Electronic Companion for "Enhancing the Benefits of Dual Sourcing with Upstream Visibility"

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EC.1. Proof of Proposition 1

Throughout this proof, we drop the subscripts i and k. We will consider four different cases, each conditioning on a different region of the domain $[0,\infty)^2$.

Case 1. If the order is restricted so that $q_1+q_2 \le d$, we can write

$$\begin{aligned} C(\mathbf{q}) =& \mathbb{E}[c_o(\mathcal{R}(\mathbf{q})-d)^+ + c_u(d-\mathcal{R}(\mathbf{q}))^+] \\ =& \mathbb{E}[c_u(d-\mathcal{R}(\mathbf{q}))] = \mathbb{E}[c_u(d-\mathcal{R}_1(q_1)-\mathcal{R}_2(q_2))] \\ =& c_u(d-\hat{\rho}_1 q_1 - \hat{\rho}_2 q_2) \end{aligned}$$

The second equality follows from conditioning on $q_1+q_2 \le d$, and the last equality follows by the linearity of expectation. Therefore, in this domain it is optimal to order an aggregate quantity equal to d.

Case 2. Now consider restricting the order so that $q_1+q_2 \ge d$, $q_1 \le d$ and $q_2 \le d$. In this case,

$$\begin{aligned} C(\mathbf{q}) =& \mathbb{E}[c_o(\mathcal{R}(\mathbf{q})-d)^+ + c_u(d-\mathcal{R}(\mathbf{q}))^+] \\ =& c_o(q_1+q_2-d)\hat{\rho}_1\hat{\rho}_2 + c_u(d-q_1)\hat{\rho}_1(1-\hat{\rho}_2) + c_u(d-q_2)\hat{\rho}_2(1-\hat{\rho}_1) + c_u d(1-\hat{\rho}_1)(1-\hat{\rho}_2). \end{aligned}$$

The first term comes from conditioning on both orders being fulfilled, the second term conditions on only supplier 1 fulfilling the order, and the third term conditions on only supplier 2 fulfilling the order. Because this expression is linear in both q_1 and q_2, the optimal point is either (d,d) or must satisfy $q_1+q_2=d$.

Case 3. Suppose $q_j \ge d$. In this case, $\frac{\partial C(\mathbf{q})}{\partial q_j} = c_o\hat{\rho}_2\hat{\rho}_1 + c_o\hat{\rho}_j(1-\hat{\rho}_{-j})$ which is non-negative. Therefore, it is never optimal for q_j to be greater than d.

Case 4. Suppose that $q_1 + q_2 = d$. In this case, we can write the cost function as a one-dimensional function of q_1, given by:

$$C(\mathbf{q}) = c_u(d - q_1)\hat{\rho}_1(1-\hat{\rho}_2) + c_u q_1 \hat{\rho}_2(1-\hat{\rho}_1) + c_u d(1-\hat{\rho}_1)(1-\hat{\rho}_2).$$

Notice this function is linear in q_1. Therefore, the optimal order is either $(0,d)$ or $(d,0)$. Combining these results, we find that $\mathbf{q}^* \in \{(0,d),(d,0),(d,d)\}$.

We now characterize when each of the three candidate orders is optimal. Since the preceding cases show that there exists an optimal order in $\{(d,d),(d,0),(0,d)\}$, it is enough to compare the expected costs of these three orders. Let $\gamma := \frac{c_u}{c_o+c_u}$. The expected costs are $C(d,0) = c_u d(1-\hat{\rho}_1)$, $C(0,d) = c_u d(1-\hat{\rho}_2)$, and $C(d,d) = c_o d\hat{\rho}_1\hat{\rho}_2 + c_u d(1-\hat{\rho}_1)(1-\hat{\rho}_2)$. First,

$$\begin{aligned} C(d,d) - C(d,0) &= c_o d\hat{\rho}_1\hat{\rho}_2 + c_u d(1-\hat{\rho}_1)(1-\hat{\rho}_2) - c_u d(1-\hat{\rho}_1) \\ &= d\hat{\rho}_2\big((c_o+c_u)\hat{\rho}_1 - c_u\big). \end{aligned}$$

Therefore, $C(d,d) \le C(d,0)$ if and only if $\hat{\rho}_2 = 0$ or $\hat{\rho}_1 \le \gamma$. Similarly,

$$C(d,d) - C(0,d) = d\hat{\rho}_1\big((c_o+c_u)\hat{\rho}_2 - c_u\big),$$

so $C(d,d) \le C(0,d)$ if and only if $\hat{\rho}_1 = 0$ or $\hat{\rho}_2 \le \gamma$. Finally, $C(d,0) \le C(0,d)$ if and only if $\hat{\rho}_1 \ge \hat{\rho}_2$, and $C(0,d) \le C(d,0)$ if and only if $\hat{\rho}_2 \ge \hat{\rho}_1$.

Thus, (d,d) is optimal if and only if

$$(\hat{\rho}_2 = 0 \text{ or } \hat{\rho}_1 \le \gamma) \quad \text{and} \quad (\hat{\rho}_1 = 0 \text{ or } \hat{\rho}_2 \le \gamma),$$

which is equivalently

$$D_{dd} = [0,\gamma]^2 \cup ([0,1]\times\{0\}) \cup (\{0\}\times[0,1]).$$

Similarly, $(d,0)$ is optimal if and only if

$$C(d,0) \le C(0,d) \quad \text{and} \quad C(d,0) \le C(d,d),$$

which gives

$$D_{d0} = \{(\hat{\rho}_1,\hat{\rho}_2) : \hat{\rho}_1 \ge \hat{\rho}_2 \text{ and } (\hat{\rho}_1 \ge \gamma \text{ or } \hat{\rho}_2 = 0)\}.$$

Likewise, $(0,d)$ is optimal if and only if

$$C(0,d) \le C(d,0) \quad \text{and} \quad C(0,d) \le C(d,d),$$

which gives

$$D_{0d} = \{(\hat{\rho}_1, \hat{\rho}_2) : \hat{\rho}_2 \ge \hat{\rho}_1 \text{ and } (\hat{\rho}_2 \ge \gamma \text{ or } \hat{\rho}_1 = 0)\}.$$

This proves the stated characterization of the optimality regions. □

EC.2. Proof of Theorem 1

Suppose, by contradiction, that the orders converge and there exists at least one buyer, g, whose order converges to (d,d) and one buyer, h, whose order converges to $(d,0)$ under the tie-breaking convention described in the main text. Following (3), at the start of period k we have that

$$\hat{\rho}_{(g,1),k} = \frac{1}{\sum_{l=1}^{k-1} \mathbf{1}\{q_{(g,1),l} > 0\}} \sum_{l=1}^{k-1} \mathbf{1}\{q_{(g,1),l} > 0, R_{(g,1),l} = q_{(g,1),l}\}$$

and similarly for buyer h. We will show that for every $\epsilon > 0$, there exists a K' such that for $k > K'$, $|\hat{\rho}_{(g,1),k} - \hat{\rho}_{(h,1),k}| < \epsilon$.

To see this, first notice that because both buyers' orders converge, let K be such that for $k > K$, $\mathbf{q}_{g,k} = (d,d)$ and $\mathbf{q}_{h,k} = (d,0)$. Now, re-write $\hat{\rho}_{(g,1),k}$ as

$$\hat{\rho}_{(g,1),k} = \frac{\sum_{l=1}^{K-1} \mathbf{1}\{q_{(g,1),l} > 0, R_{(g,1),l} = q_{(g,1),l}\} + \sum_{l=K}^{k-1} \mathbf{1}\{R_{(g,1),l} = d\}}{\sum_{l=1}^{K-1} \mathbf{1}\{q_{(g,1),l} > 0\} + (k-K)}.$$

Only the second terms depend on k. For $l \ge K$, the order profile $\mathbf{q}_l$ is fixed (converged), meaning the distribution of inventory allocation to any buyer ordering d from supplier 1 is stationary. While the outcomes $R_{(g,1),l}$ and $R_{(h,1),l}$ are distinct random variables, the sequence $\{\mathbf{1}\{R_{(i,1),l} = d\}\}_{l=K}^{k-1}$ for any buyer $i \in \{g,h\}$ is a sequence of independent and identically distributed (IID) random variables, all sharing the same common expected value, which is determined by the converged order profile $\mathbf{q}_\infty$ and the true inventory distribution $f_{X_1}(\cdot)$. By the Strong Law of Large Numbers, both perceived fulfillment likelihoods converge to this common expected value. Thus, $\hat{\rho}_{(g,1),\infty} = \hat{\rho}_{(h,1),\infty}$.

Now apply the tie-breaking convention. Since buyer g selects (d,d) in all sufficiently late periods, both of buyer g's perceived fulfillment probabilities are strictly below $c_u/(c_u+c_o)$ in all sufficiently late periods. In particular, $\hat{\rho}_{(g,1),k} < c_u/(c_u+c_o)$ for all sufficiently large k. Thus $\hat{\rho}_{(g,1),\infty} \le c_u/(c_u+c_o)$. By the no-boundary assumption in Theorem 1, the limit cannot equal $c_u/(c_u+c_o)$. Hence $\hat{\rho}_{(g,1),\infty} < c_u/(c_u+c_o)$.

On the other hand, since buyer h selects $(d,0)$ in all sufficiently late periods, the tie-breaking convention implies $\hat{\rho}_{(h,1),k} \geq c_u/(c_u+c_o)$ for all sufficiently large k. Therefore $\hat{\rho}_{(h,1),\infty} \geq c_u/(c_u+c_o)$. Again using the no-boundary assumption, this implies $\hat{\rho}_{(h,1),\infty} > c_u/(c_u+c_o)$.

We have shown that $\hat{\rho}_{(g,1),\infty} < c_u/(c_u+c_o)$ and $\hat{\rho}_{(h,1),\infty} > c_u/(c_u+c_o)$, which contradicts $\hat{\rho}_{(g,1),\infty} = \hat{\rho}_{(h,1),\infty}$. Therefore, a convergent limiting profile cannot contain both a double-ordering buyer and a buyer who orders $(d,0)$. The same argument, using supplier 2, rules out a limiting profile containing both a double-ordering buyer and a buyer who orders $(0,d)$. Hence any convergent limiting profile must be either a truthful profile, in which every buyer orders either $(d,0)$ or $(0,d)$, or a double-ordering profile, in which every buyer orders (d,d). □

EC.3. Proof of Corollary 1

Let

$$\gamma := \frac{c_u}{c_u+c_o}.$$

We first consider the double-ordering profile, in which every buyer orders (d,d). Suppose that all buyers follow this profile in all sufficiently large periods. Since any finite initial history has vanishing weight in the empirical beliefs, the limiting perceived fulfillment probability for supplier j is the probability that a buyer receives her full order from supplier j when all n buyers order d from that supplier. Under the random-priority fulfillment rule, a given buyer is equally likely to be in any priority position $m \in \{1,\dots,n\}$. Conditional on being in position m, the buyer receives her full order from supplier j if and only if $X_j \geq md$. Hence, for every buyer i,

$$\hat{\rho}^D_{(i,j),\infty} = \frac{1}{n}\sum_{m=1}^{n}\Big(1 - F_{X_j}(md)\Big), \qquad j = 1,2.$$

By Proposition 1 and the tie-breaking convention, the buyer selects the double-ordering profile (d,d) if and only if both perceived fulfillment probabilities are strictly below the critical fractile γ. Therefore, the double-ordering profile is self-consistent as a limiting profile if and only if

$$\frac{1}{n}\sum_{m=1}^{n}\Big(1 - F_{X_j}(md)\Big) < \gamma, \qquad j = 1,2.$$

We now consider a truthful profile in which n_1 buyers order $(d,0)$ and $n_2 = n - n_1$ buyers order $(0,d)$. For each supplier j with $n_j > 0$, define

$$\hat{\rho}^T_j(n_j) := \frac{1}{n_j}\sum_{m=1}^{n_j}\Big(1 - F_{X_j}(md)\Big).$$

Suppose first that the truthful profile is self-consistent as a limiting profile. Consider any supplier j with $n_j > 0$. Buyers assigned to supplier j continue to order from supplier j in all sufficiently large periods, and therefore continue to update their beliefs about supplier j. By the same random-priority argument as above, their limiting perceived fulfillment probability for supplier j is $\hat{\rho}_j^T(n_j)$. Since these buyers select a truthful order from supplier j in the limit, the tie-breaking convention implies that the perceived fulfillment probability of the supplier from which they order must be at least the critical fractile γ. Hence, for each $j \in \{1,2\}$ with $n_j > 0$,

$$\hat{\rho}_j^T(n_j) \geq \gamma,$$

or equivalently,

$$\frac{1}{n_j}\sum_{m=1}^{n_j}\Big(1 - F_{X_j}(md)\Big) \geq \frac{c_u}{c_u + c_o}.$$

Conversely, suppose that, for each $j \in \{1,2\}$ with $n_j > 0$,

$$\frac{1}{n_j}\sum_{m=1}^{n_j}\Big(1 - F_{X_j}(md)\Big) \geq \frac{c_u}{c_u + c_o}.$$

We show that the corresponding truthful profile is self-consistent. For buyers assigned to supplier 1, choose the initial beliefs, or equivalently a finite pre-limit history, so that their stale beliefs about supplier 2 satisfy

$$\hat{\rho}_{(i,2),\infty} \leq \hat{\rho}_1^T(n_1).$$

Then their limiting belief vector lies in the region where $(d,0)$ is selected under Proposition 1 and the tie-breaking convention.

