Additional Materials for "3D Printer Rentals: Technical Service Design and Pricing for Customers of Varying Expertise" by Yue Zhang and Brian Tomlin

Rationale behind Proposition 3(v) (the OEM profit can decrease in Δe)

We note that two-item is optimal in this case, i.e., both segments are retained as prosumers. By 19, and the optimal profit, given by 19, is $\Pi(\boldsymbol{\alpha}_M^*, \mathbf{r}_M^*) = (1-\tau)\Phi(\underline{e} - \frac{\tau}{1-\tau}\Delta e) + \tau\Phi(\bar{e})$. The fact that the profit delivered by the elementary segment is driven by the pseudo expertise $\underline{e} - \frac{\tau}{1-\tau}\Delta e$, in contrast to the true expertise $\underline{e}$ under user-based strategy (see (13)). This means that, as Δe increases, the gain in profit from the advanced segment cannot, in some circumstances, offset the reduction in profit from the elementary segment. Therefore the optimal profit can decrease in Δe. To understand this more deeply, consider the necessary condition in Proposition 3(v) for the optimal profit to initially decrease in Δe, that is, $e_m > e_f - c'(0)$. When this holds, the elementary segment's pseudo expertise $\underline{e} - \frac{\tau}{1-\tau}\Delta e$ is sufficiently high for small Δe that the optimal two-item package offers no technical service to either segment, and hence $\Phi(\cdot)$ is linear over the region $[\underline{e} - \frac{\tau}{1-\tau}\Delta e, \bar{e}]$. Note that $\Pi(\boldsymbol{\alpha}_M^*, \mathbf{r}_M^*) = (1-\tau)\Phi(e_m - (\frac{1}{2} + \frac{\tau}{1-\tau})\Delta e) + \tau\Phi(e_m + \frac{1}{2}\Delta e)$. As Δe grows with e_m fixed, the profit gain from the advanced segment (initially when Δe is small) cannot offset the profit reduction from the elementary segment, and thus the OEM profit can decrease in Δe. This decreasing behavior never occurs in the user-based strategy because there the profit delivered by the elementary segment is driven by their true expertise $\underline{e}$ with $\Pi\left(\{\alpha_S^*(e), r_S^*(e) | e \in \{\underline{e}, \bar{e}\}\right) = (1-\tau)\Phi(e_m - \frac{1}{2}\Delta e) + \tau\Phi(e_m + \frac{1}{2}\Delta e)$. Hence, in the region where $\Phi(\cdot)$ is linear, an increase in Δe has no impact on the profit.

Rationale behind Proposition 4 (iii) (V_C can decrease in Δe)

When $\Phi(\underline{e} - \frac{\tau}{1-\tau}\Delta e) < 0$, recall that the value of customization is the profit delivered by the elementary segment in the user-based strategy. In this case, V_C decreases in the range Δe, because the profit from the elementary segment decreases in its own expertise, and this expertise declines in Δe for any given e_m. When $\Phi(\underline{e} - \frac{\tau}{1-\tau}\Delta e) \geq 0$, V_C increases in Δe when Δe is small but decreases in Δe when Δe exceeds the threshold in Proposition 4 (iii). Recall that in the population-based strategy a two-item package is optimal if $\Phi(\underline{e} - \frac{\tau}{1-\tau}\Delta e) \geq 0$ and that the OEM effectively treats the elementary segment as if their expertise is $\underline{e} - \frac{\tau}{1-\tau}\Delta e$, whereas in the user-based strategy the OEM acts on their true expertise $\underline{e}$. Therefore, at a high enough Δe, the optimal technical service level for the elementary segment in the population-based strategy has reached its maximum (one), whereas the optimal technical service in the user-based strategy is less than the maximum. Therefore, as Δe increases further, the profit gain from the elementary segment no longer changes under the population-based strategy but continues to decrease under the user-based strategy, resulting in a reduced value of customization.

Imperfect Self-assessment of Innate Expertise

The main model can also be extended to allow inaccurate self-assessment of expertise levels by customers. Specifically, for a customer of type e, instead of perfectly knowing his/her innate expertise level e, the customer's self-assessment can be an imperfect estimation of the true expertise and hence contain errors. Denote the imperfect estimate as $\tilde{e} := e + \varsigma$, where ς is a random error term. As a result, the consumer-or-prosumer choice (see §4) for this customer will be based on $\tilde{e}$ rather than e, i.e., the resulting utility given by (1) in Lemma 1 now becomes

$$\tilde{U}_{\mathcal{P}}(\boldsymbol{\alpha}, \mathbf{r}|e) := \begin{cases} \varepsilon(\alpha_H, \tilde{e}) - r_H & \text{if } \tilde{e} < e_f - \frac{\Delta r}{\Delta \alpha} \cdot \mathbb{1}_{\{\Delta\alpha > 0\}}, \\ \varepsilon(\alpha_L, \tilde{e}) - r_L & \text{otherwise.} \end{cases}$$

For user-based strategy, instead of solving the original problem (3), the OEM now needs to also account for customer's imperfect self-assessment in maximizing the gain from him/her:

$$\begin{aligned} \max_{(\boldsymbol{\alpha}(e), \mathbf{r}(e)) \in \mathcal{G}} \quad & E_\varsigma \left\{ (r_H - c(\alpha_H)) \mathbb{1}_{\{\tilde{e} < e_f - \frac{\Delta r}{\Delta\alpha} \cdot \mathbb{1}_{\{\Delta\alpha>0\}}\}} + (r_L - c(\alpha_L)) \mathbb{1}_{\{\tilde{e} \geq e_f - \frac{\Delta r}{\Delta\alpha} \cdot \mathbb{1}_{\{\Delta\alpha>0\}}\}} \right\}, \\ \text{s.t.} \quad & \tilde{U}_{\mathcal{P}}(\boldsymbol{\alpha}(e), \mathbf{r}(e)|e) \geq U_{\mathcal{C}}. \end{aligned}$$

For population-based strategy, the OEM's original problem (4) now becomes

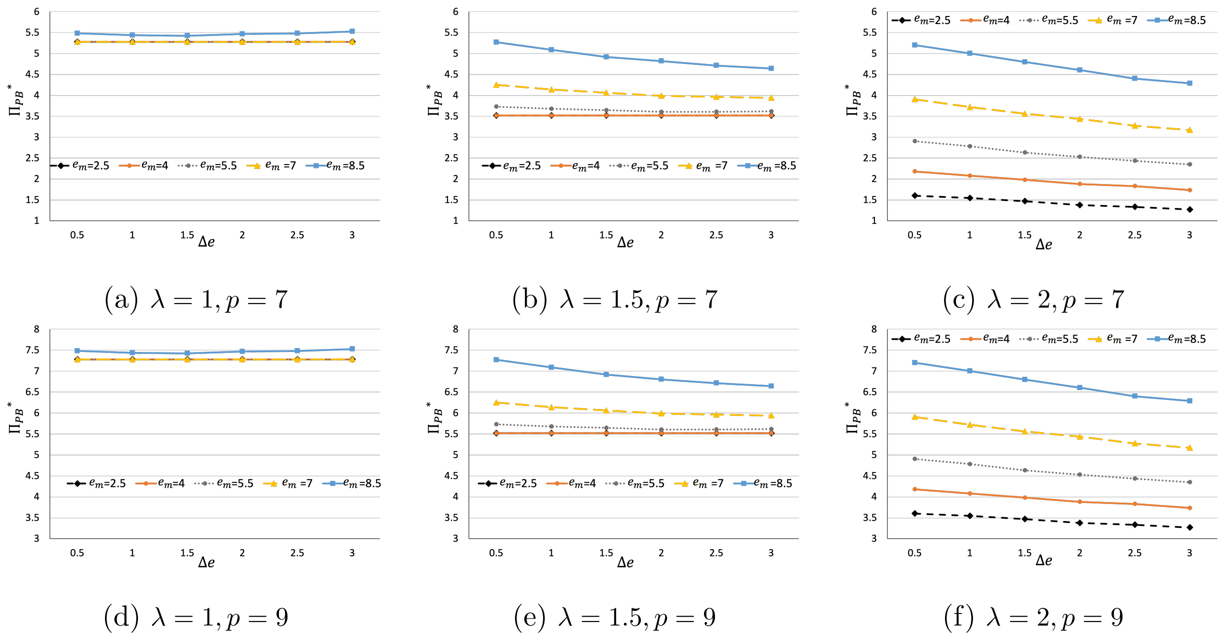
$$\max_{(\boldsymbol{\alpha}, \mathbf{r}) \in \mathcal{G}} E_\varsigma \left\{ (r_H - c(\alpha_H)) \cdot \tilde{d}_{\mathcal{P}_H}(\boldsymbol{\alpha}, \mathbf{r}) + (r_L - c(\alpha_L)) \cdot \tilde{d}_{\mathcal{P}_L}(\boldsymbol{\alpha}, \mathbf{r}) \right\},$$

where $\tilde{d}_{\mathcal{P}_H}(\boldsymbol{\alpha}, \mathbf{r}) := \int_{e_o}^{\max\{e_o,\ e_f - \frac{\Delta r}{\Delta\alpha} \cdot \mathbb{1}_{\{\Delta\alpha>0\}}\}} E_\varsigma \left(\mathbb{1}_{\{\tilde{U}_{\mathcal{P}}(\boldsymbol{\alpha}, \mathbf{r}|e) \geq U_{\mathcal{C}}\}} \right) dZ(e)$ and $\tilde{d}_{\mathcal{P}_L}(\boldsymbol{\alpha}, \mathbf{r}) := \int_{\mathcal{E}} E_\varsigma \left(\mathbb{1}_{\{\tilde{U}_{\mathcal{P}}(\boldsymbol{\alpha}, \mathbf{r}|e) \geq U_{\mathcal{C}}\}} \right) dZ(e) - \tilde{d}_{\mathcal{P}_H}$. Both problems can be analyzed in a similar way as for the main model. Moreover, if $\mathrm{E}(\varsigma) = 0$, i.e., the customer population is overall unbiased in self-assessing, all our results and insights from the main model will carry over.

Additional Plots for Online Appendix B

Numerical Study for Truncated Normal Distribution of Population Expertise

Consider when the customer population expertise follows a truncated normal distribution on $[\underline{e}, \bar{e}]$. Like uniform, truncated normal also implies a continuum of expertise levels, i.e., it is not as concentrated as a two-point distribution. Meanwhile, it is more heavily distributed close to its mean and thus is not as spread out as uniform distribution. Let μ and σ be the mean and standard deviation for the untruncated normal distribution. We consider $\mu = e_m$ and $\sigma = \Delta e/6$, which



(a) $\lambda = 1, p = 7$ (b) $\lambda = 1.5, p = 7$ (c) $\lambda = 2, p = 7$

(d) $\lambda = 1, p = 9$ (e) $\lambda = 1.5, p = 9$ (f) $\lambda = 2, p = 9$

Figure 6: Impact of Δe and e_m on OEM Profit under Optimal Population-based Package (additional plots)

implies $[\mu - 3\sigma, \mu + 3\sigma] \subset [\underline{e}, \bar{e}]$. The rest of numerical design is the same as specified in §9.3 and Online Appendix B. As before, we plot the optimal OEM profit in the population-based strategy and the value of customization, as a function of the range Δe for each of the five levels of e_m. The plots are presented in Figures 8-9. The observations from these plots are consistent with what we have discussed in §9.3 and Online Appendix B.

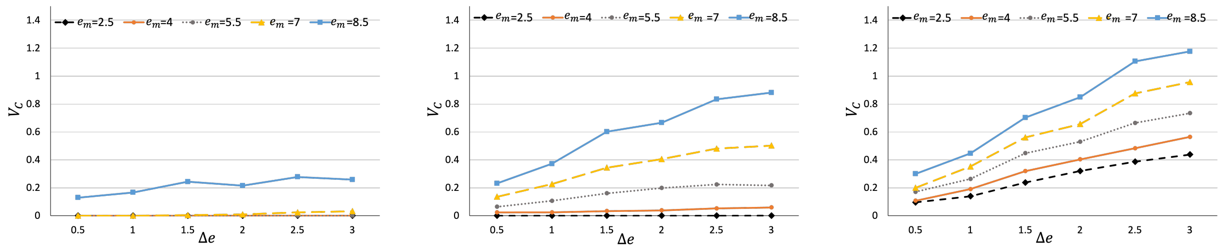
Analyses for §9.2 (Uniformly Distributed Expertise with $\bar{e} = e_f$)

Without loss of generality, we solve for the optimal package when customer expertise is uniformly distributed on $[\underline{e}, \bar{e}]$ with maximal expertise range, i.e., $\Delta e = \delta_{\max}(e_o)$ with $\underline{e} = e_o, \bar{e} = e_f$. All results for the more general case with $\underline{e} > e_o$ can be obtained by replacing the e_o in the results for maximal expertise range with $\underline{e}$.

