Online Appendix for "Shared Decision-Making under Bounded Rationality: Why Personalization Doesn't Always Help"

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The appendices are organized as follows. In Appendix A, we derive the exact expressions for the expected utilities under SM, PM, PC, and PH. In Appendix B, we present proofs for the results in the present paper. In Appendix C, we provide supplementary results for Section 5.

Throughout the proofs, $\phi(z)$ and $\Phi(z)$ denote the probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution at point z, respectively. $f(x;\mu,\sigma)$ represents normal pdf with mean μ and standard deviation σ at point x. Let $\Delta\mu_X$, $\Delta\mu_Y$, and $\Delta\mu$ denote:

$$\Delta\mu_X = \mu_{XA} - \mu_{XB}, \quad \Delta\mu_Y = \mu_{YA} - \mu_{YB}, \text{ and } \Delta\mu = \bar{w}\Delta\mu_X + (1-\bar{w})\Delta\mu_Y, \tag{A.1}$$

where $\bar{w}$ is given with (2). Recall that we assume $\Delta\mu_X > 0$, $\Delta\mu_Y < 0$, and $\Delta\mu < 0$. Furthermore, let us introduce $\bar{\sigma}_i$ and $\bar{\sigma}$ as:

$$\bar{\sigma}_i = \sqrt{w_i^2 r_X + (1-w_i)^2 r_Y} \text{ for } i = 1,2, \text{ and } \bar{\sigma} = \sqrt{\bar{w}^2 r_X + (1-\bar{w})^2 r_Y}, \tag{A.2}$$

where r_X and r_Y are defined as:

$$r_X := \frac{\sigma_X^4}{\sigma_X^2 + \sigma_{d,X}^2}, \quad r_Y := \frac{\sigma_Y^4}{\sigma_Y^2 + \sigma_{d,Y}^2}. \tag{A.3}$$

We finally turn to patients' treatment choice model under PC or PH. Under PC, the doctor shares population-average health outcomes ($\hat{X}_t = \mu_{Xt}$ and $\hat{Y}_t = \mu_{Yt}$), while under PH, the doctor shares individualized outcome predictions in (15). The patient's perceived utility $\hat{U}_{it}(.)$ is defined for PC and PH in (17). Here, γ_{it} represents type-i patient's random error, assumed to be identically normally distributed with mean zero and variance $\sigma_{p,i}^2/2$, where $\sigma_{p,1}$ and $\sigma_{p,2}$ capture levels of patient irrationality. We assume that γ_{it}'s are independent across treatments and patients, and from X_t's, Y_t's, $\mathcal{E}_t^X$'s, and $\mathcal{E}_t^Y$'s. The probability that a type-i patient selects treatment A is given by:

$$P_i(w_i, \hat{x}_t, \hat{y}_t) = \mathbb{P}_\gamma\left(\hat{U}_A(w_i, \hat{x}_A, \hat{y}_A) \geq \hat{U}_B(w_i, \hat{x}_B, \hat{y}_B)\right) = \Phi\left(\frac{w_i(\hat{x}_A - \hat{x}_B) + (1-w_i)(\hat{y}_A - \hat{y}_B)}{\sigma_{p,i}}\right), \tag{A.4}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

Throughout the appendix, EU^{SM}, EU^{PC}, EU^{PM}, and EU^{PH} represent the expected utility of SM, PC, PM, and PH, respectively. Finally, $\mathcal{I}\{A\}$ denotes the indicator random variable associated with event A that has value 1 if event A occurs and has value 0 otherwise.

Appendix A: Derivation of the Expected Utilities under SM, PM, PC, and PH

Lemma A1 *The expected utilities of SM, PC, PM, and PH, denoted by EU^{SM}, EU^{PC}, EU^{PM}, and EU^{PH}, respectively, are equal to:*

$$EU^{SM} = \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{A.5}$$

$$EU^{PM} = \Delta\mu\Phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right) + \bar{\sigma}\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right) + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{A.6}$$

$$EU^{PC} = pEU_1^{PC} + (1-p)EU_2^{PC} + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \tag{A.7}$$

$$EU^{PH} = pEU_1^{PH} + (1-p)EU_2^{PH} + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}. \tag{A.8}$$

In (A.7) *and* (A.8)*, EU_i^{PC} and EU_i^{PH} are equal to:*

$$EU_i^{PC} = \Delta\mu_i\Phi\left(\frac{\Delta\mu_i}{\sigma_{p,i}}\right), \tag{A.9}$$

$$EU_i^{PH} = \Delta\mu_i\Phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right) + \frac{\alpha_d\bar{\sigma}_i^2}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right), \tag{A.10}$$

where $\Delta\mu_1$, $\Delta\mu_2$, $\Delta\mu$, $\bar{\sigma}_i$ and $\bar{\sigma}$ are given with (A.1) *and* (A.2)*, respectively, and $\Phi(.)$ and $\phi(.)$ are cdf and pdf of the standard normal distribution, respectively.*

Proof: *Part I:* Since under SM, doctors always choose treatment B, the expected utility of SM is equal to:

$$\begin{aligned} EU^{SM} &= p\mathbb{E}_{X_B,Y_B}\left[w_1X_B + (1-w_1)Y_B\right] + (1-p)\mathbb{E}_{X_B,Y_B}\left[w_2X_B + (1-w_2)Y_B\right] \\ &= \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, \end{aligned} \tag{A.11}$$

where the subscripts in the expectations indicate what variable the expectation is taken over. The first term in the first equality corresponds to the event that the doctor sees a type-1 patient, whereas the second term corresponds to the event that the doctor sees a type-2 patient. Furthermore, recalling that μ_{XB} and μ_{YB} denote the expectations of X_B and Y_B, respectively, the second equality follows from (2).

Part II: The expected utility of PM is equal to:

$$EU^{PM} = p\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_1X_A + (1-w_1)Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) \geq 0\} \tag{A.12}$$

$$+ (w_1X_B + (1-w_1)Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) < 0\}\Big] \tag{A.13}$$

$$+ (1-p)\mathbb{E}_{X_t,Y_t,S_t^X,S_t^Y}\Big[(w_2X_A + (1-w_2)Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) \geq 0\} \tag{A.14}$$

$$+ (w_2X_B + (1-w_2)Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) < 0\}\Big] \tag{A.15}$$

$$\begin{aligned} &= \mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(\bar{w}X_A + (1-\bar{w})Y_A)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) \geq 0\} \\ &\quad + (\bar{w}X_B + (1-\bar{w})Y_B)\mathcal{I}\{\bar{w}(\hat{X}_A - \hat{X}_B) + (1-\bar{w})(\hat{Y}_A - \hat{Y}_B) < 0\}\Big], \end{aligned} \tag{A.16}$$

where the term in (A.12) corresponds to the event that the doctor sees a type-1 patient and treatment A is selected, the term in (A.13) corresponds to the event that the doctor sees a type-1 patient and treatment B is selected, whereas the term in (A.14) corresponds to the event that the doctor sees a type-2 patient and treatment A is selected, and the term in (A.15) corresponds to the event that the doctor sees a type-2 patient and treatment B is selected. Furthermore, the second equality follows from the linearity of expectation and from (2). For algebraic convenience, we will first derive the expression for the expected utility difference between PM and SM by combining (A.5) and (A.16), and then find the exact expression for EU^{PM}:

$$\begin{aligned}
EU^{PM}-EU^{SM} &= \mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\left[(\bar{w}(X_A-X_B)+(1-\bar{w})(Y_A-Y_B))\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y,X_t}\left[(X_A-X_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&\quad+(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y,Y_t}\left[(Y_A-Y_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{X_t}\left[(X_A-X_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\middle|S_t^X,S_t^Y\right]\right]\\
&\quad+(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{Y_t}\left[(Y_A-Y_B)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\middle|S_t^X,S_t^Y\right]\right]\\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{X_t}\left[(X_A-X_B)\mid S_t^X,S_t^Y\right]\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&\quad+(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\mathbb{E}_{Y_t}\left[(Y_A-Y_B)\mid S_t^X,S_t^Y\right]\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&= \bar{w}\mathbb{E}_{S_t^X,S_t^Y}\left[\left(\mathbb{E}[X_A\mid S_A^X]-\mathbb{E}[X_B\mid S_B^X]\right)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right]\\
&\quad+(1-\bar{w})\mathbb{E}_{S_t^X,S_t^Y}\left[\left(\mathbb{E}[Y_A\mid S_A^Y]-\mathbb{E}[Y_B\mid S_B^Y]\right)\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}\right],
\end{aligned}\tag{A.17}$$

where the second equality follows from linearity of the expectation, the third equality follows from the law of iterated expectations, the fourth equality follows from the fact that when S_t^X and S_t^Y are given, $\mathcal{I}\left\{\bar{w}(\hat{X}_A-\hat{X}_B)+(1-\bar{w})(\hat{Y}_A-\hat{Y}_B)\geq 0\right\}$ is not random anymore, and the fifth equality follows from the independence of S_t^X and S_t^Y. In (A.17), using (14), (15) and (16), we can write $\hat{X}_A-\hat{X}_B$ and $\hat{Y}_A-\hat{Y}_B$ as:

$$\begin{aligned}
\hat{X}_A-\hat{X}_B &= \alpha_d\frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}(S_A^X-S_B^X)+\left(1-\alpha_d\frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}\right)\Delta\mu_X=\Delta\mu_X+\alpha_d\sqrt{r_X}Z_X,\\
\hat{Y}_A-\hat{Y}_B &= \alpha_d\frac{\sigma_Y^2}{\sigma_Y^2+\sigma_{d,Y}^2}(S_A^Y-S_B^Y)+\left(1-\alpha_d\frac{\sigma_Y^2}{\sigma_Y^2+\sigma_{d,Y}^2}\right)\Delta\mu_Y=\Delta\mu_Y+\alpha_d\sqrt{r_Y}Z_Y,
\end{aligned}\tag{A.18}$$

where r_X and r_Y are given with (A.3), and $\Delta\mu_X$ and $\Delta\mu_Y$ are defined in (A.1), and Z_X and Z_Y are standard normal random variables. Similarly, $\mathbb{E}[X_A\mid S_A^X]-\mathbb{E}[X_B\mid S_B^X]$ and $\mathbb{E}[Y_A\mid S_A^Y]-\mathbb{E}[Y_B\mid S_B^Y]$ are equal to:

$$\begin{aligned}
\mathbb{E}[X_A\mid S_A^X]-\mathbb{E}[X_B\mid S_B^X] &= \frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}(S_A^X-S_B^X)+\left(1-\frac{\sigma_X^2}{\sigma_X^2+\sigma_{d,X}^2}\right)\Delta\mu_X=\Delta\mu_X+\sqrt{r_X}Z_X,\\
\mathbb{E}[Y_A\mid S_A^Y]-\mathbb{E}[Y_B\mid S_B^Y] &= \frac{\sigma_Y^2}{\sigma_Y^2+\sigma_{d,Y}^2}(S_A^Y-S_B^Y)+\left(1-\frac{\sigma_Y^2}{\sigma_Y^2+\sigma_{d,Y}^2}\right)\Delta\mu_Y=\Delta\mu_Y+\sqrt{r_Y}Z_Y.
\end{aligned}\tag{A.19}$$

Thus, $EU^{PM}-EU^{SM}$ in (A.17) could be rewritten as:

$$\begin{aligned}
&EU^{PM}-EU^{SM}\\
&=\bar{w}\mathbb{E}_{Z_X,Z_Y}\left[(\Delta\mu_X+\sqrt{r_X}Z_X)\mathcal{I}\left\{\bar{w}(\Delta\mu_X+\alpha_d\sqrt{r_X}Z_X)+(1-\bar{w})(\Delta\mu_Y+\alpha_d\sqrt{r_Y}Z_Y)\geq 0\right\}\right]\\
&+(1-\bar{w})\mathbb{E}_{Z_X,Z_Y}\left[(\Delta\mu_Y+\sqrt{r_Y}Z_Y)\mathcal{I}\left\{\bar{w}(\Delta\mu_X+\alpha_d\sqrt{r_X}Z_X)+(1-\bar{w})(\Delta\mu_Y+\alpha_d\sqrt{r_Y}Z_Y)\geq 0\right\}\right]\\
&=\mathbb{E}_{Z_X,Z_Y}\left[(\Delta\mu+\bar{w}\sqrt{r_X}Z_X+(1-\bar{w})\sqrt{r_Y}Z_Y)\mathcal{I}\left\{\Delta\mu+\alpha_d(\bar{w}\sqrt{r_X}Z_X+(1-\bar{w})\sqrt{r_Y}Z_Y)\geq 0\right\}\right]\\
&=\mathbb{E}_Z\left[(\Delta\mu+\bar{\sigma}Z)\mathcal{I}\left\{\Delta\mu+\alpha_d\bar{\sigma}Z\geq 0\right\}\right]\\
&=\int_{-\frac{\Delta\mu}{\alpha_d\bar{\sigma}}}^{\infty}(\Delta\mu+\bar{\sigma}z)\phi(z)dz
\end{aligned}$$

$$= \Delta\mu\Phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right) + \bar{\sigma}\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right), \tag{A.20}$$

where $\Delta\mu$ and $\bar{\sigma}$ are given with (A.1) and (A.2), respectively, Z is a standard normal random variable, and $\phi(z)$ is the pdf of the standard normal distribution at point z. The first equality follows from substituting the expressions in (A.18) and (A.19) for $\hat{X}_A - \hat{X}_B$, $\hat{Y}_A - \hat{Y}_B$, $\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X]$, and $\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y]$ in (A.17), the second equality follows from linearity of the expectation and (A.1), the third equality follows from plugging $\bar{\sigma}Z$ for $\bar{w}\sqrt{r_X}Z_X + (1-\bar{w})\sqrt{r_Y}Z_Y$, which are equal in distribution, the fourth equality follows from the definition of the expectation, and the fifth equality follows from algebra. Finally, combining (A.5) and (A.20), we conclude that the expected utility of PM is given with (A.6).

Part III: The expected utility of PC is equal to:

$$\begin{aligned}
EU^{PC} &= p\mathbb{E}_{X_t,Y_t}\Big[(w_1X_A + (1-w_1)Y_A)\,P_1(w_1,\mu_{Xt},\mu_{Yt}) + (w_1X_B + (1-w_1)Y_B)\,(1 - P_1(w_1,\mu_{Xt},\mu_{Yt}))\Big] \\
&\quad + (1-p)\mathbb{E}_{X_t,Y_t}\Big[(w_2X_A + (1-w_2)Y_A)\,P_2(w_2,\mu_{Xt},\mu_{Yt}) + (w_2X_B + (1-w_2)Y_B)\,(1 - P_2(w_2,\mu_{Xt},\mu_{Yt}))\Big] \\
&= p\Big((w_1\mu_{XA} + (1-w_1)\mu_{YA})\,P_1(w_1,\mu_{Xt},\mu_{Yt}) + (w_1\mu_{XB} + (1-w_1)\mu_{YB})\,(1 - P_1(w_1,\mu_{Xt},\mu_{Yt}))\Big) \\
&\qquad + (1-p)\mathbb{E}_{X_t,Y_t}\Big[(w_2\mu_{XA} + (1-w_2)\mu_{YA})\,P_2(w_2,\mu_{Xt},\mu_{Yt}) \\
&\qquad\qquad + (w_2\mu_{XB} + (1-w_2)\mu_{YB})\,(1 - P_2(w_2,\mu_{Xt},\mu_{Yt}))\Big] \\
&= p\Big(w_1(\mu_{XA} - \mu_{XB}) + (1-w_1)(\mu_{YA} - \mu_{YB})\Big)P_1(w_1,\mu_{Xt},\mu_{Yt}) \\
&\qquad + (1-p)\Big(w_2(\mu_{XA} - \mu_{XB}) + (1-w_2)(\mu_{YA} - \mu_{YB})\Big)P_2(w_2,\mu_{Xt},\mu_{Yt}) \\
&\qquad + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB} && \text{(A.21)} \\
&= p\Delta\mu_1\Phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) + (1-p)\Delta\mu_2\Phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}, && \text{(A.22)}
\end{aligned}$$

where $P_i(w_i,\hat{x}_t,\hat{y}_t)$ is given with (A.4). The second equality follows from (A.1), the third equality follows from algebraic manipulations, and the last equality follows from (3), and (4), and substituting the expression in (A.4) for $P_i(w_i,\hat{x}_t,\hat{y}_t)$.

Part IV: The expected utility of PH is equal to:

$$\begin{aligned}
EU^{PH} &= p\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_1X_A + (1-w_1)Y_A)\,P_1(w_1,\hat{X}_t,\hat{Y}_t) \\
&\qquad + (w_1X_B + (1-w_1)Y_B)\,\Big(1 - P_1(w_1,\hat{X}_t,\hat{Y}_t)\Big)\Big] \\
&\quad + (1-p)\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_2X_A + (1-w_2)Y_A)\,P_2(w_2,\hat{X}_t,\hat{Y}_t) \\
&\qquad + (w_2X_B + (1-w_2)Y_B)\,\Big(1 - P_2(w_2,\hat{X}_t,\hat{Y}_t)\Big)\Big] \\
&= p\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_1(X_A - X_B) + (1-w_1)(Y_A - Y_B))\,P_1(w_1,\hat{X}_t,\hat{Y}_t) \\
&\quad + (1-p)\mathbb{E}_{S_t^X,S_t^Y,X_t,Y_t}\Big[(w_2(X_A - X_B) + (1-w_2)(Y_A - Y_B))\,P_2(w_2,\hat{X}_t,\hat{Y}_t)\Big] \\
&\quad + \bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}. && \text{(A.23)}
\end{aligned}$$

