Online Supplement

"Tax-induced Corporate In-kind Donations: Equilibrium Analysis and Policy Design"

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Appendix A. Numerical Calibration

This appendix details the calibration of parameter values used in the numerical study. Specifically, we calibrate the corporate income tax rate r, unit acquisition cost c, donation utility rate δ, and goodwill enhancement coefficient β. We vary K to reflect different capacity levels and γ to capture the government's different levels of social welfare concern. Finally, λ is treated as an exogenous parameter for the equilibrium analysis under a given policy (Section 4), but we optimize λ for the government's policy design problem (Section 5). This calibration draws on the broader food sector and is intended to establish empirically plausible magnitudes rather than to estimate a specific differentiated packaged-food category, for which product-level data are limited.

Corporate Income Tax Rate (r). According to the Tax Foundation's 2025 global corporate tax statistics, the statutory corporate income tax rate is 25.57% in the United States on average (combining federal and state-level rates), 25% in China and the United Kingdom, 30% in Australia, 30.06% in Germany, 34% in Brazil, 35% in Colombia, 36.13% in France, etc.[1] We set the baseline at $r = 0.3$ and use the range $r \in [0, 0.5]$ to encompass plausible tax rates across countries.

Unit Product Acquisition Cost (c). We calibrate c using industry-reported gross margins (GM) in the food sector, defined as $\text{GM} = (p-c)/p$. According to CSIMarket (2026), the food processing industry reports an average gross margin of approximately 33.02%.[2] GFM Wholesale (2025) further reports gross margins of 30-40% for high-turnover staples and 50-60% for specialty and premium categories.[3] To map these empirical margins to the value of c, we leverage the firm's optimal price in Scenario N, $p^{N*} = (1+c)/2$, yielding an implied gross margin of $(1-c)/(1+c)$. Setting this implied margin equal to the empirically observed levels yields $c \approx 0.5$ for $\text{GM} \approx 33\%$ and $c \approx 0.33$ for $\text{GM} \approx 50\%$. The empirically plausible range of c is thus approximately $c \in [0.3, 0.5]$. We use $c = 0.4$ as our baseline value.

Donation Utility Rate (δ). The parameter δ represents the perceived utility a consumer derives from a donated product relative to purchasing the product. We calibrate δ using revealed-preference evidence on the economic value of food pantry services. Using a travel-cost model with sample data from a food bank system in Colorado (2005-2017), Byrne and Just (2023) estimate that the welfare value of a typical pantry visit is between \$40 and \$60 per household. Food pantries typically provide approximately one week's worth of groceries per household per visit.[4] Combining this benchmark with Feeding America's estimate of \$56.07 in weekly food spending per person in 2011 (the midpoint year of 2005-2017) yields a retail-equivalent value of roughly \$112-\$168 for a two- to three-person household.[5] Thus, the perceived utility of donated products ranges from approximately $40/168 \approx 24\%$ to $60/112 \approx 54\%$ of their retail-equivalent value. We vary $\delta \in [0.2, 0.6]$ and use $\delta = 0.4$ as our baseline value.

Goodwill Enhancement Coefficient (β). Since there is limited direct empirical evidence on goodwill premiums induced by in-kind donations, we draw on the broader literature on consumer responses to corporate sustainability as a proxy to calibrate β. Using a meta-analysis of 80 studies worldwide, Li and Kallas (2021) report an average WTP premium of 29.5% for sustainably produced food products. To map this premium to β, we note that the goodwill term $\beta\psi$ represents the donation-induced premium relative to Scenario N, so $\beta\psi = 0.295 p^{N*}$, where $p^{N*} = \max\{(1+c)/2, 1-K\}$ is the optimal price in Scenario N. To estimate the donation volume ψ, we draw on ReFED (2026), which reports that surplus food accounts for approximately 29% of total food supply in the United States.[6] Accordingly, we set $\psi \approx 0.29K$. Substituting the capacity values used in our numerical figures ($K = 0.305$ for the non-binding capacity case and $K = 0.26$ for the binding capacity case), we obtain $\beta \approx 2.9$. However, not all sustainability-related premiums in the meta-analysis can be attributed to

[1] https://taxfoundation.org/data/all/global/corporate-tax-rates-by-country-2025/

[2] https://csimarket.com/Industry/industry_Profitability_Ratios.php?ind=505

[3] https://gourmetfoodmarketplace.com/page/gross-margins-by-category-a-guide-to-maximizing-retail-profitability-2025-update

[4] https://wesleypantry.org/about.html; https://www.wellstoncenter.org/services

[5] https://www.feedingamerica.org/sites/default/files/research/map-the-meal-gap/2011/2011-mapthemealgap-exec-summary.pdf

[6] https://refed.org/food-waste/refed-us-food-waste-report-2026/#the_problem

donation-induced goodwill, suggesting that the premium attributable specifically to donations is likely smaller. We thus consider the range $\beta \in [0,3]$ and adopt $\beta = 1.5$ as our baseline value.

Insights into tax incentive policies in the United States, Colombia, France. We now provide insights into the tax incentive policies of three countries with food donation-specific policies: the United States, Colombia, and France. Table EC.1 summarizes the corporate tax rate (r), the current tax incentive rate (λ_{actual}), as well as, for sufficient and insufficient capacity levels (K), the policy-effective threshold ($\hat{\lambda}_0$) above which the firm donates, the CS-loss interval (λ_ℓ, λ_h) in which the tax incentive policy leads to a loss in consumer surplus (CS), and the worst-case CS-loss ($\max \Delta CS \doteq \max_{\lambda \in [\hat{\lambda}_0, \bar{\lambda}]} \big(CS^N - CS^T(\lambda)\big)/CS^N$).

Table EC.1 Cross-country analysis for calibrated baseline parameters and sensitivity ranges

			Sufficient capacity ($K = 0.305$)			*Insufficient capacity* ($K = 0.26$)		
Country	r	λ_{actual}	$\hat{\lambda}_0$	$(\lambda_\ell, \lambda_h)$	$\max \Delta CS$	$\hat{\lambda}_0$	$(\lambda_\ell, \lambda_h)$	$\max \Delta CS$
United States	25.6%	1.00	0.502	(2.078, 3.198)	0.22%	1.412	(1.412, 2.227)	0.16%
$\beta = 0$			3.782	(4.206, 4.526)	0.22%	4.250	(4.250, 4.483)	0.16%
$\beta = 0.8$			2.036	(3.071, 3.818)	0.22%	2.736	(2.736, 3.280)	0.16%
$\beta = 1.6$			0.283	(1.936, 3.110)	0.22%	1.223	(1.223, 2.076)	0.16%
$\delta = 0.2$			0.063	(0.249, 10.162)	38.9%	1.033	(1.033, 9.737)	33.2%
$\delta = 0.28$			0.239	(0.546, 9.985)	19.7%	1.185	(1.185, 8.688)	14.4%
$\delta = 0.36$			0.414	(1.064, 6.375)	5.2%	1.336	(1.336, 4.567)	2.6%
Colombia	35%	1.06	0.320	(1.326, 2.041)	0.22%	0.901	(0.901, 1.421)	0.16%
$\beta = 0$			2.413	(2.684, 2.888)	0.22%	2.711	(2.711, 2.860)	0.16%
$\beta = 0.8$			1.299	(1.959, 2.436)	0.22%	1.746	(1.746, 2.092)	0.16%
$\beta = 1.6$			0.180	(1.235, 1.984)	0.22%	0.780	(0.780, 1.325)	0.16%
$\delta = 0.2$			0.040	(0.159, 6.484)	38.9%	0.659	(0.659, 6.212)	33.2%
$\delta = 0.28$			0.152	(0.348, 6.370)	19.7%	0.756	(0.756, 5.543)	14.4%
$\delta = 0.36$			0.264	(0.679, 4.067)	5.2%	0.852	(0.852, 2.914)	2.6%
France	36.1%	1.66	0.305	(1.262, 1.942)	0.22%	0.857	(0.857, 1.352)	0.16%
$\beta = 0$			2.297	(2.554, 2.749)	0.22%	2.581	(2.581, 2.722)	0.16%
$\beta = 0.8$			1.237	(1.865, 2.319)	0.22%	1.662	(1.662, 1.992)	0.16%
$\beta = 1.6$			0.172	(1.176, 1.889)	0.22%	0.742	(0.742, 1.261)	0.16%
$\delta = 0.2$			0.038	(0.151, 6.172)	38.9%	0.628	(0.628, 5.913)	33.2%
$\delta = 0.28$			0.145	(0.331, 6.064)	19.7%	0.719	(0.719, 5.276)	14.4%
$\delta = 0.36$			0.252	(0.646, 3.872)	5.2%	0.811	(0.811, 2.774)	2.6%

Note. The baseline uses $\beta = 1.5$ and $\delta = 0.4$; the β rows fix $\delta = 0.4$ and the δ rows fix $\beta = 1.5$.

For each country, the first row corresponds to the results based on the calibrated baseline parameter values ($\beta = 1.5$, $\delta = 0.4$, $c = 0.4$). Interestingly, the actual tax incentive rates in these example countries all fall into potentially critical ranges, with different implications for each. In the United States, the current tax incentive rate never harms consumer surplus since it does not fall into the CS-loss intervals, but it is effective only in settings with sufficient capacity ($1.00 > 0.502$); when capacity is insufficient, the current policy is not effective ($1.00 < 1.412$). The Colombian incentive rate of 106% is effective in promoting donations under both capacity regimes, but our model suggests it would decrease consumer surplus in the case of insufficient capacity ($1.06 \in (0.901, 1.421)$). In France, the high incentive rate of 166% also effectively incentivizes donations under both capacity regimes, but it reduces consumer surplus in the case of sufficient capacity ($1.66 \in (1.262, 1.942)$).

These conclusions are robust to the calibrated ranges of β and δ, as illustrated in the second to seventh rows for each country. However, these two parameters have different effects on the reported measures. A larger β makes donating more profitable, so the firm starts to donate at a lower incentive rate ($\hat{\lambda}_0$). The CS-loss interval ((λ_l, λ_h)) shifts downward: for example, in the United States under sufficient capacity, it moves from $(4.206, 4.526)$ at $\beta = 0$ to $(1.936, 3.110)$ at $\beta = 1.6$. The magnitude of the worst-case CS-loss, however, stays at 0.22% across all β values and across all countries with different r values. Both β and r influence consumer surplus only through the donation volume. At the incentive rate that maximizes the CS-loss, the donation volume remains unchanged as either parameter varies. As a result, the worst-case CS-loss is independent of β and r, despite the fact that the incentive rate that delivers the lowest donation volume depends on these two parameters.

Unlike β, the donation utility rate δ governs the magnitude of the worst-case CS-loss. Lowering δ from 0.4 (baseline value) to 0.2 raises the worst-case loss from 0.22% to 38.9% under sufficient capacity,

and from 0.16% to 33.2% under insufficient capacity; it also widens the CS-loss interval (for the United States, from $(2.078, 3.198)$ at $\delta = 0.4$ to $(0.249, 10.162)$ at $\delta = 0.2$). CS-loss is more likely to occur when donated products are poor substitutes for retail purchases with low δ values. This suggests that government interventions that raise δ, such as cold-chain investment or better packaging for donated products, could be an effective strategy from a CS perspective.

While our intention here is not to provide a comprehensive evaluation of the effectiveness of current donation tax policies, these findings suggest that stronger tax incentives do not necessarily lead to better social outcomes. The analysis also illustrates the practical relevance of our model and demonstrates the robustness of our insights across a range of parameter specifications.

Appendix B. Price-based Donation Tax Incentives

Donation tax policies in some jurisdictions stipulate that the tax benefit is calculated based on the fair market value (FMV). FMV is the price at which the taxpayer sells products of the same or similar type and quality at the time of donation.[7] In this appendix, we examine the price-based donation tax policy. Under this policy, the firm's profit function in Scenario N is the same as in the base model, while that in Scenario T is as follows:

$$\pi^T = (1-r)(p\min\{D_b, Q\} - cQ) + r\lambda p\psi.$$

Similar to the base model, we assume $\lambda \in (0, \bar{\lambda}^{(p)})$ and $\delta \in (0, \bar{\delta}^{(p)})$, where $\bar{\lambda}^{(p)} = \frac{(1-r)(1+(\beta-\delta)K)}{r(1+K+2(\beta-\delta)K)}$ and $\bar{\delta}^{(p)} = \min\{\max\{\frac{[1-c-\sqrt{4cK+(1-c)^2}](2K+c-1)^+ + 2(1-c)K}{4K^2} + \beta, 0\}, 1\}$. The condition on β in Assumption 1 also applies here.

Proposition EC.1. *Under the price-based tax incentive policy, there exists a threshold of λ, $\hat{\lambda}_0^{(p)}$, such that the firm's optimal price and acquisition quantity decisions p^* and Q^*, and the associated donation fulfillment rate R^* are as follows:*

(a) Scenario N: The firm's optimal decisions are the same as Proposition 1(a).

(b) Scenario T: If $\lambda < \hat{\lambda}_0^{(p)}$, the firm's optimal decisions are the same as those in Scenario N; If $\lambda \geq \hat{\lambda}_0^{(p)}$, $p^{T*} = \frac{1+(\beta-\delta)K-r[1+\lambda-K(\delta-\beta+\lambda)]}{2[1-r(\lambda+1)]}$, $Q^{T*} = K$, $R^{T*} = \frac{(1-r)[K(2+\beta-\delta)-1]+(1-K)r\lambda}{(1-r)[1+(2-K)(\beta-\delta)]+(1-K)(2\delta-2\beta-1)r\lambda}$.

Proposition EC.1 shows that the firm's optimal decisions under the price-based tax policy follow the same structure as in the base model (see Proposition 1). There also exists a policy-effective threshold of the tax incentive rate, above which the firm donates.

