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Derivations of Results

EC.1. Results in Section 2: Model for Peaker Pricing

EC.1.1. Proof of Lemma 1

This is a well-known result, and we omit some details. For a more comprehensive treatment, see e.g., **?**.

If firm i submits an offer p^i for its capacity, the quantity it supplies is given by

$$q^i(D,p^i,p^j) = \begin{cases} \min\{D,k\} & \text{if } p^i < p^j, \\ \min\{RD,k\} & \text{if } p^i > p^j, \\ \frac{D}{2} & \text{if } p^i = p^j, \end{cases} \tag{EC.1}$$

where $RD = \max\{0, D-k\}$ is the residual demand for firm i. In the discriminatory (or pay-as-bid) auction the plants are always paid their own offer price, and therefore the profits are given by

$$\pi^i(D,p^i,p^j) = (p^i - c)q^i(D,p^i,p^j). \tag{EC.2}$$

If $D \le k$, prices are set at marginal cost in the equilibrium. Suppose instead that $p^i > c = 0$: then firm j will undercut since it can capture the entire market demand. Hence $p^i = p^j = 0$.

If $D > k$, the competitive equilibrium above does not exist, since by eq. (EC.1), any deviating firm would serve a positive amount and thus gain from deviating. Suppose that p^i is the highest offer. Now firm j's profit is strictly increasing up to p^i. Therefore, in any pure strategy equilibrium, the offers are the same for both firms. If both firms offer p, by the profit function (EC.2) they are equally sharing the demand. Since $D > k$, either can gain by just undercutting p to capture k. Therefore, no pure strategy equilibrium exists.

We can derive the (unique) mixed strategy equilibrium by setting expected profits equal for all $p < \overline{P}$:

$$\begin{aligned} \pi^i(D,p) = {} & H_o^j(p)[(p-c)(D-K) - z(C + (p-c)(D-K))] \\ & + (1 - H_o^j(p))[(p-c)K - z(C + (p-c)K)]. \end{aligned} \tag{EC.3}$$

Solving this, we have for $i = 1,2$:

$$H_o^i(p) = \frac{(p-c)K - \pi^j(D)}{(p-c)(2K-D)}. \tag{EC.4}$$

It is obvious that both firms should have the same lower bound of price range. Otherwise, the one with the lower price range should deviate by increasing the lower end of the range to make more profit. Let the lower bound be $\underline{p}$. Setting $H_o^i(\underline{p}) = 0$, we have $\pi^i(D) = (\underline{p} - c)K$ for $i = 1,2$.

On the other hand, we need to have $H_o^i(\overline{P}) = 1$ for $i = 1,2$. Therefore, $\pi^i(D) = (\overline{P} - c)(D-K)$. Substitute this back to $\underline{p}$ and we have the desired lower bound. □

EC.1.2. Proof of Lemma 2

The generators wish to share monopoly profits. With demand D, they would make $D\overline{P}/2$ each. If $D \leq k$, a deviation yields a gain of $D\overline{P}/2$ by capturing the whole market demand. If, on the other hand, $D > k$, the firm can only capture the demand up to k, gaining $(k - D/2)\overline{P}$. Thus the deviation gain is increasing in D up to k and decreasing after it.

With $D \leq k$, if the punishment $0 \leq M \leq k\overline{P}/2$, the deviation gain will be equal to it at some demand, $\underline{D}^*$, so that deviation gain exceed the punishment for higher demand realizations. Similarly for $D > k$, as the deviation gain is decreasing in demand, it will exceed the punishment when demand is close to k and will be equal to it at $\overline{D}^*$. Since the gains at $\underline{D}^*$ and $\overline{D}^*$ are equal to M, we have $(k - \overline{D}^*/2)\overline{P} = \underline{D}^*\overline{P}/2$ or equivalently $\underline{D}^* + \overline{D}^* = 2k$. But since $\overline{D} \leq 2k$, we also need $\overline{D}^* \leq \overline{D}$ or equivalently $\underline{D}^* \geq 2k - \overline{D}$.

Let us derive the optimal punishment. The infinite punishment involves pricing at the one-shot Nash equilibrium strategies for every period after deviation. In any period, a residual demand event arises with probability α and periods are discounted with δ. The punishment is

$$M = \sum_{t=1}^{\infty} \left[\alpha \delta^t \int_0^{\overline{D}} (\pi_c(D) - \pi_o(D))\, dF(D) \right], \tag{EC.5}$$

where π_c is the collusive profit and π_o is the one-shot profit for a single firm. It is clear that M does not depend on the current demand realization. We can write the punishment as

$$M = \alpha \frac{\delta}{1-\delta} \int_0^{\overline{D}} (\pi_c(D) - \pi_o(D))\, dF(D) \tag{EC.6}$$

$$= \frac{\gamma}{1-\gamma} \int_0^{\overline{D}} (\pi_c(D) - \pi_o(D))\, dF(D) \tag{EC.7}$$

where we have combined the ‘discounting’ parameters in one: $\gamma = \alpha\delta/(1 - \delta + \alpha\delta)$. (Clearly $\gamma \in (0,1)$.)

For $D \leq k$, we have increasing collusive profits until the cutoff demand, then constant after it, that is, $\pi_c(D) = \min\{D\overline{P}/2, \underline{D}^*\overline{P}/2\}$ and $\pi_o(D) = 0$. For $D > k$, we have to consider different cases. Throughout, we have the one-shot profits $\pi_o(D) = (D - k)\overline{P}$. Over the upper cutoff point, collusive profits are again $D\overline{P}/2$, under it they would be $\overline{D}^*\overline{P}/2$, with $\overline{D}^* = 2k - \underline{D}^*$. However, this equilibrium is not always sustainable when the firms are pricing below the price cap $\overline{P}$. Specifically, if the profits made in this equilibrium are lower than those of deviating by raising the price to $\overline{P}$ (which in this symmetric case are equal to the mixed strategy equilibrium profits, see the proof of Proposition 1), the firms would rather choose not to collude, and end up playing the mixed strategy equilibrium. In fact, collusion will never happen when the mixed strategy equilibrium profits are larger than the collusive profits; colluding in these cases would only lower the total punishment and consequently not be 'most-collusive'. Here, this happens when $(D-k)\overline{P} > DP(D)/2$, where the price $P(D) = \underline{D}^*\overline{P}/(2k-D) \leq \overline{P}$ is the highest sustainable collusive price, derived from the deviation gain being constant below $\overline{D}^*$. Thus the firms will not collude when

$$\frac{k\overline{P}}{\overline{P} - P(D)/2} \leq D \leq \overline{D}^* \tag{EC.8}$$

where we note that the left hand side is always greater than k. To see when the mixed strategies are preferred, we can solve the resulting second degree equation for

$$D = \frac{6k - \underline{D}^* \pm \sqrt{(6k - \underline{D}^*)^2 - 32k^2}}{4} \tag{EC.9}$$

where the mixed equilibrium will be played in between the roots $\{\hat{D}, \check{D}\}$ (when they exist). The range exists when the discriminant $(6k - \underline{D}^*)^2 - 32k^2$ is positive; this happens in the relevant range when $\underline{D}^* \leq (6 - 4\sqrt{2})k \approx 0.34k$. The range $[\hat{D}, \check{D}]$ will always be constrained between k and $\overline{D}^*$. For it to exist, it is further required that $\hat{D} \leq \overline{D}$, that is, $3k/2 - \underline{D}^*/4 \leq \overline{D}$. In the range, we have (expected) one-shot profits $\pi_c(D) = \pi_o(D) = (D-k)\overline{P}$.

