Online Supplement for "Increasing mHealth usage through strategic payer incentives for providers and patients"

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Due to space constraints, we provide some of the details in an **e-companion** (Agnihothri et al., 2026). The Table of Contents below summarizes the location of our contents.

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A1. Analytical Proofs

Proof of Lemma 1: We use (2) to simplify the average patient cost per period as

$$c_p(\alpha,\mu) = p + (\mu+\alpha)(c_{up} - r_p) + \frac{\lambda^2}{2\lambda + (\mu+\alpha)q} \cdot c_{sp}$$

The first and second derivatives of $c_p(\alpha)$ are given by

$$\frac{\partial c_p}{\partial \alpha} = c_{up} - r_p - \frac{\lambda^2 q}{(2\lambda + (\mu+\alpha)q)^2} \cdot c_{sp},$$

$$\frac{\partial^2 c_p}{\partial \alpha^2} = \frac{2\lambda^2 q^2}{(2\lambda + (\mu+\alpha)q)^3} \cdot c_{sp} > 0.$$

Therefore, c_p is a convex function of α. Equating its derivative to zero, and ensuring α is non-negative, we obtain the optimal reward-based upload rate as

$$\left(\frac{\lambda}{q}\left(\sqrt{\frac{qc_{sp}}{c_{up} - r_p}} - 2\right) - \mu\right)^+.$$

It is increasing in r_p since $1/\sqrt{c_{up} - r_p}$ is increasing in r_p. As μ increases, it is linearly decreasing when it is positive and remains zero once it reaches zero.

If $\alpha^* > 0$, its derivative with respect to q is

$$\frac{\partial \alpha^*}{\partial q} = \frac{\lambda}{q^2}\left(2 - \sqrt{\frac{qc_{sp}}{4(c_{up} - r_p)}}\right).$$

Suppose $\mu = 0$. Then $\lim_{\alpha^* \to 0} \partial\alpha^*/\partial q = \lambda/q^2 \cdot \left(2 - 2/\sqrt{4}\right) = \lambda/q^2 > 0$. However, if $c_{sp} > 16(c_{up} - r_p)$ then $\partial\alpha^*/\partial q|_{q=1} < 0$. Furthermore, since the derivative $\partial\alpha^*/\partial q$ is positive until a certain threshold and it stays negative beyond this threshold. Therefore, α^* increases in q until this threshold and then decreases in q. Also, this threshold is the point at which $\partial\alpha^*/\partial q$ equals zero, i.e. $q = 16(c_{up} - r_p)/c_{sp}$. As long as

$$\mu < \frac{\lambda}{q}\left(\sqrt{\frac{qc_{sp}}{c_{up} - r_p}} - 2\right)\Bigg|_{q = \frac{16(c_{up} - r_p)}{c_{sp}}} = \frac{\lambda}{8} \cdot \frac{c_{sp}}{c_{up} - r_p},$$

α^* initially increases and then decreases in q. Note that if μ does not satisfy this condition, then $\alpha^* = 0 \ \forall q$ Q.E.D.

Proof of Lemma 2: For a panel size of N, the average number of "sick visit equivalents" per period is

$$N \cdot \frac{\frac{\mu+\alpha}{\lambda+(\mu+\alpha)q} \cdot \frac{1}{\gamma} + \frac{\lambda}{\lambda+(\mu+\alpha)q}}{\tau} = N \cdot \frac{\lambda}{\gamma} \cdot \frac{\lambda\gamma + \mu + \alpha}{2\lambda + (\mu+\alpha)q}$$

Note that the average number of mobile interventions in a renewal cycle is $(\mu+\alpha)/(\lambda + (\mu+\alpha)q)$ since α is the upload rate and $1/(\lambda + (\mu+\alpha)q)$ is the expected time taken to get to state $\mathcal{N}$ from state $\mathcal{I}$. Hence, we obtain

$$N^* \cdot \frac{\lambda}{\gamma} \cdot \frac{\lambda\gamma + \mu + \alpha}{2\lambda + (\mu+\alpha)q} = k \Rightarrow N^* = \frac{\gamma k}{\lambda} \cdot \frac{2\lambda + (\mu+\alpha)q}{\lambda\gamma + \mu + \alpha}$$

Clearly, N^* is linearly increasing in q. The derivative of N^* with respect to μ is

$$\frac{\gamma k(\gamma q-2)}{(\lambda\gamma+\mu+\alpha)^2} > (<)\ 0 \Leftrightarrow \gamma q > (<)\ 2.$$

Therefore, if $\gamma q > (<)\ 2$, N is strictly increasing (decreasing) in α. Q.E.D.

Proof of Lemma 3: Differentiating the provider's net revenue partially with respect to μ, after some algebra, we obtain $\partial\pi_d(r_d,\mu,\alpha)/\partial\mu = \gamma k/\lambda \cdot \mathcal{G}(r_d,\mu,\alpha)/(\lambda\gamma+\mu+\alpha)^2$. Since $\mathcal{G}$ is strictly decreasing in μ (for any given α), π_d is strictly concave in μ. Also, $\mu^* = 0$ if $\mathcal{G} \le 0$ and is the unique solution to $\mathcal{G} = 0$ otherwise.

As α increases, $\mathcal{G}(r_d,\mu,\alpha)$ decreases for any given μ, and hence, μ^* decreases. Part (ii) follows with a similar logic. As r_d increases, $\mathcal{G}$ for any given μ increases (remains the same, decreases) if $\gamma q > 2$ ($\gamma q = 2, \gamma q < 2$), which implies μ^* increases (remains the same, decreases). Q.E.D.

Proof of Lemma 4: We first note that $\mu^e + \alpha^e \not< \hat{\alpha}$ because the patient will then be better by increasing α to $\hat{\alpha} - \mu^e$. We next separately show the result in each case.

Case (i): $\mathcal{G}(r_d,0,\hat{\alpha}) \le 0$ and $\mathcal{G}(r_d,\hat{\alpha},0) \le 0$. There are three possible sub-cases: (a) $\mathcal{G}(r_d,0,\hat{\alpha}) \le 0$ and $\mathcal{G}(r_d,\hat{\alpha},0) < 0$, (b) $\mathcal{G}(r_d,0,\hat{\alpha}) < 0 = \mathcal{G}(r_d,\hat{\alpha},0)$, and (c) $\mathcal{G}(r_d,\hat{\alpha},0) = 0 = \mathcal{G}(r_d,\hat{\alpha},0)$. In sub-case (a), $(\mu^e,\alpha^e) = (0,\hat{\alpha})$ is the only possible equilibrium. Note that although the provider will choose $\mu < \hat{\alpha}$ when $\alpha = 0$, these two rates do not form an equilibrium since $\mu + \alpha < \hat{\alpha}$ here. In sub-case (b), both $(0,\hat{\alpha})$ and $(\hat{\alpha},0)$ are possible equilibria but $(0,\hat{\alpha})$ is the Pareto dominant equilibrium since it provides a strictly higher net revenue for the provider. Likewise, in sub-case (c), any (μ,α) with $\mu+\alpha = \hat{\alpha}, \mu \ge 0, \alpha \ge 0$ is an equilibrium but $(0,\hat{\alpha})$ results in the highest net revenue.

Case (ii): Then $\mathcal{G}(r_d,0,\hat{\alpha}) \le 0 < \mathcal{G}(r_d,\hat{\alpha},0)$. We find that there are two possible equilibria: (a) $(\hat{\mu},0)$ where $\mathcal{G}(r_d,\hat{\mu},0) = 0$ and (b) $(0,\hat{\alpha})$. Equilibrium (a) represents the provider "being the driver" for uploads while equilibrium (b) represents the patient "being the driver" for uploads. Equilibrium (a) is the result if it results in a strictly higher net revenue than equilibrium (b); otherwise equilibrium (b) is the Pareto-dominant equilibrium.

Case (iii): Then $\mathcal{G}(r_d,0,\hat{\alpha}) > 0$, and $\mathcal{G}(r_d,\hat{\alpha},0) \ge 0$. The provider wants to have reminders for any $0 < \alpha \le \hat{\alpha}$. Hence, the only possible equilibrium is $(\hat{\mu},0)$. Also, $\mathcal{G}(r_d,\hat{\alpha},0) \ge 0 \Rightarrow \hat{\mu} \ge \hat{\alpha}$.

Case (iv): Then $\mathcal{G}(r_d,0,\hat{\alpha}) > 0 > \mathcal{G}(r_d,\hat{\alpha},0)$. The inequality implies the provider wants to have reminders when $\alpha = \hat{\alpha}$ but wants to have reminder rate below $\hat{\alpha}$ when $\alpha = 0$. Therefore, $\mu^e + \alpha^e \not> \hat{\alpha}$. The reasoning is as follows. If the total rate exceeds $\hat{\alpha}$ then α has to be zero (otherwise patient would be better by lowering α). However, if α is zero, the provider chooses a reminder rate below $\hat{\alpha}$. The inequalities $\mu^e + \alpha^e \not< \hat{\alpha}$ and $\mu^e + \alpha^e \not> \hat{\alpha}$ imply $\mu^e + \alpha^e = \hat{\alpha}$. Finally, the inequality for case (iv) implies $\mathcal{G}(r_d,\mu,\hat{\alpha}-\mu)$ is strictly decreasing with the function being positive when $\mu = 0$ and negative when $\mu = \hat{\alpha}$. Therefore, there exists a unique $0 < \mu^e < \hat{\alpha}$ when the function becomes zero.