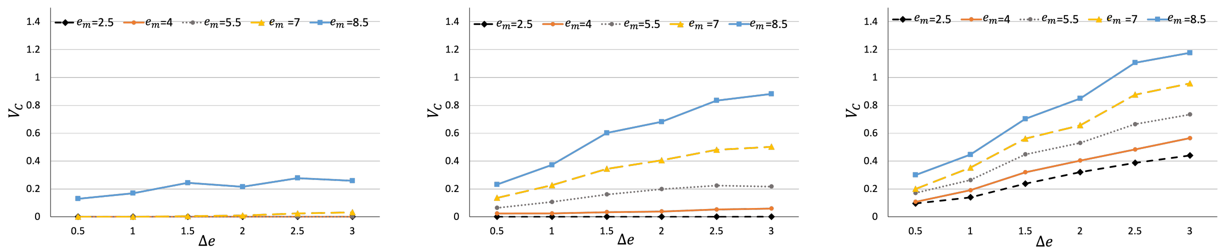
We will first solve for the optimal single-item package, and then the optimal two-item package, followed by a comparison of them which leads to the finding of the optimal package. To prepare for the analyses, we need to introduce a few definitions. Define

$$\delta_{\min}(e) := e - e_o, \tag{44}$$

which measures the gap between an expertise level e and that of a novice. Note that $\delta_{\min}(e_f) =$



(a) $\lambda = 1, p = 7$ (b) $\lambda = 1.5, p = 7$ (c) $\lambda = 2, p = 7$



(d) $\lambda = 1, p = 9$ (e) $\lambda = 1.5, p = 9$ (f) $\lambda = 2, p = 9$

Figure 7: Impact of Δe and e_m on the Value of Customization (additional plots)

$\delta_{\max}(e_o)$. Next, define three disjoint regions for the technical service level:

$$\mathcal{A}_I := \{\alpha \in [0,1] \mid p - c(\alpha) \geq 2\Delta e \cdot (1-\alpha)\}, \tag{45}$$

$$\mathcal{A}_{II} := \{\alpha \in [0,1] \mid 0 \leq p - c(\alpha) < 2\Delta e \cdot (1-\alpha)\}, \tag{46}$$

$$\mathcal{A}_{III} := \{\alpha \in [0,1] \mid p - c(\alpha) < 0\}. \tag{47}$$

As we will show in the analyses below, the optimal technical service level (regardless of the package being single- or two-item) never arises in $\mathcal{A}_{III}$, while $\mathcal{A}_I$ and $\mathcal{A}_{II}$ both may include candidates for the optimal technical service level. Note also that $0 \in \mathcal{A}_I \cup \mathcal{A}_{II}$ and $1 \in \mathcal{A}_I \cup \mathcal{A}_{III}$. In the following, we derive the explicit expressions for $\mathcal{A}_I$-$\mathcal{A}_{III}$.

The solutions to the following equation will be critical for our analysis:

$$p - c(\alpha) = 2\Delta e \cdot (1-\alpha), \tag{48}$$

whose property is presented in the following lemma.

Lemma 4 *There are at most two solutions to (48), denoted as $\hat{\alpha}_1$ and $\hat{\alpha}_2$ with $\hat{\alpha}_1 \leq \hat{\alpha}_2$.*

We will use $\hat{\alpha}_1 = \hat{\alpha}_2$ to represent the case when (48) has unique solution, and $\hat{\alpha}_1 > \hat{\alpha}_2$ to represent the case when (48) has no solution. Note that, if $\hat{\alpha}_1 \geq \hat{\alpha}_2$, there are $(\hat{\alpha}_1, \hat{\alpha}_2) = \emptyset$, $[0, \hat{\alpha}_1] \cup [\hat{\alpha}_2, x] = [0, x]$ for any $x \geq \hat{\alpha}_1$.

Lemma 5 *$\mathcal{A}_I$, $\mathcal{A}_{II}$ and $\mathcal{A}_{III}$ as defined in (45) - (47) can be derived as in Table 2, where $c^{-1}(\cdot)$ is the inverse function of $c(\cdot)$, and $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are as defined in Lemma 4.*

Table 2: Expressions for $\mathcal{A}_I$, $\mathcal{A}_{II}$ and $\mathcal{A}_{III}$

Parameter Case	$\mathcal{A}_I$	$\mathcal{A}_{II}$	$\mathcal{A}_{III}$
$p < \min\{2\Delta e, c(1)\}$	$[\hat{\alpha}_1, \hat{\alpha}_2]$	$[0, \hat{\alpha}_1) \cup (\hat{\alpha}_2, c^{-1}(p)]$	$(c^{-1}(p), 1]$
$2\Delta e \geq p > c(1)$	$[\hat{\alpha}_1, 1]$	$[0, \hat{\alpha}_1)$	$\emptyset$
$2\Delta e > p = c(1)$	$[\hat{\alpha}_1, 1]$	$[0, \hat{\alpha}_1)$	$\emptyset$
$2\Delta e < p \leq c(1)$	$[0, \hat{\alpha}_1]$	$(\hat{\alpha}_1, c^{-1}(p)]$	$(c^{-1}(p), 1]$
$2\Delta e = p < c(1)$	$[0, \hat{\alpha}_2]$	$(\hat{\alpha}_2, c^{-1}(p)]$	$(c^{-1}(p), 1]$
$2\Delta e = p = c(1)$	$[0, 1]$	$\emptyset$	$\emptyset$
$p > \max\{2\Delta e, c(1)\}$	$[0, 1]$	$\emptyset$	$\emptyset$

Uniformly Distributed Expertise: Single-item Package

We solve for the optimal single-item package. For any given single-item package (α_S, r_S), the resulting prosumer demand function is given below.

Proposition 8 *Assume the customer expertise to be uniformly distributed on $[e_o, e_f]$. Under a single-item package (α_S, r_S), the segment of prosumers is*

$$d_{\mathcal{P}}(\alpha_S, r_S) = \begin{cases} 0 & \text{if } r_S > p, \\ \frac{1}{\Delta e} \cdot \frac{p - r_S}{1 - \alpha_S} & \text{if } p - \Delta e(1 - \alpha_S) < r_S \leq p, \ \alpha_S < 1, \\ 1 & \text{if } r_S \leq p - \Delta e(1 - \alpha_S), \ \alpha_S \leq 1. \end{cases} \tag{49}$$

Define

$$\Pi_S^I(\alpha) := p - c(\alpha) - \Delta e \cdot (1 - \alpha) \quad \text{and} \quad \Pi_S^{II}(\alpha) := \frac{1}{4\Delta e} \cdot \frac{(p - c(\alpha))^2}{1 - \alpha}. \tag{50}$$

It can be verified that $\Pi_S^I(\alpha)$ is strictly concave, and there is $\Pi_S^I(\alpha) \leq \Pi_S^{II}(\alpha)$ for any $\alpha \in [0, 1)$.

Theorem 6 *Assume the customer expertise to be uniformly distributed on $[e_o, e_f]$. The optimal single-item package that solves (11) is given by*

$$(\alpha_S^*, r_S^*) = \begin{cases} (\alpha_S^I, \ r_S^I) & \text{if } \Pi_S^I(\alpha_S^I) \geq \Pi_S^{II}(\alpha_S^{II}), \\ (\alpha_S^{II}, \ r_S^{II}) & \text{otherwise}, \end{cases} \tag{51}$$

where the two candidates for the optimal single-item package are defined as follows:

$$(\alpha_S^I, r_S^I) \ := \ (\operatorname*{argmax}_{\alpha \in \mathcal{A}_I} \Pi_S^I(\alpha), \quad p - \Delta e(1 - \alpha_S^I) \ ), \tag{52}$$

$$(\alpha_S^{II}, r_S^{II}) \ := \ (\operatorname*{argmax}_{\alpha \in \mathcal{A}_{II}} \Pi_S^{II}(\alpha), \quad \frac{p + c(\alpha_S^{II})}{2} \ ). \tag{53}$$

Moreover, the OEM's resulting profit is $\Pi(\alpha_S^*, r_S^*) = \max\{\Pi_S^I(\alpha_S^I), \Pi_S^{II}(\alpha_S^{II})\}$. For a prosumer of *type e, the utility resulting from the two candidates above are:*

$$U_{\mathcal{P}}(\alpha_S^I, r_S^I|e) = U_{\mathcal{C}} + (1-\alpha_S^I)\delta_{\min}(e), \quad (54)$$

$$U_{\mathcal{P}}(\alpha_S^{II}, r_S^{II}|e) = U_{\mathcal{C}} - (1-\alpha_S^{II})\delta_{\max}(e) + \frac{p - c(\alpha_S^{II})}{2}. \quad (55)$$

By definition, it can be verified that there are $\Pi_S^I(\alpha_S^I) \geq \Delta e \cdot (1-\alpha_S^I)$ and $\Pi_S^{II}(\alpha_S^{II}) < \Delta e \cdot (1-\alpha_S^{II})$.

Corollary 3 *Assume the customer expertise to be uniformly distributed on $[e_o, e_f]$. Then $\Pi(\alpha_S^*, r_S^*)$ increases in p.*

By Corollary 3, the OEM's profit from the optimal single-item package increases when the competition from print farm lessens.

Uniformly Distributed Expertise: Two-item Package

We now consider the case when the OEM offers a two-item package, i.e., $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}$, where $\mathcal{M}$ is as defined directly before equation (12) in the main paper. To characterize $\mathcal{M}$, we first derive the prosumer demand functions for any $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}$, which are presented below.

Proposition 9 *Assume the customer expertise to be uniformly distributed on $[e_o, e_f]$. Under a two-item package $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}$, there are*

- *if* $\frac{\Delta r}{\Delta \alpha} > \Delta e$, $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$,

$$d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \begin{cases} 0 & \textit{if } \ r_L > p, \\ \frac{1}{\Delta e} \cdot \frac{p - r_L}{1-\alpha_L} & \textit{if } \ p - \Delta e \cdot (1-\alpha_L) \leq r_L \leq p, \\ 1 & \textit{if } \ r_L < p - \Delta e \cdot (1-\alpha_L). \end{cases}$$

- *If* $\frac{\Delta r}{\Delta \alpha} \leq \Delta e$,

$$d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \begin{cases} 0 & \textit{if } \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L > p, \\ \frac{1}{\Delta e} \cdot \frac{p - r_L - \frac{\Delta r}{\Delta \alpha}(1-\alpha_L)}{1-\alpha_H} & \textit{if } \alpha_H < 1, p - \Delta e \cdot (1-\alpha_H) \leq r_H \leq p, \ \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p, \\ 1 - \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha} & \textit{if } r_H \leq p - \Delta e \cdot (1-\alpha_H), \end{cases} \quad (56)$$

$$d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \begin{cases} 0 & \textit{if } r_L > p, \\ \frac{1}{\Delta e} \cdot \frac{p - r_L}{1-\alpha_L} & \textit{if } r_L \leq p, \ \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L > p, \\ \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha} & \textit{if } \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p. \end{cases} \quad (57)$$

Remark 2 *By Proposition 9, for any two-item package with either* $\frac{\Delta r}{\Delta\alpha} > \Delta e$*, or* $\frac{\Delta r}{\Delta\alpha} \leq \Delta e$ *and* $\frac{(1-\alpha_L)\Delta r}{\Delta\alpha} + r_L > p$*, there will be no prosumers who choose the premium plan, i.e., the only effective offering is the basic plan only. Hence,* $\mathcal{M}$ *reduces to* $\mathcal{M}' = \left\{(\boldsymbol{\alpha}, \mathbf{r}) \in \mathcal{M} \mid \frac{\Delta r}{\Delta\alpha} \leq \min\{\Delta e, \ \frac{p-r_L}{1-\alpha_L}\}\right\}$.

Define

$$\Pi_M^I(\boldsymbol{\alpha}) \ := \ \begin{cases} \Pi_S^I(\alpha_H) + \frac{1}{4\Delta e} \cdot \frac{(\Delta c)^2}{\Delta\alpha} & \text{if } \frac{\Delta c}{\Delta\alpha} < 2\Delta e, \\ \Pi_S^I(\alpha_L) & \text{otherwise,} \end{cases} \tag{58}$$

$$\Pi_M^{II}(\boldsymbol{\alpha}) \ := \ \begin{cases} \Pi_S^{II}(\alpha_H) + \frac{1}{4\Delta e} \cdot \frac{(\Delta c)^2}{\Delta\alpha} & \text{if } \frac{p-c(\alpha_H)}{1-\alpha_H} > \frac{p-c(\alpha_L)}{1-\alpha_L}, \\ \Pi_S^{II}(\alpha_L) & \text{otherwise.} \end{cases} \tag{59}$$

Leveraging these definitions, the optimal two-item package is characterized below.

