

Online Appendix

Humanitarian Aid Management under Corruptible Distribution and Oversight Agents
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A. Background on Corruption Events in Ukraine

[Ewins et al. \(2006\)](#) has identified variables that heighten the risk of corruption in the context of humanitarian assistance. Ukraine exhibits several of these risk factors: war, high level of pre-crisis corruption, weak anti-corruption measures, weak rule of law, high-value relief, low levels of transparency and accountability, low media coverage. In 2022, Ukraine’s score on the Corruption Perceptions Index (ranging from 0 to 100) by [Transparency International \(2023\)](#) was 33 points, compared to 69 points in the U.S. Additionally, the World Bank’s Control of Corruption Index in 2021 gave a score of -0.77 for Ukraine and 1.05 for the U.S. ([Varieties of Democracy 2022](#)).

We identified 163 corruption events within one year from the onset of the war. We categorized the 163 events by area and their impact are estimated in Table [A.1](#).

Examples of corruption events during the distribution phase are provided below:

- Diversion of resources. June 1, 2022. SSU exposes deputy head of Chernivtsi regional military administration on humanitarian aid scam ([Security Service of Ukraine 2022b](#)). The duties of the official included the distribution of ambulances that Ukraine received from Italy. He managed to carry out the ‘deal’ with the help of the director general of the medical institution. The head of the hospital re-registered 5 received ambulances to a controlled charity NGO (the founder and owner of which is he himself) to use the vehicles to provide paid transport services to patients. The pre-trial investigation is underway within the criminal proceedings under Article 191.5 (misappropriation, embezzlement of property through abuse of office) of the Criminal Code of Ukraine. Previously, the deputy head of the RMA, by his illegal actions, caused damage to the state totaling more than UAH 900,000 ([National Police of Ukraine 2022](#)).
- Sale of humanitarian aid. June 17, 2022. A resident of the city of Dnipro received a cargo of humanitarian aid from Poland – food for Ukrainians affected by Russian aggression. He decided to sell part of the humanitarian aid. To do this, he found an entrepreneur who agreed to buy 38,000 units of canned meat at half the market price. The suspect was detained while receiving UAH 1 million 350 thousand for the sale of humanitarian aid charged under Art. 201-2.3 of the Criminal Code of Ukraine ([Prosecutor General’s Office of Ukraine 2022](#)).
- Misappropriation and sale of humanitarian aid. August 30, 2022. SSU and NABU investigate possible misappropriation of humanitarian aid by top officials in Zaporizhzhia region ([Security Service of Ukraine 2022a](#)). NABU agent Evgeny Shevchenko released information about the scale of the theft of humanitarian aid ([Censor.NET 2022](#)). Top officials of the region (Chairman of the RMA, Secretary of the City Council, Deputy of the Regional Council, Deputy Chairman of the RMA) are suspected of stealing 22 sea containers, 389 railway cars, and 220 trucks with humanitarian aid in 6 months. Officials misappropriated almost all the humanitarian

Table A.1: Corruption events in Ukraine (February 24, 2022 to March 1, 2023)

Corruption area	Number of events	Impact (scale of 1 to 5)
Formation of humanitarian entities	32	3.5
Registration/use of bogus donors	1	
Creation of bogus NGO recipients	6	
Providing/receiving certificates for import of humanitarian aid	3	
Conspiracy of actors in the humanitarian chain	1	
Extortion from charitable organizations for entering data	1	
Granting illegal permits to travel outside Ukraine for transporting humanitarian aid	20	
Fundraising and funding for humanitarian aid	14	3.5
False documents about humanitarian needs	4	
Overestimation of needs	1	
Misappropriation or misuse of funds	9	
Transiting humanitarian aid cross borders	31	4.0
False documents for crossing the contact line to non-government controlled areas	2	
Extortion during customs clearance	1	
Exploiting humanitarian goods classification to evade customs taxes and fees	28	
Procurement	8	2.5
Supply of low-quality and/or expired goods	4	
Purchasing at inflated prices	4	
Upstream logistics	12	2.5
Extortion for transportation and storage of humanitarian aid	3	
Diversion of stock, vehicles, spare parts, fuel	2	
Storage of humanitarian aid in private warehouses	7	
Distribution	58	5.0
Misappropriation of humanitarian aid	9	
Sale of humanitarian aid for profit	33	
Diversion of resources away from delivering aid and service	6	
Extortion for transferring humanitarian aid to beneficiaries or coercion to pay	4	
Fraud by distribution agents	3	
Untimely distribution of humanitarian aid	3	
Accounting and auditing	8	3.0
Assets received as humanitarian aid are not recorded	2	
False or exaggerated reporting	1	
Compilation of fictitious acts of acceptance and transfer	5	
Total	163	

Notes: Impact indicates the extent to which a corruption event influences the humanitarian aid process. This encompasses the consequences on beneficiaries' protection, safety, and health, alongside the financial, reputational, and regulatory repercussions.

Data source: National Police of Ukraine (www.npu.gov.ua), Prosecutor General's Office of Ukraine (www.gp.gov.ua), Security Service of Ukraine (www.ssu.gov.ua), State Bureau of Investigation (www.dbr.gov.ua), National Anti-Corruption Bureau of Ukraine (www.nabu.gov.ua), Economic Security Bureau of Ukraine (esbu.gov.ua), State Border Guard Service of Ukraine (www.dpsu.gov.ua), and the Government Portal (www.kmu.gov.ua).

aid that came to the city and region and sold it through the ATB and Silpo retail chains. An investigation is currently underway under three articles of the CCU: 191.5 (appropriation, embezzlement of property or taking possession of it by abuse of office, committed in particularly

large amounts or by an organized group); 209.3 (legalization (laundering) of assets obtained through criminal means, committed by an organized group or in a particularly large amount); 368-5 (illegal enrichment) ([Security Service of Ukraine 2022a](#)).

- Sale of humanitarian aid. October 20, 2022. Almost 130 tons of flour arrived at one of the railway stations of Dnipropetrovsk region, from where it was delivered to local communities in the form of humanitarian aid. However, some local community leaders decided not to deliver the humanitarian aid to the people, but to sell the flour through proxies at below-market prices. [State Bureau of Investigation \(2022\)](#) documented the loading of 17 tons of flour from wagons into trucks, after which they tried to take it out for further sale without any permits.

B. Article 201 of the Criminal Code of Ukraine

Translated from source: <https://zakon.rada.gov.ua/laws/show/2155-20#Text>

THE LAW OF UKRAINE On amendments to the Criminal Code of Ukraine regarding liability for illegal use of humanitarian aid

The Verkhovna Rada of Ukraine **decrees:**

1. The Criminal Code of Ukraine (Reports of the Verkhovna Rada of Ukraine, 2001, No. 25-26, Article 131) shall be supplemented with Article 201 of the following content:

“Article 201 Illegal use for profit of humanitarian aid, charitable donations or free aid

1. The sale of goods (items) of humanitarian aid or the use of charitable donations, free aid or the conclusion of other transactions regarding the disposal of such property, for the purpose of obtaining profit, committed in a significant amount, –

shall be punished by a fine of two to three thousand tax-free minimum incomes of citizens or by corrective labor for a term of up to two years, or by restriction of freedom for a term of up to four years.

2. The same actions, committed repeatedly or by prior conspiracy by a group of persons, or by an official using an official position, or in a large amount, –

shall be punished by deprivation of liberty for a term of three to five years with deprivation of the right to hold certain positions or engage in certain activities for a term of up to three years and with confiscation of property.

3. Actions provided for by part one or two of this article, committed by an organized group or on a particularly large amount, or during a state of emergency or martial law, –

shall be punished by deprivation of liberty for a term of five to seven years, with deprivation of the right to hold certain positions or engage in certain activities for a term of up to three years, and with confiscation of property.

Note.

1. Under humanitarian aid, charitable donations and free aid should be understood the concepts defined by the Law of Ukraine “On Humanitarian Aid” and the Law of Ukraine “On Charitable Activities and Charitable Organizations”.

2. The actions provided for in this article are considered to have been committed in a significant amount, if the total value of such humanitarian aid, charitable donations or free aid exceeds three hundred and fifty times or more the tax-free minimum income of citizens; in a large amount - if the total value of goods, free assistance or monetary assistance exceeds a thousand or more times the tax-free minimum income of citizens; in a particularly large amount - if the total value of goods, free assistance or monetary assistance exceeds three thousand or more times the tax-free minimum income of citizens.”

