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Electronic Companion—"Optimizing Long-Term Production Plans in Underground and Open Pit Copper Mines" by Rafael Epstein, Marcel Goic, Andrés Weintraub, Jaime Catalán, Pablo Santibáñez, Rodolfo Urrutia, Raúl Cancino, Sergio Gaete, Augusto Aguayo, and Felipe Caro, *Operations Research*, http://dx.doi.org/10.1287/opre.1110.1003.

E-Companion to:

Optimizing Long-Term Production Plans in Underground and Open Pit Copper Mines

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CODELCO'S PRODUCTION PLANNING APPROACH

Recently Codelco has migrated from a traditional planning scheme based exclusively on cutoff grades to a methodology based on the model presented in this paper. In order to compare the results we outline the planning approach. Note that overall the steps have not changed. What has been modified is the way steps d) and e) are carried out, as explained below.

a) The available resources are determined considering the current geologic characteristics of the deposit and the mine’s history.

b) A selection of resources is performed. This is done considering Macro Options (i.e. alternative technologies, economic indicators, management directives, etc) in order to delimit the floor, perimeter and ceiling of the resources to extract during the planning horizon. The result is known as

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the Model of Mining Reserve that contains the resources with positive benefit taking into account the processing costs and the projected recovery.

c) The expansions and benches for open pit, and the sectors and columns for underground, are designed on the basis of physical and geomechanical principles.

d) The consumption strategy defines the Mining Plan, which contains for every period: tons of mineral to extract, productive sectors involved, ore grades and the level of uncertainty based on drillings and samples.

e) The Mining Plan is economically evaluated, providing profit indicators, and measures of technical risk and vulnerability. If the result is not satisfactory, the planning parameters are modified and the process is repeated from point c).

f) If an external change occurs affecting the Macro Options, it is incorporated in point b), and the process is repeated from that stage.

g) The final outcome is a plan that provides the relevant information for operating each expansion and sector, including the starting period, operation rates and termination period.

In the legacy planning approach steps d) and e) were carried out by looking separately at the different parts of the mine. This scheme allowed evaluating different options by means of alternative plans, but not in a unified and global manner. In addition, due to the high complexity of the procedure and the calculations involved, several weeks (if not months) were spent in obtaining one particular plan, preventing the planner from evaluating more options. By using the model in steps d) and e), integrated plans across mines are possible and they are obtained fast enough that the planner can evaluate multiple scenarios with little additional effort.

UNDERGROUND MINING ROUNDING HEURISTIC

In the case of underground mines, the LP relaxation does not present many fractional values. This can be explained primarily by two reasons: (i) the model formulation itself is very tight; and (ii) the copper grade in a column is typically decreasing from the bottom up, therefore larger benefits are obtained when the lower blocks are extracted first (see Lemma 2). To obtain an integer solution, we apply the following rounding procedure. First, we solve the LP relaxation and denote by $z_{int}^{LP(q)}$ its solution for the extraction variables, where q represents an iteration counter. Second, we add the following constraints:

$$z_{int} = 1 \qquad \forall i,n,t : z_{int}^{LP(q)} \geq RoundUp(q) \tag{41}$$

$$z_{int} = 0 \qquad \forall i,n,t : z_{int}^{LP(q)} \leq RoundDown(q). \tag{42}$$

Constraints (41) and (42) round up and down the z_{int} variables that have values close to one and zero in the LP solution, where $RoundUp(q)$ and $RoundDown(q)$ represent the rounding thresholds respectively. Then, we solve the LP relaxation again and the counter q is incremented by one. The procedure is repeated until the all the z_{int} variables are integer.

EXTRACTION SIMULATION ALGORITHM FOR THE OPEN PIT ROUNDING HEURISTIC

let *t = 1*
let *i = Or(s)*
let *j = Or(s)+1*
let *Buf(s)=* ϕ
while *(t≤T and j<De(s))*
  let $f_1 = IND\left(j \in SU(i,t-1)\right)$
  let $f_2 = IND\left(z_{jt} = 1 \text{ } is \text{ } feasible\right)$
  let $f_3 = IND\left(\sum_{g \leq t} z_{jg}^{LP(q)} \geq \max_{g>t} \{z_{jg}^{LP(q)}\}\right)$
  if *(f_1 = f_2 = f_3 =1)*
    let $z_{jt}^{SIM(q)} = 1$
    update *downstream flows*
    let *j = j+1*
  else
    let $z_{jt}^{SIM(q)} = 0$
    let $Buf(s) = Buf(s) \cup \{(j,t)\}$
    let *i = j - 1*
    let *t = t+1*
  end
end

Algorithm 1: Extraction simulation for an expansion $s \in \bigcup_{m \in M_O} \Lambda(m)$. Here $IND(A)$ is an indicator function that equals one if the argument A is true and is zero otherwise.