Theorem 7 *Assume the customer expertise to be uniformly distributed on* $[e_o, e_f]$*. The optimal two-item package that solves (12) is given by:*

$$(\boldsymbol{\alpha}_M^*, \mathbf{r}_M^*) = \begin{cases} \left(\boldsymbol{\alpha}_M^I, \ \mathbf{r}_M^I\right) & \text{if } \Pi_M^I(\boldsymbol{\alpha}_M^I) \geq \Pi_M^{II}(\boldsymbol{\alpha}_M^{II}), \\ \left(\boldsymbol{\alpha}_M^{II}, \ \mathbf{r}_M^{II}\right) & \text{otherwise,} \end{cases} \tag{60}$$

where the two candidates for the optimal two-item package are defined as follows:

$$\boldsymbol{\alpha}_M^I \ := \ \underset{\alpha_L \in \mathcal{A}_I \cup \mathcal{A}_{II}, \ \alpha_H \in \mathcal{A}_I}{\operatorname{argmax}} \ \Pi_M^I(\boldsymbol{\alpha}), \tag{61}$$

$$\mathbf{r}_M^I \ := \ (\ p - \Delta e \cdot (1 - \alpha_L^I) + (\Delta e \Delta\alpha^I - \frac{\Delta c^I}{2})^+, \quad p - \Delta e \cdot (1 - \alpha_H^I)\ ); \tag{62}$$

$$\boldsymbol{\alpha}_M^{II} \ := \ \underset{\alpha_L, \alpha_H \in \mathcal{A}_{II}}{\operatorname{argmax}} \ \Pi_M^{II}(\boldsymbol{\alpha}), \tag{63}$$

$$\mathbf{r}_M^{II} \ := \ (\ \frac{p + c(\alpha_L^{II})}{2}, \quad p - \frac{1-\alpha_H^{II}}{2} \cdot [\frac{p - c(\alpha_L^{II})}{1-\alpha_L^{II}} + (\frac{p-c(\alpha_H^{II})}{1-\alpha_H^{II}} - \frac{p-c(\alpha_L^{II})}{1-\alpha_L^{II}})^+]\ ), \tag{64}$$

where $\Delta\alpha^{I(II)} := \alpha_H^{I(II)} - \alpha_L^{I(II)}$ *and* $\Delta c^{I(II)} := c(\alpha_H^{I(II)}) - c(\alpha_L^{I(II)})$*. Moreover, the OEM's resulting profit is* $\Pi(\boldsymbol{\alpha}_M^*, \mathbf{r}_M^*) = \max\left\{\Pi_M^I(\boldsymbol{\alpha}_M^I), \Pi_M^{II}(\boldsymbol{\alpha}_M^{II})\right\}$.

By Theorem 7, the two candidates for the optimal two-item package are closely related to the candidates for the optimal single-item package. In particular, $\boldsymbol{\alpha}_M^{I(II)}$ is the maximizer of $\Pi_M^{I(II)}(\boldsymbol{\alpha})$, which by (58) and (59) equals to either $\Pi_S^{I(II)}(\alpha_L)$ or $\Pi_S^{I(II)}(\alpha_H)$ plus a nonnegative term $(\frac{1}{4\Delta e} \cdot \frac{(\Delta c)^2}{\Delta\alpha})$. Moreover, $r_{H(L)}^{I(II)}$ depends on $\alpha_{H(L)}^{I(II)}$ through the same function as $r_S^{I(II)}$ depends on $\alpha_S^{I(II)}$.

Uniformly Distributed Expertise: Optimal Population-based Strategy

Theorems 6 and 7 enable a comparison between the optimal single-item package and the optimal two-item package when customer expertise is uniformly distributed. In contrast to the the two-point distributed case where single-item can be strictly optimal, we show for this case that the optimal single-item package is generally dominated by the optimal two-item package. The optimality results and the condition for single-item to attain weak optimality are given in Theorem 5 and Proposition 7 in the main body of the paper.

When single-item indeed can be optimal, the prosumer's utility under the optimal population-based package can be found through (54) and (55). When single-item is suboptimal, the following corollary provides the evaluations for prosumer utility.

Corollary 4 *Assume the customer expertise to be uniformly distributed on* $[e_o, e_f]$. *(i) If* $\Pi_M^I(\boldsymbol{\alpha}_M^I) \geq \Pi_M^{II}(\boldsymbol{\alpha}_M^{II})$ *and* $\frac{\Delta c^I}{\Delta \alpha^I} < 2\Delta e$*, then* $(\boldsymbol{\alpha}^*, \mathbf{r}^*) = (\boldsymbol{\alpha}_M^I,\ \mathbf{r}_M^I)$ *with* $\mathbf{r}_M^I = (p - \Delta e \cdot (1-\alpha_H^I) - \frac{\Delta c^I}{2},\ p - \Delta e \cdot (1-\alpha_H^I))$*. For a prosumer of type e,*

$$U_{\mathcal{P}}(\boldsymbol{\alpha}^*, \mathbf{r}^*|e) = \begin{cases} U_{\mathcal{C}} + (1-\alpha_H^I)\delta_{\min}(e) & \text{if } e < e_f - \frac{\Delta c^I}{2\Delta\alpha^I}, \\ U_{\mathcal{C}} + (1-\alpha_L^I)\delta_{\min}(e) - \Delta e \Delta\alpha^I + \frac{\Delta c^I}{2} & \text{otherwise.} \end{cases} \tag{65}$$

(ii) If $\Pi_M^I(\boldsymbol{\alpha}_M^I) < \Pi_M^{II}(\boldsymbol{\alpha}_M^{II})$ *and* $\frac{p-c(\alpha_H^{II})}{1-\alpha_H^{II}} > \frac{p-c(\alpha_L^{II})}{1-\alpha_L^{II}}$*, then* $(\boldsymbol{\alpha}^*, \mathbf{r}^*) = (\boldsymbol{\alpha}_M^{II},\ \mathbf{r}_M^{II})$ *with* $\mathbf{r}_M^{II} = (\frac{p+c(\alpha_H^{II})}{2} - \frac{\Delta c^{II}}{2},\ \frac{p+c(\alpha_H^{II})}{2})$*. For a prosumer of type e,*

$$U_{\mathcal{P}}(\boldsymbol{\alpha}^*, \mathbf{r}^*|e) = \begin{cases} U_{\mathcal{C}} - (1-\alpha_H^{II})\delta_{\max}(e) + \frac{p-c(\alpha_H^{II})}{2}. & \text{if } e < e_f - \frac{\Delta c^{II}}{2\Delta\alpha^{II}}, \\ U_{\mathcal{C}} - (1-\alpha_L^{II})\delta_{\max}(e) + \frac{p-c(\alpha_L^{II})}{2}. & \text{otherwise.} \end{cases} \tag{66}$$

When $(\boldsymbol{\alpha}_M^I, \mathbf{r}_M^I)$ is optimal, by (65), the utility of the prosumers who choose the premium plan, i.e., those with lower expertise, can be obtained by replacing α_S^I with α_H^I in (54); the utility of the prosumers who choose the basic plan, however, incurs an additional negative term $(\frac{\Delta c^I}{2} - \Delta e \Delta \alpha^I)$ on top of replacing α_S^I with α_L^I in (54). When $(\boldsymbol{\alpha}_M^{II}, \mathbf{r}_M^{II})$ is optimal, by (66), the utility of the prosumers who choose the basic(premium) package, i.e., those with higher(lower) expertise, can be obtained by replacing α_S^{II} with $\alpha_L^{II}(\alpha_H^{II})$ in (55).

Proofs for Online Appendix B and Supporting Results

Proof. (Lemma 2) By (8) we have

- for $e < e_f - c'(0)$, $\Phi(e) = p - \delta_{\max}(e) \cdot (1 - \phi(\delta_{\max}(e))) - c\left(\phi(\delta_{\max}(e))\right)$.
  - if there is also $e \geq e_f - c'(1)$, we have $\phi(\delta_{\max}(e)) = (c')^{-1}(\delta_{\max}(e))$, $\Phi'(e) = 1 - \phi(\delta_{\max}(e)) \geq 0$ and $\Phi''(e) = \phi'(\delta_{\max}(e)) > 0$, where the signs are implied by the definition of $\phi(\cdot)$, and $c(\cdot)$ being strictly increasing convex.
  - if there is also $e < e_f - c'(1)$, we have $\phi(\delta_{\max}(e)) = 1$ and $\Phi(e) = p - c(1)$.
- for $e \geq e_f - c'(0)$, $\Phi(e) = p - \delta_{\max}(e)$ and is linearly increasing.

Summarizing the above lead to

$$\Phi(e)=\begin{cases} p-c(1) & \text{if } c'(1)<\delta_{\max}(e),\\ p-\delta_{\max}(e)\cdot(1-\phi(\delta_{\max}(e)))-c\left(\phi(\delta_{\max}(e))\right) & \text{if } c'(0)<\delta_{\max}(e)\le c'(1),\\ p-\delta_{\max}(e) & \text{if } \delta_{\max}(e)\le c'(0). \end{cases} \tag{67}$$

The monotonicity of $\phi(\cdot)$ and $\varphi(\cdot)$ are implied by definition and that $c(\cdot)$ being strictly increasing convex. Finally, (iii) is implied by (67) and $\Phi(\cdot)$ being increasing. The proof is complete. □

Proof. (Corollary 1) For $e \ge \delta_{\max}(c'(0))$, $\alpha_S^*(e)\equiv 0$, $r_S^*(e)\equiv p-\delta_{\max}(e)$, and $\varepsilon(\alpha_S^*(e),e)\equiv e$. So $\frac{\partial}{\partial e}r_S^*(e)=\frac{\partial}{\partial e}\varepsilon(\alpha_S^*(e),e)=1$. For $e<\delta_{\max}(c'(0))$, $\alpha_S^*=\phi(\delta_{\max}(e))$, $r_S^*=p-\delta_{\max}(e)+\phi(\delta_{\max}(e))\delta_{\max}(e)$, $\varepsilon(\alpha_S^*,e)=e+\phi(\delta_{\max}(e))\delta_{\max}(e)$. And it can be verified in this case that $\partial r_S^*(e)/\partial e=1-\phi(\delta_{\max}(e))-\delta_{\max}(e)\phi'(\delta_{\max}(e))=\partial\varepsilon(\alpha_S^*(e),e)/\partial e$. Thus, it suffices to show that r_S^* can be non-monotone in $\delta_{\max}(e)$. To see this, let $w:=\delta_{\max}(e)$, then as e increases from e_o to $\delta_{\max}(c'(0))$, w decreases from $\delta_{\max}(e_o)$ to $c'(0)$. By (9) there is $r_S^*(e)=p-w\cdot(1-\phi(w))$, which implies $\partial r_S^*/\partial w=-1+\phi(w)+w\phi'(w)$, where on the right hand side $\phi(w)-1\le 0$ and $w\phi'(w)\ge 0$. Specifically, $\partial r_S^*/\partial w|_{w=c'(0)}=c'(0)\phi'(c'(0))-1$, which can be negative if $c'(0)$ is very close to zero. On the other hand, if $\delta_{\max}(e_o)\ge c'(1)$, $\partial r_S^*/\partial w$ can be positive as w approaches $c'(1)$ from below. In other words, depending on the specific form of $c(\cdot)$ and the value of $\delta_{\max}(e_o)$, the sign of $\partial r_S^*/\partial w$ can change as w varies on $[c'(0),\delta_{\max}(e_o)]$. The proof is thus complete. □

Proof. (Lemma 3) (i) can be obtained via simple algebra.

