Online Appendix to

"Omnichannel Operations in On-Demand Delivery Platform with Buy-Online-and-Pick-up-in-Store"

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In this Online Appendix, we provide supporting proofs the main model, estimation details, robustness analyses, the details of model extensions, and additional practical examples. Detailed proofs for the direct-to-consumer-channel and endogenous-revenue-sharing extensions are provided in the Online Supplement (available in the full author manuscript at: https://papers.ssrn.com/abstract_id=4862038).

Appendix A: Proofs of Results.

Let us first derive the platform's optimal decisions in the conventional model and the omnichannel model, respectively.

Conventional Model.
Given $\rho = d_d/s$, by Equations (3) and (5), we have:

$$p_d = V - \frac{d_d \gamma R}{nw} - \frac{d_d V}{\lambda}. \tag{A.1}$$

There is a one-to-one relationship between p_d and d_d. Thus, we can treat (d_d, w), rather than (p_d, w), as the decision variables. Substituting the delivery-channel price above into Equation (6), we update the profit function of the platform π_p with decision variables d_d and w as:

$$\pi_p^C = d_d \left(-c_d - \frac{d_d \gamma R}{nw} - \frac{d_d V}{\lambda} + V \right) - d_d w. \tag{A.2}$$

Then, π_p^C is concave with respect to d_d and w. By the first-order conditions, we can solve for the platform's optimal decisions as follows.

Proposition A.1. *In the conventional model, the platform's optimal pricing decisions are:*

$$p_d^C = V - \sqrt{\frac{d_d^C \gamma R}{n}} - \frac{d_d^C V}{\lambda} \text{ and } w^C = \frac{\sqrt{\lambda \gamma R}\left(\sqrt{8nV(V-c_d) + 9\lambda\gamma R} - 3\sqrt{\lambda\gamma R}\right)}{4nV},$$

where $d_d^C = \frac{\lambda\left(\sqrt{8nV(V-c_d)+9\lambda\gamma R} - 3\sqrt{\lambda\gamma R}\right)^2}{16nV^2}$.

Following the platform's decisions, the merchant's equilibrium profit is derived by $\pi_m^C = d_d^C(c_d - \phi)$.

Omnichannel Model.
In the omnichannel model, the merchant and the platform make decisions sequentially. Thus, we begin by characterizing the platform's decision as follows. If the merchant chooses not to adopt BOPS, the resulting subgame equilibrium is identical to that of the base model. If the merchant adopts BOPS, the platform has two options:

O1. The platform shuts down the delivery channel by setting an extremely high p_d and provides only BOPS service. In this case, the profit function is:

$$\pi_p^{O1} = \max_{p_s} \left\{ d_s(p_s - c_s) \right\}. \tag{A.3}$$

Given the demand function (9), we can determine the optimal price $p_s^{O1} = \frac{1}{2}(V - k + c_s)$, and the corresponding profit is $\pi_p^{O1} = \frac{\beta\lambda(c_s + k - V)^2}{4V}$.

O2. The platform provides both delivery and BOPS services. If $p_d + \gamma\rho^2 - p_s - k < 0$, then no one chooses the BOPS option (i.e., $d_s = 0$), which is essentially the conventional model. If $p_d + \gamma\rho^2 - p_s - k \geq 0$, we have $d_d = (1-\beta)\ \lambda(V - p_d - \rho^2\ \gamma)/V$ and $d_s = \beta\ \lambda\ (V - k - p_s)/V$, where $\rho = d_d/s = \sqrt{d_d\ R/n/w}$. The last two equations can be equivalently expressed as:

$$p_d = -\frac{d_d\gamma R}{nw} + \frac{d_d V}{(\beta-1)\lambda} + V, \tag{A.4}$$

$$p_s = -k - \frac{d_s V}{\beta\lambda} + V. \tag{A.5}$$

Substituting them into the objective function (10), while considering $p_d + \gamma\rho^2 - p_s - k \geq 0$, the optimization problem is updated as follows:

$$\pi_p^{O2} = \max_{d_d, d_s, w}\left\{ d_s\left(-c_s - \frac{d_s V}{\beta\lambda} - k + V\right) + d_d\left(V - w - c_d - \frac{d_d\gamma R}{nw} + \frac{d_d V}{(\beta-1)\lambda}\right)\right\} \tag{A.6}$$

$$s.t., \quad d_d \leq \frac{(1-\beta)}{\beta}\ d_s. \tag{A.7}$$

It can be shown that the profit function is concave in d_d, d_s, and w, and that its Hessian matrix with respect to all decision variables is negative definite. Thus, the optimal solution $(d_d^{O2}, d_s^{O2}, w^{O2})$ can be summarized as follows, where we let $\gamma_0 = \frac{nV(k+c_s-c_d)^2}{(\beta-1)\lambda R(k+c_s-V)}$.

- If $\gamma \geq \frac{2}{9}\gamma_0$, then the constraint is not binding, and therefore $(d_d^{O2}, d_s^{O2}, w^{O2}) = (d_d^*, d_s^*, w^*)$ where $d_d^* = \frac{(1-\beta)\lambda\left(\sqrt{8nV(V-c_d)+9(1-\beta)\lambda\gamma R} - 3\sqrt{(1-\beta)\lambda\gamma R}\right)^2}{16nV^2}$, $d_s^* = \frac{\beta\lambda(V-k-c_s)}{2V}$, $w^* = \sqrt{d_d^*\gamma R/n}$, and the corresponding channel prices and platform profit are denoted by p_d^*, p_s^*, and π_p^* respectively;
- Otherwise, the constraint is binding, and therefore $(d_d^{O2}, d_s^{O2}, w^{O2}) = (d_d^{**}, d_s^{**}, w^{**})$ where $d_d^{**} = \frac{4(1-\beta)\lambda n(V-\beta(c_s+k)+(\beta-1)c_d)^2}{\left(\sqrt{8nV(V-\beta(c_s+k)+(\beta-1)c_d)+9(1-\beta)^3\lambda\gamma R} + 3(1-\beta)^{3/2}\sqrt{\lambda\gamma R}\right)^2}$, $d_s^{**} = \frac{\beta}{(1-\beta)} d_d^{**}$, $w^{**} = \sqrt{d_d^{**}\gamma R/n}$, and the corresponding channel prices and platform profit are denoted by p_d^{**}, p_s^{**}, and π_p^{**} respectively.

Following the platform's decisions, the merchant's profit under each option $i = 1, 2$ is expressed as $\pi_m^{Oi} = d_d^{Oi}(c_d - \phi) + d_s^{Oi}(c_s - \phi)$. By comparing the expected profit between adopting BOPS (π_m^O) and not adopting it (π_m^C), the merchant makes the adoption decision accordingly. Lemma 2 shows that the platform will never shut down the delivery channel, implying that $\pi_m^O = \pi_m^{O2}$. Therefore, $\pi_m^O = \pi_m^{O2}(d_d^*, d_s^*) \triangleq \pi_m^*$ if $\gamma < \frac{2}{9}\gamma_0$; otherwise, $\pi_m^O = \pi_m^{O2}(d_d^{**}, d_s^{**}) \triangleq \pi_m^{**}$. Since $\partial\gamma_0/\partial k > 0$, the condition of $\gamma < \frac{2}{9}\gamma_0$ is equivalent to $k > k_1$, where $k_1 = \frac{4nV(c_d-c_s)+3\sqrt{(\beta-1)\lambda\gamma R(8nV(c_d-V)+9(\beta-1)\lambda\gamma R)}+9(\beta-1)\lambda r R}{4nV}$. Moreover, we derive that both $\partial\pi_m^*/\partial k$ and $\partial\pi_m^{**}/\partial k$ are negative. Since $\pi_m^* > \pi_m^C$ when k is sufficiently large, it follows that there exists a threshold $K'(V, R, \lambda, n, \gamma, \beta, c_d, c_s)$ such that $\pi_m^O \geq \pi_m^C$ iff $k \leq K'(\cdot)$. This implies that the merchant will adopt BOPS if and only if $k \leq K'(\cdot)$.

Proof of Lemma 2. Note that $\partial\gamma_0/\partial\beta > 0$, then the condition of $\gamma < \frac{2}{9}\gamma_0$ is equal to $\beta > \beta_0$, where $\beta_0 = 1 + \frac{2nV(c_s-c_d+k)^2}{9\lambda\gamma R(c_s+k-V)}$. For $j \in \{p, m\}$, recall that $d_d^* = d_d^{O1}$, which immediately implies that $\pi_j^* > \pi_j^{O1}$ holds for $\beta < \beta_0$. Additionally, note that $\partial\pi_j^{**}/\partial\beta < 0$ and $\partial\pi^{O1}/\partial\beta > 0$. Since $\pi_j^{**}\big|_{\beta=1} = \pi_j^{O1}\big|_{\beta=1}$, it follows that $\pi_j^{O1} < \pi_j^{O2}$ for both $j \in \{p, m\}$ and all $\beta \in (0, 1)$. Therefore, Lemma 2 is established. □

Therefore, we can focus on option (O2) and obtain the following result:

Proposition A.2. *In the omnichannel model, the platform's optimal decision (p_d^O, p_s^O, w^O) is given as follows:*

(i) if $\gamma < \frac{2}{9}\gamma_0$, then $p_d^O = -\sqrt{\frac{d_d^{**}\gamma R}{n}} + \frac{d_d^{**}V}{(\beta-1)\lambda} + V$, $p_s^O = -k - \frac{d_s^{**}V}{\beta\lambda} + V$ and $w^O = w^{**}$;

(ii) if $\gamma \geq \frac{2}{9}\gamma_0$, then $p_d^O = -\sqrt{\frac{d_d^*\gamma R}{n}} + \frac{d_d^*V}{(\beta-1)\lambda} + V$, $p_s^O = -k - \frac{d_s^*V}{\beta\lambda} + V$ and $w^O = w^*$.