2. This Law enters into force on the day following its publication.

President of Ukraine
Kyiv,
March 24, 2022
No. 2155-IX

V. ZELENSKY

C. Key Notations Used in the Paper

Table A.2: Key notations

Problem primitives	
F	= government agency’s financing for humanitarian aid and anti-corruption effort
N	= number of managers or sites
s	= cost per inspector inspecting a site
θ	= moral utility parameter of a manager
η	= moral utility parameter of an inspector
$z(g)$	= probability that a manager’s misappropriation fraction g is uncovered upon inspection
α	= monetary recovery rate
ν	= probability that a bribery is uncovered
p_D	= penalty rate to a manager who is found misappropriating
p_I	= penalty rate to an inspector who is found to have accepted a bribe
p_B	= penalty rate to a manager who is found to have given a bribe
λ	= manager’s bargaining power
Decision variables	
$g \in [0, 1]$	= fraction of allocated goods that a manager decides to misappropriate
$M \in \{0, 1, \dots, N\}$	= number of inspectors that the government agency decides to hire
$\beta \geq 0$	= reward rate for honest inspectors chosen by the government agency
Other notations	
$\pi(g; M, \beta)$	= a manager’s expected utility
$H(M) = \frac{F-sM}{N}$	= value of humanitarian aid received by a manager
$A(M, \beta)$	= expected amount of aid received by beneficiaries

D. Proofs

Proof of Lemma 1: We first prove that there exists a unique equilibrium bribe that maximizes the objective in (1). The objective is maximized if and only if any strictly increasing transformation of it is maximized. We apply logarithm since the objective is positive due to constraints (2)-(3):

$$\begin{aligned} \max_{b \geq 0} & (1 - \lambda) \log \left[b(1 - \nu - \nu p_I - \eta) - \beta gH(M) \right] \\ & + \lambda \log \left[(1 - \nu)(gH(M) - b) - \nu(p_D gH(M) + p_B b) + p_D gH(M) \right] \end{aligned} \quad (\text{A.1})$$

The objective in (A.1) is strictly concave in b , and thus the optimal solution can be uniquely determined by the first-order condition:

$$\frac{(1 - \lambda)(1 - \nu - \nu p_I - \eta)}{b(1 - \nu - \nu p_I - \eta) - \beta gH(M)} = \frac{\lambda(1 - \nu + \nu p_B)}{(1 - \nu)(gH(M) - b) - \nu(p_D gH(M) + p_B b) + p_D gH(M)}, \quad (\text{A.2})$$

which simplifies to

$$b^*(g, M, \beta) = \left[\frac{\lambda \beta}{1 - \nu - \nu p_I - \eta} + \frac{(1 - \lambda)(1 - \nu)(1 + p_D)}{1 - \nu + \nu p_B} \right] gH(M). \quad (\text{A.3})$$

We next derive conditions such that constraints (2) and (3) are satisfied. Constraint (2) says that $b(1 - \nu - \nu p_I - \eta) - \beta gH(M) > 0$. Evaluating the constraint at $b^*(g, M, \beta)$, we get

$$\begin{aligned} & b^*(g, M, \beta)(1 - \nu - \nu p_I - \eta) - \beta gH(M) > 0 \\ \Leftrightarrow & \left[\frac{\lambda \beta}{1 - \nu - \nu p_I - \eta} + \frac{(1 - \lambda)(1 - \nu)(1 + p_D)}{1 - \nu + \nu p_B} \right] gH(M)(1 - \nu - \nu p_I - \eta) > \beta gH(M) \\ \Leftrightarrow & \beta < \frac{(1 - \nu)(1 - \nu - \nu p_I - \eta)(1 + p_D)}{1 - \nu + \nu p_B} = \hat{\beta}. \end{aligned} \quad (\text{A.4})$$

Constraint (3) says that $(1 - \nu)(gH(M) - b) - \nu(p_D gH(M) + p_B b) + p_D gH(M) > 0$. Evaluating the constraint at $b^*(g, M, \beta)$ and re-arranging the terms, the condition simplifies to $\beta < \hat{\beta}$ as well. Let $\bar{\beta} = \max\{\hat{\beta}, 0\}$. If $\beta < \bar{\beta}$, then a solution to (1)-(3) exists and it is given in Equation (12). If $\beta \geq \bar{\beta}$, then there is no bribery between the manager and the inspector. $\square$

Proof of Proposition 1: The manager's expected payoff $\pi(g; M, \beta)$ is the sum of linear and concave terms, and therefore concave in g . The interior maximizer $g^*(M, \beta)$ must satisfy the first-order condition: $g^*(M, \beta) = \frac{N(1-\theta)}{2M\chi(\beta)}$. The optimal decision $g^*(M, \beta)$ is constrained within the range $[0, 1]$. Because $\theta < 1$, $g^*(M, \beta) > 0$. Whether $g^*(M, \beta) < 1$ or not depends on the magnitude of M . If $M \leq \frac{N(1-\theta)}{2\chi(\beta)} = \widehat{M}$, then $g^*(M, \beta) = 1$. If $M > \widehat{M}$, then $g^*(M, \beta) = \frac{N(1-\theta)}{2M\chi(\beta)} < 1$.

We next show how $g^*(M, \beta)$ changes in M and β . When $M > \widehat{M}$, $\frac{\partial g^*(M, \beta)}{\partial M} = -\frac{N(1-\theta)}{2M^2\chi(\beta)} < 0$ and $g^*(M, \beta)$ strictly decreases in M . Furthermore, if $\beta < \bar{\beta}$, then $\frac{\partial g^*(M, \beta)}{\partial \beta} = -\frac{N(1-\theta)}{2M\chi(\beta)^2} \frac{\partial \chi(\beta)}{\partial \beta} < 0$ because $\frac{\partial \chi(\beta)}{\partial \beta} = (1 - \nu + \nu p_B) \frac{\lambda}{1 - \nu - \nu p_I - \eta} > 0$. Note that for $\beta < \bar{\beta}$ to hold, $\hat{\beta} > 0$ in (4) and therefore $1 - \nu - \nu p_I - \eta > 0$. Therefore, $g^*(M, \beta)$ also strictly decreases in β if $\beta < \bar{\beta}$. $\square$

Proof of Proposition 2: The value of $g^*(M, \beta)$ in (18) depends on the ranges of M . We show that $A(M, \beta)$ is concave in M within each range $M \leq \widehat{M}$ and $M > \widehat{M}$. We then determine the optimal number of inspectors M^* for each given value of $\beta < \bar{\beta}$.

Throughout this proof, we consider the case in which the bribery region is non-empty, i.e., $\widehat{\beta} > 0$. Note that this condition is needed for Setting 1 to occur. Then, $\overline{\beta} = \widehat{\beta} > 0$. For analytical convenience, we temporarily allow the functions $r(\beta)$ in (14) and $\chi(\beta)$ in (16) to be defined for $\beta < 0$, where they maintain the same functional forms as in (14) and (16). The monotonicity property of $\chi(\beta)$ discussed after (16) continues to hold under this extended definition. That is, if $1 - \nu - \nu p_I - \eta > 0$, then $\chi(\beta)$ strictly increases in β for $\beta < \overline{\beta}$. This extension to $\beta < 0$ facilitates the definition of β_0 and $\widehat{\beta}_0$ below. The feasible reward rate remains restricted to $\beta \geq 0$.

Case 1: Reward rate $\beta \in [0, \overline{\beta})$. Bribery occurs in this case, and the value of humanitarian aid reaching beneficiaries is given in (20), rewritten here using $NH(M) = F - sM$:

$$A(M, \beta) = (F - sM) \left[1 - g^*(M, \beta) + \frac{M}{N} \nu \alpha (g^*(M, \beta))^2 \right]. \quad (\text{A.5})$$

For each $\beta \in [0, \overline{\beta})$, we consider two ranges of M : $[0, \widehat{M}]$ and $(\widehat{M}, N]$, where $\widehat{M} = \frac{N(1-\theta)}{2\chi(\beta)}$. These two ranges correspond to the two branches in (18). Note that the second range of M is bounded by N (since $M \leq N$), and this range $(\widehat{M}, N]$ is non-empty if and only if $\widehat{M} < N$ or equivalently $\chi(\beta) > \frac{1-\theta}{2}$. Because $\chi(\overline{\beta}) = 1 + p_D > \frac{1-\theta}{2}$ and $\chi(\beta)$ strictly increases in β for $\beta < \overline{\beta}$, there exists $\beta_0 < \overline{\beta}$ defined as

$$\chi(\beta_0) = \frac{1-\theta}{2}, \quad (\text{A.6})$$

such that the range $(\widehat{M}, N]$ is non-empty if and only if the reward rate $\beta \in (\beta_0, \overline{\beta})$. Note that β_0 can be negative due to our extended definition of $\chi(\beta)$.