Now we can write out $\pi_c(D)$ and $\pi_o(D)$:

$$\pi_c(D) = \begin{cases} D\overline{P}/2 & \text{if } \underline{D} \leq D \leq \underline{D}^* \\ \underline{D}^*\overline{P}/2 & \text{if } \underline{D}^* < D \leq k \\ DP(D)/2 & \text{if } k < D \leq \hat{D} \\ (D-k)\overline{P} & \text{if } \hat{D} < D \leq \check{D} \\ DP(D)/2 & \text{if } \check{D} < D \leq \overline{D}^* \\ D\overline{P}/2 & \text{if } \overline{D}^* < D \leq \overline{D} \end{cases} \tag{EC.10}$$

$$\pi_o(D) = \begin{cases} 0 & \text{if } \underline{D} \leq D \leq k \\ (D-k)\overline{P} & \text{if } k < D \leq \overline{D}. \end{cases} \tag{EC.11}$$

Combining these into the equation (EC.7), we have the optimal punishment. The prices for each case follow from the discussion. □

EC.1.3. Proof of Proposition 1

The most-collusive strategies have been established in Lemma 2. When $\underline{D}^* = 0$, the deviation gain and the punishment are both zero so this is a solution corresponding to repeating the static equilibrium strategies. A sufficient condition for the existence of another solution is that $h = W - M$ is either (i) negative at k (and possibly increasing at zero) or (ii) decreasing at zero and positive at k (or (iii) negative somewhere in between, which we don't consider here). In case (i), the firms will collude at monopoly prices for all demand realizations, but in case (ii) they will be able to sustain monopoly prices up to the largest solution to the equation.

For point 1, The firms will always collude when $h(k) \leq 0$, that is

$$h(k) = k\overline{P}/2 - \frac{\gamma}{1-\gamma}\left[\int_0^k D\overline{P}/2\, dF(D) + \int_k^{\overline{D}} (k - D/2)\overline{P}\, dF(D)\right] \leq 0 \tag{EC.12}$$

$$\iff \gamma \geq \gamma_H^* = \frac{x}{1+x} \tag{EC.13}$$

where

$$x = \frac{k/2}{\int_0^k D/2\, dF(D) + \int_k^{\overline{D}} (k - D/2)\, dF(D)}, \tag{EC.14}$$

which we note does not depend on the price cap. The expression can be interpreted as the maximum deviation gain divided by the average deviation gain, which approaches one when the demand is concentrated around k. If more extreme demand realisations become more frequent, x increases. Consequently, the threshold for the discount factor γ increases as well.

For point 2, we evaluate $h'(\underline{D}^*)$.

$$\begin{aligned}
h'(\underline{D}^*) &= \frac{d}{d\underline{D}^*}\left[\underline{D}^*\overline{P}/2 - \frac{\gamma}{1-\gamma}\int_0^{\overline{D}} (\pi_c(D) - \pi_o(D))\, dF(D')\right] \\
&= \frac{d}{d\underline{D}^*}\Bigg[\underline{D}^*\overline{P}/2 - \frac{\gamma}{1-\gamma}\Bigg[\int_0^{\underline{D}^*} D\overline{P}/2\; dF(D) + [F(k) - F(\underline{D}^*)]\,\underline{D}^*\overline{P}/2 \\
&+ \int_k^{\hat{D}} [DP(D)/2 - (D-k)\overline{P}]\; dF(D) \\
&+ \int_{\check{D}}^{\overline{D}^*} [DP(D)/2 - (D-k)\overline{P}]\; dF(D) + \int_{\overline{D}^*}^{\overline{D}} [D\overline{P}/2 - (D-k)\overline{P}]\; dF(D)\Bigg]\Bigg] && \text{(EC.15)} \\
&= \overline{P}/2 - \frac{\gamma}{1-\gamma}\Bigg[\underline{D}^*\overline{P}/2f(\underline{D}^*) + (F(k) - F(\underline{D}^*))\overline{P}/2 - f(\underline{D}^*)\underline{D}^*\overline{P}/2 \\
&+ \int_k^{\hat{D}} \frac{D\overline{P}/4}{k - D/2}\; dF(D) + \underline{D}^*\frac{d\hat{D}}{d\underline{D}^*}\frac{\hat{D}\overline{P}/4}{k - \hat{D}/2}f(\hat{D}) - (\hat{D} - k)\overline{P}f(\hat{D})\frac{d\hat{D}}{d\underline{D}^*} \\
&+ \int_{\check{D}}^{2k - \underline{D}^*} \frac{D\overline{P}/4}{k - D/2}\; dF(D) + \underline{D}^*\left(\frac{2k - \underline{D}^*}{2\underline{D}^*}f(2k - \underline{D}^*) - \frac{d\check{D}}{d\underline{D}^*}\frac{\check{D}\overline{P}/4}{k - \check{D}/2}f(\check{D})\right) \\
&+ (\check{D} - k)\overline{P}f(\check{D})\frac{d\check{D}}{d\underline{D}^*} + \overline{P}(k - \underline{D}^*)f(2k - \underline{D}^*) \\
&- \int_{2k - \underline{D}^*}^{\overline{D}} \frac{D\overline{P}/2}{k - D/2}\; dF(D) + \overline{P}\underline{D}^*/2f(2k - \underline{D}^*)\Bigg] && \text{(EC.16)}
\end{aligned}$$

where we have simply used the fundamental theorem of calculus to take derivatives over the integral expressions. For $\underline{D}^* = 0$, most of the terms disappear and this expression simplifies to the following result:

$$h'(0) \leq 0 \iff F(k) \geq \frac{1-\gamma}{\gamma} \iff \gamma \geq \frac{1}{1 + F(k)}. \tag{EC.17}$$

That is, if the demand cannot be fully satisfied by one of firms often enough compared to the inverse of the discount factor, the difference is decreasing at zero.