Similarly, for buyers assigned to supplier 2, choose the initial beliefs, or equivalently a finite pre-limit history, so that their stale beliefs about supplier 1 satisfy

$$\hat{\rho}_{(i,1),\infty} < \hat{\rho}_2^T(n_2).$$

The strict inequality is used only to respect the tie-breaking convention, which selects supplier 1 when the two perceived fulfillment probabilities are equal and both are at least γ. Therefore, the limiting belief vector of each buyer assigned to supplier 2 lies in the region where $(0,d)$ is selected.

Thus, under the stated inequalities, the limiting beliefs induced by the truthful profile are consistent with the selected truthful-ordering regions. Hence the truthful profile is self-consistent as a limiting profile. This proves the claimed characterization. □

EC.4. Proof of Theorem 2

We define the critical ratio

$$r := \frac{c_o}{c_o + c_u}.$$

The proof proceeds in four parts: first, we characterize the single buyer's optimal best response and prove that the orders are constrained to $q_j \le d$; second, we define the NE fixed points using the continuous unconstrained best response function $\hat{q}_j$ and the clipped best response B_j; third, we characterize the three resulting NE types; and fourth, we prove the existence implications.

Throughout the proof, when no confusion arises, we suppress the dependence of A_j and $\mathcal{A}_j$ on the homogeneous order placed by the other buyers at supplier j. The expected cost function for a buyer ordering $\mathbf{q} = (q_1, q_2)$ is

$$C(\mathbf{q}) = c_o \mathbb{E}[(R(\mathbf{q}) - d)^+] + c_u \mathbb{E}[(d - R(\mathbf{q}))^+],$$

where

$$R(\mathbf{q}) = \min\{q_1, A_1\} + \min\{q_2, A_2\}$$

is the received quantity.

Part 1: Characterization of Optimal Order and Domain Constraints

A. Proof that Optimal Order $q_j \le d$. We show that ordering any quantity q_j greater than demand d is suboptimal. Without loss of generality, consider $q_1 > d$. The marginal cost of the q_1-th unit is $\partial C / \partial q_1$.

For the underage term, since $q_1 > d$, the first d units ordered from supplier 1 are already sufficient to satisfy demand d whenever supplier 1 can fulfill them, even if $A_2 = 0$. Therefore, a marginal unit ordered beyond d cannot reduce the expected shortage cost:

$$\frac{\partial}{\partial q_1} \left[c_u \mathbb{E}[(d - R(\mathbf{q}))^+] \right] = 0 \qquad \text{for } q_1 > d.$$

For the overage term, the marginal unit contributes c_o to the cost if it is received. Since the unit itself is beyond total demand d, it can only contribute to overage. Thus

$$\frac{\partial C}{\partial q_1} = \frac{\partial}{\partial q_1} \left[c_o \mathbb{E}[(R(\mathbf{q}) - d)^+] \right] = c_o \mathbb{P}(A_1 \ge q_1) = c_o (1 - F_{A_1}(q_1)).$$

Since $c_o > 0$ and $1 - F_{A_1}(q_1) \ge 0$, we have

$$\frac{\partial C}{\partial q_1} \ge 0 \qquad \text{for all } q_1 > d.$$

Thus, the minimum cost is achieved weakly before the boundary $q_j = d$. We need only analyze the domain $\mathbf{q} \in [0, d]^2$.

B. Derivation of the Unconstrained Best Response $\hat{q}_j$. We analyze the region $q_1+q_2>d$ and $q_j \le d$. On this domain, the marginal cost with respect to q_1 is

$$\frac{\partial C}{\partial q_1} = (1 - F_{A_1}(q_1))\left[c_o - (c_o + c_u)F_{A_2}(d - q_1)\right].$$

The first factor, $1 - F_{A_1}(q_1)$, is nonnegative. The second factor is increasing in q_1, because $F_{A_2}(d - q_1)$ is decreasing in q_1. Hence, the derivative crosses zero at most once, from negative to positive. The unique point satisfying the first-order condition is given by

$$q_1^\dagger = d - F_{A_2}^{-1}(r).$$

The same argument applies symmetrically for supplier 2.

The solution $q_j^\dagger$ therefore satisfies

$$F_{A_{-j}}(d - q_j^\dagger) = r.$$

However, strictly using the censored distribution A_{-j}, which is always nonnegative, creates a definition problem if the unconstrained solution lies outside the range $[0,d]$. If $q_j^\dagger > d$, then $d - q_j^\dagger < 0$, and since $A_{-j} \ge 0$, $F_{A_{-j}}(x) = 0$ for all $x < 0$. We overcome this by defining the best response using the uncensored available inventory $\mathcal{A}_{-j}$, which supports negative values. For any $x \ge 0$, the distributions agree: $F_{\mathcal{A}_{-j}}(x) = F_{A_{-j}}(x)$. Thus, this extension preserves the correctness of the solution within the feasible region. We define the continuous unconstrained best response function as

$$\hat{q}_j(q_{-j}) := d - F^{-1}_{\mathcal{A}_{-j}(q_{-j})}(r). \tag{EC.1}$$

C. Definition of the Clipped and Truthful Best Responses.

- **Clipped Best Response.** In the domain where $q_1 + q_2 > d$ and $q_j \le d$, the derivative derived above has a single-crossing structure. Consequently, if the unconstrained solution $\hat{q}_j(q_{-j})$ lies outside the feasible range $[0,d]$, the constrained minimum is found at the nearest boundary. Thus, the feasible hedging best response is obtained by clipping $\hat{q}_j(q_{-j})$ to $[0,d]$:

$$B_j(q_{-j}) = \min\{d, \max\{0, \hat{q}_j(q_{-j})\}\}.$$

- **Truthful Best Response.** Consider the domain where the total order equals demand, $q_1 + q_2 = d$. Since the total order is d, the received quantity can never exceed d. Consequently, the overage cost is zero, and the cost function simplifies to purely the expected underage cost:

$$C(\mathbf{q}) = c_u \mathbb{E}[d - R(\mathbf{q})] = c_u(d - \mathbb{E}[R(\mathbf{q})]).$$

Minimizing cost is therefore equivalent to maximizing the expected received quantity $\mathbb{E}[R(\mathbf{q})]$.

Using the identity

$$\mathbb{E}[\min\{Q, X\}] = \int_0^Q (1 - F_X(x))\, dx,$$

we can write the truthful objective in terms of q_1, substituting $q_2 = d - q_1$, as

$$\max_{q_1 \in [0,d]} \quad \int_0^{q_1} (1 - F_{A_1}(x))\, dx + \int_0^{d-q_1} (1 - F_{A_2}(x))\, dx.$$

Taking the derivative with respect to q_1 using the Leibniz integral rule gives

$$\frac{d}{dq_1}\mathbb{E}[R] = (1 - F_{A_1}(q_1)) - (1 - F_{A_2}(d - q_1)) = F_{A_2}(d - q_1) - F_{A_1}(q_1).$$

The second derivative is

$$-f_{A_1}(q_1) - f_{A_2}(d - q_1) < 0,$$

so the truthful objective is strictly concave. Hence, the truthful minimizer is unique. If the minimizer is interior, the first-order condition gives the balancing condition

$$F_{A_1(q_1)}(q_1) = F_{A_2(q_2)}(q_2), \qquad q_1 + q_2 = d. \tag{EC.2}$$

Let

$$\mathbf{q}^T = (q_1^T, q_2^T)$$

denote the unique optimal truthful best response. If the balancing condition has an interior solution, then $\mathbf{q}^T$ is that solution. If not, the strict concavity of the truthful objective implies that the optimum is at one of the two corners, $(d, 0)$ or $(0, d)$.

Part 2: Properties of the NE Fixed Points

A homogeneous NE is a fixed point of the best response mapping.

A. Uniqueness of the Unconstrained Intersection. Let $(\tilde{q}_1, \tilde{q}_2)$ denote a solution, if one exists, to the system of unconstrained best response functions:

$$\tilde{q}_1 = \hat{q}_1(\tilde{q}_2), \qquad \tilde{q}_2 = \hat{q}_2(\tilde{q}_1).$$

We now show that such a solution is unique in the interior hedging region.

Recall that $\hat{q}_j(q_{-j})$ is defined implicitly by

$$F_{\mathcal{A}_{-j}(q_{-j})}(d - \hat{q}_j(q_{-j})) = r.$$

Expanding the definition of available inventory, we have

$$\mathcal{A}_{-j}(q_{-j}) = X_{-j} - (M-1)q_{-j},$$

where M is the focal buyer's random rank under the lottery mechanism, uniformly distributed on $\{1,\dots,n\}$. Therefore, $\hat{q}_j(q_{-j})$ satisfies

$$\mathbb{E}_M\left[F_{X_{-j}}\left(d-\hat{q}_j(q_{-j})+(M-1)q_{-j}\right)\right] = r.$$

Let

$$y := d-\hat{q}_j(q_{-j})+(M-1)q_{-j}.$$

Differentiating both sides with respect to q_{-j} gives

$$\mathbb{E}_M\left[f_{X_{-j}}(y)\left(-\frac{d\hat{q}_j}{dq_{-j}}+(M-1)\right)\right] = 0,$$

and hence

$$\frac{d\hat{q}_j}{dq_{-j}} = \frac{\mathbb{E}_M[(M-1)f_{X_{-j}}(y)]}{\mathbb{E}_M[f_{X_{-j}}(y)]}.$$

We now show $\frac{d\hat{q}_j}{dq_{-j}} > 1$ on the relevant interior region under Assumption 1. Let

$$s := d-\hat{q}_j(q_{-j}), \qquad q := q_{-j}.$$

Since M is uniform on $\{1,\dots,n\}$,

$$\begin{aligned}
&\mathbb{E}_M[(M-1)f_{X_{-j}}(s+(M-1)q)] - \mathbb{E}_M[f_{X_{-j}}(s+(M-1)q)]\\
&= \frac{1}{n}\sum_{m=1}^{n}(m-2)f_{X_{-j}}(s+(m-1)q).
\end{aligned}$$

The term with $m=2$ has coefficient 0, and all terms with $m \geq 3$ have nonnegative coefficients. Let $\bar{m} \in \{2,\dots,n-1\}$ be the index in Assumption 1. Since the inequality in Assumption 1 is strict on the relevant interior region, we have

$$f_{X_{-j}}(s+\bar{m}q) > f_{X_{-j}}(s).$$

Therefore,

$$\begin{aligned}
\frac{1}{n}\sum_{m=1}^{n}(m-2)f_{X_{-j}}(s+(m-1)q) &\geq \frac{1}{n}\left(-f_{X_{-j}}(s)+(\bar{m}-1)f_{X_{-j}}(s+\bar{m}q)\right)\\
&> \frac{1}{n}\left(-f_{X_{-j}}(s)+(\bar{m}-1)f_{X_{-j}}(s)\right)\\
&= \frac{\bar{m}-2}{n}f_{X_{-j}}(s) \;\geq\; 0.
\end{aligned}$$

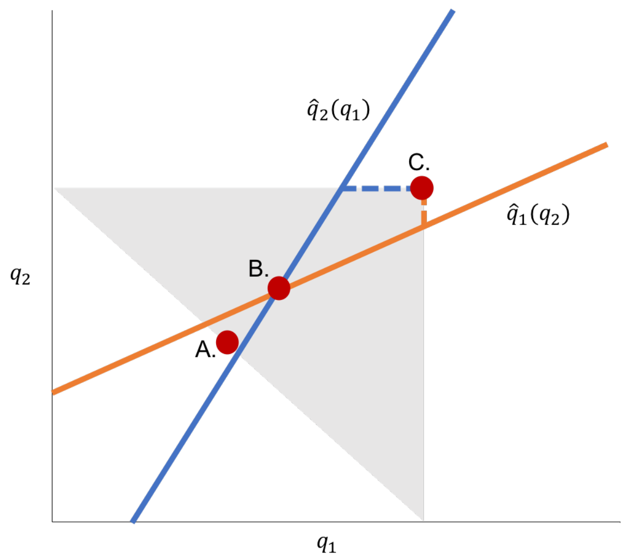

Figure EC.1 Point A corresponds to the truthful best response, point B is a solution to the unconstrained best response system, and point C is a boundary fixed point generated by clipping the best responses.

