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Online Appendix for "Quantifying the Impact of Layout on Productivity: An Analysis from Robotic-Cell Manufacturing"

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Appendix A

Theorem 1: In a robotic cell with circular layout, the recognition version of problem $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$ is NP-complete.

Proof:

We use the following NP-complete problem for our reduction (Garey and Johnson 1979).

PARTITION: Given a set $A = \{a_1, a_2, \ldots, a_{n-1}, a_n\}$ and $a_i \in Z^+$ for each $i = 1, 2, \ldots, n$, where $\sum_{a_i \in A} a_i = 2B$, does there exist a partition of A into subsets A_1 and A_2 such that $\sum_{a_k \in A_1} a_k = \sum_{a_j \in A_2} a_j = B$?

Proof: Given an arbitrary instance of PARTITION, we now describe a polynomial-time construction of an instance of $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$.

- $\{M_i, i \in M\}, m = |M| = 28n - 2$, is the set of machines in the robotic cell. The travel time of the robot between any two adjacent machines is $\delta = 3B$ and the load/unload time is $\epsilon = 0$. The set of machines is partitioned into five disjoint sets of machines, referred to as a-machines, b-machines, x-machines, y-machines, and z-machines. We now define these five sets.

- z-machines: M_{7i+1} (for convenience of exposition, also denoted as M_{z_i}), $i = 0, 1, \ldots, n$.
  x-machines: M_{7i+2} (also denoted as $M_{x_{i+1}}$), $i = 0, 1, \ldots, n-1$.
  y-machines: M_{7i+3} (also denoted as $M_{y_{i+1,1}}$), M_{7i+5} (also denoted as $M_{y_{i+1,2}}$), M_{7i+6} (also denoted as $M_{y_{i+1,3}}$), $i = 0, 1, \ldots, n-1$.
  a-machines: M_{7i+4} (also denoted as $M_{a_{i+1,1}}$) and M_{7i+7} (also denoted as $M_{a_{i+1,2}}$), $i = 0, 1, \ldots, n-1$.
  b-machines: M_{7n+1+i} (also denoted as M_{b_i}), $i = 1, 2, \ldots, 21n-3$.

- The processing times of the z-machines M_1 (i.e., M_{z_0}) and M_{7n+1} (i.e., M_{z_n}) are $p_{z_0} = p_{z_n} = 14n\delta + B$. The processing time of the z-machine M_{7i+1} (i.e., M_{z_i}) is $p_{z_i} = 14n\delta$, $i = 1, 2, \ldots, n-1$.

- The processing times of the x-machines M_{7i+2} (i.e., $M_{x_{i+1}}$) are $p_{x_{i+1}} = (m+2)\delta = 28n\delta$, $i = 0, 1, \ldots, n-1$.

- The processing times of the y-machines M_{7i+3} (i.e., $M_{y_{i+1,1}}$), M_{7i+5} (i.e., $M_{y_{i+1,2}}$), and M_{7i+6} (i.e., $M_{y_{i+1,3}}$) are $p_{y_{i+1,j}} = 14n\delta$, $j = 1, 2, 3$; $i = 0, 1, \ldots, n-1$.

- The processing times of the a-machines M_{7i+4} (i.e., $M_{a_{i+1,1}}$) and M_{7i+7} (i.e., $M_{a_{i+1,2}}$) are $p_{a_{i+1,j}} = a_{i+1}, j = 1, 2$; $i = 0, 1, \ldots, n-1$.

- The processing times of the b-machines M_{7n+1+i} (i.e., M_{b_i}) are $p_{b_i} = 0$, $i = 1, 2, \ldots, 21n-3$.

Decision Question (DQ): "Does there exist a 1-unit cycle π_r with cycle time $T(\pi_r) \leq 14n\delta + 2B + (m+2)\delta = 42n\delta + 2B$?"

Note that the cycle time of a given 1-unit cycle can be computed in time polynomial in the size of the instance (e.g., via linear programming; see Crama et al. 2000, Dawande et al. 2007). Thus, a "yes" answer to DQ can be verified in polynomial time. Consequently, DQ is in NP. We show that there exists a 1-unit cycle π_r with $T(\pi_r) \leq 42n\delta + 2B$ if and only if there exists a solution to PARTITION.

The terms *primary travel time* and *secondary travel time*, and the *machine usages* $\mathcal{U}_1$ and $\mathcal{U}_2$ are frequently used in this proof. Therefore, before proceeding further, the reader may want to refer to Section 3.1 for the definitions of these terms. Table 7 illustrates these terms for four 1-unit cycles.

CYCLE	*Total Primary Travel Time*	*Total Secondary Travel Time*	*Machine Usages*
$\pi_u = (M_1^l, M_2^l, M_3^l, M_4^l, M_5^l)$, where $m = 4$	$6\delta + 10\epsilon$	0	$\mathcal{U}_1$: M_1, M_2, M_3, M_4 $\mathcal{U}_2$: None
$\pi_d = (M_1^l, M_5^l, M_4^l, M_3^l, M_2^l)$, where $m = 4$	$6\delta + 10\epsilon$	10δ	$\mathcal{U}_1$: None $\mathcal{U}_2$: M_1, M_2, M_3, M_4
$\pi_c = (M_1^l, M_3^l, M_5^l, M_2^l, M_4^l)$, where $m = 4$	$6\delta + 10\epsilon$	6δ	$\mathcal{U}_1$: None $\mathcal{U}_2$: M_1, M_2, M_3, M_4
$\pi = (M_1^l, M_2^l, M_4^l, M_6^l, M_7^l, M_5^l, M_3^l)$, where $m = 6$	$8\delta + 14\epsilon$	10δ	$\mathcal{U}_1$: M_1, M_6 $\mathcal{U}_2$: M_2, M_3, M_4, M_5

Table 7: Travel time and machine usages in four 1-unit cycles. Graphical representations of cycles π_u, π_d, and π_c are provided in Figure 3.

If part: Suppose that there exists a solution to PARTITION. Without loss of generality, assume that $a_1 + a_2 + \cdots + a_g + a_h + a_{h+1} + \cdots + a_n = B$ and $a_{g+1} + a_{g+2} + \cdots + a_{h-1} = B$, where $g < h < n$.

Consider the 1-unit cycle $\pi_r = (\pi_{r_1}, \pi_{r_2}, \pi_{r_3})$, where

$$\begin{aligned}
\pi_{r_1} &= (M^l_{z_0}, \phi^1_1, M^l_{z_1}, \phi^1_2, \ldots, M^l_{z_{g-1}}, \phi^1_g, M^l_{z_g}, \phi^2_{g+1}, \ldots, M^l_{z_{h-2}}, \phi^2_{h-1}, M^l_{z_{h-1}}, \phi^1_h, \ldots, M^l_{z_{n-1}}, \phi^1_n, M^l_{z_n}),\\
\pi_{r_2} &= (M^l_{x_1}, \phi^4_1, M^l_{x_2}, \phi^4_2, \ldots, M^l_{x_g}, \phi^4_g, M^l_{x_{g+1}}, \phi^3_{g+1}, \ldots, M^l_{x_{h-1}}, \phi^3_{h-1}, M^l_{x_h}, \phi^4_h, \ldots, M^l_{x_n}, \phi^4_n), \text{ and}\\
\pi_{r_3} &= (M^l_{7n+2}, M^l_{7n+3}, \ldots, M^l_m, M^l_{m+1}),
\end{aligned}$$

with the subsequences ϕ^1_k, ϕ^2_k, ϕ^3_k, and ϕ^4_k, defined, respectively, as follows: $\phi^1_k = (M^l_{y_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}})$, $\phi^2_k = (M^l_{y_{k,1}}, M^l_{y_{k,3}})$, $\phi^3_k = (M^l_{a_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}})$, and $\phi^4_k = (M^l_{a_{k,1}}, M^l_{y_{k,3}})$.

The cycle time $T(\pi_r)$ can be viewed as the time consisting of four components: V_p, V_s, W_p, and W_s, where (i) $V_p = (m+2)\delta$ is the primary travel time, (ii) V_s is the robot's secondary travel time and (iii) W_p and W_f denote, respectively, the total partial and full waiting times of the robot at the machines. Thus, $T(\pi_r) = V_p + V_s + W_p + W_f$. We refer the reader to Section 3.1 for definitions of these components of the cycle time.

Note that ϕ^1_k (resp., ϕ^3_k) indicates that the usage at machine $M_{a_{k,2}}$ (resp., $M_{a_{k,1}}$) is $\mathcal{U}_1$. It is easy to verify that π_r is a feasible 1-unit cycle. There is no partial waiting time at any machine and $W_p = 0$. The robot experiences a total full-waiting time of $W_f = \sum_{i=1}^n a_i = 2B$. The total primary time is $V_p = 28n\delta$ and the total secondary travel time is $V_c = 14n\delta$. Thus, the total cycle time of π_r is $T(\pi_r) = 42n\delta + 2B$, as required. □

The following example illustrates the *if* part.

Example 2: Consider an instance of problem $RF^\circ_m|(free, A, cyclic\text{-}1)|C_t$ with $n = 5$.

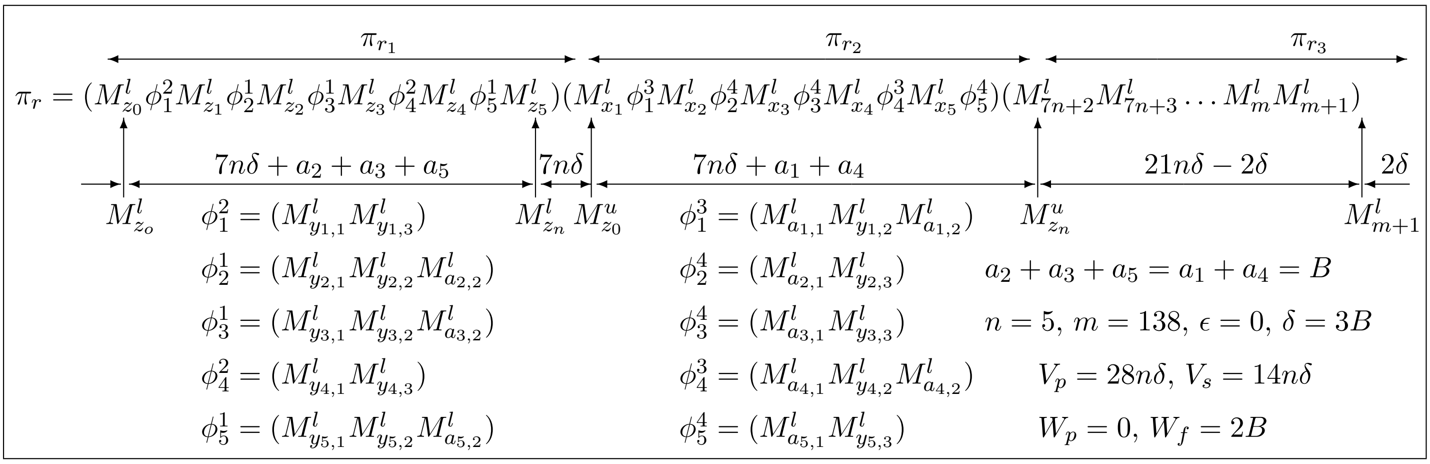


Figure 5: 1-unit cycle π_r for Example 2 with $n = 5$; $T(\pi_r) = 42n\delta + 2B$.

Figure 5 illustrates the following solution corresponding to PARTITION: $a_2 + a_3 + a_5 = B$ and $a_1 + a_4 = B$.

The cycle time is $T(\pi_r) = 42n\delta + 2B$. For this example, the sequence of moves of the robot in π_r is illustrated in Figure 6. The machines have the following usages:

(i) Machines $M_{a_{2,2}}$, $M_{a_{3,2}}$ and $M_{a_{5,2}}$, corresponding respectively to elements a_2, a_3, and a_5 of the partition set $\{a_2, a_3, a_5\}$, have usage $\mathcal{U}_1$.

(ii) Machines $M_{a_{1,1}}$ and $M_{a_{4,1}}$, corresponding respectively to the elements a_1 and a_4 of the partition set $\{a_1, a_4\}$, have usage $\mathcal{U}_1$.

(iii) The b-machines $\{M_{7n+2}, M_{7n+3}, \dots, M_m\}$ have usage $\mathcal{U}_1$.

(iv) All other machines have usage $\mathcal{U}_2$. □

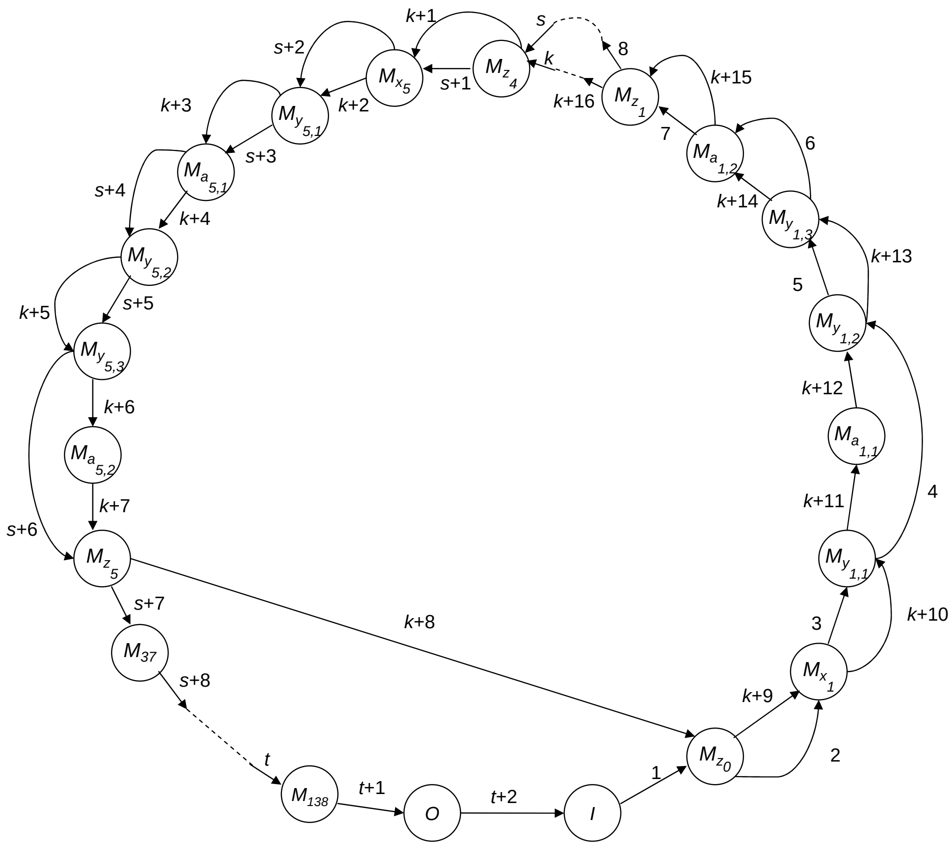


Figure 6: The moves of the robot in π_r: for clarity, only the loading and unloading of the machines in $\{M_0, M_1, \dots, M_8\}$, $\{M_{29}, M_{30}, \dots, M_{36}\}$ and $\{M_{37}, M_{38}, \dots, M_{138}, M_{139}, M_0\}$ are indicated ($k > 8, s > k + 16, t > s + 8$).