For algebraic convenience, we will first derive the expression for the expected utility difference between PH and SM by combining (A.5) and (A.23), and then find the exact expression for EU^{PH}:

$$EU^{PH} - EU^{SM}$$

$$
\begin{aligned}
&= p\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[\left(w_1(X_A - X_B) + (1-w_1)(Y_A - Y_B)\right) P_1(w_1, \hat{X}_t, \hat{Y}_t)\right] \\
&\quad + (1-p)\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[\left(w_2(X_A - X_B) + (1-w_2)(Y_A - Y_B)\right) P_2(w_2, \hat{X}_t, \hat{Y}_t)\right] \\
&= p\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[\left(w_1(X_A - X_B) + (1-w_1)(Y_A - Y_B)\right) \Phi\left(\frac{w_1(\hat{X}_A - \hat{X}_B) + (1-w_1)(\hat{Y}_A - \hat{Y}_B)}{\sigma_{p,1}}\right)\right] \\
&+ (1-p)\mathbb{E}_{S_t^X, S_t^Y, X_t, Y_t}\left[\left(w_2(X_A - X_B) + (1-w_2)(Y_A - Y_B)\right) \Phi\left(\frac{w_2(\hat{X}_A - \hat{X}_B) + (1-w_2)(\hat{Y}_A - \hat{Y}_B)}{\sigma_{p,2}}\right)\right],
\end{aligned} \tag{A.24}
$$

where the second equality follows from plugging the expression in (A.4) for $P_i(w_i, \hat{X}_t, \hat{Y}_t)$. Following the same steps as in (A.17) and (A.20) (i.e., applying the law of iterated expectations, plugging the expressions in (A.18) and (A.19) for $\hat{X}_A - \hat{X}_B$, $\hat{Y}_A - \hat{Y}_B$, $\mathbb{E}[X_A \mid S_A^X] - \mathbb{E}[X_B \mid S_B^X]$, and $\mathbb{E}[Y_A \mid S_A^Y] - \mathbb{E}[Y_B \mid S_B^Y]$), we can write $EU^{PH} - EU^{SM}$ as:

$$
EU^{PH} - EU^{SM} = p\mathbb{E}_Z\left[(\Delta\mu_1 + \bar{\sigma}_1 Z)\,\Phi\left(\frac{\Delta\mu_1 + \alpha_d\bar{\sigma}_1 Z}{\sigma_{p,1}}\right)\right] + (1-p)\mathbb{E}_Z\left[(\Delta\mu_2 + \bar{\sigma}_2 Z)\,\Phi\left(\frac{\Delta\mu_2 + \alpha_d\bar{\sigma}_2 Z}{\sigma_{p,2}}\right)\right]
$$

Rearranging above expression, we have:

$$
\begin{aligned}
&EU^{PH} - EU^{SM} \\
&= p\int_{-\infty}^{\infty}(\Delta\mu_1 + \bar{\sigma}_1 z)\,\Phi\left(\frac{\Delta\mu_1 + \alpha_d\bar{\sigma}_1 z}{\sigma_{p,1}}\right)\phi(z)dz + (1-p)\int_{-\infty}^{\infty}(\Delta\mu_2 + \bar{\sigma}_2 z)\,\Phi\left(\frac{\Delta\mu_2 + \alpha_d\bar{\sigma}_2 z}{\sigma_{p,2}}\right)\phi(z)dz \\
&= p\left(\Delta\mu_1\Phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right) + \frac{\alpha_d\bar{\sigma}_1^2}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right)\right) \\
&\qquad + (1-p)\left(\Delta\mu_2\Phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right) + \frac{\alpha_d\bar{\sigma}_2^2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right)\right),
\end{aligned} \tag{A.25}
$$

where the first equality follows from the definition of expectation and the last equality follows from algebra. Finally, combining (A.5) and (A.25), we conclude that the expected utility of PH is given with (A.8). ■

Appendix B: Proofs of Results

B.1. Proof of Section 4 Results

Proof of Proposition 1: First recall (A.5) and note that EU^{SM} is equal to $\bar{w}\mu_{XB} + (1-\bar{w})\mu_{YB}$.

Recalling (A.17), for $\sigma_{p,1} = \sigma_{p,2} = \sigma_{d,X} = \sigma_{d,Y} = 0$, $EU^{PM} - EU^{SM}$ is equal to:

$$
\begin{aligned}
EU^{PM} - EU^{SM} &= \bar{w}\mathbb{E}_{X_A, X_B, Y_A, Y_B}\left[(X_A - X_B)\mathcal{I}\left\{\bar{w}(X_A - X_B) + (1-\bar{w})(Y_A - Y_B) \geq 0\right\}\right] \\
&\quad + (1-\bar{w})\mathbb{E}_{X_A, X_B, Y_A, Y_B}\left[(Y_A - Y_B)\mathcal{I}\left\{\bar{w}(X_A - X_B) + (1-\bar{w})(Y_A - Y_B) \geq 0\right\}\right] \\
&= \mathbb{E}_{X_A, X_B, Y_A, Y_B}\left[(\bar{w}(X_A - X_B) + (1-\bar{w})(Y_A - Y_B))\mathcal{I}\left\{\bar{w}(X_A - X_B) + (1-\bar{w})(Y_A - Y_B) \geq 0\right\}\right] \\
&= \mathbb{E}_{X_A, X_B, Y_A, Y_B}\left[(\bar{w}(X_A - X_B) + (1-\bar{w})(Y_A - Y_B))^+\right].
\end{aligned} \tag{A.26}
$$

Recalling (A.21), for $\sigma_{p,1} = \sigma_{p,2} = \sigma_{d,X} = \sigma_{d,Y} = 0$, $EU^{PC} - EU^{SM}$ is equal to:

$$
\begin{aligned}
EU^{PC} - EU^{SM} &= p\Big(w_1(\mu_{XA} - \mu_{XB}) + (1-w_1)(\mu_{YA} - \mu_{YB})\Big)\mathcal{I}\{w_1(\mu_{XA} - \mu_{XB}) + (1-w_1)(\mu_{YA} - \mu_{YB}) \geq 0\} \\
&\quad + (1-p)\Big(w_2(\mu_{XA} - \mu_{XB}) + (1-w_2)(\mu_{YA} - \mu_{YB})\Big)\mathcal{I}\{w_2(\mu_{XA} - \mu_{XB}) + (1-w_2)(\mu_{YA} - \mu_{YB}) \geq 0\} \\
&= \mathbb{E}_W\left[(W(\mu_{XA} - \mu_{XB}) + (1-W)(\mu_{YA} - \mu_{YB}))\mathcal{I}\{W(\mu_{XA} - \mu_{XB}) + (1-W)(\mu_{YA} - \mu_{YB}) \geq 0\}\right] \\
&= \mathbb{E}_W\left[(W(\mu_{XA} - \mu_{XB}) + (1-W)(\mu_{YA} - \mu_{YB}))^+\right],
\end{aligned} \tag{A.27}
$$

where W is a random variable that equals w_1 with probability p and w_2 with probability $1-p$.

Finally, recalling (A.24), for $\sigma_{p,1}=\sigma_{p,2}=\sigma_{d,X}=\sigma_{d,Y}=0$, $EU^{PH}-EU^{SM}$ is equal to:

$$\begin{aligned}EU^{PH}-EU^{SM} &= p\mathbb{E}_{X_A,X_B,Y_A,Y_B}\left[(w_1(X_A-X_B)+(1-w_1)(Y_A-Y_B))\mathcal{I}\{w_1(X_A-X_B)+(1-w_1)(Y_A-Y_B)\geq 0\}\right]\\ &\quad+(1-p)\mathbb{E}_{X_A,X_B,Y_A,Y_B}\left[(w_2(X_A-X_B)+(1-w_2)(Y_A-Y_B))\mathcal{I}\{w_2(X_A-X_B)+(1-w_2)(Y_A-Y_B)\geq 0\}\right],\\ &=\mathbb{E}_{X_A,X_B,Y_A,Y_B,W}\left[(W(X_A-X_B)+(1-W)(Y_A-Y_B))\mathcal{I}\{W(X_A-X_B)+(1-W)(Y_A-Y_B)\geq 0\}\right]\\ &=\mathbb{E}_{X_A,X_B,Y_A,Y_B,W}\left[(W(X_A-X_B)+(1-W)(Y_A-Y_B))^+\right] \qquad \text{(A.28)}\end{aligned}$$

where W is a random variable that equals w_1 with probability p and w_2 with probability $1-p$.