Proof of Lemma 1. Note that π_p^C is independent of k. We next discuss the monotonicity of π_p^O with respect to $k \in (0, V - c_s)$. Since the condition of $\gamma < \frac{2}{9}\gamma_0$ in Proposition A.2 is equivalent to $k > k_1(\cdot)$. Then, $\pi_p^O = \pi_p^*$ if $k \leq k_1$; otherwise, $\pi_p^O = \pi_p^{**}$. Because $\frac{\partial\pi_p^*}{\partial k} < 0$, $\frac{\partial\pi_p^{**2}}{\partial k^2} > 0$, $\frac{\partial\pi_p^{**}}{\partial k} = 0$ if and only if (iff) $k = \frac{V-c_d}{\beta} + c_d - c_s$, and $\frac{V-c_d}{\beta} + c_d - c_s > V - c_s$ (due to $c_d < c_s$), we can find that both π_p^* and π_p^{**} are decreasing in $k \in (0, V - c_s)$. Then, let us first compare π_p^* and π_p^C. Note that $\pi_p^C = max_{p_h, w}\{d_h(p_h - c_h - w)\}$ and $\pi_p^O = max_{p_h, p_b, w}\{d_h(p_h - c_h - w) + d_b(p_b - c_b)\}$. If $k \leq k_1$, the constraint $(d_d \leq (1-\beta)d_s/\beta)$ is not binding. Then, let (p_d, w) equal the conventional model equilibrium outcome (p_d^C, w^C), and thus there must exist p_s^c such that (p_d^C, w^C, p_s^c) is a feasible solution to the platform's optimization problem in the omnichannel model. Thus, $\pi_p^* \geq \pi_p^C$. As for π_p^{**}, we have $\frac{\partial \pi_p^{**}|_{k=V-c_s}}{\partial \beta} \leq 0$ and $\left(\pi_{p\ \ k=V-c_s}^{**}\right)|_{\beta=0} = \pi_p^C$; thus, $\pi_p^{**}|_{k=V-c_s} \leq \pi_p^C$. Combining the previous results that both π_p^* and π_p^{**} are decreasing in $k \in (0, V-c_s)$, we can show that there exists $K''(V, R, \lambda, n, \gamma, \beta, c_d, c_s)$ such that $\pi_p^O \geq \pi_p^C$ iff $k \leq K''(\cdot)$.

Moreover, since the merchant adopts BOPS when $k \leq K'(\cdot)$, we conclude that BOPS is profitable for both the platform and the merchant if and only if $k \leq K(\cdot)$, where $K(\cdot) = \min\{K'(\cdot), K''(\cdot)\}$. Otherwise, either the platform chooses not to offer the omnichannel feature, or—if it is offered—the merchant chooses not to adopt BOPS, resulting in a reversion to the conventional delivery-only equilibrium. □

Proof of Propositions 1 and 2. We first prove part (i) of Proposition 1. Note that, $w^C = \sqrt{d_d^C \gamma R/n}$ and $w^O = \sqrt{d_d^O \gamma R/n}$. So $w^O < w^C$ iff $d_d^O < d_d^C$. In the proof of Lemma 1, we know that $\gamma < \frac{2}{9}\gamma_0 \Leftrightarrow k > k_1$.

- When $k \leq k_1$, $d_d^O = d_d^*$. Note that d_d^* is the same as d_d^C with λ being replaced by $(1-\beta)\lambda$. Since $\partial d_d^C/\partial\lambda > 0$ and $(1-\beta)\lambda < \lambda$, we can conclude that d_d^C is larger than d_d^* for all $\beta \in (0,1)$.
- When $k > k_1$, $d_d^O = d_d^{**}$, where $\partial d_d^{**}/\partial k < 0$. Since $\partial d_d^*/\partial k = 0$, and d_d^O is continuous with k , we can conclude that the maximum of d_d^{**} for $k > k_1$ is lower than d_d^*. Since $d^* < d^C$, we have $d^{**} < d^C$.

In sum, $d_d^O < d_d^C$ for all k. Thus, we have $w^O < w^C$. In addition, since $\frac{\partial d_d^O}{\partial \lambda} > 0$, we obtain $\frac{\partial w^O}{\partial \lambda} = \frac{\partial}{\partial \lambda}\sqrt{\frac{d_d^O \gamma R}{n}} > 0$.
Next, we prove Proposition 1(ii) and Proposition 2. Note that the optimal delivery-channel price in the conventional model is $p_d^C = \frac{\sqrt{\lambda\gamma R}\sqrt{8nV(V-c_d)+9\lambda\gamma R}+4nV(c_d+V)-3\lambda\gamma R}{8nV}$; in the omnichannel model, the optimal delivery-channel price is $p_d^O = p_d^* = \frac{\sqrt{(1-\beta)\lambda\gamma R}\sqrt{8nV(V-c_d)+9(1-\beta)\lambda\gamma R}+4nV(c_d+V)-3(1-\beta)\lambda\gamma R}{8nV}$ if $\gamma \geq \frac{2}{9}\gamma_0$, and $p_d^O = p_d^{**} = -\left(\sqrt{8nV(-\beta(k+c_s)+(\beta-1)c_d+V)-9(\beta-1)^3\lambda\gamma R} - 3(1-\beta)^{3/2}\sqrt{\lambda\gamma R}\right)^2/(16nV) + 4\sqrt{\gamma R(\lambda-\beta\lambda)}\left(3(1-\beta)^{3/2}\sqrt{\lambda\gamma R} - \sqrt{8nV(-\beta(k+c_s)+(\beta-1)c_d+V)-9(\beta-1)^3\lambda\gamma R}\right)/(16nV) + 16nV^2/(16nV)$ otherwise. Then, we get $\frac{\partial p_d^C}{\partial \gamma} = \frac{2n^2V^2(c_d-V)^2}{3nV\lambda\sqrt{8nV(V-c_d)+9\lambda\gamma R}\left(\sqrt{8nV(V-c_d)+9\lambda\gamma R}+8nV(V-c_d)+18\lambda\gamma R\right)} > 0$. Also, $\frac{\partial p_d^*}{\partial \gamma} > 0$ owing to $p^C_{d\ |\lambda=\lambda(1-\beta)} = p_d^*$. In addition, $\frac{\partial p_d^{**}}{\partial \gamma} = \frac{(\beta-1)^2(3\beta-1)\gamma\sqrt{R}\left(6\sqrt{\mathcal{P}}\sqrt{\lambda\gamma R}-\mathcal{P}-9\lambda\gamma R\right)}{16\sqrt{\mathcal{P}}nV\sqrt{\lambda\gamma}}$, where $\mathcal{P} = 9\lambda\gamma R + \frac{8nV(-\beta(c_s+k)+(\beta-1)c_d+V)}{(1-\beta)^3} > 0$. Then, $\left(6\sqrt{\mathcal{P}}\sqrt{\lambda\gamma R} - \mathcal{P} - 9\lambda\gamma R\right)$ can be simplified to $-\frac{64n^2V^2(-\beta(c_s+k)+(\beta-1)c_d+V)^2}{(\beta-1)^6\left(6\sqrt{\mathcal{P}}\sqrt{\lambda\gamma R}+\mathcal{P}+9\lambda\gamma R\right)} < 0$. Thus, $\frac{\partial p_d^{**}}{\partial \gamma} < 0$ if and only if $\beta > 1/3$. In summary, $\frac{\partial p_d^C}{\partial \gamma}$ is always positive, whereas $\frac{\partial p_d^O}{\partial \gamma}$ is negative iff $\beta \geq 1/3$ and $\gamma < \frac{2}{9}\gamma_0$.
Recall $p_d^O = p_d^{**}$ if $\gamma < \frac{2}{9}\gamma_0$; otherwise, $p_d^O = p_d^*$. Note that $(p_d^{**} - p_d^C)_{|\gamma=0} = 1/2(c_s - c_d + k)\beta > 0$ (or $p_d^C < p_d^O$ for $\gamma = 0$.) Since $\partial p_d^C/\partial\lambda > 0$ and $p^C_{d\ |\lambda=\lambda(1-\beta)} = p_d^*$, we have $p_d^C > p_d^*$ (or $p_d^C > p_d^O$ for $\gamma \geq \frac{2}{9}\gamma_0$).

- *Case (i)*: $\beta \geq 1/3$. Because $p_d^C > p_d^*$, the continuity of p_d^O implies that $p_d^C > p_d^{**}$ when $\gamma = \frac{2}{9}\gamma_0$. In addition, since p_d^{**} decreases with γ, p_d^C increases with γ, and $p_d^C < p_d^{**}$ when $\gamma = 0$, there must exist a threshold $\Gamma(V, R, \lambda, n, k, \beta, c_d, c_s) > 0$ such that be $p_d^C < p_d^O$ iff $\gamma < \Gamma(\cdot)$.
- *Case (ii)*: $\beta < 1/3$. Since $p_d^C < p_d^{**}$ when $\gamma = 0$ and $p_d^C > p_d^{**}$ when $\gamma = \frac{2}{9}\gamma_0$, there must be at least one interaction between the functions $p_d^C(\gamma)$ and $p_d^{**}(\gamma)$ for $\gamma \in (0, \frac{2}{9}\gamma_0)$. But there cannot be only two intersection points between the two functions, as both are continuously increasing in $\gamma \in (0, \frac{2}{9}\gamma_0)$ and $p_d^C < (>) p_d^{**}$ when $\gamma = 0 (= \frac{2}{9}\gamma_0)$. Suppose there are more than two intersection points between the two functions. It is easy to find that $\frac{\partial^2 p_d^C}{\partial \gamma^2} < 0$, $\frac{\partial^2 p_d^{**}}{\partial \gamma^2} < 0$, $\frac{\partial^3 p_d^C}{\partial \gamma^3} > 0$, and $\frac{\partial^3 p_d^{**}}{\partial \gamma^3} < 0$. Based on the assumption, there exist three intersection points γ_1, γ_2, and γ_3 $(0 < \gamma_1 < \gamma_2 < \gamma_3 < \frac{2}{9}\gamma_0)$ where $p_d^C = p_d^{**}$. Then, We construct an auxiliary function $h(\gamma) = \frac{\partial p_d^{**}}{\partial \gamma} - \frac{\partial p_d^C}{\partial \gamma}$, and thus $\frac{\partial^2 h(\cdot)}{\partial \gamma^2} < 0$. Since $(p_d^{**} - p_d^C)_{|\gamma=0} > 0$ and $(p_d^{**} - p_d^C)_{|\gamma=\frac{2}{9}\gamma_0} < 0$, there must exist $\gamma_I \in (0, \gamma_1)$ and $\gamma_F \in (\gamma_3, \frac{2}{9}\gamma_0)$ such that $h(\gamma_I) > 0$ and $h(\gamma_F) < 0$. In addition, according to Rolle's Theorem, there must exist $\gamma_4 \in (\gamma_1, \gamma_2)$ and $\gamma_5 \in (\gamma_2, \gamma_3)$ such that $h(\gamma_4) = 0$ and $h(\gamma_5) = 0$. Furthermore, $h(\gamma) > 0$ for all $\gamma \in (\gamma_4, \gamma_5)$ due to the concavity of $h(\gamma)$. Therefore, $h(\gamma_4) = 0 < \alpha h(\gamma_i) + (1-\alpha)h(\gamma_j)$ for all $\alpha \in (0,1)$, $\gamma_i \in (\gamma_I, \gamma_4)$, and $\gamma_j \in (\gamma_4, \gamma_5)$, which contradicts the fact that $h(\gamma)$ is concave. Thus, the initial assumption is not correct. In other words, there exists a unique intersection point, $\gamma = \Gamma(\cdot)$, between $p_d^C(\gamma)$ and $p_d^{**}(\gamma)$, with $p_d^C < p_d^O$ iff $\gamma < \Gamma(\cdot)$.