Algorithm 1 traverses the extraction network from the upper left corner down to the lower right exit node (see Figure 5). At a given node (j,t), the algorithm must decide whether to move down (i.e., extract bench $j+1$) or to the right (i.e., transition to period $t+1$). The latter occurs by default unless three conditions are met, which are given by the flags f_1, f_2, and f_3 respectively. Flag f_1 checks whether node j is accessible from the node reached in the previous period, i.e., node $(i,t-1)$. Flag f_2 checks constraints (31)-(32) and the availability of downstream capacity to process the material from bench j in period t. The last flag f_3 checks whether in the q-th LP solution the amount extracted from bench j up to period t is greater than the maximum of the fractional values in the remaining periods. The latter measures the desirability of extracting bench j in period t. This rule contrasts with the usual rounding criterion that simply picks the largest fractional value and rounds it up to one. Figure 7 shows four examples that illustrate the two criteria for a given bench j. Each example shows a different solution of the LP relaxation. In examples (a), (b) and (c), the rounding criterion based on f_3 yields the same outcome as the criterion that rounds up the largest fractional value. However, in example (d), f_3 would allow the extraction to start in the second period, whereas the criterion that considers the largest fractional value would delay the extraction until the fourth period just as in example (b). In other words, the criterion based on f_3 attempts to start extracting the bench earlier and will wait only if the fractional values of the LP solution are considerably skewed.

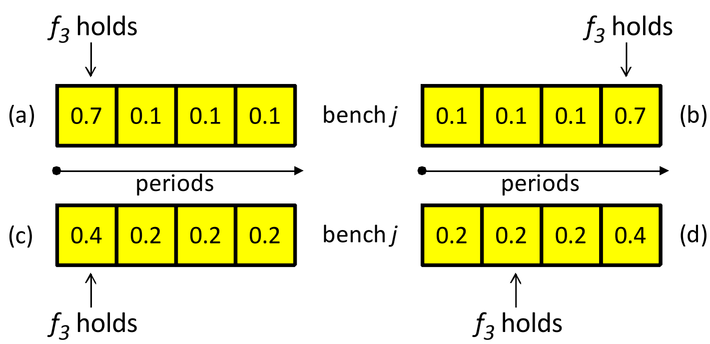

Figure 7: Examples of the rounding criterion for open pit mining.

CASE STUDY AT EL TENIENTE[2]

El Teniente is the largest underground mine in the world. It is located in the Andes Mountains at 2,200 meters above the sea level and 80 kilometers south of Santiago, Chile's capital. Its origins date back to pre-colonial times but officially its exploitation started in 1904. It currently has nearly 2,400 kilometers of underground tunnels. Our case study considered all the sectors that were in operation during the 2002 planning cycle. The horizon was divided in six periods, the first two of 1 and 4 years respectively, and the last four of 5 years each, spanning 25 years overall. The annual discount rate was 10% and all sunk costs were excluded. A fixed price of 85¢ per pound of fine copper was used throughout the horizon.[3]

Table 4 shows the results of the legacy planning approach explained in section 0 and the solution based on the optimization model. The comparison considered the same investments in both cases. Hence, the differences come from the selection of reserves. The mining plan obtained with the model-based approach increased El Teniente's NPV by 5.1%.

	Model-based vs. Legacy Percentage Difference
Revenues	2.4%
Production Costs	1.2%
Investments	0.0%
NPV	5.1%

Table 4: Legacy vs. model-based planning approach at El Teniente.

A detailed comparison of the solutions showed that both provided production schedules that were very similar in terms of total tons extracted per period and sector, but the model obtained higher grades of copper and molybdenum, which translated into larger revenues, as seen in Table 4. This improvement came mostly from considering the mine as a whole—instead of sector by sector—and from timing the production. The legacy approach that looked at each sector separately failed to maximize the throughput of the downstream processes. In contrast, the model-based solution consistently reached the capacity of the intermediate processes and it provided an appropriate ore blend from all sectors so that the concentration plant received a constant supply. Moreover, the model was solved with an explicit

[2] A preliminary version of this case study without the financial impact was presented in Epstein et al. (2003).
[3] As of February 2010 the price was $3.4/lb.

constraint to limit the total amount of arsenic produced as a byproduct each period. The legacy approach had difficulties incorporating this kind of environmental restrictions.