Also, $(\mu^e, \hat{\alpha}-\mu^e)$ is an equilibrium since neither the patient nor the provider wants to deviate from this point.

Proof of Lemma 5: We prove by contradiction that case (iv) does not occur at optimality. Suppose there is a capitation rate r_d such that the conditions of case (iv) and the provider's participation constraint (in the payer's problem) hold. Clearly, since $\mathcal{G}(r_d,0,\hat{\alpha}) > \mathcal{G}(r_d,0,\hat{\alpha})$, we have $\gamma q > 2$.

Consider a slightly lower capitation rate r_d'. Note that the change in capitation rate does not affect $\hat{\alpha}$. The conditions of case (iv) still hold. However, the equilibrium reminder rate reduces and becomes $\mu^{e\prime}$. Then $\mathcal{G}(r_d', \mu^{e\prime}, \hat{\alpha}-\mu^{e\prime}) = 0 \Rightarrow r_d' - \mu^{e\prime} c_r = r_d - \mu^e c_r$ so that the provider's net revenue is still greater than $\underline{\pi}_d$. Therefore, reduction of capitation rate still results in the same outcome for the provider and patient but it increases the payer's profit.

Finally, we note that in all the other cases of the provider-patient equilibrium, either $\alpha^e = 0$ or $\mu^e = 0$, which proves the lemma. Q.E.D.

Proof of Lemma 6: The conditions $\hat{r}_d(0,\xi) \geq \underline{r}_d(0,\xi)$ and $\hat{r}_d(\xi,0) \geq \underline{r}_d(\xi,0)$ are equivalent since $\hat{r}_d(\xi,0) = \hat{r}_d(0,\xi) + \xi c_r$ and $\underline{r}_d(\xi,0) = \underline{r}_d(0,\xi) + \xi c_r$. Also, since $\underline{r}_d(\xi,0) = \underline{r}_d(0,\xi) + \xi c_r$, $\mathcal{G}(\underline{r}_d(0,\xi),0,\xi) = \mathcal{G}(\underline{r}_d(\xi,0),\xi,0)$. There are two possible cases:

(i) $\hat{r}_d(0,\xi) < \underline{r}_d(0,\xi)$: Then $\mathcal{G}(\underline{r}_d(0,\xi),0,\xi) = \mathcal{G}(\underline{r}_d(\xi,0),\xi,0) > 0$. We next show that achieving a combined rate of ξ is impossible for the payer. Any $r_d < \underline{r}_d(0,\xi)$ would be infeasible since $\pi_d(r_d,\mu,\alpha) \leq \pi_d(r_d,0,\xi) < \underline{\pi}_d$ for any $\mu+\alpha=\xi$. Consider r_d so that $\underline{r}_d(0,\xi) \leq r_d < \underline{r}_d(\xi,0)$. Since $r_d \geq \underline{r}_d(0,\xi) > \hat{r}_d(0,\xi)$, we have $\mathcal{G}(r_d,0,\xi) > 0$. There are then two possibilities. Either $\mathcal{G}(r_d,\xi,0) > 0$ which implies $\mu^e > \xi$ and $\alpha^e = 0$ resulting in a combined rate strictly greater than ξ, or $\mathcal{G}(r_d,\xi,0) \leq 0$ which implies $0 < \mu^e \leq \xi$ and $\alpha^e = \xi - \mu^e$, where $\mathcal{G}(r_d,\mu^e,\alpha^e) = 0$. Although it results in a combined rate of ξ, this is infeasible because $\pi_d < \underline{\pi}_d$ as $\mathcal{G}(r_d,\mu^e,\alpha^e) = 0 < \mathcal{G}(\underline{r}_d(0,\xi),0,\xi) = \mathcal{G}(\underline{r}_d(\mu^e,\alpha^e),\mu^e,\alpha^e) \Leftrightarrow r_d < \underline{r}_d(\mu^e,\alpha^e)$. Finally, we note for any $r_d \geq \underline{r}_d(\xi,0)$, $\mathcal{G}(r_d,\xi,0) > 0$ which implies $\mu^e > \xi$.

(ii) $\hat{r}_d(0,\xi) \geq \underline{r}_d(0,\xi)$: Then $\mathcal{G}(\underline{r}_d(0,\xi),0,\xi) = \mathcal{G}(\underline{r}_d(\xi,0),\xi,0) \leq 0$. First, we consider reminder-based strategy with $(\xi,0)$ as the provider-patient equilibrium. For that to be achieved, $r_d^* = \hat{r}_d(\xi,0)$ because otherwise $\mu^e \neq \xi$ when $\alpha^e = 0$. Also, $\hat{r}_d(\xi,0) \geq \underline{r}_d(\xi,0)$ so that $\pi_d \geq \underline{\pi}_d$. Since $\alpha^e = 0$, the payer does not reward the patients so that $r_p^* = 0$. Note that since $\xi \geq \hat{\alpha}(0)$, $\mathcal{G}(\hat{r}_d(\xi,0),0,\hat{\alpha}(0)) > \mathcal{G}(\hat{r}_d(\xi,0),\hat{\alpha}(0),0) \geq \mathcal{G}(\hat{r}_d(\xi,0),\xi,0) = 0$ which corresponds to case (v) of provider-patient equilibrium, and hence, $(\xi,0)$ is the unique equilibrium.

Next, we consider reward-based strategy. The payer sets $r_d = \underline{r}_d(0,\xi)$ resulting in $(0,\xi)$ for the provider-patient equilibrium, and the corresponding π_d equals $\underline{\pi}_d$. Since the payer sets the least possible capitation rate for the provider to participate, the payer cannot do better under the reward-based strategy. Solving the equation $\hat{\alpha}(r_p)=\xi$ yields $r_p^*=c_{up}-\lambda^2 qc_{sp}/(2\lambda+\xi q)^2$. Note that $\mathcal{G}(\underline{r}_d(0,\xi),\xi,0)<\mathcal{G}(\underline{r}_d(0,\xi),0,\xi)\le\mathcal{G}(\hat{r}_d(0,\xi),0,\xi)=0$ which corresponds to case (i) of provider-patient equilibrium, and hence, $(0,\xi)$ is the unique equilibrium.

Finally, we consider the feasibility condition $\hat{r}_d(0,\xi)\ge\underline{r}_d(0,\xi)$. Before we examine it in detail, we first derive the expressions for $\hat{r}_d(0,\xi)$ and $\underline{r}_d(0,\xi)$. Equating $\mathcal{G}(\hat{r}_d(0,\xi),0,\xi)$ to zero, we obtain

$$\lambda\Big((\gamma q-2)\hat{r}_d(0,\xi)-2\lambda\gamma(c_{ud}+c_r)-\lambda\gamma c_m+\lambda c_{sd}\Big)-\lambda\Big(2\gamma qc_{ud}+(\gamma q+2)c_r\Big)\xi-q(c_{ud}+c_r)\xi^2=0$$
$$\Leftrightarrow\ \hat{r}_d(0,\xi)=\frac{q(c_{ud}+c_r)\xi^2+\lambda\big(2\gamma qc_{ud}+(\gamma q+2)c_r\big)\xi+\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)}{\lambda(\gamma q-2)} \tag{A1}$$

Equating $\pi_d(\underline{r}_d(0,\xi),0,\xi)$ from (6) to $\underline{\pi}_d$, we obtain

$$\frac{\gamma k}{\lambda}\cdot\left(\frac{\Big(\underline{r}_d(0,\xi)-\xi c_{ud}\Big)\Big(2\lambda+\xi q\Big)-\lambda\Big(\xi c_m+\lambda c_{sd}\Big)}{\lambda\gamma+\xi}\right)=\underline{\pi}_d$$
$$\Leftrightarrow\ \underline{r}_d(0,\xi)=\xi c_{ud}+\frac{\lambda\underline{\pi}_d(\lambda\gamma+\xi)+\lambda\gamma k(\xi c_m+\lambda c_{sd})}{\gamma k(2\lambda+\xi q)}$$
$$\Leftrightarrow\ \underline{r}_d(0,\xi)=\xi c_{ud}+\frac{\lambda^2\gamma(\underline{\pi}_d+kc_{sd})+\lambda\xi(\underline{\pi}_d+\gamma kc_m)}{\gamma k(2\lambda+\xi q)} \tag{A2}$$