We now compare the price-based policy with the cost-based policy as examined in the base model. We conduct a numerical study by varying the parameters around the calibrated values in Appendix A. Specifically, we use $c \in \{0.3, 0.4, 0.5\}$, $\delta \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$, and $\beta \in \{0, 0.2, 0.4, \ldots, 3\}$, while varying $K \in \{0.1, 0.2, \ldots, 1\}$ and $\gamma \in \{0, 0.1, \ldots, 1\}$ across their full ranges and fixing $r = 0.3$. Each parameter combination is filtered through the feasibility conditions of our model, resulting in 13,607 numerical instances in total. For each instance, we optimize λ over the policy-effective region under the constraint $\lambda \in [0, \bar{\lambda}]$ to obtain the government's optimal price-based policy. We then compare the resulting government objectives among different policy scenarios. Table EC.2 summarizes the comparison results, separated into two cases based on capacity.

Table EC.2 Comparison of three policy scenarios

Case	Total	No incentive	Cost-based	Price-based
Overall	13,607	3,478 (25.6%)	5,855 (43.0%)	4,274 (31.4%)
$K \geq (1-c)/2$ (high capacity)	8,206	3,469 (42.3%)	4,726 (57.6%)	11 (0.1%)
$K < (1-c)/2$ (low capacity)	5,401	9 (0.2%)	1,129 (20.9%)	4,263 (78.9%)

Table EC.2 reveals a striking dichotomy between the two capacity cases. In the high-capacity case, the cost-based policy is the government's preferred instrument in 57.6% of feasible combinations, while the price-based policy is preferred in only 0.1%. When capacity is not restrictive, the firm has full flexibility to inflate its price under both policy regimes. Under the price-based regime, however, the tax

[7] https://www.thetaxadviser.com/newsletters/2016/jul/charitable-donations-of-food-inventory.html

deduction $\lambda p\psi$ directly rewards this inflation because raising price generates a proportional tax saving, thereby turning this policy into a self-harming welfare instrument for the government.

In the low-capacity case, the ranking reverses: the price-based policy dominates in 78.9% of feasible combinations, compared with only 20.9% for the cost-based policy. In this case, the firm has limited intrinsic incentive to donate; since its acquisition quantity is constrained by K regardless of the policy, the amount of unsold products available for donations is naturally restricted. At the same time, the firm's price is high in the low-capacity case, so the price-based deduction, being scaled by this elevated p rather than by c, generates greater donation incentives per unit of forgone tax revenue. The government can therefore induce a meaningful donation volume at a lower fiscal cost, making the price-based policy more attractive in the low-capacity case.

These findings deliver a clear policy implication: the ranking of cost-based versus price-based tax policies depends critically on the firm's capacity. Cost-based donation tax incentives are well-suited when firms have ample production capacity, where preventing price inflation is the primary concern. Price-based tax incentives, by contrast, are superior when firm capacity is limited, where the greater donation incentive outweighs the modest price distortion.

Appendix C. Beta Distribution of Consumer Willingness-to-Pay

Our base model assumes that consumer willingness-to-pay (WTP) is uniformly distributed over $[0,1]$. To verify whether our main findings hinge on this distributional assumption, we now consider a more flexible specification in which v follows a Beta distribution, $v \sim \text{Beta}(a,b)$, with PDF:

$$f(v;a,b) = \frac{v^{a-1}(1-v)^{b-1}}{B(a,b)}, \quad v \in [0,1],$$

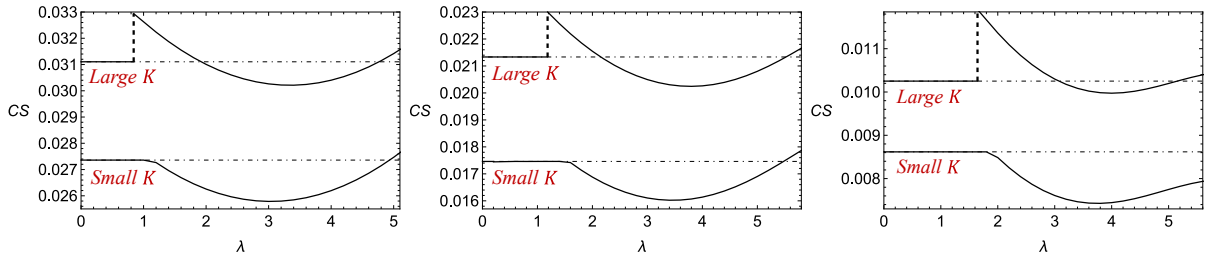
where $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$. This parameterization generates flexible shapes including right-skewed distributions with substantial mass near zero, reflecting populations in which a large fraction of consumers have low WTP. Our base model corresponds to the special case with $a=b=1$.

With a Beta distribution, the model remains the same except that the buying and donation demands are now specified using the Beta CDF $F(\cdot;a,b)$. Specifically, given the indifference point v^*, the buying demand becomes $D_b = 1 - F(v^*;a,b)$ and the donation demand becomes $D_d = F(v^*;a,b)$. Consumer surplus is correspondingly modified to weigh individual surpluses by the Beta PDF:

$$CS^j = \int_0^{v^*} R\delta v f(v;a,b)dv + \int_{v^*}^1 (v - p + \beta\psi) f(v;a,b)dv.$$

The firm's unconstrained optimal price in Scenario N is characterized by the first-order condition $1 - F(p;a,b) = (p-c)f(p;a,b)$, and the corresponding unconstrained optimal quantity is $Q_{\text{unc}}^{N*} = 1 - F(p^{N*};a,b)$. Because the Beta distribution does not admit a closed-form solution in both scenarios except for the special case of $a=b=1$, we compute the equilibrium outcomes numerically. For simplicity, we fix $a=1$ and vary $b \in \{1.5, 2, 3\}$, where larger b corresponds to a more right-skewed distribution with greater mass concentrated at low values of WTP. It is worth noting that the capacity threshold separating the two cases of equilibrium depends on the value of b. Within the $\text{Beta}(1,b)$ family, the first-order condition yields $p^{N*} = (1+bc)/(1+b)$ and $Q_{\text{unc}}^{N*} = \big(b(1-c)/(1+b)\big)^b$. This quantity serves as the capacity threshold separating two regimes: the non-binding capacity case ($K \geq Q_{\text{unc}}^{N*}$), in which the firm's capacity does not constrain its optimal output in Scenario N, and the binding capacity case ($K < Q_{\text{unc}}^{N*}$), in which capacity is binding. As the Beta distribution becomes more right-skewed (larger b), the capacity threshold Q_{unc}^{N*} decreases, reflecting the fact that a population with lower average WTP generates less demand and hence a smaller unconstrained optimal quantity.

We plot the consumer surplus against the tax incentive rate at different values of b and K in Figure EC.1. In the figure, we adjust K for each given b so that both capacity regimes are illustrated within each panel. Figure EC.1 shows that our main qualitative insights continue to hold across a range of Beta parameter values: (i) consumer surplus exhibits a U-shaped pattern in λ, (ii) introducing the donation tax incentive may reduce consumer surplus relative to no policy, and (iii) the policy-effective threshold behaves analogously to the base model.



(a) $b = 1.5$ (b) $b = 2$ (c) $b = 3$

Figure EC.1 **Impact of λ on the consumer surplus with $v \sim \text{Beta}(a, b)$; solid lines: CS^T, dash-dotted lines: CS^N ($a = 1$, $r = 0.3$, $c = 0.4$, $\delta = 0.4$, $\beta = 1.5$, $K = 0.225$ (Large K) and $K = 0.2$ (Small K) in (a), $K = 0.17$ (Large K) and $K = 0.14$ (Small K) in (b), and $K = 0.105$ (Large K) and $K = 0.08$ (Small K) in (c))**

Appendix D. Donation Seekers with Retail Fallback

In the base model, consumers who seek a donation but are not served exit the market. We relax this assumption by allowing an unsuccessful donation seeker to return to the retail channel to purchase. A consumer who buys directly obtains $v - p + \beta\tilde{\psi}$; a consumer who seeks a donation receives δv with probability $\tilde{R}$ and, with probability $1 - \tilde{R}$, returns to the retail channel and obtains $\max\{v - p + \beta\tilde{\psi}, 0\}$. Consumers choose their initial channel *ex ante*, anticipating the option to return, and the firm chooses (Q, p) anticipating the resulting returning demand, so donation volume and returning demand are jointly determined in equilibrium.

Given (Q, p), an RE equilibrium satisfies: (1) $v^* - p + \beta\tilde{\psi} = \tilde{R}(\tilde{\psi})\delta v^* + [1 - \tilde{R}(\tilde{\psi})](v^* - p + \beta\tilde{\psi})$; (2) $\psi^* = \arg\max_{\psi \in [0, (Q - \tilde{D}_b)^+]} \pi^T(Q, p, \psi)$; (3) $\tilde{\psi} = \psi^*$; (4) $\tilde{v} = v^*$. Condition (1) differs from its base-model counterpart only in the second term on the right-hand side, which captures the option to return. Solving it gives $v^* = (p - \beta\psi)/(1 - \delta)$, and consumers partition into three groups: those with $v \in [v^*, 1]$ purchase directly; those with $v \in [p - \beta\psi, v^*)$ seek a donation and return to retail if unserved; and those with $v \in [0, p - \beta\psi)$ seek a donation and exit if unserved. We focus on the case where there always exist some customers who do not seek for donations initially; that is, $v^* < 1$, which is equivalent to $\lambda < \lambda_f \doteq \frac{(1-r)[1+\beta K-\delta-(2-K)\delta(1+\beta-\delta)]}{(1-\delta)cr}$.

PROPOSITION EC.2. *For any $\lambda \in (0, \lambda_f)$, allowing unserved donation seekers to return to the retail channel leaves buying demand, equilibrium donation volume, the firm's optimal price and acquisition quantity, firm profit, aggregate consumer surplus, and hence the government's optimal incentive rate unchanged relative to the base model.*

The equivalence reflects two offsetting effects. The fallback option makes donation seeking more attractive and raises the indifference threshold, drawing some consumers away from direct purchase; but since the donation volume is unchanged in the RE equilibrium, the enlarged pool of donation seekers faces a lower fulfillment rate. The measure of consumers switching away from direct purchase exactly equals the measure of unsuccessful seekers who return to the retail channel, leaving the buying demand D_b unchanged. In welfare terms, consumers with $v \geq p - \beta\psi$ benefit from the fallback option, while those below the threshold are worse off due to the reduced donation fulfillment rate, and these two effects offset each other in aggregate.

Appendix E. Two-Period Model with Goodwill Persistence

We extend the base model to a two-period model. In each period, a group of consumers with valuations drawn from $U[0, 1]$ arrives. Consumers are myopic and make decisions based on their utilities in the current period, whereas the firm is forward-looking and accounts for both periods when making its first-period decisions. Let G_t denote the goodwill stock in period t, which evolves as $G_t = \rho G_{t-1} + \psi_t, \forall t = 1, 2$, where $\rho \in [0, 1]$ is the rate at which goodwill carries over to the next period and $G_0 = g_0 \geq 0$ is an exogenous initial stock accumulated before period 1. The expected utility of a consumer with valuation v in period t is

$$U_t = \max\Big\{v - p_t + \beta\big(\rho G_{t-1} + \tilde{\psi}_t\big),\ \tilde{R}_t(\tilde{\psi}_t)\,\delta v\Big\}, \qquad t = 1, 2.$$

Setting $\rho = 0$ and $g_0 = 0$ reproduces the base model in each period; the new term $\beta\rho G_{t-1}$ captures the goodwill that past donations carry into the current period. Given (Q_t, p_t) and the inherited stock G_{t-1}, the period-t RE equilibrium satisfies (1) $v_t^* - p_t + \beta(\rho G_{t-1} + \tilde{\psi}_t) = \tilde{R}_t(\tilde{\psi}_t)\delta v_t^*$, (2) $\psi_t^* = \arg\max_{\psi_t \in [0,(Q_t - \tilde{D}_{b,t})^+]} \sum_{t=1}^{2} \pi_t^T$, (3) $\tilde{\psi}_t = \psi_t^*$, and (4) $\tilde{v}_t = v_t^*$.

The firm's after-tax profit in period t is $\pi_t^T = (1-r)\big(p_t \min\{D_{b,t}, Q_t\} - cQ_t\big) + r\lambda_t c\psi_t$, where λ_t is the incentive rate in period t. Let $\pi_2^{T*}(\psi_1)$ denote the firm's optimal second-period profit given the stock $G_1 = \rho g_0 + \psi_1$. In period 1, the firm chooses (Q_1, p_1) to maximize $\pi_1^T + \pi_2^{T*}(\psi_1)$. The government commits to the incentive rates before period 1 and chooses them to maximize the weighted sum of social welfare and tax revenue across both periods,

$$\max_{\lambda_1, \lambda_2 \geq 0} GO^T = \sum_{t=1}^{2} \Big(\gamma SW_t^T + (1-\gamma) TR_t^T \Big),$$

where $SW_t = CS_t + \pi_t^T$, $CS_t = \int_{v_t^*}^{1} (v - p_t^* + \beta G_t) dv + R_t^* \int_0^{v_t^*} \delta v dv$, and $TR_t = r(p_t^* \min\{D_{b,t}^*, Q_t^*\} - cQ_t^* - \lambda_t c\psi_t^*)$.

The model does not admit a tractable closed-form solution. The first-period decisions must be evaluated against the sum of profits across both periods, and in each period, the capacity constraint and the conditions characterizing the firm's optimal operational and donation decisions further complicate the analysis of this two-period model. We therefore resort to numerical studies.

We first consider the case in which a single incentive rate applies to both periods ($\lambda_1 = \lambda_2 = \lambda$), and then extend the analysis to allow the two periods to carry different rates to study the timing of policy.