Moreover, when $\bar{m} = 2$, the strict inequality above still implies the left-hand side is strictly positive. Hence

$$\mathbb{E}_M[(M-1)f_{X_{-j}}(y)] > \mathbb{E}_M[f_{X_{-j}}(y)],$$

and therefore

$$\frac{d\hat{q}_j}{dq_{-j}} > 1.$$

This strict slope property implies that the unconstrained system has at most one solution in the interior hedging region. To see this, suppose there are two distinct solutions (q_1, q_2) and (q_1', q_2'), with $q_1' > q_1$ without loss of generality. Since $q_2 = \hat{q}_2(q_1)$ and $q_2' = \hat{q}_2(q_1')$, and since $\hat{q}_2' > 1$, we have

$$q_2' - q_2 > q_1' - q_1.$$

Similarly, since $q_1 = \hat{q}_1(q_2)$ and $q_1' = \hat{q}_1(q_2')$, and since $\hat{q}_1' > 1$, we have

$$q_1' - q_1 > q_2' - q_2.$$

These two strict inequalities cannot both hold. Therefore, the unconstrained system has at most one solution in the interior hedging region.

B. Uniqueness of the Truthful Intersection. From above, an interior homogeneous truthful candidate $q = (q_1, d - q_1)$ must satisfy

$$F_{A_1(q_1)}(q_1) = F_{A_2(d-q_1)}(d - q_1).$$

Define $\phi_j(q) := F_{A_j(q)}(q)$ for $q \in [0, d]$

and $h(q_1) := \phi_1(q_1) - \phi_2(d - q_1)$. We now show that h is strictly increasing. Recall that $A_j(q) = (X_j - (M - 1)q)^+$, where M is uniformly distributed on $\{1, \ldots, n\}$. Conditional on $M = m$, for $q \geq 0$,

$$\{A_j(q) \leq q\} = \{(X_j - (m-1)q)^+ \leq q\} = \{X_j \leq mq\}.$$

Therefore,

$$\phi_j(q) = F_{A_j(q)}(q) = \frac{1}{n} \sum_{m=1}^{n} F_{X_j}(mq).$$

Differentiating gives

$$\phi_j'(q) = \frac{1}{n} \sum_{m=1}^{n} m f_{X_j}(mq).$$

Thus, for $q_1 \in (0, d)$,

$$\begin{aligned} h'(q_1) &= \phi_1'(q_1) + \phi_2'(d - q_1) \\ &= \frac{1}{n} \sum_{m=1}^{n} m f_{X_1}(mq_1) + \frac{1}{n} \sum_{m=1}^{n} m f_{X_2}(m(d - q_1)). \end{aligned}$$

By nonnegativity of densities, $h'(q_1) \geq 0$. Moreover, under Assumption 1, $h'(q_1) > 0$ for every $q_1 \in (0, d)$. To see this, let $\bar{m}$ denote the index in Assumption 1. For any $q \in (0, d]$, Assumption 1 with $s = 0$ gives

$$f_{X_j}(\bar{m}q) > f_{X_j}(0) \geq 0,$$

so the $\bar{m}$-th term in the sum defining $\phi_j'(q)$ is strictly positive. Hence h is strictly increasing on $(0, d)$.

It follows immediately that the balancing equation $h(q_1) = 0$ has at most one solution in $(0, d)$. Therefore, if an interior homogeneous truthful candidate exists, it is unique and is characterized by

$$q_1 + q_2 = d, \qquad F_{A_1(q_1)}(q_1) = F_{A_2(q_2)}(q_2).$$

It remains only to identify the boundary cases. For fixed $y = (y_1, y_2)$, define the focal buyer's expected receipt from truthful orders by

$$G_y(t) := \mathbb{E}[\min\{t, A_1(y_1)\}] + \mathbb{E}[\min\{d - t, A_2(y_2)\}], \qquad t \in [0, d].$$

As shown in Part 1C, G_y is strictly concave in t, and

$$\frac{\partial G_y(t)}{\partial t} = F_{A_2(y_2)}(d-t) - F_{A_1(y_1)}(t).$$

Evaluating this derivative at a homogeneous truthful candidate, i.e., at $y = (t, d-t)$, gives

$$\left.\frac{\partial G_y(t')}{\partial t'}\right|_{t'=t,\, y=(t,d-t)} = F_{A_2(d-t)}(d-t) - F_{A_1(t)}(t) = -h(t).$$

Therefore the self-consistent truthful candidate is unique and can be written as $q^T = (q_1^T, q_2^T)$, where $q_2^T = d - q_1^T$ and

$$q_1^T = \begin{cases} 0, & \text{if } h(0) \geq 0, \\ \text{the unique } q \in (0,d) \text{ such that } h(q) = 0, & \text{if } h(0) < 0 < h(d), \\ d, & \text{if } h(d) \leq 0. \end{cases}$$

Thus, the interior truthful intersection is unique if it exists; otherwise the truthful candidate is one of the two boundary points $(0,d)$ or $(d,0)$.

C. Boundary Fixed Points of the Clipped Best Response. The feasible hedging best response is the clipped map

$$B_j(y) = \min\{d, \max\{0, \hat{q}_j(y)\}\}.$$

A boundary hedging equilibrium is a fixed point of the clipped best response map with total order exceeding demand and with at least one supplier order equal to d.

First suppose $q_2 = d$. Then buyer 1's best response to $q_2 = d$ must be $q_1 = B_1(d)$. For the outcome to be an over-ordering equilibrium, we require $B_1(d) > 0$. Buyer 2 must also be willing to order d in response to $q_1 = B_1(d)$, which requires $B_2(B_1(d)) = d$. Thus, a boundary equilibrium of the form (x, d) exists if and only if $B_1(d) > 0$ and $B_2(B_1(d)) = d$, in which case the equilibrium is $q^D = (B_1(d), d)$. The symmetric argument shows that a boundary equilibrium of the form (d, x) exists if and only if $B_2(d) > 0$ and $B_1(B_2(d)) = d$, in which case the equilibrium is $q^D = (d, B_2(d))$.

It remains to show that these two alternatives cannot generate two distinct strict boundary equilibria. Suppose the first alternative holds with $x := B_1(d) < d$. Then $B_2(x) = d$. Since B_2 is nondecreasing and $x < d$, we also have $B_2(d) = d$. If the second alternative also generated a boundary equilibrium, it would require $B_1(B_2(d)) = B_1(d) = d$, contradicting $B_1(d) = x < d$. Thus, if the first alternative holds strictly, the second cannot generate a distinct boundary equilibrium. If instead $B_1(d) = d$, then the first alternative implies $B_2(d) = d$, and both alternatives identify the same full double-ordering equilibrium (d, d). The symmetric argument applies when the second alternative holds strictly. Hence, there is at most one boundary hedging equilibrium.

Part 3: Characterization of the Three Nash Equilibria

We now characterize all homogeneous Nash equilibria. First, no equilibrium can have total order strictly less than demand. If $q_1 + q_2 < d$, then increasing one of the orders slightly creates no overage and strictly increases the expected received quantity (by Assumption 1, $f_{X_j} > 0$ on $(0, 2d]$, so $\mathbb{P}(X_j > q_j) > 0$ for $q_j < d$), thereby reducing underage cost. Hence, any homogeneous NE must have total order at least d.

Case 1: Partial Quantity Hedging (Hedge).

The equilibrium Hedge is the fixed point of the unclipped system in the interior of the hedging domain. Therefore, Hedge exists and is given by $(\tilde{q}_1, \tilde{q}_2)$ if and only if

$$\tilde{q}_1 + \tilde{q}_2 > d \qquad \text{and} \qquad 0 < \tilde{q}_j < d \quad \text{for } j = 1, 2.$$

Equivalently, $(\tilde{q}_1, \tilde{q}_2)$ must be an interior fixed point of the unconstrained best response system in the over-ordering region. The red point labeled "B" in Figure EC.1 illustrates this equilibrium solution.

Case 2: Boundary Quantity Hedging (Double).

The equilibrium Double is a Nash equilibrium where at least one supplier order is constrained at the upper limit d. Formally, it is a fixed point of the clipped best response map B such that $q_1 + q_2 > d$ and $q_j = d$ for at least one supplier j. The red point labeled "C" in Figure EC.1 illustrates this equilibrium solution.

From Part 2C, such an equilibrium exists if and only if $B_1(d) > 0$ and $B_2(B_1(d)) = d$, or $B_2(d) > 0$ and $B_1(B_2(d)) = d$. If the first condition holds, the equilibrium is $q^D = (B_1(d), d)$. If the second condition holds, the equilibrium is $q^D = (d, B_2(d))$. In particular, the full double-ordering equilibrium (d, d) exists if and only if $B_1(d) = d$ and $B_2(d) = d$.

Case 3: Truthful Ordering (True).

The equilibrium True is the unique truthful candidate $\mathbf{q}^T = (q_1^T, q_2^T)$, where $\mathbf{q}^T$ minimizes $C(\mathbf{q} \mid \mathcal{R}(\mathbf{q} \mid \mathbf{q}_{-i}))$ over

$$\{(q_1, q_2) : q_1 + q_2 = d,\ q_j \in [0, d]\}.$$

If $\mathbf{q}^T$ is interior, then it is characterized by the balancing condition

$$F_{A_1(q_1^T)}(q_1^T) = F_{A_2(q_2^T)}(q_2^T).$$

The red point labeled "A" in Figure EC.1 illustrates this equilibrium solution.

We now characterize when the truthful candidate is a Nash equilibrium. Deviations with total order weakly below d cannot improve on $\mathbf{q}^T$: such deviations create no overage, and any order with total strictly less than d can be increased to a truthful order with weakly larger expected receipt and no overage penalty. By construction, $\mathbf{q}^T$ is a best response to itself among truthful orders; equivalently, it minimizes the focal buyer's cost over

$$\{(q_1, q_2) : q_1 + q_2 = d,\ q_j \in [0, d]\}$$

when the other buyers order $\mathbf{q}^T$. Therefore, no unilateral deviation with total order at most d is profitable.

It remains to rule out deviations with total order greater than d. In the hedging region, the clipped best response in coordinate j, holding the other coordinate fixed at q^T_{-j}, is $B_j(q^T_{-j})$. If $B_j(q^T_{-j}) > q^T_j$ for some j, then increasing the order to supplier j from q^T_j moves into the over-ordering region and initially lowers cost. Hence, $\mathbf{q}^T$ is not a Nash equilibrium.

Conversely, suppose $B_j(q^T_{-j}) \le q^T_j$ for $j = 1, 2$. Consider any deviation with total order greater than d. Since $q^T_1 + q^T_2 = d$, at least one component of the deviation must exceed its truthful counterpart. For any such component j, the single-crossing property of the marginal cost implies that, once $q'_j \ge q^T_j \ge B_j(q^T_{-j})$, reducing q'_j weakly decreases cost as long as the deviation remains in the over-ordering region. Repeating this argument brings the deviation back to the truthful region without increasing cost. Since $\mathbf{q}^T$ is optimal in the truthful region, no over-ordering deviation is profitable. Therefore, $\mathbf{q}^T$ is a Nash equilibrium if and only if $B_j(q^T_{-j}) \le q^T_j$ for $j = 1, 2$.

When $\mathbf{q}^T$ is interior, this condition has a simpler equivalent form. Let

$$V := F_{A_1(q^T_1)}(q^T_1) = F_{A_2(q^T_2)}(q^T_2).$$

For supplier 1, $B_1(q^T_2) \le q^T_1$ is equivalent to $\hat{q}_1(q^T_2) \le q^T_1$. Using the definition of $\hat{q}_1$, this is equivalent to $F_{A_2(q^T_2)}(q^T_2) \le r$. Similarly, $B_2(q^T_1) \le q^T_2$ is equivalent to $F_{A_1(q^T_1)}(q^T_1) \le r$. Since the interior truthful solution satisfies the balancing condition, both coordinate-wise inequalities reduce to $V \le r$. Thus, when $\mathbf{q}^T$ is interior, True exists if and only if

$$F_{A_1(q^T_1)}(q^T_1) = F_{A_2(q^T_2)}(q^T_2) \le \frac{c_o}{c_o + c_u}.$$

Part 4: Existence Implication We prove that HEDGE existence implies the existence of both DOUBLE and TRUE.

1. **HEDGE $\implies$ DOUBLE.**

   Suppose HEDGE exists. Let $(\tilde{q}_1, \tilde{q}_2)$ denote the corresponding interior hedging equilibrium. Then

$$\tilde{q}_j = \hat{q}_j(\tilde{q}_{-j}), \qquad 0 < \tilde{q}_j < d, \qquad \tilde{q}_1 + \tilde{q}_2 > d.$$

   Without loss of generality, suppose $\tilde{q}_1 \geq \tilde{q}_2$. We show that the second boundary condition in Part 2C holds.

   Let $z := B_2(d)$. Since B_2 is nondecreasing and $B_2(\tilde{q}_1) = \tilde{q}_2 > 0$, we have $z = B_2(d) \geq \tilde{q}_2 > 0$. If $z = d$, then using the slope property of $\hat{q}_1$,

$$\hat{q}_1(d) - \hat{q}_1(\tilde{q}_2) \geq d - \tilde{q}_2.$$

   Since $\hat{q}_1(\tilde{q}_2) = \tilde{q}_1$, this gives

$$\hat{q}_1(d) \geq \tilde{q}_1 + d - \tilde{q}_2 \geq d,$$

   where the last inequality uses $\tilde{q}_1 \geq \tilde{q}_2$. Hence

$$B_1(B_2(d)) = B_1(d) = d.$$

   Therefore, $B_2(d) > 0$ and $B_1(B_2(d)) = d$.