Therefore, Case 1 can be split into two sub-cases: in the case of $\beta \in [0, \beta_0]$, $\widehat{M} \geq N$ and the second range $(\widehat{M}, N]$ does not exist; in the case of $\beta \in (\beta_0, \overline{\beta})$, we need to consider both ranges of M , which is the focus of our following analysis.

Range 1a of M : $0 \leq M \leq \widehat{M} = \frac{N(1-\theta)}{2\chi(\beta)}$. In this range, $g^*(M, \beta) = 1$ and $A(M, \beta)$ in (A.5) becomes:

$$A(M) = \frac{M}{N} \nu \alpha (F - sM), \quad (\text{A.7})$$

independent of β . Clearly, $A(M)$ is strictly concave in M in Range 1a, and its derivative is

$$A'(M) = \frac{\nu \alpha}{N} (F - 2sM). \quad (\text{A.8})$$

The unconstrained optimal number of inspectors in this range, denoted as M_{1a}^* , is obtained by setting $A'(M) = 0$ and it is therefore

$$M_{1a}^* = \frac{F}{2s}. \quad (\text{A.9})$$

Because of the concavity, the optimal M within this Range 1a of M is

$$\min \{M_{1a}^*, \widehat{M}\}. \quad (\text{A.10})$$

Note that we do not need to bound the solution (A.10) by N , because $F \leq sN$ (Assumption 1) already ensures $M_{1a}^* = \frac{F}{2s} < N$.

Range 1b of M : $\widehat{M} < M \leq N$. In this range, $g^*(M, \beta) = \frac{N(1-\theta)}{2M\chi(\beta)} < 1$, and $A(M, \beta)$ in (A.5) becomes

$$\begin{aligned} A(M, \beta) &= (F - sM) \left[1 - \frac{N(1-\theta)}{2M\chi(\beta)} + \nu\alpha \frac{N(1-\theta)^2}{4M\chi(\beta)^2} \right] \\ &= F - sM - F \frac{N(1-\theta)}{2M\chi(\beta)} \left[1 - \nu\alpha \frac{(1-\theta)}{2\chi(\beta)} \right] + s \frac{N(1-\theta)}{2\chi(\beta)} - \nu\alpha \frac{N(1-\theta)^2}{4\chi(\beta)^2} s. \end{aligned} \quad (\text{A.11})$$

The objective $A(M, \beta)$ in (A.11) is strictly concave in M if and only if

$$1 - \nu\alpha \frac{(1-\theta)}{2\chi(\beta)} > 0. \quad (\text{A.12})$$

This condition indeed holds because $\alpha \in (0, 1]$, $\theta \in (0, 1)$, and $\chi(\beta) > \nu$, established by

$$\chi(\beta) > \chi(0) = (1 - \lambda(1 - \nu))(1 + p_D) \geq \nu(1 + p_D) \geq \nu, \quad (\text{A.13})$$

where the first inequality is due to $\chi(\beta)$ strictly increasing in β , the equality follows from straightforward algebra after setting $\beta = 0$, and $(1 - \lambda(1 - \nu)) \geq \nu$ follows from $(1 - \nu) \geq \lambda(1 - \nu)$.

The derivative of $A(M, \beta)$ with respect to M is

$$\frac{\partial A(M, \beta)}{\partial M} = -s + \frac{FN(1-\theta)}{2M^2\chi(\beta)} \left[1 - \nu\alpha \frac{(1-\theta)}{2\chi(\beta)} \right]. \quad (\text{A.14})$$

Because $A(M, \beta)$ in (A.11) is strictly concave in M , the unconstrained optimal M in this range, denoted as M_{1b}^* and obtained by solving $\frac{\partial A(M, \beta)}{\partial M} = 0$, is

$$M_{1b}^* = \sqrt{\frac{FN(1-\theta)}{2s\chi(\beta)} \left[1 - \nu\alpha \frac{(1-\theta)}{2\chi(\beta)} \right]}. \quad (\text{A.15})$$

Because of $F \leq sN$ (Assumption 1) and $\frac{N(1-\theta)}{2\chi(\beta)} < N$ (the condition for Range 1b of M to exist), it follows that $M_{1b}^* < N$. Therefore, the optimal M within this Range 1b of M is

$$\max \{M_{1b}^*, \widehat{M}\}. \quad (\text{A.16})$$

Finding M^* : If $\beta \in [0, \beta_0]$, then $\widehat{M} \geq N$ and Range 1b $(\widehat{M}, N]$ is empty. In this case, M_{1a}^* is in the interior of Range 1a $[0, \widehat{M}]$ since $M_{1a}^* < N$ as discussed after (A.10). Thus, the optimal number of inspectors is

$$M^* = M_{1a}^*, \quad \text{for } \beta \in [0, \beta_0]. \quad (\text{A.17})$$

Now consider $\beta \in (\beta_0, \widehat{\beta}_0] \cap [0, \widehat{\beta}_0]$, where $\widehat{\beta}_0$ is defined as

$$\chi(\widehat{\beta}_0) = \frac{sN(1-\theta)}{2F}. \quad (\text{A.18})$$

Note that $\widehat{\beta}_0 > \beta_0$ as $F < sN$, compared to (A.6). Since $\beta \leq \widehat{\beta}_0$, we have $\widehat{M} = \frac{N(1-\theta)}{2\chi(\beta)} \geq \frac{N(1-\theta)}{2\chi(\widehat{\beta}_0)} = \frac{F}{s}$, and thus $F - s\widehat{M} \leq 0$. Then, $A(M, \beta) \leq 0$ in Range 1b expressed in (A.11), while $A(M, \beta)$ in Range 1a expressed in (A.7) can take positive values, and therefore

$$M^* = M_{1a}^*, \quad \text{for } \beta \in (\beta_0, \widehat{\beta}_0]. \quad (\text{A.19})$$

Next, consider $\beta \in L_1 \equiv (\widehat{\beta}_0, \bar{\beta}) \cap [0, \bar{\beta})$. In this case, Range 1b is non-empty, $F - s\widehat{M} > 0$, and the optimal M may occur in either of the two ranges of M . At the boundary of the two ranges, i.e., at $M = \widehat{M}$, $A(M, \beta)$ is continuous in M as $g^*(M, \beta)$ in (18) is continuous in M . However, $A(M, \beta)$ is

not smooth in M because its directional derivatives at $M = \widehat{M}$ are

$$\lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \beta)}{\partial M} = \frac{\nu\alpha}{N}(F - 2s\widehat{M}), \quad (\text{A.20})$$

$$\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \beta)}{\partial M} = -s + \frac{F}{\widehat{M}} \left[1 - \frac{\nu\alpha}{N}\widehat{M} \right] = -s + \frac{F}{\widehat{M}} - \frac{\nu\alpha F}{N}. \quad (\text{A.21})$$

Taking the difference, we have

$$\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \beta)}{\partial M} - \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \beta)}{\partial M} = (F - s\widehat{M}) \left(\frac{1}{\widehat{M}} - \frac{2\nu\alpha}{N} \right) > 0, \quad (\text{A.22})$$

where the inequality follows from $F - s\widehat{M} > 0$ and the fact that $\chi(\beta) > \nu$ shown in (A.13) implies that $\frac{2\chi(\beta)}{N(1-\theta)} > \frac{2\nu\alpha}{N}$ or equivalently $\frac{1}{\widehat{M}} > \frac{2\nu\alpha}{N}$.

The relationship between the two directional derivatives in (A.22) leads to the following three scenarios where we can characterize the optimal number of inspectors M^* .

- (S-i) If $\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \beta)}{\partial M} > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \beta)}{\partial M} > 0$, then $A(M, \beta)$ strictly increases in M for $M \in [0, \widehat{M} + \epsilon)$ for some $\epsilon > 0$, and thus $M^* = M_{1b}^*$, while M_{1a}^* is infeasible (outside of Range 1a).
- (S-ii) If $0 > \lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \beta)}{\partial M} > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \beta)}{\partial M}$, then $A(M, \beta)$ strictly decreases in M for $M \in (\widehat{M} - \epsilon, N]$ for some $\epsilon > 0$, and thus $M^* = M_{1a}^*$, while M_{1b}^* is infeasible (outside of Range 1b).
- (S-iii) If $\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \beta)}{\partial M} > 0 > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \beta)}{\partial M}$, then $M_{1a}^* \leq \widehat{M}$ and $M_{1b}^* \geq \widehat{M}$, i.e., both M_{1a}^* and M_{1b}^* are in their respective range and M^* is the one that attains the higher $A(M, \beta)$.