(ii) Let $\underline{k}:=\kappa(e_f)$ and $\psi:=\kappa(e_o)-\underline{k}$. Then we have,

$$\begin{aligned}
\frac{\partial}{\partial k}\ell(\alpha,k) &= \left((1-\alpha)\cdot\frac{1}{k}+\alpha\cdot\frac{1}{\underline{k}}\right)^{-2}(1-\alpha)\cdot\frac{1}{k^2}=\left((1-\alpha)+\alpha\cdot\frac{k}{\underline{k}}\right)^{-2}(1-\alpha)\ge 0.\\
\frac{\partial}{\partial k}(k-\ell(\alpha,k)) &= 1-\left((1-\alpha)+\alpha\cdot\frac{k}{\underline{k}}\right)^{-2}(1-\alpha)\ge 0.\\
\frac{\partial^2}{\partial k^2}\ell(\alpha,k) &= 2\left((1-\alpha)\cdot\frac{1}{k}+\alpha\cdot\frac{1}{\underline{k}}\right)^{-3}(1-\alpha)^2\cdot\frac{1}{k^4}-2\left((1-\alpha)\cdot\frac{1}{k}+\alpha\cdot\frac{1}{\underline{k}}\right)^{-2}(1-\alpha)\cdot\frac{1}{k^3}\\
&= 2\left((1-\alpha)\cdot\frac{1}{k}+\alpha\cdot\frac{1}{\underline{k}}\right)^{-3}(1-\alpha)\cdot\frac{1}{k^3}\left(-\alpha\cdot\frac{1}{\underline{k}}\right)\le 0.\\
\ell(\alpha,\underline{k}) &= \underline{k}.\\
\ell(\alpha,\underline{k}+\psi) &= \left((1-\alpha)\cdot\frac{1}{\underline{k}+\psi}+\alpha\cdot\frac{1}{\underline{k}}\right)^{-1}=\frac{(\underline{k}+\psi)\underline{k}}{\underline{k}+\psi\alpha}=\underline{k}+\psi\frac{1-\alpha}{1+\frac{\psi}{\underline{k}}\alpha}.
\end{aligned}$$

(iii)

$$\begin{aligned}\frac{\partial}{\partial\alpha}\ell(\alpha,k) &= -k\left(1+\alpha(\frac{k}{\underline{k}}-1)\right)^{-2}(\frac{k}{\underline{k}}-1)\leq 0.\\ \frac{\partial^2}{\partial\alpha^2}\ell(\alpha,k) &= 2k\left(1+\alpha(\frac{k}{\underline{k}}-1)\right)^{-3}(\frac{k}{\underline{k}}-1)^2\geq 0.\end{aligned}$$

And the proof is complete. □

Proof. (Corollary 2) The result is immediately implied by $\Phi(\cdot)$ being increasing convex (Lemma 2 in Appendix B) and Proposition 1 (iv). The proof is thus complete. □

Proof. (Theorem 5) By (58)-(59) in Online Appendix B, it can be verified that

$$\begin{aligned}\Pi_M^I(\boldsymbol{\alpha}) &= f(\alpha_L)+\frac{[(2\Delta e\Delta\alpha-\Delta c)^+]^2}{4\Delta e\Delta\alpha}, && (68)\\ \Pi_M^{II}(\boldsymbol{\alpha}) &= g(\alpha_L)+\frac{(1-\alpha_H)(1-\alpha_L)}{4\Delta e\Delta\alpha}\cdot\left[\left(\frac{p-c(\alpha_H)}{1-\alpha_H}-\frac{p-c(\alpha_L)}{1-\alpha_L}\right)^+\right]^2.\end{aligned}$$

Combined with the definitions in (61) and (63), there are $\Pi_M^I(\boldsymbol{\alpha}_M^I)=\max_{\alpha_L\in\mathcal{A}_I\cup\mathcal{A}_{II},\ \alpha_H\in\mathcal{A}_I}\Pi_M^I(\boldsymbol{\alpha})\geq\max_{\alpha_L\in\mathcal{A}_I}f(\alpha_L)=f(\alpha_S^I), \Pi_M^{II}(\boldsymbol{\alpha}_M^{II})=\max_{\alpha_L,\alpha_H\in\mathcal{A}_{II}}\Pi_M^{II}(\boldsymbol{\alpha})\geq\max_{\alpha_L\in\mathcal{A}_{II}}g(\alpha_L)=g(\alpha_S^{II})$. Therefore, by Theorem 7, we must have $\Pi(\boldsymbol{\alpha}_M^*,\mathbf{r}_M^*)\geq\max\left\{\Pi_M^I(\boldsymbol{\alpha}_M^I),\Pi_M^{II}(\boldsymbol{\alpha}_M^{II})\right\}\geq\max\{f(\alpha_S^I),g(\alpha_S^{II})\}=\Pi(\alpha_S^*,r_S^*)$. So single-item attains optimality only if $\Pi(\alpha_S^*,r_S^*)=\Pi(\boldsymbol{\alpha}_M^*,\mathbf{r}_M^*)$. □

Proof. (Proposition 7) (i) When $(\boldsymbol{\alpha}_M^*,\mathbf{r}_M^*)=(\boldsymbol{\alpha}_M^I,\mathbf{r}_M^I)$, if $\frac{\Delta c^I}{\Delta\alpha^I}\geq 2\Delta e$, by (58), there must be $\Pi_M^I(\boldsymbol{\alpha}_M^I)=f(\alpha_L^I)$. Recall that $f(\alpha)\leq g(\alpha)$ for any $\alpha\in[0,1)$, so we must have $\alpha_L^I\in\mathcal{A}_I$ (otherwise if $\alpha_L^I\in\mathcal{A}_{II}$, $\Pi_M^I(\boldsymbol{\alpha}_M^I)=f(\alpha_L^I)\leq g(\alpha_L^I)\leq\Pi_M^{II}(\boldsymbol{\alpha}_M^{II})$, contradicting the optimality of $(\boldsymbol{\alpha}_M^I,\mathbf{r}_M^I)$). Furthermore, by Theorem 6, $f(\alpha_L^I)\leq\max\{f(\alpha_S^I),g(\alpha_S^{II})\}=\Pi(\alpha_S^*,r_S^*)$. Therefore, the optimal single-item package is also the optimal package, i.e., the sufficient direction is proven. Next suppose $\frac{\Delta c^I}{\Delta\alpha^I}<2\Delta e$, by (68) there is $\Pi_M^I(\boldsymbol{\alpha}_M^I)=f(\alpha_L^I)+\frac{[2\Delta e\Delta\alpha^I-\Delta c^I]^2}{4\Delta e\Delta\alpha^I}>\max\{\max_{\alpha_L\in\mathcal{A}_I}f(\alpha_L),\max_{\alpha_L\in\mathcal{A}_{II}}g(\alpha_L)\}=\Pi(\alpha_S^*,r_S^*)$, where the inequality is implied by (58), (59), and the optimality of $(\boldsymbol{\alpha}_M^I,\mathbf{r}_M^I)$. Hence the necessary direction is also shown. (ii) When $(\boldsymbol{\alpha}_M^*,\mathbf{r}_M^*)=(\boldsymbol{\alpha}_M^{II},\mathbf{r}_M^{II})$, the sufficient and necessary condition can be shown following the same structure as above, and we omit them for brevity. Thus the proof is complete. □

Proof. (Lemma 4) Define

$$v(\alpha):=p-c(\alpha)-2\Delta e\cdot(1-\alpha). \tag{69}$$

Then there are $v'=-c'+2\Delta e$, $v''=-c''$. Recall that $c'>0$ and $c''>0$, therefore $v''<0$, i.e., v is strictly concave. As a result, $v(\alpha) = 0$ can have no more than two solutions. □

Proof. (Lemma 5) First of all, any $\alpha \in \mathcal{A}_I \cup \mathcal{A}_{II}$ must imply $\alpha \le c^{-1}(p)$, where $c^{-1}(\cdot)$ is the inverse function of $c(\cdot)$. Because otherwise, there is $c(\alpha) > p$, i.e., $\alpha \in \mathcal{A}_{III}$. Secondly, $c^{-1}(p) \le 1$ if and only if $p \le c(1)$. Thirdly, by (69), note that $v(0) = p - 2\Delta e$, $v(1) = p - c(1)$.

- If $c'(1) < 2\Delta e$,

  there is $v' > 0$ for $\alpha \in [0, 1]$, i.e., v strictly increases in α on $[0, 1]$, which implies $v(1) > v(0)$, i.e., $c(1) < 2\Delta e$.

  - $p < c(1)$, i.e. $v(1) < 0$,

    there are $v(\cdot) < 0$ on $[0, 1]$, i.e., $\mathcal{A}_I = \emptyset$, $\mathcal{A}_{II} = [0, c^{-1}(p)]$.
  - $c(1) < p \le 2\Delta e$

    there are $v(0) \le 0, v(1) > 0$. And there is $\hat{\alpha}_1 = \hat{\alpha}_2$, and $\mathcal{A}_I = [\hat{\alpha}_1, 1]$ and $\mathcal{A}_{II} = [0, \hat{\alpha}_1)$.
  - $c(1) = p < 2\Delta e$, there are $v(0) < 0, v(1) = 0$. And there is $\hat{\alpha}_1 = \hat{\alpha}_2 = 1$, and $\mathcal{A}_I = \{1\}$ and $\mathcal{A}_{II} = [0, 1)$.
  - $p > 2\Delta e$,

    there are $v(0) > 0$ and thus $v(\cdot) > 0$ on $[0, 1]$, i.e., $\mathcal{A}_I = [0, 1]$, $\mathcal{A}_{II} = \emptyset$.

- If $c'(0) \le 2\Delta e$ and $c'(1) \ge 2\Delta e$,

  there are $v'(0) \ge 0$ and $v'(1) \le 0$, i.e., v first increases and then decreases on $[0, 1]$.

  - $2\Delta e < p \le c(1)$

    then $v(0) > 0, v(1) \le 0$. As a result, there is $\hat{\alpha}_1 = \hat{\alpha}_2$, and $\mathcal{A}_I = [0, \hat{\alpha}_1]$ and $\mathcal{A}_{II} = (\hat{\alpha}_1, c^{-1}(p)]$.
  - $2\Delta e = p < c(1)$

    then $v(0) = 0, v(1) < 0$. As a result, there is $\hat{\alpha}_1 = 0$, $\mathcal{A}_I = [0, \hat{\alpha}_2]$ and $\mathcal{A}_{II} = (\hat{\alpha}_2, c^{-1}(p)]$
  - $2\Delta e = p = c(1)$

    then $v(0) = 0, v(1) = 0$, $\hat{\alpha}_1 = 0, \hat{\alpha}_2 = 1$. $\mathcal{A}_I = [0, 1]$ and $\mathcal{A}_{II} = \emptyset$.
  - $p < \min\{2\Delta e, c(1)\}$,

    then $v(0) < 0, v(1) < 0$, $\mathcal{A}_I = [\hat{\alpha}_1, \hat{\alpha}_2]$, $\mathcal{A}_{II} = [0, \hat{\alpha}_1) \cup (\hat{\alpha}_2, c^{-1}(p)]$;
  - $c(1) < p \le 2\Delta e$,

    then $v(0) \le 0, v(1) > 0$, and there is $\hat{\alpha}_1 = \hat{\alpha}_2$. As a result, $\mathcal{A}_I = [\hat{\alpha}_1, 1]$ and $\mathcal{A}_{II} = [0, \hat{\alpha}_1)$.

- $c(1) = p < 2\Delta e$,

  then $v(0) < 0, v(1) = 0$, and there is $\hat{\alpha}_2 = 1$. $\mathcal{A}_I = [\hat{\alpha}_1, 1]$ and $\mathcal{A}_{II} = [0, \hat{\alpha}_1)$.

- $p > \max\{2\Delta e, c(1)\}$,

  then $v(0) > 0, v(1) > 0$. Thus, $\mathcal{A}_I = [0, 1]$, $\mathcal{A}_{II} = \emptyset$.

- If $c'(0) > 2\Delta e$,

  there is $v'(\cdot) < 0$ on $[0, 1]$, i.e., v strictly decreases in α on $[0, 1]$, which implies $v(1) < v(0)$, i.e., $c(1) > 2\Delta e$.

  - $2\Delta e < p \leq c(1)$,

    then $v(0) > 0, v(1) \leq 0$. As a result, there is $\hat{\alpha}_1 = \hat{\alpha}_2$, and $\mathcal{A}_I = [0, \hat{\alpha}_1]$ and $\mathcal{A}_{II} = (\hat{\alpha}_1, c^{-1}(p)]$.

  - $2\Delta e = p < c(1)$,

    then $v(0) = 0, v(1) < 0$. As a result, there is $\hat{\alpha}_1 = \hat{\alpha}_2 = 0$, and $\mathcal{A}_I = \{0\}$ and $\mathcal{A}_{II} = (0, c^{-1}(p)]$.

  - $p < \min\{2\Delta e, c(1)\}$,

    then $v(0) < 0, v(1) < 0$. Thus, $\mathcal{A}_I = \emptyset$, $\mathcal{A}_{II} = [0, c^{-1}(p)]$.

  - $p > \max\{2\Delta e, c(1)\}$,

    then $v(0) > 0, v(1) > 0$. Thus, $\mathcal{A}_I = [0, 1]$, $\mathcal{A}_{II} = \emptyset$.

By consolidating the above cases and noting that $\mathcal{A}_I \cup \mathcal{A}_{II} \cup \mathcal{A}_{III} = [0, 1]$, we can obtain the results as in Table 2. □

Proof. (Proposition 8) Note that in this case, $U_{\mathcal{P}}(e) \geq U_{\mathcal{C}}$ if and only if $\varepsilon(\alpha_S, e) - e_f \geq r_S - p$, which is equivalent to

$$(1 - \alpha_S)\delta_{\max}(e) \leq p - r_S. \tag{70}$$

- For $\alpha_S = 1$, (70) is equivalent to $r_S \leq p$.