Combining the results in the two cases above, we can conclude Proposition 1(ii). □

Proof of Proposition 3. Note that $\gamma < \frac{2}{9}\gamma_0$ is equivalent to $\lambda < \frac{2nV(k+c_s-c_d)^2}{9(\beta-1)\gamma R(k+c_s-V)}$. Then, we have: $p_d^O - p_s^O = k - \frac{\sqrt{d_d^{**}\ \gamma\ R}}{\sqrt{n}}$ if $\lambda < \frac{2nV(k+c_s-c_d)^2}{9(\beta-1)\gamma R(k+c_s-V)}$; otherwise, $p_d^O - p_s^O = p_d^* - \frac{1}{2}(-k+c_s+V)$. Since $\frac{\partial d_d^{**}}{\partial\lambda} > 0$ and $\frac{\partial p_d^*}{\partial\lambda} > 0$, we can conclude that $p_d^O - p_s^O$ decreases in $\lambda < \frac{2nV(k+c_s-c_d)^2}{9(\beta-1)\gamma R(k+c_s-V)}$ and increases in $\lambda > \frac{2nV(k+c_s-c_d)^2}{9(\beta-1)\gamma R(k+c_s-V)}$. Note that $(p_d^O - p_s^O)|_{\lambda=0} = k$, $(p_d^O - p_s^O)|_{\lambda=\frac{2nV(k+c_s-c_d)^2}{9(\beta-1)\gamma R(k+c_s-V)}} = (-c_s+c_d+2k)/3$, and $(p_d^O - p_s^O)|_{\lambda\to\infty} = (-3c_s+2c_d-3k+V)/6$. Therefore:

- When $k \geq \frac{c_s-c_d}{2}$, then $p_d^O - p_s^O$ is always non-negative.
- When $k < \frac{c_s-c_d}{2}$ and $(-3c_s+2c_d-3k+V) \leq 0$, the equation $p_d^O(\lambda) - p_s^O(\lambda) = 0$ has only one solution, i.e., $\lambda = \frac{2k^2nV}{\gamma R(1-\beta)(V+3k-c_d(1-\beta)-(c_s+4k)\beta)} \triangleq \lambda_1(V,R,\gamma,n,k,\beta,c_d,c_s)$; and thus $p_s^O > p_d^O$ iff $\lambda > \lambda_1(\cdot)$.
- When $k < \frac{c_s-c_d}{2}$ and $(-3c_s+2c_d-3k+V) > 0$, the equation $p_d^O(\lambda) - p_s^O(\lambda) = 0$ has two solutions, i.e., $\lambda = \lambda_1(\cdot)$ and $\lambda = \frac{2(c_s-c_d-k)^2nV}{\gamma R(V-3c_s+2c_d+3k)(1-\beta)} \triangleq \lambda_2'(V,R,\gamma,n,k,\beta,c_d,c_s)$; and thus $p_s^O > p_d^O$ iff $\lambda_1(\cdot) < \lambda < \lambda_2'(\cdot)$.

Set $\lambda_2(V,R,\gamma,n,k,\beta,c_d,c_s) = \frac{2(c_s-c_d-k)^2nV}{\gamma R(V-3c_s+2c_d+3k)^+(1-\beta)}$, where $(x)^+ = \max\{0,x\}$. Then, we can conclude that $p_s^O > p_d^O$ iff $k < \frac{c_s-c_d}{2}$ and $\lambda_1(\cdot) < \lambda < \lambda_2(\cdot)$. $\square$

Proof of Proposition 4. Lemma 1 ensures that, under the model assumption $k < K(V,R,\lambda,n,\gamma,\beta,c_d,c_s,\phi)$, both platform and merchant profits in the omnichannel model exceed those in the conventional model. Hence, parts (i) and (ii) of Proposition 4 follow immediately.

For part (iii), courier welfare in the conventional and omnichannel models can be expressed as $\Omega^C = \frac{d_d^{C\,3/2}}{2}\sqrt{\frac{\gamma R}{n}}$ and $\Omega^O = \frac{d_d^{O\,3/2}}{2}\sqrt{\frac{\gamma R}{n}}$. From the proof of Proposition 1, we have $d_d^O < d_d^C$, which directly implies $\Omega^C > \Omega^O$.

Lastly, we prove Proposition 4(iv). Consumer surplus in the omnichannel model can be expressed as $\Psi^O = \frac{d_d^{**2}V}{2(1-\beta)^2\lambda}$ if $\lambda < \frac{2(c_s-c_d+k)^2nV}{9\gamma R(V-c_s-k)(1-\beta)}$, and $\Psi^O = \frac{(c_b+k-V)^2\beta\lambda}{8V} + \frac{d_d^{*2}V}{2\lambda(1-\beta)\lambda}$ otherwise. While, consumer surplus in the conventional model can be expressed as $\Psi^C = \frac{d_d^{C\,2}V}{2\lambda}$. Therefore,

- When $\lambda < \frac{2(c_s-c_d+k)^2nV}{9\gamma R(V-c_s-k)(1-\beta)}$, the sign of $\Psi^O - \Psi^C$ is equivalent to that of $\frac{d_d^{**}}{1-\beta} - d_d^C$. Let $f_1 = -3\sqrt{\gamma R} + \sqrt{9\gamma R + 8nV(-c_d+V)/\lambda}$, and $f_2 = -3\sqrt{\gamma R}(1-\beta)^{\frac{3}{2}} + \sqrt{-9\gamma R(-1+\beta)^3 + 8nV\big(V+c_d(-1+\beta)-(c_s+k)\beta\big)/\lambda}$. It is straightforward to see that $f_1 > 0$ and $f_2 > 0$. Then $\frac{d_d^{**}}{1-\beta} - d_d^C = \frac{-f_1^2+f_2^2}{16nV^2}\lambda^2$. Therefore, the sign of $\Psi^O - \Psi^C$ is equivalent to the sign of $f_2 - f_1$.

Let $f_3 = \frac{\partial(f_2-f_1)}{\partial\lambda} \cdot \sqrt{9\gamma R\lambda^4 + 8nV(-c_d+V)\lambda^3} \cdot \sqrt{-9\gamma R(-1+\beta)^3\lambda^4 + 8nV\big(V+c_d(-1+\beta)-(c_s+k)\beta\big)\lambda^3}$. As $c_d+k > c_s$, f_3 increases with λ, and $f_3 < 0$ when $\lambda \to 0$, $f_3 > 0$ when $\lambda \to +\infty$. Thus, $f_2 - f_1$ decreases first and then increases with λ.

- When $\lambda \geq \frac{2(c_s-c_d+k)^2nV}{9\gamma R(V-c_s-k)(1-\beta)}$, $\Psi^O - \Psi^C = \frac{\lambda\left(64(c_d+k-V)^2V^2\beta - \frac{f_4(\lambda)^4}{n^2} + \frac{(1-\beta)\ f_4((1-\beta)\lambda)^4}{n^2}\right)}{512V^3}$, where $f_4(\lambda) = -3\sqrt{\gamma R\lambda} + \sqrt{8nV(V-c_d)+9\gamma R\lambda}$. Thus, the monotonicity of $(\Psi^O - \Psi^C)/\lambda$ in λ is equivalent to the monotonicity of $f_4((1-\beta)\lambda)\ (1-\beta)^{1/4} - f_4(\lambda) \triangleq g(\lambda)$ in λ.

The term $\lambda^2\ \frac{\partial}{\partial\lambda}\left(\frac{\partial g(\lambda)}{\partial\lambda}\cdot\frac{2\sqrt{\lambda}}{3\gamma R}\right)$ increases with λ, and tends to $-\infty$ as $\lambda \to 0$ and to $+\infty$ as $\lambda \to \infty$. Hence $\frac{\partial g(\lambda)}{\partial\lambda}\cdot\frac{2\sqrt{\lambda}}{3\gamma R}$ first decreases and then increases with λ. Noting that $\frac{\partial g(\lambda)}{\partial\lambda}\cdot\frac{2\sqrt{\lambda}}{3\gamma R}$ equals $\frac{1-(1-\beta)^{1/4}}{\sqrt{\gamma R}}$ as $\lambda \to 0$ and diverges to 0 as $\lambda \to \infty$, we conclude that $g(\lambda)$ first increases and then decreases in λ. Therefore, $(\Psi^O - \Psi^C)/\lambda$ first increases and then decreases with λ.

At $\lambda = 0$, we have $(\Psi^O - \Psi^C) = 0$, whereas $\lim_{\lambda\to\infty}(\Psi^O - \Psi^C) = \frac{(c_d+k-V)^2\beta\lambda}{8V}$. Therefore, $(\Psi^O - \Psi^C) > 0$ if and only if $\lambda > \Lambda(V,R,n,\gamma,\beta,c_d,c_s,\phi)$. This completes the proof of Proposition 4(iv). $\square$

Appendix B: Merchant's Direct-to-Consumer Channel

In this section, we consider the case where consumers can place orders through the merchants' direct-to-consumer (D2C) channel by calling the merchant directly.

LEMMA B.1. *In equilibrium under the conventional model, a positive demand exists in the D2C channel if and only if the pickup hassle cost $k < K_m(V,R,\lambda,n,\gamma,\beta,c_d,h,p_m)$.*

Since this extension aims to test the robustness of the main conclusions after considering the D2C channel, we will focus on the parameter regime where $k < K_m(\cdot)$ for the remainder of this extension.

Lemma B.2. *In the omnichannel model, the equilibrium demand for the delivery channel remains positive.*

Lemma B.2 shows that Lemma 2 continues to hold with a D2C channel.