If the function h is decreasing at zero and positive at k, the existence of a fixed point is guaranteed, since clearly the difference is continuous. We note here that this is a sufficient condition, but since the function h, if increasing at zero, will not in general be concave, it is not a necessary condition. Therefore, there may be lower values of the discount factor for which a fixed point exists.

The difference between deviation gain W and punishment M, $h = W - M$, must be increasing at the largest fixed point (if it exists in the interior of the interval). As the discount factor γ decreases, the punishment decreases and consequently h increases. Hence we must have a lower $\underline{D}^*$ to keep h at zero. □

EC.2. Results in Section 3: Revenue Targets

EC.2.1. Proof of Proposition 2

With low demand ($D \leq k$), it is easy to see that marginal cost pricing is the unique equilibrium. With $D > k$, consider the final period before the target, with the targets of the firms are $\Pi_T^1 \geq \Pi_T^2$ and penalty function $C(\Pi_t^i - x)^+$. Like in the stationary case, a pure strategy equilibrium cannot exist, so we consider mixed strategies with CDF's $H_{T,nc}^i(p)$. Setting the expected utilities equal for all p, we have:

$$u^i(T,p) = H_{T,nc}^j(p)[p(D-k) - C(\Pi_T^i - p(D-k))^+] + (1 - H_{T,nc}^j(p))[pk - C(\Pi_T^i - pk)^+]. \quad \text{(EC.18)}$$

We can solve these to get

$$H_{T,nc}^1(p) = \frac{pk - C(\Pi_T^2 - pk)^+ - u^2(T)}{p(2k-D) + C(\Pi_T^2 - p(D-k))^+ - C(\Pi_T^2 - pk)^+}; \quad \text{(EC.19)}$$

$$H_{T,nc}^2(p) = \frac{pk - C(\Pi_T^1 - pk)^+ - u^1(T)}{p(2k-D) + C(\Pi_T^1 - p(D-k))^+ - C(\Pi_T^1 - pk)^+}. \quad \text{(EC.20)}$$

It is obvious that both firms should have the same lower bound of price range. Otherwise, the one with lower price range should deviate increase the lower end of the range to make more profit. Let the lower bound be $\underline{p}$. Setting $H_{T,nc}^i(\underline{p}) = 0$, we have for $i = 1,2$:

$$u^i(T) = \underline{p}k - C(\Pi_T^i - \underline{p}k)^+. \quad \text{(EC.21)}$$

On the other hand, each CDF needs to be equal to one at the price cap. However, if the targets Π_T^i differ, the limits of $H_{T,nc}^i(p)$ as p approaches v are different. Hence only one of the limits can be equal to one, and the other firm's function will have a mass point at $p = \overline{P}$. Otherwise, one of the firms would have $H_{T,nc}^i(\overline{P}) > 1$. Comparing the limits of the distributions, we find that $\lim_{p \uparrow \overline{P}} H_{T,nc}^1(p) = 1$. Therefore we have

$$u^2(T, \Pi_T^2, D) = \overline{P}(D-k) - C(\Pi_T^2 - \overline{P}(D-k))^+ \quad \text{(EC.22)}$$

$$\underline{p} = \frac{1}{k}\left(\overline{P}(D-k)\right) \quad \text{(EC.23)}$$

$$u^1(T, \Pi_T^1, D) = \overline{P}(D-k) - C(\Pi_T^1 - \overline{P}(D-k))^+. \quad \text{(EC.24)}$$

The lower bound for prices is the same as in the case with no revenue targets and the mixed strategy distributions are (weakly) stochastically dominated by their non-constrained counterparts; the game with targets is hence less competitive.

If just firm 1 is below its target, we have as a special case

$$H^1_{T,nc}(p) = \frac{pk - u^2(T)}{p(2k-D)}; \tag{EC.25}$$

$$H^2_{T,nc}(p) = \frac{pk - C(\Pi^1_T - pk)^+ - u^1(T)}{p(2k-D) + C(\Pi^1_T - p(D-k))^+ - C(\Pi^1_T - pk)^+}. \tag{EC.26}$$

Again comparing the limits of the distributions, we have $\lim_{p\uparrow\overline{P}} H^1_{T,nc}(p) = 1$. The expected utilities are hence

$$u^2_T(D) = \overline{P}(D-k) \tag{EC.27}$$

$$u^1_T(\Pi^1_T, D) = \overline{P}(D-k) - C(\Pi^1_T - \overline{P}(D-k))^+. \tag{EC.28}$$

The equilibrium profits for the firm over the price cap are thus not affected; the price distribution is changed though.

Next, consider the second to last period before the target. Now, since the expected utilities of the firms in the last period only depend on their own distance to their target, the expected utilities are

$$\begin{aligned} u^1(T-1,p) =& H^2_{T-1,nc}(p)\left(p(D-k) + \delta\mathbb{E}[u^1(T, \Pi^1_{T-1} - p(D-k))]\right) \\ &+ (1 - H^2_{T-1,nc}(p))\left(pk + \delta\mathbb{E}[u^1(T, \Pi^1_{T-1} - pk)]\right); \end{aligned} \tag{EC.29}$$

$$\begin{aligned} u^2(T-1,p) =& H^1_{T-1,nc}(p)\left(p(D-k) + \delta\mathbb{E}[u^2(T, \Pi^2_{T-1} - p(D-k))]\right) \\ &+ (1 - H^1_{T-1,nc}(p))\left(pk + \delta\mathbb{E}[u^2(T, \Pi^2_{T-1} - pk)]\right). \end{aligned} \tag{EC.30}$$

Again, we can solve these to get

$$H^1_{T-1,nc}(p) = \frac{pk + \delta\mathbb{E}[u^2(T, \Pi^2_{T-1} - pk)] - u^2(T-1)}{p(2k-D) - \delta\mathbb{E}[u^2(T, \Pi^2_{T-1} - p(D-k))] + \delta\mathbb{E}[u^2(T, \Pi^2_{T-1} - pk)]}; \tag{EC.31}$$

$$H^2_{T-1,nc}(p) = \frac{pk + \delta\mathbb{E}[u^1(T, \Pi^1_{T-1} - pk)] - u^1(T-1)}{p(2k-D) - \delta\mathbb{E}[u^1(T, \Pi^1_{T-1} - p(D-k))] + \delta\mathbb{E}[u^1(T, \Pi^1_{T-1} - pk)]}. \tag{EC.32}$$

The expected utilities are for firm i

$$u^i(T-1) = \underline{p}k + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - \underline{p}k)]. \tag{EC.33}$$

Suppose $\lim_{p\uparrow\overline{P}} H^i_{T-1,nc}(p) = 1$ for either i. Then we have

$$u^i(T-1) = \underline{p}k + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - \underline{p}k)] = \overline{P}(D-k) + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - \overline{P}(D-k))], \tag{EC.34}$$

which has a unique solution $\underline{p}k = \overline{P}(D-k)$. Hence the expected utilities of each firm are

$$u^i(T-1) = \overline{P}(D-k) + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - \overline{P}(D-k))]. \tag{EC.35}$$

These are again independent of the other firm's distance from its target.