It trivially follows from (A.26) and (A.27) that $EU^{PM}-EU^{SM}\geq 0$ and $EU^{PC}-EU^{SM}\geq 0$ hold. Comparing (A.26) with (A.28), we obtain $EU^{PM}-EU^{SM}\leq EU^{PH}-EU^{SM}\Leftrightarrow EU^{PM}\leq EU^{PH}$ by Jensen's inequality. Similarly, comparing (A.27) with (A.28), we obtain $EU^{PC}-EU^{SM}\leq EU^{PH}-EU^{SM}\Leftrightarrow EU^{PC}\leq EU^{PH}$ by Jensen's inequality. ■

B.2. Proof of Section 5.1 Results

B.2.1. Proof of Section 5.1.1 Results

Proof of Lemma 1: The derivative of EU^{PM} in (A.6) with respect to α_d is:

$$\frac{\partial EU^{PM}}{\partial \alpha_d}=\frac{\Delta\mu^2}{\alpha_d^2\bar{\sigma}}\left(\frac{1}{\alpha_d}-1\right)\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right).$$

It is obvious that $\frac{\partial EU^{PM}}{\partial \alpha_d}>0$ for all $\alpha_d<1$, and $\frac{\partial EU^{PM}}{\partial \alpha_d}<0$ for all $\alpha_d>1$. Hence, EU^{PM} is a unimodal function, which reaches its maximum at $\alpha_d=1$. ■

Proof of Proposition 2: The derivatives of EU^{PM} in (A.6) with respect to r_X and r_Y are equal to:

$$\frac{\partial EU^{PM}}{\partial r_X}=\frac{\partial EU^{PM}}{\partial \bar{\sigma}}\frac{\partial \bar{\sigma}}{\partial r_X}=\frac{\bar{w}^2}{2\bar{\sigma}}\frac{\partial EU^{PM}}{\partial \bar{\sigma}} \text{ and } \frac{\partial EU^{PM}}{\partial r_Y}=\frac{\partial EU^{PM}}{\partial \bar{\sigma}}\frac{\partial \bar{\sigma}}{\partial r_Y}=\frac{(1-\bar{w})^2}{2\bar{\sigma}}\frac{\partial EU^{PM}}{\partial \bar{\sigma}}, \qquad \text{(A.29)}$$

where

$$\frac{\partial EU^{PM}}{\partial \bar{\sigma}}=\left(\frac{\Delta\mu^2}{\alpha_d\bar{\sigma}^2}\left(\frac{1}{\alpha_d}-1\right)+1\right)\phi\left(\frac{\Delta\mu}{\alpha_d\bar{\sigma}}\right). \qquad \text{(A.30)}$$

Let $\alpha_d>1$. The proof consists of two parts: In part I, we will investigate the monotonicity/unimodality of EU^{PM} with respect to r_X and r_Y, whereas in part II, we will analyze when PM outperforms SM.

Part I: Observe from (A.30) that when $\alpha_d>1$, $\frac{\partial EU^{PM}}{\partial\bar{\sigma}}<0$ for all $\bar{\sigma}\in\left[0,\sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)}\right)$, and $\frac{\partial EU^{PM}}{\partial\bar{\sigma}}>0$ for all $\bar{\sigma}>\sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)}$. Hence, EU^{PM} is a U-shaped function of $\bar{\sigma}$, which reaches its minimum at $\bar{\sigma}=\sqrt{\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)}$. Hence, recalling the definition of $\bar{\sigma}$ in (A.2), it follows that for a given r_X, we have:

$$\begin{cases} EU^{PM} \text{ increases in } r_Y \text{ for all } r_Y\geq 0, & \text{if } r_X>\frac{1}{\bar{w}^2}\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right),\\ EU^{PM} \text{ decreases in } r_Y \text{ for all } r_Y\in\left[0,\frac{1}{(1-\bar{w})^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)-\bar{w}^2r_X\right)\right), & \\ \text{and } EU^{PM} \text{ increases in } r_Y \text{ for all } r_Y\in\left(\frac{1}{(1-\bar{w})^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)-\bar{w}^2r_X\right),\infty\right), & \text{if } r_X<\frac{1}{\bar{w}^2}\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right).\end{cases}$$

Similarly, for a given r_Y, we have:

$$\begin{cases} EU^{PM} \text{ increases in } r_X \text{ for all } r_X \geq 0, & \text{if } r_Y > \frac{1}{(1-\bar{w})^2}\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right), \\ EU^{PM} \text{ decreases in } r_X \text{ for all } r_X \in \left[0, \frac{1}{\bar{w}^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)-(1-\bar{w})^2 r_Y\right)\right), & \\ \text{and } EU^{PM} \text{ increases in } r_X \text{ for all } r_X \in \left(\frac{1}{\bar{w}^2}\left(\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right)-(1-\bar{w})^2 r_Y\right), \infty\right), & \text{if } r_Y < \frac{1}{(1-\bar{w})^2}\frac{\Delta\mu^2}{\alpha_d}\left(1-\frac{1}{\alpha_d}\right). \end{cases}$$

Part II: One can easily confirm from (A.5) and (A.6) that when $\bar{\sigma}=0$, we have $EU^{PM}-EU^{SM}=0$. Furthermore, as $\bar{\sigma}$ tends to ∞, $EU^{PM}-EU^{SM}$ goes to ∞. Since EU^{PM} is a U-shaped function of $\bar{\sigma}$, i.e., it first decreases, then increases in $\bar{\sigma}$ (recall part I), it follows that there exists a unique threshold $\psi>0$ such that if $0<\bar{\sigma}<\psi$, we have $EU^{PM}<EU^{SM}$, and if $\bar{\sigma}>\psi$, we have $EU^{PM}>EU^{SM}$. This, together with the definition of $\bar{\sigma}$ in (A.2), implies that for a given r_X, we have:

$$\begin{cases} EU^{PM} > EU^{SM} \text{ for all } r_Y \geq 0, & \text{if } r_X > \frac{\psi^2}{\bar{w}^2}, \\ EU^{PM} < EU^{SM} \text{ for all } r_Y \in \left[0, \frac{\psi^2-\bar{w}^2 r_X}{(1-\bar{w})^2}\right), & \\ \text{and } EU^{PM} > EU^{SM} \text{ for all } r_Y \in \left(\frac{\psi^2-\bar{w}^2 r_X}{(1-\bar{w})^2}, \infty\right), & \text{if } r_X < \frac{\psi^2}{\bar{w}^2}. \end{cases}$$

Similarly, for a given r_Y, we have:

$$\begin{cases} EU^{PM} > EU^{SM} \text{ for all } r_X \geq 0, & \text{if } r_Y > \frac{\psi^2}{(1-\bar{w})^2}, \\ EU^{PM} < EU^{SM} \text{ for all } r_X \in \left[0, \frac{\psi^2-(1-\bar{w})^2 r_Y}{\bar{w}^2}\right), & \\ \text{and } EU^{PM} > EU^{SM} \text{ for all } r_X \in \left(\frac{\psi^2-(1-\bar{w})^2 r_Y}{\bar{w}^2}, \infty\right), & \text{if } r_Y < \frac{\psi^2}{(1-\bar{w})^2}. \end{cases}$$

$$\psi_X(r_Y) = \frac{\left(\psi^2-(1-\bar{w})^2 r_Y\right)^+}{\bar{w}^2}, \qquad \psi_Y(r_X) = \frac{\left(\psi^2-\bar{w}^2 r_X\right)^+}{(1-\bar{w})^2},$$

where x^+ denotes $\max\{x,0\}$. Using these definitions of $\psi_X(r_Y)$ and $\psi_Y(r_X)$, we conclude that, for a fixed r_Y, $EU^{PM}\leq EU^{SM}$ if and only if $r_X \leq \psi_X(r_Y)$, and, for a fixed r_X, $EU^{PM}\leq EU^{SM}$ if and only if $r_Y\leq\psi_Y(r_X)$. Thus, $EU^{PM}\leq EU^{SM}$ when health-outcome heterogeneity (σ_X and σ_Y) is sufficiently low relative to doctor errors ($\sigma_{d,X}$ and $\sigma_{d,Y}$). ■

B.2.2. Proof of Section 5.1.2 Results

For the proof of Lemma 2, we first need the following lemma to establish the structure of EU_i^{PH} given with (A.10) with respect to α_d.

Lemma A2 *(a) EU_i^{PH} given with (A.10) is a quasiconcave function of α_d in* $\left[1/2, \min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2}, \frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$.
(b) If $|\Delta\mu_i| > \sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$, EU_i^{PH} decreases in α_d if $\alpha_d \geq 1$.