   Now suppose $z < d$. Since $z > 0$, clipping is not binding at either boundary and $z = \hat{q}_2(d)$. Again using the slope property,

$$z - \tilde{q}_2 = \hat{q}_2(d) - \hat{q}_2(\tilde{q}_1) \geq d - \tilde{q}_1.$$

   Applying the slope property to $\hat{q}_1$ gives

$$\hat{q}_1(z) - \hat{q}_1(\tilde{q}_2) \geq z - \tilde{q}_2.$$

   Combining these two inequalities and using $\hat{q}_1(\tilde{q}_2) = \tilde{q}_1$, we obtain

$$\hat{q}_1(z) \geq \tilde{q}_1 + (d - \tilde{q}_1) = d.$$

   Thus $B_1(z) = d$, or equivalently, $B_1(B_2(d)) = d$. Hence, in both cases, $B_2(d) > 0$ and $B_1(B_2(d)) = d$.

   By the characterization in Part 2C, DOUBLE exists. The case $\tilde{q}_2 > \tilde{q}_1$ is symmetric and implies the first boundary condition.

2. **HEDGE $\Longrightarrow$ TRUE.**

Suppose again that HEDGE exists, and let $(\tilde{q}_1, \tilde{q}_2)$ denote the corresponding interior hedging equilibrium.

We first show that the truthful candidate $\mathbf{q}^T$ cannot be a boundary point. Suppose, toward a contradiction, that $\mathbf{q}^T = (d, 0)$. Since the truthful objective is concave, optimality at the right endpoint requires the left derivative to be nonnegative:

$$F_{A_2(0)}(0) - F_{A_1(d)}(d) \geq 0.$$

On the other hand, since HEDGE exists, we have

$$r = F_{A_1(\tilde{q}_1)}(d - \tilde{q}_2) = F_{A_2(\tilde{q}_2)}(d - \tilde{q}_1),$$

with $0 < \tilde{q}_j < d$. Because $\tilde{q}_1 < d$ and $d - \tilde{q}_2 < d$,

$$F_{A_1(\tilde{q}_1)}(d - \tilde{q}_2) < F_{A_1(d)}(d).$$

Similarly, because $\tilde{q}_2 > 0$ and $d - \tilde{q}_1 > 0$,

$$F_{A_2(\tilde{q}_2)}(d - \tilde{q}_1) > F_{A_2(0)}(0).$$

Therefore,

$$F_{A_2(0)}(0) < r < F_{A_1(d)}(d),$$

contradicting the boundary optimality condition

$$F_{A_2(0)}(0) \geq F_{A_1(d)}(d).$$

Hence, $\mathbf{q}^T \neq (d, 0)$. The argument ruling out $\mathbf{q}^T = (0, d)$ is symmetric. Therefore, if HEDGE exists, the truthful candidate $\mathbf{q}^T$ is interior.

Let $\mathbf{q}^T = (q_1^T, q_2^T)$ denote this interior truthful candidate, and define

$$V := F_{A_1(q_1^T)}(q_1^T) = F_{A_2(q_2^T)}(q_2^T).$$

By Part 3, TRUE exists if and only if $V \leq r$. We prove this condition.

Suppose, toward a contradiction, that $V > r$. Then, by the definition of the unconstrained best responses,

$$\hat{q}_1(q_2^T) > q_1^T \qquad \text{and} \qquad \hat{q}_2(q_1^T) > q_2^T.$$

Define the composition

$$G(q_1) := \hat{q}_1(\hat{q}_2(q_1)).$$

Since each $\hat{q}_j$ has derivative at least one, $G'(q_1) \geq 1$ wherever the derivative exists. Hence, $G(q_1) - q_1$ is nondecreasing. Moreover,

$$G(q_1^T) = \hat{q}_1(\hat{q}_2(q_1^T)) > \hat{q}_1(q_2^T) > q_1^T.$$

Therefore, $G(q_1) - q_1 > 0$ for all $q_1 \geq q_1^T$. Thus, no fixed point of the unconstrained system can have first coordinate at least q_1^T.

Now consider the interior hedging fixed point $(\tilde{q}_1, \tilde{q}_2)$. Since

$$\tilde{q}_1 + \tilde{q}_2 > d = q_1^T + q_2^T,$$

at least one component of $(\tilde{q}_1, \tilde{q}_2)$ exceeds its truthful counterpart. If $\tilde{q}_2 > q_2^T$, then the monotonicity of $\hat{q}_1$ and the inequality $\hat{q}_1(q_2^T) > q_1^T$ imply

$$\tilde{q}_1 = \hat{q}_1(\tilde{q}_2) > \hat{q}_1(q_2^T) > q_1^T.$$

If instead $\tilde{q}_2 \leq q_2^T$, then

$$\tilde{q}_1 > d - \tilde{q}_2 \geq d - q_2^T = q_1^T.$$

Hence, in all cases, $\tilde{q}_1 > q_1^T$. But $(\tilde{q}_1, \tilde{q}_2)$ is a fixed point of the unconstrained system, so $G(\tilde{q}_1) = \tilde{q}_1$, contradicting the fact that no fixed point can have first coordinate at least q_1^T. Therefore, the supposition $V > r$ is false. Hence, $V \leq r$. By the truthful-equilibrium characterization in Part 3, this implies that $\mathbf{q}^T$ is a Nash equilibrium. Thus, TRUE exists.

□

EC.5. Proof of Proposition 2

Part 1: Convergence to double ordering. Recall from the proof of Theorem 2 that (d, d) is a Nash equilibrium of the UV setting if and only if buyers prefer ordering d from both suppliers rather than reducing their order size. The necessary and sufficient condition is

$$F_{A_j(d)}(0) \geq \frac{c_o}{c_o + c_u}, \qquad j = 1, 2. \tag{EC.3}$$

From Corollary 1, the buyers' orders in the Learning setting can converge to double ordering if and only if

$$1 - F_{A_j(d)}(d) < \frac{c_u}{c_o + c_u} \iff F_{A_j(d)}(d) > \frac{c_o}{c_o + c_u}, \qquad j = 1, 2, \tag{EC.4}$$

where the strict inequality reflects the tie-breaking convention.

We show that the UV condition (EC.3) implies the strict Learning condition (EC.4). Fix a supplier j. By the definition of $A_j(d)$,

$$F_{A_j(d)}(d) - F_{A_j(d)}(0) = \frac{1}{n}\sum_{m=1}^{n}\Big[F_{X_j}(md) - F_{X_j}\big((m-1)d\big)\Big] = \frac{1}{n}\Big[F_{X_j}(nd) - F_{X_j}(0)\Big],$$

where the second equality follows by telescoping. Under Assumption 1, setting $s = 0$ yields $f_{X_j}(\bar{m}q) > f_{X_j}(0) \geq 0$ for all $q \in (0, d]$, where $\bar{m} \in \{2, \ldots, n-1\}$ is the index in Assumption 1. Hence f_{X_j} is strictly positive on $(0, \bar{m}d] \supseteq (0, d]$, so $F_{X_j}(d) > F_{X_j}(0)$. Since $F_{X_j}(nd) \geq F_{X_j}(d)$, it follows that

$$F_{A_j(d)}(d) - F_{A_j(d)}(0) \;\geq\; \frac{1}{n}\Big[F_{X_j}(d) - F_{X_j}(0)\Big] \;>\; 0.$$

Therefore, if (d, d) is a Nash equilibrium in the UV setting, so that (EC.3) holds, then

$$F_{A_j(d)}(d) \;>\; F_{A_j(d)}(0) \;\geq\; \frac{c_o}{c_o + c_u}, \qquad j = 1, 2,$$

which is exactly the strict condition (EC.4) for the Learning setting to converge to double ordering.

Part 2: Convergence to Truthful Ordering. By Corollary 1, truthful ordering can emerge as a limiting order profile in the Learning setting if and only if for each supplier $j \in \{1, 2\}$ with $n_j > 0$ buyers:

$$\frac{1}{n_j}\sum_{m=1}^{n_j} F_{X_j}(md) \leq c_o/(c_o + c_u). \tag{EC.5}$$

We now verify the existence of a truthful equilibrium in the UV setting. For a truthful split $q_1 + q_2 = d$, write $q = q_1$ and define

$$\Phi_1(q) := F_{A_1(q)}(q) = \frac{1}{n}\sum_{k=1}^{n} F_{X_1}(kq), \qquad \Phi_2(q) := F_{A_2(d-q)}(d-q) = \frac{1}{n}\sum_{k=1}^{n} F_{X_2}(k(d-q)).$$

The function Φ_1 is nondecreasing in q, while Φ_2 is nonincreasing in q. Hence the truthful candidate q^T from Theorem 2 minimizes

$$\max\{\Phi_1(q), \Phi_2(q)\}$$

over $q \in [0, d]$: when the crossing is interior, the maximum decreases as q moves toward the crossing from either side, and when there is no interior crossing the minimizer is the corresponding boundary point.

Therefore, to prove that True exists, it is enough to exhibit any truthful split q^* such that

$$\Phi_1(q^*) \le r \quad \text{and} \quad \Phi_2(q^*) \le r, \qquad r := \frac{c_o}{c_o + c_u}.$$

Indeed, for such a split,

$$\max\{\Phi_1(q^T), \Phi_2(q^T)\} \le \max\{\Phi_1(q^*), \Phi_2(q^*)\} \le r.$$

Thus the truthful candidate q^T satisfies the over-ordering deterrence conditions

$$B_1(q_2^T) \le q_1^T, \qquad B_2(q_1^T) \le q_2^T,$$

and Theorem 2 implies that True exists.

We construct a candidate order based on the Learning profile:

$$q_1^* = \frac{n_1}{n} d, \quad q_2^* = \frac{n_2}{n} d.$$

We wish to verify the following condition:

$$\frac{1}{n} \sum_{k=1}^{n} F_{X_j}(k q_j^*) \le c_o/(c_o + c_u) \quad \text{for } j = 1, 2. \tag{EC.6}$$

We compare the LHS of (EC.6) to the Learning condition (EC.5). Substituting $q_j^* = \frac{n_j}{n} d$:

$$\text{LHS} = \frac{1}{n} \sum_{k=1}^{n} F_{X_j}\left(k \frac{n_j}{n} d\right).$$

By the assumption of the proposition and (EC.5), the LHS of (EC.6) satisfies

$$\frac{1}{n} \sum_{k=1}^{n} F_{X_j}\left(k \frac{n_j}{n} d\right) \le \frac{1}{n_j} \sum_{m=1}^{n_j} F_{X_j}(md) \le c_o/(c_o + c_u).$$

Hence, (EC.6) holds and True exists. □

EC.6. Proof of Proposition 3

Let $r \triangleq c_o/(c_o + c_u)$. From Corollary 1 we know that convergence to double ordering (all buyers ordering (d, d)) is possible if and only if

$$\frac{1}{n} \sum_{m=1}^{n} \left(1 - F_{X_j}(md)\right) < 1 - r \text{ for } j = 1, 2,$$

which is equivalent to

$$\frac{1}{n}\sum_{m=1}^{n} F_{X_j}(md) > r \text{ for } j = 1, 2.$$

From the argument in the proof of Proposition 2, we know that TRUE exists if and only if there exists an order (q_1, q_2) with $q_1 + q_2 = d$ such that $F_{A_j(q_j)}(q_j) \leq r$ for $j = 1, 2$. This can be written as $\frac{1}{n}\sum_{m=1}^{n} F_{X_j}(mq_j) \leq r$. Therefore, we require $\min_{q\in[0,d]} \max\{\frac{1}{n}\sum_{m=1}^{n} F_{X_1}(mq), \frac{1}{n}\sum_{m=1}^{n} F_{X_2}(m(d-q))\} \leq r$. Combining these two conditions gives the inequalities in the proposition. □

EC.7. Proof of Proposition 4

We will show that buyers always weakly prefer TRUE to DOUBLE when DOUBLE consists of the order (d, d). Let $r \triangleq \frac{c_o}{c_o+c_u}$. Suppose, as in the statement of the proposition, that X_j is divisible by d. Then, when all buyers order d from supplier j, each buyer either receives d units (in full) or 0 units from that supplier.

In TRUE, each buyer orders (q_1^T, q_2^T) with $q_1^T + q_2^T = d$, so each buyer's total receipt satisfies $R_1 + R_2 \leq d$ and hence there is no overage. Therefore, the expected cost per buyer is

$$C(\mathbf{q}^T) = c_u \, \mathbb{E}\big[d - (R_1 + R_2)\big] = c_u\Big(d - \mathbb{E}[R_1] - \mathbb{E}[R_2]\Big).$$

For each supplier $j \in \{1, 2\}$, since $R_j = \min\{q_j^T, A_j(q_j^T)\} \geq 0$, we have

$$\mathbb{E}[R_j] \ \geq \ q_j^T \cdot \mathbb{P}\big(A_j(q_j^T) \geq q_j^T\big) = q_j^T\big(1 - F_{A_j(q_j^T)}(q_j^T)\big).$$

By the existence condition for TRUE (see Theorem 2),

$$F_{A_j(q_j^T)}(q_j^T) \leq r,$$

for $j = 1, 2$, so $\mathbb{E}[R_j] \geq (1 - r)q_j^T$ for $j = 1, 2$. Summing yields $\mathbb{E}[R_1 + R_2] \geq (1 - r)(q_1^T + q_2^T) = (1 - r)d$, and thus

$$C(\mathbf{q}^T) \leq c_u\big(d - (1 - r)d\big) = c_u r d. \tag{EC.7}$$

Now we will lower bound the buyers' expected costs under DOUBLE. Define $p_j := \mathbb{P}(R_j = d)$, the probability that a buyer receives its full order from supplier j. Under the divisibility assumption, $R_j \in \{0, d\}$ and hence $p_j = 1 - F_{A_j(d)}(0)$.