Taken together, M^* must be one of the unconstrained solutions: either M_{1a}^* or M_{1b}^* . Next, we find a threshold of β at which M^* jumps from M_{1a}^* to M_{1b}^* .

The unconstrained maximum of (A.7) can be obtained by evaluating (A.7) at $M_{1a}^* = \frac{F}{2s}$:

$$A_{1a}^* = \frac{M_{1a}^*}{N} \nu\alpha(F - sM_{1a}^*) = \nu\alpha \frac{F^2}{4sN}. \quad (\text{A.23})$$

The unconstrained maximum of (A.11) can be obtained by evaluating (A.5) at M_{1b}^* :

$$A_{1b}^*(\beta) = (F - sM_{1b}^*) \left[1 - g^*(M_{1b}^*, \beta) + \frac{M_{1b}^*}{N} \nu\alpha (g^*(M_{1b}^*, \beta))^2 \right], \quad (\text{A.24})$$

where $g^*(M, \beta) = \frac{N(1-\theta)}{2M\chi(\beta)}$ and M_{1b}^* is defined in (A.15), which also depends on β . We take derivative with respect to β and obtain

$$A_{1b}'(\beta) = \frac{\partial A(M_{1b}^*, \beta)}{\partial M} \frac{\partial M_{1b}^*}{\partial \beta} + \frac{\partial A(M_{1b}^*, \beta)}{\partial \beta} = \frac{\partial A(M_{1b}^*, \beta)}{\partial \beta} \quad (\text{Envelope theorem}) \quad (\text{A.25})$$

$$\begin{aligned} &= (F - sM_{1b}^*) \left[-1 + \frac{M_{1b}^*}{N} \nu\alpha 2g^*(M_{1b}^*, \beta) \right] \frac{\partial g^*(M_{1b}^*, \beta)}{\partial \beta} \\ &= -(F - sM_{1b}^*) \left[1 - \nu\alpha \frac{(1-\theta)}{\chi(\beta)} \right] \frac{\partial g^*(M_{1b}^*, \beta)}{\partial \beta} > 0, \end{aligned} \quad (\text{A.26})$$

where $F - sM_{1b}^* > 0$ because $F - s\widehat{M} > 0$ implies that the unconstrained maximum of (A.11) must be positive; $1 - \nu\alpha \frac{(1-\theta)}{\chi(\beta)} > 0$ (established in (A.12)); $\frac{\partial g^*(M_{1b}^*, \beta)}{\partial \beta} < 0$ since $g^*(M, \beta) = \frac{N(1-\theta)}{2M\chi(\beta)}$ and $\chi(\beta)$ strictly increases in β for $\beta < \bar{\beta}$.

Because A_{1a}^* is a constant and $A_{1b}^*(\beta)$ strictly increases in β , it follows from the single-crossing

property of monotone function that there exists at most one value of $\beta \in L_1$, denoted as β_1 , at which $A_{1a}^* = A_{1b}^*(\beta_1)$, That is, β_1 solves

$$\nu\alpha \frac{F^2}{4sN} = (F - sM_{1b}^*(\beta_1)) \left[1 - g^*(M_{1b}^*(\beta_1), \beta_1) + \frac{M_{1b}^*(\beta_1)}{N} \nu\alpha (g^*(M_{1b}^*(\beta_1), \beta_1))^2 \right]. \quad (\text{A.27})$$

If $A_{1a}^* = A_{1b}^*(\beta_1)$ does not have a solution satisfying $\beta_1 \in L_1$, we have either $A_{1a}^* > A_{1b}^*(\beta)$ for all $\beta \in L_1$, in which case we set $\beta_1 = \bar{\beta}$, or $A_{1a}^* < A_{1b}^*(\beta)$ for all $\beta \in L_1$, in which case we set $\beta_1 = \hat{\beta}_0$.⁵

Finally, we prove (23):

$$M^* = \begin{cases} M_{1a}^*, & \text{if } \beta \leq \beta_1, \\ M_{1b}^*, & \text{if } \beta > \beta_1. \end{cases}$$

Since $\beta_1 \geq \hat{\beta}_0$ by the above definition of β_1 , the cases in (A.17) and (A.19) are consistent with (23).

We focus on proving (23) when $\beta \in L_1$. If β is such that scenario (S-iii) occurs, i.e., both M_{1a}^* and M_{1b}^* are in their respective range, then (23) clearly holds.

If β is such that scenarios (S-i) or (S-ii) occurs, then we need to rule out the possible situation where $A_{1a}^* > A_{1b}^*(\beta)$, but A_{1a}^* is unattainable because M_{1a} lies outside of $[0, \widehat{M}]$, as well as the reverse situation with $A_{1b}^*(\beta) > A_{1a}^*$ and unattainable $A_{1b}^*(\beta)$. To rule out these situations, it suffices to show the following:

- (i) In scenario (S-i) where M_{1b}^* is optimal, we have $A_{1b}^*(\beta) > A_{1a}^*$, i.e., $\beta > \beta_1$.
- (ii) In scenario (S-ii) where M_{1a}^* is optimal, we have $A_{1b}^*(\beta) < A_{1a}^*$, i.e., $\beta < \beta_1$.

We prove these two statements in sequence.

In scenario (S-i), M_{1a}^* is infeasible, i.e., $\widehat{M} < M_{1a}^* = \frac{F}{2s}$. Consider an auxiliary function:

$$\tilde{A}(M, g) = (F - sM) \left(1 - g + \frac{M}{N} \nu\alpha g^2 \right), \quad M \in \left[\widehat{M}, \frac{F}{2s} \right]. \quad (\text{A.28})$$

Its derivative with respect to g is

$$\frac{\partial \tilde{A}}{\partial g} = (F - sM) \left(-1 + 2 \frac{M}{N} \nu\alpha g \right) \leq (F - sM) \left(-1 + \frac{F}{sN} \nu\alpha g \right) < 0, \quad M \in \left[\widehat{M}, \frac{F}{2s} \right],$$

where the last inequality follows from Assumption 1. Then, we have

$$A_{1b}^*(\beta) \geq A(M_{1a}^*, \beta) = \tilde{A}(M_{1a}^*, g^*(M_{1a}^*, \beta)) > \tilde{A}(M_{1a}^*, 1) = A_{1a}^*, \quad M \in (\widehat{M}, M_{1a}^*],$$

where the first inequality is because $A_{1b}^*(\beta)$ is the maximum in scenario (S-i), the equality is by definition, and the second inequality is due to $\tilde{A}(M, g)$ strictly decreasing in g and $g^*(M, \beta) < 1$ when $M > \widehat{M}$.

In Scenario (S-ii), we show that the $A(M)$ in (A.7) always exceeds $A(M, \beta)$ in (A.11) for $M < \widehat{M}$ by showing that their difference increases as M decreases (note that the difference is zero at $M = \widehat{M}$). In fact, we show a stronger result that the difference of their derivatives in (A.14) and (A.8) is positive

⁵If $\hat{\beta}_0 \geq 0$, then at $\beta = \hat{\beta}_0$ we have $F = s\widehat{M}$, so we are in scenario (S-ii) and our later proof will show $A_{1a}^* > A_{1b}^*(\hat{\beta}_0)$. By continuity, $A_{1a}^* > A_{1b}^*(\beta)$ when β is in a neighborhood of $\hat{\beta}_0$. Thus, in the case of $A_{1a}^* < A_{1b}^*(\beta)$ for all $\beta \in L_1$, we must have $\hat{\beta}_0 < 0$.

and increases as M decreases. The difference, (A.14) – (A.8) can be written as

$$J(M) = -s + \frac{F\widehat{M}}{M^2} \left(1 - \frac{\nu\alpha}{N}\widehat{M}\right) - \frac{\nu\alpha}{N}(F - 2sM).$$

Note that $J(\widehat{M})$ is exactly the directional derivative difference in (A.22), and thus $J(\widehat{M}) > 0$. Now we show that $J(M)$ decreases in M for $M < \widehat{M}$. Differentiating $J(M)$, we have

$$J'(M) = -\frac{2F\widehat{M}(1 - \frac{\nu\alpha}{N}\widehat{M})}{M^3} + \frac{2s\nu\alpha}{N}. \quad (\text{A.29})$$

At $M = \widehat{M}$, $J'(\widehat{M}) < 0$ because

$$J'(\widehat{M}) = -\frac{2F(1 - \frac{\nu\alpha}{N}\widehat{M})}{\widehat{M}^2} + \frac{2s\nu\alpha}{N} < -\frac{2s\widehat{M}(1 - \frac{\nu\alpha}{N}\widehat{M})}{\widehat{M}^2} + \frac{2s\nu\alpha}{N} = -2s\left(\frac{1}{\widehat{M}} - \frac{2\nu\alpha}{N}\right) < 0,$$

where the first inequality is due to $F > s\widehat{M}$, and the last inequality follows from the inequality in (A.22). Therefore, when M decreases, the first term in (A.29) becomes more negative, so does $J'(M)$. Therefore, $J'(M) < 0$ for $M < \widehat{M}$, implying $J(M) > 0$. This completes the proof of (23).