Proposition B.1. *As a platform switches from the conventional model to the omnichannel model by adopting BOPS:*

(i) The platform reduces courier wages (i.e., $w^O < w^C$) if $\gamma < \Gamma_m(V, R, \lambda, n, \beta, c_d, c_s, k, h, p_m)$; otherwise, $w^O = w^C$;

(ii) The platform increases the delivery-channel price (i.e., $p_d^O > p_d^C$) if and only if $\gamma < \Gamma_m(V, R, \lambda, n, \beta, c_d, c_s, k, h, p_m)$.

Proposition B.1 demonstrates that the key findings from Proposition 1—namely, that the platform must decrease courier wages and may increase delivery-channel prices after implementing BOPS—remain valid in this model extension.

Proposition B.2. *In response to a surge in market size, the platform's adjustment of delivery-channel prices may vary between the conventional and omnichannel models.*

(i) Conventional model: p_d^C is always increasing in λ;

(ii) Omnichannel model: p_d^O is decreasing in λ if and only if $\gamma < \Gamma_m(V, R, \lambda, n, \beta, c_d, c_s, k, h, p_m)$ and $\beta > \boldsymbol{B}_m(V, c_s, k, h, p_m)$.

Proposition B.2 confirms the insight in Proposition 2, i.e., surge pricing is an optimal strategy in response to a demand surge in the conventional model, but not necessarily in the omnichannel model.

Proposition B.3. *In the omnichannel model, the BOPS-channel price is higher than the delivery-channel price (i.e., $p_s^O > p_d^O$) if and only if $k < K'_m(V, c_d, c_s, h, p_m)$ and $\Lambda_m(V, R, \gamma, n, \beta, c_d, c_s, k, h, p_m) < \lambda < \Lambda'_m(V, R, \gamma, n, \beta, c_d, c_s, k, h, p_m)$.*

Proposition B.3 shows that the key finding in Proposition 3, i.e., the platform may need to charge a higher price in the BOPS channel than in the delivery channel when it offers BOPS service, is still valid.

Proposition B.4. *If the BOPS option is available to consumers in equilibrium, then, compared to the equilibrium outcome in the conventional model:*

(i) Platform profit is higher (i.e., $\pi_p^O > \pi_p^C$);

(ii) Merchant profit is higher (i.e., $\pi_m^O > \pi_m^C$);

(iii) Courier welfare is lower (i.e., $\Omega^O \leq \Omega^C$);

(iv) Suppose $\lambda \geq \frac{2nV(-c_d+c_s+k)^2}{9(1-\beta)\gamma R(V-c_s-k)}$. Then, consumer surplus is higher (i.e., $\Psi^O \geq \Psi^C$) if and only if $h \geq \frac{1}{2}(c_s - k - 2p_m + V)$.

Proposition B.4 shows that the key finding in Proposition 4—(i) higher platform and merchant profits, (ii) lower courier welfare, and (iii) the conditional improvement in consumer surplus—remain valid under the new setting.

Appendix C: Model Parameter Estimation

C.1. Parameter Calibration

Data Preparation. For our model estimation, we focus on orders placed between 10:00 am and 6:59 pm, which account for 82.03% of total transactions.[16] To account for temporal fluctuations in demand, we employ a sliding window method to construct a dynamic pool of active consumers, varying at a day-hour level. Within hour h of day t ($h \in \mathcal{H} :=$ {10-11 am, 11-12 pm, ..., 6-7 pm} and $t \in \mathcal{T} :=$ {7/1/2015, 7/2/2015, ..., 8/31/2015}), any consumers who placed at least one order within hour h between day $t-3$ and day $t+3$

[16] Working during late-night hours (8 pm to 12 am) involves overtime and driving safety concerns for couriers, which can potentially confound the calibration of courier participation. Additionally, morning orders (12 am to 9 am) account for only 4.60% of total transactions. Moreover, these orders stem from a small group of 6,510 unique consumers who collectively placed 13,593 orders over two months, introducing significant noise into the estimation of congestion and price effects.

are selected to form the pool for hour h on day t; we use $\mathcal{I}_{ht}$ to denote the set of consumers in the pool and use $\widehat{\lambda}_{ht}$ to denote its size (i.e., $\widehat{\lambda}_{ht} = |\mathcal{I}_{ht}|$). [17] To capture labor-supply dynamics at the hourly level, we apply the same method to identify a pool of couriers who completed at least one delivery during hour h on day t. We use $\mathcal{J}_{ht}$ to denote the set of couriers in the pool and $\widehat{n}_{ht}$ to denote the pool size ($\widehat{n}_{ht} = |\mathcal{J}_{ht}|$) within hour h on day t.

Model Estimation. We begin by estimating consumer-related parameters, i.e., the maximum valuation of a transaction, V (CNY per order), and consumer sensitivity to congestion, γ (CNY per unit of delivery congestion), using maximum likelihood estimation (MLE). For consumer i in the pool of hour h on day t ($i = 1, \ldots, \widehat{\lambda}_{ht}$), we let v_{iht} denote this consumer's valuation, which we assume is identically and independently distributed from $U[0, V]$. In addition, delivery-channel prices are highly variable in the data, yet they are observed only for consumers who place an order. To address this censoring issue, we assume that the delivery-channel price is identically and independently distributed according to a log-normal distribution, $\log\mathcal{N}(\mu_p, \sigma_p^2)$, where μ_p and σ_p^2 are unknown parameters to estimate; we use f_p and F_p to denote its probability density function and cumulative distribution function, respectively. Hence, the parameters to estimate are denoted by $\boldsymbol{\theta} = (V, \gamma, \mu_p, \sigma_p)^T$.

Within hour h on day t, we use p_{iht} and ρ_{ht} to denote the delivery-channel price and congestion encountered by consumer i. The likelihood that consumer i placed an order at price p_{iht} (observed) is given by:

$$\mathbb{P}_{v_{iht}, p_{iht}}(v_{iht} - p_{iht} - \gamma\rho_{ht}^2 \geq 0) = \mathbb{P}_{v_{iht}}(v_{iht} - p_{iht} - \gamma\rho_{ht}^2 \geq 0 | p_{iht})\ f_p(p_{iht}) = \left[(V - p_{iht} - \gamma\rho_{ht}^2)/V\right]^+ f_p(p_{iht}).$$

Meanwhile, the probability that consumer i did not place an order is (p_{iht} is unobserved):

$$\mathbb{P}_{v_{iht}, p_{iht}}(v_{iht} - p_{iht} - \gamma\rho_{ht}^2 < 0) = \int_0^\infty \mathbb{P}_{v_{iht}}(v_{iht} - p - \gamma\rho_{ht}^2 < 0 | p)\ dF_p(p) = \int_0^\infty \min\left\{(p + \gamma\rho_{ht}^2)/V, 1\right\}\ dF_p(p).$$

Additionally, we use $Y_{iht} = 1$ to indicate that consumer i placed an order, and $Y_{iht} = 0$ to indicate that he/she did not place an order within an hour h on day t. Hence, the likelihood function of $\boldsymbol{\theta}$ given data $\mathcal{Z} = \{p_{iht}, \rho_{ht}, Y_{iht}\}$ can be characterized as:

$$\mathcal{L}(\boldsymbol{\theta}|\mathcal{Z}) = \prod_{t\in\mathcal{T}}\prod_{h\in\mathcal{H}}\prod_{i\in\mathcal{I}_{ht}} \left\{ \left[\int_0^\infty \min\left\{\frac{p + \gamma\rho_{ht}^2}{V}, 1\right\}\ dF_p(p)\right]^{1-Y_{iht}} \left[\left(\frac{V - p_{iht} - \gamma\rho_{ht}^2}{V}\right)^+ \ f_p(p_{iht})\right]^{Y_{iht}} \right\}.$$

Therefore, the MLE of $\boldsymbol{\theta}$ can be obtained by $\widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}}\ \log[\mathcal{L}(\boldsymbol{\theta}|\mathcal{Z})]$.

Second, we estimate omni-consumers' pickup hassle cost, k (CNY per order), in a nonparametric manner. In practice, k captures consumers' opportunity cost due to the time spent on a pickup trip. Let τ denote the expected pickup time; then, k can be denoted by $\varkappa \cdot \tau$, where $\varkappa$ is the opportunity cost per unit of time. We assume that the average travel speed is 5 km per hour in the city[18]. Using the widely adopted Haversine formula, the delivery distance (which is also the pickup distance) x can be calculated accurately from our dataset as
$x = 6371 \cdot 2 \cdot \arcsin\sqrt{\sin^2\left(\frac{Lat_2 - Lat_1}{2}\right) + \cos(Lat_1)\cdot\cos(Lat_2)\cdot\sin^2\left(\frac{Lon_2 - Lon_1}{2}\right)}$, where Lat_j and Lon_j ($j = 1, 2$) denote the latitude and longitude in radians of the consumer and merchant, respectively. For each potential consumer i, the location is assumed to remain fixed within a specific time window. Then, the pickup time for a potential consumer who lives x_i km away from the restaurant is $\frac{2x}{5}$. We further note that the average hourly income in Hangzhou is 40.29 CNY/hour,[19] implying $\varkappa = 40.29$. Consequently, the pickup hassle cost within hour h on day t is estimated by $\widehat{k}_{ht} = \varkappa \cdot \frac{2}{5} \cdot \frac{1}{\widehat{\lambda}_{ht}} \sum_{i\in\mathcal{I}_{ht}} x_i$.

Third, we nonparametrically calibrate the platform's revenue shares, c_s and c_d (CNY per order), as well as the merchant's menu price and production cost, p_m and ϕ (CNY per order). Meal-delivery platforms

[17] In addition, for each active consumer in the pool, we keep only one of their orders in each hour per day in our sample, excluding 4.19% of total transactions.

[18] See Zhang, Tang, Liu, and Cheng, "Reducing Traffic Incidents in Meal Delivery: Penalize the Platform or Its Independent Drivers?" *Manufacturing & Service Operations Management*, forthcoming.