From Theorem 2, the necessary and sufficient condition for the existence of DOUBLE is $F_{A_j(d)}(0) \geq r$ for $j = 1, 2$ (see the discussion following Theorem 2), hence $p_j \leq 1 - r$ for $j = 1, 2$.

A buyer incurs overage cost $c_o d$ if it receives d from both suppliers, incurs underage cost $c_u d$ if it receives 0 from both suppliers, and incurs zero cost otherwise. Therefore, the expected per-buyer cost under DOUBLE is

$$C((d,d)) = d\Big[c_o\, p_1 p_2 + c_u(1-p_1)(1-p_2)\Big].$$

Over the domain $0 \leq p_1, p_2 \leq 1 - r$, the bracketed term is decreasing in each p_j (because the partial derivative with respect to p_1 is $-c_u + (c_o + c_u)p_2 \leq 0$ when $p_2 \leq 1 - r = \frac{c_u}{c_o + c_u}$, and similarly for p_2). Hence the minimum is attained at $p_1 = p_2 = 1 - r$, giving

$$C((d,d)) \geq d\Big[c_o(1-r)^2 + c_u r^2\Big] = d \cdot \frac{c_o c_u}{c_o + c_u} = c_u r d. \tag{EC.8}$$

Finally, combining (EC.7) and (EC.8) yields $C(\mathbf{q}^T) \leq C((d,d))$.

Lastly, we note that although this proof invokes conditions from Theorem 2, Assumption 1 is not necessary for this proposition to hold, as the part of Theorem 2 it uses does not depend on this assumption. □

EC.8. Proof of Theorem 3

First, we characterize the cost-minimizing truthful order in the following lemma:

LEMMA EC.1. *If the homogeneous order that minimizes total buyer cost on the domain $\{q_1, q_2 : q_1 + q_2 = d\}$ is interior (i.e., $q_1 \in (0, d)$), then it is given by the unique solution to*

$$F_{X_1}(nq_1) = F_{X_2}(n(d - q_1)), \quad q_2 = d - q_1. \tag{EC.9}$$

Proof of Lemma EC.1. Recall from the proof of Theorem 2 that along the domain $\{q_1, q_2 : q_1 + q_2 = d\}$, $C(\mathbf{q})$ is equivalent to

$$-c_u \sum_{m_1=1}^{n} \sum_{m_2=1}^{n} \frac{1}{n^2} \left(\int_0^{q_1} \big(1 - F_{X_1}(x + (m_1 - 1)q_1)\big)\, dx + \int_0^{d-q_1} \big(1 - F_{X_2}(x + (m_2 - 1)(d - q_1))\big)\, dx \right).$$

The derivative of the inner term with respect to q_1 is

$$\begin{aligned}&1+(-1+m_1)F_{X_1}((-1+m_1)q_1)-m_1F_{X_1}(m_1q_1)-\\&1-(m_2-1)F_{X_2}((-1+m_2)(d-q_1))+m_2F_{X_2}(m_2(d-q_1)),\end{aligned}$$

which simplifies to $(-1+m_1)F_{X_1}((-1+m_1)q_1)-m_1F_{X_1}(m_1q_1)-(m_2-1)F_{X_2}((-1+m_2)(d-q_1))+m_2F_{X_2}(m_2(d-q_1))$. Now consider taking the sum with respect to m_1. We can simplify

$$\sum_{m_1=1}^{n}(-1+m_1)F_{X_1}((-1+m_1)q_1)-m_1F_{X_1}(m_1q_1)$$

to $-nF_{X_1}(nq_1)$. Similarly,

$$\sum_{m_2=1}^{n}-(m_2-1)F_{X_2}((-1+m_2)(d-q_1))+m_2F_{X_2}(m_2(d-q_1))$$

simplifies to $nF_{X_2}(n(d-q_1))$.

Therefore, the derivative simplifies to $-c_u(-F_{X_1}(nq_1)+F_{X_2}(n(d-q_1)))$. The second derivative is positive; therefore setting the first derivative equal to zero results in a global minimum, characterized by the first-order condition: $F_{X_1}(nq_1)=F_{X_2}(n(d-q_1))$.

We note that although this proof invokes Theorem 2, the validity of the cost function referenced above from Theorem 2 does not depend on Assumption 1 and therefore this lemma does not adopt that assumption. □

We are now prepared to prove Theorem 3.

Proof of Theorem 3. The proof of Theorem 3 proceeds in six steps.

Step 1: From per-buyer cost to aggregate cost. Each of the n identical buyers orders (q_1,q_2) with $q_1+q_2=d$. Because no buyer can receive more than they ordered in total, the overage cost is zero and each buyer's expected cost is purely an expected underage cost. Under the random lottery, the aggregate receipt at supplier j across all n buyers is $\sum_{i=1}^{n}R_{(i,j)}=\min\{nq_j,X_j\}$, so the aggregate shortage from supplier j is $\max\{0,\,nq_j-X_j\}$. Multiplying by c_u and summing the two suppliers' contributions, the aggregate expected cost is

$$C(\mathbf{q})\;=\;c_u\,\mathbb{E}\big[\max\{0,\,nq_1-X_1\}\big]\;+\;c_u\,\mathbb{E}\big[\max\{0,\,nq_2-X_2\}\big].$$

Setting $Q \triangleq nq_1 \in [0, nd]$ as the aggregate order to supplier 1 (so $nq_2 = nd - Q$), the aggregate cost depends on the order profile only through Q:

$$C(Q) \triangleq C_{(1)}(Q) + C_{(2)}(nd - Q), \qquad C_{(j)}(Q) \triangleq c_u \, \mathbb{E}\big[\max\{0, Q - X_j\}\big].$$

Because the n buyers are identical and order homogeneously, each buyer's expected cost equals $C(Q)/n$ by symmetry; consequently the per-buyer cost-minimizing truthful order q_1^* and the aggregate cost-minimizing order $Q^* \triangleq \arg\min_{Q \in [0,nd]} C(Q)$ are related by $Q^* = nq_1^*$, and we may work with $C(Q)$ in place of the per-buyer cost. Let $\bar{Q} \triangleq nq_1^T$ denote the aggregate order in True, where q_1^T is the truthful equilibrium order from Theorem 2.

Step 2: Closed form for $C_{(j)}(Q)$. For $X_j \sim \text{Uniform}[a_j, b_j]$, the density is $f_{X_j}(x) = 1/(b_j - a_j)$ for $x \in [a_j, b_j]$ and zero elsewhere. We evaluate $C_{(j)}(Q)$ on three regimes.

(i) $Q < a_j$. Then $Q < X_j$, so $\max\{0, Q - X_j\} = 0$ a.s. and $C_{(j)}(Q) = 0$.

(ii) $Q \in [a_j, b_j]$. Then $\max\{0, Q - X_j\} = (Q - X_j)\mathbf{1}\{X_j \le Q\}$, so

$$\mathbb{E}[\max\{0, Q - X_j\}] = \int_{a_j}^{Q} \frac{Q - x}{b_j - a_j}\, dx = \frac{(Q - a_j)^2}{2(b_j - a_j)}$$

and

$$C_{(j)}(Q) = c_u \frac{(Q - a_j)^2}{2(b_j - a_j)}.$$

(iii) $Q > b_j$. Then $Q > X_j$ a.s., so $\max\{0, Q - X_j\} = Q - X_j$ and $\mathbb{E}[\max\{0, Q - X_j\}] = Q - \mathbb{E}[X_j] = Q - (a_j + b_j)/2$, giving

$$C_{(j)}(Q) = c_u\big(Q - (a_j + b_j)/2\big).$$

Defining $k_j \triangleq 1/(b_j - a_j)$, we summarize:

$$C_{(j)}(Q) = \begin{cases} 0 & Q < a_j, \\ \dfrac{c_u k_j}{2}(Q - a_j)^2 & a_j \le Q \le b_j, \\ c_u\big(Q - \frac{a_j + b_j}{2}\big) & Q > b_j. \end{cases} \tag{EC.10}$$

Step 3: C is convex with a uniform curvature bound. Differentiating (EC.10):

$$C'_{(j)}(Q) = \begin{cases} 0 & Q < a_j, \\ c_u k_j (Q - a_j) & a_j \le Q \le b_j, \\ c_u & Q > b_j, \end{cases} \qquad C''_{(j)}(Q) = \begin{cases} 0 & Q \notin [a_j, b_j], \\ c_u k_j & a_j < Q < b_j. \end{cases}$$

The first derivative is continuous on $\mathbb{R}$ (the values 0 at $Q = a_j$ and c_u at $Q = b_j$ match on both sides) and non-decreasing, so each $C_{(j)}$ is convex. Hence $C(Q) = C_{(1)}(Q) + C_{(2)}(nd - Q)$ is convex on $[0, nd]$, and wherever C'' exists,

$$C''(Q) \;=\; C''_{(1)}(Q) + C''_{(2)}(nd - Q) \;\le\; c_u(k_1 + k_2), \tag{EC.11}$$

with equality precisely when $Q \in (a_1, b_1)$ and $nd - Q \in (a_2, b_2)$.

Step 4: Bounding the cost gap via the curvature. Because C is convex and C' is continuous and non-decreasing, the minimizer Q^* in $[0, nd]$ satisfies $C'(Q^*) = 0$ if it is interior. By the fundamental theorem of calculus applied to C,

$$C(Q) - C(Q^*) \;=\; \int_{Q^*}^{Q} C'(t)\, dt,$$

and applied to C' (which is absolutely continuous on $[0, nd]$ with bounded C''),

$$C'(t) \;=\; C'(Q^*) + \int_{Q^*}^{t} C''(s)\, ds \;=\; \int_{Q^*}^{t} C''(s)\, ds.$$

Substituting the second identity into the first,

$$C(Q) - C(Q^*) \;=\; \int_{Q^*}^{Q} \int_{Q^*}^{t} C''(s)\, ds\, dt. \tag{EC.12}$$

The integrand satisfies $0 \le C''(s) \le c_u(k_1 + k_2)$ everywhere it is defined (convexity and (EC.11)). Replacing $C''(s)$ by its constant upper bound in (EC.12) and evaluating,

$$C(Q) - C(Q^*) \;\le\; \frac{c_u(k_1 + k_2)}{2}\,(Q - Q^*)^2.$$

Applying this at $Q = \bar{Q}$ and recalling $C(Q^*) = \min_{Q \in [0, nd]} C(Q)$:

$$C(\bar{Q}) - \min_{Q \in [0, nd]} C(Q) \;\le\; \frac{c_u(k_1 + k_2)}{2}\,(\bar{Q} - Q^*)^2. \tag{EC.13}$$

Step 5: Closed forms for Q^* and $\bar{Q}$. We work under the assumption of Theorem 3, in which F_{X_j} is linear over all relevant arguments. Both the cost-min FOC and the equilibrium balancing condition then reduce to balancing the uniform CDF $F_{X_j}(x) = (x - a_j)/(b_j - a_j)$ at scaled aggregate quantities.

Closed form for Q^*. We first derive bounds on nd using the assumption of Theorem 3. Summing the lower-bound inequalities $q_j^T \ge a_j$ over j with $q_1^T + q_2^T = d$ gives $d \ge a_1 + a_2$, and summing the upper-bound inequalities $nq_j^T \le b_j$ gives $d \le (b_1 + b_2)/n$. Multiplying by n and using $n \ge 1$,

$$a_1 + a_2 \;\le\; nd \;\le\; b_1 + b_2, \tag{EC.14}$$

so the interval $\mathcal{I} \triangleq [a_1, b_1] \cap [nd - b_2,\ nd - a_2]$ is non-empty.