Proof of Lemma A2: (a) The derivative of EU_i^{PH} in (A.10) with respect to α_d is:

$$\frac{\partial EU_i^{PH}}{\partial \alpha_d} = \frac{\bar{\sigma}_i^2}{\left(\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2\right)^{3/2}}\left[\Delta\mu_i^2\left(\frac{\alpha_d^2\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}-\alpha_d\right)+\sigma_{p,i}^2\right]\phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}}\right). \tag{A.31}$$

Let us represent $G_1(\bar{\sigma}_i,\alpha_d)$ by:

$$G_1(\bar{\sigma}_i,\alpha_d) = \Delta\mu_i^2\left(\theta(\bar{\sigma}_i,\alpha_d)-\alpha_d\right)+\sigma_{p,i}^2, \tag{A.32}$$

where $\theta(\bar{\sigma}_i,\alpha_d)=\frac{\alpha_d^2\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}$ and for notational brevity, we suppress the dependence of $G_1(\bar{\sigma}_i,\alpha_d)$ and $\theta(\bar{\sigma}_i,\alpha_d)$ on $\sigma_{p,i}$. The derivative of $G_1(\bar{\sigma}_i,\alpha_d)$ with respect to α_d is:

$$\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial\alpha_d}=\Delta\mu_i^2\left(\frac{2}{\alpha_d}\theta(\bar{\sigma}_i,\alpha_d)(1-\theta(\bar{\sigma}_i,\alpha_d))-1\right) \tag{A.33}$$

$$\leq\Delta\mu_i^2\left(\frac{2}{\alpha_d}\frac{1}{2}\left(1-\frac{1}{2}\right)-1\right)=\Delta\mu_i^2\left(\frac{1}{2\alpha_d}-1\right), \tag{A.34}$$

where the first inequality follows since $\theta(\bar{\sigma}_i,\alpha_d)(1-\theta(\bar{\sigma}_i,\alpha_d))$ is concave in $\theta(\bar{\sigma}_i,\alpha_d)$ and reaches its maximum value at $\theta(\bar{\sigma}_i,\alpha_d)=1/2$. Furthermore, we have $\Delta\mu_i^2\left(\frac{1}{2\alpha_d}-1\right)\leq 0$ for all $\alpha_d\geq 1/2$. Hence, $\frac{\partial G_1(\bar{\sigma}_i,\alpha_d)}{\partial\alpha_d}\leq 0$ follows for all $\alpha_d\in\left[1/2,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$, which means that $G_1(\bar{\sigma}_i,\alpha_d)$ decreases in α_d when $\alpha_d\in\left[1/2,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$. This confirms that EU_i^{PH} is quasiconcave in α_d when $\alpha_d\in\left[1/2,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$.

(b) Setting α_d equal to 1 in (A.31), the derivative of EU_i^{PH} at $\alpha_d=1$ is:

$$\left[\frac{\partial EU_i^{PH}}{\partial\alpha_d}\right]_{\alpha_d=1}=\frac{\bar{\sigma}_i^2\sigma_{p,i}^2}{\left(\bar{\sigma}_i^2+\sigma_{p,i}^2\right)^{3/2}}\left[\frac{-\Delta\mu_i^2}{\bar{\sigma}_i^2+\sigma_{p,i}^2}+1\right]\phi\left(\frac{\Delta\mu_i}{\sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}}\right)<0, \tag{A.35}$$

where the inequality follows from $|\Delta\mu_i|>\sqrt{\bar{\sigma}_i^2+\sigma_{p,i}^2}$. Since the derivative of EU_i^{PH} at $\alpha_d=1$ is negative by (A.35) and EU_i^{PH} is a quasiconcave function of α_d for $\alpha_d\in\left[1/2,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$ by part (a) of this lemma, it follows that EU_i^{PH} decreases in α_d if $\alpha_d\geq 1$. ■

Now, we are ready to prove Lemma 2 using Lemma A2.

Proof of Lemma 2: It follows from Lemma A2(b) that if $|\Delta\mu_1|>\sqrt{\bar{\sigma}_1^2+\sigma_{p,1}^2}$, and $|\Delta\mu_2|>\sqrt{\bar{\sigma}_2^2+\sigma_{p,2}^2}$ hold, both EU_1^{PH} and EU_2^{PH} decrease with α_d for all $\alpha_d\in\left[1,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$. This implies that EU^{PH} given with (A.8) decreases with α_d for all $\alpha_d\in\left[1,\min\left\{\frac{\sigma_X^2+\sigma_{d,X}^2}{\sigma_X^2},\frac{\sigma_Y^2+\sigma_{d,Y}^2}{\sigma_Y^2}\right\}\right]$, and thus, EU^{PH} reaches its maximum value when $\alpha_d<1$, i.e., $\alpha_d^*<1$. ■

For the proof of Proposition 3, we first need the following lemma to establish the structure of EU_i^{PH} given with (A.10) with respect to $\bar{\sigma}_i$ given with (A.2), r_X and r_Y.

Lemma A3 *Consider EU_i^{PC} and EU_i^{PH} given with (A.9) and (A.10), respectively. Then:*

(a) EU_i^{PH} is quasiconvex in $\bar{\sigma}_i$.

(b) Let $|\Delta\mu_i|>\sqrt{2\sigma_{p,i}^2/\alpha_d}$ and let τ_i denote the unique value of $\bar{\sigma}_i$ that satisfies the following equation:[1]

$$\Delta\mu_i^2\alpha_d\left(\frac{\alpha_d\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}-1\right)+\alpha_d^2\bar{\sigma}_i^2+2\sigma_{p,i}^2=0. \tag{A.36}$$

Then:

(i) EU_i^{PH} decreases in $\bar{\sigma}_i$ if $\bar{\sigma}_i<\tau_i$, and it increases in $\bar{\sigma}_i$ if $\bar{\sigma}_i>\tau_i$.

(ii) There exists a unique threshold $\eta_i>0$ such that $EU_i^{PH}<EU_i^{PC}$ if $0<\bar{\sigma}_i<\eta_i$, and $EU_i^{PH}>EU_i^{PC}$ if $\bar{\sigma}_i>\eta_i$. Moreover, $EU_i^{PH}=EU_i^{PC}$ when $\bar{\sigma}_i=0$ or $\bar{\sigma}_i=\eta_i$.

Proof of Lemma A3: (a) The derivatives of EU_i^{PH} in (A.10) with respect to r_X and r_Y are equal to

$$\frac{\partial EU_i^{PH}}{\partial r_X}=\frac{\partial EU_i^{PH}}{\partial\bar{\sigma}_i}\frac{\partial\bar{\sigma}_i}{\partial r_X}=\frac{w_i^2}{2\bar{\sigma}_i}\frac{\partial EU_i^{PH}}{\partial\bar{\sigma}_i}\text{ and }\frac{\partial EU_i^{PH}}{\partial r_Y}=\frac{\partial EU_i^{PH}}{\partial\bar{\sigma}_i}\frac{\partial\bar{\sigma}_i}{\partial r_Y}=\frac{(1-w_i)^2}{2\bar{\sigma}_i}\frac{\partial EU_i^{PH}}{\partial\bar{\sigma}_i}, \tag{A.37}$$

[1] We show the existence and uniqueness of τ_i in the proof.

where

$$\frac{\partial EU_i^{PH}}{\partial \bar{\sigma}_i} = \frac{\alpha_d \bar{\sigma}_i}{\left(\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2\right)^{3/2}} \left[\Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2\right] \phi\left(\frac{\Delta\mu_i}{\sqrt{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2}}\right). \tag{A.38}$$

One can easily confirm in (A.38) that $\Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2$ increases in $\bar{\sigma}_i$, and thus, EU_i^{PH} is quasi-convex in $\bar{\sigma}_i$.

(b) *Part (i):* Let the expression in square brackets in (A.38) be denoted by

$$G_i(\bar{\sigma}_i) = \Delta\mu_i^2 \alpha_d \left(\frac{\alpha_d \bar{\sigma}_i^2}{\alpha_d^2 \bar{\sigma}_i^2 + \sigma_{p,i}^2} - 1\right) + \alpha_d^2 \bar{\sigma}_i^2 + 2\sigma_{p,i}^2.$$

Under $|\Delta\mu_i| > \sqrt{2\sigma_{p,i}^2/\alpha_d}$, we have $G_i(0) = 2\sigma_{p,i}^2 - \alpha_d \Delta\mu_i^2 < 0$. Furthermore, as established in part (a), $G_i(\bar{\sigma}_i)$ is strictly increasing in $\bar{\sigma}_i$, and $G_i(\bar{\sigma}_i) \to \infty$ as $\bar{\sigma}_i \to \infty$. Hence, there exists a unique $\tau_i > 0$ such that $G_i(\tau_i) = 0$, or equivalently, such that (A.36) holds. For every $\bar{\sigma}_i > 0$, all terms multiplying $G_i(\bar{\sigma}_i)$ in (A.38) are strictly positive. Therefore,

$$\frac{\partial EU_i^{PH}}{\partial \bar{\sigma}_i} < 0 \quad \text{for } 0 < \bar{\sigma}_i < \tau_i, \text{ and } \frac{\partial EU_i^{PH}}{\partial \bar{\sigma}_i} > 0 \quad \text{for } \bar{\sigma}_i > \tau_i.$$

At $\bar{\sigma}_i = 0$, $\partial EU_i^{PH}/\partial \bar{\sigma}_i = 0$. Thus, EU_i^{PH} decreases in $\bar{\sigma}_i$ up to τ_i and increases thereafter.

Part (ii): One can easily confirm from (A.9) and (A.10) that when $\bar{\sigma}_i = 0$, we have $EU_i^{PH} - EU_i^{PC} = 0$. Furthermore, as $\bar{\sigma}_i$ tends to ∞, $EU_i^{PH} - EU_i^{PC}$ goes to ∞. Since EU_i^{PH} first decreases, then increases in $\bar{\sigma}_i$ (recall part I), it follows that there exists a unique threshold $\eta_i > 0$ such that if $0 < \bar{\sigma}_i < \eta_i$, we have $EU_i^{PH} < EU_i^{PC}$, and if $\bar{\sigma}_i > \eta_i$, we have $EU_i^{PH} > EU_i^{PC}$. ■

Now, we are ready to prove Proposition 3 using Lemma A3.