Single-rate policy. Table EC.3 illustrates how the optimal single incentive rate λ^* varies with the initial goodwill stock g_0. We observe that the firm may donate even in the absence of government intervention (i.e., $\psi^N > 0$). The reason is that first-period donations generate an intertemporal benefit by increasing the goodwill stock carried into the second period, which in turn raises consumers' willingness to pay and stimulates future demand. In most cases, this intertemporal goodwill effect is sufficiently strong to induce the firm to donate without policy support.

Table EC.3 Optimal policy under different g_0 and γ ($r = 0.3$, $c = 0.4$, $\delta = 0.4$, $\beta = 1.5$, $\rho = 0.6$)

	Sufficient capacity ($K = 0.305$)						Insufficient capacity ($K = 0.26$)					
g_0	ψ^N	$\gamma \leq 0.52$	$\gamma = 0.60$	$\gamma = 0.64$	$\gamma = 0.68$	$\gamma \geq 0.71$	ψ^N	$\gamma \leq 0.52$	$\gamma = 0.62$	$\gamma = 0.66$	$\gamma = 0.70$	$\gamma \geq 0.73$
0.00	0.05	N	T(0.76)	T(0.76)	T(7.78)	T(7.78)	0.01	N	T(0.25)	T(1.50)	T(7.49)	T(7.49)
0.05	0.04	N	T(0.44)	T(0.88)	T(8.05)	T(8.04)	0	N	N	N	T(7.76)	T(7.76)
0.10	0.03	N	T(0.10)	T(0.74)	T(1.01)	T(8.31)	0	N	N	N	N	T(8.02)
0.15	0.02	N	N	N	T(1.14)	T(8.57)	0	N	N	N	N	T(8.28)
0.20	0.01	N	N	N	N	T(8.83)	0	N	N	N	N	T(8.54)
0.25	0	N	N	N	N	T(9.09)	0	N	N	N	N	T(8.81)

Note. "N" denotes no incentive; "T(·)" denotes offering a tax incentive at the optimal rate $\lambda^* = \cdot$.The column ψ^N reports the total donation volume without policy (independent of γ).

Within the intervention region, λ^* is non-monotonic in g_0, which arises because of two counteracting effects. On the one hand, a larger g_0 increases first-period demand and thereby raises the profitability of the sales channel, increasing the opportunity cost of diverting products to donation. The government therefore needs to provide a stronger incentive to induce the firm to maintain its donation level, leading to the initial increase in λ^*. On the other hand, as g_0 becomes larger, the marginal effectiveness of the incentive declines, while the accumulated goodwill stock already generates a substantial share of the attainable consumer-surplus benefit. As a result, the additional welfare gain from a higher incentive becomes smaller, prompting the government to reduce λ^*. Eventually, when g_0 is sufficiently large, the marginal benefit of inducing additional donations no longer justifies the associated fiscal cost. The government therefore withdraws the incentive, yielding $\lambda^* = 0$ and Scenario N.

Taken together, both the firm's self-motivated donations and the government's eventual withdrawal of incentives at high g_0 act to limit further goodwill accumulation. This dynamic pattern also sharpens the policy reading of our static base model: the case for donation tax incentives is strongest when the firm's accumulated reputation is modest, and weakens as reputation builds up.

Two-rate policy. We further extend the analysis to allow the two periods to carry different incentive rates. Table EC.4 reports the resulting optimal incentive rates and donation volumes at the calibrated parameter values.

Table EC.4 Optimal two-period policy ($r=0.3$, $c=0.4$, $\delta=0.4$, $\beta=1.5$, $g_0=0$)

	$\rho=0.3$					$\rho=0.7$				
γ	λ_1^*	λ_2^*	ψ_1^*	ψ_2^*	Pattern	λ_1^*	λ_2^*	ψ_1^*	ψ_2^*	Pattern
Panel A: Sufficient capacity ($K=0.305$). $\psi^N=0.020$ at $\rho=0.3$ and 0.063 at $\rho=0.7$.										
≤ 0.30	0	0	0.020	0	no incentive	0	0	0.063	0	no incentive
0.40	0.02	0	0.021	0	front-loaded	0	0	0.063	0	no incentive
0.50	0.06	0	0.022	0	front-loaded	0	0	0.063	0	no incentive
0.55	0.06	0	0.022	0	front-loaded	0.30	0	0.075	0	front-loaded
0.60	0.06	0	0.022	0	front-loaded	1.12	0	0.109	0	front-loaded
0.70	7.78	0	0.305	0	front-loaded	7.78	0	0.305	0	front-loaded
≥ 0.80	7.78	8.58	0.305	0.305	both periods	7.78	9.65	0.305	0.305	both periods
Panel B: Insufficient capacity ($K=0.26$). $\psi^N=0$ at $\rho=0.3$ and 0.019 at $\rho=0.7$.										
≤ 0.40	0	0	0	0	no incentive	0	0	0.019	0	no incentive
0.50	0	0	0	0	no incentive	0.02	0	0.020	0	front-loaded
0.55	0	0	0	0	no incentive	0.14	0	0.025	0	front-loaded
0.60	0	0	0	0	no incentive	0.44	0	0.037	0	front-loaded
0.70	7.49	0	0.260	0	front-loaded	7.49	0	0.260	0	front-loaded
≥ 0.80	7.49	8.18	0.260	0.260	both periods	7.49	9.09	0.260	0.260	both periods

Goodwill persistence makes donation partly self-sustaining: even absent any incentive, the firm may donate in the first period purely to build the stock that raises its second-period price. The optimal policy then takes one of three forms as the government's weight on social welfare rises. At low γ no incentive is offered and the firm's self-sustaining donation is the entire outcome. At intermediate γ the incentive is front-loaded, with only the first period subsidized: a first-period donation also raises second-period willingness to pay, whereas a second-period donation has no such spillover. This front-loaded region speaks directly to the timing of donation policy: a government that places moderate weight on social welfare optimally concentrates the incentive early and lets accumulated goodwill carry over to the second period. At high γ, tax incentives are offered in both periods, and $\lambda_2^* > \lambda_1^*$, since inducing the same donation volume requires a higher rate in later periods.

Extending the horizon beyond two periods provides a stationary interpretation of the goodwill term in our base model. Retaining the law of motion $G_t = \rho G_{t-1} + \psi_t, \rho \in [0,1)$, suppose consumers' willingness to pay increases by $\beta_G G_t$, where $\beta_G > 0$ captures the marginal effect of the goodwill stock on willingness to pay. In a stationary equilibrium, the firm makes the same decisions each period, so that the goodwill stock remains constant over time. Hence, $G^* = \rho G^* + \psi^*$ which is equivalent to $G^* = \frac{\psi^*}{1-\rho}$, and the consumer's utility from purchasing becomes $v - p + \beta_G G^* = v - p + \frac{\beta_G}{1-\rho}\psi^*$. Defining $\beta \doteq \frac{\beta_G}{1-\rho}$ reduces this to $v - p + \beta\psi^*$, which is precisely the goodwill term in our base model. The goodwill coefficient we calibrate therefore captures two distinct forces: the marginal effect of goodwill on consumers' willingness to pay and the persistence of the goodwill stock. Setting $\rho = 0$ gives $\beta = \beta_G$ and recovers the base model, in which goodwill generated in one period does not carry over to the next.

Appendix F. Technical Proofs

Proofs of Lemma 1 and Proposition 1

According to the equilibrium requirements of RE, we obtain the firm's donation quantity as $\psi^* = (Q - D_b)^+$. Given $D_b = 1 - v^*$ and $D_d = v^*$, the donation fulfillment rate is given by $R = (Q - 1 + v^*)^+ / v^*$. Further, we know $v^* + \beta\psi^* - p = R\delta v^*$. To solve for v^* given fixed values of p and Q, we consider two cases: First, if $Q - 1 + v^* > 0$, we have $v^* = \frac{p + (\beta - \delta)(1-Q)}{1 + \beta - \delta}$, which holds if $p > 1 - Q$; second, if $Q - 1 + v^* \leq 0$, we have $v^* = p$ by solving $v^* - p = 0$, which holds if $p \leq 1 - Q$. Therefore, the buying

demand is given by

$$D_b(Q,p) = 1 - v^* = \begin{cases} 1-p, & p \le 1-Q, \\ \frac{1-p+(\beta-\delta)Q}{1+\beta-\delta}, & p > 1-Q. \end{cases}$$

(a) In **Scenario N**, the firm's profit is given by

$$\pi^N(Q,p) = (1-r)(p\min\{D_b, Q\} - cQ) = \begin{cases} \pi_1^N(Q,p) \doteq (1-r)(p-c)Q, & p \le 1-Q, \\ \pi_2^N(Q,p) \doteq (1-r)(pD_b(Q,p) - cQ), & p > 1-Q. \end{cases}$$

To maximize π^N, we take a two-step approach by first solving for the optimal p for a given Q and then solving for the optimal Q.

When $p \le 1-Q$, we obtain $\frac{\partial \pi_1(Q,p)}{\partial p} = (1-r)Q > 0$. Thus, given the condition $p \le 1-Q$, it is (locally) optimal for the firm to set the price at $p_1^*(Q) = 1-Q$, and thus we have $D_b(Q, p_1^*(Q)) = Q$. When $p > 1-Q$, we obtain $\frac{\partial^2 \pi_2(Q,p)}{\partial p^2} = -2(1-r) < 0$, so there exists a stationary point of the price, denoted by $p_2^{N*}(Q) = \frac{1+(\beta-\delta)Q}{2}$, and we have $D_b(Q, p_2^{N*}(Q)) = \frac{1+(\beta-\delta)Q}{2(1+\beta-\delta)}$. This shows that $\pi^N(Q,p)$ increases in p for $p \le 1-Q$ and is concave in p for $p > 1-Q$. By comparing $p_2^{N*}(Q)$ and $p_1^*(Q) = 1-Q$, we have two cases. If $p_2^{N*}(Q) \le 1-Q$, or equivalently, $Q \le Q_0^N \doteq \frac{1}{2+\beta-\delta}$, the optimal price is $p^{N*}(Q) = 1-Q$ and in this case all products are sold without donation; if $p_2^{N*}(Q) > 1-Q$, or equivalently, $Q > Q_0^N$, the optimal price is $p^{N*}(Q) = p_2^{N*}(Q) = \frac{1+(\beta-\delta)Q}{2}$ and in this case the firm donates a positive amount of products.

Next, we solve for the optimal Q. Substituting the optimal price $p^{N*}(Q)$ into the profit function, we obtain

$$\pi^N(Q, p^{N*}(Q)) = \begin{cases} \pi_1^N(Q, 1-Q), & Q \le Q_0^N, \\ \pi_2^N(Q, p_2^{N*}(Q)), & Q > Q_0^N. \end{cases}$$

The firm aims to maximize $\pi^N(Q, p^{N*}(Q))$ by choosing the optimal Q, subject to the capacity constraint $0 \le Q \le K$. To solve this problem, we first obtain the second derivative as follows:

$$\frac{\partial^2 \pi^N(Q, p^{N*}(Q))}{\partial Q^2} = \begin{cases} -2(1-r), & Q \le Q_0^N, \\ \frac{(\delta-\beta)^2(1-r)}{2(1+\beta-\delta)}, & Q > Q_0^N. \end{cases}$$

It can also be shown that $\frac{\partial \pi^N(Q,p^{N*}(Q))}{\partial Q}|_{Q \nearrow Q_0^N} = \frac{\partial \pi^N(Q,p^{N*}(Q))}{\partial Q}|_{Q \searrow Q_0^N}$. Thus, the objective function $\pi^N(Q, p^{N*}(Q))$ is concave in Q for $Q \le Q_0^N$, convex in Q for $Q > Q_0^N$, and smooth everywhere. Solving $\frac{\partial \pi_1^N(Q, p_1^*(Q))}{\partial Q} = 0$, we have a stationary point $Q = \frac{1-c}{2}$. Given these results and the constraint $Q \le K$, we consider the following cases.

i) If $K \le Q_0^N \le \frac{1-c}{2}$ or $K \le \frac{1-c}{2} \le Q_0^N$, then $\pi(Q, p^{N*}(Q)) = \pi_1^N(Q, 1-Q)$ always increases in Q for $Q \in [0, K]$. Thus, the optimal quantity is $Q^{N*} = K$, and the corresponding price is $p^{N*} = 1-K$.

ii) If $\frac{1-c}{2} \le K \le Q_0^N$, then $\pi^N(Q, p^{N*}(Q)) = \pi_1^N(Q, 1-Q)$ for all $Q \in [0, K]$. Given the concavity of $\pi_1(Q, 1-Q)$, the optimal quantity is obtained at the stationary point $Q^{N*} = \frac{1-c}{2}$. The corresponding price is $p^{N*} = 1 - Q^{T*} = \frac{1+c}{2}$.

iii) If $Q_0^N \le K \le \frac{1-c}{2}$ or $Q_0^N \le \frac{1-c}{2} \le K$, $\pi^N(Q, p^{N*}(Q))$ always increases in Q for $Q \in [0, K]$. Thus, the optimal quantity is $Q^{N*} = K$. The corresponding price is $p^{N*} = p_2^{N*}(K) = \frac{1+(\beta-\delta)K}{2}$.

iv) If $\frac{1-c}{2} \le Q_0^N \le K$, the optimal quantity can be $\frac{1-c}{2}$ or K, depending on the relative sizes of $\pi_1^N(\frac{1-c}{2}, \frac{1+c}{2})$ and $\pi_2^N(K, p_2^{N*}(K))$. Let $\Delta^N \doteq \pi_2^N(K, p_2^{N*}(K)) - \pi_1^N(\frac{1-c}{2}, \frac{1+c}{2})$. We have

$$\begin{aligned} \frac{\partial \Delta^N}{\partial \beta} &= \frac{(1-r)[K^2(1+\beta-\delta)^2 - (1-K)^2]}{4(1+\beta-\delta)^2} \\ &\ge \frac{(1-r)[(Q_0^N)^2(1+\beta-\delta)^2 - (1-Q_0^N)^2]}{4(1+\beta-\delta)^2} = 0, \end{aligned}$$

where the inequality follows from the condition $K \ge Q_0^N$ and the second equality follows from the expression of Q_0^N. Thus, Δ^N increases in β, and there exists a threshold of β, denoted

by $\hat{\beta}_{s0} \doteq \frac{\left(\sqrt{4cK+(1-c)^2}+2K+c-1\right)^2}{4K^2} + \delta - 1$, such that $\pi_1^N(\frac{1-c}{2}, \frac{1+c}{2}) > \pi_2^N(K, p_2^{N*}(K))$ if $\lambda < \hat{\lambda}_{s0}$ and $\pi_1^N(\frac{1-c}{2}, p_1^*(\frac{1-c}{2})) \le \pi_2^N(K, p_2^{N*}(K))$ otherwise.