The cost-minimizer Q^* satisfies $C'(Q^*) = 0$ if it is interior, which using the form of $C'_{(j)}$ from Step 3 gives the FOC

$$F_{X_1}(Q^*) \;=\; F_{X_2}(nd - Q^*). \tag{EC.15}$$

We claim $Q^* \in \mathcal{I}$. If $Q^* < a_1$, then $F_{X_1}(Q^*) = 0$ while $nd - Q^* > nd - a_1 \geq a_2$ by (EC.14), so $F_{X_2}(nd - Q^*) > 0$ — contradicting (EC.15). The remaining boundary violations ($Q^* > b_1$, $nd - Q^* < a_2$, $nd - Q^* > b_2$) are excluded by symmetric arguments using the other inequality in (EC.14). Hence $Q^* \in \mathcal{I}$, where F_{X_j} takes the linear form $F_{X_j}(x) = (x - a_j)/(b_j - a_j)$. Substituting into (EC.15),

$$\frac{Q^* - a_1}{b_1 - a_1} = \frac{nd - Q^* - a_2}{b_2 - a_2},$$

which solves to

$$Q^* \;=\; \frac{nd\,(b_1 - a_1) \,-\, (a_2 b_1 - a_1 b_2)}{b_1 + b_2 - a_1 - a_2}. \tag{EC.16}$$

Closed form for $\bar{Q}$. By Theorem 2, q_1^T satisfies $F_{A_1(q_1^T)}(q_1^T) = F_{A_2(q_2^T)}(q_2^T)$ with $q_2^T = d - q_1^T$, where $F_{A_j(q)}(q) = \mathbb{E}_M[F_{X_j}(Mq)]$ and M is the focal buyer's rank (uniform on $\{1, \ldots, n\}$). Under the assumption of Theorem 3, every Mq_j^T lies in $[a_j, b_j]$, where F_{X_j} is linear. Linearity lets the expectation pass through:

$$F_{A_j(q_j^T)}(q_j^T) \;=\; F_{X_j}\!\big(\mathbb{E}_M[Mq_j^T]\big) \;=\; F_{X_j}\!\Big(\tfrac{n+1}{2} q_j^T\Big). \tag{EC.17}$$

The balancing condition therefore reduces to an aggregate-level equation with the same structure as (EC.15), but with the scaling factor $\frac{n+1}{2}$ in place of n:

$$F_{X_1}\!\Big(\tfrac{n+1}{2} q_1^T\Big) \;=\; F_{X_2}\!\Big(\tfrac{n+1}{2} q_2^T\Big).$$

Substituting the linear-piece formula and solving for q_1^T (using $q_2^T = d - q_1^T$), then setting $\bar{Q} = nq_1^T$,

$$\bar{Q} \;=\; \frac{nd\,(b_1 - a_1) \,-\, \frac{2n}{n+1}(a_2 b_1 - a_1 b_2)}{b_1 + b_2 - a_1 - a_2}. \tag{EC.18}$$

Comparison. The two FOCs are structurally similar, differing only in the scaling: the cost-min weights the aggregate quantity by n (counting all n buyers), while the equilibrium weights by $(n+1)/2$ (the expected rank of the focal buyer). The gap between $\bar{Q}$ and Q^* arises entirely from this scaling discrepancy.

Step 6: Combining. Subtracting (EC.16) from (EC.18),

$$\bar{Q}-Q^{*} = \frac{\left(1-\frac{2n}{n+1}\right)(a_2b_1-a_1b_2)}{b_1+b_2-a_1-a_2} = -\frac{n-1}{n+1}\frac{a_2b_1-a_1b_2}{b_1+b_2-a_1-a_2},$$

so

$$|\bar{Q}-Q^{*}| = \frac{n-1}{n+1}\frac{|a_2b_1-a_1b_2|}{b_1+b_2-a_1-a_2}. \tag{EC.19}$$

Substituting (EC.19) into (EC.13) and using $\frac{c_u(k_1+k_2)}{2}=\frac{c_u}{2}\left(\frac{1}{b_1-a_1}+\frac{1}{b_2-a_2}\right)$ yields (9).

Finally, we note that although this proof invokes Theorem 2, the validity of the cost function referenced above from Theorem 2 does not depend on Assumption 1 and therefore this theorem does not adopt that assumption. □

EC.9. Proof of Theorem 4

We compare the supplier's expected profit under the two demand scenarios: Truthful ordering ($D_L = nd/2$) and Double ordering ($D_H = nd$). Using the identity $\min(D,X) = D-(D-X)^{+}$, we rewrite the objective function as:

$$\Pi(D) = p[D-\mathbb{E}(D-X)^{+}] - s\mathbb{E}[(D-X)^{+}] \tag{EC.20}$$

$$= pD-(p+s)\mathbb{E}[(D-X)^{+}]. \tag{EC.21}$$

The supplier prefers Truthful ordering if and only if $\Pi(D_L)\geq\Pi(D_H)$. This inequality can be written as:

$$pD_L-(p+s)\mathbb{E}[(D_L-X)^{+}] \geq pD_H-(p+s)\mathbb{E}[(D_H-X)^{+}] \tag{EC.22}$$

$$(p+s)\left(\mathbb{E}[(D_H-X)^{+}]-\mathbb{E}[(D_L-X)^{+}]\right) \geq p(D_H-D_L). \tag{EC.23}$$

Recall that for a non-negative random variable, the expected shortage at demand Q can be expressed as the integral of the CDF: $\mathbb{E}[(Q-X)^{+}] = \int_0^Q F_X(x)dx$. Therefore, the difference in expected shortage is:

$$\mathbb{E}[(D_H-X)^{+}]-\mathbb{E}[(D_L-X)^{+}] = \int_{D_L}^{D_H} F_X(x)dx. \tag{EC.24}$$

Substituting this back into the inequality:

$$(p+s)\int_{nd/2}^{nd} F_X(x)dx \geq p\left(nd - \frac{nd}{2}\right) \tag{EC.25}$$

$$\int_{nd/2}^{nd} F_X(x)dx \geq \frac{p}{p+s}\frac{nd}{2}. \tag{EC.26}$$

Dividing both sides by the integration range $nd/2$, we obtain the condition that the average probability of shortage over the interval must exceed the critical revenue-to-penalty ratio:

$$\frac{2}{nd}\int_{\frac{nd}{2}}^{nd} F_X(x)dx \geq \frac{p}{p+s}. \tag{EC.27}$$

□

EC.10. Proof of Theorem 5

Step 1: Derivation of the Optimal Capacity Factor.

Let D be the target demand, and recall that $X = \mu Y$, where $Y \sim U[1-\delta, 1+\delta]$. Using the identity

$$(D-X)^+ = D - \min(D, X),$$

we can rewrite the supplier's expected profit as

$$\Pi(\mu) = p\mathbb{E}[\min(D,X)] - k\mu - s\mathbb{E}[(D-X)^+] \tag{EC.28}$$

$$= p\mathbb{E}[\min(D,X)] - k\mu - s\left(D - \mathbb{E}[\min(D,X)]\right) \tag{EC.29}$$

$$= (p+s)\mathbb{E}[\min(D,X)] - k\mu - sD \tag{EC.30}$$

$$= (p+s)\mathbb{E}[\min(D,\mu Y)] - k\mu - sD. \tag{EC.31}$$

For an interior solution, differentiating with respect to μ gives

$$\frac{\partial \Pi}{\partial \mu} = (p+s)\frac{\partial}{\partial \mu}\mathbb{E}[\min(D,\mu Y)] - k.$$

Since Y has a continuous distribution, the event $\{\mu Y = D\}$ has probability zero. Therefore,

$$\frac{\partial}{\partial \mu}\mathbb{E}[\min(D,\mu Y)] = \mathbb{E}\left[Y\mathbf{1}\{\mu Y < D\}\right] = \mathbb{E}\left[Y\mathbf{1}\{Y < D/\mu\}\right].$$

Thus the first-order condition is

$$\mathbb{E}\left[Y\mathbf{1}\{Y < D/\mu\}\right] = \frac{k}{p+s}.$$

For an interior solution, $D/\mu \in [1-\delta, 1+\delta]$. Using the density $f_Y(y) = 1/(2\delta)$ on this interval, the first-order condition becomes

$$\frac{1}{2\delta}\int_{1-\delta}^{D/\mu} y\,dy = \frac{k}{p+s}.$$

Equivalently,

$$\frac{1}{4\delta}\left[\left(\frac{D}{\mu}\right)^2 - (1-\delta)^2\right] = \frac{k}{p+s}.$$

Solving for D/μ, we obtain

$$\frac{D}{\mu} = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}.$$

Define

$$\alpha := \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}.$$

Then the optimal capacity for target demand D is

$$\mu^*(D) = \frac{D}{\alpha}.$$

It remains to verify that this first-order condition identifies the global maximizer. First, since $k > 0$, $\alpha > 1-\delta$. Moreover, since $p + s > k$,

$$\alpha^2 = (1-\delta)^2 + \frac{4\delta k}{p+s} < (1-\delta)^2 + 4\delta = (1+\delta)^2.$$

Hence $1-\delta < \alpha < 1+\delta$, so the solution is indeed interior.

Next, for μ such that $D/\mu \in (1-\delta, 1+\delta)$,

$$\frac{\partial^2}{\partial\mu^2}\mathbb{E}[\min(D, \mu Y)] = -\frac{D^2}{\mu^3} f_Y(D/\mu) < 0.$$

Thus $\Pi(\mu)$ is strictly concave on the interior region. For $\mu < D/(1+\delta)$, supply is below demand almost surely, so

$$\frac{\partial\Pi}{\partial\mu} = (p+s)\mathbb{E}[Y] - k = p + s - k > 0.$$

For $\mu > D/(1-\delta)$, demand is met almost surely, so

$$\frac{\partial\Pi}{\partial\mu} = -k < 0.$$

Therefore the derivative is positive below the interior region, strictly decreasing in the interior region, and negative above the interior region. Hence the interior solution satisfying the first-order condition is the unique global maximizer.

We define the strategic capacity levels as

$$\mu_H^* = \frac{nd}{\alpha}, \qquad \mu_L^* = \frac{nd}{2\alpha}.$$

Finally, the assumption

$$p + s \geq \frac{4k}{2-\delta}$$

implies

$$\frac{4\delta k}{p+s} \leq \delta(2-\delta).$$

Therefore,

$$\alpha^2 = (1-\delta)^2 + \frac{4\delta k}{p+s} \leq (1-\delta)^2 + \delta(2-\delta) = 1.$$

Hence $\alpha \leq 1$, so the optimal capacity level is weakly larger than the target demand:

$$\mu^*(D) = \frac{D}{\alpha} \geq D.$$

Step 2: Stability of Buyer Equilibria. We verify the existence of the relevant equilibria at these capacity levels. We drop the subscript j since the suppliers are identical.

1. **At Bloat Capacity (μ_H^*):** We first show that under Bloat capacity, double ordering cannot be a stable limiting outcome of the Learning dynamics. Let

$$r := \frac{c_o}{c_o + c_u}.$$

Under Bloat capacity, each supplier chooses capacity for target demand nd. By Step 1, this capacity is

$$\mu_H^* = \frac{nd}{\alpha}, \qquad \alpha = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}.$$

Hence, under Bloat capacity,

$$X = \frac{nd}{\alpha}Y, \qquad Y \sim U[1-\delta, 1+\delta].$$

Let F_X^H denote the CDF of X under Bloat capacity.

By Corollary 1, double ordering can be a stable limiting outcome only if

$$\frac{1}{n}\sum_{m=1}^{n} F_X^H(md) > r.$$

Since F_X^H is nondecreasing,

$$\frac{1}{n}\sum_{m=1}^{n} F_X^H(md) \le F_X^H(nd).$$

We now bound $F_X^H(nd)$. Since $X = ndY/\alpha$, we have

$$\begin{aligned} F_X^H(nd) &= \mathbb{P}(X \le nd) \\ &= \mathbb{P}\left(\frac{nd}{\alpha}Y \le nd\right) \\ &= \mathbb{P}(Y \le \alpha). \end{aligned}$$

Because it is assumed that $p > k$, $0 < k/(p+s) < 1$. Therefore

$$1-\delta < \alpha = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2} < 1+\delta.$$

Thus α lies in the support of Y, and the uniform CDF formula gives

$$F_X^H(nd) = \frac{\alpha - (1-\delta)}{2\delta}.$$

Multiplying numerator and denominator by $\alpha + (1-\delta)$, we obtain

$$F_X^H(nd) = \frac{(\alpha-(1-\delta))\,(\alpha+(1-\delta))}{2\delta\,(\alpha+(1-\delta))}.$$

Using $(a-b)(a+b) = a^2 - b^2$, this becomes

$$F_X^H(nd) = \frac{\alpha^2 - (1-\delta)^2}{2\delta\,(\alpha+1-\delta)}.$$

By the definition of α,

$$\alpha^2 - (1-\delta)^2 = \frac{4\delta k}{p+s}.$$

Thus

$$F_X^H(nd) = \frac{4\delta k/(p+s)}{2\delta\,(\alpha+1-\delta)} = \frac{2k/(p+s)}{\alpha+1-\delta}.$$

Since $k > 0$, we have $\alpha > 1-\delta$. Hence $\alpha+1-\delta > 2(1-\delta)$. It follows that

$$F_X^H(nd) < \frac{2k/(p+s)}{2(1-\delta)} = \frac{k}{(1-\delta)(p+s)}.$$

Therefore, under the theorem assumption

$$r = \frac{c_o}{c_o+c_u} \ge \frac{k}{(1-\delta)(p+s)},$$

we have $F_X^H(nd) < r$.