Proof of Proposition 3: It follows from part (ii) of Lemma A3(b) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 < \eta_1$ and $\bar{\sigma}_2 < \eta_2$, we have $EU_1^{PH} < EU_1^{PC}$ and $EU_2^{PH} < EU_2^{PC}$. Recalling (A.2), this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, we have $EU_1^{PH} < EU_1^{PC}$ and $EU_2^{PH} < EU_2^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \eta_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \eta_2^2$. Then, from (A.7) and (A.8), it follows that $EU^{PH} < EU^{PC}$ for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \eta_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \eta_2^2$, i.e., for sufficiently low r_X and r_Y (or, equivalently, when σ_X and σ_Y are sufficiently low relative to $\sigma_{d,X}$ and $\sigma_{d,Y}$). ■

Proof of Proposition 4:

The proof for $EU^{PM} - EU^{SM}$: It follows from Part I of the proof of Proposition 2 that for $\alpha_d > 1$, EU^{PM} first decreases and then increases in r_X and r_Y. The rest directly follows from the fact that r_X and r_Y increase in σ_X and σ_Y; and decrease in $\sigma_{d,X}$ and $\sigma_{d,Y}$, respectively.

The proof for $EU^{PH} - EU^{PC}$: First, it follows from part (i) of Lemma A3(b) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 < \tau_1$ and $\bar{\sigma}_2 < \tau_2$, EU_1^{PH} and EU_2^{PH} decrease with $\bar{\sigma}_1$ and $\bar{\sigma}_2$, respectively. Recalling from (A.2) that both $\bar{\sigma}_1$ and $\bar{\sigma}_2$ increase with r_X and r_Y, this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, both EU_1^{PH} and EU_2^{PH} decrease with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \tau_2^2$. Then, from (A.8), it follows that EU^{PH} decreases with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y < \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y < \tau_2^2$, i.e., for sufficiently low r_X and r_Y. Second, it follows from part (i) of Lemma A3(b) that if $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$ hold, and if $\bar{\sigma}_1 > \tau_1$ and $\bar{\sigma}_2 > \tau_2$, EU_1^{PH}

and EU_2^{PH} increase with $\bar{\sigma}_1$ and $\bar{\sigma}_2$, respectively. Recalling from (A.2) that both $\bar{\sigma}_1$ and $\bar{\sigma}_2$ increase with r_X and r_Y, this means that when $|\Delta\mu_1| > \sqrt{2\sigma_{p,1}^2/\alpha_d}$ and $|\Delta\mu_2| > \sqrt{2\sigma_{p,2}^2/\alpha_d}$, both EU_1^{PH} and EU_2^{PH} increase with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \tau_2^2$. Then, from (A.8), it follows that EU^{PH} increases with r_X and r_Y for all $r_X, r_Y \geq 0$ satisfying $w_1^2 r_X + (1-w_1)^2 r_Y > \tau_1^2$ and $w_2^2 r_X + (1-w_2)^2 r_Y > \tau_2^2$, i.e., for sufficiently large r_X and r_Y. The rest directly follows from the fact that r_X and r_Y increase in σ_X and σ_Y; and decrease in $\sigma_{d,X}$ and $\sigma_{d,Y}$, respectively. ■

Proof of Corollary 1: It follows directly from Proposition 4. ■

B.3. Proof of Section 5.2 Results

We now present the proof of Proposition 5, which follows from a sequence of lemmas stated and proved below.

Lemma A4 *(a)* $\frac{\partial EU^{PC}}{\partial \sigma_{p,1}} < 0$ *and* $\frac{\partial EU^{PC}}{\partial \sigma_{p,2}} < 0$ *for all* $\sigma_{p,1}, \sigma_{p,2} > 0$.
(b) For $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}}$ *and* $\frac{\partial EU^{PH}}{\partial \sigma_{p,2}}$*, we have:*

(i) If $\alpha_d \geq 1$ *or if* $\alpha_d < 1$ *and* $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$*, we have* $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} < 0$ *for all* $\sigma_{p,1} > 0$*, whereas if* $\alpha_d < 1$ *and* $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$*, we have* $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} < 0$ *if and only if* $\sigma_{p,1} > \sqrt{\left(\alpha_d\bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d\bar{\sigma}_1^2} - \alpha_d^2\bar{\sigma}_1^2\right)^+}$.

(ii) If $\alpha_d \geq 1$ *or if* $\alpha_d < 1$ *and* $\Delta\mu_2^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2$*, we have* $\frac{\partial EU^{PH}}{\partial \sigma_{p,2}} < 0$ *for all* $\sigma_{p,2} > 0$*, whereas if* $\alpha_d < 1$ *and* $\Delta\mu_2^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_2^2$*, we have* $\frac{\partial EU^{PH}}{\partial \sigma_{p,2}} < 0$ *if and only if* $\sigma_{p,2} > \sqrt{\left(\alpha_d\bar{\sigma}_2^2 \frac{\Delta\mu_2^2}{\Delta\mu_2^2 + \alpha_d\bar{\sigma}_2^2} - \alpha_d^2\bar{\sigma}_2^2\right)^+}$.

Proof of Lemma A4: (a) The derivatives of EU^{PC} with respect to $\sigma_{p,1}$ and $\sigma_{p,2}$ are:

$$\frac{\partial EU^{PC}}{\partial \sigma_{p,1}} = -p\frac{\Delta\mu_1^2}{\sigma_{p,1}^2}\phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) < 0, \text{ and } \frac{\partial EU^{PC}}{\partial \sigma_{p,2}} = -(1-p)\frac{\Delta\mu_2^2}{\sigma_{p,2}^2}\phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) < 0.$$

(b) We will prove only part (i). The proof of part (ii) follows the same lines and is therefore omitted. The derivative of EU^{PH} with respect to $\sigma_{p,1}$ is:

$$\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} = p\frac{\sigma_{p,1}}{\left(\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2\right)^{3/2}}\left[\Delta\mu_1^2\left(\frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2} - 1\right) - \alpha_d\bar{\sigma}_1^2\right]\phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right). \tag{A.39}$$

We consider the following two cases:

Case 1: $\alpha_d \geq 1$. One can easily confirm in (A.39) that when $\alpha_d \geq 1$, $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} < 0$ holds for all $\sigma_{p,1} > 0$.

Case 2: $\alpha_d < 1$. One can easily confirm in (A.39) that $\Delta\mu_1^2\left(\frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2} - 1\right) - \alpha_d\bar{\sigma}_1^2$ decreases in $\sigma_{p,1}$, and thus, EU^{PH} is quasiconcave in $\sigma_{p,1}$. Now, we consider the following two subcases:

Case 2a: $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$. First note that, if $\Delta\mu_1^2 \leq \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ holds, $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} \leq 0$ at $\sigma_{p,1} = 0$. Then, EU^{PH} decreases in $\sigma_{p,1}$ for all $\sigma_{p,1} \geq 0$ since EU^{PH} is quasiconcave in $\sigma_{p,1}$.

Case 2b: $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$. First note that, if $\Delta\mu_1^2 > \frac{\alpha_d^2}{1-\alpha_d}\bar{\sigma}_1^2$ holds, the expression in square brackets in (A.39) is positive at $\sigma_{p,1} = 0$, and hence $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}} > 0$ for sufficiently small positive $\sigma_{p,1}$. Furthermore, $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}}$ tends to zero from negative values as $\sigma_{p,1}$ tends to ∞. Then, since EU^{PH} is quasiconcave in $\sigma_{p,1}$, noting that $\frac{\partial EU^{PH}}{\partial \sigma_{p,1}}$ becomes equal to zero at $\sigma_{p,1} = \sqrt{\alpha_d\bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d\bar{\sigma}_1^2} - \alpha_d^2\bar{\sigma}_1^2}$, we conclude that EU^{PH} increases with $\sigma_{p,1}$ for all $\sigma_{p,1} \in \left[0, \sqrt{\alpha_d\bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d\bar{\sigma}_1^2} - \alpha_d^2\bar{\sigma}_1^2}\right)$, and it decreases with $\sigma_{p,1}$ for all $\sigma_{p,1} > \sqrt{\alpha_d\bar{\sigma}_1^2 \frac{\Delta\mu_1^2}{\Delta\mu_1^2 + \alpha_d\bar{\sigma}_1^2} - \alpha_d^2\bar{\sigma}_1^2}$. ■

We also need to establish the monotonicity of EU^{PC} and EU^{PH} in (A.8) and (A.7) with respect to d_p.