- For $\alpha_S < 1$, (70) is equivalent to $e \geq e_f - \frac{p - r_S}{1 - \alpha_S}$.

  - For $r_S \leq p - \Delta e(1 - \alpha_S)$, there is $e_f - \frac{p - r_S}{1 - \alpha_S} \leq e_o$. Hence (70) always hold, i.e. $d_{\mathcal{P}} = 1$.
  - For $p - \Delta e(1 - \alpha_S) < r_S \leq p$, $d_{\mathcal{P}} = \frac{1}{\Delta e} \cdot \frac{p - r_S}{1 - \alpha_S}$.
  - For $r_S > p$, then $e_f - \frac{p - r_S}{1 - \alpha_S} > e_f$, and thus (70) does not hold, i.e. $d_{\mathcal{P}} = 0$.

□

Proof. (Theorem 6) For simplicity, define $f(\cdot) := \Pi_S^I(\cdot)$ and $g(\cdot) := \Pi_S^{II}(\cdot)$. By (11) and (49), the OEM's profit function under any given single-item package (α_S, r_S) can be obtained as

$$\Pi(\alpha_S, r_S) = \begin{cases} 0 & \text{if } r_S > p, \\ \frac{1}{\Delta e} \cdot \frac{(r_S - c(\alpha_S))(p - r_S)}{1-\alpha_S} & \text{if } p - \Delta e(1-\alpha_S) < r_S \le p, \alpha_S < 1, \\ r_S - c(\alpha_S) & \text{if } r_S \le p - \Delta e(1-\alpha_S), \alpha_S \le 1. \end{cases}$$

- For any given α_S that satisfies $p - c(\alpha_S) \ge 2\Delta e(1-\alpha_S)$, i.e.,

$$h(\alpha_S) := p - c(\alpha_S) - 2\Delta e(1-\alpha_S) \ge 0, \tag{71}$$

  there is $p - \Delta e(1-\alpha_S) \ge \frac{p+c(\alpha_S)}{2}$, and hence $\Pi(\alpha_S, r_S)$ is decreasing in r_S on $[p-\Delta e(1-\alpha_S), p]$. Therefore, on $[p - \Delta e(1-\alpha_S), p]$, $\Pi(\alpha_S, r_S)$ is maximized at $r_S = p - \Delta e(1-\alpha_S)$ with

$$\Pi(\alpha_S, r_S)|_{r_S = p-\Delta e(1-\alpha_S)} = f(\alpha_S).$$

  On the other hand, on $[0, p - \Delta e(1-\alpha_S)]$, $\Pi(\alpha_S, r_S)$ is increasing in r_S and thus is also maximized at $r_S = p - \Delta e(1-\alpha_S)$. Note that $f(\alpha_S) = h(\alpha_S) + \Delta e(1-\alpha_S) \ge \Delta e(1-\alpha_S) \ge 0$, where the first inequality is due to (71). To summarize, for any given α_S that satisfies (71), $\max_{r_S \ge 0} \Pi(\alpha_S, r_S) = f(\alpha_S)$ with $\text{argmax}_{r_S \ge 0} \Pi(\alpha_S, r_S) = p - \Delta e(1-\alpha_S)$.

- For any given α_S that satisfies $0 \le p - c(\alpha_S) \le 2\Delta e(1-\alpha_S)$, i.e.,

$$h(\alpha_S) \le 0 \text{ and } p - c(\alpha_S) \ge 0, \tag{72}$$

  there is $p - \Delta e(1-\alpha_S) \le \frac{p+c(\alpha_S)}{2} \le p$. Therefore, on $[p - \Delta e(1-\alpha_S), p]$, $\Pi(\alpha_S, r_S)$ is maximized at $r_S = \frac{p+c(\alpha_S)}{2}$ with

$$\Pi(\alpha_S, r_S)\Big|_{r_S = \frac{p+c(\alpha_S)}{2}} = g(\alpha_S).$$

  By definition, there must be

  $g(\alpha_S) \ge \max\left\{\Pi(\alpha_S, r_S)|_{r_S = p-\Delta e(1-\alpha_S)},\ \Pi(\alpha_S, r_S)|_{r_S = p}\right\} = \max\{f(\alpha_S), 0\}.$

  On the other hand, on $[0, p - \Delta e(1-\alpha_S)]$, $\Pi(\alpha_S, r_S)$ is increasing in r_S and thus is maximized at $r_S = p - \Delta e(1-\alpha_S)$ with $\Pi(\alpha_S, r_S)|_{r_S = p-\Delta e(1-\alpha_S)} = f(\alpha_S)$. In addition, for $r_S \in (p, +\infty)$, $\Pi(\alpha_S, r_S) \equiv 0$. To summarize, for any given α_S that satisfies (72), $\max_{r_S \ge 0} \Pi(\alpha_S, r_S) = g(\alpha_S)$ with $\text{argmax}_{r_S \ge 0} \Pi(\alpha_S, r_S) = \frac{p+c(\alpha_S)}{2}$.

- For any given α_S that satisfies $p - c(\alpha_S) < 0$, there is $\frac{p+c(\alpha_S)}{2} > p$. Therefore, on $[p - \Delta e(1-\alpha_S), p]$, $\Pi(\alpha_S, r_S)$ is maximized at $r_S = p$ with $\Pi(\alpha_S, r_S)|_{r_S = p} = 0$. For $r_S \in (p, +\infty)$, $\Pi(\alpha_S, r_S) \equiv 0$. On the other hand, on $[0, p - \Delta e(1-\alpha_S)]$, $\Pi(\alpha_S, r_S)$ is increasing in r_S and thus is maximized at $r_S = p - \Delta e(1-\alpha_S)$ with $\Pi(\alpha_S, r_S)|_{r_S = p-\Delta e(1-\alpha_S)} = f(\alpha_S) < 0$, where the inequality is due to $p - c(\alpha_S) < 0$. Therefore, for any α_S such that $p - c(\alpha_S) < 0$, $\max_{r_S \geq 0} \Pi(\alpha_S, r_S) = 0$ with $\text{argmax}_{r_S \geq 0} \Pi(\alpha_S, r_S) = p$.

By summarizing the above scenarios, we have

$$\max_{r_S \geq 0} \Pi(\alpha_S, r_S) = \begin{cases} f(\alpha_S) & \text{for } \alpha_S \in \mathcal{A}_I, \\ g(\alpha_S) & \text{for } \alpha_S \in \mathcal{A}_{II}, \\ 0 & \text{for } \alpha_S \in \mathcal{A}_{III}. \end{cases}$$

Since we have established in the analyses above that $f(\alpha_S) \geq 0$ for $\alpha_S \in \mathcal{A}_I$ and $g(\alpha_S) \geq 0$ for $\alpha_S \in \mathcal{A}_{II}$, any α_S such that $\alpha_S \in \mathcal{A}_{III}$ must be suboptimal (note that $\mathcal{A}_I \cup \mathcal{A}_{II} \neq \emptyset$ with $0 \in \mathcal{A}_I \cup \mathcal{A}_{II}$). The optimal package and resulting profit can be obtained as in Theorem 6. □

Proof. (Corollary 3) Consider arbitrary p_1, p_2 such that $0 \leq p_1, p_2 \leq e_f$ with $p_1 \leq p_2$. Let $f(\cdot|p_i)$, $\alpha_S^I(p_i)$, $\alpha_S^{II}(p_i)$ denote the relevant notations under the corresponding p_i. Then there are, $f(\alpha_S^I(p_1)|p_1) \leq f(\alpha_S^I(p_1)|p_2) \leq f(\alpha_S^I(p_2)|p_2)$, where the first inequality is by simply algebra, and the second is by the definition of $\alpha_S^I(p_2)$. Similarly, we also have $g(\alpha_S^{II}(p_1)|p_1) \leq g(\alpha_S^{II}(p_1)|p_2) \leq g(\alpha_S^{II}(p_2)|p_2)$ - this can be seen by examining all the cases in Table 2: in the first three cases, for $\alpha_S \in \mathcal{A}_{II}$, we have $p - c(\alpha_S) \geq 0$ and thus $g(\alpha_S|p)$ increases in p; in the last case, $\mathcal{A}_{II} = \emptyset$, i.e., α_S^{II} becomes irrelevant. Therefore we must have $\max\{f(\alpha_S^I(p_1)|p_1), g(\alpha_S^{II}(p_1)|p_1)\} \leq \max\{f(\alpha_S^I(p_2)|p_2), g(\alpha_S^{II}(p_2)|p_2)\}$, which implies that $\Pi(\alpha_S^*, r_S^*)$ increases in p. □

Proof. (Proposition 9) For any two-item package $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}'$, there are $0 < \Delta\alpha \leq 1, \Delta r > 0$. In this case, the utility of a prosumer of type e is given by (1).

- If $\frac{\Delta r}{\Delta \alpha} > \Delta e$,

  then $e_f - \frac{\Delta r}{\Delta \alpha} < e_o$, then no prosumers choose premium plan, i.e., $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$. Also, note that $U_{\mathcal{P}}(e) \geq U_{\mathcal{C}}$ if and only if $e \geq e_f - \frac{p - r_L}{1 - \alpha_L}$. Therefore, if there is further

  - $r_L > p$, then $e_f - \frac{p - r_L}{1 - \alpha_L} > e_f$, i.e, $U_{\mathcal{P}}(e) < U_{\mathcal{C}}$ for all $e \in [e_o, e_f]$, and thus $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$.
  - $p - \Delta e \cdot (1 - \alpha_L) \leq r_L \leq p$, then $e_o \leq e_f - \frac{p - r_L}{1 - \alpha_L} \leq e_f$, and thus $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \frac{p - r_L}{1 - \alpha_L}$.
  - $r_L < p - \Delta e \cdot (1 - \alpha_L)$, then $e_f - \frac{p - r_L}{1 - \alpha_L} < e_o$, and thus $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 1$.

- If $\frac{\Delta r}{\Delta \alpha} \leq \Delta e$,

  then $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_o$, then a prosumer of type e will choose premium plan if and only if $e < e_f - \frac{\Delta r}{\Delta \alpha}$.

- For $e < e_f - \frac{\Delta r}{\Delta \alpha}$, i.e., for customers who will choose premium plan conditioning that they decide to be prosumers, $U_{\mathcal{P}}(e) \geq U_{\mathcal{C}}$ is equivalent to

$$(1-\alpha_H)\delta_{\max}(e) \leq p - r_H. \tag{73}$$

  * For $\alpha_H = 1$, (73) is equivalent to $r_H \leq p$. Therefore,
    if $r_H \leq p$, $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \left(\Delta e - \frac{\Delta r}{\Delta \alpha}\right) = 1 - \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha}$;
    if $r_H > p$, $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$.
  * For $\alpha_H < 1$, (73) is equivalent to $e \geq e_f - \frac{p - r_H}{1-\alpha_H}$.
    · For $r_H \leq p - \Delta e \cdot (1-\alpha_H)$, there is $e_f - \frac{p-r_H}{1-\alpha_H} \leq e_o$. (Combined with $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_o$, we have $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_f - \frac{p-r_H}{1-\alpha_H}$, which is equivalent to $\frac{(1-\alpha_H)\Delta r}{\Delta \alpha} + r_H = \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p$.)
    Hence (73) always hold with $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 1 - \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha}$;
    · For $p - \Delta e \cdot (1-\alpha_H) \leq r_H \leq p$, there is $e_o \leq e_f - \frac{p-r_H}{1-\alpha_H} \leq e_f$.
    If there is further $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p$ (which is equivalent to $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_f - \frac{p-r_H}{1-\alpha_H}$), customers of type e will choose to be a prosumer and also prefer premium plan over the basic plan if and only if $e \in \left[e_f - \frac{p-r_H}{1-\alpha_H}, e_f - \frac{\Delta r}{\Delta \alpha}\right]$, with $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \left(\frac{p-r_H}{1-\alpha_H} - \frac{\Delta r}{\Delta \alpha}\right) = \frac{1}{\Delta e} \cdot \frac{p - r_L - \frac{\Delta r}{\Delta \alpha}(1-\alpha_L)}{1-\alpha_H}$;
    if instead $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L > p$ (i.e., $e_f - \frac{\Delta r}{\Delta \alpha} < e_f - \frac{p-r_H}{1-\alpha_H}$), then $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$.
    · For $r_H > p$, there is $e_f - \frac{p-r_H}{1-\alpha_H} > e_f$, then $d_{\mathcal{P}_H}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$.
- For $e \geq e_f - \frac{\Delta r}{\Delta \alpha}$, i.e., for customers who will not choose premium plan if they decide to be prosumers, $U_{\mathcal{P}}(e) \geq U_{\mathcal{C}}$ is equivalent to

$$e \geq e_f - \frac{p - r_L}{1-\alpha_L}. \tag{74}$$

  * For $r_L \leq p - \Delta e \cdot (1-\alpha_L)$, there is $e_f - \frac{p-r_L}{1-\alpha_L} \leq e_o$. (Combined with $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_o$, we have $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_f - \frac{p-r_L}{1-\alpha_L}$, which is equivalent to $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p$.)
    Hence (74) always hold with $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha}$;
  * For $p - \Delta e \cdot (1-\alpha_L) \leq r_L \leq p$, there is $e_o \leq e_f - \frac{p-r_L}{1-\alpha_L} \leq e_f$.
    If there is further $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L \leq p$ (which is equivalent to $e_f - \frac{\Delta r}{\Delta \alpha} \geq e_f - \frac{p-r_L}{1-\alpha_L}$), customers of type $e > e_f - \frac{\Delta r}{\Delta \alpha}$ will all choose to be a prosumer and prefer the basic plan, i.e., $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta \alpha}$.
    if instead $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L > p$ (i.e., $e_f - \frac{\Delta r}{\Delta \alpha} < e_f - \frac{p-r_L}{1-\alpha_L}$), then customers of type $e \geq e_f - \frac{p-r_L}{1-\alpha_L}$ will choose to be prosumers with basic plan only, i.e., $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \frac{1}{\Delta e} \cdot \frac{p-r_L}{1-\alpha_L}$. (Note that $\frac{(1-\alpha_L)\Delta r}{\Delta \alpha} + r_L > p$ implies $r_L > p - \frac{(1-\alpha_L)\Delta r}{\Delta \alpha} \geq p - (1-\alpha_L)\Delta e \geq p - e_f$.)