The feasibility condition $\hat{r}_d(0,\xi)\ge\underline{r}_d(0,\xi)$ implies

$$\frac{q(c_{ud}+c_r)\xi^2+\lambda\big(2\gamma qc_{ud}+(\gamma q+2)c_r\big)\xi+\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)}{\lambda(\gamma q-2)}$$
$$\ge\ \xi c_{ud}+\frac{\lambda^2\gamma(\underline{\pi}_d+kc_{sd})+\lambda\xi(\underline{\pi}_d+\gamma kc_m)}{\gamma k(2\lambda+\xi q)}$$
$$\Leftrightarrow\ q(c_{ud}+c_r)\xi^2+\lambda(\gamma q+2)(c_{ud}+c_r)\xi+\lambda^2\big(2\gamma(c_{ud}+c_r)+c_m-c_{sd}\big)$$
$$\ge\ \frac{\lambda^2(\gamma q-2)}{\gamma k}\cdot\frac{\lambda\gamma(\underline{\pi}_d+kc_{sd})+\xi(\underline{\pi}_d+\gamma kc_m)}{2\lambda+\xi q}$$

Clearly, the left side of the inequality is strictly increasing in ξ. The derivative of right side is

$$\frac{\lambda^2(\gamma q-2)}{\gamma k}\cdot\frac{\partial}{\partial\xi}\left(\frac{\lambda\gamma(\underline{\pi}_d+kc_{sd})+\xi(\underline{\pi}_d+\gamma kc_m)}{2\lambda+\xi q}\right)=\frac{\lambda^2(\gamma q-2)}{\gamma k}\cdot\frac{2\lambda(\underline{\pi}_d+\gamma kc_m)-\lambda\gamma q(\underline{\pi}_d+kc_{sd})}{(2\lambda+\xi q)^2}$$
$$=\ \frac{-\lambda^3(\gamma q-2)}{\gamma k(2\lambda+\xi q)^2}\cdot\Big((\gamma q-2)\underline{\pi}_d+\gamma k(qc_{sd}-2c_m)\Big)$$

If $(\gamma q-2)\underline{\pi}_d+\gamma k(qc_{sd}-2c_m)\ge 0$, then the right side is decreasing in ξ. As $\xi\to\infty$, the left side goes to infinity while the right side remains finite. Hence, there is a "single crossing" and there exists $x_{min}\ge\hat{\alpha}(0)$ such that the feasibility condition is satisfied for any $\xi\ge x_{min}$.

If $(\gamma q-2)\underline{\pi}_d+\gamma k(qc_{sd}-2c_m)<0$, then both left and right sides are strictly increasing in ξ. However, there is "no crossing" in this case as we show next. Specifically, the left side at $\xi=0$ exceeds the right side as $\xi\to\infty$, i.e.

$$\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)>\lim_{\xi\to\infty}\frac{\lambda^2(\gamma q-2)}{\gamma k}\cdot\frac{\lambda\gamma(\underline{\pi}_d+kc_{sd})+\xi(\underline{\pi}_d+\gamma kc_m)}{2\lambda+\xi q}$$

$$\Leftrightarrow 2\gamma(c_{ud}+c_r)+\gamma c_m - c_{sd} > \frac{\gamma q-2}{\gamma qk}\cdot(\underline{\pi}_d+\gamma k c_m)$$
$$\Leftarrow \gamma k(2c_m - qc_{sd}) > (\gamma q-2)\underline{\pi}_d,$$

which is true. Therefore, the inequality holds for any non-negative ξ, and $x_{min}=\hat{\alpha}(0)$. Q.E.D.

Proof of Lemma 7: (i) Consider reminder-based strategy. For this strategy, $r_d^*=\hat{r}_d(\xi,0)$ because, when $\alpha^e=0$, any $r_d>\hat{r}_d(\xi,0)$ implies $\mu^e<\xi$ while any $r_d<\hat{r}_d(\xi,0)$ implies $\mu^e>\xi$. Since $\hat{r}_d(\xi,0)\geq \underline{r}_d(\xi,0)$, the provider's participation constraint is satisfied. Also, $r_p^*=0$ since $\alpha^e=0$.

If $\hat{r}_d(0,\hat{\alpha}(0))>\hat{r}_d(\xi,0)$ then $\mathcal{G}(\hat{r}_d(\xi,0),\hat{\alpha}(0),0)>\mathcal{G}(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))>0$ so that case (v) of provider-patient equilibrium applies, and hence $(\xi,0)$ is the unique equilibrium.

However, suppose $\hat{r}_d(0,\hat{\alpha}(0))\leq\hat{r}_d(\xi,0)$. We then have $\mathcal{G}(\hat{r}_d(\xi,0),\hat{\alpha}(0),0)\geq 0\geq\mathcal{G}(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))$ which corresponds to case (ii) if $\xi>\hat{\alpha}(0)$ or case (iii) if $\xi=\hat{\alpha}(0)$. In case (iii), $(0,\hat{\alpha}(0))$ is the equilibrium. In case (ii) too, $(0,\hat{\alpha}(0))$ becomes the equilibrium since $\pi_d(\hat{r}_d(\xi,0),\xi,0)<\pi_d(\hat{r}_d(\xi,0),\xi-\hat{\alpha}(0),\hat{\alpha}(0))<\pi_d(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))$, where the first inequality results from the provider unnecessarily doing additional reminders (at the rate of $\hat{\alpha}(0)$ for every patient) and the second inequality results because $\mathcal{G}(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))\leq\mathcal{G}(\hat{r}_d(0,\hat{\alpha}(0)),0,\hat{\alpha}(0))=0$ since $\hat{r}_d(0,\hat{\alpha}(0))\leq\hat{r}_d(\xi,0)$ and $\gamma q<2$. Therefore, $(\xi,0)$ is not an equilibrium in this case.

We then combine the conditions for $(\xi,0)$ equilibrium as $\hat{r}_d(0,\hat{\alpha}(0))>\hat{r}_d(\xi,0)\geq\underline{r}_d(\xi,0)$. The lower bound y_{min} arises from the condition that $\xi\geq\hat{\alpha}(0)$. We show next the inequality $\hat{r}_d(\xi,0)\geq\underline{r}_d(\xi,0)$ results in an upper bound y_{max}. Similar to the proof of Lemma 6, it implies

$$\begin{aligned}
&\frac{q(c_{ud}+c_r)\xi^2+\lambda\big(2\gamma q c_{ud}+(\gamma q+2)c_r\big)\xi+\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)}{\lambda(\gamma q-2)}\\
\geq\ &\xi c_{ud}+\frac{\lambda^2\gamma(\underline{\pi}_d+kc_{sd})+\lambda\xi(\underline{\pi}_d+\gamma k c_m)}{\gamma k(2\lambda+\xi q)}\\
\Leftrightarrow\ &q(c_{ud}+c_r)\xi^2+\lambda(\gamma q+2)(c_{ud}+c_r)\xi+\lambda^2\big(2\gamma(c_{ud}+c_r)+c_m-c_{sd}\big)\\
\leq\ &\frac{\lambda^2(\gamma q-2)}{\gamma k}\cdot\frac{\lambda\gamma(\underline{\pi}_d+kc_{sd})+\xi(\underline{\pi}_d+\gamma k c_m)}{2\lambda+\xi q}
\end{aligned}$$

As $\xi\to\infty$, the left hand side goes to infinity while the right hand side stays negative (since $\gamma q<2$), thereby violating the inequality. Therefore, there exists y_{max} such that the inequality is satisfied only if y does not exceed an upper bound, which implies the existence of y_{max}.

(ii) Consider reward-based strategy. In this strategy, we show that $r_d^*=\max(\underline{r}_d(0,\xi),\hat{r}_d(0,\xi))$. Note that this is the least possible r_d; $r_d<\underline{r}_d(0,\xi)$ would result in $\pi_d<\underline{\pi}_d$ (for the $(0,\xi)$ equilibrium) while $r_d<\hat{r}_d(0,\xi)$ implies $\mathcal{G}(r_d,0,\xi)>0$ so that $\mu^e>0$ and $(0,\xi)$ is not an equilibrium. Similar to the proof of Lemma 6, we can show that $r_p^*=c_{up}-\lambda^2 qc_{sp}/(2\lambda+\xi q)^2$. We show that these values of r_d^* and r_p^* indeed result in the $(0,\xi)$ equilibrium. We first note that $\hat{\alpha}(r_p^*)=\xi$. We next examine the value of r_d^* more closely.

Suppose $r_d^* = \hat{r}_d(0,\xi)$. Then $\mathcal{G}(r_d^*,\xi,0) > \mathcal{G}(r_d^*,0,\xi) = 0$ which corresponds to case (ii) of provider-patient equilibrium. However, $\pi_d(\hat{r}_d(0,\xi),0,\xi) = \pi_d(\hat{r}_d(\xi,0),\xi,0)$ (since $\hat{r}_d(\xi,0) = \hat{r}_d(0,\xi) + \xi c_r$) $= \max_\mu \pi_d(\hat{r}_d(\xi,0),\mu,0)$ (since the derivative of net revenue, $\pi_d(\hat{r}_d(\xi,0),\mu,0)$, with respect to μ at ξ becomes zero as $\mathcal{G}(\hat{r}_d(\xi,0),\xi,0) = 0$) $> \max_\mu \pi_d(\hat{r}_d(0,\xi),\mu,0)$ (since $\hat{r}_d(\xi,0) > \hat{r}_d(0,\xi)$). As $\pi_d(\hat{r}_d(0,\xi),0,\xi) > \pi_d(\hat{r}_d(0,\xi),\mu,0)\ \forall\mu$, $(0,\xi)$ is the resulting equilibrium.