Lemma A5 *Let us set* $\beta_1 = \frac{\alpha_d \bar{\sigma}_1^2}{\alpha_d^2 \bar{\sigma}_1^2 + \sigma_{p,1}^2}$, *and* $\beta_2 = \frac{\alpha_d \bar{\sigma}_2^2}{\alpha_d^2 \bar{\sigma}_2^2 + \sigma_{p,2}^2}$. *Then:*
(a) EU^{PC} *increases in* d_p.
(b) EU^{PH} *increases in* d_p *if we have either (i)* $\beta_1 \le 2$, *and* $\beta_2 \le 2$, *or (ii)* $\beta_1 > 2$, *or* $\beta_2 > 2$, *and* d_p *is sufficiently large.*

Proof of Lemma A5: We introduce a new parameter $a \ge 0$, and express $\mu_{XA} - \mu_{XB}$ and $\mu_{YA} - \mu_{YB}$ in terms of a in the following form:

$$\mu_{XA} - \mu_{XB} = \Delta\mu + \frac{a}{\bar{w}}, \text{ and } \mu_{YA} - \mu_{YB} = \Delta\mu - \frac{a}{1-\bar{w}}, \tag{A.40}$$

where $\bar{w}$ is given with (2) and $\Delta\mu$ is given with (A.1). Since $\Delta\mu < 0$, $\mu_{XA} > \mu_{XB}$ and $\mu_{YA} < \mu_{YB}$ by assumption, we need to have $\Delta\mu + \frac{a}{\bar{w}} > 0 \Leftrightarrow a > -\bar{w}\Delta\mu$. Replacing $\mu_{XA} - \mu_{XB}$ and $\mu_{YA} - \mu_{YB}$ in (3) and (4) with the expressions in (A.40), $\Delta\mu_1$, $\Delta\mu_2$ and d_p could be written in terms of $\Delta\mu$ and a as:

$$\Delta\mu_1 = \Delta\mu + \frac{(w_1 - w_2)(1-p)}{\bar{w}(1-\bar{w})}a, \;\; \Delta\mu_2 = \Delta\mu - \frac{(w_1 - w_2)p}{\bar{w}(1-\bar{w})}a, \text{ and } d_p = \frac{(w_1 - w_2)}{\bar{w}(1-\bar{w})}a. \tag{A.41}$$

Since we have $\Delta\mu_1 > 0$ and $\Delta\mu_2 < 0$ by assumption, we need to have $\Delta\mu + \frac{(w_1-w_2)(1-p)}{\bar{w}(1-\bar{w})}a > 0 \Leftrightarrow a > \frac{-\bar{w}(1-\bar{w})\Delta\mu}{(w_1-w_2)(1-p)}$. Note that we analyze the effect of a change in d_p by varying a while keeping p, w_1, w_2, and $\bar{w}$ constant.

(a) When we replace $\Delta\mu_1$ and $\Delta\mu_2$ in EU^{PC} in (A.7) with the expressions in (A.41), and take the derivative of EU^{PC} with respect to a, we have:

$$\frac{\partial EU^{PC}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\left[\Phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) + \frac{\Delta\mu_1}{\sigma_{p,1}}\phi\left(\frac{\Delta\mu_1}{\sigma_{p,1}}\right) - \Phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right) - \frac{\Delta\mu_2}{\sigma_{p,2}}\phi\left(\frac{\Delta\mu_2}{\sigma_{p,2}}\right)\right] \ge 0,$$

where $\Delta\mu_1$ and $\Delta\mu_2$ are given with (A.41), and the inequality follows from $\Delta\mu_2 < 0 < \Delta\mu_1$.

(b) The derivative of EU^{PH} in (A.8) with respect to a is:

$$\begin{aligned}\frac{\partial EU^{PH}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\Bigg[&\left(\Phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right) + \frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\left(1 - \frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}\right)\phi\left(\frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2 + \sigma_{p,1}^2}}\right)\right) \\ &- \left(\Phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right) + \frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\left(1 - \frac{\alpha_d\bar{\sigma}_2^2}{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}\right)\phi\left(\frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2 + \sigma_{p,2}^2}}\right)\right)\Bigg],\end{aligned}$$

where $\Delta\mu_1$ and $\Delta\mu_2$ are given with (A.41). Setting $z_1 = \frac{\Delta\mu_1}{\sqrt{\alpha_d^2\bar{\sigma}_1^2+\sigma_{p,1}^2}}$ and $z_2 = \frac{\Delta\mu_2}{\sqrt{\alpha_d^2\bar{\sigma}_2^2+\sigma_{p,2}^2}}$, and recalling that $\beta_1 = \frac{\alpha_d\bar{\sigma}_1^2}{\alpha_d^2\bar{\sigma}_1^2+\sigma_{p,1}^2}$, and $\beta_2 = \frac{\alpha_d\bar{\sigma}_2^2}{\alpha_d^2\bar{\sigma}_2^2+\sigma_{p,2}^2}$, we can rewrite $\frac{\partial EU^{PH}}{\partial a}$ as:

$$\frac{\partial EU^{PH}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\Big[H(z_1, \beta_1) - H(z_2, \beta_2)\Big], \tag{A.42}$$

where $H(z,\beta)$ is equal to:

$$H(z,\beta) = \Phi(z) + z(1-\beta)\phi(z). \tag{A.43}$$

To investigate the sign of $\frac{\partial EU^{PH}}{\partial a}$ we consider the following three cases:

Case 1: $\beta_1 \le 1$ *and* $\beta_2 \le 1$. Since $\Delta\mu_2 < 0 < \Delta\mu_1$, one can directly confirm in (A.43) and (A.42) that we have $\frac{\partial EU^{PH}}{\partial a} > 0$.

Case 2: $\beta_1 \le 2$, $\beta_2 \le 2$, *and* $\beta_1 > 1$ *or* $\beta_2 > 1$. First, note that when $\alpha_d \ge 0.5$, $\beta_1 \le 2$ and $\beta_2 \le 2$ immediately holds, i.e., $\alpha_d \ge 0.5$ is a sufficient condition for $\beta_1 \le 2$ and $\beta_2 \le 2$. Without loss of generality, assume that $\beta_1 < \beta_2$. Then, $\frac{\partial EU^{PH}}{\partial a}$ in (A.42) could be rewritten as:

$$\frac{\partial EU^{PH}}{\partial a} = \frac{p(1-p)(w_1 - w_2)}{\bar{w}(1-\bar{w})}\Big[H(z_1, \beta_2) - H(z_2, \beta_2) + z_1(\beta_2 - \beta_1)\phi(z_1)\Big]. \tag{A.44}$$

Since for $1 < \beta \leq 2$, $H(z,\beta)$ increases in z as per Claim 1(a) proved at the end of this lemma and $z_2 < 0 < z_1$ holds, we have $H(z_1,\beta_2) > H(z_2,\beta_2)$. Then, in (A.44), $\frac{\partial EU^{PH}}{\partial a} > 0$ follows (recall that $\beta_2 > \beta_1$ and $z_1 > 0$).

Case 3: $\beta_1 > 2$ or $\beta_2 > 2$. Without loss of generality, we again assume that $\beta_1 < \beta_2$. Then, $\frac{\partial EU^{PH}}{\partial a}$ in (A.42) could be rewritten as:

$$\frac{\partial EU^{PH}}{\partial a} = \frac{p(1-p)(w_1-w_2)}{\bar{w}(1-\bar{w})}\Big[(H(z_1,\beta_2)-1/2)-(H(z_2,\beta_2)-1/2)+z_1(\beta_2-\beta_1)\phi(z_1)\Big]. \tag{A.45}$$

As per part (ii) of Claim 1(b) proved at the end of this lemma, for a given $\beta > 2$, $H(z,\beta)-1/2 \geq 0$ for $z \geq \zeta_2(\beta)$, i.e., for sufficiently large z, whereas $H(z,\beta)-1/2 < 0$ for $z < \zeta_1(\beta)$, i.e., for sufficiently low z. Then, if $z_1 \geq \zeta_2(\beta_2)$ and $z_2 \leq \zeta_1(\beta_2)$, i.e., a is sufficiently large, we have $H(z_1,\beta_2)-1/2 > 0 > H(z_2,\beta_2)-1/2$, and thus, in (A.45), $\frac{\partial EU^{PH}}{\partial a} > 0$ follows (recall that $\beta_2 > \beta_1$ and $z_1 > 0$).

Claim 1: Consider $H(z,\beta)$ given with (A.43).

(a) For a given $\beta \in (1,2]$, $H(z,\beta)$ increases in z for all $z \in \mathbb{R}$.

(b) For a given $\beta \in (2,\infty)$, we have:

(i) $H(z,\beta)$ increases in z for all $z \in \left(-\infty, -\sqrt{\frac{\beta-2}{\beta-1}}\,\right] \cup \left[\sqrt{\frac{\beta-2}{\beta-1}}, \infty\right)$, whereas it decreases in z for all $z \in \left(-\sqrt{\frac{\beta-2}{\beta-1}}, \sqrt{\frac{\beta-2}{\beta-1}}\right)$.

(ii) There exist two thresholds $\zeta_1(\beta)$ and $\zeta_2(\beta)$ with $\zeta_1(\beta) < 0 < \zeta_2(\beta)$ such that $H(z,\beta)-1/2 < 0$ for all $z \in (-\infty,\zeta_1(\beta)) \cup (0,\zeta_2(\beta))$ and $H(z,\beta)-1/2 \geq 0$ for all $z \in [\zeta_1(\beta),0] \cup [\zeta_2(\beta),\infty)$.