In order to facilitate the understanding of the proof of the *Only If part*, it is beneficial to summarize the properties of the cycle π_r defined above.

Properties of π_r: The main properties of cycle $\pi_r = (\pi_{r_1}, \pi_{r_2}, \pi_{r_3})$ can be described as follows: (i) the sequence of the loading/unloading of the machines in the cycle can be partitioned into three smaller subsequences: π_{r_1}, π_{r_2} and π_{r_3}. The subsequence (π_{r_1}, π_{r_2}) specifies the sequence of loading/unloading of machines in the set $\{M_1, M_2, \ldots, M_{7n+1}\}$. The robot performs both loading and unloading on this set of machines before moving to do the same for the other machines (defined in π_{r_3}) not in this set, (ii) in π_{r_1}, the z-machines are loaded and the x-machines are unloaded, (iii) in π_{r_2}, the x-machines are loaded, (iv) in π_{r_3}, all machine usages are $\mathcal{U}_1$, and (v) the total secondary travel time is $V_s = 14n\delta < (m+2)\delta$. The total primary travel time is $V_p = 28n\delta$, with $W_p = 0$ and $W_f = 2B$.

$T(\pi_o)$	the cycle time of 1-unit cycle π_o, $T(\pi_o) = V_p + V_s + W_p + W_f$.
V_p	the primary travel time.
V_s	the robot's secondary travel time.
W_p	the total partial waiting times of the robot at the machines.
W_f	the total full waiting times of the robot at the machines.
$\mathcal{A}_k$	the robot's travel path between M_k^l and M_k^u, $k \in M$.
L_k	the robot's travel time in $\mathcal{A}_k$ excluding the partial and full waiting times.
$\mathcal{S}_M$	the machine set, $\mathcal{S}_M = \{M_{z_0}, M_{z_i}, M_{x_i}, M_{y_{i,j}} \mid i = 1, 2 \ldots, n;\ j = 1, 2, 3\}$.

Table 8: Summary of notation.

Only If part: Suppose there exists a cycle π_o such that $T(\pi_o) \leq 42n\delta + 2B$. We first argue through a series of claims that π_o can only take the form $\pi_o = (\pi_{o_1}, \pi_{o_2}, \pi_{o_3})$, where

$$\begin{aligned}
\pi_{o_1} &= (M_{z_0}^l, \phi_1^{s_1}, M_{z_1}^l, \phi_2^{s_2}, \ldots, M_{z_{n-1}}^l, \phi_n^{s_n}, M_{z_n}^l), \text{ with } s_i \in \{1, 2\}, i = 1, 2, \ldots, n, \\
\pi_{o_2} &= (M_{x_1}^l, \phi_1^{t_1}, M_{x_2}^l, \phi_2^{t_2}, \ldots, M_{x_n}^l, \phi_n^{t_n}), \text{ with } t_i = 4 \text{ (resp., 3) if } s_i = 1 \text{ (resp., 2)}, i = 1, 2, \ldots, n, \\
\pi_{o_3} &= (M_{7n+2}^l, M_{7n+3}^l, \ldots, M_m^l, M_{m+1}^l).
\end{aligned}$$

We then show that $T(\pi_o) \leq 42n\delta + 2B$ implies that there exists a solution to PARTITION.

We require additional notation and definition for the rest of the proof. Let $\mathcal{S}_M = \{M_{z_0}, M_{z_i}, M_{x_i}, M_{y_{i,j}} \mid i = 1, 2 \ldots, n;\ j = 1, 2, 3\}$. Let $\mathcal{A}_k, k \in M$, denote the robot's travel path between M_k^l and M_k^u and let L_k denote the robot's travel time in $\mathcal{A}_k$ excluding the partial and full waiting times. A *primary pass* refers to the robot's travel between unloading M_r and loading the next machine $M_{r+1}, r = 0, 1, \ldots, m$. Any other pass of the robot across any two machines M_i and $M_j, i \neq j$ (while the robot is either traveling directly from M_i to M_j or while it is traveling between any other pair of machines) is referred to as a *secondary pass* across M_i and M_j. Table 8 summarizes some of the notation that is used repeatedly in the remainder of the proof.

Next, we establish some properties of cycle π_o. Note that, since $V_p = 28n\delta$, we have $V_s \leq 14n\delta + 2B$ in π_o.

- **Claim 1**: *In cycle π_o, the machines in $\mathcal{S}_M$ must have usage $\mathcal{U}_2$.*

  Proof of Claim 1: There are exactly $5n + 1$ machines in set $\mathcal{S}_M$. Note that the primary robot travel time for any 1-unit cycle is $V_p = 28n\delta$. If a machine in $\mathcal{S}_M$ has usage $\mathcal{U}_1$, then it contributes at least $14n\delta$ towards the full waiting time. Thus, if two or more machines have usage $\mathcal{U}_1$, then the cycle time $T(\pi_o) \geq 28n\delta + 28n\delta = 56n\delta$, which exceeds the upper bound $42n\delta + 2B$ (note that $\delta = 3B$). Suppose exactly one machine has usage $\mathcal{U}_1$. Each machine with usage $\mathcal{U}_2$ contributes at least $\delta = 3B$ to the secondary travel time. Thus, $T(\pi_o) \geq 28n\delta + 5n\delta + 14n\delta = 47n\delta > 42n\delta + 2B$, which contradicts the required upper bound on $T(\pi_o)$. The result follows. □

- **Claim 2**: *In π_o, the number of secondary passes across any two adjacent machines, M_i and M_{i+1}, is even, $i = 0, 1, \ldots, m$.*

  Proof of Claim 2: Let there exist an odd number of secondary passes across consecutive machines M_i and M_{i+1}. The only situation that results in secondary travel is when there exist two machines M_j and M_k, $j \neq k$, with each having usage $\mathcal{U}_2$ and M_j^l is immediately followed by M_k^u.

  Depending on the relationship between the three indices i, j, k, and the sign of the quantity $[\frac{(m+2)}{2} - (\max\{i,j,k\} - \min\{i,j,k\})]$, there are twelve different cases. For brevity, we provide a detailed proof under one case; the proofs under the other cases are similar. Consider the case $j \leq i$, $k \geq i+1$, and $k - j < \frac{(m+2)}{2}$. Then, there exists at least one secondary pass across the machines M_i and M_{i+1}. In order to complete cycle π_o, the robot must return to machine M_j for unloading. There are two possibilities. If the robot returns to M_j by traversing the intermediate set of machines $\{M_{j+1}, M_{j+2}, \ldots, M_i, M_{i+1}, \ldots, M_k\}$, then the number of secondary passes across M_i and M_{i+1} is even and, therefore, contradicts our assumption. Otherwise, if the robot returns to M_j by traversing the intermediate set of machines $\{M_{k+1}, M_{k+2}, \ldots, M_m, M_{m+1}, M_0, \ldots, M_{j-1}\}$, then the number of secondary passes across M_i and M_{i+1} is odd. In the latter case, the robot travels across all $(m+2)$ machines between the activities M_j^l and M_j^u (an example of this is the odd-even cycle in Figure 3(c)). Therefore, when the robot completes cycle π_o, the total secondary time is at least $(m+2)\delta$ and $V_s \geq (m+2)\delta = 28n\delta$. This implies that $T(\pi_o) \geq 28n\delta + 28n\delta > 42n\delta + 2B$, which contradicts the required upper bound on $T(\pi_o)$. □

Remark 1: As a consequence of Claim 2, the number of secondary passes across two adjacent machines, M_i and M_{i+1}, $i = 0, 1, \ldots, m$, in π_o is even. Hence, first, by starting with M_0 and scanning the machines in a forward pass until M_{m+1}, we group the machines in the set $\{M_0, M_1, M_2, \ldots, M_m, M_{m+1}\}$ into disjoint subsets $\mathcal{Q}_j$, $j = 1, 2, \ldots, t$, of consecutive machines, say $\mathcal{Q}_j = \{M_{j_1}, M_{j_1+1}, \ldots, M_{j_1+n_j}\}$, such that: (i) the subsets $\mathcal{Q}_j, j = 1, 2, \ldots, t$, are arranged so that their first machine indices are in ascending order. That is, $0 \leq j_1 < (j+1)_1, j = 1, 2, \ldots, t-1$, and $t_1 \leq m$, (ii) there exist at least two secondary passes across each pair of adjacent machines in subset $\mathcal{Q}_j$, and (iii) there are no secondary passes across the pair of machines (M_{j_1-1}, M_{j_1}), $j_1 \geq 1$, and $(M_{j_1+n_j}, M_{j_1+n_j+1})$, $j_1 + n_j \leq m$. Using Claim 1 and Claim 2, $t \geq 1$. The definition of $\mathcal{Q}_j$ implies properties P_1-P_7 below:

P_1: $|\mathcal{Q}_j| = n_j + 1 \geq 2$ and the secondary travel time across the machines in $\mathcal{Q}_j$ is at least $2n_j\delta$.

P_2: If $\mathcal{Q}_j = \{M_{j_1}, M_{j_1+1}\}$, i.e., $|\mathcal{Q}_j| = 2$, then either $j_1 = 0$ or $j_1 + 1 = m + 1$.

P_3: Machines $M_{j_1+n_j+1}$, $M_{j_1+n_j+2}$, $\ldots$, $M_{(j+1)_1-1}$, $j = 1, 2, \ldots, t-1$, which are located between the sets $\mathcal{Q}_j$ and $\mathcal{Q}_{j+1}$, have usage $\mathcal{U}_1$.

P_4: Machines M_1, M_2, $\ldots$, M_{j_1-1}, which are located between M_0 and M_{j_1}, and machines $M_{t_1+n_t+1}$, $M_{t_1+n_t+2}$, $\ldots$, M_m, which are located between $M_{t_1+n_t}$ and M_{m+1}, have usage $\mathcal{U}_1$.

P_5: There is no secondary pass between sets $\mathcal{Q}_i$ and $\mathcal{Q}_j$, $i \neq j$ (i.e., between a machine in $\mathcal{Q}_i$ and a machine in $\mathcal{Q}_j$), except possibly between sets $\mathcal{Q}_t$ and $\mathcal{Q}_1$.

P_6: $|\mathcal{Q}_1| \leq 7n + 1$. To see this, suppose $|\mathcal{Q}_1| > 7n + 1$. Since the number of secondary passes across all the machines in $\mathcal{Q}_1$ is at least two, the total secondary travel time in the cycle π_o is $V_s \geq 2(7n + 1)\delta$. This implies that $T(\pi_o) \geq 28n\delta + 2(7n + 1)\delta > 42n\delta + 2B$, which contradicts the required upper bound on $T(\pi_o)$.

Using Property P_6 and the fact that $t \geq 1$, we have $1_1 + n_1 + 1 \in M$. Therefore, without loss of generality, we assume in the rest of the proof that the 1-unit cycle π_o starts with the loading of machine $M_{1_1+n_1+1}$, i.e., with $M^l_{1_1+n_1+1}$. Hence, $\pi_o = (M^l_{1_1+n_1+1}, M^l_{1_1+n_1+2}, \ldots, M^l_{2_1-1}$, $\hat{\pi}^2, M^l_{2_1+n_2+1}, \ldots, \hat{\pi}^3, \ldots, \hat{\pi}^{t-1}, \ldots, \psi)$, where $\hat{\pi}^j$, $j = 2, 3 \ldots, t-1$, is a subsequence of π_o, such that both loading and unloading of machines in $\mathcal{Q}_j$ are performed in $\hat{\pi}^j$. Furthermore, ψ is a subsequence of π_o, in which loading and unloading of all the machines in $\{\mathcal{Q}_t, \mathcal{Q}_1\}$ are performed, where $\{\mathcal{Q}_t, \mathcal{Q}_1\} = \{M_{t_1}, M_{t_1+1}, \ldots, M_{m+1}, M_0, \ldots, M_{1_1}, M_{1_1+1}, \ldots, M_{1_1+n_1}\}$. Thus, we have the following property.

P$_7$: In π_o, the robot performs both loading and unloading activities for all the machines in $\mathcal{Q}_j$ before it performs the same for the machines in $\mathcal{Q}_{j+1}$, $j = 2, 3, \ldots, t-1$. Consequently, $M^l_{j_1}, j = 2, 3, \ldots, t$, precedes $M^u_{j_1}$ and $M^l_{j_1+n_j}, j = 1, 3, \ldots, t-1$, precedes $M^u_{j_1+n_j}$.