* For $r_L > p$, there is $e_f - \frac{p-r_L}{1-\alpha_L} > e_f$, then $d_{\mathcal{P}_L}(\boldsymbol{\alpha}_M, \mathbf{r}_M) = 0$.

By summarizing the above cases, we can obtain the customers' choice as in Proposition 9. $\square$

Proof. (Theorem 7) Firstly, note that via simple algebra we have

$$\mathcal{M}' = \left\{(\boldsymbol{\alpha}, \mathbf{r}) \in \mathcal{M} \mid \Delta r \leq \Delta e \cdot \Delta\alpha,\ r_H \leq p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r\right\}.$$

Note that for any $(\boldsymbol{\alpha}, \mathbf{r}) \in \mathcal{M}'$, there are $\frac{1-\alpha_H}{\Delta\alpha}\Delta r \leq \frac{1-\alpha_H}{\Delta\alpha} \cdot \Delta e \cdot \Delta\alpha = \Delta e(1-\alpha_H)$, and this implies $p - \Delta e \cdot (1-\alpha_H) \leq p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r$. Combined with (12) and Proposition 9, the OEM's profit function on the region $\mathcal{M}'$ can be obtained as: for any $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}'$,

$$\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) = \begin{cases} \frac{1}{\Delta e} \cdot \left(\frac{(r_H - c(\alpha_H)) \cdot \left(p - r_L - \frac{\Delta r}{\Delta\alpha}(1-\alpha_L)\right)}{1-\alpha_H} + \frac{(r_L - c(\alpha_L))\Delta r}{\Delta\alpha}\right) & \text{if } \alpha_H < 1, p - \Delta e \cdot (1-\alpha_H) \leq r_H \leq p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r, \\ (r_H - c(\alpha_H)) \cdot \left(1 - \frac{1}{\Delta e} \cdot \frac{\Delta r}{\Delta\alpha}\right) + \frac{1}{\Delta e} \cdot \frac{(r_L - c(\alpha_L))\Delta r}{\Delta\alpha} & \text{if } r_H \leq p - \Delta e \cdot (1-\alpha_H). \end{cases}$$

We will solve the optimization by first fixing the value of α_L. Note that it is never optimal to consider $\alpha_L \in \mathcal{A}_{III}$, since that will results in negative profit for the OEM. Therefore it suffices to consider $\alpha_L \in \mathcal{A}_I \cup \mathcal{A}_{II}$. Hence we will first suppose that such a $0 \leq \alpha_L < 1$ is given, and later optimize over α_L. We will separate the analyses into two parts based on whether α_H is less than or equal to one.

- For $\alpha_H = 1$,

  $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}'$ implies $\Delta r \leq \Delta e \cdot (1-\alpha_L)$ and $r_H \leq p$, and $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{\alpha_H=1} = (r_H - c(1)) \cdot \left(1 - \frac{1}{\Delta e} \cdot \frac{\Delta r}{1-\alpha_L}\right) + \frac{1}{\Delta e} \cdot \frac{(r_H - \Delta r - c(\alpha_L))\Delta r}{1-\alpha_L}$. Recall that $1 \in \mathcal{A}_I \cup \mathcal{A}_{III}$. As a result, we only consider the case of $1 \in \mathcal{A}_I$ for the analysis of $\alpha_H = 1$, because if $1 \in \mathcal{A}_{III}$, i.e., $c(1) > p$, offering any premium plan with $\alpha_H = 1$ can only cause the OEM to loose money, and must be suboptimal.

  Therefore, for any given α_L and Δr s.t. $(\boldsymbol{\alpha}_M, \mathbf{r}_M) \in \mathcal{M}'$, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{\alpha_H=1}$ is maximized at $r_H = p$. Furthermore, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{\alpha_H=1,\ r_H=p} = \frac{1}{\Delta e(1-\alpha_L)}\left[-(\Delta r)^2 + (c(1) - c(\alpha_L)) \cdot \Delta r\right] + p - c(1)$.

  - If $c'(0) > 2\Delta e$, then since $c''(\cdot) \geq 0$, there must be $c(\alpha) - 2\alpha\Delta e$ is increasing on $\alpha \in [0,1]$. Hence we have $c(\alpha_L) - 2\alpha_L\Delta e \leq c(1) - 2\Delta e$, which implies $\Delta e(1-\alpha_L) \leq \frac{c(1)-c(\alpha_L)}{2}$. As a result, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{\alpha_H=1,\ r_H=p}$ is maximized at $\Delta r = \Delta e \cdot (1-\alpha_L)$ (i.e., $r_L = p - \Delta e \cdot (1-\alpha_L)$) with $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\Delta e \cdot (1-\alpha_L)} = f(\alpha_L)$.
  - If $c'(1) < 2\Delta e$, then since $c''(\cdot) \geq 0$, there must be $c(\alpha) - 2\alpha\Delta e$ is decreasing on $\alpha \in [0,1]$. Hence we have $c(\alpha_L) - 2\alpha_L\Delta e \geq c(1) - 2\Delta e$, which implies $\Delta e(1-\alpha_L) \geq \frac{c(1)-c(\alpha_L)}{2}$. As a

result, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p}$ is maximized at $\Delta r = \frac{c(1)-c(\alpha_L)}{2}$ (i.e., $r_L = p - \frac{c(1)-c(\alpha_L)}{2}$) with $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\frac{c(1)-c(\alpha_L)}{2}} = \frac{1}{4\Delta e}\cdot\frac{(c(1)-c(\alpha_L))^2}{1-\alpha_L} + p - c(1)$.

- If $c'(0) \le 2\Delta e \le c'(1)$, then $c(\alpha) - 2\alpha\Delta e$ first decreases and then increases in α on $[0,1]$.
  * If $c(1) \ge 2\Delta e$, then we have $c(\alpha_L) - 2\alpha_L\Delta e \le c(1) - 2\Delta e$ for $\alpha_L \in [0,1]$, which implies $\Delta e(1-\alpha_L) \le \frac{c(1)-c(\alpha_L)}{2}$. With similar analysis as for the case of $c'(0) > 2\Delta e$, OEM's profit in this case is maximized at $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\Delta e\cdot(1-\alpha_L)} = f(\alpha_L)$.
  * If $c(1) < 2\Delta e$, then there exists $\tilde{\alpha} \in [0,1)$ such that $c(\tilde{\alpha}) - 2\tilde{\alpha}\Delta e = c(1) - 2\Delta e$.

    For $\alpha_L \in [0,\tilde{\alpha}]$, $c(\alpha_L) - 2\alpha_L\Delta e \ge c(1) - 2\Delta e$, which implies $\frac{c(1)-c(\alpha_L)}{1-\alpha_L} \le 2\Delta e$. Following similar analysis as for the case of $c'(1) < 2\Delta e$, given such α_L, the OEM's profit is maximized at $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\frac{c(1)-c(\alpha_L)}{2}} = \frac{1}{4\Delta e}\cdot\frac{[c(1)-c(\alpha_L)]^2}{1-\alpha_L} + p - c(1)$.

    For $\alpha_L \in [\tilde{\alpha},1]$, $c(\alpha_L) - 2\alpha_L\Delta e \le c(1) - 2\Delta e$, which implies $\Delta e(1-\alpha_L) \le \frac{c(1)-c(\alpha_L)}{2}$. With similar analysis as for the case of $c'(0) > 2\Delta e$, OEM's profit in this case is maximized at $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\Delta e\cdot(1-\alpha_L)} = f(\alpha_L)$.

We can further consolidate the above cases into the following:

- If $c(1) \ge 2\Delta e$, the OEM's profit in this case is maximized at
  $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\Delta e\cdot(1-\alpha_L)} = f(\alpha_L)$.
- If $c(1) < 2\Delta e$, redefine (with slight abuse of notation) $\tilde{\alpha} := \operatorname{argmin}\{\alpha \in [0,1] \mid c(\alpha) - 2\alpha\Delta e = c(1) - 2\Delta e\}$.

  For $\alpha_L \in [0,\tilde{\alpha}]$, the OEM's profit is maximized at
  $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\frac{c(1)-c(\alpha_L)}{2}} = \frac{1}{4\Delta e}\cdot\frac{[c(1)-c(\alpha_L)]^2}{1-\alpha_L} + p - c(1) \ge f(\alpha_L)$, where the inequality can be verified through simple algebra. Note that $\frac{1}{4\Delta e}\cdot\frac{[c(1)-c(\alpha_L)]^2}{1-\alpha_L} + p - c(1) = \left(f(\alpha_H) + \frac{1}{4\Delta e}\cdot\frac{(\Delta c)^2}{\Delta\alpha}\right)\mid_{\alpha_H=1}$. Also, recall that we only need to consider $\alpha_L \in \mathcal{A}_I \cup \mathcal{A}_{II}$ to avoid negative profit for the OEM, and that $\frac{c(1)-c(\alpha_L)}{1-\alpha_L} \le 2\Delta e$ for $\alpha_L \in [0,\tilde{\alpha}]$. Therefore, we must have $\max_{\alpha_L\in[0,\tilde{\alpha}]\cap(\mathcal{A}_I\cup\mathcal{A}_{II})}\left(\frac{1}{4\Delta e}\cdot\frac{[c(1)-c(\alpha_L)]^2}{1-\alpha_L} + p - c(1)\right) \le \max_{\alpha_L\in\mathcal{A}_I\cup\mathcal{A}_{II},\ \alpha_H\in\mathcal{A}_I}\left(f(\alpha_H) + \frac{1}{4\Delta e}\cdot\frac{(\Delta c)^2}{\Delta\alpha}\right)$ with $1 \in \mathcal{A}_I$.

  For $\alpha_L \in [\tilde{\alpha},1]$, the OEM's profit is maximized at
  $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)\mid_{\alpha_H=1,\ r_H=p,\ r_L=p-\Delta e\cdot(1-\alpha_L)} = f(\alpha_L)$.

- For $\alpha_H \in (\alpha_L, 1)$

  $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) =$

  $$\begin{cases} \frac{1}{\Delta e} \cdot \frac{-r_H^2 + (p+c(\alpha_H)) \cdot r_H + \frac{1-\alpha_H}{\Delta\alpha} \cdot \left[-(\Delta r)^2 + \Delta c \cdot \Delta r\right] - p \cdot c(\alpha_H)}{1-\alpha_H}, & \text{if } p - \Delta e \cdot (1-\alpha_H) \le r_H \le p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r; \\ r_H - c(\alpha_H) + \frac{1}{\Delta\alpha\Delta e} \cdot \left[-(\Delta r)^2 + \Delta c \cdot \Delta r\right], & \text{if } r_H \le p - \Delta e \cdot (1-\alpha_H). \end{cases}$$

  which is increasing in r_H for any given $\Delta r \le \Delta e \cdot \Delta\alpha$ and $r_H \le p - \Delta e \cdot (1-\alpha_H)$. Therefore, we only need to consider the region $p - \Delta e \cdot (1-\alpha_H) \le r_H \le p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r$.