We conclude this proof by proving claims (i) and (ii) of Proposition 2. We have already shown that M_{1a}^* and $A(M_{1a}^*, \beta)$ are constant with respect to β , thereby proving claim (i). We have also shown above that $A(M_{1b}^*, \beta)$ increases in β (see (A.26)). Therefore, the only claim left to prove is that M_{1b}^* decreases in β . The derivative of M_{1b}^* with respect to β is

$$\frac{\partial M_{1b}^*}{\partial \beta} = \frac{1}{2M_{1b}^*} \frac{FN(1-\theta)}{4s} \times \frac{-2\chi(\beta)\frac{\partial\chi(\beta)}{\partial\beta} + 2\frac{\partial\chi(\beta)}{\partial\beta}\nu\alpha(1-\theta)}{\chi(\beta)^3}. \quad (\text{A.30})$$

It then follows that $\frac{\partial M_{1b}^*}{\partial \beta} \leq 0$ if and only if

$$-2\frac{\partial\chi(\beta)}{\partial\beta} \left[\chi(\beta) - \nu\alpha(1-\theta) \right] \leq 0, \quad (\text{A.31})$$

which always holds because $\frac{\partial\chi(\beta)}{\partial\beta} \geq 0$ and $\chi(\beta) \geq \nu\alpha(1-\theta)$. $\square$

Proof of Proposition 3: Because Setting 1 requires $\widehat{\beta} > 0$, $\overline{\beta} = \widehat{\beta}$ throughout this proof. As discussed before Proposition 3, the optimal combination of anti-corruption levers depends on whether $\beta_1 < \overline{\beta}$ or $\beta_1 = \overline{\beta}$. We show that there exist two values of F at which $\beta_1 = \overline{\beta}$. We set $\beta_1 = \overline{\beta}$ in (A.27) and substitute $g^*(M_{1b}^*(\overline{\beta}), \overline{\beta}) = \frac{N(1-\theta)}{2M_{1b}^*(\overline{\beta})(1+p_D)}$ and get

$$\nu\alpha \frac{F^2}{4sN} = (F - sM_{1b}^*(\overline{\beta})) \left[1 - \frac{N(1-\theta)}{2M_{1b}^*(\overline{\beta})(1+p_D)} + \nu\alpha \frac{N(1-\theta)^2}{4M_{1b}^*(\overline{\beta})(1+p_D)^2} \right]. \quad (\text{A.32})$$

Let $K = \frac{\sqrt{sN(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{2(1+p_D)}$. Then, $M_{1b}^*(\overline{\beta}) = \frac{K}{s}\sqrt{F}$ and $\frac{N(1-\theta)}{2M_{1b}^*(\overline{\beta})(1+p_D)} - \nu\alpha \frac{N(1-\theta)^2}{4M_{1b}^*(\overline{\beta})(1+p_D)^2} = \frac{K}{\sqrt{F}}$. The right-hand-side of (A.32) becomes $(F - K\sqrt{F}) \left[1 - \frac{K}{\sqrt{F}} \right] = F - 2K\sqrt{F} + K^2 = (\sqrt{F} - K)^2$. Therefore, (A.32) simplifies to

$$\nu\alpha \frac{F^2}{4sN} = (\sqrt{F} - K)^2 \Leftrightarrow \sqrt{\nu\alpha} \frac{F}{2\sqrt{sN}} = \sqrt{F} - K \Leftrightarrow \frac{\nu\alpha}{4sN} F^2 + \left[K\sqrt{\frac{\nu\alpha}{sN}} - 1 \right] F + K^2 = 0, \quad (\text{A.33})$$

which is quadratic in F . The two roots are

$$\begin{aligned} & 2sN \frac{1 - K\sqrt{\frac{\nu\alpha}{sN}} \pm \sqrt{1 - 2K\sqrt{\frac{\nu\alpha}{sN}}}}{\nu\alpha} = \\ & 2sN \frac{1 - \frac{\sqrt{\nu\alpha(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{2(1+p_D)} \pm \sqrt{1 - \frac{\sqrt{\nu\alpha(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{(1+p_D)}}}{\nu\alpha}. \end{aligned} \quad (\text{A.34})$$

Note that $1 - \frac{\sqrt{\nu\alpha(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{(1+p_D)} \geq 0$. We next show that the smallest of the two roots is $\leq sN$, whereas the largest is $\geq sN$. This implies that for our problem setting, only the smallest of the two roots is relevant. The amount sN is in between the two roots if at $F = sN$, $(\sqrt{sN} - K)^2 \geq \frac{\nu\alpha(sN)^2}{4sN}$ (from (A.33)), i.e., if $(1 - \frac{\sqrt{(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{2(1+p_D)})^2 \geq \frac{\nu\alpha}{4}$. For notational convenience, let $y = \nu\alpha$ and $G(y) = (1 - \frac{\sqrt{(1-\theta)(2(1+p_D)-y(1-\theta))}}{2(1+p_D)})^2 - \frac{y}{4}$. Note that $0 \leq y \leq 1$. We want to show that $G(y) \geq 0$. To do so, we first show that $G(y)$ decreases in y and then that at $y = 1$, $G(y) \geq 0$.

$$\frac{\partial G(y)}{\partial y} = \frac{(1-\theta)^2}{2(1+p_D)\sqrt{(1-\theta)(2(1+p_D)-y(1-\theta))}} - \frac{(1-\theta)^2}{4(1+p_D)^2} - \frac{1}{4}. \quad (\text{A.35})$$

The second derivative of $G(y)$ with respect to y is $\frac{\partial^2 G(y)}{\partial y^2} = \frac{(1-\theta)^4}{4(1+p_D)[(1-\theta)(2(1+p_D)-y(1-\theta))]^{3/2}} \geq 0$. Because $\frac{\partial^2 G(y)}{\partial y^2} \geq 0$, it follows that $\frac{\partial G(y)}{\partial y}$ increases in y . To show that $\frac{\partial G(y)}{\partial y} \leq 0$, $\forall y \in [0, 1]$, it then suffices to show that $\frac{\partial G(y)}{\partial y} \leq 0$ at $y = 1$. That is,

$$\begin{aligned} & \frac{(1-\theta)^2}{2(1+p_D)\sqrt{(1-\theta)(2(1+p_D)-(1-\theta))}} - \frac{(1-\theta)^2}{4(1+p_D)^2} - \frac{1}{4} \leq 0 \\ \Leftrightarrow & \left[(1+p_D)^2 + (1-\theta)^2 \right] \sqrt{(1-\theta)(2(1+p_D)-(1-\theta))} \geq 2(1+p_D)(1-\theta)^2 \\ \Leftrightarrow & \left[(1+p_D) - (1-\theta) \right] \left[\left((1+p_D)^2 + (1-\theta)^2 \right)^2 \right. \\ & \left. + (1+p_D) \left((1+p_D)^3 + (1-\theta)(1+p_D)^2 + 3(1-\theta)^2(1+p_D) - (1-\theta)^3 \right) \right] \geq 0, \end{aligned} \quad (\text{A.36})$$

where the last inequality holds because $1+p_D \geq 1-\theta$. Therefore, $G(y)$ decreases in y . At $y = 1$, $G(1) = (1 - \frac{\sqrt{(1-\theta)(2(1+p_D)-(1-\theta))}}{2(1+p_D)})^2 - \frac{1}{4}$. We show that $G(1) \geq 0$. This holds because $(1 - \frac{\sqrt{(1-\theta)(2(1+p_D)-(1-\theta))}}{2(1+p_D)})^2 \geq \frac{1}{4} \Leftrightarrow [(1+p_D) - (1-\theta)]^2 \geq 0$. Thus, sN lies between the two roots. We denote the smaller root as F_1 . That is,

$$F_1 = 2sN \frac{1 - \frac{\sqrt{\nu\alpha(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{2(1+p_D)} - \sqrt{1 - \frac{\sqrt{\nu\alpha(1-\theta)(2(1+p_D)-\nu\alpha(1-\theta))}}{(1+p_D)}}}{\nu\alpha}. \quad (\text{A.37})$$

Because the coefficient of F^2 in (A.33) is positive, it follows that $A_{1a}^* > A_{1b}^*(\beta)$ if $F \leq F_1$, in which case $\beta_1 = \bar{\beta}$ and the optimal anti-corruption levers are $(M_{1a}^*, \beta^* \in [0, \bar{\beta} - \epsilon])$. If $F_1 \leq F \leq sN$, then $\beta_1 < \bar{\beta}$ and the optimal anti-corruption levers are $(M_{1b}^*, \bar{\beta} - \epsilon)$. $\square$

Proof of Proposition 4: The last claim of Proposition 4 states that $\beta^* = \bar{\beta} < \alpha$. This directly follows from the fact that $\alpha - \bar{\beta} \geq \nu\alpha > 0$, which is necessary for this setting to exist.