Proof: The derivative of $H(z,\beta)$ with respect to z is:

$$\frac{\partial H(z,\beta)}{\partial z} = \Big[2-\beta-z^2(1-\beta)\Big]\phi(z). \tag{A.46}$$

(a) One can easily confirm in (A.46) that when $1 < \beta \leq 2$, $\frac{\partial H(z,\beta)}{\partial z} \geq 0$ for all $z \in \mathbb{R}$.

(b) *Part (i):* It is straightforward to establish from (A.46) that the roots of $\frac{\partial H(z,\beta)}{\partial z}$ are $z = -\sqrt{\frac{\beta-2}{\beta-1}}$ and $z = \sqrt{\frac{\beta-2}{\beta-1}}$. Furthermore, as z tends to $-\infty$ or ∞, $H(z,\beta)$ goes to nonnegative values. Hence, $\frac{\partial H(z,\beta)}{\partial z} > 0$ for all $z < -\sqrt{\frac{\beta-2}{\beta-1}}$ and $z > \sqrt{\frac{\beta-2}{\beta-1}}$, whereas $\frac{\partial H(z,\beta)}{\partial z} < 0$ for all z with $-\sqrt{\frac{\beta-2}{\beta-1}} < z < \sqrt{\frac{\beta-2}{\beta-1}}$.

Part (ii): First, note that $H(z,\beta)-1/2$ goes to -1/2 as z tends to $-\infty$, it goes to 1/2 as z tends to ∞, whereas it is equal to 0 at $z=0$. Furthermore, part (i) implies that for $z \in (-\infty,0]$, $H(z,\beta)-1/2$ first increases, then decreases in z, whereas for $z \in [0,\infty)$, $H(z,\beta)-1/2$ first decreases, then increases in z. Thus, for a given $\beta > 2$, there exist two thresholds $\zeta_1(\beta)$ and $\zeta_2(\beta)$ with $\zeta_1(\beta) < 0 < \zeta_2(\beta)$ such that $H(z,\beta)-1/2 < 0$ for all $z \in (-\infty,\zeta_1(\beta)) \cup (0,\zeta_2(\beta))$ and $H(z,\beta)-1/2 \geq 0$ for all $z \in [\zeta_1(\beta),0] \cup [\zeta_2(\beta),\infty)$. ■

Now, we are ready to prove Proposition 5 using Lemmas A4 and A5.

Proof of Proposition 5: We argue separately for the two inequalities.

(i) $EU^{PC} < EU^{SM}$. Lemma A4(a) shows that EU^{PC} is strictly decreasing in both $\sigma_{p,1}$ and $\sigma_{p,2}$, while Lemma A5(a) implies monotonicity in d_p. As $\sigma_{p,1},\sigma_{p,2} \to \infty$, we have $\lim_{\sigma_{p,1},\sigma_{p,2}\to\infty}(EU^{PC}-EU^{SM}) = \Delta\mu/2 < 0$, where $\Delta\mu < 0$ is given with (A.1). Hence, as $\sigma_{p,1},\sigma_{p,2} \to \infty$, PC yields strictly lower expected utility than SM for any $d_p > 0$. By continuity, there exists a finite threshold such that $EU^{PC} < EU^{SM}$ whenever $\sigma_{p,1},\sigma_{p,2}$ exceed this threshold.

(ii) $EU^{PH} < EU^{PM}$. By Lemma A4(b), for each $i \in \{1,2\}$ the function EU^{PH} is strictly decreasing in $\sigma_{p,i}$ either globally or at least beyond some finite threshold $\sigma^{\dagger}_{p,i}$. Let $\beta_i = \frac{\alpha_d\bar{\sigma}_i^2}{\alpha_d^2\bar{\sigma}_i^2+\sigma_{p,i}^2}$; then Lemma A5(b) implies that EU^{PH} is increasing in d_p

whenever $(\beta_1,\beta_2)\le(2,2)$, i.e., $\sigma_{p,1}$ and $\sigma_{p,2}$ are sufficiently large. As $\sigma_{p,1},\sigma_{p,2}\to\infty$, we have $\lim_{\sigma_{p,1},\sigma_{p,2}\to\infty}(EU^{PH}-EU^{PM})=\Delta\mu\left(1/2-\Phi\left(\frac{\Delta\mu}{\alpha_d\bar\sigma}\right)\right)-\bar\sigma\phi\left(\frac{\Delta\mu}{\alpha_d\bar\sigma}\right)<0$, where $\Delta\mu<0$ is given with (A.1). Since EU^{PH} is strictly decreasing in each $\sigma_{p,i}$ (at least beyond $\sigma^{\dagger}_{p,i}$) and continuous, there exists a finite, nondecreasing threshold function $S(d_p)$ such that if $\min\{\sigma_{p,1},\sigma_{p,2}\}\ge S(d_p)$—i.e., patient errors are sufficiently large relative to d_p—then $EU^{PH}<EU^{PM}$.
Combining (i) and (ii) yields the claim.

■

Appendix C: Proofs and Supplementary Results for Section 5

C.1. Supplementary Analysis for Section 5.1

Here, we compare PH and PC when patients have weak preferences between treatments.

Lemma A6 *If $|\Delta\mu_1|<\sqrt{2\sigma^2_{p,1}/\alpha_d}$ and $|\Delta\mu_2|<\sqrt{2\sigma^2_{p,2}/\alpha_d}$ hold, then we have $EU^{PH}\ge EU^{PC}$.*

Proof of Lemma A6: First, recalling (A.38), if $|\Delta\mu_i|<\sqrt{2\sigma^2_{p,i}/\alpha_d}$ holds, the expression in square brackets in (A.38) is positive at $\bar\sigma_i=0$. Since this expression increases in $\bar\sigma_i$, we have $\partial EU_i^{PH}/\partial\bar\sigma_i>0$ for all $\bar\sigma_i>0$. Then, EU_i^{PH} increases in $\bar\sigma_i$ for all $\bar\sigma_i\ge0$ since EU_i^{PH} is quasiconvex in $\bar\sigma_i$ as per Lemma A3(a). This implies that EU_i^{PH} increases with both r_X and r_Y (recall from (A.2) that $\bar\sigma_i$ is an increasing function of both r_X and r_Y). When $r_X=r_Y=0$, one can easily confirm from (A.9) and (A.10) that $EU_i^{PH}-EU_i^{PC}=0$. Since $EU_i^{PH}-EU_i^{PC}=0$ for $r_X=r_Y=0$, and EU_i^{PH} increases with r_X and r_Y, we have $EU_i^{PH}-EU_i^{PC}\ge0$ for all $r_X\ge0$ and $r_Y\ge0$.

As a result, if $|\Delta\mu_1|<\sqrt{2\sigma^2_{p,1}/\alpha_d}$ and $|\Delta\mu_2|<\sqrt{2\sigma^2_{p,2}/\alpha_d}$ hold, we have $EU_1^{PH}\ge EU_1^{PC}$ and $EU_2^{PH}\ge EU_2^{PC}$. Then, recalling (A.7) and (A.8), we conclude that $EU^{PH}\ge EU^{PC}$ for all $r_X,r_Y\ge0$. ■

Lemma A6 demonstrates that if patients have weak preferences between treatments (small $|\Delta\mu_1|$ and $|\Delta\mu_2|$), then PH outperforms PC. The reason for this is as follows. When patients have weak preferences between treatments, it becomes less clear a priori which treatment is most appropriate. In these cases, personalizing outcome predictions by providing patient-specific information can significantly reduce this uncertainty and improve patient outcomes.

C.2. Supplementary Analysis for Section 5.2

In this subsection, we provide the formal analysis supporting the discussion in Section 5.2. We focus on the conditions under which avoiding preference-based personalization and patient participation yields the greatest utility improvement.

Lemma A7 *The utility gaps $EU^{PC}-EU^{SM}$ and $EU^{PH}-EU^{PM}$ reach their minimum when (i) d_p is low, and (ii) $\sigma_{p,1}$, $\sigma_{p,2}$ are high.*

Proof of Lemma A7: The proof follows directly from Lemmas A4 and A5.

For PC, Lemma A4(a) shows that $\frac{\partial EU^{PC}}{\partial\sigma_{p,i}}<0$ for all $\sigma_{p,i}>0$, and Lemma A5(a) establishes that EU^{PC} increases in d_p. Since EU^{SM} does not depend on patient errors, the performance gap $EU^{SM}-EU^{PC}$ widens as $\sigma_{p,i}$ increase and d_p decreases.

For PH, Lemma A4(b) implies that EU^{PH} decreases in $\sigma_{p,i}$ either globally or once $\sigma_{p,i}$ is sufficiently large (equivalently, when $\beta_1\le2$ and $\beta_2\le2$). Moreover, Lemma A5(b) ensures that if $\sigma_{p,i}$ are sufficiently large (equivalently, when $\beta_1\le2$ and $\beta_2\le2$), then EU^{PH} also increases in d_p. As EU^{PM} is independent of patient error, the gap $EU^{PM}-EU^{PH}$ is therefore largest when d_p is small and $\sigma_{p,1},\sigma_{p,2}$ are large. ■