  Specifically, for $\Delta r \le \Delta e \cdot \Delta\alpha$ and $r_H \in \left[p - \Delta e \cdot (1-\alpha_H),\ p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r\right]$,

  $$\begin{aligned} &\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \\ = \ & \frac{1}{\Delta e} \cdot \frac{-\left(r_H - \frac{p+c(\alpha_H)}{2}\right)^2 + \frac{(p-c(\alpha_H))^2}{4} + \frac{1-\alpha_H}{\Delta\alpha} \cdot \left[-\left(\Delta r - \frac{\Delta c}{2}\right)^2 + \frac{(\Delta c)^2}{4}\right]}{1-\alpha_H}. \end{aligned}$$

  - $\alpha_H \in \mathcal{A}_I \cap (\alpha_L, 1)$,

    i.e., $\frac{p-c(\alpha_H)}{1-\alpha_H} \ge 2\Delta e$, we have $\frac{p+c(\alpha_H)}{2} \le p - \Delta e \cdot (1-\alpha_H)$.

    For any given $\Delta r \le \Delta e \cdot \Delta\alpha$, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)$ is maximized at $r_H = p - \Delta e \cdot (1-\alpha_H)$ with

    $$\begin{aligned} &\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e \cdot (1-\alpha_H)} \\ = \ & \frac{1}{\Delta e} \cdot \frac{-\left(\frac{p-c(\alpha_H)}{2} - \Delta e(1-\alpha_H)\right)^2 + \frac{(p-c(\alpha_H))^2}{4} + \frac{1-\alpha_H}{\Delta\alpha} \cdot \left[-\left(\Delta r - \frac{\Delta c}{2}\right)^2 + \frac{(\Delta c)^2}{4}\right]}{1-\alpha_H}. \end{aligned}$$

    * If $\frac{\Delta c}{\Delta\alpha} \ge 2\Delta e$, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e \cdot (1-\alpha_H)}$ is maximized at $\Delta r = \Delta e \cdot \Delta\alpha$, i.e, $r_L = p - \Delta e \cdot (1-\alpha_L)$ with

      $$\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e \cdot (1-\alpha_H), r_L = p - \Delta e \cdot (1-\alpha_L)} = f(\alpha_L).$$

    * If $\frac{\Delta c}{\Delta\alpha} \le 2\Delta e$, $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e \cdot (1-\alpha_H)}$ is maximized at $\Delta r = \frac{\Delta c}{2}$, i.e, $r_L = p - \Delta e \cdot (1-\alpha_H) - \frac{\Delta c}{2} (= p - \Delta e \cdot (1-\alpha_L) + \Delta e \Delta\alpha - \frac{\Delta c}{2})$ with

      $$\begin{aligned} &\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e \cdot (1-\alpha_H), r_L = p - \Delta e \cdot (1-\alpha_H) - \frac{\Delta c}{2}} \\ = \ & f(\alpha_H) + \frac{1}{4\Delta e} \cdot \frac{(\Delta c)^2}{\Delta\alpha} = f(\alpha_L) + \frac{(2\Delta e \cdot \Delta\alpha - \Delta c)^2}{4\Delta e \cdot \Delta\alpha} \ge f(\alpha_L). \end{aligned}$$

  - $\alpha_H \in \mathcal{A}_{II} \cap (\alpha_L, 1)$,

    i.e., $0 \le \frac{p-c(\alpha_H)}{1-\alpha_H} \le 2\Delta e$, we have $\frac{p+c(\alpha_H)}{2} \ge p - \Delta e \cdot (1-\alpha_H)$.

    * On the region $\Delta r \le \frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H}$, there is $p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r \ge \frac{p+c(\alpha_H)}{2}$, and $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)$ is maximized at $r_H = \frac{p+c(\alpha_H)}{2}$ with

      $$\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = \frac{p+c(\alpha_H)}{2}} = \frac{1}{\Delta e} \cdot \frac{\frac{(p-c(\alpha_H))^2}{4} + \frac{1-\alpha_H}{\Delta\alpha} \cdot \left[-\left(\Delta r - \frac{\Delta c}{2}\right)^2 + \frac{(\Delta c)^2}{4}\right]}{1-\alpha_H}.$$

      · If $\frac{p-c(\alpha_H)}{1-\alpha_H} \le \frac{p-c(\alpha_L)}{1-\alpha_L}$, there is $\frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H} \le \frac{\Delta c}{2}$. Thus $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = \frac{p+c(\alpha_H)}{2}}$ is maximized at $\Delta r = \frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H}$, i.e., $r_L = \frac{p+c(\alpha_H)}{2} - \frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H} = \frac{p}{2} +$

$\frac{(1-\alpha_L)c(\alpha_H)-p\cdot\Delta\alpha}{2(1-\alpha_H)}$ with

$$\begin{aligned}
&\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=\frac{p+c(\alpha_H)}{2},\ r_L=\frac{p}{2}+\frac{(1-\alpha_L)c(\alpha_H)-p\cdot\Delta\alpha}{2(1-\alpha_H)}}\\
=\ &\frac{1}{4\Delta e}\cdot\frac{p-c(\alpha_H)}{1-\alpha_H}\cdot\left[p-c(\alpha_L)-\frac{p-c(\alpha_H)}{1-\alpha_H}\cdot\Delta\alpha+\Delta c\right]\\
\leq\ &\frac{1}{4\Delta e}\cdot\frac{(p-c(\alpha_L))^2}{1-\alpha_L}=g(\alpha_L),
\end{aligned}$$

where the inequality can be shown as following: let $x=\frac{p-c(\alpha_H)}{1-\alpha_H}$ and $y=\frac{p-c(\alpha_L)}{1-\alpha_L}$, then there is $\Delta c=(1-\alpha_L)y-(1-\alpha_H)x$, and as a result, there are

$$\begin{aligned}
&\frac{p-c(\alpha_H)}{1-\alpha_H}\cdot\left[p-c(\alpha_L)-\frac{p-c(\alpha_H)}{1-\alpha_H}\cdot\Delta\alpha+\Delta c\right]\\
=\ &(1-\alpha_L)(2xy-x^2)\leq(1-\alpha_L)y^2=\frac{(p-c(\alpha_L))^2}{1-\alpha_L}.
\end{aligned}$$

Moreover, since $\alpha_H\in\mathcal{A}_{II}\cap(\alpha_L,1)$, i.e., $x\leq 2\Delta e$, there is $\frac{1}{4\Delta e}\cdot(1-\alpha_L)(2xy-x^2)=\frac{1}{4\Delta e}\cdot(1-\alpha_L)\cdot(-(x-y)^2+y^2)\leq\frac{1}{4\Delta e}\cdot(1-\alpha_L)\cdot(-(2\Delta e-y)^2+y^2)=f(\alpha_L)$, where the inequality holds when $y=\frac{p-c(\alpha_L)}{1-\alpha_L}\geq 2\Delta e$.

· If $\frac{p-c(\alpha_H)}{1-\alpha_H}\geq\frac{p-c(\alpha_L)}{1-\alpha_L}$, there is $\frac{\Delta\alpha}{2}\cdot\frac{p-c(\alpha_H)}{1-\alpha_H}\geq\frac{\Delta c}{2}$. Thus $\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=\frac{p+c(\alpha_H)}{2}}$ is maximized at $\Delta r=\frac{\Delta c}{2}$, i.e., $r_H=\frac{p+c(\alpha_H)}{2}(=p-\frac{1-\alpha_H}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}-\frac{1-\alpha_H}{2}\cdot\left(\frac{p-c(\alpha_H)}{1-\alpha_H}-\frac{p-c(\alpha_L)}{1-\alpha_L}\right)$, $r_L=\frac{p+c(\alpha_L)}{2}$ with

$$\begin{aligned}
&\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=\frac{p+c(\alpha_H)}{2},\ r_L=\frac{p+c(\alpha_L)}{2}}\\
=\ &\frac{1}{4\Delta e}\cdot\left(\frac{(p-c(\alpha_H))^2}{1-\alpha_H}+\frac{(\Delta c)^2}{\Delta\alpha}\right)\\
=\ &g(\alpha_H)+\frac{1}{4\Delta e}\cdot\frac{(\Delta c)^2}{\Delta\alpha}=g(\alpha_L)+\frac{(1-\alpha_H)(1-\alpha_L)(x-y)^2}{4\Delta e\Delta\alpha}\geq g(\alpha_L),
\end{aligned}$$

where the inequality is due to $\frac{(p-c(\alpha_H))^2}{1-\alpha_H}+\frac{(\Delta c)^2}{\Delta\alpha}=(1-\alpha_H)x^2+\frac{(\Delta c)^2}{\Delta\alpha}\geq(1-\alpha_L)y^2$, which can be verified by plugging in $\Delta c=(1-\alpha_L)y-(1-\alpha_H)x$.

∗ On the region $\Delta r\in[\frac{\Delta\alpha}{2}\cdot\frac{p-c(\alpha_H)}{1-\alpha_H},\ \Delta e\cdot\Delta\alpha]$, there is $p-\frac{1-\alpha_H}{\Delta\alpha}\Delta r\leq\frac{p+c(\alpha_H)}{2}$, and $\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)$ is maximized at $r_H=p-\frac{1-\alpha_H}{\Delta\alpha}\Delta r$ with

$$\begin{aligned}
&\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=p-\frac{1-\alpha_H}{\Delta\alpha}\Delta r}=\frac{\Delta r}{\Delta e\Delta\alpha}\cdot\left[-\Delta r\cdot\frac{1-\alpha_L}{\Delta\alpha}+p-c(\alpha_L)\right]\\
=\ &\frac{(1-\alpha_L)}{\Delta e\cdot(\Delta\alpha)^2}\left[-\left(\Delta r-\frac{\Delta\alpha}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}\right)^2+\left(\frac{\Delta\alpha}{2}\right)^2\left(\frac{p-c(\alpha_L)}{1-\alpha_L}\right)^2\right].
\end{aligned}$$

· If $\frac{p-c(\alpha_H)}{1-\alpha_H}\leq\frac{p-c(\alpha_L)}{1-\alpha_L}\leq 2\Delta e$, then $\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=p-\frac{1-\alpha_H}{\Delta\alpha}\Delta r}$ is maximized at $\Delta r=\frac{\Delta\alpha}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}$, i.e., $r_H=p-\frac{1-\alpha_H}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}=\frac{p}{2}\cdot\left(1+\frac{\Delta\alpha}{1-\alpha_L}\right)+\frac{1-\alpha_H}{1-\alpha_L}\cdot\frac{c(\alpha_L)}{2}$, $r_L=p-\frac{1-\alpha_L}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}=\frac{p+c(\alpha_L)}{2}$ with

$$\Pi(\boldsymbol{\alpha}_M,\mathbf{r}_M)\mid_{r_H=p-\frac{1-\alpha_H}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L},\ r_L=\frac{p+c(\alpha_L)}{2}}=g(\alpha_L)\geq f(\alpha_L),$$

where the inequality can be verified via simple algebra.

· If $\frac{p-c(\alpha_H)}{1-\alpha_H} \leq 2\Delta e \leq \frac{p-c(\alpha_L)}{1-\alpha_L}$, then $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r}$ is maximized at $\Delta r = \Delta e \cdot \Delta\alpha$, i.e., $r_H = p - \Delta e(1-\alpha_H), r_L = p - \Delta e(1-\alpha_L)$ with

$$\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e(1-\alpha_H),\ r_L = p - \Delta e(1-\alpha_L)} = f(\alpha_L).$$

· If $\frac{p-c(\alpha_L)}{1-\alpha_L} \leq \frac{p-c(\alpha_H)}{1-\alpha_H} \leq 2\Delta e$, then $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r}$ is maximized at $\Delta r = \frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H}$, i.e., $r_H = \frac{p+c(\alpha_H)}{2}, r_L = \frac{p}{2} - \frac{p\cdot\Delta\alpha - c(\alpha_H)(1-\alpha_L)}{2(1-\alpha_H)}$ with

$$\begin{aligned} & \Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = \frac{p+c(\alpha_H)}{2},\ r_L = \frac{p}{2} - \frac{p\cdot\Delta\alpha - c(\alpha_H)(1-\alpha_L)}{2(1-\alpha_H)}} \\ = & \frac{1-\alpha_L}{4\Delta e} \cdot \frac{p-c(\alpha_H)}{1-\alpha_H} \cdot \left[\frac{2(p-c(\alpha_L))}{1-\alpha_L} - \frac{p-c(\alpha_H)}{1-\alpha_H}\right] \\ \leq & g(\alpha_H) + \frac{1}{4\Delta e} \cdot \frac{(\Delta c)^2}{\Delta\alpha}, \end{aligned}$$

where the inequality can be verified with simple algebra after plugging in $x = \frac{p-c(\alpha_H)}{1-\alpha_H}$, $y = \frac{p-c(\alpha_L)}{1-\alpha_L}$, and $\Delta c = (1-\alpha_L)y - (1-\alpha_H)x$.