When $\beta = \bar{\beta}$, none of the inspectors can be bribed, which leads to $\chi(\beta) = 1+p_D$ and $\widehat{M} = \frac{N(1-\theta)}{2(1+p_D)}$.

The value of humanitarian aid reaching beneficiaries is given in (22), and re-written below.

$$A(M, \bar{\beta}) = (F - sM) \left[1 - g^*(M) + \frac{M}{N} (\alpha - \bar{\beta}) (g^*(M))^2 \right]. \quad (\text{A.38})$$

We first show that $A(M, \bar{\beta})$ is concave in M within each range $M \leq \widehat{M}$ and $M > \widehat{M}$. We then determine M^* . Note that $\widehat{M}$ is always bounded above by N because $1 + p_D > \frac{1-\theta}{2}$.

Range 2a of M : $0 \leq M \leq \widehat{M} = \frac{N(1-\theta)}{2(1+p_D)}$. In this range, $g^*(M) = 1$ and $A(M, \bar{\beta})$ in (22) becomes:

$$A(M, \bar{\beta}) = \frac{M}{N} (\alpha - \bar{\beta}) (F - sM). \quad (\text{A.39})$$

Because $(\alpha - \bar{\beta}) \geq \nu\alpha$ and $\nu\alpha > 0$ it holds that $(\alpha - \bar{\beta}) > 0$. The objective $A(M, \bar{\beta})$ is strictly concave in M and the unconstrained optimal number of inspectors is in this range, denoted as M_{2a}^* , is obtained by setting is obtained below.

$$M_{2a}^* = \frac{F}{2s}. \quad (\text{A.40})$$

Note that $M_{2a}^* = M_{1a}^*$ of Setting 1. The optimal M within Range 2a of M is then

$$\min \{M_{2a}^*, \widehat{M}\}. \quad (\text{A.41})$$

Range 2b of M : $\widehat{M} < M \leq N$. In this range, $g^*(M) = \frac{N(1-\theta)}{2M(1+p_D)} < 1$, and $A(M, \bar{\beta})$ becomes

$$A(M, \bar{\beta}) = F - sM - F \frac{N(1-\theta)}{2M(1+p_D)} \left[1 - (\alpha - \bar{\beta}) \frac{(1-\theta)}{2(1+p_D)} \right] + s \frac{N(1-\theta)}{2(1+p_D)} - (\alpha - \bar{\beta}) \frac{N(1-\theta)^2}{4(1+p_D)^2} s. \quad (\text{A.42})$$

This objective is strictly concave in M because $1 - (\alpha - \bar{\beta}) \frac{1-\theta}{2(1+p_D)} > 0$. Therefore, the unconstrained optimal M in this range, denoted as M_{2b}^* , solves $\frac{\partial A(M, \bar{\beta})}{\partial M} = 0$ and it is given below.

$$M_{2b}^* = \sqrt{\frac{FN(1-\theta)}{2s(1+p_D)} \left[1 - (\alpha - \bar{\beta}) \frac{(1-\theta)}{2(1+p_D)} \right]}. \quad (\text{A.43})$$

Note that $M_{2b}^* \leq N$ for $F \leq sN$ (Assumption 1). The optimal number of inspectors is

$$\max \{M_{2b}^*, \widehat{M}\}.$$

Finding M^* : If $F \leq \frac{sN(1-\theta)}{2(1+p_D)}$, then $\widehat{M} \geq \frac{F}{s}$ and therefore $F - s\widehat{M} \leq 0$. Then, $A(M, \bar{\beta}) \leq 0$ in Range 2b expressed in (A.42), while $A(M, \bar{\beta})$ in Range 2a expressed in (A.39) can take positive values, and

$$M^* = M_{2a}^*, \quad \text{for } F \leq \frac{sN(1-\theta)}{2(1+p_D)}. \quad (\text{A.44})$$

Consider $F > \frac{sN(1-\theta)}{2(1+p_D)}$, in which case $F - s\widehat{M} > 0$ and the optimal M may occur in either of the two ranges of M . At $M = \widehat{M} = \frac{N(1-\theta)}{2(1+p_D)}$, $A(M, \bar{\beta})$ is continuous in M as $g^*(M)$ in (18) is continuous in M . However, $A(M, \bar{\beta})$ is not smooth in M because its directional derivatives at $M = \widehat{M}$ are

$$\lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M} = \frac{(\alpha - \bar{\beta})}{N} (F - 2s\widehat{M}), \quad (\text{A.45})$$

$$\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} = -s + \frac{F}{\widehat{M}} \left[1 - \frac{(\alpha - \bar{\beta})}{N} \widehat{M} \right] = -s + \frac{F}{\widehat{M}} - \frac{(\alpha - \bar{\beta})F}{N}. \quad (\text{A.46})$$

Taking the difference, we have

$$\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} - \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M} = (F - s\widehat{M}) \left(\frac{1}{\widehat{M}} - \frac{2(\alpha - \bar{\beta})}{N} \right) > 0, \quad (\text{A.47})$$

because $F - s\widehat{M} > 0$ and $(1 + p_D) > (\alpha - \bar{\beta})(1 - \theta)$ implies that $\frac{1}{\widehat{M}} > \frac{2(\alpha - \bar{\beta})}{N}$.

The relationship between the two directional derivatives in (A.47) leads to the following three scenarios where we can characterize the optimal number of inspectors M^* .

- (S-i) If $\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M} > 0$, then $A(M, \bar{\beta})$ strictly increases in M for $M \in [0, \widehat{M} + \epsilon)$ for some $\epsilon > 0$, and thus $M^* = M_{2b}^*$, while M_{2a}^* is infeasible (outside of Range 2a).
- (S-ii) If $0 > \lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M}$, then $A(M, \bar{\beta})$ strictly decreases in M for $M \in (\widehat{M} - \epsilon, N]$ for some $\epsilon > 0$, and thus $M^* = M_{2a}^*$, while M_{2b}^* is infeasible (outside of Range 2b).
- (S-iii) If $\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} > 0 > \lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M}$, then $M_{2a}^* \leq \widehat{M}$ and $M_{2b}^* \geq \widehat{M}$, i.e., both M_{2a}^* and M_{2b}^* are in their respective range and M^* is the one that attains the higher $A(M, \bar{\beta})$.

Therefore, M^* must be one of the unconstrained solutions: either M_{2a}^* or M_{2b}^* . We write conditions (S-i)-(S-iii) in terms of the financing F . Condition (S-i) holds if

$$\lim_{M \rightarrow \widehat{M}^-} \frac{\partial A(M, \bar{\beta})}{\partial M} > 0 \Leftrightarrow F > \frac{sN(1 - \theta)}{(1 + p_D)}. \quad (\text{A.48})$$

Therefore, if $F > \frac{2sN(1 - \theta)}{2(1 + p_D)}$, then $M^* = M_{2b}^*$.

Condition (S-ii) holds if

$$\lim_{M \rightarrow \widehat{M}^+} \frac{\partial A(M, \bar{\beta})}{\partial M} < 0 \Leftrightarrow F < \frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}. \quad (\text{A.49})$$

Then, if $F < \frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}$, $M^* = M_{2a}^*$. Note that $\frac{sN(1 - \theta)}{2(1 + p_D)} < \frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}$, which implies that the optimal number of inspectors in (A.44) is subsumed by this case.

Condition (S-iii) holds if

$$\frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)} < F < \frac{sN(1 - \theta)}{(1 + p_D)}. \quad (\text{A.50})$$

We find a threshold of F in this range at which M^* jumps from M_{2a}^* to M_{2b}^* .