Defining a threshold $\hat{\beta}_{i0} \doteq \frac{1-(2-\delta)K}{K}$ such that $K = Q_0^N$ when $\beta = \hat{\beta}_{i0}$, we have $K \ge Q_0^T$ if and only if $\beta \ge \hat{\beta}_{i0}$. Combining the above cases, when $0 < K < \frac{1-c}{2}$, there exists a threshold $\hat{\beta}_{i0}$ such that $(Q^{N*}, p^{N*}) = (K, 1-K)$ if $\beta < \hat{\beta}_{i0}$ and $(Q^{N*}, p^{N*}) = (K, \frac{1+(\beta-\delta)K}{2})$ otherwise; when $K \ge \frac{1-c}{2}$, there exists a threshold $\hat{\beta}_{s0}$ such that $(Q^{N*}, p^{N*}) = (\frac{1-c}{2}, \frac{1+c}{2})$ if $\beta < \hat{\beta}_{s0}$ and $(Q^{N*}, p^{N*}) = (K, \frac{1+(\beta-\delta)K}{2})$ otherwise.

In this paper, we focus on the case where $\beta < \max\{\hat{\beta}_{s0}, \hat{\beta}_{i0}\}$, such that donation does not arise without policy support. So the optimal results in Scenario N are as follows:

$$(Q^{N*}, p^{N*}) = \begin{cases} (K, 1-K), & K < \frac{1-c}{2}, \\ (\frac{1-c}{2}, \frac{1+c}{2}), & K \ge \frac{1-c}{2}. \end{cases}$$

The firm's optimal profit is given by

$$\pi^{N*} = \begin{cases} K(1-r)(1-c-K), & K < \frac{1-c}{2}, \\ \frac{1}{4}(1-r)(1-c)^2, & K \ge \frac{1-c}{2}. \end{cases}$$

Given the firm's optimal decisions the consumer surplus is as follows:

$$CS^N = \begin{cases} \frac{K^2}{2}, & K < \frac{1-c}{2}, \\ \frac{1}{8}(1-c)^2, & K \ge \frac{1-c}{2}. \end{cases}$$

(b) In **Scenario T**, the firm's profit is given by

$$\begin{aligned} \pi^T(Q,p) =& (1-r)(p\min\{D_b, Q\} - cQ) + r\lambda c(Q - D_b)^+ \\ =& \begin{cases} \pi_1^T(Q,p) \doteq (1-r)(p-c)Q, & p \le 1-Q, \\ \pi_2^T(Q,p) \doteq (1-r)(pD_b(Q,p) - cQ) + r\lambda c(Q - D_b(Q,p)), & p > 1-Q. \end{cases} \end{aligned}$$

To maximize π^T, we take a two-step approach by first solving for the optimal p for a given Q and then solving for the optimal Q.

When $p \le 1-Q$, we obtain $\frac{\partial \pi_1^T(Q,p)}{\partial p} = (1-r)Q > 0$. Thus, given the condition $p \le 1-Q$, it is (locally) optimal for the firm to set the price at $p_1^*(Q) = 1-Q$, so we have $D_b(Q, p_1^*(Q)) = Q$. When $p > 1-Q$, we obtain $\frac{\partial^2 \pi_2(Q,p)}{\partial p^2} = -\frac{2(1-r)}{1-\delta} < 0$, so there exists a stationary price, denoted by $p_2^{T*}(Q) = \frac{(1-r)(1+(\beta-\delta)Q)+rc\lambda}{2(1-r)}$, and we have $D_b(Q, p_2^{T*}(Q)) = \frac{(1-r)(1+(\beta-\delta)Q)-c\lambda r}{2(1+\beta-\delta)(1-r)}$. This shows that $\pi^T(Q,p)$ increases in p for $p \le 1-Q$ and is concave in p for $p > 1-Q$. By comparing $p_2^{T*}(Q)$ and $p_1^*(Q) = 1-Q$, we have two cases. If $p_2^{T*}(Q) \le 1-Q$, or equivalently, $Q \le Q_0^T \doteq \frac{1-r-rc\lambda}{(1-r)(2+\beta-\delta)}$, the optimal price is $p^{T*}(Q) = 1-Q$ and all products are sold without donation; if $p_2^{T*}(Q) > 1-Q$, or equivalently, $Q > Q_0^T$, the optimal price is $p^{T*}(Q) = p_2^{T*}(Q) = \frac{(1-r)(1+(\beta-\delta)Q)+rc\lambda}{2(1-r)}$ and the firm donates a positive amount of products.

Next, we solve for the optimal Q. Substituting $p^{T*}(Q)$ into the profit function, we obtain

$$\pi^T(Q, p^{T*}(Q)) = \begin{cases} \pi_1^T(Q, 1-Q), & Q \le Q_0^T, \\ \pi_2^T(Q, p_2^{T*}(Q)), & Q > Q_0^T. \end{cases}$$

The firm aims to maximize $\pi^T(Q, p^{T*}(Q))$ by choosing the optimal Q, subject to the capacity constraint $0 \le Q \le K$. To solve this problem, we first obtain the second derivative as follows:

$$\frac{\partial^2 \pi^T(Q, p^{T*}(Q))}{\partial Q^2} = \begin{cases} -2(1-r), & Q \le Q_0^T, \\ \frac{(\delta-\beta)^2(1-r)}{2(1+\beta-\delta)}, & Q > Q_0^T. \end{cases}$$

It can also be shown that $\frac{\partial \pi^T(Q, p^{T*}(Q))}{\partial Q}|_{Q \nearrow Q_0^T} = \frac{\partial \pi^T(Q, p^{T*}(Q))}{\partial Q}|_{Q \searrow Q_0^T}$. Thus, the objective function $\pi^T(Q, p^{T*}(Q))$ is concave in Q for $Q \le Q_0^T$, convex in Q for $Q > Q_0^T$, and smooth everywhere. Solving

$\frac{\partial \pi_1^T(Q,p_1^*(Q))}{\partial Q}=0$, we have a stationary point $Q=\frac{1-c}{2}$. Given these results and the constraint $Q\le K$, we consider the following cases.

i) If $K\le Q_0^T\le\frac{1-c}{2}$ or $K\le\frac{1-c}{2}\le Q_0^T$, then $\pi(Q,p^{T*}(Q))=\pi_1^T(Q,1-Q)$ always increases in Q for $Q\in[0,K]$. Thus, the optimal quantity is $Q^{T*}=K$, and the corresponding price is $p^{T*}=1-K$.

ii) If $Q_0^T\le K\le\frac{1-c}{2}$ or $Q_0^T\le\frac{1-c}{2}\le K$, then $\pi^T(Q,p^{T*}(Q))=\pi_1^T(Q,1-Q)$ for any $Q\in[0,Q_0^T]$ and $\pi^T(Q,p^{T*}(Q))=\pi_2^T(Q,p_2^{T*}(Q))$ for any $Q\in[Q_0^T,K]$. In addition, $\pi^T(Q,p^{T*}(Q))$ always increases in Q for $Q\in[0,K]$. Thus, the optimal quantity is $Q^{T*}=K$. The corresponding price is $p^{T*}=p_2^{T*}(K)=\frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)}$.

iii) If $\frac{1-c}{2}\le K\le Q_0^T$, then $\pi^T(Q,p^{T*}(Q))=\pi_1^T(Q,1-Q)$ for all $Q\in[0,K]$. Given the concavity of $\pi_1^T(Q,1-Q)$, the optimal quantity is obtained at the stationary point $Q^{T*}=\frac{1-c}{2}$. The corresponding price is $p^{T*}=1-Q^{T*}=\frac{1+c}{2}$.

iv) If $\frac{1-c}{2}\le Q_0^T\le K$, then $\pi^T(Q,p^{T*}(Q))=\pi_1^T(Q,1-Q)$ for any $Q\in[0,Q_0^T]$ and $\pi^T(Q,p^{T*}(Q))=\pi_2^T(Q,p_2^{T*}(Q))$ for any $Q\in[Q_0^T,K]$. In this case, the optimal quantity can be $\frac{1-c}{2}$ or K, depending on the relative sizes of $\pi_1^T(\frac{1-c}{2},\frac{1+c}{2})$ and $\pi_2^T(K,p_2^{T*}(K))$. Let $\Delta^T\doteq\pi_2^T(K,p_2^{T*}(K))-\pi_1^T(\frac{1-c}{2},\frac{1+c}{2})$. We have

$$\begin{aligned}\frac{\partial\Delta^T}{\partial\lambda}=&\frac{cr[\lambda cr+(1-r)((2+\beta-\delta)K-1)]}{2(1+\beta-\delta)(1-r)}\\ \ge&\frac{cr[\lambda cr+(1-r)((2+\beta-\delta)Q_0^T-1)]}{2(1+\beta-\delta)(1-r)}=0,\end{aligned}$$

where the inequality follows from the condition $K\ge Q_0^T$ and the second equality follows from the expression of Q_0^T. Thus, Δ^T increases in λ, and there exists a threshold of λ, denoted by $\hat\lambda_{s0}\doteq\frac{(1-r)[1-(2+\beta-\delta)K+(2K+c-1)\sqrt{1+\beta-\delta}]}{cr}$, such that $\pi_1^T(\frac{1-c}{2},\frac{1+c}{2})>\pi_2^T(K,p_2^{T*}(K))$ if $\lambda<\hat\lambda_{s0}$ and $\pi_1^T(\frac{1-c}{2},p_1^*(\frac{1-c}{2}))\le\pi_2^T(K,p_2^{T*}(K))$ otherwise.

It is convenient to use K and λ to characterize the firm's optimal decisions. Defining a threshold $\hat\lambda_{i0}\doteq\frac{(1-r)[1-(2+\beta-\delta)K]}{cr}$ such that $K=Q_0^T$ when $\lambda=\hat\lambda_{i0}$, we have $K\ge Q_0^T$ if and only if $\lambda\ge\hat\lambda_{i0}$. Combining the above cases, when $0<K<\frac{1-c}{2}$, there exists a threshold $\hat\lambda_{i0}$ such that $(Q^{T*},p^{T*})=(K,1-K)$ if $\lambda<\hat\lambda_{i0}$ and $(Q^{T*},p^{T*})=(K,\frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)})$ otherwise; when $K\ge\frac{1-c}{2}$, there exists a threshold $\hat\lambda_{s0}$ such that $(Q^{T*},p^{T*})=(\frac{1-c}{2},\frac{1+c}{2})$ if $\lambda<\hat\lambda_{s0}$ and $(Q^{T*},p^{T*})=(K,\frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)})$ otherwise. Since $\hat\lambda_{s0}\ge\hat\lambda_{i0}$ if and only if $K\ge\frac{1-c}{2}$, we now summarize these results as follows:

$$(Q^{T*},p^{T*})=\begin{cases}(K,1-K), & K<\frac{1-c}{2}\text{ and }\lambda<\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}=\hat\lambda_{i0},\\ (\frac{1-c}{2},\frac{1+c}{2}), & K\ge\frac{1-c}{2}\text{ and }\lambda<\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}=\hat\lambda_{s0},\\ (K,\frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)}), & \lambda\ge\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}.\end{cases}$$

The associated donation fulfillment rate is given by

$$R^{T*}=\begin{cases}0, & \lambda<\max\{\hat\lambda_{s0},\hat\lambda_{i0}\},\\ \frac{(1-r)[K(2+\beta-\delta)-1]+rc\lambda}{(1-r)[1+(2-K)(\beta-\delta)]+rc\lambda}, & \lambda\ge\max\{\hat\lambda_{s0},\hat\lambda_{i0}\},\end{cases}$$

and the firm's optimal profit is given by

$$\pi^{T*}=\begin{cases}K(1-r)(1-c-K), & K<\frac{1-c}{2}\text{ and }\lambda<\hat\lambda_{i0},\\ \frac{1}{4}(1-r)(1-c)^2, & K\ge\frac{1-c}{2}\text{ and }\lambda<\hat\lambda_{s0},\\ \frac{c^2\lambda^2r^2+(1-r)^2[1+(\beta-\delta)K]^2-2c(1-r)\{K[2(1+\beta-\delta)+(\delta-\beta)(\lambda+2)r-2(\lambda+1)r]+r\lambda\}}{4(1+\beta-\delta)(1-r)}, & \lambda\ge\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}.\end{cases}$$

To ensure $D_b(Q^{T*},p^{T*})>0$, we require $\lambda<\bar\lambda\doteq\frac{(1-r)[1+(\beta-\delta)K]}{cr}$. Also, to ensure $\bar\lambda>\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}$, we require $\delta<\bar\delta\doteq\min\{\frac{(1-c)[2(2K+c-1)^++1-c]}{[(2K+c-1)^++1-c]^2}+\beta,1\}$. If $\beta<\max\{\hat\beta_{s0},\hat\beta_{i0}\}$, we have $\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}>0$; otherwise, $\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}\le0$, implying that the firm will always donate.