In π_o, exactly one of the following two cases occurs: (i) there exists at most one secondary pass between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ (i.e., between a machine in $\mathcal{Q}_1$ and a machine in $\mathcal{Q}_t$) and (ii) there exist at least two secondary passes between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$. This distinction is required later in our proof. Figure 8, which is placed at the end of the proof of Theorem 1, provides an example of these two cases.

Figure 8(a) illustrates Case (i), where there exists at most one secondary pass between $\mathcal{Q}_t$ and $\mathcal{Q}_1$. Recall that both loading and unloading of the machines in sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ are performed in the subsequence ψ. Thus, in this case, the robot completes both loading and unloading of all the machines in $\mathcal{Q}_t$ before moving to $\mathcal{Q}_1$. Hence, ψ can be decomposed into two subsequences, namely $\hat{\pi}^t$ and $\hat{\pi}^1$, where the robot performs both loading and unloading of the machines in $\mathcal{Q}_t$ (resp., $\mathcal{Q}_1$) in $\hat{\pi}^t$ (resp., $\hat{\pi}^1$).

Figure 8(b) illustrates Case (ii), where the robot travels between the two sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ in performing the loading and unloading of the machines in $\mathcal{Q}_1$. Thus, in this case, ψ cannot be decomposed into $\hat{\pi}^t$ and $\hat{\pi}^1$ as above. In Figure 8(b), the number of secondary passes between the sets $\mathcal{Q}_t$ and $\mathcal{Q}_1$ is three.

- **Claim 3**: *There exists at least one machine $M_{k^*} \in \mathcal{S}_M$ such that either $M_{k^*} \in \mathcal{Q}_1$ or $M_{k^*} \in \mathcal{Q}_2$, $p_{k^*} \geq 14n\delta$, and $M^l_{k^*}$ precedes $M^u_{k^*}$ in π_o.*

  Proof of Claim 3: Observe that $M^l_{1_1+n_1}$ precedes $M^u_{1_1+n_1}$ in the subsequence ψ. We consider the following two cases: (i) $M_{1_1+n_1} \in \mathcal{S}_M$ and (ii) $M_{1_1+n_1} \notin \mathcal{S}_M$.

  (i) $M_{1_1+n_1} \in \mathcal{S}_M$: $p_{1_1+n_1} \geq 14n\delta$. Hence, $k^* = 1_1 + n_1$. The result follows.

  (ii) $M_{1_1+n_1} \notin \mathcal{S}_M$: From Property P$_6$, $|\mathcal{Q}_1| \leq 7n + 1$. This implies that $M_{1_1+n_1}$ is an a-machine. Note that both the machines that are adjacent to an a-machine are in $\mathcal{S}_M$. Hence, $M_{1_1+n_1+1} \in \mathcal{S}_M$, $p_{1_1+n_1+1} \geq 14n\delta$ and has usage $\mathcal{U}_2$ (Claim 1). Consequently, $M_{1_1+n_1+1}$

belongs to $\mathcal{Q}_2$. Thus, for $k^* = 1_1 + n_1 + 1 = 2_1$, we have $M_{k^*} \in \mathcal{Q}_2$, $p_{k^*} \geq 14n\delta$. Since $M^l_{2_1}$ precedes $M^u_{2_1}$, the result follows. $\square$

As a consequence of Claim 3, in the rest of the proof, we assume that there exists a machine M_{k^*} such that, $M_{k^*} \in \mathcal{Q}_1$ or $M_{k^*} \in \mathcal{Q}_2$, $p_{k^*} \geq 14n\delta$, and $M^l_{k^*}$ precedes $M^u_{k^*}$ in π_o. The robot's travel in $\mathcal{A}_{k^*}$ (recall that $\mathcal{A}_{k^*}$ denotes the robot's travel path between $M^l_{k^*}$ and $M^u_{k^*}$ and L_{k^*} is the corresponding travel time) can be decomposed into primary and secondary passes across adjacent machines. Let $\mathcal{A}^s_{k^*}$ denote the set of secondary passes in $\mathcal{A}_{k^*}$. In Claim 4, we prove that the secondary travel time in π_o is at least L_{k^*}. To do this, we show that for each primary pass between a pair of adjacent machines in $\mathcal{A}_{k^*}$, there exists a corresponding secondary pass, across the same pair of machines, which is not in $\mathcal{A}_{k^*}$. Let $\bar{\mathcal{A}}^s_{k^*}$ denote the set of these corresponding secondary passes. For clarity, we denote the total travel time for the secondary passes in $\mathcal{A}^s_{k^*} \cup \bar{\mathcal{A}}^s_{k^*}$ as V^*_s. An example, placed at the end of the proof of Theorem 1, illustrates $\mathcal{A}_{k^*}$ on the 1-unit cycle given in Figure 8(a).

- **Claim 4**: $L_{k^*} = V^*_s \leq V_s$.

Proof of Claim 4: We provide the proof under two cases: (i) $M_{k^*} \in \mathcal{Q}_1$ and (ii) $M_{k^*} \in \mathcal{Q}_2$.

Case (i): Let $M_{k^*} \in \mathcal{Q}_1$. There are two possibilities: (a) there exists at most one secondary pass between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ and (b) there exist at least two secondary passes between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$. Recall that the loading and unloading of the machines in sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ are performed in the subsequence ψ. Also, note that under the first possibility, the robot completes both loading and unloading of all the machines in $\mathcal{Q}_t$ before moving to $\mathcal{Q}_1$. Hence, ψ can be decomposed into two subsequences $\hat{\pi}^t$ and $\hat{\pi}^1$, where the robot performs both loading and unloading of the machines in $\mathcal{Q}_t$ (resp., $\mathcal{Q}_1$) in $\hat{\pi}^t$ (resp., $\hat{\pi}^1$). However, under the second possibility, the robot travels between the machines in the two sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ in performing the loading and unloading of the machines in $\mathcal{Q}_1$.

(a) Observe that the subsequence $\hat{\pi}^1$ can be decomposed into primary and secondary passes across adjacent machines in set $\mathcal{Q}_1$. The following two properties are immediate: (i) for any pair of adjacent machines, there is exactly one primary pass and (ii) the total number of secondary passes across each pair of adjacent machines in $\hat{\pi}^1$ is even and is at least two (from Claim 2).

The robot's travel in $\mathcal{A}_{k^*}$, $k^* \in M$, can be decomposed into primary and secondary passes across adjacent machines in $\mathcal{Q}_1$. From Property P_6, $|\mathcal{Q}_1| \leq 7n+1$. Consequently, in $\mathcal{A}_{k^*}$, the total number of passes (primary and secondary) across a pair of adjacent machines is even. Consider the unique primary pass in $\mathcal{A}_{k^*}$ across two adjacent machines, say M_{i_r} and M_{i_r+1}. Thus, the total number of secondary passes across M_{i_r} and M_{i_r+1} in $\mathcal{A}_{k^*}$ is odd. However, as noted earlier, the total number of secondary passes in $\hat{\pi}^1$ across M_{i_r} and M_{i_r+1} is even. Thus, there exists a secondary pass in $\hat{\pi}^1$ across M_{i_r} and M_{i_r+1} which does not belong to $\mathcal{A}_{k^*}$. We conclude that for each primary pass in $\mathcal{A}_{k^*}$, there is at least one secondary pass in $\hat{\pi}^1$ that does not belong to $\mathcal{A}_{k^*}$. The result follows.

(b) Note that $\mathcal{A}_{k^*}$ can be decomposed into primary (resp., secondary) passes across adjacent machines in $\mathcal{Q}_1$ and $\mathcal{Q}_t$. Since there exist at least two secondary passes between $\mathcal{Q}_1$ and $\mathcal{Q}_t$, it is clear that $M_{m+1} \in \mathcal{Q}_t$ and $M_0 \in \mathcal{Q}_1$. Furthermore, since the total number of secondary passes across the machines in $\mathcal{Q}_1 \cup \mathcal{Q}_t$ is at least $2(|\mathcal{Q}_1| + |\mathcal{Q}_t| - 2)\delta$, we have (using an argument similar to that in Property P_6), $(|\mathcal{Q}_1| + |\mathcal{Q}_t|) \leq 7n + 2$. Consequently, in $\mathcal{A}_{k^*}$, the total number of passes (primary and secondary) across a pair of adjacent machines in $\mathcal{Q}_1 \cup \mathcal{Q}_t$ is even. The result now follows from the argument in Case (i)-(a).

Case (ii): Let $M_{k^*} \in \mathcal{Q}_2$. Then, $\mathcal{A}_{k^*}$ can be decomposed into primary and secondary passes across adjacent machines in $\mathcal{Q}_2$. As in Property P_6, we have $|\mathcal{Q}_2| \leq 7n + 1$. Consequently, in $\mathcal{A}_{k^*}$, the total number of passes (primary and secondary) across a pair of adjacent machines in $\mathcal{Q}_2$ is even. The result now follows from the argument given in Case (i)-(a). □

Recall that for cycle π_o, (i) V_p and V_s denote, respectively, the primary and secondary robot travel times and (ii) W_p and W_f denote, respectively, the total partial and full waiting times of the robot at the machines. $T(\pi_o) = V_p + V_s + W_p + W_f$ (see Table 8). Note that the elapsed time between the loading of M_{k^*} and the unloading of M_{k^*} is $(L_{k^*} + W_p^* + W_f^*)$, where we let W_p^* and W_f^* be, respectively, the total partial waiting and the total full waiting times between $M_{k^*}^l$ and $M_{k^*}^u$.

- **Claim 5**: $(V_s + W_p + W_f) \geq (V_s^* + W_p^* + W_f^*) \geq 14n\delta$.

Proof of Claim 5: We consider two cases: (i) $M_{k^*} \in \mathcal{Q}_1$ and (ii) $M_{k^*} \in \mathcal{Q}_2$. We further divide Case (i) into two subcases: (a) there exists at most one secondary pass between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ and (b) there exist at least two secondary passes between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$. We provide a proof for Case (i)-(a) below. The proofs for Case (i)-(b) and Case (ii) are similar and, therefore, not explicitly provided.

Case (i)-(a): Recall that $M_{k^*} \in \mathcal{Q}_1$ and activity $M^l_{k^*}$ precedes activity $M^u_{k^*}$ in $\hat{\pi}^1$. The processing time of M_{k^*} is $p_{k^*} \geq 14n\delta$ (Claim 3). From Claim 4, we have $L_{k^*} = V^*_s \leq V_s$. The elapsed time between the loading of M_{k^*} and the unloading of M_{k^*} is $(L_{k^*} + W^*_p + W^*_f) \geq 14n\delta$. Thus, $(V_s + W_p + W_f) \geq (V^*_s + W^*_p + W^*_f) = (L_{k^*} + W^*_p + W^*_f) \geq 14n\delta$. □

- **Claim 6**: *There cannot exist $t > 1$ subsets, $\mathcal{Q}_j$, $j = 1, 2, \ldots, t$, which are defined as in Remark 1.*

  Proof of Claim 6: Suppose $t > 1$. We provide the proof under two cases.

  (i) Suppose $M_{k^*} \in \mathcal{Q}_1$. From Claim 5, $(V^*_s + W^*_p + W^*_f) \geq 14n\delta$. When there exists at most one secondary pass between $\mathcal{Q}_1$ and $\mathcal{Q}_t$, the robot's travel in each subsequence $\hat{\pi}^j, j = 2, 3, \ldots, t$, contributes at least an additional amount δ towards the secondary travel time other than that included in V^*_s. Using this observation together with Claim 4, we have $L_{k^*} + (t-1)\delta = V^*_s + (t-1)\delta \leq V_s$. Thus, $(V_s + W_p + W_f) \geq (V^*_s + W^*_p + W^*_f) + (t-1)\delta \geq 14n\delta + (t-1)\delta$. Since $V_p = 28n\delta$, $T(\pi_o) \geq 28n\delta + 14n\delta + (t-1)\delta > 42n\delta + 2B$, which exceeds the required upper bound.

  When there exist at least two secondary passes between $\mathcal{Q}_1$ and $\mathcal{Q}_t$, it is clear that $M_{m+1} \in \mathcal{Q}_t$ and $M_0 \in \mathcal{Q}_1$. Recall that the total number of secondary passes across the machines in $\mathcal{Q}_1 \cup \mathcal{Q}_t$ is at least $2(|\mathcal{Q}_1| + |\mathcal{Q}_t| - 2)\delta$ and $(|\mathcal{Q}_1| + |\mathcal{Q}_t|) \leq 7n + 2$ (refer to the proof of Claim 4). Note that $M_{7n+1} \in \mathcal{S}_M$ has usage $\mathcal{U}_2$ (Claim 1). Since $|\mathcal{Q}_t| \geq 2$, we have $|\mathcal{Q}_1| \leq 7n$. Hence $M_{7n+1} \notin \mathcal{Q}_1$. As a result, $M_{7n+1} \in \mathcal{Q}_2$ and subsequence $\hat{\pi}^2$ contributes at least an additional amount δ towards the secondary travel time other than that included in V^*_s. This observation and Claim 4 together imply that $L_{k^*} + \delta = V^*_s + \delta \leq V_s$. Thus, $(V_s + W_p + W_f) \geq (V^*_s + W^*_p + W^*_f) + \delta \geq 14n\delta + \delta$. Hence, $T(\pi_o) \geq 28n\delta + 14n\delta + \delta > 42n\delta + 2B$, which exceeds the required upper bound.