Suppose $r_d^* = \underline{r}_d(0,\xi) > \hat{r}_d(0,\xi)$. Then $\mathcal{G}(\underline{r}_d(0,\xi),0,\xi) < \mathcal{G}(\hat{r}_d(0,\xi),0,\xi) = 0$. If $\mathcal{G}(\underline{r}_d(0,\xi),\xi,0) < 0$, then we have case (i) of provider-patient equilibrium, and if $\mathcal{G}(\underline{r}_d(0,\xi),\xi,0) = 0$, then we have case (iii). In both these cases, $(0,\xi)$ is the equilibrium. If $\mathcal{G}(\underline{r}_d(0,\xi),\xi,0) > 0$ then we have case (ii) which implies possible $(0,\xi)$ and $(\hat{\mu},0)$ equilibria where $\hat{\mu} > \xi$. However, we still have the $(0,\xi)$ equilibrium. The reasoning is as follows. We have $\pi_d(\underline{r}_d(0,\xi),0,\xi) = \underline{\pi}_d = \pi_d(\underline{r}_d(\xi,0),\xi,0) > \pi_d(\underline{r}_d(\xi,0),\mu,0)\ \forall\mu > \xi$, where the last inequality follows from $\mathcal{G}(\underline{r}_d(\xi,0),\xi,0) < 0$ since $\underline{r}_d(\xi,0) > \hat{r}_d(\xi,0) \Leftrightarrow \underline{r}_d(0,\xi) > \hat{r}_d(0,\xi)$ which is true as $r_d^* = \underline{r}_d(0,\xi) > \hat{r}_d(0,\xi)$. Also, we have $\pi_d(\underline{r}_d(\xi,0),\mu,0) > \pi_d(\underline{r}_d(0,\xi),\mu,0)\ \forall\mu$ since $\underline{r}_d(\xi,0) = \underline{r}_d(0,\xi) + \xi c_r > \underline{r}_d(0,\xi)$. Together, these inequalities imply $\pi_d(\underline{r}_d(0,\xi),0,\xi) > \pi_d(\underline{r}_d(0,\xi),\mu,0)\ \forall\mu \geq \xi$, and hence, the $(0,\xi)$ equilibrium. Q.E.D.

Proof of Lemma 8: Consider reminder-based strategy. Then $r_d^* = \underline{r}_d(\xi,0)$ since r_d does not affect $\mathcal{G}$. Only $\xi = \hat{\xi}$ is possible because for any r_d, $\mathcal{G}(r_d,\xi,0) > 0\ \forall\xi < \hat{\xi}$ while $\mathcal{G}(r_d,\xi,0) < 0\ \forall\xi > \hat{\xi}$. Hence $(\xi,0)$ equilibrium occurs only if $\xi = \hat{\xi}$. Also, if $\hat{\xi} = \hat{\alpha}(0)$, then $\mathcal{G}(r_d^*,\hat{\alpha}(0),0) = \mathcal{G}(r_d^*,0,\hat{\alpha}(0)) = 0$ which corresponds to case (iv) of provider-patient equilibrium, and hence, the outcome is $(0,\hat{\alpha}(0))$ equilibrium. Therefore, we require $\hat{\xi} > \hat{\alpha}(0)$ which implies $\mathcal{G}(r_d^*,\hat{\alpha}(0),0) = \mathcal{G}(r_d^*,0,\hat{\alpha}(0)) > 0$, and therefore, corresponds to case (v) that results in $(\hat{\xi},0)$ equilibrium.

Consider reward-based strategy. Again, $r_d^* = \underline{r}_d(0,\xi)$ since r_d does not affect $\mathcal{G}$. Similar to the proof of Lemma 6, we can show that $r_p^* = c_{up} - \lambda^2 q c_{sp}/(2\lambda + \xi q)^2$. That implies $\hat{\alpha}(r_p^*) = \xi$. For any $\xi \geq \hat{\xi}$, we then have $\mathcal{G}(r_d^*,\xi,0) = \mathcal{G}(r_d^*,0,\xi) \leq 0$ which corresponds to case (i) or (iv) of provider-patient equilibrium, both of which result in $(0,\xi)$ equilibrium. Also, note that any $\xi < \hat{\xi}$ implies $\mathcal{G}(r_d^*,\xi,0) = \mathcal{G}(r_d^*,0,\xi) > 0$ which corresponds to case (v) that does not result in $(\xi,0)$ equilibrium. Q.E.D.

Proof of Lemma 9: Suppose $\gamma q > 2$. If $\xi^* = \hat{\alpha}(0)$ then $\xi = \hat{\alpha}(0)$ for both strategies since the feasible set $\mathcal{X}$ is identical for both of them. Also, the profits are given by

$$\pi_{id}(\hat{\alpha}(0)) = \frac{\gamma k}{\lambda} \cdot \left(\frac{\Big(p - \hat{r}_d(\hat{\alpha}(0),0)\Big)\Big(2\lambda + \hat{\alpha}(0)q\Big) - \lambda^2 c_{si}}{\lambda\gamma + \hat{\alpha}(0)} \right),$$

$$\pi_{ip}(\hat{\alpha}(0)) = \frac{\gamma k}{\lambda} \cdot \left(\frac{\Big(p - \underline{r}_d(0,\hat{\alpha}(0))\Big)\Big(2\lambda + \hat{\alpha}(0)q\Big) - \lambda^2 c_{si}}{\lambda\gamma + \hat{\alpha}(0)} \right).$$

Note that $r_p^* = 0$ in the payer's profit under reward-based strategy. We then find $\pi_{id}(\hat{\alpha}(0)) \leq \pi_{ip}(\hat{\alpha}(0))$ since $\hat{r}_d(\hat{\alpha}(0),0) = \hat{r}_d(0,\hat{\alpha}(0)) + c_r\hat{\alpha}(0) \geq \underline{r}_d(0,\hat{\alpha}(0))$ since $\hat{\alpha}(0) \in \mathcal{X}$.

Suppose $\gamma q < 2$. Then $\xi = \hat{\alpha}(0)$ is feasible under reward-based strategy. If $\hat{\alpha}(0) \notin \mathcal{Y}$ then only reward-based strategy is feasible. However, if $\hat{\alpha}(0) \in \mathcal{Y}$ then $\hat{r}_d(0,\hat{\alpha}(0)) > \underline{r}_d(0,\hat{\alpha}(0))$, and the capitation rates under reminder-based and rewards-based strategies are $\hat{r}_d(\hat{\alpha}(0),0)$ and $\hat{r}_d(0,\hat{\alpha}(0))$ respectively. Since $\hat{r}_d(\hat{\alpha}(0),0) = \hat{r}_d(0,\hat{\alpha}(0)) + c_r\hat{\alpha}(0) \geq \hat{r}_d(0,\hat{\alpha}(0))$, we obtain $\pi_{id}(\hat{\alpha}(0)) \leq \pi_{ip}(\hat{\alpha}(0))$.

Suppose $\gamma q = 2$. Then $\xi = \hat{\alpha}(0)$ is infeasible under reminder-based strategy. Q.E.D.

Proof of Theorem 1: (i) Suppose $\gamma q > 2$. The payer's profit under reminder-based strategy is given by

$$
\begin{aligned}
\pi_{id}(\xi) &= \frac{\gamma k}{\lambda}\cdot\left(\frac{\Big(p-\hat{r}_d(\xi,0)\Big)\Big(2\lambda+\xi q\Big)-\lambda^2 c_{si}}{\lambda\gamma+\xi}\right)\\
&= \frac{\gamma k}{\lambda}\cdot\left(\frac{\left(p-\frac{q(c_{ud}+c_r)\xi^2+2\lambda\gamma q(c_{ud}+c_r)\xi+\lambda^2\left(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\right)}{\lambda(\gamma q-2)}\right)\Big(2\lambda+\xi q\Big)-\lambda^2 c_{si}}{\lambda\gamma+\xi}\right)\\
&= \frac{\gamma k}{\lambda^2(\gamma q-2)}\cdot\\
&\frac{\Big(\lambda p(\gamma q-2)-\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)-2\lambda\gamma q(c_{ud}+c_r)\xi-q(c_{ud}+c_r)\xi^2\Big)\Big(2\lambda+\xi q\Big)-\lambda^3(\gamma q-2)c_{si}}{\lambda\gamma+\xi}
\end{aligned}
$$