We now derive the consumer surplus given the firm's optimal decisions in Scenario T. In the case of $\lambda<\max\{\hat\lambda_{s0},\hat\lambda_{i0}\}$, we obtain

$$CS^T=\begin{cases}\frac{K^2}{2}, & K<\frac{1-c}{2},\\ \frac{1}{8}(1-c)^2, & K\ge\frac{1-c}{2}.\end{cases}$$

In the case of $\lambda \geq \max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$, $CS^T = \frac{c^2r^2\lambda^2 - 2(1+\beta-\delta)(1-r)cr\lambda + (1-r)^2\{1+(\beta-\delta)[2-K(4+(2+K)(\beta-\delta))]\}}{8(1+\beta-\delta)(1-r)^2}$. □

Proof of Proposition 2

For the effect of δ, recall that the feasible region of δ is $0 < \delta < \bar{\delta}$. There are two cases to consider depending on K. First, when $K \geq \frac{1-c}{2}$, $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\} = \hat{\lambda}_{s0}$ and $\bar{\delta} = \frac{(1-c)(4K+c-1)}{4K^2} + \beta$. In this case, we obtain $\frac{\partial \hat{\lambda}_{s0}}{\partial \delta} = \frac{(1-r)\left(2K\sqrt{1+\beta-\delta}+1-c-2K\right)}{2cr\sqrt{1+\beta-\delta}} > 0$, since $2K\sqrt{1+\beta-\delta}+1-c-2K > [2K\sqrt{1+\beta-\delta}+1-c-2K]_{\delta=\bar{\delta}} = 0$. Second, when $K < \frac{1-c}{2}$, $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\} = \hat{\lambda}_{i0}$ and $\bar{\delta} = 1$. In this case, we obtain $\frac{\partial \hat{\lambda}_{i0}}{\partial \delta} = \frac{K(1-r)}{cr} > 0$. Combining these two cases, we have shown that $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$ increases with δ.

For the effect of K, we obtain $\frac{\partial \hat{\lambda}_{s0}}{\partial K} = -\frac{(1-r)\left(1-\sqrt{1+\beta-\delta}\right)^2}{cr} < 0$, and $\frac{\partial \hat{\lambda}_{i0}}{\partial K} = -\frac{(2+\beta-\delta)(1-r)}{cr} < 0$. Thus, $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$ decreases with K.

For the effect of r, we obtain $\frac{\partial \hat{\lambda}_{s0}}{\partial r} = \frac{\left(1-\sqrt{1+\beta-\delta}\right)[(1-\sqrt{1+\beta-\delta})K-1]-c\sqrt{1+\beta-\delta}}{cr^2}$, which is convex in β due to $\frac{\partial^3 \hat{\lambda}_{s0}}{\partial r \partial \beta^2} = \frac{1+c+2K}{4cr^2(1+\beta-\delta)^{3/2}} > 0$. It is easy to show that $\frac{\partial \hat{\lambda}_{s0}}{\partial r}|_{\beta=0} < 0$ due to $(1-\sqrt{1+\beta-\delta})K - 1 < 0$, and $\frac{\partial \hat{\lambda}_{s0}}{\partial r}|_{\beta=\hat{\beta}_{s0}} = 0$. Thus, we have $\frac{\partial \hat{\lambda}_{s0}}{\partial r} < 0$. In addition, $\frac{\partial \hat{\lambda}_{i0}}{\partial r} = -\frac{1-(2+\beta-\delta)K}{cr^2} < 0$ for $\beta < \hat{\beta}_{i0}$. Thus, $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$ decreases with r.

For the effect of c, we obtain $\frac{\partial \hat{\lambda}_{s0}}{\partial c} = -\frac{(1-r)(1-\sqrt{1+\beta-\delta})\left[1-K\left(1-\sqrt{1+\beta-\delta}\right)\right]}{c^2r}$. If $\beta \leq \delta$, $1-\sqrt{1+\beta-\delta} \geq 0$, so $\frac{\partial \hat{\lambda}_{s0}}{\partial c}$ increases with K and $\frac{\partial \hat{\lambda}_{s0}}{\partial c} < \frac{\partial \hat{\lambda}_{s0}}{\partial c}|_{K=1} \leq 0$. If $\beta > \delta$, $1-\sqrt{1+\beta-\delta} < 0$, so $\frac{\partial \hat{\lambda}_{s0}}{\partial c}$ decreases with K and $\frac{\partial \hat{\lambda}_{s0}}{\partial c} > \frac{\partial \hat{\lambda}_{s0}}{\partial c}|_{K=1} > 0$. In addition, we have $\frac{\partial \hat{\lambda}_{i0}}{\partial c} = -\frac{(1-r)[1-(2+\beta-\delta)K]}{c^2r} < 0$ for $\beta < \hat{\beta}_{i0}$. Thus, $\max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$ increases with c if and only if $K \geq \frac{1-c}{2}$ and $\beta > \delta$. □

Proof of Proposition 3

In Scenario N, both p^{N*} and R^{N*} are independent of λ. In Scenario T, in the case of $\lambda < \max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$, both p^{T*} and R^{T*} are independent of λ. In the case of $\lambda \geq \max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$, it can be easily shown that $p^{T*} = \frac{(1-r)[1+(\beta-\delta)K]+rc\lambda}{2(1-r)}$ increases with λ. In addition, we have $R^{T*} = \frac{(1-r)[K(2+\beta-\delta)-1]+rc\lambda}{(1-r)[1+(2-K)(\beta-\delta)]+rc\lambda} = 1 - \frac{(1-r)(1-K)(1+\beta-\delta)}{(1-r)[1+(2-K)(\beta-\delta)]+rc\lambda}$, which increases with λ. □

Proof of Proposition 4

(a) When $K \geq (1-c)/2$, $p^{N*} = \frac{1+c}{2}$ and $p^{T*} = \begin{cases} \frac{1+c}{2}, & \lambda < \hat{\lambda}_{s0} \\ \frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)}, & \lambda \geq \hat{\lambda}_{s0} \end{cases}$. Thus, we obtain $p^{N*} = p^{T*}$ if $\lambda < \hat{\lambda}_{s0}$. If $\lambda \geq \hat{\lambda}_{s0}$, we obtain $p^{T*} - p^{N*} = \frac{c(\lambda r + r - 1) + (\beta-\delta)K(1-r)}{2(1-r)}$, which increases with λ. Solving $p^{T*} - p^{N*} = 0$ yields $\lambda = \lambda_{s1} \doteq \frac{(1-r)[c+(\delta-\beta)K]}{cr}$, below which we obtain $p^{T*} < p^{N*}$. In addition, we also have $\lambda_{s1} - \hat{\lambda}_{s0} = \frac{(1-\sqrt{1+\beta-\delta})(1-r)(2K+c-1)}{cr} > 0$ if and only if $\beta < \delta$. By comparing λ_{s1} and $\bar{\lambda}$, there exists a threshold of δ, denoted by $\delta = \frac{1-c}{2K}$, such that $\lambda_{s1} < \bar{\lambda}$ if $\delta < \frac{1-c}{2K}$, and $\lambda_{s1} \geq \bar{\lambda}$ otherwise. In summary, we have $p^{T*} < p^{N*}$ if $\hat{\lambda}_{s0} \leq \lambda < \min\{\lambda_{s1}, \bar{\lambda}\}$ and $p^{T*} \geq p^{N*}$ otherwise.

(b) When $K < (1-c)/2$, $p^{N*} = 1-K$ and $p^{T*} = \begin{cases} 1-K, & \lambda < \hat{\lambda}_{s0} \\ \frac{(1-r)(1+(\beta-\delta)K)+rc\lambda}{2(1-r)}, & \lambda \geq \hat{\lambda}_{s0} \end{cases}$. Thus, we have $p^{T*} = p^{N*}$ if $\lambda < \hat{\lambda}_{i0}$. If $\lambda \geq \hat{\lambda}_{s0}$, $p^{T*} - p^{N*} = \frac{c(\lambda r + r - 1) + (\beta-\delta)K(1-r)}{2(1-r)} > [p^{T*} - p^{N*}]_{\lambda=\hat{\lambda}_{i0}} = 0$. □

Proof of Proposition 5

When $\lambda < \hat{\lambda}_0 = \max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$, we have $CS^T = CS^N = \max\{\frac{K^2}{2}, \frac{(1-c)^2}{8}\}$.

When $\lambda \geq \hat{\lambda}_0$, we first show the monotonicity of CS^T with respects to δ. We obtain $\frac{C(\delta)}{8(1-r)^2(1+\beta-\delta)^3}$, the sign of which depends on its numerator $C(\delta) \doteq c^2\lambda^2r^2(1-\beta-\delta) + 2\beta c\lambda r(1-r)(3+\beta-\delta-2K) - (1-r)^2\{2K[\beta(3\beta+7)\delta - 3(\beta+1)\delta^2 - \beta(\beta(\beta+4)+7) + \delta^3 + 4\delta - 2] + \beta(2\beta - 2\delta + 5) - \delta + K^2[2\delta - (\beta-\delta)^2(3+\beta-\delta)] + 1\}$. We obtain $\frac{\partial^2 C(\delta)}{\partial \delta^2} = 6K(K+2)(1-r)^2(1+\beta-\delta) > 0$, so $C(\delta)$ is convex in δ. When $\delta = 0$, $C(0) = (1-\beta)c^2\lambda^2r^2 + 2\beta c\lambda r(1-r)(3+\beta-2K) + (1-r)^2\{(\beta+3)\beta^2K^2 - (2\beta+5)\beta + 2[\beta(\beta(\beta+4)+7)+2]K - 1\}$. Because $\frac{\partial C(0)}{\partial \lambda} = 2cr[(1-\beta)c\lambda r + \beta(1-r)(\beta-2K+3)] > 0$, $C(0)$ increases with λ. When $K < (1-c)/2$, we have $C(0)|_{\lambda=\hat{\lambda}_{i0}} = 4(\beta+1)^2K^2(1-r)^2 > 0$; when $K \geq (1-c)/2$, we have $C(0)|_{\lambda=\hat{\lambda}_{s0}} > C(0)|_{\lambda=\hat{\lambda}_{i0}} > 0$ since $C(0)$ increases with λ and $\hat{\lambda}_{s0} > \hat{\lambda}_{i0}$. Thus, we have $C(0) > 0$. When

$\delta = 1+\beta$, $C(1+\beta) = -2\beta[1-r(c\lambda+1)-K(1-r)]^2 > 0$. Given that $C(\delta)$ is convex in δ, $C(0) > 0$, and $C(1+\beta) < 0$, there is at most one zero point of $C(\delta)$ for $\delta \in (0, \bar{\delta})$ (due to $\bar{\delta} < 1+\beta$). Note that $C(\bar{\delta})$ can be either positive or negative. Therefore, CS^T first increases and then decreases with δ when $C(\bar{\delta}) < 0$, or always increases with δ when $C(\bar{\delta}) > 0$.

Because CS^N is independent of δ, we now compare the values of CS^T at $\delta = 0$ or at $\delta = \bar{\delta}$ with CS^N. When $\delta = 0$, $[CS^T - CS^N]_{\delta=0} = \frac{[(1-r)(\beta K+1)-c\lambda r]^2-(\beta+1)^2(1-r)^2\min\{1-c,2K\}^2}{8(\beta+1)^2(1-r)^2}$, the sign of which depends on $E(\lambda) = (1-r)(\beta K+1) - c\lambda r - (\beta+1)(1-r)\min\{1-c, 2K\}$, due to $(1-r)(\beta K+1) - c\lambda r > 0$. $E(\lambda)$ can be either positive or negative. In particular, in the case of $K \geq (1-c)/2$, $E(\hat{\lambda}_{s0}(\delta=0)) = (1+\beta-\sqrt{1+\beta})(1-r)(2K+c-1) > 0$ and $E(\bar{\lambda}(\delta=0)) = -(\beta+1)(1-c)(1-r) < 0$. When $\delta = \bar{\delta}$, we have $[CS^T - CS^N]_{\delta=\bar{\delta}} > 0$. $\frac{\partial(CS^T-CS^N)}{\partial\lambda}|_{\delta=\bar{\delta}} = \frac{c(1-K)r}{4\beta(1-r)} > 0$, so $[CS^T - CS^N]_{\delta=\bar{\delta}}$ increases with λ. Specifically, if $K < (1-c)/2$, $[CS^T - CS^N]_{(\delta=\bar{\delta},\lambda=\hat{\lambda}_{i0})} = 0$; if $K \geq (1-c)/2$, $[CS^T - CS^N]_{\delta=\bar{\delta}} > 0$.

Solving $CS^T = CS^N$, we obtain a unique solution, denoted by δ_1, which may be negative. Given $\lambda \geq \hat{\lambda}_0$, we have $CS^T < CS^N$ when $\delta < \max\{\delta_1, 0\}$ and $CS^T \geq CS^N$ otherwise. □

Proof of Lemma 2

(a) In Scenario T, for $\lambda \geq \hat{\lambda}_0$, $\frac{\partial SW^T}{\partial\lambda} = \frac{crD(\lambda)}{4(1-r)^2(1+\beta-\delta)^2}$, which has the same sign as $D(\lambda) \doteq c\lambda r[2\beta(1-r)+(1-\delta)(3-2r)]+(1-r)\{(\beta+4)\delta-2\beta-\delta^2+K[2(\beta-\delta)^2+5\beta-6\delta-2r(\beta-\delta+1)(\beta-\delta+2)+4]+2r(\beta-\delta+1)-3\}$, increasing with λ. Because $D(\bar{\lambda}) = (1-r)(1+\beta-\delta)[\delta+K(4(\beta+1)(1-r)-\delta(5-4r))]$ is linear in δ, $D(\bar{\lambda})|_{\delta=0} = 4(1-r)(1+\beta-\delta)(1+\beta)K(1-r) > 0$, and $D(\bar{\lambda})|_{\delta=1} = (1-r)(1+\beta-\delta)[1-K+4\beta K(1-r)] > 0$, $D(\bar{\lambda}) > 0$ holds for all $\delta \in (0, \bar{\delta})$.