  (ii) Suppose $M_{k^*} \in \mathcal{Q}_2$. Since $|\mathcal{Q}_1| \geq 2$, we have $M_1 \in \mathcal{Q}_1$. As M_1 has usage $\mathcal{U}_2$, the subsequence ψ contributes at least an additional amount δ towards the secondary travel time other than that included in V^*_s. This observation and Claim 4 imply that $L_{k^*} + \delta = V^*_s + \delta \leq V_s$. Thus, $(V_s + W_p + W_f) \geq (V^*_s + W^*_p + W^*_f) + \delta \geq 14n\delta + \delta$. Hence, $T(\pi_o) \geq 28n\delta + 14n\delta + \delta > 42n\delta + 2B$, which exceeds the required upper bound. □

As a consequence of Claim 6, we have $t = 1$. From Claim 1, each machine in $\mathcal{S}_M$ has usage $\mathcal{U}_2$. Thus, all $5n + 1$ machines in $\mathcal{S}_M$ belong to $\mathcal{Q}_1$, and are both loaded and unloaded in $\hat{\pi}^1$. Recall that $M_1 = M_{z_0} \in \mathcal{S}_M$ and $M_{7n+1} = M_{z_n} \in \mathcal{S}_M$. Also, the $2n$ a-machines and the machines in $\mathcal{S}_M$ (defined in the construction at the beginning of the proof of Theorem 1) belong to the set $\{M_1, M_2, \ldots, M_{7n+1}\}$. From Property P_6, $|\mathcal{Q}_1| \leq 7n + 1$. Thus, the $7n + 1$ machines $M_1, M_2, \ldots, M_{7n+1}$, are in $\mathcal{Q}_1$ and form the subsequence $\hat{\pi}^1$. Recall that the 1-unit cycle π_o starts with the loading of machine $M_{1_1+n_1+1}$, where $(1_1 + n_1 + 1) = 7n + 2$.

- **Claim 7**: *In cycle π_o, the b-machines $\{M_{7n+2}, M_{7n+3}, \ldots, M_m\}$ must have usage $\mathcal{U}_1$.*

  Proof of Claim 7: The primary travel time in π_o is $V_p = 28n\delta$. From Claim 5, we have $(V_s^* + W_p^* + W_f^*) \geq 14n\delta$. If any b-machine has usage $\mathcal{U}_2$, then it contributes at least $\delta = 3B$ to the secondary travel time other than that included in V_s^* and $V_s \geq V_s^* + \delta$. Thus, $(V_s + W_p + W_f) \geq (V_s^* + W_p^* + W_f^*) + 3B \geq 14n\delta + 3B$. Consequently, $T(\pi_o) \geq V_p + V_s + W_p + W_f \geq 42n\delta + 3B > 42n\delta + 2B$. □

- **Claim 8**: *π_o is of the following form: $\pi_o = (M^l_{7n+2}, M^l_{7n+3}, \ldots, M^l_m, M^l_{m+1}, \hat{\pi}^1)$, where $\hat{\pi}^1$ defines the loading/unloading of the machines in $\{M_1, M_2, \ldots, M_{7n+1}\}$.*

  Proof of Claim 8: Follows from Claims 6 and 7. □

Next, in Claims 9, 10, and 11, we establish the precise structure of $\hat{\pi}^1$.

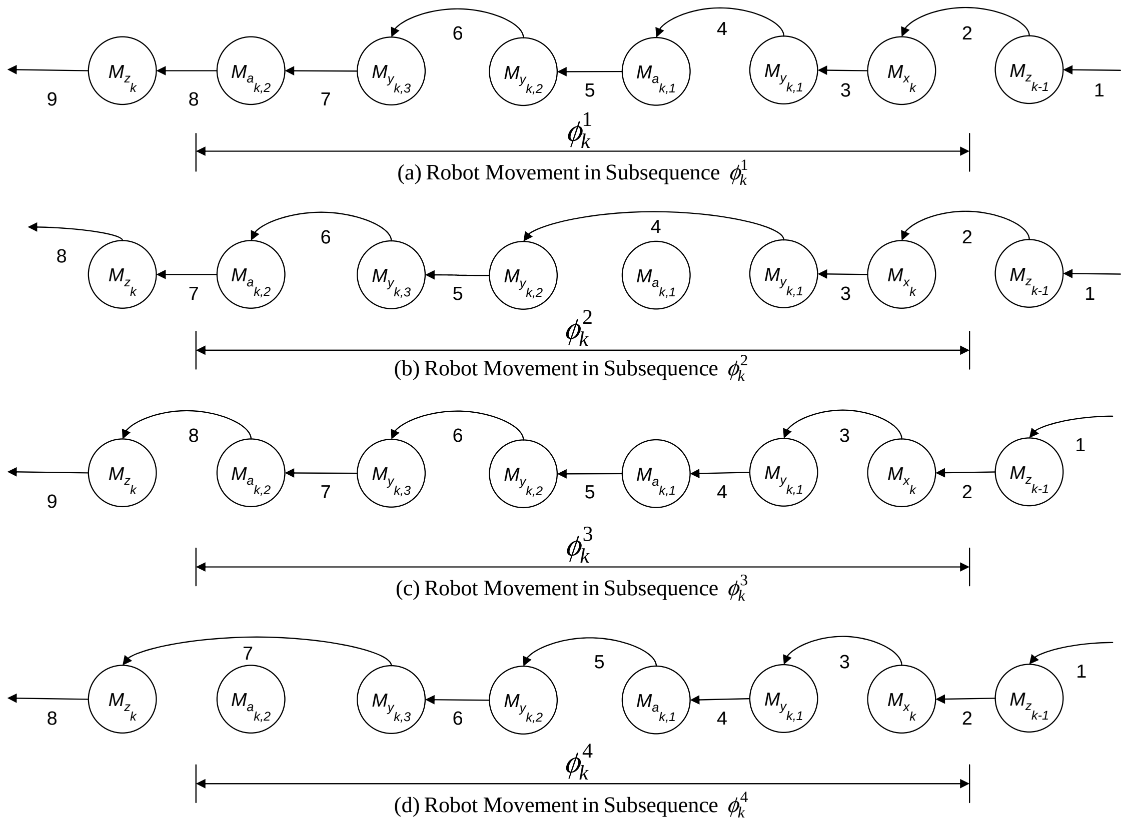
- **Claim 9**: *In subsequence $\hat{\pi}^1$, $M^u_{x_i}$ must precede $M^l_{x_i}$, $i = 1, 2, \ldots, n$.*

  Proof of Claim 9: Suppose, for machine M_{x_i}, the activity $M^u_{x_i}$ does not precede activity $M^l_{x_i}$ in $\hat{\pi}^1$. The processing time of machine M_{x_i} is $p_{x_i} = 28n\delta$. Note that the robot travel time between the loading of M_{x_i} and the unloading of M_{x_i} is L_{x_i}. From Claims 4 and 6, we have $L_{x_i} \leq V_s$. Moreover, the elapsed time between the loading of M_{x_i} and the unloading of M_{x_i} is $(L_{x_i} + W'_p + W'_f) \geq p_{x_i} = 28n\delta$, where W'_p, W'_f are, respectively, the total partial waiting and the total full waiting times between $M^l_{x_i}$ and $M^u_{x_i}$. Thus, $V_s + W_p + W_f \geq V_s + W'_p + W'_f \geq L_{x_i} + W'_p + W'_f \geq 28n\delta$. Consequently, using $\delta = 3B$, we have $T(\pi_o) = V_p + V_s + W_p + W_f \geq 28n\delta + 28n\delta > 42n\delta + 2B$. □

- **Claim 10**: *$V_s = 14n\delta$. Furthermore, $\hat{\pi}^1$ takes the following form: $\hat{\pi}^1 = (M^l_{z_0}, M^l_{y_{1,1}}, \ldots, M^l_{z_1}, M^l_{y_{2,1}}, \ldots, M^l_{z_n}, M^l_{x_1}, M^l_{a_{1,1}}, \ldots, M^l_{x_2}, M^l_{a_{2,1}}, \ldots, M^l_{x_n}, M^l_{a_{n,1}}, \ldots, M^u_{z_n})$.*

Proof of Claim 10: Since $V_p = 28n\delta$ and $T(\pi_o) \leq 42n\delta + 2B$, we have $V_s \leq 14n\delta + 2B$. From Claims 2 and 6 and the argument following Claim 6, the secondary robot travel time in $\hat{\pi}^1$ is at least $14n\delta$. Any additional secondary robot travel adds at least $\delta = 3B$ to V_s, resulting in $V_s \geq 14n\delta + 3B$. This contradicts the requirement that $V_s \leq 14n\delta + 2B$. Thus, $V_s = 14n\delta$.

From Claim 9 and $V_s = 14n\delta$, $\hat{\pi}^1$ consists of the following subsequences: $(M^l_{z_{i-1}}, M^l_{y_{i,1}})$, $i = 1, 2, \ldots, n$, and $(M^l_{x_i}, M^l_{a_{i,1}})$, $i = 1, 2, \ldots, n$. Moreover, $(M^l_{z_{i-1}}, M^l_{y_{i,1}})$ precedes $(M^l_{x_i}, M^l_{a_{i,1}})$, $i = 1, 2, \ldots, n$. Since $V_s = 14n\delta$, the only possible arrangement of these subsequences in $\hat{\pi}^1$ is $(M^l_{z_0}, M^l_{y_{1,1}}, \ldots, M^l_{z_1}, M^l_{y_{2,1}}, \ldots, M^l_{z_n}, M^l_{x_1}, M^l_{a_{1,1}}, \ldots, M^l_{x_2}, M^l_{a_{2,1}}, \ldots, M^l_{x_n}, M^l_{a_{n,1}}, \ldots, M^u_{z_n})$, for otherwise $V_s \geq 14n\delta + 3B$. □



(a) Robot Movement in Subsequence ϕ^1_k

(b) Robot Movement in Subsequence ϕ^2_k

(c) Robot Movement in Subsequence ϕ^3_k

(d) Robot Movement in Subsequence ϕ^4_k

Figure 7: Robot movement in subsequence $\phi^i, i = 1, 2, 3, 4$.

- **Claim 11**: $\hat{\pi}^1 = (M^l_{z_0}, \phi_1^{s_1}, M^l_{z_1}, \phi_2^{s_2}, \ldots, M^l_{z_{n-1}}, \phi_n^{s_n}, M^l_{z_n}, M^l_{x_1}, \phi_1^{t_1}, M^l_{x_2}, \phi_2^{t_2}, \ldots, M^l_{x_n}, \phi_n^{t_n}, M^u_{z_n})$, *where* $\phi^1_k = (M^l_{y_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}})$, $\phi^2_k = (M^l_{y_{k,1}}, M^l_{y_{k,3}})$, $\phi^3_k = (M^l_{a_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}})$, *and* $\phi^4_k = (M^l_{a_{k,1}}, M^l_{y_{k,3}})$, *with* $s_k \in \{1, 2\}$, *and* $t_k = 4$ *(resp.,* 3*) if* $s_k = 1$ *(resp.,* 2*),* $k = 1, 2, \ldots, n$.

Proof of Claim 11: From Claim 10, we have $\hat{\pi}^1 = (M^l_{z_0}, M^l_{y_{1,1}}, \ldots, M^l_{z_1}, M^l_{y_{2,1}}, \ldots, M^l_{z_n}, M^l_{x_1}, M^l_{a_{1,1}}, \ldots, M^l_{x_2}, M^l_{a_{2,1}}, \ldots, M^l_{x_n}, M^l_{a_{n,1}}, \ldots, M^u_{z_n})$. We first determine the machine loading sequence in $(M^l_{y_{k,1}}, \ldots, M^l_{z_k}), k = 1, 2, \ldots, n$. Since $V_s = 14n\delta$ (Claim 10) and the usage of machines $M_{y_{k,2}}$ and $M_{y_{k,3}}$ is $\mathcal{U}_2$ (Claim 1), the possible loading sequence in $(M^l_{y_{k,1}}, \ldots, M^l_{z_k})$ is either $(M^l_{y_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}}, M^l_{z_k}) = (\phi^1_k, M^l_{z_k})$ or $(M^l_{y_{k,1}}, M^l_{y_{k,3}}, M^l_{z_k}) = (\phi^2_k, M^l_{z_k})$. We now determine the loading sequence in $(M^l_{x_k}, M^l_{a_{k,1}}, \ldots), k = 1, 2, \ldots, n$. Since sequence $(M^l_{z_0}, \ldots, M^l_{z_n})$ must precede sequence $(M^l_{z_n}, \ldots, M^u_{z_n})$ in $\hat{\pi}^1$, the loading sequence in $(M^l_{x_k}, M^l_{a_{k,1}}, \ldots)$ should be consistent with the corresponding sequences ϕ^1_k and ϕ^2_k in $(M^l_{z_0}, \ldots, M^l_{z_n})$. The only possible sequences are either $(M^l_{x_k}, M^l_{a_{k,1}}, M^l_{y_{k,3}}) = (M^l_{x_k}, \phi^4_k)$ (compatible with ϕ^1_k) or $(M^l_{x_k}, M^l_{a_{k,1}}, M^l_{y_{k,2}}, M^l_{a_{k,2}}) = (M^l_{x_k}, \phi^3_k)$ (compatible with ϕ^2_k) (see Figure 7). Thus, $\hat{\pi}^1 = (M^l_{z_0}, \phi^{s_1}_1, M^l_{z_1}, \phi^{s_2}_2, \ldots, M^l_{z_{n-1}}, \phi^{s_n}_n, M^l_{z_n}, M^l_{x_1}, \phi^{t_1}_1, M^l_{x_2}, \phi^{t_2}_2, \ldots, M^l_{x_n}, \phi^{t_n}_n, M^u_{z_n})$. □

- **Claim 12**: *In π_o, there is no partial waiting at (i) the x-machines, (ii) the y-machines, (iii) the z-machines $M_{z_i}, i = 1, 2 \ldots, n-1$, and (iv) the a-machines that have usage $\mathcal{U}_2$.*

  Proof of Claim 12: From Claim 8, $\pi_o = (M^l_{7n+2}, M^l_{7n+3}, \ldots, M^l_m, M^l_{m+1}, \hat{\pi}^1)$. Also, Claim 11 specifies the machine loading sequence for $\hat{\pi}^1$. We have

  - For $M_k \in \{M_{z_i}, i = 1, 2 \ldots, n-1\}$, activity M^l_k precedes activity M^u_k in π_o. Since $L_k = 14n\delta$ and $p_k = 14n\delta$, there is no partial waiting at these machines.
  - For $M_k \in \{M_{x_i}, i = 1, 2 \ldots, n\}$, activity M^u_k precedes activity M^l_k. Since $L_k = (m+2)\delta = 28n\delta$ and $p_k = 28n\delta$, there is no partial waiting at these machines.
  - For an a-machine, say M_k, with usage $\mathcal{U}_2$, the processing time is $p_k \le B < \delta$. Thus, there is no partial waiting at these machines.
  - For some y-machines, activity M^l_k precedes activity M^u_k; in this case, $L_k = 14n\delta$ and $p_k = 14n\delta$. For the remaining y-machines, activity M^u_k precedes activity M^l_k; in this case, $L_k = (m+2)\delta = 28n\delta$ and $p_k = 14n\delta$. In either case, $p_k \le L_k$ and there is no partial waiting. □

  Let $[M^l_i, M^u_i]$ refer to the time interval between the instant of completion of activity M^l_i and the instant of completion of activity M^u_i.