[19] See statistical bulletin on annual average wages issued by the Hangzhou Bureau of Statistics, available at https://www.hangzhou.gov.cn/art/2024/3/1/art_1229063404_4242877.html.

typically charge different commission rates for orders fulfilled in different channels. For example, Uber Eats, DoorDash, and Grubhub set commission rates for pickups between 6-10%, whereas their delivery-commission rates range between 15-30%. [20] In the platform we cooperate with, commission rates are 20% of the menu price for delivery orders and 8% for BOPS-channel orders. In our model, c_d and c_s represent the expected delivery- and BOPS-channel costs incurred by the platform. Thus, we approximate c_d and c_s by subtracting the platform's commission revenue from the calibrated menu price: $\widehat{c}_d = (1-20\%)\widehat{p}_m$ and $\widehat{c}_s = (1-8\%)\widehat{p}_m$. In terms of gross merchandise value, the partner platform collects approximately 6–8% (commonly reported as a technology service fee). The remaining revenue is distributed between couriers and merchants. For delivery-only scenario, the platform pays a fixed wage w to couriers and remits 80% of the menu price to merchants as shared revenue. In our setting, this translates to the platform's profit share in the conventional model being: $(1-7\%)\cdot\mathbb{E}_p[p|u\geq 0] = 80\%\ \widehat{p}_m + \mathbb{E}[w_{iht}]$, where $\mathbb{E}[p|u\geq 0] = \mathbb{E}_p[p|\widehat{V} - p - \widehat{\gamma}\mathbb{E}[\rho^2]\geq 0]$.
On the merchant side, industry reports suggest that the gross profit margin of the catering industry is 60%–80%[21]. We therefore impose $(80\%\ \widehat{p}_m - \phi)/(80\%\ \widehat{p}_m) = 70\%$, and accordingly calibrate the per-order production cost ϕ as $\widehat{\phi} = 80\%\ \widehat{p}_m\ (1-70\%)$.

Fourth, we estimate the courier-related parameter, R (CNY per hour), which is the upper bound of the courier's outside option, using nonlinear least squares (NLS). Per our model, the opportunity cost of courier j within hour h on day t, denoted by r_{jht}, is assumed to be identically and independently distributed as $U[0,R]$. Then, we use $\tilde{w}_{jiht}$ to denote courier j's income for delivering consumer i's order within hour h on day t. Hence, the average income of participating couriers within hour h on day t can be written as $\frac{1}{s_{ht}}\sum_{j\in\mathcal{J}_{ht}}\sum_{i\in\mathcal{I}_{ht}}\tilde{w}_{jiht}\cdot\delta_{jiht}$, where dummy δ_{jiht} indicates whether or not courier j served consumer i's order and s_{ht} denotes the number of active couriers during the hour. On one hand, the proportion of couriers participating in delivery services within hour h on day t is equal to $\frac{s_{ht}}{\widehat{n}_{ht}}$. On the other hand, Equation (5) of our model suggests that the same proportion can also be denoted by $\frac{1}{R\cdot s_{ht}}\sum_{j\in\mathcal{J}_{ht}}\sum_{i\in\mathcal{I}_{ht}}\tilde{w}_{jiht}\cdot\delta_{jiht}$. Therefore, we construct the following loss function to estimate R using NLS:

$$\widehat{R} = \arg\max_{R}\left[-\sum_{h\in\mathcal{H}}\sum_{t\in\mathcal{T}}\left(\frac{1}{R\cdot s_{ht}}\sum_{j\in\mathcal{J}_{ht}}\sum_{i\in\mathcal{I}_{ht}}\tilde{w}_{jiht}\cdot\delta_{jiht} - \frac{s_{ht}}{\widehat{n}_{ht}}\right)^2\right].$$

Lastly, we calibrate $1-\beta$, which represents the proportion of delivery-only consumers, nonparametrically. By definition, delivery-only consumers are those who do not choose BOPS even when the price of the BOPS channel is reduced to the lowest possible channel cost, c_s—a price that yields zero platform profit. A consumer is categorized as delivery-only if their utility from choosing BOPS at this lowest price is negative. That is, $1-\beta = \mathbb{E}\left[\mathbb{P}(u_s<0|p_s=c_s)\right] = \mathbb{E}_x\left[\min\left(\frac{c_s+\varkappa\cdot\frac{2}{5}x}{V},1\right)\right]$. Then, the plug-in estimator β_{ht} of the consumer pool λ_{ht} at hour h on day t can be obtained by:

$$\widehat{\beta}_{ht} = 1 - \frac{1}{\widehat{\lambda}_{ht}}\sum_{i\in\mathcal{I}_{ht}}\min\left(\frac{\widehat{c}_s + \widehat{\varkappa}\cdot\frac{2}{5}x_{iht}}{\widehat{V}},1\right).$$

Estimation Results. Table 1 in the main text reports the global parameters estimated in our model (V, γ, R, c_d, c_s, and ϕ), along with the averages of time-varying parameters (λ, n, k, and β). Beyond the reported averages, for each hour $h\in\mathcal{H} := \{$10–11 am, 11–12 pm, ..., 6–7 pm$\}$ and each day $t\in\mathcal{T} :=$ $\{$7/1/2015, 7/2/2015, ..., 8/31/2015$\}$ in our dataset, the market size λ, labor pool size n, pickup hassle cost k, and proportion of omni-consumers β range from 1,077 to 5,264; 260 to 383; 19.48 to 24.66; and 0.43 to 0.45, respectively. Furthermore, we obtain three intermediate parameter estimates—$\mu_p = 5.80$, $\sigma_p = 1.47$, and $p_m = 113.32$—which are not reported in the main text but serve as essential inputs for the calibration.

Appendix D: Cross-Sectional Heterogeneity in Pickup Hassle Cost

In this section, we extend the analysis to consider two alternative specifications of pickup hassle cost κ among omni-consumers, both based on lognormal distributions, as described in §6.2.

[20] See the marketplace plans for Uber Eats at `https://merchants.ubereats.com/ca/en/pricing/`, DoorDash at `https://get.doordash.com/en-us/products`, and Grubhub at `https://get.grubhub.com/blog/understanding-grubhub-fees/`.

[21] See `https://img3.gelonghui.com/pdf/238c7-fe37a1cf-b801-4595-82f2-773630515c51.pdf`.

On the merchant side, we numerically solve Problem (19) to obtain the platform's equilibrium decisions (p_d^O, p_s^O, w^O) and the resulting channel demands (d_d^O, d_s^O). The merchant's profit under BOPS is then $\pi_m^O = d_d^O(c_d - \phi) + d_s^O(c_s - \phi)$. The merchant adopts BOPS if and only if this profit exceeds its profit in the conventional model with a single delivery channel, $\pi_m^C = d_d^C(c_d - \phi)$.

Finally, note that the platform's profit in the conventional model without BOPS is not affected by the pickup-hassle distribution and is therefore the same as in the main model: $\pi_p^C = d_d^C\left(p_d^C - c_d - w^C\right)$. Accordingly, BOPS is available to consumers if and only if both the platform and the merchant prefer the BOPS regime, i.e., $\pi_p^O > \pi_p^C$ and $\pi_m^O > \pi_m^C$.

The expression for courier welfare remains unchanged under heterogeneous pickup cost settings, namely $\Omega = n\int_0^{\rho w} \frac{\rho w - r}{R} dr$. By contrast, consumer surplus should be updated as follows:

- If BOPS is not available to consumers,

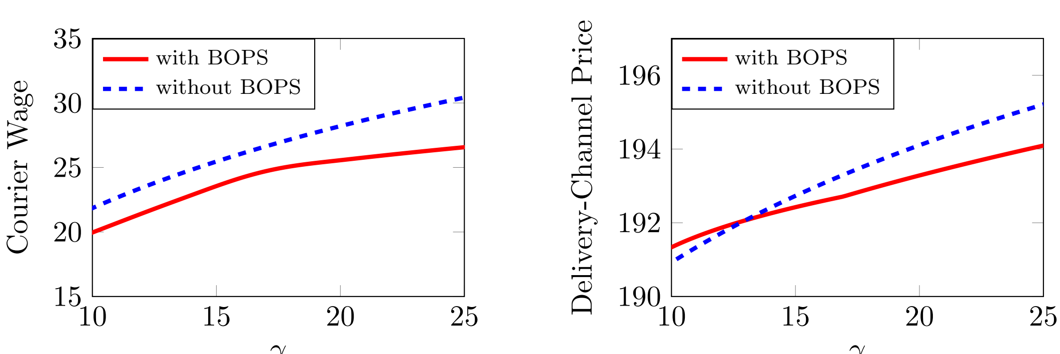
$$\Psi^C = \lambda \int_{p_d+\gamma\rho^2}^{V} \frac{v - p_d - \gamma\rho^2}{V} dv.$$

- If BOPS is available to consumers,

$$\Psi^O = \lambda \left(\int_{p_d+\gamma\rho^2-p_s}^{+\infty} f(\kappa) \left(\int_{p_d+\gamma\rho^2}^{V} \frac{v - p_d - \gamma\rho^2}{V} dv \right) d\kappa + \int_0^{p_d+\gamma\rho^2-p_s} f(\kappa) \left(\int_{p_s+\kappa}^{V} \frac{v - p_s - \kappa}{V} dv \right) d\kappa \right).$$

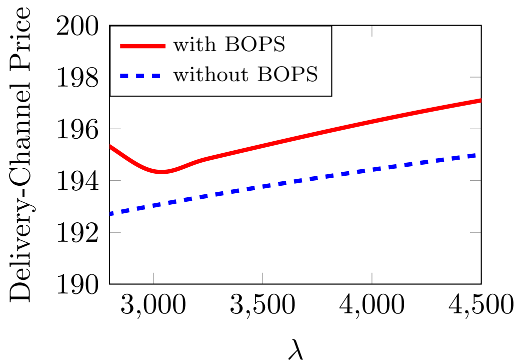
We numerically solve the game under both pickup-hassle specifications introduced in §6.2 and examine whether the main results in Propositions 1–4 continue to hold. Figures D.1–D.4 present the corresponding results under the Lognormal–Mass-Point specification, and Figures D.5–D.8 present the corresponding results under the Two-Lognormal Mixture specification. These figures collectively reinforce the key insights established in the main analysis. Figures D.1 and D.5 show that BOPS adoption tends to suppress courier wages and, under low congestion sensitivity (γ), may even raise the delivery-channel price—consistent with Proposition 1. Figures D.2 and D.6 further illustrate the nuanced impact of surge pricing: while effective under the conventional model during demand spikes, it becomes suboptimal in the omnichannel setting when the market size is limited. Instead, lowering the delivery-channel price emerges as the optimal response as the market size increases, consistent with Proposition 2. Figures D.3 and D.7 highlight that the platform may optimally set a higher BOPS-channel price than the delivery-channel price in certain conditions (i.e., low market size), corroborating the result in Proposition 3. Finally, Figures D.4 and D.8 support the findings of Proposition 4, demonstrating that BOPS introduction reduces courier welfare and may also reduce consumer surplus.