Solving $D(\lambda) = 0$, we have a threshold of λ, denoted by $\lambda_1 \doteq \frac{(1-r)\{3+2\beta+\delta^2-(\beta+4)\delta-2r(1+\beta-\delta)+K[-2(\beta-\delta)^2-5\beta+6\delta+2r(\beta-\delta+1)(\beta-\delta+2)-4]\}}{cr[2\beta(1-r)+(1-\delta)(3-2r)]}$ such that $\frac{\partial SW^T}{\partial\lambda} > 0$ if $\lambda > \max\{\lambda_1, \hat{\lambda}_0\}$ and $\frac{\partial SW^T}{\partial\lambda} \leq 0$ otherwise.

(b) In Scenario T, for $\lambda \geq \hat{\lambda}_0$, we obtain $\frac{\partial TR^T}{\partial\lambda} = \frac{crF(\lambda)}{2(1-r)^2(1+\beta-\delta)}$, the sign of which depends on $F(\lambda) \doteq [1-K(2+\beta-\delta)](1-r)^2 - c\lambda(2-r)r$, decreasing with λ. When $K \geq (1-c)/2$, $F(\hat{\lambda}_{s0}) = (1-r)\{(1-r)[1-K(2+\beta-\delta)]-(2-r)[(2K+c-1)\sqrt{1+\beta-\delta}-K(2+\beta-\delta)+1]\}$. Because $\frac{\partial^2 F(\hat{\lambda}_{s0})}{\partial\delta^2} = \frac{(1-r)(2-r)(c+2K-1)}{4(\beta-\delta+1)^{3/2}} > 0$ and both $F(\hat{\lambda}_{s0})|_{\delta=0}$ and $F(\hat{\lambda}_{s0})|_{\delta=\bar{\delta}}$ are negative, $F(\hat{\lambda}_{s0}) < 0$. When $K < (1-c)/2$, $F(\hat{\lambda}_{i0}) = -(1-r)[1-K(2+\beta-\delta)] < 0$. Therefore, we obtain $F(\lambda) < F(\hat{\lambda}_0) < 0$ for all $K \in (0,1)$. □

Proof of Lemma 3

In Scenario T, we obtain $\frac{\partial^2 GO^T(\lambda)}{\partial\lambda^2} = \frac{c^2r^2[2(1-2\gamma)r(1+\beta-\delta)-2\beta(2-3\gamma)-(4-7\gamma)(1-\delta)]}{4(1+\beta-\delta)(1-r)^2}$, which is negative when $\gamma < \gamma_0 \doteq \frac{2(2-r)(1+\beta-\delta)}{6\beta+7(1-\delta)-4r(1+\beta-\delta)}$ and positive otherwise.

(a) When $\gamma < \gamma_0$, $GO^T(\lambda)$ has an extreme value point, that is, $\lambda_0 \doteq \frac{(1-r)\{\gamma[(\beta+6)\delta-4\beta-\delta^2-5]+2(1+\beta-\delta)(1-r+2r\gamma)-K[2\beta^2(1-2\gamma)+\beta(\gamma(8\delta-11)-4\delta+6)-2(2\gamma-1)((2-\delta)(1-\delta)+r(1+\beta-\delta)(2+\beta-\delta))]\}}{cr[2\beta(2-3\gamma)+(4-7\gamma)(1-\delta)-2(1-2\gamma)r(\beta-\delta+1)]}$.

Solving $\frac{\partial\lambda_0}{\partial\gamma} = 0$, we have one zero point, denote by $\beta = \beta_2 \doteq \frac{1}{4K(1-r)}\big\{2(1-\delta)-(2-\delta)r-K[3(1-\delta+\delta r)-4r]+\{2K[(1-r)(\delta^2(2-r)+6)+\delta((9-2r)r-8)]+K^2[\delta(1-r)(\delta(1-r)+6)+8r-7]+[2(1-\delta)-(2-\delta)r]^2\}^{1/2}\big\}$. When $\beta \leq \beta_2$, $\frac{\partial\lambda_0}{\partial\gamma} \leq 0$, and $\frac{\partial\lambda_0}{\partial\gamma} > 0$ otherwise. We have $\lambda_0|_{\gamma=0} = \frac{(1-r)^2[1-K(2+\beta-\delta)]}{c(2-r)r} < \hat{\lambda}_0 = \max\{\hat{\lambda}_{s0}, \hat{\lambda}_{i0}\}$.

(a1) If $\beta \leq \beta_2$, we obtain $\lambda_0 < \hat{\lambda}_0$ since λ_0 decreases with γ. Thus, the optimal tax incentive rate is given by $\lambda^* = \hat{\lambda}_0$ in the case of $\gamma < \gamma_0$.

(a2) If $\beta > \beta_2$, λ_0 increases with γ. In this case, we obtain $\lim_{\gamma\to\gamma_0}\lambda_0 = +\infty$. By solving $\lambda_0 = \bar{\lambda}$, we have $\gamma_3 \doteq \frac{K[6(\beta-\delta)+4r(\beta-\delta+1)+4]+2}{\delta+K[10\beta-11\delta-8r(\beta-\delta+1)+8]+2}$, above which $\lambda_0 > \bar{\lambda}$ and the optimal tax incentive rate is $\lambda^* = \bar{\lambda}$. Similarly, by solving $\lambda_0 = \hat{\lambda}_0$, we have $\gamma = \gamma_2$. If $\gamma < \gamma_2$, we obtain $\lambda_0 < \hat{\lambda}_0$ and the optimal tax incentive rate is $\lambda^* = \hat{\lambda}_0$. If $\gamma \in (\gamma_2, \gamma_3)$, $\lambda_0 \in (\hat{\lambda}_0, \bar{\lambda})$ and the optimal tax incentive rate is $\lambda^* = \lambda_0$.

(b) When $\gamma = \gamma_0$, $GO^T(\lambda)$ increases with λ if $\beta > \beta_2$ and decreases otherwise. Thus, if $\beta > \beta_2$, $\lambda^* = \bar{\lambda}$; if $\beta \leq \beta_2$, then $\lambda^* = \hat{\lambda}_0$.

(c) When $\gamma > \gamma_0$, $GO^T(\lambda)$ is a convex quadratic function with respect to λ, so we need to compare $GO^T(\hat{\lambda}_0)$ with $GO^T(\bar{\lambda})$.

(c1) If $\beta \leq \beta_2$, we have $GO^T(\hat{\lambda}_0) > GO^T(\bar{\lambda})$ under $\gamma = \gamma_0$, as shown in Part (b). We then find that $[GO^T(\hat{\lambda}_0) - GO^T(\bar{\lambda})]_{\gamma=1}$ can be either positive or negative. So, there exists a threshold, denoted by γ_1

(by solving $GO^T(\hat{\lambda}_0) = GO^T(\bar{\lambda})$), such that the optimal tax incentive rate is $\lambda^* = \hat{\lambda}_0$ for $\gamma < \min\{\gamma_1, 1\}$ and $\lambda^* = \bar{\lambda}$ otherwise.

(c2) If $\beta > \beta_2$, the stationary point λ_0 increases with γ. In this case, we have $GO^T(\hat{\lambda}_{i0}) < GO^T(\bar{\lambda})$ for all $K \in (0,1)$. When $K < (1-c)/2$, $\lambda^* = \bar{\lambda}$; when $K \geq (1-c)/2$, we have $GO^T(\hat{\lambda}_{s0}) < GO^T(\bar{\lambda})$ because $GO^T(\lambda)$ is a convex quadratic function with respect to λ and $\hat{\lambda}_{s0} \in (\hat{\lambda}_{i0}, \bar{\lambda})$. Thus, the optimal tax incentive rate is $\lambda^* = \lambda_0$ for $\beta > \beta_2$.

Combining the above two cases, we obtain the results as shown in the lemma. □

Proof of Proposition 6

(a) When $K \geq (1-c)/2$, solving $GO^T(\hat{\lambda}_0) = GO^N$, we have one unique solution $\gamma = \gamma_{s1-1} \doteq \frac{2\left\{1-(1-c)\sqrt{1-\delta}+\left[\delta-2\left(1-\sqrt{1-\delta}\right)\right]K\right\}}{2+\delta-2(1-c)\sqrt{1-\delta}+3\left[\delta-2\left(1-\sqrt{1-\delta}\right)\right]K}$; solving $GO^T(\bar{\lambda}) = GO^N$, we have one unique solution $\gamma = \gamma_{s1-2} \doteq \frac{8K[r(c+\delta K-1)-\delta K+1]+2(1-c)^2 r}{4K[2c(2r-1)+\delta(1-4K)-4r(1-\delta K)+4]+(1-c)^2(4r-3)}$. As shown in Lemma 3, $\lambda^* = \hat{\lambda}_0$ if $\gamma < \gamma_{s2}$ and $\lambda^* = \bar{\lambda}$ otherwise, where $\gamma_{s2} = \gamma_1(\beta = 0) \doteq \frac{4K\left[(1-\delta)(1-r)+(2-r)\sqrt{1-\delta}-1\right]-2(2-r)(1-c)\sqrt{1-\delta}+4}{(7-4r)(2K+c-1)\sqrt{1-\delta}-2(1-\delta)[1-4K(1-r)]+6(1-K)}$ under $K \geq (1-c)/2$. Therefore, when $\gamma < \gamma_{s2}$, $GO^N > GO^T(\hat{\lambda}_0)$ if $\gamma < \min\{\gamma_{s2}, \gamma_{s1-1}\}$; when $\gamma > \gamma_{s2}$, $GO^N > GO^T(\bar{\lambda})$ if $\gamma_{s2} < \gamma < \max\{\gamma_{s2}, \gamma_{s1-2}\}$.

We now prove by contradiction that γ_{s2} will never lie between γ_{s1-1} and γ_{s1-2}. Suppose $\gamma_{s1-2} < \gamma_{s2} < \gamma_{s1-1}$. When $\gamma_{s1-2} < \gamma < \gamma_{s2}$, we have $GO^T(\hat{\lambda}_0) > GO^T(\bar{\lambda}) > GO^N$. However, $GO^T(\bar{\lambda}) < GO^N$ due to $\gamma < \gamma_{s1-1}$, which is a contradiction. Similarly, we can show that $\gamma_{s1-1} < \gamma_{s2} < \gamma_{s1-2}$ does not hold.

Combining the above results, we obtain $GO^N > \max\{GO^T(\hat{\lambda}_0), GO^T(\bar{\lambda})\}$ if $\gamma \in \Lambda = \{\gamma | (\gamma < \min\{\gamma_{s2}, \gamma_{s1-1}\}) \bigcap (\gamma_{s2} < \gamma < \max\{\gamma_{s2}, \gamma_{s1-2}\})\}$. If $\gamma_{s2} \leq \gamma_{s1-1}$, $\max\{\gamma_{s2}, \gamma_{s1-2}\} = \gamma_{s1-2}$ (since $\gamma_{s1-2} < \gamma_{s2} < \gamma_{s1-1}$ does not hold), so we obtain $\Lambda = \{\gamma | \gamma < \gamma_{s1-2}\}$; if $\gamma_{s2} > \gamma_{s1-1}$, $\max\{\gamma_{s2}, \gamma_{s1-2}\} = \gamma_{s2}$ (since $\gamma_{s1-1} < \gamma_{s2} < \gamma_{s1-2}$ does not hold), so we obtain $\Lambda = \{\gamma | \gamma < \gamma_{s1-1}\}$. Defining $\gamma_{s1} = \begin{cases} \gamma_{s1-1}, & \gamma_{s2} > \gamma_{s1-1} \\ \gamma_{s1-2}, & \gamma_{s2} \leq \gamma_{s1-2} \end{cases}$, we have $GO^T(\lambda^*) < GO^N$ for $\gamma < \gamma_{s1}$ and $GO^T(\lambda^*) \geq GO^N$ otherwise.