- **Claim 13**: *There is no partial waiting at machines M_{z_0} and M_{z_n}*

  Proof of Claim 13: Claim 11 identifies the precise sequence of loading of the machines in $\hat{\pi}^1$. For $k = 1, 2, \ldots, n$, in the subsequence ϕ^1_k (resp., ϕ^3_k), machine $M_{a_{k,2}}$ (resp., $M_{a_{k,1}}$) has usage $\mathcal{U}_1$.

Thus, the full waiting time in ϕ_k^1 (resp., ϕ_k^3) is exactly a_k. Thus, $W_f = \sum_{i=1}^{n} a_i = 2B$. Next, the processing time at each of the machines M_{z_0} and M_{z_n} is $14n\delta + B$. Let f_0 and f_n denote the total full waiting times incurred in the intervals $[M_{z_0}^l, M_{z_0}^u]$ and $[M_{z_n}^l, M_{z_n}^u]$, respectively. From Claim 12, there is no partial waiting at any of the machines in these two intervals, except at machines M_{z_0} and M_{z_n}.

Suppose $f_0 < B$. Since the robot travel time between $M_{z_0}^l$ and $M_{z_0}^u$ is $14n\delta$ (Claim 11), the partial waiting time at M_{z_0} is $w_{z_0} = B - f_0$. Thus, $W_p \geq B - f_0$. Since $V_p = 28n\delta$, $W_f = 2B$, and from Claims 10 and 11, $V_s = 14n\delta$, we have $T(\pi_o) = V_p + V_s + W_p + W_f \geq 28n\delta + 14n\delta + B - f_0 + 2B$, which violates the condition $T(\pi_o) \leq 42n\delta + 2B$.

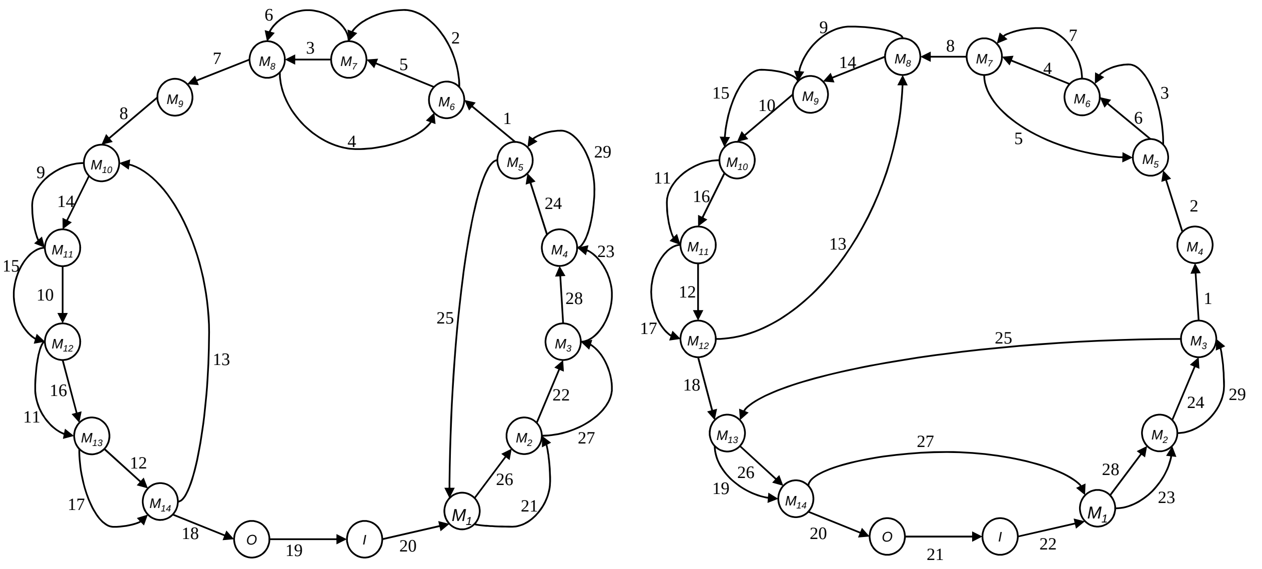
If $f_0 > B$, then $w_{z_0} = 0$. Since the robot travel time between $M_{z_n}^l$ and $M_{z_n}^u$ is $14n\delta$ and $f_n = 2B - f_0$, the partial waiting time at M_{z_n} is $w_{z_n} = B - f_n = f_0 - B$. Thus, $W_p \geq f_0 - B$. Then, $T(\pi_o) = V_p + V_s + W_p + W_f \geq 28n\delta + 14n\delta + f_0 - B + 2B$ which again violates the condition $T(\pi_o) \leq 42n\delta + 2B$. Thus, $f_0 = f_n = B$ and $w_{z_0} = w_{z_n} = 0$. □

- **Claim 14**: *If $T(\pi_o) \leq 42n\delta + 2B$, then there exists a solution to* Partition.

  Proof of Claim 14: Claims 8-11 specify completely the machine loading sequence in π_o. As mentioned earlier, $V_p = 28n\delta$ for all 1-unit cycles. From Claims 10 and 11, $V_s = 14n\delta$. From Claim 11, $W_f = \sum_{i=1}^{n} a_i = 2B$. From Claims 12 and 13, we have $W_p = 0$. Thus, $T(\pi_o) = 42n\delta + 2B$. From Claim 13, $f_0 = f_n = B$, i.e., the total full-waiting time in both intervals $[M_{z_0}^l, M_{z_0}^u]$ and $[M_{z_n}^l, M_{z_n}^u]$ is exactly B. Let A_1 (resp., A_2) denote the processing times of the subset of a-machines in the interval $[M_{z_0}^l, M_{z_0}^u]$ (resp., $[M_{z_n}^l, M_{z_n}^u]$) where the robot performs full waiting. Then, $A_1 \cap A_2 = \emptyset$ and $\sum_{a_i \in A_1} a_i = \sum_{a_i \in A_2} a_i = B$. Therefore, we have a solution to Partition. This completes the proof of Theorem 1. ■

Description of Figure 8

In π_o, one of the following two cases occurs: (i) there exists at most one secondary pass between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$ and (ii) there exist at least two secondary passes between the sets $\mathcal{Q}_1$ and $\mathcal{Q}_t$. Figure 8 illustrates the above two cases. The 1-unit cycle π_1 given in Figure 8(a), illustrates Case (i) while the 1-unit cycle π_2 given in Figure 8(b) illustrates Case (ii). The machine loading sequence in each 1-unit cycle and the corresponding grouping of the machines are given below:



(a) The moves of the robot in π_1 **(b) The moves of the robot in π_2**

Figure 8: Two cases: (a) π_1: There is no secondary pass between subsets $\mathcal{Q}_1$ and $\mathcal{Q}_t$, $t = 3$, (b) π_2: There are three secondary passes between subsets $\mathcal{Q}_1$ and $\mathcal{Q}_t$, $t = 4$.

$$
\begin{aligned}
\pi_1 &= (M_6^l, M_8^l, M_7^l, M_9^l, M_{10}^l, M_{12}^l, M_{14}^l, M_{11}^l M_{13}^l, M_{15}^l, M_1^l, M_3^l, M_5^l, M_2^l, M_4^l), t = 3. \\
\mathcal{Q}_1 &= \{M_1, M_2, M_3, M_4, M_5\} \\
\mathcal{Q}_2 &= \{M_6, M_7, M_8\} \\
\mathcal{Q}_3 &= \{M_{10}, M_{11}, M_{12}, M_{13}, M_{14}\} \\
\hat{\pi}^1 &= (M_1^l, M_3^l, M_5^l, M_2^l, M_4^l) \\
\hat{\pi}^2 &= (M_6^l, M_8^l, M_7^l) \\
\hat{\pi}^3 &= (M_{10}^l, M_{12}^l, M_{14}^l, M_{11}^l, M_{13}^l) \\
\psi &= (\hat{\pi}^3, \hat{\pi}^1) = (M_{10}^l, M_{12}^l, M_{14}^l, M_{11}^l, M_{13}^l, M_1^l, M_3^l, M_5^l, M_2^l, M_4^l)
\end{aligned}
$$

$$
\begin{aligned}
\pi_2 &= (M_4^l, M_5^l, M_7^l, M_6^l, M_8^l, M_{10}^l, M_{12}^l, M_9^l, M_{11}^l, M_{13}^l, M_{15}^l, M_1^l, M_3^l, M_{14}^l, M_2^l), t = 4. \\
\mathcal{Q}_1 &= \{M_0, M_1, M_2, M_3\} \\
\mathcal{Q}_2 &= \{M_5, M_6, M_7\} \\
\mathcal{Q}_3 &= \{M_8, M_9, M_{10}, M_{11}, M_{12}\} \\
\mathcal{Q}_4 &= \{M_{13}, M_{14}, M_{15}\} \\
\hat{\pi}^2 &= (M_5^l, M_7^l, M_6^l) \\
\hat{\pi}^3 &= (M_8^l, M_{10}^l, M_{12}^l, M_9^l, M_{11}^l) \\
\psi &= (M_{13}^l, M_{15}^l, M_1^l, M_3^l, M_{14}^l, M_2^l)
\end{aligned}
$$

An example to illustrate $\mathcal{A}_{k^*}$: Consider the 1-unit cycle π_1 in Figure 8(a). Observe that $M_1 \in \mathcal{Q}_1$ and M_1^l precedes M_1^u in $\hat{\pi}_1$. Let $k^* = 1$. The robot's travel path $\mathcal{A}_1$ between M_1^l and M_1^u consists of the following: a move from M_1 to M_2 without carrying a part (a secondary pass indicated by arrow 21), a move from M_2 to M_3 carrying a part (a primary pass indicated by arrow 22), a move from M_3 to M_4 without carrying a part (a secondary pass indicated by arrow 23), a move from M_4 to M_5 carrying a part (a primary pass indicated by arrow 24), and a move from M_5 to M_1 without carrying a part (a secondary pass indicated by arrow 25).

The secondary pass from M_5 to M_1 (indicated by arrow 25) can be decomposed into secondary passes across adjacent machines as follows: a pass across M_5 and M_4, a pass across M_4 and M_3, a pass across M_3 and M_2, and a pass across M_2 and M_1. Thus, the path $\mathcal{A}_1$ can be decomposed into primary and secondary passes across adjacent machines. Note that $\mathcal{A}_1^s$ denotes the set of secondary passes between adjacent machines in the path $\mathcal{A}_1$, i.e., the passes (M_1, M_2), (M_3, M_4), (M_5, M_4), (M_4, M_3), (M_3, M_2), and (M_2, M_1). Furthermore, $\bar{\mathcal{A}}_1^s$ is the set of the secondary passes (M_2, M_3) and (M_4, M_5), indicated by arrows 27 and 29, respectively. Note that $L_1 = V_s^* = 8\delta$.

Appendix B: Proofs of Lemmas 1-7 and Theorems 2-5

Theorem 2: For an additive-travel-time cell under a circular layout, the cycle-time $T(\pi)$ of any 1-unit cycle π satisfies

$$T(\pi) \quad \geq \quad \max\left\{(m+2)\delta + 2(m+1)\epsilon + \sum_{i=1}^{m}\min\{p_i,\delta\},\ \max_{1\leq i\leq m} p_i + 4\delta + 4\epsilon\right\} = T_{LB}.$$

Proof: Consider the first argument. Each part requires $m+1$ part-transfer activities (Section 3.1) between successive machines: from machine M_i to machine $M_{i+1}, i = 0, 1, ..., m$. The transfer between successive machines M_i and M_{i+1} requires time $\delta + 2\epsilon$ and includes the following: unloading machine M_i (requiring time ϵ), a loaded forward robot movement (requiring time δ), and loading machine M_{i+1} (requiring time ϵ). Thus, for each part, the total time for these part-transfer activities is $(m+1)\delta + 2(m+1)\epsilon$. Also, an additional time of at least δ is required for the robot to return to the input M_0 to complete the cycle. Together, the total time for these activities is at least $(m+2)\delta + 2(m+1)\epsilon$. The summation term in the first argument represents the time *between* the part-transfer activities. After the robot completes loading machine M_i, $i = 1, 2, \ldots, m$, it either waits at machine M_i for the duration of its processing (time p_i), or it moves to another machine (requiring time δ, at minimum) to begin another part-transfer activity. Thus, the time required for such activities is at least $\sum_{i=1}^{m}\min\{p_i,\delta\}$. It follows that $T(\pi) \geq (m+2)\delta + 2(m+1)\epsilon + \sum_{i=1}^{m}\min\{p_i,\delta\}$.