Figure D.1: Impact of congestion sensitivity (γ) on equilibrium courier wage (w^C, w^O) and delivery-channel price (p_d^C, p_d^O) under the Lognormal–Mass-Point specification.



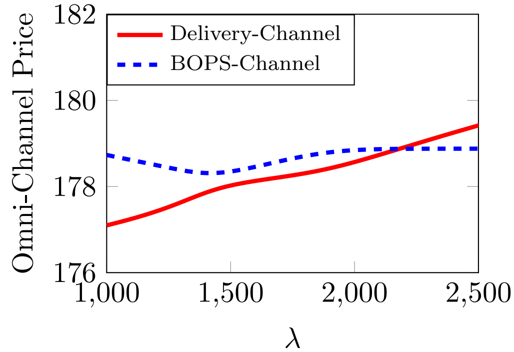
Note. The following parameter values are used: $V = 270$, $R = 30$, $c_d = 90$, $c_s = 100$, $\phi = 30$, $\lambda = 2500$, $\beta = 0.5$, $\mu_k = 4.2$, $\sigma_k = 0.32$, and $n = 333$.

Figure D.2: Impact of market size (λ) on delivery-channel prices under the Lognormal–Mass-Point specification.



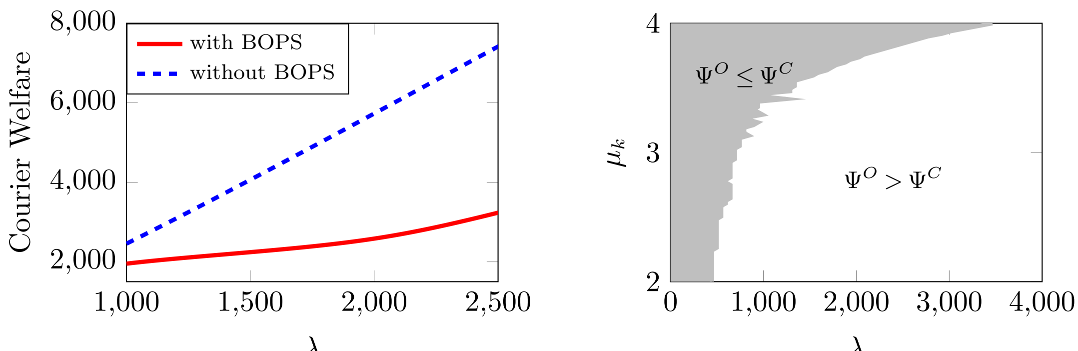
Note. The following parameter values are used: $V=270$, $\gamma=20$, $R=20$, $c_d=90$, $c_s=100$, $\phi=30$, $\beta=0.4$, $\mu_k=4.2$, $\sigma_k=0.32$, and $n=333$.

Figure D.3: Impact of market size (λ) on channel prices in omnichannel model under the Lognormal–Mass-Point specification.



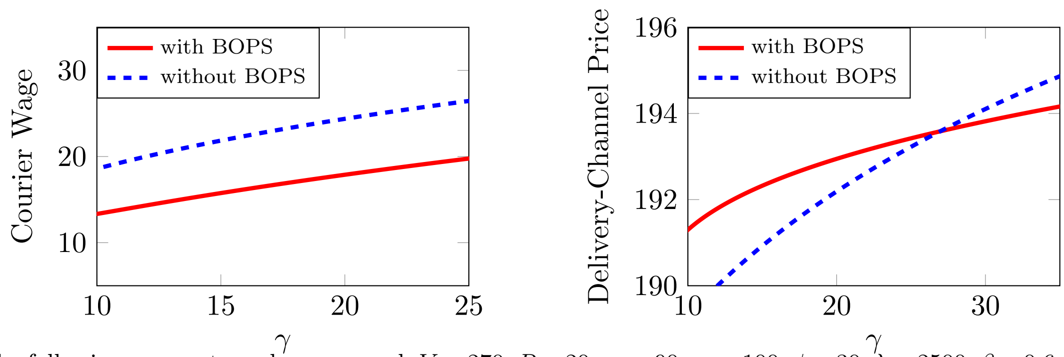
Note. The following parameter values are used: $V=270$, $\gamma=20$, $R=20$, $c_d=69$, $c_s=100$, $\phi=30$, $\beta=0.5$, $\mu_k=2.5$, $\sigma_k=0.32$, and $n=333$.

Figure D.4: Impact of parameters on equilibrium courier welfare and consumer surplus across models under the Lognormal–Mass-Point specification.



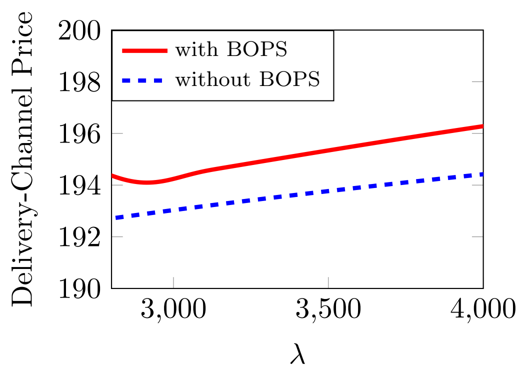
Note. The following parameter values are used: $V=270$, $\gamma=20$, $R=20$, $c_d=69$, $c_s=100$, $\phi=30$, $\beta=0.5$, $\sigma_k=0.32$, and $n=333$. The left panel uses $\mu_k=3$.

Figure D.5: Impact of congestion sensitivity (γ) on equilibrium courier wage (w^C, w^O) and delivery-channel price (p_d^C, p_d^O) under the Two-Lognormal Mixture specification.



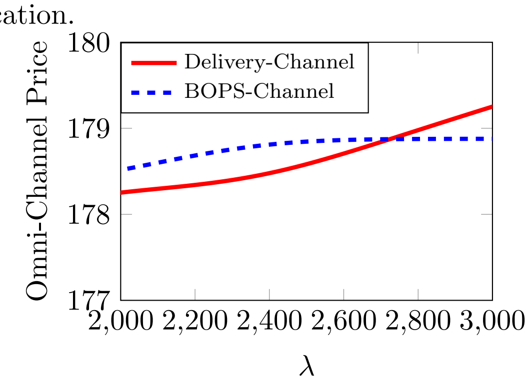
Note. The following parameter values are used: $V = 270$, $R = 20$, $c_d = 90$, $c_s = 100$, $\phi = 30$, $\lambda = 2500$, $\beta = 0.6$, $\mu_k' = 4.2$, $\mu_k'' = 5$, $\sigma_k' = \sigma_k'' = 0.32$, and $n = 333$.

Figure D.6: Impact of market size (λ) on delivery-channel prices under the Two-Lognormal Mixture specification.



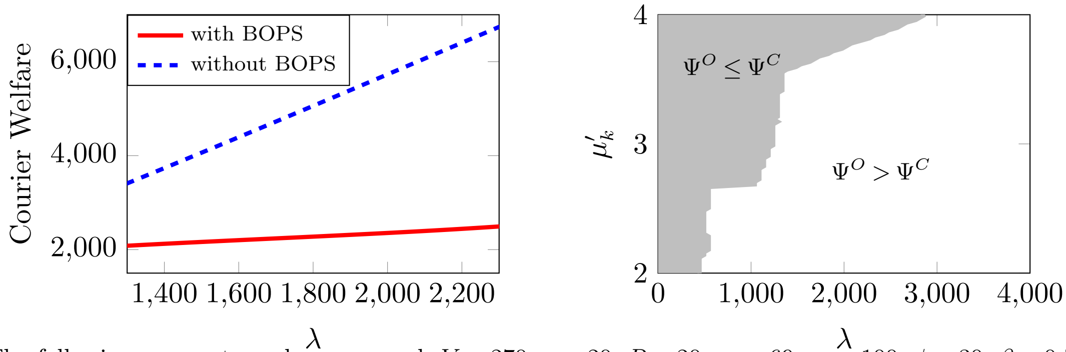
Note. The following parameter values are used: $V = 270$, $\gamma = 20$, $R = 20$, $c_d = 90$, $c_s = 100$, $\phi = 30$, $\beta = 0.4$, $\mu_k' = 4.2$, $\mu_k'' = 5$, $\sigma_k' = \sigma_k'' = 0.32$, and $n = 333$.

Figure D.7: Impact of market size (λ) on channel prices in omnichannel model under the Two-Lognormal Mixture specification.



Note. The following parameter values are used: $V = 270$, $\gamma = 20$, $R = 20$, $c_d = 69$, $c_s = 100$, $\phi = 30$, $\beta = 0.5$, $\mu_k' = 2.5$, $\mu_k'' = 3.5$, $\sigma_k' = \sigma_k'' = 0.32$, and $n = 333$.

Figure D.8: Impact of parameters on equilibrium courier welfare and consumer surplus across models under the Two-Lognormal Mixture specification.



Note. The following parameter values are used: $V=270$, $\gamma=20$, $R=20$, $c_d=69$, $c_s=100$, $\phi=30$, $\beta=0.5$, $\mu'_k=3$, $\mu''_k=4$, $\sigma'_k=\sigma''_k=0.32$, and $n=333$.

Appendix E: Heterogeneous Consumers' Congestion Sensitivity

In this section, we explore scenarios in which consumers exhibit heterogeneity in their channel choice behavior. To realize this, we assume that consumers' congestion sensitivity, $\tilde{\gamma}$, is independently and uniformly distributed within the interval $[\mu_\gamma-\delta_\gamma,\mu_\gamma+\delta_\gamma]$ and is independent of their product valuations and types. Everything else remains the same as in the main model. Therefore, we can derive the demand in each channel, as well as the merchants and platform profits, as follows.