(b) When $K < (1-c)/2$, because $GO^T(\hat{\lambda}_0) = GO^N$, the government is better off without the tax incentive (corresponding to Scenario N) if $\gamma < \gamma_{i1}$, where $\gamma_{i1} = \gamma_1(\beta = 0) \doteq \frac{2(1-Kr)-2\delta K(1-r)}{\delta(1-4K)-4(1-\delta)Kr+K+2}$ under $K < (1-c)/2$. If $\gamma \geq \gamma_{i1}$, $\lambda^* = \bar{\lambda}$ and $GO^T(\bar{\lambda}) - GO^N = \frac{1}{2}K\{\gamma(2+\delta) + K[2(1-2\gamma)\delta + \gamma + 2(1-2\gamma)(1-\delta)r] - 2\}$. Because $\frac{\partial(GO^T(\bar{\lambda}) - GO^N)}{\partial \gamma} = \frac{1}{2}K[\delta(1-4K) - 4(1-\delta)Kr + K + 2]$ increases with δ and $\frac{\partial(GO^T(\bar{\lambda}) - GO^N)}{\partial \gamma}|_{\delta=0} = \frac{1}{2}K(K+2-4Kr) > 0$, $GO^T(\bar{\lambda}) - GO^N$ increases with γ. Due to $[GO^T(\bar{\lambda}) - GO^N]_{\gamma=\gamma_{i1}} = 0$, we obtain $GO^T(\bar{\lambda}) > GO^N$ for all $\gamma \geq \gamma_{i1}$. □

Proof of Proposition 7

In the competitive fringe model, the dominant firm faces residual demand after the fringe supply $Q_f(p) = \sigma(p - \beta\tilde{\psi})$ is absorbed. The residual buying demand is $D_b = 1 - v^* - Q_f(p)$, while the donation demand remains as $D_d = v^*$. Following the same rational expectations equilibrium derivation as in the base model, the donation quantity is $\tilde{\psi}^* = (Q - \tilde{D}_b)^+$ and the indifference condition is $\tilde{v}^* + \beta\tilde{\psi}^* - p = \tilde{R}\delta\tilde{v}^*$. Solving for $\tilde{v}^*$ in two cases—with and without donation—yields the residual buying demand:

$$D_b(Q,p) = 1 - v^* - \sigma(p - \beta\psi) = \begin{cases} 1-(1+\sigma)p, & p \leq \frac{1-Q}{1+\sigma}, \\ \frac{1-(1+\sigma)p+((1+\sigma)\beta-\delta)Q}{1+(1+\sigma)\beta-\delta}, & p > \frac{1-Q}{1+\sigma}. \end{cases}$$

In Scenario N, the firm's profit is given by

$$\pi^N(Q,p) = (1-r)(p\min\{D_b, Q\} - cQ) = \begin{cases} \pi_1^N(Q,p) \doteq (1-r)(p-c)Q, & p \leq \frac{1-Q}{1+\sigma}, \\ \pi_2^N(Q,p) \doteq (1-r)(pD_b(Q,p) - cQ), & p > \frac{1-Q}{1+\sigma}. \end{cases}$$

In Scenario T, the firm's profit is given by

$$\begin{aligned} \pi^T(Q,p) =& (1-r)(p\min\{D_b, Q\} - cQ) + r\lambda c(Q - D_b)^+ \\ =& \begin{cases} \pi_1^T(Q,p) \doteq (1-r)(p-c)Q, & p \leq \frac{1-Q}{1+\sigma}, \\ \pi_2^T(Q,p) \doteq (1-r)(pD_b(Q,p) - cQ) + r\lambda c(Q - D_b(Q,p)), & p > \frac{1-Q}{1+\sigma}. \end{cases} \end{aligned}$$

The proof of the firm's optimal decisions follows the same two-step approach as in Proposition 1, that is, first optimizing p for a given Q, then optimizing Q subject to $Q \le K$, with σ entering multiplicatively throughout. The key structural properties carry over: $\pi^T(Q,p)$ increases in p for $p \le \frac{1-Q}{1+\sigma}$ and is concave in p for $p > \frac{1-Q}{1+\sigma}$; the reduced profit $\pi^T(Q, p^{T*}(Q))$ is concave–convex in Q with a smooth transition at the donation threshold $Q_0^T = \frac{1}{2+(1+\sigma)\beta-\delta}$. Following the same analysis, we obtain the results as given in the proposition. The policy-effective threshold is given by $\hat{\lambda}_0^{(\mathrm{cf})} = \max\left\{\hat{\lambda}_{s0}^{(\mathrm{cf})}, \hat{\lambda}_{i0}^{(\mathrm{cf})}\right\}$, where $\hat{\lambda}_{s0}^{(\mathrm{cf})} = \frac{(1-r)\left[1-(2+(1+\sigma)\beta-\delta)K+(2K+(1+\sigma)c-1)\sqrt{1+(1+\sigma)\beta-\delta}\right]}{(1+\sigma)cr}$, and $\hat{\lambda}_{i0}^{(\mathrm{cf})} = \frac{(1-r)\left[1-(2+(1+\sigma)\beta-\delta)K\right]}{(1+\sigma)cr}$. When $\sigma = 0$, these reduce to the thresholds $\hat{\lambda}_{s0}$ and $\hat{\lambda}_{i0}$ in Proposition 1. □

Proof of Proposition 8

In Scenario T, we obtain $\frac{\partial^2 GO^T(\lambda)}{\partial \lambda^2} = \frac{c^2 r^2\{7\gamma-4+2r(1-2\gamma)+\sigma[7\gamma-4+r(1-2\gamma)(3-\delta)]\}}{4(1-r)^2(1-\delta)}$, which is negative when $\gamma < \gamma_0^{(\mathrm{cf})} \doteq \frac{4(\sigma+1)-r[2+\sigma(3-\delta)]}{7(\sigma+1)-2r[2+\sigma(3-\delta)]}$ and positive otherwise. When $\gamma \le \gamma_0^{(\mathrm{cf})}$, $GO^T(\lambda)$ decreases with λ for $\lambda \in [\hat{\lambda}_0^{(\mathrm{cf})}, \bar{\lambda}^{(\mathrm{cf})})$ if $\sigma < \frac{2[\delta K+(1-2K)(1-\sqrt{1-\delta})]}{2(K-\delta)+(1+\delta)(1-2K)\sqrt{1-\delta}}$. When $\gamma > \gamma_0^{(\mathrm{cf})}$, $GO^T(\lambda)$ increases with λ for $\lambda \in [\hat{\lambda}_0^{(\mathrm{cf})}, \bar{\lambda}^{(\mathrm{cf})})$. Thus, the candidate solutions of λ^* in Scenario T are 0, $\hat{\lambda}_0^{(\mathrm{cf})}$ and $\bar{\lambda}^{(\mathrm{cf})}$. We now compare $GO^T(\hat{\lambda}_0^{(\mathrm{cf})})$, $GO^T(\bar{\lambda}^{(\mathrm{cf})})$ and GO^N, all of which are linear in γ.

(a) When $K \ge (1-c)/2$, since $[GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^N]_{\gamma=0} < 0$ and $[GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^N]_{\gamma=1} > 0$, there exists a threshold γ_{shatN} such that $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) > GO^N$ if and only if $\gamma > \gamma_{shatN}$. Because $[GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N]_{\gamma=0} < 0$, while $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ can increase or decrease linearly with γ and can be either positive or negative at $\gamma = 1$, there exists a threshold γ_{sbarN} such that $GO^T(\bar{\lambda}^{(\mathrm{cf})}) > GO^N$ if and only if $\gamma > \gamma_{sbarN}$; $\gamma_{sbarN} = \max\{\min\{\gamma^*_{sbarN}, 1\}, \gamma_0^{(\mathrm{cf})}\}$ if $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ increases with γ, and $\gamma_{sbarN} = 1$ if $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ decreases with γ, where γ^*_{sbarN} solves $GO^T(\bar{\lambda}^{(\mathrm{cf})}) = GO^N$. Because $[GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^T(\bar{\lambda}^{(\mathrm{cf})})]_{\gamma=\gamma_0^{(\mathrm{cf})}} > 0$, while $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^T(\bar{\lambda}^{(\mathrm{cf})})$ can increase or decrease linearly with γ and can be either positive or negative at $\gamma = 1$, there exists a threshold $\gamma_{shatbar}$ such that $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) > GO^T(\bar{\lambda}^{(\mathrm{cf})})$ if and only if $\gamma < \gamma_{shatbar}$; $\gamma_{shatbar} = \min\{\gamma^*_{sbarN}, 1\}$ if $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^T(\bar{\lambda}^{(\mathrm{cf})})$ increases with γ, and $\gamma_{shatbar} = 1$ if $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) - GO^T(\bar{\lambda}^{(\mathrm{cf})})$ decreases with γ, where $\gamma^*_{shatbar}$ solves $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) = GO^T(\bar{\lambda}^{(\mathrm{cf})})$. These three thresholds have six potential orderings.

(ai) If $\gamma_{sbarN} < \gamma_{sbarhat} < \gamma_{shatN}$, then for any $\gamma \in (\gamma_{sbarN}, \gamma_{sbarhat})$, we have $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) > GO^T(\bar{\lambda}^{(\mathrm{cf})}) > GO^N$ and simultaneously $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) < GO^N$, a contradiction; hence this ordering cannot occur. By the same reasoning, the orderings $\gamma_{sbarN} < \gamma_{shatN} < \gamma_{sbarhat}$, $\gamma_{shatN} < \gamma_{sbarhat} < \gamma_{sbarN}$, and $\gamma_{sbarhat} < \gamma_{shatN} < \gamma_{sbarN}$ are likewise contradictory and cannot occur.

(aii) If $\gamma_{shatN} < \gamma_{sbarN} < \gamma_{sbarhat}$, then $\lambda^* = 0$ for $\gamma < \gamma_{shatN}$, $\lambda^* = \hat{\lambda}_0^{(\mathrm{cf})}$ for $\gamma_{shatN} \le \gamma < \gamma_{sbarhat}$, and $\lambda^* = \bar{\lambda}^{(\mathrm{cf})}$ for $\gamma > \gamma_{sbarhat}$.

(aiii) If $\gamma_{sbarhat} < \gamma_{sbarN} < \gamma_{shatN}$, then $\lambda^* = 0$ for $\gamma < \gamma_{sbarN}$, and $\lambda^* = \bar{\lambda}^{(\mathrm{cf})}$ for $\gamma \ge \gamma_{sbarN}$.

In summary, we have $\lambda^* = 0$ for $\gamma < \gamma_{s1}^{(\mathrm{cf})} \doteq \min\{\gamma_{shatN}, \gamma_{sbarN}\}$, $\lambda^* = \hat{\lambda}_0^{(\mathrm{cf})}$ for $\gamma_{s1}^{(\mathrm{cf})} \le \gamma < \gamma_{s2}^{(\mathrm{cf})} \doteq \max\{\gamma_{sbarN}, \gamma_{sbarhat}\}$, and $\lambda^* = \bar{\lambda}^{(\mathrm{cf})}$ for $\gamma \ge \gamma_{s2}^{(\mathrm{cf})}$.

(b) When $K < (1-c)/2$, since $GO^T(\hat{\lambda}_0^{(\mathrm{cf})}) = GO^N$, the candidate solutions of λ^* in Scenario T are 0 and $\bar{\lambda}^{(\mathrm{cf})}$. When $\gamma \le \gamma_0^{(\mathrm{cf})}$, we have $\lambda^* = 0$ since GO^T decreases with λ for $\lambda \in [\hat{\lambda}_0^{(\mathrm{cf})}, \bar{\lambda}^{(\mathrm{cf})})$. When $\gamma > \gamma_0^{(\mathrm{cf})}$, we have $[GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N]_{\gamma=\gamma_0^{(\mathrm{cf})}} < 0$, while $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ may increase or decrease linearly with γ and may be either positive or negative at $\gamma = 1$. Thus, there exists a threshold $\gamma_{i1}^{(\mathrm{cf})}$ such that $GO^T(\bar{\lambda}^{(\mathrm{cf})}) > GO^N$ if and only if $\gamma > \gamma_{i1}^{(\mathrm{cf})}$; $\gamma_{i1}^{(\mathrm{cf})} = \min\{\gamma^*_{ibarN}, 1\}$ if $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ increases with γ, and $\gamma_{i1}^{(\mathrm{cf})} = 1$ if $GO^T(\bar{\lambda}^{(\mathrm{cf})}) - GO^N$ decreases with γ, where $\gamma^*_{ihatbar}$ solves $GO^T(\bar{\lambda}^{(\mathrm{cf})}) = GO^N$.

Given $\beta = 0$, we have $\frac{\partial \hat{\lambda}_0^{(\mathrm{cf})}}{\partial \sigma} = \max\{-\frac{(1-r)[\delta K+\left(1-\sqrt{1-\delta}\right)(1-2K)]}{cr(\sigma+1)^2}, -\frac{(1-r)[1-(2-\delta)K]}{cr(\sigma+1)^2}\} < 0$ and $\frac{\partial \bar{\lambda}^{(\mathrm{cf})}}{\partial \sigma} = -\frac{(1-r)(1-\delta K)}{c(\sigma+1)^2 r} < 0$. □

Proof of Proposition 9

In Scenario E, the firm's problem is

$$\max_{p \ge p_g, Q \le K} \pi^E(Q,p) = (1-r)(p\min\{D_b(Q,p), Q\} - cQ) = \begin{cases} \pi_1^E(Q,p) \doteq (1-r)(p-c)Q, & p \le 1-Q, \\ \pi_2^E(Q,p) \doteq (1-r)(pD_b(Q,p) - cQ), & p > 1-Q. \end{cases}$$

We can show that $\pi_1^E(Q,p)$ increases with p, and $\pi_2^E(Q,p)$ is concave in p, with a stationary point $p=\frac{1-\delta Q}{2}$. If $Q\le 1-p_g$, the optimal price is $p^{E*}=p_g$, and $\pi^{E*}(Q)=\pi_1^E(Q,p_g)$. If $Q>1-p_g$, $p^{E*}(Q)=1-Q$ and $\pi^{E*}(Q)=\pi_1^E(Q,1-Q)$ for $\frac{1-\delta Q}{2}<1-Q$; $p^{E*}(Q)=\frac{1-\delta Q}{2}$ and $\pi^{E*}(Q)=\pi_2^E(Q,\frac{1-\delta Q}{2})$ for $1-Q\le\frac{1-\delta Q}{2}<p_g$; $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_2^E(Q,p_g)$ for $\frac{1-\delta Q}{2}\ge p_g$.

When $K\le 1-c_g$, we have $p^{E*}=p_g$, $Q^{E*}=K$, and $R^{E*}=0$.

When $K>1-c_g$, we have the following three cases.

(a) When $p_g\ge\frac{1-\delta}{2-\delta}$, $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_1^E(Q,p_g)$ for $Q<1-p_g$; $p^{E*}(Q)=1-Q$ and $\pi^{E*}(Q)=\pi_1^E(Q,1-Q)$ for $1-p_g\le Q<\frac{1}{2-\delta}$; $p^{E*}(Q)=\frac{1-\delta Q}{2}$ and $\pi^{E*}(Q)=\pi_2^E(Q,\frac{1-\delta Q}{2})$ for $Q\ge\frac{1}{2-\delta}$. The objective function $\pi^E(Q,p^{E*}(Q))$ increases in Q for $Q<1-p_g$, is concave in Q for $1-p_g\le Q<\frac{1}{2-\delta}$ (with an extreme value point $Q=\frac{1-c}{2}$), is convex in Q for $Q\ge\frac{1}{2-\delta}$, and is smooth everywhere.