Since $\gamma q > 2$, the sign of the derivative $\partial\pi_{id}/\partial\xi$ is then the same as that of the expression

$$
\begin{aligned}
&(\lambda\gamma+\xi)\Big(\lambda pq(\gamma q-2)-\lambda^2 q\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)-4\lambda^2\gamma q(c_{ud}+c_r)\Big)\\
&-(\lambda\gamma+\xi)\Big(4\lambda q(\gamma q+1)(c_{ud}+c_r)\xi+3q^2(c_{ud}+c_r)\xi^2\Big)+\lambda^3(\gamma q-2)c_{si}\\
&-\Big(\lambda p(\gamma q-2)-\lambda^2\big(2\gamma(c_{ud}+c_r)+\gamma c_m-c_{sd}\big)-2\lambda\gamma q(c_{ud}+c_r)\xi-q(c_{ud}+c_r)\xi^2\Big)\Big(2\lambda+\xi q\Big)\\
=\;&\lambda^2 p(\gamma q-2)^2+\lambda^3\Big((c_{si}+c_{sd}-\gamma c_m)(\gamma q-2)-2\gamma(3\gamma q-2)(c_{ud}+c_r)\Big)\\
&-4\lambda^2\gamma q(\gamma q+1)(c_{ud}+c_r)\xi-\lambda q(5\gamma q+2)(c_{ud}+c_r)\xi^2-2q^2(c_{ud}+c_r)\xi^3,
\end{aligned}
$$

which equals $\mathcal{F}_d$ from (15). Also, $\mathcal{F}_d$ is strictly decreasing in ξ, and hence, π_{id} is unimodal in ξ, with π_{id} initially possibly increasing and then decreasing. Therefore, if $\mathcal{F}_d(x_{min}) \leq 0$ (note that x_{min} is the lowest value of the feasible set $\mathcal{X}$), then $\xi_{id}^* = x_{min}$; otherwise, ξ_{id}^* is the unique solution to $\mathcal{F}_d(\xi) = 0$.

The payer's profit under reward-based strategy is given by

$$
\begin{aligned}
\pi_{ip}(\xi) &= \frac{\gamma k}{\lambda}\cdot\left(\frac{\left(p-\underline{r}_d(0,\xi)-\xi\left(c_{up}-\frac{\lambda^2 qc_{sp}}{(2\lambda+\xi q)^2}\right)\right)\Big(2\lambda+\xi q\Big)-\lambda^2 c_{si}}{\lambda\gamma+\xi}\right)\\
&= \frac{\gamma k}{\lambda}\cdot\left(\frac{(p-\xi c_u)(2\lambda+\xi q)+\frac{\lambda^2 qc_{sp}\xi}{2\lambda+\xi q}-\frac{\lambda^2\gamma(\underline{\pi}_d+kc_{sd})+\lambda\xi(\underline{\pi}_d+\gamma kc_m)}{\gamma k}-\lambda^2 c_{si}}{\lambda\gamma+\xi}\right)
\end{aligned}
$$

$$= -\underline{\pi}_d + \frac{\gamma k}{\lambda} \cdot \left(\frac{(p - \xi c_u)(2\lambda + \xi q) + \frac{\lambda^2 q c_{sp} \xi}{2\lambda + \xi q} - \lambda \xi c_m - \lambda^2 (c_{sd} + c_{si})}{\lambda\gamma + \xi} \right)$$

After some algebra, we find that the derivative of the profit can be written as

$$\frac{\partial \pi_{ip}}{\partial \xi} = \frac{\gamma k}{\lambda} \cdot \frac{1}{(\lambda\gamma + \xi)^2} \cdot \begin{pmatrix} -c_u q \xi^2 - 2\lambda\gamma c_u q \xi \\ +\lambda\left((\gamma q - 2)p + \lambda(c_{sd} + c_{si} - \gamma c_m - 2\gamma c_u)\right) \\ +\lambda^2 c_{sp} q \left(\frac{2\gamma\lambda^2 - q\xi^2}{(2\lambda + \xi q)^2}\right) \end{pmatrix}$$

The third term equals $\mathcal{F}_p(\xi)/(2\lambda + \xi q)^2$. Furthermore, it is strictly decreasing as its derivative

$$-2\xi c_u q - 2\lambda\gamma c_u q - 4\lambda^3 q^2 \cdot \frac{\lambda\gamma + \xi}{(2\lambda + \xi q)^3} \cdot c_{sp} < 0.$$

Hence the equation $\partial \pi_{id}/\partial \xi = 0$ has at most one root. Hence, if $\mathcal{F}_p(x_{min}) \le 0$ then $\xi^*_{ip} = x_{min}$; otherwise ξ^*_{ip} is the unique solution to $\mathcal{F}_p(\xi) = 0$.

(ii) Suppose $\gamma q < 2$. The payer's profit under reminder-based strategy, $\pi_{id}(\xi)$, is the same as that in part (i). However, since $\gamma q < 2$, similar to the proof in part (i), the sign of the derivative $\partial \pi_{id}/\partial \xi$ is the same as that of $-\mathcal{F}_d$ given by

$$\begin{aligned} &-\lambda^2 p(\gamma q - 2)^2 - \lambda^3 \Big((c_{si} + c_{sd} - \gamma c_m)(\gamma q - 2) - 2\gamma(3\gamma q - 2)(c_{ud} + c_r) \Big) \\ &+4\lambda^2 \gamma q(\gamma q + 1)(c_{ud} + c_r)\xi + \lambda q(5\gamma q + 2)(c_{ud} + c_r)\xi^2 + 2q^2(c_{ud} + c_r)\xi^3. \end{aligned}$$

Hence, π_{id} is initially possibly decreasing and then increasing, i.e. U-shaped. Therefore, ξ^*_{id} is either y_{min} or y_{max}, and $\xi^*_{id} = \arg\max\{\pi_{id}(y_{min}), \pi_{id}(y_{max})\}$.

We next consider reward-based strategy. Comparing the rewards $\underline{r}_d(0,\xi)$ and $\hat{r}_d(0,\xi)$, we have

$$\begin{aligned} & \underline{r}_d(0,\xi) > (=,<)\ \hat{r}_d(0,\xi) \\ \Leftrightarrow \quad & \xi c_{ud} + \frac{\lambda^2 \gamma(\underline{\pi}_d + k c_{sd}) + \lambda \xi(\underline{\pi}_d + \gamma k c_m)}{\gamma k(2\lambda + \xi q)} \\ > (=,<) \quad & \frac{q(c_{ud} + c_r)\xi^2 + \lambda\big(2\gamma q c_{ud} + (\gamma q + 2)c_r\big)\xi + \lambda^2\big(2\gamma(c_{ud} + c_r) + \gamma c_m - c_{sd}\big)}{\lambda(\gamma q - 2)} \\ \Leftrightarrow \quad & \gamma q^2 k(c_{ud} + c_r)\xi^3 + \lambda\gamma q k(\gamma q + 4)(c_{ud} + c_r)\xi^2 + \lambda^2\Big((2 - \gamma q)\underline{\pi}_d + \gamma k\big(2c_m - q c_{sd} + \\ & 4(\gamma q + 1)(c_{ud} + c_r)\big)\Big)\xi + \lambda^3\gamma\Big((2 - \gamma q)\underline{\pi}_d + \gamma k\big(2c_m - q c_{sd} + 4(c_{ud} + c_r)\big)\Big) > (=,<)\ 0 \\ \Leftrightarrow \quad & (2\lambda + \xi q)^2 > (=,<)\ \lambda^2 \cdot \frac{\gamma k(q c_{sd} - 2c_m) - (2 - \gamma q)\underline{\pi}_d}{\gamma k(c_{ud} + c_r)}. \end{aligned}$$

For any $\xi \ge \hat{\alpha}(0)$, we have $(2\lambda + \xi q)^2 \ge \lambda q c_{sp}/c_{up}$, and therefore, $\underline{r}_d(0,\xi) \ge \hat{r}_d(0,\xi)$ if

$$\begin{aligned} \frac{q c_{sp}}{c_{up}} &\ge \frac{\gamma k(q c_{sd} - 2c_m) - (2 - \gamma q)\underline{\pi}_d}{\gamma k(c_{ud} + c_r)} \\ \gamma q k c_{sp}(c_{ud} + c_r) &\ge c_{up}\big(\gamma k(q c_{sd} - 2c_m) - (2 - \gamma q)\underline{\pi}_d\big) \end{aligned} \tag{A3}$$

In this case, $r_d^* = \max(\underline{r}_d(0,\xi), \hat{r}_d(0,\xi)) = \underline{r}_d(0,\xi)\ \forall \xi \geq \hat{\alpha}(0)$. The optimal solution is then obtained similar to the reward-based strategy in part (i). Specifically, if $\mathcal{F}_p(\hat{\alpha}(0)) \leq 0$, then $\xi_{ip}^* = \hat{\alpha}(0)$; otherwise ξ_{ip}^* is the unique solution to $\mathcal{F}_p(\xi) = 0$.