- $\alpha_H \in \mathcal{A}_{III} \cap (\alpha_L, 1)$, where $\mathcal{A}_{III} = \{\alpha \in [0,1] \mid p - c(\alpha) < 0\}$. We have $\frac{p+c(\alpha_H)}{2} > p \geq p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r$, and $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M)$ is maximized at $r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r$ with

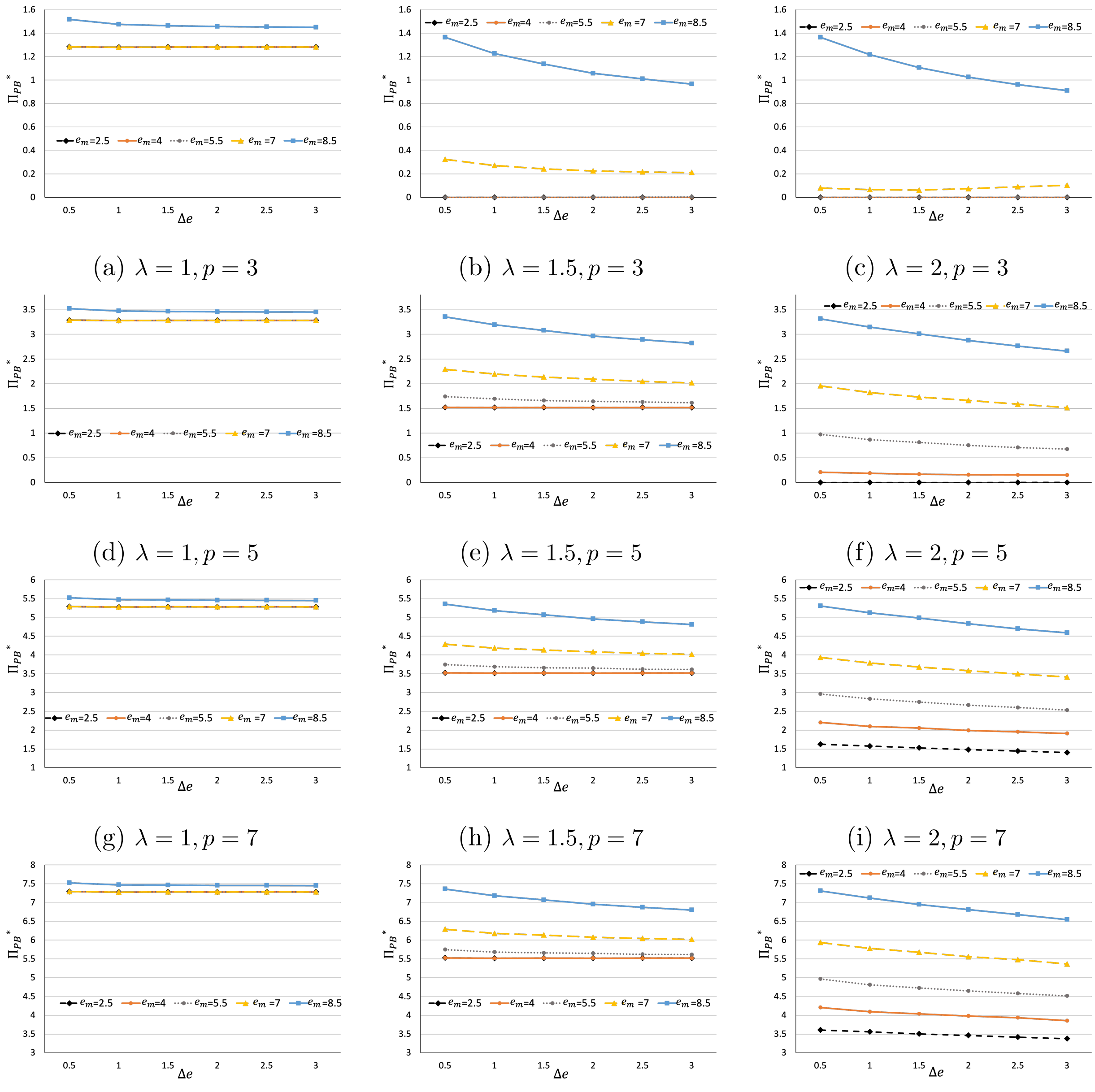
$$\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r} = \frac{1-\alpha_L}{\Delta e \cdot (\Delta\alpha)^2}\left[-\left(\Delta r - \frac{\Delta\alpha}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L}\right)^2 + \left(\frac{\Delta\alpha}{2}\right)^2\left(\frac{p-c(\alpha_L)}{1-\alpha_L}\right)^2\right].$$

  – If $\frac{p-c(\alpha_L)}{1-\alpha_L} \leq 2\Delta e$, and thus $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r}$ is maximized at $\Delta r = \frac{\Delta\alpha}{2} \cdot \frac{p-c(\alpha_L)}{1-\alpha_L}$, i.e., $r_H = p - \frac{1-\alpha_H}{2} \cdot \frac{p-c(\alpha_L)}{1-\alpha_L} = \frac{p}{2} \cdot \left(1 + \frac{\Delta\alpha}{1-\alpha_L}\right) + \frac{1-\alpha_H}{1-\alpha_L} \cdot \frac{c(\alpha_L)}{2}, r_L = p - \frac{1-\alpha_L}{2} \cdot \frac{p-c(\alpha_L)}{1-\alpha_L} = \frac{p+c(\alpha_L)}{2}$ with $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{2}\cdot\frac{p-c(\alpha_L)}{1-\alpha_L},\ r_L = \frac{p+c(\alpha_L)}{2}} = g(\alpha_L) \geq f(\alpha_L)$, where the inequality can be verified via simple algebra.

  – If $\frac{p-c(\alpha_L)}{1-\alpha_L} \geq 2\Delta e$, and thus $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \frac{1-\alpha_H}{\Delta\alpha}\Delta r}$ is maximized at $\Delta r = \Delta e \cdot \Delta\alpha$, i.e., $r_H = p - \Delta e(1-\alpha_H), r_L = p - \Delta e(1-\alpha_L)$ with $\Pi(\boldsymbol{\alpha}_M, \mathbf{r}_M) \mid_{r_H = p - \Delta e(1-\alpha_H),\ r_L = p - \Delta e(1-\alpha_L)} = f(\alpha_L)$.

Consolidating the above cases, the optimal package and profit can be obtained as in Theorem 7. □

Proof. (Corollary 4) By Theorem 5, conditions in Corollary 4 (i) and (ii) both imply that the optimal package can only be two-item. For (i), the optimal package is $(\boldsymbol{\alpha}_M^I,\ \mathbf{r}_M^I)$, and the rental price given by (62) can be further simplified as $\mathbf{r}_M^I = (p - \Delta e \cdot (1-\alpha_H^I) - \frac{\Delta c^I}{2},\ p - \Delta e \cdot (1-\alpha_H^I))$. For (ii), the optimal package is $(\boldsymbol{\alpha}_M^{II},\ \mathbf{r}_M^{II})$ and (64) can be further simplified as $\mathbf{r}_M^{II} = (\frac{p+c(\alpha_L^{II})}{2},\ \frac{p+c(\alpha_H^{II})}{2})$. The prosumer utility is obtained by plugging the optimal package into (1). □



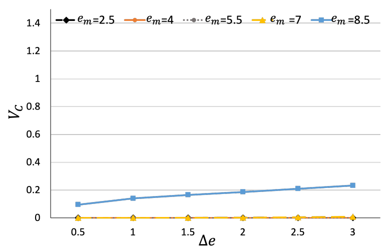
(a) $\lambda = 1, p = 3$ (b) $\lambda = 1.5, p = 3$ (c) $\lambda = 2, p = 3$

(d) $\lambda = 1, p = 5$ (e) $\lambda = 1.5, p = 5$ (f) $\lambda = 2, p = 5$

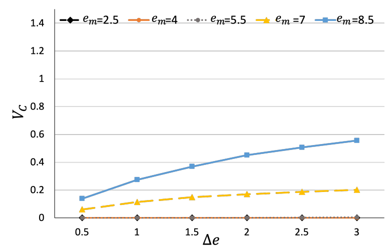
(g) $\lambda = 1, p = 7$ (h) $\lambda = 1.5, p = 7$ (i) $\lambda = 2, p = 7$

(j) $\lambda = 1, p = 9$ (k) $\lambda = 1.5, p = 9$ (l) $\lambda = 2, p = 9$

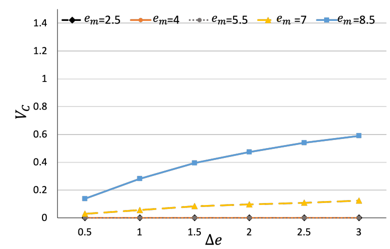
Figure 8: Impact of Δe and e_m on OEM Profit under Optimal Population-based Package (Truncated Normal)



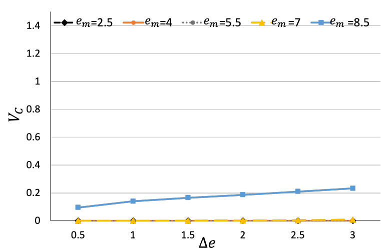
(a) $\lambda = 1, p = 3$



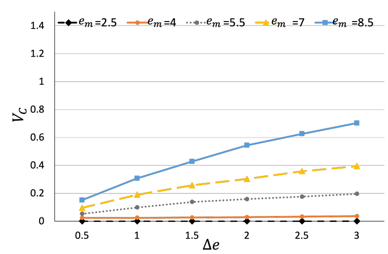
(b) $\lambda = 1.5, p = 3$



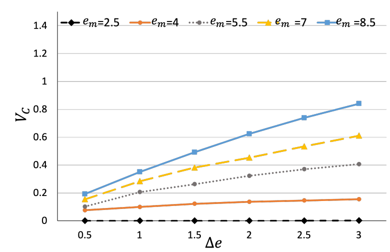
(c) $\lambda = 2, p = 3$



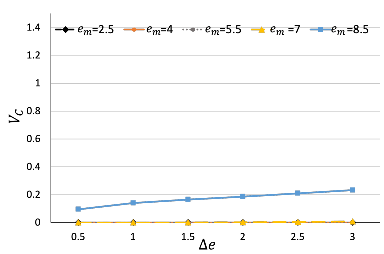
(d) $\lambda = 1, p = 5$



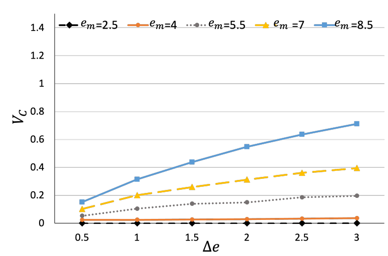
(e) $\lambda = 1.5, p = 5$



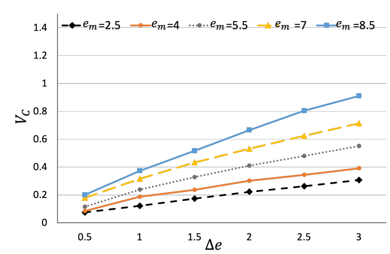
(f) $\lambda = 2, p = 5$



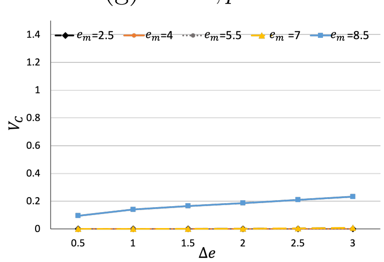
(g) $\lambda = 1, p = 7$



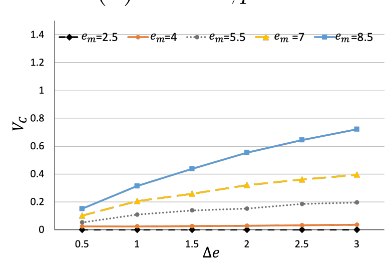
(h) $\lambda = 1.5, p = 7$



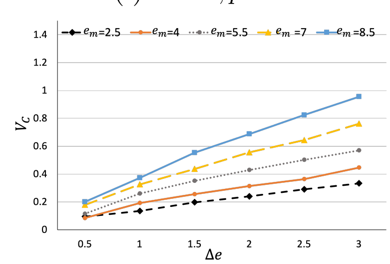
(i) $\lambda = 2, p = 7$



(j) $\lambda = 1, p = 9$



(k) $\lambda = 1.5, p = 9$



(l) $\lambda = 2, p = 9$

Figure 9: Impact of Δe and e_m on the Value of Customization (Truncated Normal)