We next show directly that no buyer can double order in any convergent limiting profile under Bloat capacity. Consider an arbitrary convergent limiting profile, and let ℓ_j denote the number of buyers who order d units from supplier j in the limit. If $\ell_j = 0$, supplier j is irrelevant for the limiting orders. If $\ell_j > 0$, then for any buyer who continues to order from supplier j, the limiting perceived reliability of supplier j is

$$\frac{1}{\ell_j}\sum_{m=1}^{\ell_j}\left(1 - F_X^H(md)\right).$$

Because F_X^H is nondecreasing and $\ell_j \le n$,

$$\frac{1}{\ell_j}\sum_{m=1}^{\ell_j} F_X^H(md) \le F_X^H(\ell_j d) \le F_X^H(nd) < r.$$

Therefore,

$$\frac{1}{\ell_j}\sum_{m=1}^{\ell_j}\left(1 - F_X^H(md)\right) > 1 - r = \frac{c_u}{c_o + c_u}.$$

Thus every supplier that a buyer continues to use in the limiting profile is perceived by that buyer as having reliability strictly above the double-ordering threshold. Under the tie-breaking convention, such a buyer selects a truthful order from a supplier with maximal perceived reliability rather than the double order (d,d). Hence no buyer can order (d,d) in any convergent limiting profile under Bloat capacity. Consequently, any convergent limiting profile under Bloat capacity must be truthful ordering.

2. **At Lean Capacity (μ_L^*):** Again let
$$r := \frac{c_o}{c_o + c_u}.$$
Under Lean capacity, each supplier chooses capacity for target demand $Q := nd/2$. By Step 1, this capacity is
$$\mu_L^* = \frac{Q}{\alpha} = \frac{nd}{2\alpha}, \qquad \alpha = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}.$$
Hence, under Lean capacity,
$$X = \frac{Q}{\alpha}Y = \frac{nd}{2\alpha}Y, \qquad Y \sim U[1-\delta, 1+\delta].$$
Let F_X^L denote the CDF of X under Lean capacity.

We first show that TRUE exists. Recall from Theorem 2 that TRUE exists if and only if

$$F_{A(d/2)}(d/2) \le r.$$

We note that although Theorem 2 is referenced for readability, this result does not depend on Assumption 1 and therefore this theorem does not adopt that assumption.

Given the random lottery mechanism, this condition can be equivalently written as

$$\frac{1}{n}\sum_{M=1}^{n} F_X^L(Md/2) \le r. \tag{EC.32}$$

Since F_X^L is nondecreasing, the largest term in the sum is the term with $M = n$. Therefore,

$$\frac{1}{n}\sum_{M=1}^{n} F_X^L(Md/2) \le F_X^L(nd/2) = F_X^L(Q).$$

We now bound $F_X^L(Q)$. Since $X = (Q/\alpha)Y$,

$$F_X^L(Q) = \mathbb{P}(X \le Q) = \mathbb{P}\left(\frac{Q}{\alpha}Y \le Q\right) = \mathbb{P}(Y \le \alpha).$$

As shown in Step 1, $1-\delta < \alpha < 1+\delta$, so the cutoff α lies in the support of Y. Hence the uniform CDF formula gives

$$\mathbb{P}(Y \le \alpha) = \frac{\alpha - (1-\delta)}{2\delta}.$$

As in the Bloat-capacity argument,

$$\frac{\alpha-(1-\delta)}{2\delta} = \frac{\alpha^2-(1-\delta)^2}{2\delta(\alpha+1-\delta)} = \frac{2k/(p+s)}{\alpha+1-\delta}.$$

Because $\alpha > 1-\delta$, $\alpha+1-\delta > 2(1-\delta)$. Thus

$$F_X^L(Q) = \mathbb{P}(Y \le \alpha) < \frac{k}{(1-\delta)(p+s)}.$$

By the theorem assumption,

$$r = \frac{c_o}{c_o+c_u} \ge \frac{k}{(1-\delta)(p+s)}.$$

Therefore, $F_X^L(Q) < r$. It follows that

$$\frac{1}{n}\sum_{M=1}^{n} F_X^L(Md/2) \le F_X^L(Q) < r.$$

Hence condition (EC.32) holds, so True exists under Lean capacity.

We now show that double ordering can emerge as a limiting order profile under Lean capacity. By Corollary 1, Learning can converge to double ordering iff

$$\frac{1}{n}\sum_{m=1}^{n} F_X(md) > r.$$

Since $F_X(md) \geq F_X((m-1)d)$, it is enough to prove

$$\frac{1}{n}\sum_{M=1}^{n} F_X^L((M-1)d) > r. \tag{EC.33}$$

The left-hand side of (EC.33) has a direct interpretation. Given orders of size d and random lottery, it is the probability that a randomly selected buyer receives zero units from the supplier.

We first verify that total realized supply is at most total double-order demand almost surely. Since $X = QY/\alpha$ and $Y \leq 1+\delta$, we have $X \leq Q(1+\delta)/\alpha$. Because $\alpha > 1-\delta$,

$$\frac{Q(1+\delta)}{\alpha} < \frac{Q(1+\delta)}{1-\delta}.$$

Since $\delta \leq 1/3$,

$$\frac{1+\delta}{1-\delta} \leq 2.$$

Therefore, $X < 2Q = nd$ almost surely.

Conditional on a realization X, exactly $\lceil X/d \rceil$ buyers receive a positive allocation under random lottery, and the remaining $n - \lceil X/d \rceil$ buyers receive zero. Therefore,

$$\frac{1}{n}\sum_{M=1}^{n} \mathbf{1}\{X \leq (M-1)d\} = 1 - \frac{1}{n}\left\lceil \frac{X}{d} \right\rceil.$$

Taking expectations yields

$$\begin{aligned}
\frac{1}{n}\sum_{M=1}^{n} F_X^L((M-1)d) &= 1 - \frac{1}{n}\mathbb{E}\left[\left\lceil \frac{X}{d} \right\rceil\right] \\
&\geq 1 - \frac{1}{n}\left(\mathbb{E}\left[\frac{X}{d}\right] + 1\right) \\
&= 1 - \frac{\mathbb{E}[X]}{nd} - \frac{1}{n}.
\end{aligned} \tag{EC.34}$$

Under Lean capacity,

$$\mathbb{E}[X] = \mu_L^* \mathbb{E}[Y] = \frac{nd}{2\alpha},$$

since $\mathbb{E}[Y] = 1$. Substituting into (EC.34), we obtain

$$\frac{1}{n}\sum_{M=1}^{n} F_X^L((M-1)d) \geq 1 - \frac{1}{2\alpha} - \frac{1}{n}.$$

Therefore, a sufficient condition for double ordering to be a limiting profile under Lean capacity is

$$r = \frac{c_o}{c_o + c_u} < 1 - \frac{1}{2\alpha} - \frac{1}{n}.$$

Finally, since $\alpha > 1 - \delta$, we have

$$\frac{1}{2\alpha} < \frac{1}{2(1-\delta)}.$$

Hence

$$1 - \frac{1}{2\alpha} - \frac{1}{n} > 1 - \frac{1}{2(1-\delta)} - \frac{1}{n}.$$

Consequently, the condition

$$\frac{c_o}{c_o + c_u} \leq 1 - \frac{1}{2(1-\delta)} - \frac{1}{n}$$

is also sufficient for double ordering to be a limiting profile under Lean capacity.

Step 3: Profit Calculation. We calculate the expected profit for the three relevant outcomes (Bloat capacity with truthful ordering, Lean capacity with double ordering, Lean capacity with truthful ordering). Throughout, let $Q := \frac{nd}{2}$.

- **Scenario 1: Bloat strategy (μ_H^*) with truthful ordering ($D = nd/2$).**

  We first compute the supplier's profit under Bloat capacity when aggregate truthful demand is split evenly across the two suppliers.

  Under Bloat capacity,

  $$\mu_H^* = \frac{nd}{\alpha} = \frac{2Q}{\alpha}.$$

  The Learning dynamics could converge to different truthful ordering profiles, which may induce different aggregate truthful demand levels at the two suppliers. However, among all truthful aggregate splits, total supplier profit is maximized when each supplier receives aggregate demand $Q = nd/2$. To see this, fix the Bloat capacity distribution and let $\pi^H(D)$ denote one supplier's expected profit when it faces aggregate demand D. Since

  $$\pi^H(D) = p\mathbb{E}[\min(D, X)] - k\mu_H^* - s\mathbb{E}[(D-X)^+],$$

and since $(D-X)^+ = D - \min(D,X)$, we can write

$$\pi^H(D) = (p+s)\mathbb{E}[\min(D,X)] - sD - k\mu_H^*.$$

The function $D \mapsto \mathbb{E}[\min(D,X)]$ is concave, and therefore $\pi^H(D)$ is concave in D. Hence, for any truthful aggregate split $(D, nd-D)$, Jensen's inequality implies

$$\pi^H(D) + \pi^H(nd-D) \le 2\pi^H\left(\frac{nd}{2}\right) = 2\pi^H(Q).$$

Thus the balanced split gives the highest total supplier profit among truthful ordering profiles. It is therefore sufficient (and conservative for the comparison to the Lean strategy) to compute Bloat profit at the balanced truthful split.

Now consider one supplier facing aggregate demand $D = Q$. Under Bloat capacity,

$$X = \mu_H^* Y = \frac{2Q}{\alpha} Y, \qquad Y \sim U[1-\delta, 1+\delta].$$

Since $Y \ge 1-\delta$, we have

$$X \ge \frac{2Q}{\alpha}(1-\delta).$$

By the theorem assumptions, $\alpha \le 1$ (see the discussion at the end of Step 1) and $\delta \le 1/3$. Therefore,

$$\frac{2Q}{\alpha}(1-\delta) \ge 2Q(1-\delta) \ge \frac{4Q}{3} > Q.$$

Thus supply exceeds truthful demand almost surely under the balanced truthful split. Consequently,

$$\min(Q,X) = Q \qquad \text{and} \qquad (Q-X)^+ = 0$$

almost surely.

The per-supplier expected profit under Bloat capacity and balanced truthful ordering is therefore

$$\Pi_{H,T} = pQ - k\mu_H^* = pQ - k\frac{2Q}{\alpha}.$$

Substituting $Q = nd/2$, we obtain

$$\Pi_{H,T} = \frac{nd}{2}\left(p - \frac{2k}{\alpha}\right).$$

- **Scenario 2: Lean strategy (μ_L^*) with double ordering.**

  Under Lean capacity, each supplier chooses capacity for target demand Q. Thus $\mu_L^* = \frac{Q}{\alpha} = \frac{nd}{2\alpha}$. Under double ordering, each supplier faces aggregate demand $nd = 2Q$.

  We first show that demand exceeds supply almost surely. Since

$$X = \mu_L^* Y = \frac{nd}{2\alpha} Y$$

and $Y \le 1 + \delta$, the maximum possible supply is

$$X_{\max} = \frac{(1+\delta)nd}{2\alpha}.$$

From Step 1, $\alpha > 1 - \delta$. Therefore,

$$X_{\max} < \frac{(1+\delta)nd}{2(1-\delta)}.$$

Since $\delta \le 1/3$, we have

$$\frac{1+\delta}{1-\delta} \le 2.$$

Hence $X_{\max} < nd$. Thus, under Lean capacity with double ordering, demand exceeds supply almost surely.

  Consequently, $\min(nd, X) = X$ and $(nd - X)^+ = nd - X$ almost surely. Since $\mathbb{E}[Y] = 1$, we have

$$\mathbb{E}[X] = \mu_L^* \mathbb{E}[Y] = \mu_L^* = \frac{nd}{2\alpha}.$$

Therefore expected sales are μ_L^*, and expected shortage is $nd - \mu_L^*$.

  The per-supplier expected profit under Lean capacity and double ordering is

$$\begin{aligned} \Pi_{L,D} &= p\mu_L^* - k\mu_L^* - s(nd - \mu_L^*) && \text{(EC.35)} \\ &= (p + s - k)\mu_L^* - snd. && \text{(EC.36)} \end{aligned}$$

Substituting $\mu_L^* = nd/(2\alpha)$, we obtain

$$\begin{aligned} \Pi_{L,D} &= (p + s - k)\frac{nd}{2\alpha} - snd && \text{(EC.37)} \\ &= \frac{nd}{2}\left[\frac{p + s - k}{\alpha} - 2s\right]. && \text{(EC.38)} \end{aligned}$$

- **Scenario 3: Lean strategy (μ_L^*) with truthful ordering.** Under Lean capacity, each supplier chooses capacity for target demand Q, so

$$\mu_L^* = \frac{Q}{\alpha} = \frac{nd}{2\alpha}.$$

Under truthful ordering, each supplier faces aggregate demand Q.