Replacing the periods T and $T-1$ by t and $t-1$ in the subscripts of the expressions above, we have an induction argument, and the reasoning holds for T periods. We can thus characterize the dynamic mixed strategy equilibrium as in the Proposition. $\square$

EC.2.2. Proof of Proposition 3

The distribution of prices is as follows:

$$H^i_{t,nc}(p) = \frac{pk + \delta\mathbb{E}[u^j(t+1, \Pi^j_t - pk)] - \overline{P}(D-k) - \delta\mathbb{E}[u^j(t+1, \Pi^j_t - \overline{P}(D-k))]}{p(2k-D) - \delta\mathbb{E}[u^j(t+1, \Pi^j_t - p(D-k))] + \delta\mathbb{E}[u^j(t+1, \Pi^j_t - pk)]}, \tag{EC.36}$$

$$H^i_{T,nc}(p) = \frac{pk - C(\Pi^j_T - pk)^+ - \overline{P}(D-k) + C(\Pi^j_T - \overline{P}(D-k))^+}{p(2k-D) + C(\Pi^j_T - p(D-k))^+ - C(\Pi^j_T - pk)^+}. \tag{EC.37}$$

1. When, at time T, $\Pi^j_T \le p(D-k)$, it is clear that

$$H^i_{T,nc}(p) = \frac{pk - \overline{P}(D-k)}{p(2k-D)} = H^i_o(p). \tag{EC.38}$$

2. When $p(D-k) < \Pi^j_T \le \overline{P}(D-k)$, we have

$$H^i_{T,nc}(p) = \frac{pk - \overline{P}(D-k)}{p(2k-D) + C(\Pi^j_T - p(D-k))^+}, \tag{EC.39}$$

where, compared to the stationary distribution, the denominator is increased, and hence $H^i_{T,nc}(p) \le H^i_o(p)$.

3. When $\overline{P}(D-k) < \Pi^j_T \le pk$,

$$H^i_{T,nc}(p) = \frac{pk - \overline{P}(D-k) + C(\Pi^j_T - \overline{P}(D-k))^+}{p(2k-D) + C(\Pi^j_T - p(D-k))^+}. \tag{EC.40}$$

Suppose the penalty is linear. Then we have $H^i_{T,nc}(p) > H^i_o(p)$ iff

$$(pk - \overline{P}(D-k))C(\Pi^j_T - p(D-k)) < p(2k-D)C(\Pi^j_T - \overline{P}(D-k)) \tag{EC.41}$$

$$(\overline{P} - p)(D-k)(pk - \Pi^j_T) < 0, \tag{EC.42}$$

which requires $pk < \Pi^j_T$, implying the next scenario. Hence it is never true for linear penalty. Consequently it is not true for convex penalties either, but may be true for concave penalties.

4. When $pk < \Pi_T^j$,

$$H^i_{T,nc}(p) = \frac{pk - C(\Pi^j_T - pk)^+ - \overline{P}(D-k) + C(\Pi^j_T - \overline{P}(D-k))^+}{p(2k-D) + C(\Pi^j_T - p(D-k))^+ - C(\Pi^j_T - pk)^+}. \tag{EC.43}$$

We have $H^i_{T,nc}(p) \leq H^i_o(p)$ iff

$$(\overline{P} - p)(D-k)C(\Pi^j_T - pk) - p(2k-D)C(\Pi^j_T - \overline{P}(D-k)) \tag{EC.44}$$

$$+(pk - \overline{P}(D-k))C(\Pi^j_T - p(D-k)) \leq 0. \tag{EC.45}$$

Plugging in $x = (\overline{P} - p)(D-k)$, $y = pk - \overline{P}(D-k)$, $z_1 = \Pi^j_T - pk$, $z_2 = \Pi^j_T - p(D-k)$, $z = \Pi^j_T - \overline{P}(D-k)$ and noting that

$$z = \frac{x}{x+y} z_1 + \frac{y}{x+y} z_2 \tag{EC.46}$$

we have the condition

$$xC(z_1) + yC(z_2) - (x+y)C(z) \geq 0, \tag{EC.47}$$

which is true iff $C(\cdot)$ is convex. $\square$

EC.2.3. Mixed strategies in dynamic collusive game

Proposition EC.1. *In the dynamic collusive game with revenue targets at time T, penalty functions $C^i_T(x) = C(\Pi^i_T - x)^+$ mixed strategies are played whenever $k < D \leq 2k$ and at least one firm would deviate to the price cap. The equilibrium in mixed strategies defined recursively as follows:*

$$H^i_{t,c}(p) = \frac{pk + \delta E^j_{at}(p) - \overline{P}(D-k) - \delta E^j_{at}(\overline{P}(D-k)/k)}{p(2k-D) - \delta E^j_{pt}(p) + \delta E^j_{at}(p)}, \tag{EC.48}$$

$$H^i_{T,c}(p) = \frac{pk - C(\Pi^j_T - pk)^+ - \overline{P}(D-k) + C(\Pi^j_T - \overline{P}(D-k))^+}{p(2k-D) + C(\Pi^j_T - p(D-k))^+ - C(\Pi^j_T - pk)^+}, \tag{EC.49}$$

with $p \in [\overline{P}(D-k)/k, \overline{P}]$, where

$$E^j_{at}(p) = \mathbb{E}[u^j(t+1, \Pi^j_t - pk, \Pi^i_t - p(D-k))|p^j > p] \tag{EC.50}$$

$$E^j_{bt}(p) = \mathbb{E}[[u^j_{t+1}(\Pi^j_t - p(D-k), \Pi^i_t - pk)|p^j \leq p]. \tag{EC.51}$$

Proof. The derivation is similar to that of the non-collusive dynamic equilibrium, with the difference that in the "collusive" mixed equilibrium, both firms' future profits affect the structure of strategies and hence profits.

For the first period before the target, consider mixed strategies with CDF's $H^i_{T,c}(p)$. Since the targets do not affect the future states of the game, the mixed strategies are exactly the same as above. Therefore we have the expected utilities

$$u^2(T,\Pi^2_T,D) = \overline{P}(D-k) - C(\Pi^2_T - \overline{P}(D-k))^+ \tag{EC.52}$$

$$\underline{p} = \frac{1}{k}\left(\overline{P}(D-k)\right) \tag{EC.53}$$

$$u^1(T,\Pi^1_T,D) = \overline{P}(D-k) - C(\Pi^1_T - \overline{P}(D-k))^+. \tag{EC.54}$$

For earlier periods, we need to take into account both firms' profit developments. That is, the (different) profit levels resulting from mixed strategies will affect the collusive prices achieved by the firms through the distance to the target.