Suppose (A3) does not hold. Then there is a threshold $\xi_{th} > \hat{\alpha}(0)$ such that $\underline{r}_d(0,\xi) > (=,<)\ \hat{r}_d(0,\xi)\ \forall \xi > (=,<)\ \xi_{th}$. If $\mathcal{F}_p(\xi_{th}) \geq 0$ then ξ_{ip}^* is the unique solution to $\mathcal{F}_p(\xi) = 0$. Note that the profit π_{ip} at this point not only exceeds $\pi_i p(\xi)\ \forall\ \xi \geq \xi_{th}$ but it also exceeds $\pi_i p(\xi)\ \forall\ \xi < \xi_{th}$ since $r_d^* = \hat{r}_d(0,\xi) > \underline{r}_d(0,\xi)$. Suppose $\mathcal{F}_p(\xi_{th}) < 0$. Since $\xi_{ip}^* \leq \xi_{th}$, we need to consider the payer's profit when $\xi \leq \xi_{th}$. As $r_d^* = \hat{r}_d(0,\xi)$, the payer's profit becomes

$$\pi_{ip}(\xi) = \frac{\gamma k}{\lambda} \cdot \left(\frac{\left(p - \hat{r}_d(0,\xi) - \xi \left(c_{up} - \frac{\lambda^2 q c_{sp}}{(2\lambda+\xi q)^2}\right)\right)(2\lambda + \xi q) - \lambda^2 c_{si}}{\lambda\gamma + \xi} \right)$$

$$= \frac{\gamma k}{\lambda} \cdot \left(\frac{\left(p - \frac{q(c_{ud}+c_r)\xi^2 + \lambda(2\gamma q c_{ud} + (\gamma q+2)c_r)\xi + \lambda^2\left(2\gamma(c_{ud}+c_r) + \gamma c_m - c_{sd}\right)}{\lambda(\gamma q - 2)} - \xi c_{up}\right)\left(2\lambda + \xi q\right) + \frac{\lambda^2 q c_{sp}\xi}{2\lambda+\xi q} - \lambda^2 c_{si}}{\lambda\gamma + \xi} \right)$$

$$= \frac{\gamma k}{\lambda} \cdot \left(\frac{\pi_{id}(\xi) + (c_r - c_{up}) \cdot \xi \cdot (2\lambda + \xi q) + \frac{\lambda^2 q c_{sp}\xi}{2\lambda+\xi q}}{\lambda\gamma + \xi} \right)$$

After some algebra, we find that the derivative of payer's profit can be written as

$$\frac{-\mathcal{F}_d(\xi) + \lambda(2 - \gamma q)\left(\frac{\lambda^2 q c_{sp}(2\lambda^2\gamma - q\xi^2)}{(2\lambda+\xi q)^2} + (c_r - c_{up})(2\lambda^2\gamma + 2\lambda\gamma q\xi + q\xi^2)\right)}{\lambda^2(2-\gamma q)(\lambda\gamma + \xi)^2}$$

$$= \frac{\mathcal{H}(\xi)}{\lambda^2(2-\gamma q)(\lambda\gamma+\xi)^2}$$

Since the denominator of the derivative is positive, it has the same sign as $\mathcal{H}(\xi)$. Furthermore, in the interval $[\hat{\alpha}(0), \xi_{th}]$, $\pi_{ip}(\xi)$ is maximized at the end points or when $\mathcal{H}(\xi) = 0$.

(iii) Suppose $\gamma q = 2$. In the reminder-based strategy, only $\hat{\xi}$ is feasible if $\hat{\xi} > \hat{\alpha}(0)$. The reward-based strategy results in payer profit that is same as in part (i), and hence, ξ_{ip}^* also has the same characterization. Q.E.D.

A2. Incorporating Measurement Errors

We use $Pr\{x|y\}$ to denote the conditional probability that the measured health state is x given that the actual health state is y. Since state S does not depend on the measurements from the uploads, two conditional probabilities become vital for incorporating the measurement errors. The first one, $Pr\{I|N\} \equiv \zeta_1 < 1$, measures the Type-1 error or false positive rate while the second one, $Pr\{N|I\} \equiv \zeta_2 < 1$ measures the Type-2 error or false negative rate. Note that $Pr\{N|N\} = 1 - \zeta_1$ and $Pr\{I|I\} = 1 - \zeta_2$.

For brevity, we only consider the combined upload rate to be $\mu + \alpha$ but note that we can perform a similar analysis when it is $\mu + \theta\alpha$ or $\max(\mu, \alpha)$. Before deriving the patient's cost, provider's net revenue, and payer's profit in the presence of measurement errors, we make a few observations. First, we note that in terms of the transitions between the (actual) health states, the only change in Figure 1 is that the transition rate from I to N becomes $(\mu+\alpha)q(1-\zeta_2)$. The rest of the analysis is similar to the main paper. The average renewal cycle time, which is the time between two consecutive arrivals to state N, denoted by $\tau(\alpha,\mu)$, and the average number of mobile interventions during this time are respectively obtained as

$$\frac{1}{\lambda}+\frac{1}{\lambda+(\mu+\alpha)q(1-\zeta_2)} \text{ and } \frac{(\mu+\alpha)\zeta_1}{\lambda}+\frac{(\mu+\alpha)(1-\zeta_2)}{\lambda+(\mu+\alpha)q(1-\zeta_2)}.$$

The patient's cost per period is

$$c_p(\alpha,\mu)=p+(\mu+\alpha)(c_{up}-r_p)+\frac{\lambda}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\cdot\frac{c_{sp}}{\tau(\alpha,\mu)}.$$

The patient's problem to minimize c_p is similar to the one in the main paper with $q(1-\zeta_2)$ replacing q. Therefore, the optimal reward-based upload rate is

$$\alpha^*(r_p,\mu)=\left(\frac{\lambda}{q(1-\zeta_2)}\left(\sqrt{\frac{q(1-\zeta_2)c_{sp}}{c_{up}-r_p}}-2\right)-\mu\right)^+.$$

We then have $\hat{\alpha}(r_p)=\alpha^*(r_p,0)$ given by

$$\hat{\alpha}(r_p)=\frac{\lambda}{q(1-\zeta_2)}\cdot\left(\sqrt{\frac{q(1-\zeta_2)c_{sp}}{c_{up}-r_p}}-2\right)^+.$$

The provider's net revenue is given by

$$\begin{aligned}\pi_d(r_d,\mu,\alpha,N) = N\Bigg(& r_d-(\mu+\alpha)c_{ud}-\mu c_r-\left(\frac{(\mu+\alpha)\zeta_1}{\lambda}+\frac{(\mu+\alpha)(1-\zeta_2)}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\right)\cdot\frac{c_m}{\tau(\alpha,\mu)}\\ &-\frac{\lambda}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\cdot\frac{c_{sd}}{\tau(\alpha,\mu)}\Bigg).\end{aligned}$$

The provider's optimization problem becomes

$$\begin{aligned}&\max_{N,\mu}\pi_d(r_d,\mu,\alpha,N)\\ \text{s. t. }&\frac{N}{\tau(\alpha,\mu)}\left(\left(\frac{(\mu+\alpha)\zeta_1}{\lambda}+\frac{(\mu+\alpha)(1-\zeta_2)}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\right)\cdot\frac{1}{\gamma}+\frac{\lambda}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\right)\leq k.\end{aligned}$$

We obtain N^* by binding the provider's capacity constraint, which yields

$$N^*=\frac{\gamma k\Big(2\lambda+(\mu+\alpha)q(1-\zeta_2)\Big)}{\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2}$$

After substituting N^*, the provider's net revenue becomes

$$\pi_d(r_d,\mu,\alpha,N)=\frac{\gamma k\Big(2\lambda+(\mu+\alpha)q(1-\zeta_2)\Big)}{\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2}.$$

$$= \frac{\begin{pmatrix} r_d - (\mu+\alpha)c_{ud} - \mu c_r - \left(\frac{(\mu+\alpha)\zeta_1}{\lambda} + \frac{(\mu+\alpha)(1-\zeta_2)}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\right)\cdot\frac{c_m}{\tau(\alpha,\mu)} \\ -\frac{\lambda}{\lambda+(\mu+\alpha)q(1-\zeta_2)}\cdot\frac{c_{sd}}{\tau(\alpha,\mu)} \end{pmatrix}}{}$$

$$= \frac{\gamma k}{\gamma\lambda^2 + (1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda + q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2}\cdot \begin{pmatrix} (r_d-(\mu+\alpha)c_{ud}-\mu c_r)(2\lambda+(\mu+\alpha)q(1-\zeta_2)) \\ -((1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2)c_m - \lambda^2 c_{sd} \end{pmatrix}.$$

While the corresponding net revenue in the main paper is a ratio of quadratic and linear functions in μ, the net revenue here is a ratio of two quadratic functions in μ. Nevertheless, we next show that μ^* is still unique.