We compute the per-supplier expected profit. Using

$$\mathbb{E}[\min(D, X)] = D - \mathbb{E}[(D - X)^+],$$

we can write

$$\Pi_{L,T} = p\left(D - \mathbb{E}[(D - X)^+]\right) - k\mu_L^* - s\mathbb{E}[(D - X)^+] \tag{EC.39}$$

$$= pD - (p + s)\mathbb{E}[(D - X)^+] - k\mu_L^*. \tag{EC.40}$$

It remains to compute the expected shortage term. Under Lean capacity,

$$X = \mu_L^* Y, \qquad Y \sim U[1 - \delta, 1 + \delta].$$

Therefore X is uniformly distributed on $[(1 - \delta)\mu_L^*, (1 + \delta)\mu_L^*]$ with density $\frac{1}{2\delta\mu_L^*}$. From Step 1, $1 - \delta < \alpha < 1 + \delta$. Since $D = Q = \alpha\mu_L^*$, this implies $(1 - \delta)\mu_L^* < D < (1 + \delta)\mu_L^*$. Thus D lies in the interior of the support of X, and

$$\mathbb{E}[(D - X)^+] = \int_{(1-\delta)\mu_L^*}^{D} \frac{D - x}{2\delta\mu_L^*}\, dx \tag{EC.41}$$

$$= \frac{\left(D - (1 - \delta)\mu_L^*\right)^2}{4\delta\mu_L^*}. \tag{EC.42}$$

Substituting $D = Q$ and $\mu_L^* = Q/\alpha$, we obtain

$$\mathbb{E}[(D - X)^+] = \frac{(Q - (1 - \delta)Q/\alpha)^2}{4\delta Q/\alpha} \tag{EC.43}$$

$$= Q\frac{\alpha\,(1 - (1 - \delta)/\alpha)^2}{4\delta} \tag{EC.44}$$

$$= Q\frac{(\alpha - (1 - \delta))^2}{4\delta\alpha}. \tag{EC.45}$$

Therefore,

$$\Pi_{L,T} = pQ - (p+s)Q\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - k\frac{Q}{\alpha} \tag{EC.46}$$
$$= Q\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha}\right]. \tag{EC.47}$$

Substituting back $Q = nd/2$, we obtain

$$\Pi_{L,T} = \frac{nd}{2}\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha}\right]. \tag{EC.48}$$

Step 4: Proof of Dominance and Incentives. We establish the supplier's optimal strategy by comparing the expected profits derived in Step 3. First, we compare the two buyer demand scenarios under Lean capacity. We then compare the Lean strategy to the Bloat strategy.

1. Lean comparison

Define

$$W_{UV} := \Pi_{L,T} - \Pi_{L,D}.$$

Using the profit expressions derived above,

$$\Pi_{L,T} = \frac{nd}{2}\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha}\right],$$

and

$$\Pi_{L,D} = \frac{nd}{2}\left[\frac{p+s-k}{\alpha} - 2s\right].$$

Therefore,

$$W_{UV} = \frac{nd}{2}\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha} - \frac{p+s-k}{\alpha} + 2s\right]$$
$$= \frac{nd}{2}\left[p + 2s - \frac{p+s}{\alpha} - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha}\right]. \tag{EC.49}$$

Hence, under Lean capacity, the supplier weakly prefers truthful ordering to double ordering if and only if

$$p + 2s \geq \frac{p+s}{\alpha} + (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha}.$$

Equivalently,

$$W_{UV} \geq 0 \quad \Longleftrightarrow \quad \frac{s}{p+s} \geq \frac{1-\alpha}{\alpha} + \frac{(\alpha-(1-\delta))^2}{4\delta\alpha}.$$

Thus W_{UV} is the supplier's willingness to pay for upstream visibility, measured relative to the Lean-capacity outcome with double ordering.

The statement in Theorem 5 uses a sufficient condition based only on model primitives. We now derive that sufficient condition. Let

$$Q := \frac{nd}{2}, \qquad a := 1-\delta, \qquad \eta := \frac{k}{p+s}.$$

From Step 1, the optimal capacity for target demand D is

$$\mu^*(D) = \frac{D}{\alpha}, \qquad \alpha = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}.$$

Define $F := \mathbb{P}(X \leq D)$ when capacity is chosen optimally for target demand D. Since $X = (D/\alpha)Y$, we have

$$F = \mathbb{P}\left(\frac{D}{\alpha}Y \leq D\right) = \mathbb{P}(Y \leq \alpha).$$

By Step 1, $1-\delta < \alpha < 1+\delta$, so the cutoff α lies in the support of Y. Since $Y \sim U[1-\delta, 1+\delta]$, we therefore have

$$F = \frac{\alpha-(1-\delta)}{2\delta} = \frac{\alpha-a}{2\delta}.$$

Equivalently, $\alpha = a + 2\delta F$.

Recall that the capacity first-order condition from Step 1 is

$$\frac{k}{p+s} = \mathbb{E}\left[Y\mathbf{1}\{Y \leq \alpha\}\right].$$

Using the uniform distribution, this gives

$$\eta = \frac{k}{p+s} = \frac{\alpha^2-a^2}{4\delta}.$$

Since $\alpha = a + 2\delta F$, we obtain

$$\eta = \frac{(a+2\delta F)^2 - a^2}{4\delta} = aF + \delta F^2 = F(a+\delta F).$$

Because $F \leq 1$, we have $a + \delta F \leq a + \delta = 1$. Therefore, $\eta = F(a+\delta F) \leq F$.

From (EC.49),

$$W_{UV} = Q\left[p + 2s - \frac{p+s}{\alpha} - (p+s)\frac{(\alpha-a)^2}{4\delta\alpha}\right].$$

Equivalently,

$$W_{UV} = Q\left[s - (p+s)\left(\frac{1-\alpha}{\alpha} + \frac{(\alpha-a)^2}{4\delta\alpha}\right)\right].$$

Using $\alpha = a + 2\delta F$, we can rewrite the inner term as

$$\begin{aligned}\frac{1-\alpha}{\alpha} + \frac{(\alpha-a)^2}{4\delta\alpha} &= \frac{\delta(1-2F)}{a+2\delta F} + \frac{\delta F^2}{a+2\delta F}\\ &= \frac{\delta(1-F)^2}{a+2\delta F}.\end{aligned} \tag{EC.50}$$

Therefore,

$$W_{UV} = Q\left[s - (p+s)\frac{\delta(1-F)^2}{a+2\delta F}\right].$$

Now define

$$g(x) := \frac{\delta(1-x)^2}{a+2\delta x}.$$

For $x \in [0,1]$,

$$g'(x) = -\frac{2\delta(1-x)(1+\delta x)}{(a+2\delta x)^2} \le 0.$$

Thus g is decreasing on $[0,1]$. Since $\eta \le F$, we have $g(F) \le g(\eta)$. That is,

$$\frac{\delta(1-F)^2}{a+2\delta F} \le \frac{\delta(1-\eta)^2}{a+2\delta\eta}.$$

It follows that

$$W_{UV} \ge Q\left[s - (p+s)\frac{\delta(1-\eta)^2}{a+2\delta\eta}\right].$$

Consequently, if

$$\frac{s}{p+s} > \frac{\delta(1-\eta)^2}{a+2\delta\eta},$$

then $W_{UV} > 0$. Substituting back $a = 1-\delta$ and $\eta = k/(p+s)$, this sufficient condition is

$$\frac{s}{p+s} > \frac{\delta\left(1-\frac{k}{p+s}\right)^2}{1-\delta+2\delta\frac{k}{p+s}}.$$

2. Bloat versus Lean capacity.

2(a). Bloat compared to Lean with double ordering.

We first compare Lean capacity with double ordering to Bloat capacity with truthful ordering. We use the balanced truthful-ordering Bloat benchmark from Scenario 1. As argued above, among truthful aggregate order profiles, balanced truthful ordering maximizes aggregate supplier profit under Bloat capacity. Thus, if Lean capacity with double ordering is more profitable than this benchmark, then it is more profitable than any truthful-ordering outcome under Bloat capacity in terms of aggregate supplier profit.

Using the expressions from Step 3, the per-supplier profits are $\Pi_{H,T} = \frac{nd}{2}\left(p - \frac{2k}{\alpha}\right)$ and $\Pi_{L,D} = \frac{nd}{2}\left[\frac{p+s-k}{\alpha} - 2s\right]$. Let $\Delta := \Pi_{L,D} - \Pi_{H,T}$. Then

$$\begin{aligned}\Delta &= \frac{nd}{2}\left[\frac{p+s-k}{\alpha} - 2s\right] - \frac{nd}{2}\left[p - \frac{2k}{\alpha}\right] \\ &= \frac{nd}{2}\left[\frac{p+s-k+2k}{\alpha} - p - 2s\right] \\ &= \frac{nd}{2}\left[\frac{p+s+k}{\alpha} - (p+2s)\right].\end{aligned}$$

Therefore, $\Pi_{L,D} > \Pi_{H,T}$ if and only if $\frac{p+s+k}{\alpha} > p + 2s$. Equivalently, $p + s + k > \alpha(p + 2s)$.

We now show that this condition holds under the theorem's stated assumptions. The sufficient condition $p + s \geq 4k/(2-\delta)$ implies

$$\frac{4\delta k}{p+s} \leq \delta(2-\delta).$$

Therefore,

$$\alpha^2 = (1-\delta)^2 + \frac{4\delta k}{p+s} \leq (1-\delta)^2 + \delta(2-\delta) = 1.$$

Hence $\alpha \leq 1$. Moreover, since $k > s$, we have $p + s + k > p + 2s$. Because $p + 2s > 0$, the inequality $\alpha \leq 1$ implies $\alpha(p + 2s) \leq p + 2s$. Combining these inequalities gives

$$\alpha(p+2s) \leq p + 2s < p + s + k.$$

Thus $p + s + k > \alpha(p + 2s)$, and hence $\Pi_{L,D} > \Pi_{H,T}$.

Therefore, even if Lean capacity induces double ordering, Lean capacity gives higher per-supplier profit than Bloat capacity with balanced truthful ordering. Since balanced truthful ordering maximizes aggregate supplier profit among truthful Bloat outcomes, Lean capacity with double ordering also dominates any truthful-ordering outcome under Bloat capacity in aggregate supplier profit.

2(b). Bloat compared to Lean with truthful ordering.

Next, we compare Lean capacity with truthful ordering to Bloat capacity with truthful ordering. Let $Q := nd/2$. Using the expressions from Step 3, the per-supplier profit under Lean capacity and truthful ordering is

$$\Pi_{L,T} = Q\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha}\right],$$

while the per-supplier profit under Bloat capacity and balanced truthful ordering is

$$\Pi_{H,T} = Q\left[p - \frac{2k}{\alpha}\right].$$

Let $\Delta := \Pi_{L,T} - \Pi_{H,T}$. Then

$$\begin{aligned}\Delta &= Q\left[p - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha} - \frac{k}{\alpha}\right] - Q\left[p - \frac{2k}{\alpha}\right]\\ &= Q\left[\frac{k}{\alpha} - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha}\right].\end{aligned}$$

By the definition of α,

$$\alpha^2 - (1-\delta)^2 = \frac{4\delta k}{p+s}.$$

Equivalently,

$$\frac{k}{\alpha} = \frac{p+s}{4\delta\alpha}\left(\alpha^2 - (1-\delta)^2\right).$$

Substituting this expression into the profit difference gives

$$\begin{aligned}\Delta &= Q\left[\frac{p+s}{4\delta\alpha}\left(\alpha^2-(1-\delta)^2\right) - \frac{p+s}{4\delta\alpha}(\alpha-(1-\delta))^2\right]\\ &= Q\frac{p+s}{4\delta\alpha}\left[\left(\alpha^2-(1-\delta)^2\right) - (\alpha-(1-\delta))^2\right].\end{aligned}$$

Now,

$$\begin{aligned}\left(\alpha^2-(1-\delta)^2\right) - (\alpha-(1-\delta))^2 &= (\alpha-(1-\delta))(\alpha+1-\delta) - (\alpha-(1-\delta))^2\\ &= (\alpha-(1-\delta))\left[\alpha+1-\delta-\alpha+1-\delta\right]\\ &= 2(1-\delta)(\alpha-(1-\delta)).\end{aligned}$$

Therefore,

$$\Delta = Q\frac{p+s}{4\delta\alpha}\cdot 2(1-\delta)(\alpha-(1-\delta)).$$

It remains to show that this expression is positive. Since $k>0$, $\delta>0$, and $p+s>0$, we have

$$\alpha^2 = (1-\delta)^2 + \frac{4\delta k}{p+s} > (1-\delta)^2.$$

Because $\alpha > 0$ and $1-\delta > 0$, it follows that $\alpha > 1-\delta$. Hence every factor in the expression for Δ is strictly positive, and therefore $\Delta > 0$.

Thus Lean capacity with truthful ordering strictly dominates Bloat capacity with balanced truthful ordering. □

EC.11. Proof of Corollary 2.

We define the willingness to pay as $W_{UV} \triangleq \Pi_{L,T} - \Pi_{L,D}$. As derived in the proof of Theorem 5 (Step 4, Part 1), we have:

$$W_{UV} = \frac{nd}{2}\left[p + 2s - \frac{p+s}{\alpha} - (p+s)\frac{(\alpha-(1-\delta))^2}{4\delta\alpha}\right] \tag{EC.51}$$

where $\alpha = \left((1-\delta)^2 + \frac{4\delta k}{p+s}\right)^{1/2}$ as in the proof of Theorem 5. □