In the conventional model, the consumer's utility from the delivery channel is: $u_d=v-p_d-\tilde{\gamma}\rho^2$. Since $v\sim U[0,V]$ and $\tilde{\gamma}\sim U[\mu_\gamma-\delta_\gamma,\mu_\gamma+\delta_\gamma]$ are independently distributed, the demand for the channel is given by: $d_d=\lambda\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde{\gamma}\rho^2}^{V}\frac{1}{V}dv d\tilde{\gamma}$. On the supply side, the number of participating couriers is described in Equation 5. The platform's profit maximization problem can be formulated as follows:

$$\pi_p^C=\max_{p_d,w}\left\{d_d\left(p_d-c_d-w\right)\right\}.$$

After the platform provides the BOPS service and merchants adopt it, omnichannel consumers consider using the BOPS option, which generates a utility as follows: $u_s=v-p_s-k$. If BOPS is available for consumers in the omnichannel model, omni-consumers will opt for the BOPS service if $u_s\geq\max\{u_d,0\}$; otherwise, they will choose the delivery option if $u_d\geq 0$. As for delivery-only consumers, they simply decide whether to use the delivery channel or not. Based on consumer decisions, we update the number of delivery requests d_d and the number of BOPS requests d_s for this subgame as follows:

$$\begin{aligned}d_d=&\lambda\beta\left(\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde{\gamma}\rho^2}^{p_s+k}\frac{1}{V}dv d\tilde{\gamma}+\int_{\mu_\gamma-\delta_\gamma}^{\frac{p_s+k-p_d}{\rho^2}}\frac{1}{2\delta_\gamma}\int_{p_s+k}^{V}\frac{1}{V}dv d\tilde{\gamma}\right)+\lambda(1-\beta)\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde{\gamma}\rho^2}^{V}\frac{1}{V}dv d\tilde{\gamma};\\ d_s=&\lambda\beta\int_{\frac{p_s+k-p_d}{\rho^2}}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_s+k}^{V}\frac{1}{V}dv d\tilde{\gamma}.\end{aligned}$$

Combining the above channel demand and courier supply shown in Equation 5, the platform's profit maximization problem in this subgame can be summarized as follows:

$$\pi_p^O=\max_{p_d,p_s,w}\left\{d_d\left(p_d-c_d-w\right)+d_s\left(p_s-c_s\right)\right\}.$$

On the merchant side, given the resulting demands (d_d^O,d_s^O) and the revenue in each channel (c_d,c_s), the merchant's profit is $\pi_m^O=d_d^O(c_d-\phi)+d_s^O(c_s-\phi)$. Finally, the merchant will adopt BOPS if and only if the profit from doing so (i.e., π_m^O) exceeds the profit in the conventional model, which involves only a single delivery channel and is given by $\pi_m^C=d_d^C(c_d-\phi)$.

Solving the model analytically becomes intractable. Thus, we employ a numerical approach to address this issue. The results are presented in Figures E.1-E.3. These figures further confirm the robustness of our main findings. Figure E.1 illustrates that adopting BOPS leads to a lower courier wage, and it may also result in an increase in the delivery-channel price when consumer average congestion sensitivity μ_γ is low; this is consistent with the result in Proposition 1. The left panel in Figure E.2 confirms the insight in Proposition 2, i.e., surge pricing is an optimal strategy in response to a demand surge in the conventional model, but not necessarily in the omnichannel model, especially when the average congestion sensitivity is low. The right panel in Figure E.2 shows that the key finding in Proposition 3, i.e., the platform may need to charge a higher price in the BOPS channel than in the delivery channel when it offers BOPS service, is still valid after considering the heterogeneous congestion sensitivity.

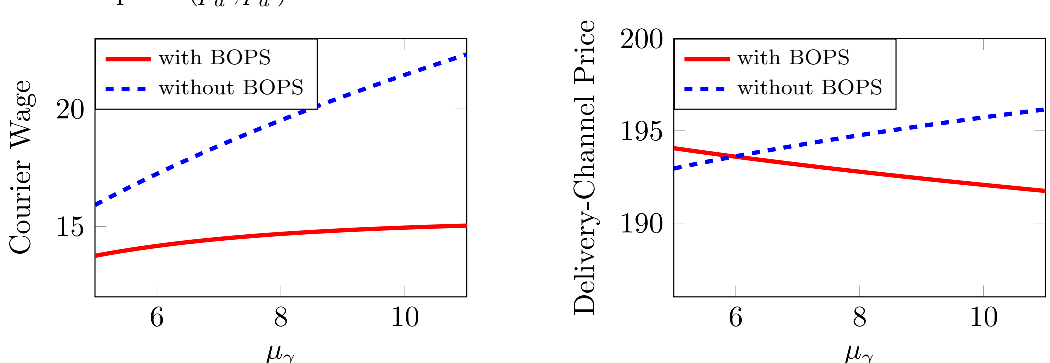
The expression for courier welfare remains unchanged under heterogeneous congestion sensitivity, that is $\Omega = n\int_0^{\rho w}\frac{\rho w - r}{R}dr$. By contrast, consumer surplus should be updated as follows:

- If BOPS is not available to consumers, $\Psi^C = \lambda\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde\gamma\rho^2}^{V}\frac{v-p_d-\tilde\gamma\rho^2}{V}dvd\tilde\gamma$.
- If BOPS is available to consumers,

$$\begin{aligned}\Psi^O =& \lambda\beta\left(\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde\gamma\rho^2}^{p_s+k}\frac{v-p_d-\tilde\gamma\rho^2}{V}dvd\tilde\gamma + \int_{\mu_\gamma-\delta_\gamma}^{\frac{p_s+k-p_d}{\rho^2}}\frac{1}{2\delta_\gamma}\int_{p_s+k}^{V}\frac{v-p_d-\tilde\gamma\rho^2}{V}dvd\tilde\gamma\right)\\ &+\lambda(1-\beta)\int_{\mu_\gamma-\delta_\gamma}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_d+\tilde\gamma\rho^2}^{V}\frac{v-p_d-\tilde\gamma\rho^2}{V}dvd\tilde\gamma + \lambda\beta\int_{\frac{p_s+k-p_d}{\rho^2}}^{\mu_\gamma+\delta_\gamma}\frac{1}{2\delta_\gamma}\int_{p_s+k}^{V}\frac{v-k-p_s}{V}dvd\tilde\gamma.\end{aligned}$$

Finally, Figure E.3 supports the findings of Proposition 4, demonstrating that the introduction of BOPS reduces courier welfare and may also reduce consumer surplus.

Figure E.1: Impact of average congestion sensitivity (μ_γ) on equilibrium courier wage (w^C, w^O) and delivery-channel price (p_d^C, p_d^O).



Note. The following parameter values are used: $V=250$, $\delta_\gamma=2$, $k=20$, $\beta=0.7$, $R=20$, $c_d=70$, $c_s=90$, $\lambda=2500$, $\phi=30$, and $n=300$.

Appendix F: Endogenous Revenue Sharing in the BOPS Channel

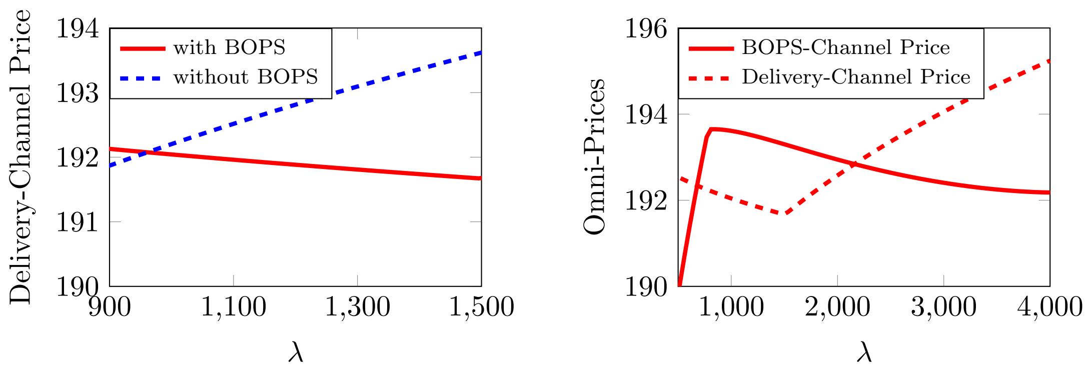
In the main model, we treated the revenue sharing in the BOPS channel as exogenously given. In this section, we endogenize the platform's decision regarding the BOPS revenue shared with the merchant (i.e., c_s). Specifically, in the omnichannel model, the platform first determines c_s, after which the merchant decides whether to adopt BOPS. Finally, the platform sets the price(s) for each available channel and the courier wage. We note that the conventional model, where c_s is irrelevant, remains the same as before.

The main model can be regarded as the subgame where the platform has set c_s, and thus both the platform and the merchant's payoff in the subgame have been derived in the main model. Specifically, if the merchant adopts BOPS, the platform's profit is $\pi_p^O(c_s)$ and the merchant's is $\pi_m^O(c_s)$; if the merchant does not adopt BOPS, the platform's profit is π_p^C and the merchant's is π_m^C. Then, given the merchant's best responses, $\pi_m^O(c_s)$ and π_m^C, the platform (if it decides to offer BOPS) chooses c_s to maximize its expected payoff with BOPS, $\pi_p^O(c_s)$, while ensuring that the merchant has an incentive to adopt BOPS (i.e., $\pi_m^O(c_s)\geq\pi_m^C$). This corresponds to solving the following optimization problem:

$$\max_{c_s}\left\{\pi_p^O(c_s)\,\middle|\,\pi_m^O(c_s)\geq\pi_m^C\right\}. \tag{F.1}$$

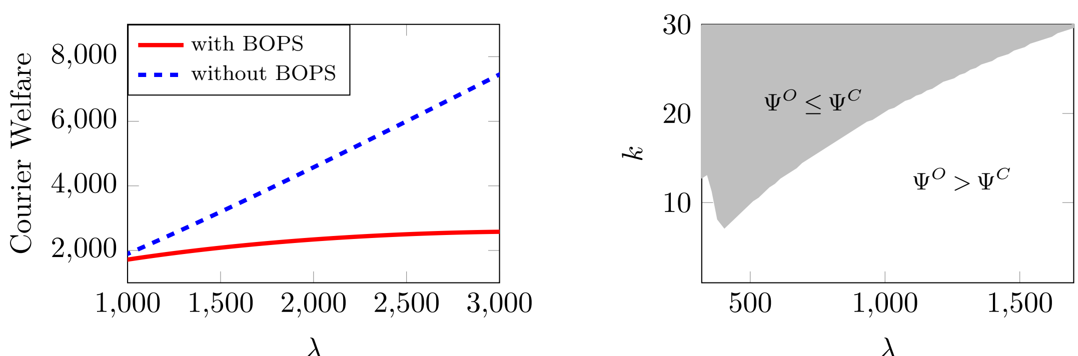
LEMMA F.1. *In equilibrium under the omnichannel model, the optimal c_s^* satisfies $\pi_m^O(c_s^*)=\pi_m^C$.*

Figure E.2: Impact of market size (λ) on equilibrium prices.