Note that for any 1-unit cycle π and any machine M_i, the instant of the loading of machine M_i can be viewed as the start of π. Also, the cell returns to the same state after executing π. Thus, $T(\pi) \geq p_i + 4\delta + 4\epsilon$, which is the minimum time between two successive loadings of M_i: processing time (p_i), transfer of the part from M_i to M_{i+1} (requiring time $\delta + 2\epsilon$), the travel time of the robot from M_{i+1} to M_{i-1} (requiring time 2δ), and transfer of a (new) part from M_{i-1} to M_i (requiring time $\delta + 2\epsilon$). Thus, $T(\pi) \geq \max_{1\leq i\leq m} p_i + 4\delta + 4\epsilon$. ■

Theorem 3: The 1-unit cycle π_c achieves a 2-approximation for problem $RF_m^\circ|(free, A, cyclic\text{-}1)|C_t$ and this bound is tight.

Proof: We divide our argument into two subcases:

Case 1: $p_{max} \leq (m+2)\delta + (m-1)\epsilon$. From (4), in this case, the cycle time of π_c satisfies

$$T(\pi_c) \quad \leq 2(m+2)\delta + 2(m+1)\epsilon.$$

From Theorem 2, if π^* is an optimum 1-unit cycle, then $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon$. Hence,

$$\frac{T(\pi_c)}{T(\pi^*)} \leq \frac{2(m+2)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon} \leq \frac{2(m+2)\delta}{(m+2)\delta} = 2.$$

Case 2: $p_{max} > (m+2)\delta + (m-1)\epsilon$.

In this case, we have from (4)

$$T(\pi_c) \leq p_{max} + (m+2)\delta + (m+3)\epsilon.$$

Also, from Theorem 2, we have $T(\pi^*) \geq \max\{(m+2)\delta + 2(m+1)\epsilon, p_{max} + 4\delta + 4\epsilon\}$. Then,

$$\begin{aligned} \frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{T(\pi^*)} = \frac{p_{max}}{T(\pi^*)} + \frac{(m+2)\delta + (m+3)\epsilon}{T(\pi^*)}, \\ &< 1 + \frac{(m+2)\delta + (m+3)\epsilon}{(m+2)\delta + 2(m+1)\epsilon} \leq 2. \end{aligned}$$

To demonstrate the tightness of this bound, consider a cell with $m \geq 2$, $\epsilon = 0$, $p_{max} = p_k = (m+2)\delta$ for some index $k \in M$, and $p_j = 0, j \in M\backslash\{k\}$. The optimal 1-unit cycle for the cell is $\pi^* = (M_1^l, M_2^l, M_3^l, \ldots, M_{k-1}^l, M_{k+1}^l, M_k^l, M_{k+2}^l, M_{k+3}^l, \ldots, M_m^l, M_{m+1}^l)$ with cycle-time $T(\pi^*) = p_{max} + 4\delta = (m+6)\delta$. Since $T(\pi_c) = 2(m+2)\delta$, the ratio $\frac{T(\pi_c)}{T(\pi^*)} = \frac{2(m+2)\delta}{(m+6)\delta} \rightarrow 2$ as $m \rightarrow \infty$. ■

Lemma 1: If $p_{max} \geq 3m\delta + 2(m-1)\epsilon$, then the reverse cycle $\pi_d = (M_1^l, M_{m+1}^l, M_m^l, M_{m-1}^l, \ldots, M_2^l)$ achieves the optimal 1-unit cycle time.

Proof: When $p_{max} \geq 3m\delta + 2(m-1)\epsilon$, it follows from (2) that $T(\pi_d) = p_{max} + 4\delta + 4\epsilon$. Since $T(\pi_d)$ equals the lower bound of Theorem 2, cycle π_d is optimal. ■

Theorem 4: When $p_j \geq \delta, j = 1, 2, \ldots, m$, the reverse cycle π_d provides an asymptotically 3/2-approximation for $RF_m^\circ|(free, A, cyclic\text{-}1)|C_t$.

Proof: If $p_{max} \geq 3m\delta + 2(m-1)\epsilon$, then π_d is optimal by Lemma 1. Thus, without loss of generality, we can assume that $p_{max} < 3m\delta + 2(m-1)\epsilon$. Then, from (2), $T(\pi_d) = (3m+4)\delta + 2(m+1)\epsilon$. Thus, we have

$$\begin{aligned} \frac{T(\pi_d)}{T(\pi^*)} &\leq \frac{(3m+4)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \sum_{i=1}^m \min\{p_i, \delta\}}, \\ &\leq \frac{(3m+4)\delta + 2(m+1)\epsilon}{2(m+1)\delta + 2(m+1)\epsilon}, \text{ since } \min\{p_i, \delta\} = \delta \ \forall i, \\ &\leq \frac{(3m+4)\delta}{2(m+1)\delta} \rightarrow \frac{3}{2} \text{ as } m \rightarrow \infty. \end{aligned}$$

■

Lemma 2: If $p_i \leq \delta\ \forall i$, then the forward cycle $\pi_u = (M_1^l, M_2^l, M_3^l, \ldots, M_m^l, M_{m+1}^l)$ is optimal.

Proof: When $p_i \leq \delta, \forall i$, $T(\pi_u) = (m+2)\delta + 2(m+1)\epsilon + \sum_{j=1}^{m} p_j = T_{LB}$, the lower bound on the cycle time of any of 1-unit cycle (Theorem 2). The result follows. ∎

Lemma 3: If $p_i < 2\delta\ \forall i$, then the forward cycle $\pi_u = (M_1^l, M_2^l, M_3^l, \ldots, M_m^l, M_{m+1}^l)$ is a 3/2-approximation.

Proof: $T(\pi_u) = (m+2)\delta + 2(m+1)\epsilon + \sum_{j=1}^{m} p_j$. Also, $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + |B_2|\delta + \sum_{j \in B_3} p_j$. Thus,

$$\begin{aligned}
\frac{T(\pi_u)}{T(\pi^*)} &\leq \frac{(m+2)\delta + 2(m+1)\epsilon + \sum_{j=1}^{m} p_j}{(m+2)\delta + 2(m+1)\epsilon + |B_2|\delta + \sum_{j \in B_3} p_j} = \frac{(m+2)\delta + 2(m+1)\epsilon + \sum_{j \in B_2} p_j + \sum_{j \in B_3} p_j}{(m+2)\delta + 2(m+1)\epsilon + |B_2|\delta + \sum_{j \in B_3} p_j}, \\
&\leq \frac{(m+2)\delta + \sum_{j \in B_2} p_j}{(m+2)\delta + |B_2|\delta} = r.
\end{aligned}$$

Note that $2\sum_{j \in B_2} p_j - 3|B_2|\delta < 4|B_2|\delta - 3|B_2|\delta < (m+2)\delta$. Thus, $r < \frac{3}{2}$. ∎

Lemma 4: If $2(m+1)\delta + 2m\epsilon \leq p_{max} \leq 3m\delta + 2(m-1)\epsilon$, then the reverse cycle π_d is a 3/2-approximation.

Proof: Using $p_{max} \leq 3m\delta + 2(m-1)\epsilon$ in (2), we have $T(\pi_d) = (3m+4)\delta + 2(m+1)\epsilon$. Also, from Theorem 2, $T(\pi^*) \geq p_{max} + 4\delta\ + 4\epsilon \geq 2(m+3)\delta + 2(m+2)\epsilon$. Thus,

$$\frac{T(\pi_d)}{T(\pi^*)} \leq \frac{(3m+4)\delta + 2(m+1)\epsilon}{2(m+3)\delta + 2(m+2)\epsilon} = \frac{3[(m+3)\delta + (m+2)\epsilon] - [5\delta + (m+4)\epsilon]}{2[(m+3)\delta + (m+2)\epsilon]} < \frac{3}{2}.$$

∎

Lemma 5: If $\frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon] \leq p_{max} \leq 3m\delta + 2(m-1)\epsilon$, then the odd-even cycle π_c is a 5/3-approximation.

Proof: From (4), we have $T(\pi_c) \leq p_{max} + (m+2)\delta + (m+3)\epsilon$. Using $T(\pi^*) \geq p_{max}$, we have

$$\begin{aligned}
\frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{p_{max}} = 1 + \frac{(m+2)\delta + (m+3)\epsilon}{p_{max}}, \\
&\leq 1 + \frac{(m+2)\delta + (m+3)\epsilon}{\frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon]}, \\
&\leq \frac{5}{3}.
\end{aligned}$$

∎

Before proving Lemma 6 and Lemma 7, we first need an intermediate result (Property A below) that uses the characteristics of the circular layout to modify the lower bound given by Theorem 2 under a certain condition.

Property A: If $[\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j] < (m+2)\delta$, then the cycle-time $T(\pi_r)$ of a 1-unit cycle π_r satisfies $T(\pi_r) \geq (m+2)\delta + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j + 2(m+1)\epsilon$.

Proof: Let I be an arbitrary instance of problem $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$. In order to simplify the proof, we modify I by truncating its processing times to obtain a transformed instance I', with processing times p' defined as follows:

- If $j \in B_1$, then $p'_j = 2\delta, j = 1, 2, \ldots, m$.
- If $j \in B_2$, then $p'_j = \delta, j = 1, 2, \ldots, m$.
- If $j \in B_3$, then $p'_j = p_j, j = 1, 2, \ldots, m$.

Let $B'_1 = \{j \in M : p'_j = 2\delta\}$, $B'_2 = \{j \in M : p'_j = \delta\}$, and $B'_3 = \{j \in M : p'_j = p_j\}$. Note that $|B'_1| = |B_1|$, $|B'_2| = |B_2|$ and $|B'_3| = |B_3|$. Let π_I (resp., $\pi_{I'}$) be an optimal 1-unit cycle for the instance I (resp., I') with corresponding cycle time $T^I(\pi_I)$ (resp., $T^{I'}(\pi_{I'})$). Let $T^{I'}(\pi_I)$ be the cycle time obtained by executing cycle π_I on instance I'.

Claim 1: $T^I(\pi_I) \geq T^{I'}(\pi_{I'})$.

Proof of Claim 1: Since $p'_j \leq p_j \; \forall j \in M$, we have $T^I(\pi_I) \geq T^{I'}(\pi_I) \geq T^{I'}(\pi_{I'})$. □

From Theorem 2, we have $T^{I'}(\pi_{I'}) \geq (m+2)\delta + 2(m+1)\epsilon + |B'_1|\delta + |B'_2|\delta + \sum_{j\in B'_3} p'_j$. Recall from the proof of Theorem 2 that the last three terms of this lower bound represent the total time for the robot's waiting and secondary activities (Section 3.1). Observe that the only change in this bound and the one proposed in the statement of Property A is that the third term is $\frac{4}{3}|B'_1|\delta$ instead of $|B'_1|\delta$; the third term represents the contribution of the machines in B'_1 towards the total time for the secondary activities. We let β denote the average per-machine contribution from the machines in B'_1 towards this secondary time and show that $\beta \geq \frac{4}{3}\delta$ under the hypothesis of Property A.

Claim 2: If $[\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j] < (m+2)\delta$, then $\beta \geq \frac{4}{3}\delta$.

Proof of Claim 2: Before starting the detailed proof, we provide its brief outline. We first show that, in $\pi_{I'}$, there cannot be an odd number of secondary passes across any two adjacent machines. Next, we transform cycle $\pi_{I'}$ into an alternate optimal 1-unit cycle $\overline{\pi}_{I'}$. Finally, we show that the optimality of cycles $\pi_{I'}$ and $\overline{\pi}_{I'}$ implies that $\beta \geq \frac{4}{3}\delta$.

Consider the optimal cycle $\pi_{I'}$ for the transformed instance I'. If there is no secondary travel in $\pi_{I'}$, then each machine has usage $\mathcal{U}_1$. Hence, at each machine $M_j, j \in B'_1$, the robot experiences a full waiting time of $p_j = 2\delta$. Thus, $\beta = 2\delta > \frac{4}{3}\delta$. Consequently, for the remainder of the proof we consider the case where there is secondary travel in $\pi_{I'}$.

Without loss of generality, assume that cycle $\pi_{I'}$ starts with M_1^l. We first observe that, in $\pi_{I'}$, the number of secondary passes across any two adjacent machines M_i and M_{i+1} ($i = 0, 1, \ldots, m,$) must be even. The proof is similar to that provided for Claim 2 of Theorem 1. However, for better readability, we explicitly provide the proof with slight modifications below.

Let there exist an odd number of secondary passes across consecutive machines M_i and M_{i+1}, $i = 0, 1, \ldots, m,$. The only situation that results in secondary travel is when there exist two machines M_j and $M_k, j \neq k$, with each having usage $\mathcal{U}_2$ and M_j^l is immediately followed by M_k^u. Depending on the relationship between the three indices i, j, k, and the sign of the quantity $[\frac{(m+2)}{2} - (\max\{i, j, k\} - \min\{i, j, k\})]$, there are twelve different cases. For brevity, we provide a detailed proof under one case; the proofs under the other cases are similar. Consider the case $j \leq i$, $k \geq i+1$, and $k - j < \frac{(m+2)}{2}$. Then, there exists at least one secondary pass across the machines M_i and M_{i+1}. In order to complete cycle $\pi_{I'}$, the robot must return to machine M_j for unloading. There are two possibilities. If the robot returns to M_j by traversing intermediary machines in $\{M_j, M_{j+1}, \ldots, M_i, M_{i+1}, \ldots, M_k\}$, then the number of secondary passes across M_i and M_{i+1} is even and, therefore, contradicts our assumption. Otherwise, if the robot returns to M_j by traversing intermediate machines $\{M_{k+1}, M_{k+2}, \ldots, M_m, M_{m+1}, M_0, \ldots, M_{j-1}\}$, then the number of secondary passes across M_i and M_{i+1} is odd. In the latter case, the robot travels across all $(m+2)$ machines between the activities M_j^l and M_j^u (an example of this is the odd-even cycle in Figure 3(c)). Therefore, when the robot completes the 1-unit cycle $\pi_{I'}$, the total secondary time is at least $(m+2)\delta$. It follows that $T^{I'}(\pi_{I'}) \geq 2(m+2)\delta + 2(m+1)\epsilon$. Hence, the total secondary and full waiting time for any cycle that has an odd number of secondary passes across any two adjacent machines is at least $(m+2)\delta$, which is strictly larger than the lower bound stated in Property A. Consequently, we assume that in $\pi_{I'}$, the number of secondary passes across any two adjacent machines is even. Next, we use this observation to transform cycle $\pi_{I'}$ into an alternate optimal 1-unit cycle $\overline{\pi}_{I'}$ and show that $\beta \geq \frac{4}{3}\delta$.