Since γk is a positive constant, we only need to consider the function

$$\frac{\big(r_d-(\mu+\alpha)c_{ud}-\mu c_r\big)\big(2\lambda+(\mu+\alpha)q(1-\zeta_2)\big) - \big((1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2\big)c_m - \lambda^2 c_{sd}}{\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2}.$$

The numerator of its partial derivative with respect to μ (the denominator is always positive) determines whether provider's net revenue increases or decreases. For this reason, we refer to it as $\mathcal{G}(r_d,\mu,\alpha)$, and it is given by

$$\begin{aligned} &\Big(\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2\Big) \\ \cdot &\begin{pmatrix} (r_d-(\mu+\alpha)c_{ud}-\mu c_r)q(1-\zeta_2)-(2\lambda+(\mu+\alpha)q(1-\zeta_2))(c_{ud}+c_r) \\ -((1+\zeta_1-\zeta_2)\lambda+2q\zeta_1(1-\zeta_2)(\mu+\alpha))c_m \end{pmatrix} \\ - &\begin{pmatrix} (r_d-(\mu+\alpha)c_{ud}-\mu c_r)\big(2\lambda+(\mu+\alpha)q(1-\zeta_2)\big) \\ -\big((1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2\big)c_m-\lambda^2c_{sd} \end{pmatrix} \\ \cdot &\Big((1+\zeta_1-\zeta_2)\lambda+2q\zeta_1(1-\zeta_2)(\mu+\alpha)\Big) \end{aligned}$$

This function $\mathcal{G}$ can be seen as the product of first and second terms minus the product of third and fourth terms in which the first term $\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu+\alpha)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu+\alpha)^2>0$, the third term is positive (since net revenue is positive), and the fourth term $(1+\zeta_1-\zeta_2)\lambda+2q\zeta_1(1-\zeta_2)(\mu+\alpha)$ is also positive. Therefore, for $\mathcal{G}$ to be non-negative, the second term needs to be positive which implies

$$(r_d-\mu c_r)q(1-\zeta_2) > 2\lambda(c_{ud}+c_r). \tag{A4}$$

After some algebra, $\mathcal{G}$ simplifies to

$$\begin{aligned} &\lambda^2\Big((r_d-\mu c_r)\big(\gamma q(1-\zeta_2)-2(1+\zeta_1-\zeta_2)\big)-\lambda\big(2\gamma(c_{ud}+c_r)+(\gamma c_m-c_{sd})(1+\zeta_1-\zeta_2)\big)\Big) \\ -\ &(\mu+\alpha)\cdot\lambda\cdot\Big(q(1-\zeta_2)\big(\gamma\lambda(2c_{ud}+c_r+2\zeta_1c_m)+4\zeta_1(r_d-\mu c_r)-2\lambda\zeta_1c_{sd}\big)+2\lambda c_r(1+\zeta_1-\zeta_2)\Big) \\ -\ &(\mu+\alpha)^2\cdot q\cdot(1-\zeta_2)\cdot\Big((r_d-\mu c_r)\zeta_1q(1-\zeta_2)+\lambda c_{ud}(1-\zeta_1-\zeta_2)+\lambda c_r(1+3\zeta_1-\zeta_2)\Big) \\ -\ &(\mu+\alpha)^3q^2\zeta_1(1-\zeta_2)^2c_r. \end{aligned}$$

The partial derivative of $\mathcal{G}$ with respect to μ, $\partial G/\partial\mu$, is

$$-2\lambda q(1-\zeta_2)\cdot\Big(\gamma\lambda(c_{ud}+c_r+\zeta_1c_m)+2\zeta_1(r_d-\mu c_r)-\lambda\zeta_1c_{sd}\Big)$$

$$
\begin{aligned}
&-2(\mu+\alpha)q(1-\zeta_2)\cdot\Big((r_d-\mu c_r)\zeta_1 q(1-\zeta_2)+\lambda c_{ud}(1-\zeta_1-\zeta_2)+\lambda c_r(1+\zeta_1-\zeta_2)\Big)\\
&-2(\mu+\alpha)^2q^2(1-\zeta_2)^2\zeta_1 c_r\\
=\ &-2\lambda q(1-\zeta_2)\cdot\Big(\gamma\lambda(c_{ud}+c_r+\zeta_1 c_m)+2\zeta_1(r_d+\alpha c_r)-\lambda\zeta_1 c_{sd}\Big)\\
&-2(\mu+\alpha)q(1-\zeta_2)\cdot\Big((r_d+\alpha c_r)\zeta_1 q(1-\zeta_2)+\lambda c_{ud}(1-\zeta_1-\zeta_2)+\lambda c_r(1-\zeta_1-\zeta_2)\Big)
\end{aligned}
$$

The derivative $\partial\mathcal{G}/\partial\mu$ is linear in μ, and is strictly decreasing for any non-negative $\mathcal{G}$ since $(r_d+\alpha c_r)\zeta_1 q(1-\zeta_2)+\lambda c_{ud}(1-\zeta_1-\zeta_2)+\lambda c_r(1-\zeta_1-\zeta_2)>\zeta_1\Big((r_d-\mu c_r)q(1-\zeta_2)-2\lambda(c_{ud}+c_r)\Big)>0$ from (A4). Therefore, the equation $\partial\mathcal{G}/\partial\mu=0$ has at most a single positive root. There are two possibilities: (i) there is no positive root, then $\partial\mathcal{G}/\partial\mu<0$ so that $\mathcal{G}$ is strictly decreasing in μ and μ^* is unique ($\mu^*=0$ or μ^* solves $\mathcal{G}(\mu)=0$, and (ii) there is a positive root, then $\partial\mathcal{G}/\partial\mu>0$ until this root and then $\partial\mathcal{G}/\partial\mu<0$ beyond this root so that $\mathcal{G}$ is a $\cap$-shaped function of μ. In this case, since we find $\lim_{\mu\to\infty}\mathcal{G}(\mu)=-\infty<0$, μ^* is the largest root of $\mathcal{G}(\mu)=0$ if the root exists and yields a higher net revenue than $\mu=0$; otherwise, $\mu^*=0$. Therefore, in both cases (i) and (ii), μ^* is unique.

The payer's profit is given by

$$
\begin{aligned}
&N^*\left(p-(\mu^e+\alpha^e)r_p-r_d-\frac{\lambda}{\lambda+(\mu^e+\alpha^e)q(1-\zeta_2)}\cdot\frac{c_{si}}{\tau(\alpha^e,\mu^e)}\right)\\
=\ &\frac{\gamma k\cdot\Big(\big(p-r_d-(\mu^e+\alpha^e)r_p\big)\cdot\big(2\lambda+(\mu^e+\alpha^e)q(1-\zeta_2)\big)-\lambda^2 c_{si}\Big)}{\gamma\lambda^2+(1+\zeta_1-\zeta_2)(\mu^e+\alpha^e)\cdot\lambda+q\zeta_1(1-\zeta_2)\cdot(\mu^e+\alpha^e)^2}
\end{aligned}
$$

Lemma 5 holds because the logic in its proof still applies: if there are both reminders and rewards at provider-patient equilibrium, the payer would benefit by reducing r_d that results in less reminders. Therefore, either reminder-based strategy or reward-based strategy is optimal for the payer.

We extend the results in §9 of the main paper to the case when there are measurement errors in mHealth. We assume $\zeta_1=0.01$ and $\zeta_2=0.05$. Details are presented in the eCompanion (see §EC 1). The discussion is largely similar to §9. The switch between RmBS and RwBS strategy is similar to what we discussed in the main paper.

A3. Reward-based Upload Rate and Reminder Rate are Substitutes

In §'s 3-7, we consider the combined upload rate $\xi=\alpha+\mu$, where the rates α and μ can be seen as *perfect complements*. We now consider two extensions. The first examines *imperfect complements/partial substitutes*, which we discuss in §A8. The second examines perfect substitutes and is presented here.

We first note that, similar to §4 and §5, we obtain the patient's cost and provider's net revenue by replacing $\mu+\alpha$ terms with $\max(\mu,\alpha)$. Since the patient cost depends only on $\max(\mu,\alpha)$, we assume without any loss of generality that the patient prefers to choose $\alpha=0$ instead of any other value of $\alpha\le\mu$. We note that we make this assumption for brevity and easier exposition, and it does not affect our analysis. As in §5, the derivative of the provider's net revenue $\partial\pi_d(r_d,\mu,\alpha)/\partial\mu$ is a positive multiple of the function $\mathcal{G}(r_d,\mu,0)$. We note that $\hat{\mu}$ (defined in §6) is the unique solution to $\mathcal{G}(r_d,\mu,0)=0$. Furthermore, if necessary, we use $\hat{\mu}(r_d)$ to explicitly denote the reminder rate as a function of capitation rate r_d.

Lemma A1 characterizes the optimal α chosen by the patient (for a given reminder rate μ), optimal μ chosen by the provider (for a given upload rate α), and the resulting provider-patient equilibrium (μ^e,α^e). All proofs are provided in the eCompanion (see §EC 2.1).