(a1) When $K\le\frac{1}{2-\delta}$, we have $Q^{E*}=1-p_g$ and $p^{E*}=p_g$ if $p_g<\frac{1+c}{2}$; $Q^{E*}=\min\{\frac{1-c}{2},K\}$ and $p^{E*}=\max\{\frac{1+c}{2},1-K\}$ if $p_g\ge\frac{1+c}{2}$.

(a2) When $K>\frac{1}{2-\delta}$, we have $\frac{1-c}{2}<\frac{1}{2}<\frac{1}{2-\delta}$. If $p_g<\frac{1+c}{2}$, we need to compare $\pi_1^E(1-p_g,p_g)=(1-r)(p_g-c)(1-p_g)$ and $\pi_2^E(K,\frac{1-\delta K}{2})=\frac{(1-r)[(1-\delta K)^2-4c(1-\delta)K]}{4(1-\delta)}$. Because $\pi_1^E(1-p_g,p_g)$ increases with p_g for $p_g<\frac{1+c}{2}$ and $\pi_2^E(K,\frac{1-\delta K}{2})$ decreases with K, we have $\pi_1^E(1-p_g,p_g)\ge\pi_1^E(1-p_g,p_g)|_{p_g=\frac{1-\delta}{2-\delta}}=\pi_2^E(K,\frac{1-\delta K}{2})|_{K=\frac{1}{2-\delta}}>\pi_2^E(K,\frac{1-\delta K}{2})$. Thus, the optimal decisions are $Q^{E*}=1-p_g$ and $p^{E*}=p_g$. Similarly, if $p_g\ge\frac{1+c}{2}$, we have $\pi_2^E(\frac{1-c}{2},\frac{1+c}{2})=\frac{(1-r)(1-c)^2}{4}>\pi_2^E(K,\frac{1-\delta K}{2})$. So the optimal decisions are $Q^{E*}=\frac{1-c}{2}$ and $p^{E*}=\frac{1+c}{2}$.

(b) When $\frac{1-\delta}{2}\le p_g<\frac{1-\delta}{2-\delta}$, $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_1^{E*}(Q,p_g)$ for $Q<1-p_g$; $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_2^E(Q,p_g)$ for $1-p_g\le Q<\frac{1-2p_g}{\delta}$; $p^{E*}(Q)=\frac{1-\delta Q}{2}$ and $\pi^{E*}(Q)=\pi_2^E(Q,\frac{1-\delta Q}{2})$ for $Q\ge\frac{1-2p_g}{\delta}$. The objective function $\pi^E(Q,p^{E*}(Q))$ increases in Q for $Q<1-p_g$, decreases in Q for $1-p_g\le Q<\frac{1-2p_g}{\delta}$, is convex in Q for $Q\ge\frac{1-2p_g}{\delta}$, and is smooth everywhere.

(b1) When $K\le\frac{1-2p_g}{\delta}$, we have $Q^{E*}=1-p_g$ and $p^{E*}=p_g$.

(b2) When $K>\frac{1-2p_g}{\delta}$, we have $Q^{E*}=1-p_g$ and $p^{E*}=p_g$, due to $\pi_1^E(1-p_g,p_g)>\pi_2^E(K,\frac{1-\delta K}{2})$.

(c) When $p_g<\frac{1-\delta}{2}$, $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_1^E(Q,p_g)$ for $Q<1-p_g$; $p^{E*}=p_g$ and $\pi^{E*}(Q)=\pi_2^E(Q,p_g)$ for $Q\ge 1-p_g$. The objective function $\pi^E(Q,p^{E*}(Q))$ increases in Q for $Q<1-p_g$, decreases in Q for $Q\ge 1-p_g$, and is smooth everywhere. Thus, we have $Q^{E*}=1-p_g$ and $p^{E*}=p_g$. Combining the above cases, we obtain the results as shown in the proposition. □

Proof of Proposition 10

When $p_g\ge\max\{1-K,\frac{1+c}{2}\}$, the optimal decisions in Scenario E are the same as those in Scenario N, so the government's optimal donation tax policy decision is the same as in Proposition 6. When $p_g<\max\{1-K,\frac{1+c}{2}\}$,

$$GO^E=\min\left\{(p_g-c)K(r+\gamma-2r\gamma)+\frac{\gamma(1-p_g)^2}{2},\frac{1-p_g}{2}[(1-2c)\gamma+2c(2\gamma-1)r+p_g(\gamma+(2-4\gamma)r)]\right\}.$$

(a) When $K<\frac{1-c}{2}$, we have $GO^E=(p_g-c)K(r+\gamma-2r\gamma)+\frac{\gamma(1-p_g)^2}{2}$. Comparing GO^E with GO^N, we obtain $GO^E>GO^N$ if $\gamma>\gamma_{EN2}\doteq\frac{2Kr}{1-K-p_g+4Kr}$ and $GO^E\le GO^N$ otherwise. Because γ_{i1} decreases with δ and $\gamma_{EN1}<\gamma_{i1}|_{\delta=\bar\delta}$, where γ_{i1} denotes the threshold at which $GO^N=GO^T(\bar\lambda)$ (as established in the proof of Proposition 6), it follows that $\gamma_{EN2}<\gamma_{i1}$.

Next, we compare GO^E with $GO^T(\bar\lambda)$. Note that $GO^T(\bar\lambda)-GO^E$ is linear in γ and $[GO^T(\bar\lambda)-GO^E]_{\gamma=0}=-K[rp_g+(1-r)(1-\delta K)]<0$. If $GO^T(\bar\lambda)-GO^E$ increases with γ, then there exists a threshold, $\gamma_{ET\bar\lambda}=\frac{2K[rp_g+(1-r)(1-\delta K)]}{K[\delta+2(2r-1)p_g+4(1-r)]-(1-p_g)^2-4\delta K^2(1-r)}$, such that $GO^T(\bar\lambda)<GO^E$ if $\gamma<\min\{\gamma_{ET\bar\lambda},1\}$ and $GO^T(\bar\lambda)\ge GO^E$ otherwise. We further claim that $\gamma_{ET\bar\lambda}>\gamma_{i1}>\gamma_{EN2}$. Suppose otherwise and $\gamma_{ET\bar\lambda}\le\gamma_{i1}$. Then for any $\gamma\in[\gamma_{ET\bar\lambda},\gamma_{i1}]$, we must have $GO^N>GO^T(\bar\lambda)>GO^E$, which contradicts the established result that $GO^N<GO^E$ whenever $\gamma>\gamma_{EN2}$ (recall that $\gamma_{EN2}<\gamma_{i1}$). Hence $\gamma_{ET\bar\lambda}>\gamma_{i1}$. If $GO^T(\bar\lambda)-GO^E$ decreases with γ, then $GO^T(\bar\lambda)<GO^E$ holds for all $\gamma\in(0,1)$. For notational convenience, we set $\gamma_{ET\bar\lambda}=1$ in this case. Combining the two cases and letting $\gamma_{ET2}=\min\{\gamma_{ET\bar\lambda},1\}$, we obtain that $GO^T(\bar\lambda)<GO^E$ if $\gamma<\gamma_{ET2}$ and $GO^T(\bar\lambda)\ge GO^E$ otherwise, where $\gamma_{ET2}=\min\{\gamma_{ET\bar\lambda},1\}$.

Thus, when $K<\frac{1-c}{2}$, the government is better off with no intervention if $\gamma\le\gamma_{EN2}$, with direct market entry if $\gamma_{EN2}<\gamma\le\gamma_{ET2}$, and with the tax incentive if $\gamma>\gamma_{ET2}$.

(b) When $K \geq \frac{1-c}{2}$, solving $GO^E = GO^N$ yields $\gamma = \gamma_{EN1} \doteq \max\{\frac{2r[(1-c)^2-4K(p_g-c)]}{c[4r(c+4K-2)-3c-8K+6]+4p_g[p_g+2K(1-2r)-2]+4r+1}, \frac{2r(1+c-2p_g)}{4(1+c)r-3c+2(1-4r)p_g+1}\}$. Hence $GO^N > GO^E$ if $\gamma < \gamma_{EN1}$, and $GO^N \leq GO^E$ otherwise. Solving $GO^E = GO^T(\lambda^*)$ gives a single root γ_{ET1}, such that $GO^E > GO^T(\lambda^*)$ when $\gamma < \gamma_{ET1}$, and $GO^E \leq GO^T(\lambda^*)$ otherwise.

We further claim that $\gamma_{ET1} > \gamma_{s1} > \gamma_{EN1}$, and establish the two inequalities in turn. Recall that $\gamma_1(\beta = 0)$ denotes the threshold at which $GO^T(\hat{\lambda}_0) = GO^T(\bar{\lambda})$ (as characterized in Lemma 3(a)), and that $\gamma_1(\beta = 0)$ is decreasing in δ.

Step 1: $\gamma_{s1} > \gamma_{EN1}$. When $\gamma \geq \gamma_1(\beta = 0)$, the optimal tax rate is $\bar{\lambda}$, so γ_{s1} solves $GO^N = GO^T(\bar{\lambda})$. Since $\gamma_1(\beta = 0)$ is decreasing in δ and $\gamma_{EN1} < \gamma_1(\beta = 0)|_{\delta=\bar{\delta}}$, it follows that $\gamma_{s1} > \gamma_{EN1}$. When $\gamma < \gamma_1(\beta = 0)$, the optimal tax rate is $\hat{\lambda}_0$, so $\gamma_{s1} = \gamma_{s1-1}$, which solves $GO^N = GO^T(\hat{\lambda}_0)$. The minimum value of γ_{s1-1} over the feasible parameter range is $\frac{\sqrt{(1-K)p_f[2-(1+K)p_f]}+p_f(p_f+5K-2)+2}{p_f(p_f+6K-2)+4}$, which exceeds γ_{EN1}. Thus, we obtain $\gamma_{s1} > \gamma_{EN1}$.

Step 2: $\gamma_{ET1} > \gamma_{s1}$. Suppose otherwise and $\gamma_{ET1} \leq \gamma_{s1}$. Then for any $\gamma \in [\gamma_{ET1}, \gamma_{s1}]$, we must have $GO^N > GO^T(\lambda^*) > GO^E$, which contradicts the established result that $GO^N < GO^E$ whenever $\gamma > \gamma_{EN1}$ (recall that $\gamma_{EN1} < \gamma_{s1}$). Thus, we obtain $\gamma_{ET1} > \gamma_{s1}$.

Thus, when $K \geq \frac{1-c}{2}$, the government is better off in Scenario N if $\gamma \leq \gamma_{EN1}$, in Scenario E if $\gamma_{EN1} < \gamma \leq \gamma_{ET1}$, and in Scenario T if $\gamma > \gamma_{ET1}$.

Combining the above results, we obtain the results as shown in the proposition. □

Proof of Proposition EC.1

Under a price-based tax policy, the firm's tax benefit from donation is $r\lambda p\psi$, so the deduction scales with the firm's posted price p rather than the exogenous cost c. In Scenario N, the tax incentive is irrelevant, so the firm's optimal decisions are the same as in the base model. In Scenario T, the buying demand remains as in the base model, and the firm's profit is given by

$$\begin{aligned}\pi^T(Q,p) &= (1-r)(p\min\{D_b, Q\} - cQ) + r\lambda p(Q - D_b)^+ \\ &= \begin{cases} \pi_1(Q,p) = (1-r)(p-c)Q, & p \leq 1-Q, \\ \pi_2(Q,p) = (1-r)(pD_b(Q,p) - cQ) + r\lambda p(Q - D_b(Q,p)), & p > 1-Q. \end{cases}\end{aligned}$$

The proof of the firm's optimal decisions follows the same two-step approach as in Proposition 1 by first optimizing p for a given Q and then optimizing Q subject to $Q \leq K$. The key structural properties carry over: $\pi^T(Q,p)$ increases in p for $p \leq 1-Q$ and is concave in p for $p > 1-Q$; the reduced profit $\pi^T(Q, p^{T*}(Q))$ is concave–convex in Q with a smooth transition at the donation threshold $Q_0^T = \frac{1-r-r\lambda}{(1-r)(2+\beta-\delta)-r\lambda}$. Following the same analysis, we obtain the results stated in the proposition. The policy-effective threshold is given by $\hat{\lambda}_0^{(p)} = \max\{\hat{\lambda}_{s0}^{(p)}, \hat{\lambda}_{i0}^{(p)}\}$, where $\hat{\lambda}_{s0}^{(p)} = \frac{(1-r)\{(1+\beta-\delta)[(2K+c-1)\sqrt{4cK+(1-c)^2}-2K(2c+K-1)-(1-c)^2]+2(1-K)^2\}}{2(1-K)^2 r}$ and $\hat{\lambda}_{i0}^{(p)} = \frac{(1-r)[1-(2+\beta-\delta)K]}{(1-K)r}$. □

Proof of Proposition EC.2

When $v^* < 1$, the buying demand comprises direct buyers and unsuccessful donation seekers who return to the retail channel, $D_b = (1-v^*) + (1-R)(v^* - p + \beta\psi)$. Substituting $R = \psi/v^*$ and $v^* = (p - \beta\psi)/(1-\delta)$ reduces the returning term to $\delta\psi$, so $D_b = 1 - p + (\beta - \delta)\psi$, the same as the base model. Thus, from $\psi = (Q - D_b)^+$ we obtain the same $\psi = \max\{p + Q - 1, 0\}/(1 + \beta - \delta)$, and the demand and donation volume remain unchanged. The total consumer surplus $CS = \int_{v^*}^1 (v - p + \beta\psi)dv + \int_{p-\beta\psi}^{v^*} \big[R\delta v + (1 - R)(v - p + \beta\psi)\big]dv + \int_0^{p-\beta\psi} R\delta v dv$ can be similarly shown to be the same as in the base model. In equilibrium, $v^* < 1$ is equivalent to $\lambda < \lambda_f \doteq \frac{(1-r)\{1+\beta K-\delta[\beta(2-K)+3-K]+\delta^2(2-K)\}}{cr(1-\delta)}$. □

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