First, by starting with M_0 and scanning the machines in $\pi_{I'}$ in a forward pass until M_{m+1}, we group the machines in the set $\{M_0, M_1, M_2, \ldots, M_m, M_{m+1}\}$ into disjoint subsets $\mathcal{Q}_j$, $j = 1, 2, \ldots, t$, of consecutive machines, say $\mathcal{Q}_j = \{M_{j_1}, M_{j_1+1}, \ldots, M_{j_1+n_j}\}$, such that: (i) the subsets $\mathcal{Q}_j, j = 1, 2, \ldots, t$, are arranged so that their first machine indices are in ascending order. That is, $0 \leq j_1 < (j+1)_1, j =$

$1, 2, \ldots, t-1$, and $t_1 \leq m$, (ii) there exist at least two secondary passes across each pair of adjacent machines in subset $\mathcal{Q}_j$, and (iii) there are no secondary passes across the pair of machines (M_{j_1-1}, M_{j_1}), $j_1 \geq 1$, and $(M_{j_1+n_j}, M_{j_1+n_j+1})$, $j_1 + n_j \leq m$. The definition of $\mathcal{Q}_j$ implies the properties P_1-P_4 below:

P_1: $|\mathcal{Q}_j| = n_j + 1 \geq 2$ and the secondary travel time across the machines in $\mathcal{Q}_j$ is at least $2n_j\delta$.

P_2: If $\mathcal{Q}_j = \{M_{j_1}, M_{j_1+1}\}$, then either $j_1 = 0$ or $j_1 + 1 = m + 1$.

P_3: Machines $M_{j_1+n_j+1}$, $M_{j_1+n_j+2}$, $\ldots$, $M_{(j+1)_1-1}$, $j = 1, 2, \ldots, t-1$, which are located between the sets $\mathcal{Q}_j$ and $\mathcal{Q}_{j+1}$, have usage $\mathcal{U}_1$.

P_4: Machines M_1, M_2, $\ldots$, M_{j_1-1}, which are located between M_0 and M_{j_1}, and machines $M_{t_1+n_t+1}$, $M_{t_1+n_t+2}$, $\ldots$, M_m, which are located between $M_{t_1+n_t}$ and M_{m+1}, have usage $\mathcal{U}_1$.

Next, we transform cycle $\pi_{I'}$ into an alternate 1-unit cycle $\overline{\pi}_{I'}$ with $T(\overline{\pi}_{I'}) \leq T(\pi_{I'})$. The transformation involves rearranging the loading sequence of the machines in each subset $\mathcal{Q}_j$, $j = 1, 2, \ldots, t$, while keeping the relative loading sequence of the machines that are not in the set $\mathcal{Q}_j, j = 1, 2, \ldots, t$, unaffected. We now provide the details of the construction which, when completed, results in $\overline{\pi}_{I'}$.

- Step 1: Initialize: Set $j = 1$. If $j_1 \geq 1$, then set $i = j_1$ and $\psi = (M_1^l, M_2^l, \ldots, M_i^l)$. Otherwise, set $i = 1$ and $\psi = (M_i^l)$. Steps 2, 3 and 4, perform the transformation of the loading sequence of the machines in $\mathcal{Q}_j$.

- Step 2: If $i \leq (j_1 + n_j - 2)$ and if at least two of the three indices i, $i+1$, and $i+2$, in $\mathcal{Q}_j$, are in B_1', then set $\psi = (\psi, M_{i+2}^l, M_{i+1}^l, M_{i+3}^l)$ and $i = i + 3$ (refer to Figure 9(b)). Otherwise, set $\psi = (\psi, M_{i+1}^l)$ and $i = i + 1$.

- Step 3: If $i \leq (j_1 + n_j - 2)$. then go to Step 2.

- Step 4: If $i \leq (j_1 + n_j)$, then set $\psi = (\psi, M_{i+1}^l, M_{i+2}^l, \ldots, M_{j_1+n_j}^l, M_{j_1+n_j+1}^l)$. If $j = t$, then go to Step 7.

- Step 5: If $j < t$ and $((j+1)_1 - j_1 + n_j) > 1$, then set $\psi = (\psi, M_{j_1+n_j+2}^l, M_{j_1+n_j+3}^l, \ldots, M_{(j+1)_1}^l)$. Set $j = j + 1$.

- Step 6: If $j \leq t$, then set $i = j_1$ and go to Step 2.

- Step 7: If $t_1 + n_t < m$, then set $\psi = (\psi, M_{t_1+n_t+2}^l, M_{t_1+n_t+3}^l, \ldots, M_{m+1}^l)$. Set $\overline{\pi}_{I'} = \psi$ and END.

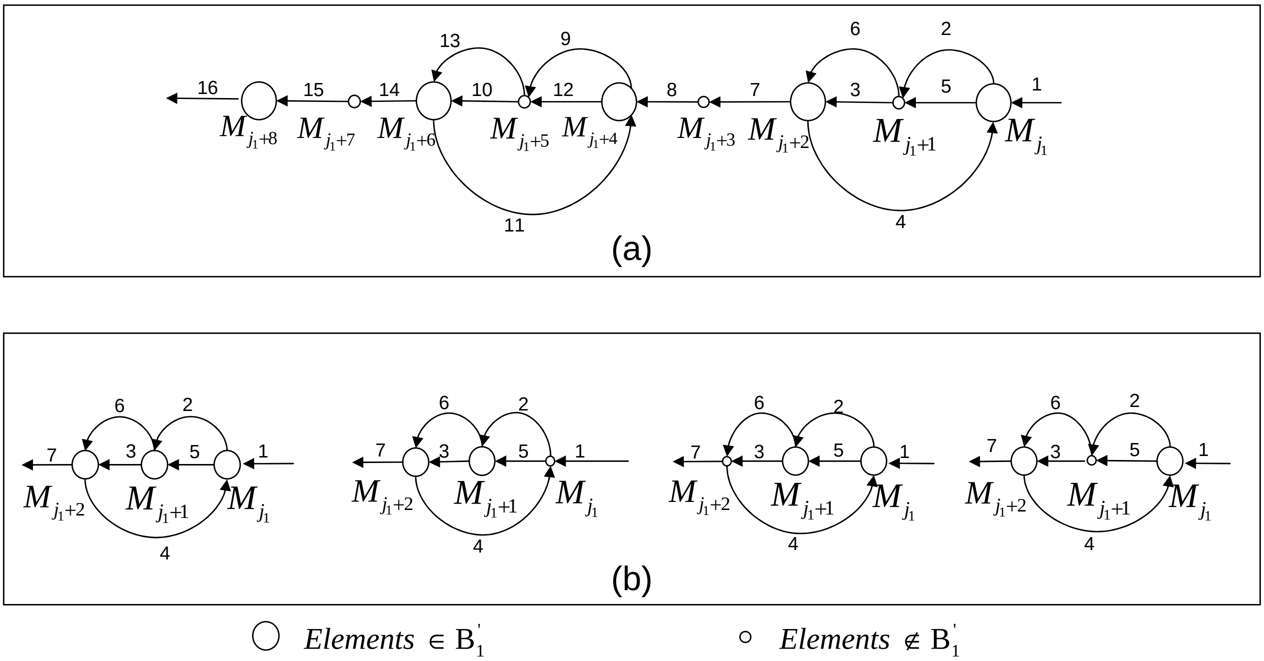


Figure 9: Examples for robot move sequences in $\overline{\pi}_{I'}$: (a) An Example for the transformed loading sequence of the machines in $\mathcal{Q}_j = \{M_{j_1}, M_{j_1+1}, \ldots, M_{j_1+n_j}\}$, where $n_j = 8$, (b) Possible arrangements of the machines in $\overline{\pi}_{I'}$ and their corresponding machine loading sequences.

We now show that $T(\overline{\pi}_{I'}) \leq T(\pi_{I'})$. Note that $T^{I'}(\pi_{I'}) \geq (m+2)\delta + 2(m+1)\epsilon + \sum_{j=1}^{t} 2n_j\delta + f$, where f is the total full waiting at machines not in $\mathcal{Q}_j$, $j = 1, 2, \ldots, t$. The above transformation reduces (or keeps the same) the secondary travel time in the resulting cycle $\overline{\pi}_{I'}$ by decomposing the set $\mathcal{Q}_j$ into smaller subsets with exactly three machines (Figure 9(a)). Since the processing time at each machine is at most 2δ, the robot does not experience any partial waiting in $\pi_{I'}$ or in $\overline{\pi}_{I'}$. As a result of the above transformation, we obtain the following upper bound: $T^{I'}(\overline{\pi}_{I'}) \leq \{(m+2)\delta + 2(m+1)\epsilon + \sum_{j=1}^{t}(4\lfloor\frac{(n_j+1)}{3}\rfloor\delta + 2a_j\delta) + f\}$, where $n_j + 1 \equiv a_j \pmod 3$ (i.e., $a_j \in \{0, 1, 2\}$ is the integral remainder left after dividing $n_j + 1$ by 3). The quantity $\sum_{j=1}^{t}(4\lfloor\frac{(n_j+1)}{3}\rfloor\delta + 2a_j\delta) + f$ represents the upper bound on the sum of total secondary travel time and the full waiting time in $\overline{\pi}_{I'}$. Note that $\sum_{j=1}^{t} 2n_j\delta = \sum_{j=1}^{t}(6\lfloor\frac{(n_j+1)}{3}\rfloor\delta + (2a_j - 2)\delta) \geq \sum_{j=1}^{t}(4\lfloor\frac{(n_j+1)}{3}\rfloor\delta + 2a_j\delta)$. Thus, $T^{I'}(\overline{\pi}_{I'}) \leq T^{I'}(\pi_{I'})$.

Since $\pi_{I'}$ is an optimal 1-unit cycle for the instance I', we have $T^{I'}(\overline{\pi}_{I'}) = T^{I'}(\pi_{I'})$. Hence, $\overline{\pi}_{I'}$ is an alternate optimal 1-unit cycle for the problem instance I'. Observe that $|\mathcal{Q}_j| \leq 5, j = 1, 2, \ldots, t$. If not, then $\sum_{j=1}^{t}(6\lfloor\frac{(n_j+1)}{3}\rfloor\delta + (2a_j - 2)\delta) > \sum_{j=1}^{t}(4\lfloor\frac{(n_j+1)}{3}\rfloor\delta + 2a_j\delta)$. Therefore $T^{I'}(\overline{\pi}_{I'}) < T^{I'}(\pi_{I'})$, which contradicts the optimality of cycle $\pi_{I'}$. If $|\mathcal{Q}_j| = 2$, $\beta = 2\delta > \frac{4}{3}\delta$. Finally, note that for $3 \leq |\mathcal{Q}_j| \leq 5$, we have $\beta \geq \frac{2n_j}{n_j+1}\delta \geq \frac{4}{3}\delta$.

This completes the proof of Property A. ■

We will use this lower bound in the proofs of Lemma 6 and Lemma 7.

Lemma 6: If $2\delta \leq p_{max} \leq (m+2)\delta + 2(m+1)\epsilon$, then the better of π_s^E and π_c achieves a 3/2-approximation.

Proof: We illustrate the bound under two cases: $|B_1| \leq \frac{(m+2)}{4}$ and $|B_1| > \frac{(m+2)}{4}$. Recall that $B_1 = \{j | p_j \geq 2\delta\}$.

Case 1-(a): $(|B_1| > \frac{(m+2)}{4})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j] < (m+2)\delta)$.

Consider cycle π_c. Then, from (4), $T(\pi_c) \leq 2(m+2)\delta + 2(m+1)\epsilon$. From Property A, $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j$. Thus,

$$\begin{aligned} \frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{2(m+2)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j}, \\ &\leq \frac{2(m+2)\delta}{(m+2)\delta + \frac{4}{3}|B_1|\delta} < \frac{2(m+2)\delta}{(m+2)\delta + \frac{1}{3}(m+2)\delta} = \frac{3}{2}. \end{aligned}$$

Case 1-(b): $(|B_1| > \frac{(m+2)}{4})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j] \geq (m+2)\delta)$.

Consider cycle π_c. Again, from (4), $T(\pi_c) \leq 2(m+2)\delta + 2(m+1)\epsilon$. From Theorem 2, $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j$. Thus,

$$\begin{aligned} \frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{2(m+2)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j}, \\ &\leq \frac{2(m+2)\delta}{2(m+2)\delta - \frac{1}{3}|B_1|\delta} \leq \frac{2(m+2)\delta}{2(m+2)\delta - \frac{1}{3}m\delta} \leq \frac{6}{5}. \end{aligned}$$

Case 2-(a): $(|B_1| \leq \frac{(m+2)}{4})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j] < (m+2)\delta)$.