Let us next consider the second to last period before the target. Now, since the expected utilities of the firms in the last period only depend on both their own distance to their target and the competitor's distance, the expected utilities are

$$u^i(T-1,p) = H^j_{T-1,c}(p)\left(p(D-k) + \delta\mathbb{E}[u^i(T,\Pi^i_{T-1} - p(D-k),\Pi^j_{T-1} - p^j k))|p^j \le p]\right) \tag{EC.55}$$

$$+ (1 - H^j_{T-1,c}(p))\left(pk + \delta\mathbb{E}[u^i(T,\Pi^i_{T-1} - pk,\Pi^j_{T-1} - p^j(D-k))|p^j > p]\right). \tag{EC.56}$$

Here, the inner expectations are with respect to the bid of the other player which follows the distribution H^j_{T-1}. They determine the distance of the competitor from its target. Denoting the expectations E^i_a and E^i_b for brevity, we can write H^j as:

$$H^j_{T-1,c}(p) = \frac{pk + \delta E^i_a - u^i(T-1)}{p(2k-D) - \delta E^i_b + \delta E^i_a}. \tag{EC.57}$$

Notice that since the expectations depend on H^j_{T-1}, this defines the distribution implicitly. The expected utilities are for firm i

$$u^i(T-1) = \underline{p}k + \delta E^i_a(\underline{p}). \tag{EC.58}$$

Suppose $\lim_{p\uparrow\overline{P}} H^i_{T-1,c}(p) = 1$ for either i. Then we have

$$u^i(T-1) = \underline{p}k + \delta E^i_a(\underline{p}) = \overline{P}(D-k) + \delta E^i_a(\overline{P}(D-k)/k), \tag{EC.59}$$

which has a unique solution $\underline{p}k = \overline{P}(D-k)$. We further note that at $E^i_a(\underline{p})$, the other firm is confined to bid only $\underline{p}$ and hence there is no need for the expectation. Therefore we can write the expected utilities of each firm as

$$u^i(T-1) = \overline{P}(D-k) + \delta\mathbb{E}[u^i(T,\Pi^i_{T-1} - \overline{P}(D-k), u^j(T,\Pi^j_{T-1} - \overline{P}(D-k)^2/k))]. \tag{EC.60}$$

Replacing the periods T and $T-1$ by t and $t-1$ in the subscripts of the expressions above, we have an induction argument, and the reasoning holds for T periods. We can thus characterize the dynamic mixed strategy equilibrium as in the Proposition. $\square$

EC.2.4. Proof of Proposition 4

It is clear that there is always at least an equilibrium (the non-collusive one). For the collusive equilibrium, we need to show that it is always optimal for the plants to choose the highest possible collusive price, due to their expected utility being non-decreasing in price. We will show this by induction.

In the last period before the target T, the expected utility $\mathbb{E}[u^i(T,D,\Pi_T)]$ is non-decreasing in price p up to the most-collusive price. This is easy to see if the plants can make the target by colluding: then the game is like the one without targets. The same is true for the case where they cannot make it by colluding or deviating. In the middle range, where they can make it by deviating only, the prices decrease to reduce the deviation gain, and the firms just miss the target: below it expected profits increase in price. Asymmetric cases can be handled similarly.

Suppose that $\mathbb{E}[u^i(\tau+1,D,\Pi_\tau)] = Dp/2 + \delta\mathbb{E}_u[\tau+1,\Pi_{\tau+1}(\Pi_\tau,D/2,p)]$ is non-decreasing in p, where $\mathbb{E}_u[\tau+1,\Pi_{\tau+1}]$ is the expected continuation utility at $\tau+1$. That is, suppressing some notation,

$$
\begin{aligned}
D/2 + \delta\mathbb{E}'_u[\tau+1,\Pi_{\tau+1}] &\geq 0 \\
\mathbb{E}'_u[\tau+1,\Pi_{\tau+1}] &\geq -D/2\delta.
\end{aligned}
$$

Now, in period τ,

$$
\begin{aligned}
\mathbb{E}_u[\tau,\Pi_{\tau+1}(\Pi_\tau,D_\tau/2,p_\tau)] &= \mathbb{E}[\max_{p(p_\tau)}\{D_\tau p(p_\tau)/2 + \delta\mathbb{E}_u[\tau+1,\Pi_{\tau+1}(\Pi_\tau,D_\tau/2,p_\tau)]\}] \\
&= \mathbb{E}[D_\tau p^*(p_\tau)/2 + \delta\mathbb{E}_u(\tau+1,\Pi_{\tau+1}(\Pi_\tau,D_\tau/2,p^*(p_\tau)))]. \qquad \text{(EC.61)}
\end{aligned}
$$

Differentiating, we have

$$
\begin{aligned}
\mathbb{E}'_u[\tau,\Pi_{\tau+1}(\Pi_\tau,D_\tau/2,p_\tau)] &= \mathbb{E}[p'^*(p_\tau)D_\tau/2 + \delta D_\tau p'^*(p_\tau)/2\mathbb{E}'_u[\tau+1,\Pi_{\tau+1}(\Pi_\tau,D_\tau/2,p^*)]] \\
&\geq \mathbb{E}[p'^*(p_\tau)D_\tau/2 - p'^*(p_\tau)D_\tau/2] = 0 \qquad \text{(EC.62)}
\end{aligned}
$$

where the inequality follows from the induction assumption. Hence we have the induction step. With Proposition EC.1, it follows that the equilibrium strategies in the text are optimal.

Let us denote the stationary price $p_b(D)$. To see that there exist τ_l,Π_l,D_l s.t. $p_c(\tau_l,D_l,\Pi_l) < p_b(D)$, where p_b is the stationary base-case price (without targets), consider the last period T before the target with D_l,Π_l such that $D_l\overline{P} \geq \Pi_l \geq D_l\overline{P}/2$. There is an increased incentive to deviate compared to the base case, but the punishment is the same. The price is hence never higher than in the stationary case and sometimes strictly lower.