The unconstrained maximum of (A.39) can be obtained by evaluating (A.39) at $M_{2a}^* = \frac{F}{2s}$:

$$A_{2a}^* = \frac{M_{2a}^*}{N} (\alpha - \bar{\beta}) (F - sM_{2a}^*) = (\alpha - \bar{\beta}) \frac{F^2}{4sN}. \quad (\text{A.51})$$

The unconstrained maximum of (A.42) can be obtained by evaluating (A.42) at M_{2b}^* :

$$A_{2b}^* = (F - sM_{2b}^*) \left[1 - g^*(M_{2b}^*) + \frac{M_{2b}^*}{N} (\alpha - \bar{\beta}) (g^*(M_{2b}^*))^2 \right], \quad (\text{A.52})$$

where $g^*(M) = \frac{N(1 - \theta)}{2M(1 + p_D)}$ and M_{2b}^* is defined in (A.43).

We show that there exist two values of F at which $A_{2a}^* = A_{2b}^*$, i.e., at which

$$(\alpha - \bar{\beta}) \frac{F^2}{4sN} = (F - sM_{2b}^*) \left[1 - \frac{N(1 - \theta)}{2M_{2b}^*(1 + p_D)} + (\alpha - \bar{\beta}) \frac{N(1 - \theta)^2}{4M_{2b}^*(1 + p_D)^2} \right]. \quad (\text{A.53})$$

Note that the equation above is the same as (A.32), with the only difference that $\nu\alpha$ is replaced by $(\alpha - \bar{\beta})$. Therefore, the same steps as those outlined in the proof of Proposition 3 apply for this setting as well. We omit those details for brevity. The threshold of F at which $A_{2a}^* = A_{2b}^*$ that is also relevant for our setting (i.e., the one that is $\leq sN$) is denoted as F_2 and given below.

$$F_2 = 2sN \frac{1 - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{2(1 + p_D)} - \sqrt{1 - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{(1 + p_D)}}}{(\alpha - \bar{\beta})}. \quad (\text{A.54})$$

We confirm that F_2 belongs to the relevant range for condition (S-iii), which is $[\frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}, \frac{sN(1 - \theta)}{(1 + p_D)}]$.

First, we show that $F_2 \leq \frac{sN(1 - \theta)}{(1 + p_D)}$. This holds if $2 - \frac{(\alpha - \bar{\beta})(1 - \theta)}{1 + p_D} - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{1 + p_D} \leq 2\sqrt{1 - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{(1 + p_D)}}$. Squaring both sides and simplifying, we obtain $4(1 + p_D)^2 \geq (\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))$, which holds because $2(1 + p_D) \geq (\alpha - \bar{\beta})(1 - \theta)$ and $2(1 + p_D) \geq (2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))$. Next, we show that $F_2 \geq \frac{sN(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}$. This holds if

$$\begin{aligned} & 2 - \frac{(\alpha - \bar{\beta})(1 - \theta)}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)} - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{1 + p_D} \\ & \geq 2\sqrt{1 - \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)(2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta))}}{(1 + p_D)}}. \end{aligned} \quad (\text{A.55})$$

Upon squaring both sides and simplifying, we get

$$\begin{aligned} & \left(\frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)}}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)} + \frac{\sqrt{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}}{1 + p_D} \right)^2 \geq \frac{4}{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)} \\ & \Leftrightarrow \frac{\sqrt{(\alpha - \bar{\beta})(1 - \theta)}}{\sqrt{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}} + \frac{2(1 + p_D) - (\alpha - \bar{\beta})(1 - \theta)}{1 + p_D} \geq 2 \\ & \Leftrightarrow \left(1 + p_D - (\alpha - \bar{\beta})(1 - \theta) \right)^2 \geq 0. \end{aligned} \quad (\text{A.56})$$

Combining the three conditions together, it holds that $M^* = M_{2a}^*$ if $F \leq F_2$ and $M^* = M_{2b}^*$ if $F_2 \leq F \leq sN$. $\square$

Proof of Proposition 5: We consider each parameter separately. Recall that $\epsilon \rightarrow 0$ and hence $\chi(\bar{\beta} - \epsilon) \rightarrow 1 + p_D$. Furthermore, $\bar{\beta} = \max\{\hat{\beta}, 0\}$. We derive the comparative statics of the unconstrained threshold when $\hat{\beta} > 0$. The corresponding comparative statics of $\bar{\beta}$ are weak once the zero lower-bound becomes binding.

Impact of F : in Setting 1 ($\nu\alpha > \alpha - \bar{\beta}$), the optimal number of inspectors is M_{1a}^* if $F \leq F_1$ and M_{1b}^* if $F \geq F_1$. Because $\frac{dM_{1a}^*}{dF} = \frac{1}{2s} > 0$ and $\frac{dM_{1b}^*(\bar{\beta} - \epsilon)}{dF} = \frac{1}{2} \sqrt{\frac{N(1 - \theta)}{2s\chi(\bar{\beta} - \epsilon)F}} \left[1 - \nu\alpha \frac{(1 - \theta)}{2\chi(\bar{\beta} - \epsilon)} \right] > 0$, M^* increases in F , with an upward jump at F_1 . The optimal reward rate is either $\beta^* \in [0, \bar{\beta} - \epsilon]$ or $\beta^* = \bar{\beta} - \epsilon$, independent of F . Similarly, in Setting 2 ($\nu\alpha \leq \alpha - \bar{\beta}$), $\frac{dM_{2a}^*}{dF} = \frac{1}{2s} > 0$ and $\frac{dM_{2b}^*}{dF} = \frac{1}{2} \sqrt{\frac{N(1 - \theta)}{2s(1 + p_D)F}} \left[1 - (\alpha - \bar{\beta}) \frac{(1 - \theta)}{2(1 + p_D)} \right] > 0$ and $\beta^* = \bar{\beta}$ is independent of F .

Impact of η : in Setting 1, M_{1a}^* and $M_{1b}^*(\bar{\beta} - \epsilon)$ are independent of η for $\epsilon \rightarrow 0$. The threshold F_1 is independent of η and therefore M^* is a constant in η . From (4), $\frac{d\hat{\beta}}{d\eta} = -\frac{(1-\nu)(1+p_D)}{1-\nu+\nu p_B} < 0$. Therefore, the optimal reward rate weakly decreases in η when it is $\bar{\beta} - \epsilon$. When $\beta^* \in [0, \bar{\beta} - \epsilon]$, the optimal reward rate is not unique. As η increases, the upper bound $\bar{\beta} - \epsilon$ weakly decreases. In Setting 2, M_{2a}^* is a constant in η and $\frac{dM_{2b}^*}{d\eta} = \sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-(\alpha-\bar{\beta})(1-\theta)]}} \times \frac{(1-\theta)}{4(1+p_D)} \frac{d\bar{\beta}}{d\eta} \leq 0$. The threshold F_2 weakly increases in η because F_2 increases in $(\alpha - \bar{\beta})$ and $(\alpha - \bar{\beta})$ weakly increases in η . Therefore, the switch between the two candidates, if any, is from M_{2b}^* to M_{2a}^* , resulting in a downward jump. The optimal reward rate $\bar{\beta}$ weakly decreases in η . Lastly, as η increases, a transition between the two settings can only occur from Setting 1 to Setting 2. At the boundary, $\nu\alpha = \alpha - \bar{\beta}$ and therefore the limiting values of F_1 and F_2 coincide, as do the limiting values of $M_{1b}^*(\bar{\beta} - \epsilon)$ and M_{2b}^* . Recall that $M_{1a}^* = M_{2a}^*$. There is therefore no jump in M^* at $\nu\alpha = \alpha - \bar{\beta}$.

Impact of ν : in Setting 1, M_{1a}^* is independent of ν , $\frac{dM_{1b}^*(\bar{\beta}-\epsilon)}{d\nu} = -\sqrt{\frac{FN(1-\theta)}{s[2\chi(\bar{\beta}-\epsilon)-\nu\alpha(1-\theta)]}} \times \frac{\alpha(1-\theta)}{4\chi(\bar{\beta}-\epsilon)} < 0$ and F_1 increases in ν . Therefore, M^* weakly decreases in ν , with a possible downward jump from M_{1b}^* to M_{1a}^* . The optimal reward rate weakly decreases in ν because $\frac{d\hat{\beta}}{d\nu} = -\frac{p_B(1-\nu-\nu p_I-\eta)+(1+p_I)(1-\nu)(1-\nu+\nu p_B)}{(1-\nu+\nu p_B)^2}(1+p_D) < 0$ when $\hat{\beta} > 0$. Similarly, in Setting 2, M_{2a}^* is independent of ν , $\frac{dM_{2b}^*}{d\nu} = \sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-(\alpha-\bar{\beta})(1-\theta)]}} \times \frac{(1-\theta)}{4(1+p_D)} \frac{d\bar{\beta}}{d\nu} \leq 0$, F_2 weakly increases in ν (and any potential jump in M^* is downward) and $\bar{\beta}$ weakly decreases in ν . There is no jump in M^* at $\nu\alpha = \alpha - \bar{\beta}$.