Consider cycle π_s^E. Since the total secondary time for this cycle is $4\delta|B_1|$ (Section 3.2), the cycle time $T(\pi_s^E)$ satisfies the following upper bound

$$T(\pi_s^E) \leq (m+2)\delta + 2(m+1)\epsilon + 4\delta|B_1| + \sum_{j \in B_2} p_j + \sum_{j \in B_3} p_j.$$

Also, from Property A, we have

$$T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j.$$

Thus,

$$\begin{aligned} \frac{T(\pi_s^E)}{T(\pi^*)} &\leq \frac{(m+2)\delta + 2(m+1)\epsilon + 4|B_1|\delta + \sum_{j \in B_2} p_j + \sum_{j \in B_3} p_j}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j}, \\ &\leq \frac{(m+2) + 4|B_1| + 2|B_2|}{(m+2) + \frac{4}{3}|B_1| + |B_2|} = r. \end{aligned}$$

If $(4|B_1| + 3|B_2|) \leq (m+2)$, then we show that $r \leq \frac{3}{2}$.

$$r = \frac{(m+2) + 4|B_1| + 2|B_2|}{(m+2) + \frac{4}{3}|B_1| + |B_2|} = 1 + \frac{\frac{8}{3}|B_1| + |B_2|}{(m+2) + \frac{4}{3}|B_1| + |B_2|}.$$

Note that $4|B_1| \leq (m+2) \leq (m+2) + 3|B_2|$. Hence, we have $8|B_1| + 3|B_2| \leq [(m+2) + 4|B_1|\,]$ and $2[(m+2) + 4|B_1|\,] \leq 3(m+2) + 4|B_1| + 3|B_2|$. Then,

$$r = 1 + \frac{8|B_1| + 3|B_2|}{3(m+2) + 4|B_1| + 3|B_2|} \leq 1 + \frac{1}{2} = \frac{3}{2}.$$

Hence, $r \leq \frac{3}{2}$; so, π_s^E is a 3/2-approximation.

Otherwise, i.e., $(4|B_1| + 3|B_2|) > (m+2)$, for cycle π_c, we have

$$\begin{aligned} \frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{2(m+2)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j}, \\ &\leq \frac{2(m+2)}{(m+2) + \frac{4}{3}|B_1| + |B_2|} < \frac{2(m+2)}{\frac{4}{3}(m+2)} = \frac{3}{2}. \end{aligned}$$

<u>Case 2-(b)</u>: $(|B_1| \leq \frac{(m+2)}{4})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j] \geq (m+2)\delta)$.

Consider cycle π_c. Then, as in Case 1(b), we have

$$\begin{aligned} \frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{2(m+2)\delta + 2(m+1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j}, \\ &\leq \frac{2(m+2)\delta}{2(m+2)\delta - \frac{1}{3}|B_1|\delta} \leq \frac{2(m+2)\delta}{2(m+2)\delta - \frac{1}{12}(m+2)\delta} \leq \frac{24}{23}. \end{aligned}$$

■

Lemma 7: If $(m+2)\delta + 2(m+1)\epsilon < p_{max} < \frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon]$, then the better of π_s^E and π_c achieves a 5/3-approximation.

Proof: As in the proof of the previous result, we divide our argument into 4 subcases. From our previous discussion, we recall the following four bounds:

– From (4), $T(\pi_c) \leq p_{max} + (m+2)\delta + (m+3)\epsilon$.

– From Theorem 2, $T(\pi^*) \geq p_{max} + 4\delta + 4\epsilon$.

– From Property A, $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j$.

– From Theorem 2, $T(\pi^*) \geq (m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j \in B_3} p_j$.

We will use these bounds in the subsequent analysis.

Case 1-(a): $(|B_1| > \frac{3(m+2)}{8})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j] < (m+2)\delta)$.

Consider cycle π_c.

$$\begin{aligned}\frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{T(\pi^*)},\\ &\leq 1 + \frac{(m-2)\delta + (m-1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j},\\ &\leq 1 + \frac{(m-2)\delta + (m-1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta},\\ &< 1 + \frac{(m-2)\delta + (m-1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{(m+2)}{2}\delta} < 1 + \frac{2}{3} < \frac{5}{3}.\end{aligned}$$

Case 1-(b): $(|B_1| > \frac{3(m+2)}{8})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j] \geq (m+2)\delta)$.

Consider cycle π_c.

$$\begin{aligned}\frac{T(\pi_c)}{T(\pi^*)} &\leq \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{T(\pi^*)},\\ &\leq \frac{p_{max} + 4\delta + 4\epsilon}{p_{max} + 4\delta + 4\epsilon} + \frac{(m-2)\delta + (m-1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j},\\ &\leq 1 + \frac{(m-2)\delta + (m-1)\epsilon}{2(m+2)\delta + 2(m+1)\epsilon - \frac{1}{3}|B_1|\delta},\\ &\leq 1 + \frac{(m-2)\delta + (m-1)\epsilon}{2(m+2)\delta + 2(m+1)\epsilon - \frac{1}{3}m\delta} \leq 1 + \frac{3}{5} \leq \frac{8}{5}.\end{aligned}$$

Case 2-(a): $(|B_1| \leq \frac{3(m+2)}{8})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j] < (m+2)\delta)$. First, we obtain the bound offered by cycle π_s^E.

$$T(\pi_s^E) \leq (m+2)\delta + 2(m+1)\epsilon + 4|B_1|\delta + \sum_{j\in B_2} p_j + \sum_{j\in B_3} p_j.$$

$$\begin{aligned}\frac{T(\pi_s^E)}{T(\pi^*)} &\leq \frac{(m+2)\delta + 2(m+1)\epsilon + 4|B_1|\delta + \sum_{j\in B_2} p_j + \sum_{j\in B_3} p_j}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j},\\ &\leq \frac{(m+2)\delta + 2(m+1)\epsilon + 4|B_1|\delta + 2|B_2|\delta}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta} = r.\end{aligned}$$

If $(4|B_1| + 3|B_2|)\delta \leq \frac{3}{2}(m+2)\delta + 3(m+1)\epsilon$, then we show that $r \leq \frac{5}{3}$.

$$r = 1 + \frac{[\frac{8}{3}|B_1| + |B_2|\,]\delta}{[(m+2) + \frac{4}{3}|B_1| + |B_2|\,]\delta + 2(m+1)\epsilon}.$$

Since $4|B_1|\delta + 3|B_2|\delta \le \frac{3}{2}(m+2)\delta + 3(m+1)\epsilon$, we have $[24|B_1|\delta + 9|B_2|\delta] \le [6(m+2)\delta + 12(m+1)\epsilon + 8|B_1|\delta + 6|B_2|\delta]$. This is the same as

$$\frac{8|B_1|\delta + 3|B_2|\delta}{3(m+2)\delta + 6(m+1)\epsilon + 4|B_1|\delta + 3|B_2|\delta} \le \frac{2}{3}.$$

Hence, we have

$$r \le 1 + \frac{2}{3} = \frac{5}{3}.$$

So, π_s^E is 5/3-approximation. Otherwise, i.e., if $(4|B_1| + 3|B_2|)\delta > \frac{3}{2}(m+2)\delta + 3(m+1)\epsilon$, then, for cycle π_c, using $T(\pi^*) \ge p_{max}$, we have

$$\begin{aligned}
\frac{T(\pi_c)}{T(\pi^*)} &\le \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{T(\pi^*)} = 1 + \frac{(m+2)\delta + (m+3)\epsilon}{T(\pi^*)}, \\
&\le 1 + \frac{(m+2)\delta + (m+3)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + \frac{4}{3}|B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j}, \\
&\le 1 + \frac{(m+2)\delta + (m+3)\epsilon}{\frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon]} \le \frac{5}{3}.
\end{aligned}$$

<u>Case 2-(b)</u>: $(|B_1| \le \frac{3(m+2)}{8})$ and $([\frac{4}{3}|B_1|\delta + |B_2|\delta + \sum\limits_{j\in B_3} p_j] \ge (m+2)\delta)$. For cycle π_c, we have

$$\begin{aligned}
\frac{T(\pi_c)}{T(\pi^*)} &\le \frac{p_{max} + (m+2)\delta + (m+3)\epsilon}{T(\pi^*)} = \frac{p_{max} + 4\delta + 4\epsilon}{p_{max} + 4\delta + 4\epsilon} + \frac{(m-2)\delta + (m-1)\epsilon}{T(\pi^*)}, \\
&\le 1 + \frac{(m-2)\delta + (m-1)\epsilon}{(m+2)\delta + 2(m+1)\epsilon + |B_1|\delta + |B_2|\delta + \sum_{j\in B_3} p_j}, \\
&\le 1 + \frac{(m-2)\delta + (m-1)\epsilon}{2(m+2)\delta + 2(m+1)\epsilon - \frac{1}{3}|B_1|\delta}, \\
&\le 1 + \frac{(m-2)\delta + (m-1)\epsilon}{2(m+2)\delta + 2(m+1)\epsilon - \frac{1}{8}(m+2)\delta} < 1 + \frac{8}{15} = \frac{23}{15}.
\end{aligned}$$

■

Theorem 5: For $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$, Algorithm LCYCLE is a 5/3-approximation and this bound is asymptotically tight.

Proof:

- From Lemma 2, cycle π_u is optimal when $|B_3| = m$.
- From Lemma 3, cycle π_u provides a 3/2-approximation when $|B_1| = 0$.
- When $2\delta \le p_{max} \le (m+2)\delta + 2(m+1)\epsilon$, the better of π_s^E and π_c provides a 3/2-approximation (Lemma 6).

- When $p_{max} \geq 3m\delta + 2(m-1)\epsilon$, cycle π_d is optimal (Lemma 1).
- When $3m\delta + 2(m-1)\epsilon > p_{max} \geq 2(m+1)\delta + 2m\epsilon$, cycle π_d provides 3/2-approximation (Lemma 4).
- When $2(m+1)\delta + 2m\epsilon > p_{max} \geq \frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon]$, cycle π_c provides a 5/3-approximation (Lemma 5).
- When $\frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon] > p_{max} \geq (m+2)\delta + 2(m+1)\epsilon$, the better of π_s^E and π_c provides a 5/3-approximation (Lemma 7).

Thus, Algorithm LCYCLE is a 5/3-approximation for $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$.

To demonstrate the tightness of this bound, consider a cell with $m \geq 2$, $\epsilon = 0$, and processing times defined as follows:

- $p_{3i-2} = 4\delta$, $i = 1, 2, \ldots, \frac{(m+2)}{8}$,
- $p_{3i-1} = p_{max} = \frac{3(m+2)}{2}\delta$, $i = 1, 2, \ldots \frac{(m+2)}{8}$,
- $p_{3i} = 4\delta$, $i = 1, 2, \ldots, \frac{(m+2)}{8}$,
- $p_i = 0$, $i = \frac{3(m+2)}{8} + 1, \frac{3(m+2)}{8} + 2, \ldots, m$.

Consider the 1-unit cycle $\pi^* = (M_1^l, M_3^l, M_2^l, \ldots, M_{3i-2}^l, M_{3i}^l, M_{3i-1}^l, \ldots, M_{\frac{3(m+2)}{8}-2}^l, M_{\frac{3(m+2)}{8}}^l, M_{\frac{3(m+2)}{8}-1}^l, M_{\frac{3(m+2)}{8}+1}^l, M_{\frac{3(m+2)}{8}+2}^l, \ldots, M_m^l, M_{m+1}^l)$. The cycle time of π^* is $T(\pi^*) = p_{max} + 4\delta = \frac{(3m+14)}{2}\delta$, which is the lower bound on the cycle time of any 1-unit cycle (Theorem 2). Thus, π^* is an optimal 1-unit cycle. Since $p_{max} = \frac{3}{2}(m+2)\delta = \frac{3}{2}[(m+2)\delta + 2(m+1)\epsilon]$, the solution from Algorithm LCYCLE is π_c. The cycle time for π_c is $T(\pi_c) = p_{max} + (m+2)\delta = \frac{5(m+2)}{2}\delta$. Hence, the ratio $\frac{T(\pi_c)}{T(\pi^*)} = \frac{(5m+10)\delta}{(3m+14)\delta} \to \frac{5}{3}$ as $m \to \infty$. ■

Corollary 2: For $RF_m^{\circ}|(free, A, cyclic\text{-}1)|C_t$, Algorithm MODIFIED-LCYCLE is a 5/3-approximation, and this bound is asymptotically tight.

Proof: The proof of the 5/3 approximation guarantee for Algorithm MODIFIED-LCYCLE is immediate from Theorem 5.

To show the tightness of the 5/3 approximation guarantee of Algorithm MODIFIED-LCYCLE, consider the example in the proof of Theorem 5. We compute the cycle times of four cycles: π_u, π_c, π_d, and π_s^E.

Note that $E = \{0, \frac{3(m+2)}{8}+1, \frac{3(m+2)}{8}+2, \ldots, m, m+1\}$ and $\pi_s^E = (M^l_{\frac{3(m+2)}{8}+1}, M^l_{\frac{3(m+2)}{8}}, M^l_{\frac{3(m+2)}{8}-1}, \ldots,$ $M^l_3, M^l_2, M^l_1, M^l_{\frac{3(m+2)}{8}+2}, M^l_{\frac{3(m+2)}{8}+3}, \ldots, M^l_{m-1}, M^l_m, M^l_{m+1})$.
Thus,

- $T(\pi_u) = (m+2)\delta + \sum_{i=1}^m p_i = 2(m+2)\delta + \frac{3(m+2)^2}{16}\delta$,
- $T(\pi_c) = p_{max} + (m+2)\delta = \frac{5(m+2)}{2}\delta$,
- $T(\pi_d) = (3m+4)\delta$, and
- $T(\pi_s^E) = (m+2)\delta + (\frac{3(m+2)}{8})4\delta = \frac{5(m+2)}{2}\delta$.

Since $T(\pi_c) \leq \min\{T(\pi_u), T(\pi_d), T(\pi_s^E)\}$, the solution from Algorithm MODIFIED-LCYCLE is $T(\pi_c) = p_{max} + (m+2)\delta = \frac{5(m+2)}{2}\delta$. As shown above, $\frac{T(\pi_c)}{T(\pi^*)} \rightarrow \frac{5}{3}$ as $m \rightarrow \infty$. ■

References

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