Now suppose that $p_b(D) < \overline{P}$ for some D. To see that there exists τ_h,Π_h,D_h s.t. $p_c(\tau_h,D_h,\Pi_h) \geq p_b(D)$, suppose $\overline{D} \leq k$. Let us fix a (small) distance x_0 from the target. Consider period T: the collusive profit is $p_c(T,D,x_0)D/2 - C(x_0 - p_cD/2)^+ + M$ and the deviation profit is $p_c(T,D,x_0)D - C(x_0 - p_cD)^+$. Clearly the price $p_c(T,D)$ is never higher than the base case price $p_b(D)$. Now consider the period $T-1$. Collusive profit is $p_c(T-1,D,x_0)D/2 + \mathbb{E}[p_c(T,D_T,x_0 - p_c(T-1,D,x_0)D/2)D_T/2] - \mathbb{E}[C(x_0 - p_c(T-1,D,x_0)D/2 - \delta p_c(T,D_T,x_0 - p_cD_T/2)D_T/2)^+] + \delta M$ and deviation profit is $p_c(T-1,D,x_0)D - \delta C(x_0 - p_c(T-1,D,x_0)D)^+$. For $D>0$ and small enough x_0, the penalties over C disappear in both periods, and the price in period T is equal to $p_b(D)$. In period $T-1$, there is a positive probability that D_T is small enough so that $p_c(T,D_T,x_0 - p_cD_T/2) < p_b(D_T)$. But by decreasing x_0, this probability becomes negligible. Hence the price in period T is virtually equal to $p_b(D)$, and converges to $p_b(D)$ in earlier periods $T-t$ as t increases for any $x \le x_0$. Now fix another slightly higher distance $x_1 > x_0$. The collusive and deviation profits can be written as above. By the argument above, since prices converge to $p_b(D)$ for $x \le x_0$, they will also do so for $x \le x_1$. Now fix x and D such that a deviating firm will not reach the target. Collusive profits in period $T-t$ are $p_c(T-t,D,x)D/2 + \sum_t \delta^{T-t}\mathbb{E}[p_c(T-t,D_{T-t},x_{T-t})D_{T-t}/2] - \mathbb{E}[C(x - p_c(T-1,D,x)D/2 - \sum_t \delta^{T-t}p_c(T-t,D_{T-t},x_{T-t})D_{T-t}/2)^+] + \delta^{T-t}M$. Suppose $p_b(D) < \overline{P}$. Increasing t, by the convergence shown above, this will converge to $p_c(T-t,D,x)D/2 + \sum_t \delta^{T-t}q\mathbb{E}[p_b(D_{T-t})] + \delta^{T-t}M$, for q (arbitrarily) close to 1. Deviation profits, which guarantee not reaching the target, are $p_c(T-t,D,x)D - \delta^{T-t}C(x - p_c(T-1,D,x)D)^+$. Given that $M = \frac{\delta}{1-\delta}\mathbb{E}[p_b(D_{T-t})]$, the price will be higher than the stationary price if $\sum_t \delta^{T-t}q\mathbb{E}[p_b(D_{T-t})] + \delta^{T-t}M + \delta^{T-t}C(x - p_c(T-1,D,x)D)^+ > M$, that is, $C(x - p_c(T-1,D,x)D)^+ > \frac{\mathbb{E}[p_b(D_{T-t})](q-\delta)z(\delta)}{\delta^{T-t}}$, which will always hold as q converges to 1. Thus, for low enough k, there exists a demand-time state such that the collusive price will exceed the stationary price. □

EC.2.5. Proof of Proposition 5

The proof is by induction. Consider first a single-period game, that is, $T=1$. Suppose that $\Pi^i_T \neq \Pi^j_T$. Then, for a given (τ,D,Π), the $p(\tau,D,\Pi) = \min\{p^i,p^j\}$, and $\mathbb{E}_u[\tau,D,\Pi]$ for the firms is lower than if the targets were equal. If mixed strategies are played, the utility is similarly lower.

Now consider $T=\tau$. Suppose that the above holds for $T=\tau-1$. That is, for games of shorter length, equal targets are optimal. If $\Pi^i_\tau \neq \Pi^j_\tau$, for a given (τ,D,Π), $p(\tau,D,\Pi) = \min\{p^i,p^j\}$, and the prices are lower than those with equal targets in all future periods as well. Furthermore, punishments are lower for unequal targets. Thus $\mathbb{E}_u[\tau,D,\Pi]$ is lower as well. □

EC.2.6. Proof of Proposition 6

Suppose there is a single equal target at time T, the expected stationary punishment is $\Delta\pi_{c,s}$, and the expected punishment in the first period is $\Delta\pi_{c,1}$. Now consider expanding the horizon from the beginning by adding a another target before the current one (the original T becomes $2T$). Suppose that $\Delta\pi_{c,1} > \Delta\pi_{c,s}$, that is, targets increase expected profits. The incentives of the original target period are unaffected. Note that the non-collusive equilibrium is unaffected. At time T in the first target period, deviation incentives are not changed, but punishment is increased. Collusive prices hence (weakly) increase. The induction argument is simple: if prices increase in period $\tau < T$, punishment increases. Hence prices also increase in $\tau - 1$.

This argument applies to adding further targets. If, on the other hand, the target is such that punishment decreases, the effect is the opposite. $\square$

EC.3. Results for Section 3.2: Plant retirement

Again we distinguish between the dynamic non-collusive and collusive game. Given probabilities of retirement $\theta(\Pi_t^i)$, in the non-collusive game, the mixed strategies in the last period are derived from

$$\begin{aligned} u^i(T,p) =& H^i_{T,nc}(p)\big[p(D-k) - C(\Pi_T^i - p(D-k))^+ \\ &+ \big(1-\theta(\Pi_T^i - p(D-k))\big)\mathbb{E}\big[\theta(\Pi_T^j - p^j k)R_m + (1-\theta(\Pi_T^j - pk))R_{nc}|p^j \le p\big]\big] \\ &+ (1-H^i_{T,nc}(p))\big[pk - C(\Pi_T^i - pk)^+ \\ &+ \big(1-\theta(\Pi_T^i - pk)\big)\mathbb{E}\big[\theta(\Pi_T^j - p^j(D-k))R_{nc} + (1-\theta(\Pi_T^j - p^j(D-k))\big)R_m|p^j > p\big]\big], \end{aligned} \tag{EC.63}$$

where R_{nc} is the expected discountednon-collusive profit if both plants continue and $R_m = \sum_t \delta^t \mathbb{E}[\overline{P}\min\{D,k\}]$ is the expected discounted monopoly profit if the plant continues alone.

First, let us establish conditions for expected profit to be increasing and concave so that highest collusive prices are optimal and the existence of a mixed strategy equilibrium is guaranteed.

Lemma EC.1. *The profit function* $x + \mathbb{E}[u(t,\Pi_t^i,\Pi_t^j,x)] = \mathbb{E}[u^j(t+1,\Pi_t^j - f^j(x),\Pi_t^i - f^i(x))]$ *is increasing if* $\theta'(\Pi_t^i - x) \le \overline{\theta}_1$.

Proof. We will show the result for $f^i(x) = f^j(x) = x$ (the collusive profits case; the mixed profits case is similar. We have assumed θ to be continuous and concave increasing.