Impact of p_I or p_B : in Setting 1, M_{1a}^* , $M_{1b}^*(\bar{\beta} - \epsilon)$ and F_1 are independent of p_I and p_B . The threshold $\hat{\beta} > 0$ decreases in p_I and in p_B because $\frac{d\hat{\beta}}{dp_I} = -\frac{\nu(1-\nu)(1+p_D)}{1-\nu+\nu p_B} < 0$ and $\frac{d\hat{\beta}}{dp_B} = -\frac{\nu(1-\nu)(1-\nu-\nu p_I-\eta)(1+p_D)}{(1-\nu+\nu p_B)^2} < 0$. Therefore, the optimal reward rate weakly decreases in p_I and p_B . In Setting 2, M_{2a}^* is independent of p_I and p_B , $\frac{dM_{2b}^*}{dp_I} = \sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-(\alpha-\bar{\beta})(1-\theta)]}} \times \frac{(1-\theta)}{4(1+p_D)} \frac{d\bar{\beta}}{dp_I} \leq 0$ and $\frac{dM_{2b}^*}{dp_B} = \sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-(\alpha-\bar{\beta})(1-\theta)]}} \times \frac{(1-\theta)}{4(1+p_D)} \frac{d\bar{\beta}}{dp_B} \leq 0$, F_2 weakly increases in either p_I or p_B . Therefore, M^* and the β^* weakly decrease in p_I and in p_B . There is no jump in M^* at $\nu\alpha = \alpha - \bar{\beta}$.

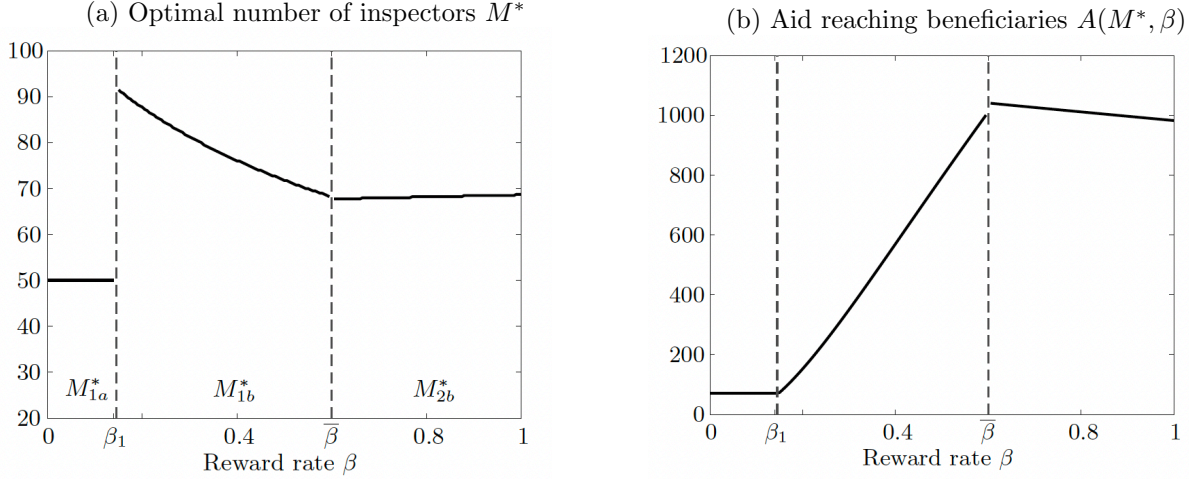
Impact of p_D : in Setting 1, M_{1a}^* is independent of p_D and $\frac{dM_{1b}^*(\bar{\beta}-\epsilon)}{dp_D} = -\sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-\nu\alpha(1-\theta)]}} \times \frac{1}{2(1+p_D)} \left(1 - \frac{\nu\alpha(1-\theta)}{1+p_D}\right) < 0$. The threshold F_1 decreases in p_D . Therefore, although M^* decreases in p_D in each regime, there exists a possible upward jump in M^* from M_{1a}^* to $M_{1b}^*(\bar{\beta} - \epsilon)$ as p_D increases. When $\hat{\beta} > 0$, $\frac{d\hat{\beta}}{dp_D} = \frac{(1-\nu)(1-\nu-\nu p_I-\eta)}{1-\nu+\nu p_B} > 0$ and therefore the optimal reward rate weakly increases in p_D . Similarly, in Setting 2, M_{2a}^* is independent of p_D , $\frac{dM_{2b}^*}{dp_D} = -\sqrt{\frac{FN(1-\theta)}{s[2(1+p_D)-(\alpha-\bar{\beta})(1-\theta)]}} \times \frac{1}{2(1+p_D)} \left(1 - \frac{\alpha(1-\theta)}{1+p_D} + \frac{(1-\theta)}{2} \frac{d\bar{\beta}}{dp_D}\right) \leq 0$ and F_2 decreases in p_D . Therefore, M^* weakly decreases in p_D in each regime, but there may exist an upward jump from M_{2a}^* to M_{2b}^* . The optimal reward rate weakly increases in p_D . $\square$

E. Additional Examples of How M^* Changes in β

We set the total financing available to be $F = 10,000$ and the penalty rate to be $p_D = p_B = p_I = 2$. All other parameters are the same as in Section 5. Figure A.1a shows how the optimal number of inspectors changes in β , while Figure A.1b shows how the total amount of aid reaching beneficiaries

changes in β at the optimal number of inspectors. We also highlight the thresholds $\bar{\beta}$ and β_1 in both panels. In this example, $0 \leq \beta_1 \leq \bar{\beta}$. Therefore, consistent with Proposition 2, $M^* = M_{1a}^*$ if $\beta \leq \beta_1$ and $M^* = M_{1b}^*$ if $\beta_1 < \beta < \bar{\beta}$. As we proved in Proposition 2, M_{1a}^* and $A(M_{1a}^*, \beta)$ are constant with respect to β . Similar to Figure 5a, $M^* = M_{2b}^*$ if $\beta \geq \bar{\beta}$.

Figure A.1: Impact of reward rate β on the optimal number of inspectors and humanitarian aid flow



F. Additional Numerical Examples

We set $F = \$350,000$ and $F = \$700,000$. That is, the financing is five times and ten times the financing of Section 5. All other parameters are the same as in Section 5. Note that in this case, the assumption that $F < sN$ is violated. However, these numerical examples show that our key insights remain intact even when $F \geq sN$. Table A.3 shows the optimal number of inspectors, the amount of salary paid to inspectors, the amount misappropriated, the recovered amount, the amount of reward paid to inspectors and the overall amount of aid reaching beneficiaries for $\beta = 0$ vs $\beta = \beta^* = \bar{\beta}$ for the two budgets. The numbers in parenthesis show the % of the financing. As shown in Section 5, we find that at the optimal β , fewer inspectors are needed, the amount misappropriated decreases, and the amount recovered and paid in reward increase. As F increases, a higher fraction of F reaches the beneficiaries. Moreover, the fraction of F that is spent on inspectors' reward is still minimal.

Table A.3: Comparison of $\beta = 0$ vs optimal β under different levels of total financing

	$F = \$350,000$		$F = \$700,000$	
	$\beta = 0$	Optimal β	$\beta = 0$	Optimal β
Optimal num. of inspectors, M	549	345	707	488
Prob. of site inspection, M/N	0.777	0.488	1	0.69
Inspection cost (salary), sM	54,900 (15.7%)	34,500 (9.9%)	70,700 (10.1%)	48,800 (7%)
Fraction misappropriated, g	0.161	0.103	0.125	0.072
Amount misappropriated	47,504 (13.6%)	32,327 (9.2%)	78,663 (11.2%)	47,172 (6.7%)
Recovered amount	1,188 (0.3%)	1,616 (0.5%)	1,967 (0.3%)	2,359 (0.3%)
Reward amount	0 (0%)	369 (0.1%)	0 (0%)	539 (0.2%)
Amount reaching beneficiaries, $A(M, \beta)$	248,784 (71%)	284,420 (81.3%)	552,604 (79%)	605,848 (86.4%)