PROOFS

Proof of Proposition 1

Consider an instance I of the precedence constraint knapsack problem (PCKP) where the capacity of the knapsack is c, the precedence constraints are given by a directed, non-cyclic graph $G_I = (V_I, A_I)$, and the weight w_i of each item $i \in V_I$ is equal to its profit p_i. We will show that we can solve I as a special case of the underground mining problem given by Equations (1)-(22). Consider a single period t, a single product k, and a single downstream process v with finite capacity $CP_{vt} = c$. Let $I_U = V_I$, and to each item $i \in V_I$ associate a column with a single block and let $TON_{i0k} = w_i$. Let $EMIN(i) = \{0\}$, $\Delta = T = 1$, $WIN(s) = 1$, $TC_{vkk} = 1$, $\delta_s = \infty$, $\gamma_{i0} = 0$, $E_{kt} = \infty$, $p_{kt} = 1$, and set all the cost parameters equal to zero. Set the minimum incorporated area and extraction requirements equal to zero, and set the maximum values equal to infinity. Finally, let the precedence among columns be the same as in the knapsack problem, i.e., let $SEC_U = A_I$. Note that Equation (12) guarantees that $e_{i0t} = z_{i0t}, \forall i \in I_U$. Therefore, extracting a column becomes a binary decision—equivalent to including an item in the knapsack—and solving the underground model given by Equations (1)-(22) provides a solution to instance I of the PCKP. The PCKP where the profits are equal to the weights is strongly NP-hard (see Theorem 13.2.1 in Kellerer et al. 2004). Since we have shown a polynomial reduction, the underground production planning problem is strongly NP-hard as well.

Proof of Lemma 2

We show the result for $T = 1$. The extension to multiple periods follows a similar argument. For simplicity, we omit the t subscript. Let $\{e^*_{in}\}_{n \in N(i)}$ be the optimal solution of the LP relaxation for the extraction variables of column i. Suppose that for a given block $n \in N(i)$, the following holds:

$$e^*_{in+1} > 0 \;\; and \;\; e^*_{in} < 1. \tag{43}$$

Due to the decreasing grades, the tonnage from block n that reaches the concentration plant is more profitable than an equivalent amount from block $n+1$. Except for this difference at the commercial level, the blocks are identical. Hence, shifting production from block $n+1$ to block n is feasible since the downstream processes and the other constraints remain unaffected. In other words, if (43) holds, it is possible to shift a certain amount $\varepsilon > 0$ from e^*_{in+1} to e^*_{in} and the objective value would improve. This means that $\{e^*_{in}\}_{n \in N(i)}$ cannot be optimal, but that would be a contradiction. So condition (43) cannot hold, and e^*_{in+1} can only be positive if e^*_{in} equals one. Therefore, $e^*_{in} \in \{0,1\}$ for all blocks except the last one extracted (i.e., except for the highest n such that $e^*_{in} > 0$). From constraints (11) and (13) we have that $e_{in} \le z_{in} \le e_{in-1}$ $\forall n > 0$. This implies that the optimal solution of the LP relaxation must satisfy $z_{in} \in \{0,1\}$ for all blocks except the first and last one extracted, which concludes the proof.

Proof of Proposition 3

Note that the open pit extraction sequence given by Equation (31) plays the same sequencing role as Equation (14) in the underground formulation. Moreover, if there is a single period t, then Equations (26) and (27) guarantee that $e_{it} = z_{it}, \forall i \in I_O$. Therefore, the same reduction from the proof of Proposition 1 applies to open pit where the items in the knapsack represent benches instead of columns. Hence, open pit problem is strongly NP-Hard.

REFERENCES

Epstein, R., F. Caro, J. Catalán, P. Santibáñez, A. Weintraub, S. Gaete. 2003. Optimizing long-term planning for underground copper mines. In Copper 2003 - Cobre 2003, 5th Internat. Copper Conf., Vol I. Plenary Lectures, Economics and Appl. of Copper Proc., Canadian Instit. of Mining, Metallurgy and Petroleum, Santiago, Chile, 265–279.

Kellerer, H., U. Pferschy, D. Pisinger. 2004. *Knapsack problems*. Springer, 546 pages.