The expected utility of the plants is not always increasing in the price they set. In the last period, this can be seen from differentiating the expected profits

$$\begin{aligned} \mathbb{E}[u^i(T,D,\Pi_T,p)] =& Dp/2 - C(\Pi_T^i - Dp/2)^+ \\ &+ (1-\theta(\Pi_T^i - Dp/2))(\theta(\Pi_T^j - Dp/2)R_m + (1-\theta(\Pi_T^j - Dp/2))R_c). \end{aligned} \tag{EC.64}$$

The derivative with respect to p is

$$\frac{D}{2} + \frac{D}{2}C'(x^i)^+ + \frac{D}{2}\theta'(x^i)\left[\theta(x^j)R_m + (1-\theta(x^j))R_c\right] - \frac{D}{2}\theta'(x^j)\left[(R_m - R_c)(1-\theta(x^i))\right], \tag{EC.65}$$

where we have set $x^i = \Pi_T^i - Dp/2$. Note that the functions are not differentiable at zero, so this does not cover the cases where the firm reaches its target in the last period. However, it is easy to see that the lowest value for the change is achieved when firm i has reached its target but j has not. Then we have simply

$$\frac{D}{2}\left(1-\theta'(x^j)\left[R_m - R_c\right]\right), \tag{EC.66}$$

which is negative for high enough $\theta'(x^j)$. With concave $\theta(x)$, this can be prevented by having $\lim_{x\downarrow 0}\theta'(x) \le \frac{1}{R_m - R_c}$. Then we can follow the steps in the proof of Proposition 4 to show that this condition can be used to guarantee the function to be increasing in earlier periods as well. □

The following proposition describes the dynamic non-collusive (punishment) equilibrium.

Proposition EC.2. *In the dynamic non-collusive game with revenue targets at time T and penalty functions $C(x)^+$ and retirement probability $\theta(\Pi_T^i)$, mixed strategies are played whenever $k < D \le 2k$. The equilibrium in mixed strategies is defined recursively as follows:*

$$H^i_{t,nc}(p) = \frac{pk + \delta E^i_{at}(p) - \overline{P}(D-k) - \delta E^j_{bt}(\overline{P})}{p(2k-D) - \delta E^j_{bt}(p) + \delta E^j_{at}(p)}, \tag{EC.67}$$

$$H^i_{T,nc}(p) = \frac{pk - C(\Pi_T^j - pk)^+ - \overline{P}(D-k) + C(\Pi_T^j - \overline{P}(D-k))^+ + E^j_{cT}(p) - E^j_{dT}(\overline{P})}{p(2k-D) + C(\Pi_T^j - p(D-k))^+ - C(\Pi_T^j - pk)^+ + E^j_{cT}(p) - E^j_{dT}(p)}, \tag{EC.68}$$

with $p \in [\overline{P}(D-k)/k, \overline{P}]$, where

$$E^j_{at}(p) = \mathbb{E}[u^j(t+1, \Pi_t^j - pk, t+1, \Pi_t^i - p^i(D-k))|p^i > p] \tag{EC.69}$$

$$E^j_{bt}(p) = \mathbb{E}[u^j(t+1, \Pi_t^j - p(D-k), t+1, \Pi_t^i - p^i k)|p^i \le p] \tag{EC.70}$$

$$E^j_{ct}(p) = \left(1-\theta(\Pi_T^j - p(D-k))\right)\mathbb{E}\left[\theta(\Pi_T^i - p^i k)R_m + (1-\theta(\Pi_T^i - pk))R_{nc}|p^i \le p\right] \tag{EC.71}$$

$$E^j_{dt}(p) = \left(1-\theta(\Pi_T^j - pk)\right)\mathbb{E}\left[\theta(\Pi_T^i - p^i(D-k))R_m + \left(1-\theta(\Pi_T^i - p^i(D-k))\right)R_{nc}|p^i > p\right]. \tag{EC.72}$$

In the collusive game, the mixed strategies in the last period are derived from

$$\begin{aligned}
u^i(T,p) = & H^i_{T,c}(p)\big[p(D-k) - C(\Pi_T^i - p(D-k))^+ \\
& + \left(1-\theta(\Pi_T^i - p(D-k))\right)\mathbb{E}\left[\theta(\Pi_T^j - p^j k)R_m + (1-\theta(\Pi_T^j - pk))R_c|p^j \le p\right]\big] \\
& + (1 - H^i_{T,c}(p))\big[pk - C(\Pi_T^i - pk)^+ \\
& + \left(1-\theta(\Pi_T^i - pk)\right)\mathbb{E}\left[\theta(\Pi_T^j - p^j(D-k))R_m + \left(1-\theta(\Pi_T^j - p^j(D-k))\right)R_c|p^j > p\right]\big],
\end{aligned} \tag{EC.73}$$

where R_c is the expected collusive profit if poth plants continue and R_m is the monopoly profit if the plant continues alone. In previous periods, the expected profit is as follows:

$$u^i(T-1,p) = H^j_{T-1,c}(p)\left(p(D-k) + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - p(D-k), \Pi^j_{T-1} - p^j k)|p^j \le p]\right) \tag{EC.74}$$

$$+ (1 - H^j_{T-1,c}(p))\left(pk + \delta\mathbb{E}[u^i(T, \Pi^i_{T-1} - pk, \Pi^j_{T-1} - p^j(D-k))|p^j > p]\right). \tag{EC.75}$$

The following proposition describes the collusive mixed strategies.

PROPOSITION EC.3. *In the dynamic collusive game with revenue targets at time T and penalty functions $C(x)^+$, and retirement probability $\theta(\Pi_T^i)$, mixed strategies are played whenever $k < D \le 2k$ and at least one firm would deviate to the price cap. The equilibrium in mixed strategies is defined recursively as follows:*

$$H_{t,c}^i(p) = \frac{pk + \delta E_a^i(p) - \overline{P}(D-k) - \delta E_{bt}^j(\overline{P})}{p(2k-D) - \delta E_{bt}^j(p) + \delta E_{at}^j(p)}, \tag{EC.76}$$

$$H_{T,c}^i(p) = \frac{pk - C(\Pi_T^j - pk)^+ - \overline{P}(D-k) + C(\Pi_T^j - \overline{P}(D-k))^+ + E_{cT}^j(p) - E_{dT}^j(\overline{P})}{p(2k-D) + C(\Pi_T^j - p(D-k))^+ - C(\Pi_T^j - pk)^+ + E_{cT}^j(p) - E_{dT}^j(p)}, \tag{EC.77}$$

with $p \in [\overline{P}(D-k)/k, \overline{P}]$, where

$$E_{at}^j(p) = \mathbb{E}[u^j(t+1, \Pi_t^j - pk, \Pi_t^i - p^i(D-k))|p^i > p] \tag{EC.78}$$

$$E_{bt}^j(p) = \mathbb{E}[u^j(t+1, \Pi_t^j - p(D-k), \Pi_t^i - p^i k)|p^i \le p] \tag{EC.79}$$

$$E_{dt}^j(p) = \big(1 - \theta(\Pi_T^j - p(D-k))\big)\mathbb{E}\big[\theta(\Pi_T^i - p^i k)R_m + (1 - \theta(\Pi_T^i - pk))R_c|p^i \le p\big] \tag{EC.80}$$

$$E_{ct}^j(p) = \big(1 - \theta(\Pi_T^j - pk)\big)\mathbb{E}\big[\theta(\Pi_T^i - p^i(D-k))R_m + (1 - \theta(\Pi_T^i - p^i(D-k))\big)R_c|p^i \le p\big]. \tag{EC.81}$$