Lemma A1. *(i) If $\hat{\alpha}>\mu$ then $\alpha^*=\hat{\alpha}$ else $\alpha^*=0$.*

(ii) If $\mathcal{G}(r_d,\alpha,0)\le 0$ then $\mu^*=0$. If $\mathcal{G}(r_d,\alpha,0)>0$ then there are two possibilities. If $\pi_d(r_d,\hat{\mu},\alpha)>\pi_d(r_d,0,\alpha)$ then $\mu^*=\hat{\mu}$ else $\mu^*=0$.

(iii) The equilibrium rates are given as follows:

(a) If $\mathcal{G}(r_d,\hat{\alpha},0)\le 0$ then $\mu^e=0$ and $\alpha^e=\hat{\alpha}$.

(b) If $\mathcal{G}(r_d,\hat{\alpha},0)>0$ then there are two possibilities. If $\pi_d(r_d,\hat{\mu},0)>\pi_d(r_d,0,\hat{\alpha})$ then $\mu^e=\hat{\mu}$ and $\alpha^e=0$, else $\mu^e=0$ and $\alpha^e=\hat{\alpha}$.

We use the result from Lemma A1 and obtain in Lemmas A2-A4 the feasibility conditions for the reminder-based and reward-based strategies as well as the payer's optimal capitation rate and reward for any feasible ξ in these two strategies, under different ranges of values of mHTP. We also compare these results with those obtained when $\xi=\mu+\alpha$ (Lemmas 6-8) in Corollaries A1-A3.

A3.1. mHTP Above Two

Lemma A2. *(i) If $\hat{r}_d(\xi,0)\ge \underline{r}_d(\xi,0)$ and $\pi_d(\hat{r}_d(\xi,0),\xi,0)>\pi_d(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))$, then reminder-based strategy is feasible. The optimal values are given by $r_d^*=\hat{r}_d(\xi,0)$ and $r_p^*=0$.*

(ii) If either $\hat{r}_d(\xi,0)\ge \underline{r}_d(0,\xi)$, or $\hat{r}_d(\xi,0)<\underline{r}_d(0,\xi)$ and $\pi_d\big(\underline{r}_d(0,\xi),\hat{\mu}(\underline{r}_d(0,\xi)),0\big)\le \pi_d(\underline{r}_d(0,\xi),0,\xi)$, then reward-based strategy is feasible. The optimal values are given by $r_d^*=\underline{r}_d(0,\xi)$ and $r_p^*=c_{up}-\lambda^2 q c_{sp}/(2\lambda+\xi q)^2$.

Corollary A1. *In comparison to $\xi=\mu+\alpha$, $\xi=\max(\mu,\alpha)$ reduces the feasibility of reminder-based strategy while it increases the feasibility of reward-based strategy. The optimal capitation rate and reward (for a ξ feasible in both cases) remain the same in both strategies.*

A3.2. mHTP Below Two

LEMMA A3. *(i) If* $\hat{r}_d(\xi,0) \geq \underline{r}_d(\xi,0)$ *and* $\pi_d(\hat{r}_d(\xi,0),\xi,0) > \pi_d(\hat{r}_d(\xi,0),0,\hat{\alpha}(0))$*, then reminder-based strategy is feasible. The optimal values are given by* $r_d^* = \hat{r}_d(\xi,0)$ *and* $r_p^* = 0$*.*

(ii) The reward-based strategy is feasible for any ξ*. The optimal reward* $r_p^* = c_{up} - \lambda^2 q c_{sp}/(2\lambda + \xi q)^2$*. The optimal capitation rate is given as follows:*

(a) If $\hat{r}_d(\xi,0) \leq \underline{r}_d(0,\xi)$ *then* $r_d^* = \underline{r}_d(0,\xi)$*.*

(b) If $\hat{r}_d(\xi,0) > \underline{r}_d(0,\xi)$ *and* $\underline{r}_d(\hat{\mu}(\underline{r}_d(0,\xi)),0) \geq \underline{r}_d(0,\xi)$ *then* $r_d^* = \underline{r}_d(0,\xi)$*.*

(c) If $\hat{r}_d(\xi,0) > \underline{r}_d(0,\xi) > \underline{r}_d(\hat{\mu}(\underline{r}_d(0,\xi)),0)$ *then there exists unique* $\tilde{r}_d$ *such that* $\hat{r}_d(\xi,0) > \tilde{r}_d > \underline{r}_d(0,\xi)$ *and* $\pi_d(\tilde{r}_d,\hat{\mu}(\tilde{r}_d),0) = \pi_d(\tilde{r}_d,0,\xi)$*, and* $r_d^* = \tilde{r}_d$*.*

COROLLARY A2. *A change from* $\xi = \mu + \alpha$ *to* $\xi = \max(\mu,\alpha)$ *has the following impacts:*

(i) The optimal capitation rate and reward (for a ξ *feasible in both cases) remain the same in the reminder-based strategy.*

(ii) It maintains the feasibility of reward-based strategy (for any ξ*). For a* ξ *feasible in both cases, optimal reward remains the same, and if* $\hat{r}_d(\xi,0) \leq \underline{r}_d(0,\xi)$ *then optimal capitation rate remains the same too.*

A3.3. mHTP Equals Two

LEMMA A4. *(i) The reminder-based strategy is feasible if* $\xi = \hat{\xi} > \hat{\alpha}(0)$ *and* $\underline{r}_d(\hat{\xi},0) < \underline{r}_d(0,\hat{\alpha}(0))$*. In this case,* $r_d^* = \underline{r}_d(\xi,0)$ *and* $r_p^* = 0$*.*

(ii) The reward-based strategy is feasible if $\xi \geq \hat{\xi}$*, or* $\xi < \hat{\xi}$ *and* $\underline{r}_d(0,\xi) \leq \underline{r}_d(\hat{\xi},0)$*. In this case,* $r_d^* = \underline{r}_d(0,\xi)$ *and* $r_p^* = c_{up} - \lambda^2 q c_{sp}/(2\lambda + \xi q)^2$*.*

COROLLARY A3. *In comparison to* $\xi = \mu + \alpha$*,* $\xi = \max(\mu,\alpha)$ *reduces the feasibility of reminder-based strategy while it increases the feasibility of reward-based strategy. The optimal capitation rate and reward (for a* ξ *feasible in both cases) remain the same in both strategies.*

In summary, we note that there may be minor differences in the provider-patient equilibrium (Lemma A1) and the feasibility of ξ as well as r_d^* and r_p^* for a given ξ (Lemmas A2-A4 and Corollaries A1-A3) depending on whether the rates α and μ are complements or substitutes. Nevertheless, we find that the main results including how different parameters impact which strategy, reminder-based or reward-based, is optimal remain qualitatively similar in both cases (see §9 and the online supplement for further details). We extend the numerical results in §9 of the main paper to the case when $\xi = \max(\mu,\alpha)$ in the eCompanion (see §EC 2.2).

A4. Incorporating Fee-for-Service (FFS) Payment for Sick Visits

We consider the same model setup as the main paper with just one additional element: the provider gets r_f for every office visit by a patient, i.e., whenever the patient gets sick. We find that in a pure FFS system, the provider has no incentive to send any reminders (regardless of the value of r_f), and the reminder-based strategy becomes infeasible to the payer. Hence, the payer has to rely only on patient rewards. We provide further details in the **e-companion** (see §EC 3).

A5. Additional Numerical Analysis

The payer chooses RwBS over RmBS beyond a particular threshold for the provider's cost of sending reminders (c_r). If λ is low, RwBS is better than RmBS for the payer, and the use of mHealth is influenced by direct patient rewards. The payer switches to RmBS from RwBS beyond a particular threshold for λ. We provide further details in the **e-companion** (see §EC 4).

A6. Multiple patient types

We extend our analysis to the case of multiple patient types. We use two patient types for easier exposition but our analysis can be easily extended to more than two patient types. We provide further details in the **e-companion** (see §EC 5).

A7. Different Transition Rates to Health States

In the main paper, we assume the transition rates from $\mathcal{N}$ to $\mathcal{I}$ and $\mathcal{I}$ to $\mathcal{S}$ both equal λ. We extend our analysis to consider different transition rates. We provide further details in the **e-companion** (see §EC 6).

A8. Imperfect Reminders

In the main paper, we assume that all the reminders would result in uploads from patients. We relax this assumption here. Specifically, we let θ $(0 < \theta < 1)$ be the fraction of reminders that result in uploads, and therefore, mobile intervention if the patient is in state I. We show that we can transform the problem with imperfect reminders to one with perfect reminders, and there is no loss of generality in the assumption of perfect reminders. We provide further details in the **e-companion** (see §EC 7).

References

Agnihothri, Saligrama, Arvind Sainathan, Balaraman Rajan. 2026. Increasing mhealth usage through strategic payer incentives for providers and patients. *eCompanion* URL `https://www.researchgate.net/publication/404460417_Agnihothri-MSOM-2025-0182-ResearchGate_eCompanion`.