces some consumers to abandon bracketing (e.g., when $\rho_1 = \rho_2 = 0.4$, $c = 0.1$, and $m_{n1} = 0.85$, $S_b^1 = 0$).

Observation F.2. Consistent with Theorem 3, Table F.2 demonstrates that $f^* = 0$ is not necessarily optimal when $c = 0$.

Observation F.3. Consistent with Theorem 4, Table F.2 demonstrates that bracketing remains advantageous despite high costs, as evidenced by either S_b^1 or S_b^2 remaining strictly positive as c rises.

Observation F.4. As reverse logistics costs increase, the e-tailer can pass on part of the cost through higher prices. This finding is consistent with Theorem 5. Table F.2 shows that, as the reverse logistics cost c rises from 0.06 to 0.1, the table reveals two optimal response patterns: in some cases, the e-tailer primarily passes the cost through higher prices rather than a higher return service fee, for instance with $\rho_1 = \rho_2 = 0.2$ and $(m_{n1}, m_{n2}) = (0.55, 0.85)$, (p_1, p_2, f) moves from $(0.463, 0.159, 0.052)$ to $(0.494, 0.183, 0.049)$ (p^* increases while f^* falls); in other cases, the price and return service fee increase in tandem, for instance with $\rho_1 = \rho_2 = 0.4$ and $(m_{n1}, m_{n2}) = (0.55, 0.65)$, (p_1, p_2, f) moves from $(0.388, 0.162, 0.141)$ to $(0.408, 0.183, 0.146)$ (both p^* and f^* increase).

Observation F.5. Consistent with Theorem 6, Table F.2 reveals that reducing product fit uncertainty (increasing m_n) could undermine the e-tailer's overall profit. For instance, when $c = 0.1$ and $\rho_1 = \rho_2 = 0.2$, increasing the matching probabilities from $(m_{n1}, m_{n2}) = (0.55, 0.65)$ to $(0.75, 0.85)$ results in the profit decline from 0.176 to 0.161.

Observation F.6. A fixed return fee can induce bracketing spillover. For instance, when matching is highly asymmetric, for example, when $(m_{n1}, m_{n2}) = (0.55, 0.94)$, $c = 0.1$ and $\rho_1 = \rho_2 = 0.2$, the high-matching product is not bracketed in isolation ($S_b^1 = 0.095$ and $S_b^2 = 0$), whereas under multi-product purchasing the bracketing mass becomes sizeable ($S_b^{12} = 0.187$), indicating that bracketing spills over from the low-matching product to the entire basket.

Appendix G: Parameter Calibration

In Figure 2, the matching probability is set within the range $0.55 \le m_{nj} < m_b$, with $m_b = 0.95$, $c = 0.1$, and $\rho_j \in \{0.2, 0.4\}$. This range is chosen to capture the heterogeneity in return rates across product categories. Industry evidence suggests that return rates vary widely across categories and firms. For instance, apparel products often exhibit return rates of approximately 30–40% (Red Stag Fulfillment 2025). Survey evidence reported by Coresight Research further indicates that most US apparel brands and retailers experience return rates between 5% and 50% (Zheng 2023). These observations motivate a wide range for m_{nj}, covering environments from high to relatively low return risk. In practice, although bracketing can substantially increase the likelihood of finding a suitable product, consumers may still fail to obtain a truly satisfactory match due to subjective preferences and residual uncertainty. Consequently, the post-bracketing matching probability is unlikely to reach unity; we set it to $m_b = 0.95$.

We set the probability of multi-product purchasing to $\rho_{12} = 0.4$, which is consistent with the evidence that 35%–45% of online orders contain multiple distinct products (BusinessDojo 2025). To avoid an overly imbalanced split between single-product purchases and distinguish between cases in which consumers are more likely to purchase a high-value product versus a low-value product, we consider two configurations: $(\rho_1, \rho_2) = (0.2, 0.4)$ and $(\rho_1, \rho_2) = (0.4, 0.2)$. BusinessDojo (2025) indicates that the incidence of multi product purchasing may increase to 40%–55% owing to store format or recommendation support; therefore, we examine $\rho_{12} = 0.6$. To isolate the effect of a higher ρ_{12} without changing the structure of single product purchasing, we compare $(\rho_1, \rho_2) = (0.2, 0.2)$ with $(\rho_1, \rho_2) = (0.4, 0.4)$.

BusinessDojo (2025) suggests that the average price of a single item in online retail typically falls within the range of USD 20 to 50. Within a given e-tailer (e.g., Zara and H&M), the assortment usually includes a large share of basic items and a smaller set of higher-priced core products. Hence, the internal price tiers tend to be relatively stable and do not exhibit extreme dispersion. Motivated by this feature, we normalise the upper bounds of consumer valuations for the two product types to 1 and 0.6, respectively, to reflect a moderate within-retailer value (and price) gap between higher-valuation and relatively lower-valuation products. We interpret the lower valuation type as representing a broader mid- to low-value segment, rather than implying that the assortment comprises only two discrete price points.

We set the unit reverse-logistics cost to $c = 0.1$. Industry evidence suggests that reverse logistics costs often amount to 20–65% of a product's original value (Dopson, 2025). In our model, return risk reduces consumers' effective willingness to pay; therefore, equilibrium prices are endogenously below the middle of the valuation range. Under our normalised scale (valuation upper bounds equal to 1 and 0.6), a representative equilibrium price level is $p_1 \approx 0.5$ for the high valuation product and $p_2 \approx 0.3$ for the lower-valuation product. Therefore, $c = 0.1$ corresponds to approximately 20% and 33% of the transaction price, respectively.

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