Note. The following parameter values are used: $V = 250$, $\mu_\gamma = 10$, $\delta_\gamma = 2$, $k = 20$, $\beta = 0.44$, $R = 20$, $c_d = 70$, $c_s = 90$, $\phi = 30$, and $n = 300$.

Figure E.3: Impact of market parameters on equilibrium courier welfare and consumer surplus across models.



Note. The following parameter values are used: $V = 250$, $\mu_\gamma = 20$, $\delta_\gamma = 2$, $\beta = 0.44$, $R = 20$, $c_d = 70$, $c_s = 90$, $\phi = 30$, and $n = 300$. The left panel uses $k = 20$.

Thus, the only difference between the main model and this extension lies in the value of c_s. In the main model, c_s is exogenously given, whereas here, c_s^* is endogenously determined by the equation in Lemma F.1. The dependence of c_s^* on other model parameters adds another layer of complexity to the analysis.

Lemma F.2. *The BOPS option is offered by the platform and adopted by the merchant if the pickup hassle cost $k < K_c(V, R, \lambda, n, \gamma, \beta, c_d, \phi)$.*

This lemma establishes that the key condition in Lemma 1 continues to hold under this extension with c_s endogenously determined. Given the widespread prevalence of BOPS adoption in practice, we henceforth focus on the parameter regime where $k < K_c(V, R, \lambda, n, \gamma, \beta, c_d, \phi)$ holds.

Proposition F.1. *Compared to the conventional model, in the omnichannel model with BOPS:*
(i) The courier wage decreases (i.e., $w^O < w^C$);
(ii) The delivery-channel price increases (i.e., $p_d^O > p_d^C$) if $\beta > B_c(V, R, \lambda, n, k, \gamma, c_d, c_s)$.

Proposition F.1 reinforces the results established in Proposition 1, indicating that following the introduction of BOPS, the platform continues to reduce courier wages and may raise delivery-channel prices.

Proposition F.2. *In the omnichannel model, the BOPS-channel price is higher than the delivery-channel price (i.e., $p_s^O > p_d^O$) if $k < K_c(V, R, n, \gamma, \beta, c_d)$ and $\Lambda_c'(V, R, n, \gamma, \beta, c_d) < \lambda < \Lambda_c''(V, R, n, \gamma, k, \beta, c_d)$.*

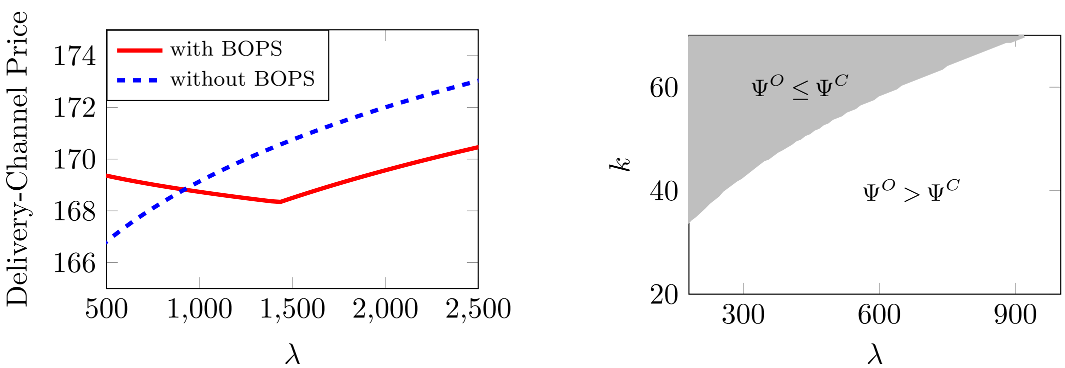
Proposition F.2 verifies that the insight from Proposition 3 still holds—namely, the platform may set a higher price in the BOPS channel than in the delivery channel when both are available.

Proposition F.3. *If the BOPS option is available to consumers in equilibrium, then, compared to the equilibrium outcome in the conventional model:*
(i) Platform profit is higher (i.e., $\pi_p^O > \pi_p^C$*);*
(ii) Merchant profit is higher (i.e., $\pi_m^O > \pi_m^C$*);*
(iii) Courier welfare is lower (i.e., $\Omega^O < \Omega^C$*).*

Proposition F.3 corroborates Proposition 4(i)–(iii), showing that BOPS adoption consistently raises platform and merchant profits while reducing courier welfare.

Due to model complexity, we employ a numerical approach to verify the remaining results of Propositions 2 and 4(iv). The outcomes are illustrated in Figure F.1. The left panel in Figure F.1 supports the insight established in Proposition 2—that surge pricing emerges as an optimal response to demand shocks in the conventional model, but not necessarily under the omnichannel setting. The right panel provides additional evidence for Proposition 4(iv), illustrating that BOPS adoption may decrease consumer surplus.

Figure F.1: Impact of market parameters on delivery-channel price (p_d^C, p_d^O) and consumer surplus (Ψ^C, Ψ^O).



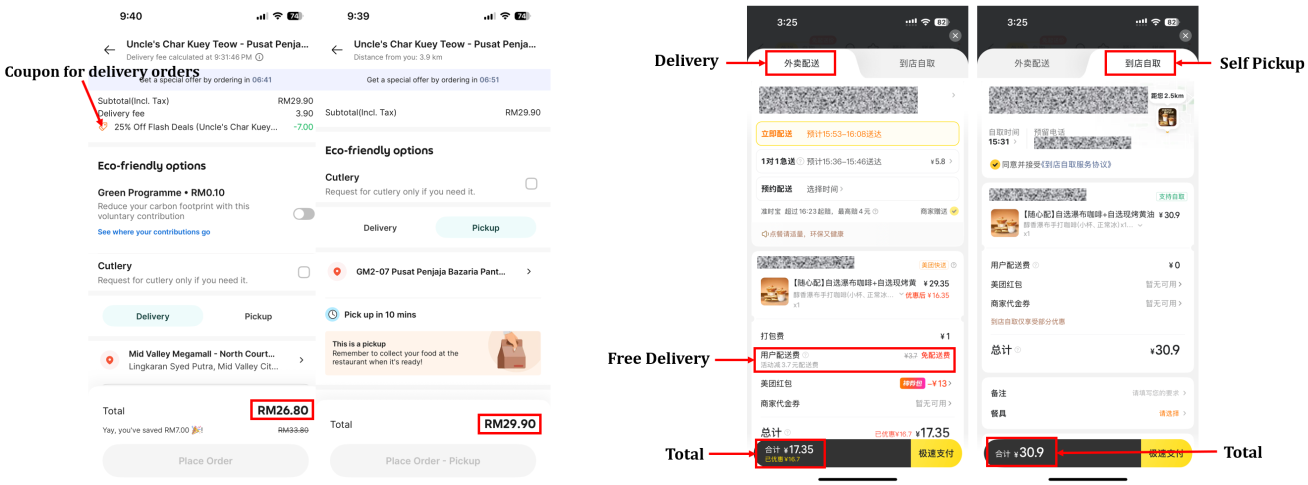
Note. The following parameter values are used: $V = 250$, $\gamma = 20$, $\beta = 0.44$, $R = 20$, $c_d = 70$, $c_s = 90$, $\phi = 30$, and $n = 300$. The left panel uses $k = 50$.

Appendix G: Practical Examples

Figure G.1 shows screenshots from Grab and Meituan, two leading delivery platforms in Singapore and China, respectively. The screenshots display the same product ordered at the same time, with the BOPS option showing a higher total price compared to the delivery option.

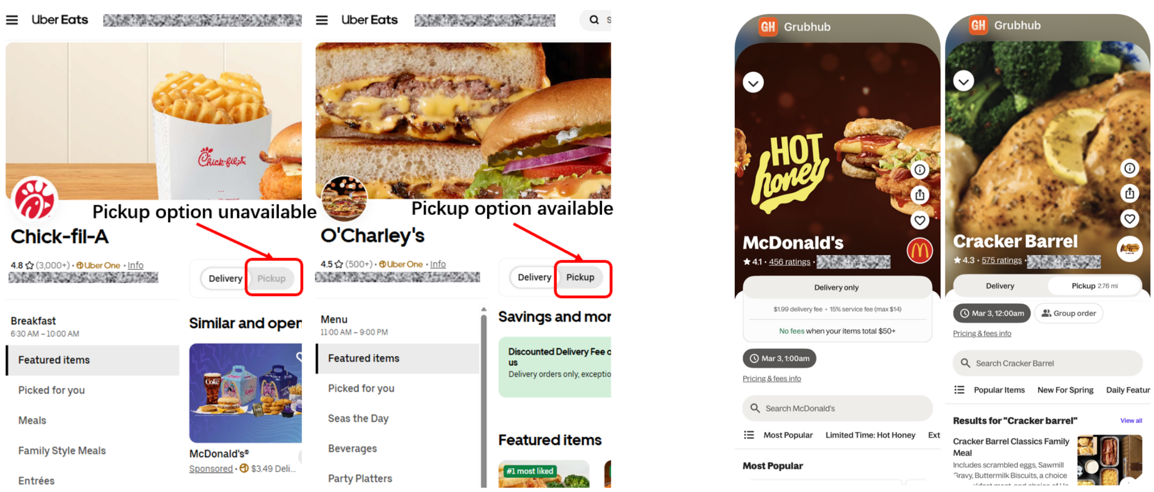
Figure G.2 shows screenshots from Uber Eats and Grubhub, two leading delivery platforms in the US. The figure illustrates that large merchants typically do not offer BOPS services on third-party delivery platforms. For example, on the left, Chick-fil-A (on Uber Eats) and on the right, McDonald's (on Grubhub) do not provide BOPS options. In contrast, smaller merchants tend to offer both delivery and BOPS services. This is exemplified by Square Donuts (on Uber Eats, left) and Cracker Barrel (on Grubhub, right), which provide both options.

Figure G.1: Higher total price for the BOPS option in Grab and Meituan



(a) Grab (b) Meituan

Figure G.2: BOPS-channel availability in Uber Eats and Grubhub



(a) Uber Eats (b) Grubhub