The proofs of the two above Propositions closely follow the steps of the proof of Proposition EC.1 and are hence omitted. Note that here the expected profits refer to the expected collusive profits so the equilibrium needs to be solved in conjunction with the full collusive equilibrium.

The final period payoffs are now

$$R_c^i(\Pi_T^i) = -C\left(\Pi_{T+1}^i\right)^+ + \left(1 - \theta(\Pi_{T+1}^i)\right)\big)\big(\theta(\Pi_{T+1}^j)R_m + (1 - \theta(\Pi_{T+1}^j))R_c\big), \tag{EC.82}$$

$$R_{nc}^i(\Pi_T^i) = -C\left(\Pi_{T+1}^i\right)^+ + \left(1 - \theta(\Pi_{T+1}^i)\right)\big)\big(\theta(\Pi_{T+1}^j)R_m + (1 - \theta(\Pi_{T+1}^j))R_{nc}\big). \tag{EC.83}$$

We can again use the mixed strategy result to write the most-collusive strategies recursively through continuation payoffs:

$$\mathbb{E}[u^i(\tau, D, \Pi_\tau)] = \begin{cases} Dp_c/2 + \delta\mathbb{E}[u^i(\tau+1, \Pi_{\tau+1}(\Pi_\tau, D/2, p_c))] & \text{if } \max_i \zeta^i = 0 \\ (D-k)^+\overline{P} + \delta\mathbb{E}[u^i(\tau+1, \Pi_{\tau+1,M}(\Pi_\tau))] & \text{o.w.} \end{cases} \quad \tau \in \{1, \ldots, T\} \tag{EC.84}$$

$$\zeta^i(\tau, D, \Pi_\tau) = \mathbb{1}(Dp_c/2 + \delta\mathbb{E}[u^i(\tau+1, \Pi_{\tau+1}(\Pi_\tau, D/2, p_c))] \le \tag{EC.85}$$

$$(D-k)\overline{P} + \delta\mathbb{E}[u^i(\tau+1, \Pi_{\tau+1,M}(\Pi_\tau))]),\ \forall\, i,$$

$$p_c = \min_i \ \arg\max_{p(\tau, D, \Pi_\tau)} \mathbb{E}[u^i(\tau, D, \Pi_\tau, p)] \tag{EC.86}$$

$$\mathbb{E}[u^i(\tau, D, \Pi_\tau, p)] = Dp/2 + \delta\mathbb{E}[u^i(\tau+1, \Pi_{\tau+1}(\Pi_\tau, D/2, p))],\ \forall\, i \tag{EC.87}$$

$$u^i(T+1, D, \Pi) = R_c^i(\Pi_{T+1}^i) \tag{EC.88}$$

$$\mathbb{E}[u^i(\tau, D, \Pi_\tau, p)] \ge \mathbb{E}(u_{dev}^i(\tau, D, \Pi_\tau, p))\ \forall\, i \tag{EC.89}$$

$$\mathbb{E}[u_{dev}^i(\tau, D, \Pi_\tau, p)] = \min\{k, D\}p + \left[u_{t+1,nc}^i(\Pi_{\tau+1}(\Pi_\tau, \min\{k, D\}, p))\right] \tag{EC.90}$$

$$u_{t,nc}^i(\Pi_t) = \delta\mathbb{E}[\overline{P}(D-k)^+ + u_{t+1,nc}^i(\Pi_{t+1,M}(\Pi_t))] \tag{EC.91}$$

$$u_{T+1,nc}^i(\Pi_{T+1}) = R_{nc}^i(\Pi_{T+1}^i). \tag{EC.92}$$

Here $\Pi_\tau = (\Pi^i_\tau, \Pi^j_\tau)$ denotes the state of the game, updated as $\Pi_{\tau+1}(\Pi_\tau, x, p) = (\Pi^i_\tau - xp, \Pi^j_\tau - (D - x)p)$. With mixed strategies, the update is $\Pi_{\tau+1,M}(\Pi_\tau) = (\Pi^i_\tau - \overline{P}(D-k)^+, \Pi^j_\tau - \overline{P}(D-k)^{+2}/k)$.

EC.3.1. Proof of Proposition 7

Letting the distances from the targets $\Pi^i \uparrow \infty$, deviations will happen iff

$$DG + (1-\theta_\infty)\theta_\infty R_m + (1-\theta_\infty)^2 R_{nc} > (1-\theta_\infty)\theta_\infty R_m + (1-\theta_\infty)^2 R_c \tag{EC.93}$$

$$DG + (1-\theta_\infty)^2 R_{nc} > (1-\theta_\infty)^2 R_c, \tag{EC.94}$$

where $DG = \min\{D/2, k - D/2\}p$ denotes the current period deviation gain and $\theta = 0$ corresponds to the case of no retirement. As the distances are large, there is no chance of meeting the target and θ is not affected by Π^i. Clearly, if $\theta_\infty > 0$, the RHS is decreased more, and collusion becomes more difficult to sustain. We can see that as $\theta_\infty \uparrow 1$, collusion cannot be sustained, and pricing will be non-collusive.

To see that prices will be highest with $\theta(x) = 0$, we will use induction.

In the last period before the target, deviation becomes more attractive with retirement. This is because firm 1 will deviate iff

$$DG + (1-\theta^1_d)\theta^2_d R_m + (1-\theta^1_d)(1-\theta^2_d) R_{nc} > (1-\theta^1_c)\theta^2_c R_m + (1-\theta^1_c)(1-\theta^2_c) R_c, \tag{EC.95}$$

where in the shorthand, the superscripts denote the firm and subscripts deviation or collusive retirement probabilities. We can write this as the deviation incentives without retirement and an adjustment term:

$$DG > R_c - R_{nc} + (1-\theta^1_c)\theta^2_c(R_m - R_c) - (1-\theta^1_d)\theta^2_d(R_m - R_{nc}) - \theta^1_c R_c + \theta^1_d R_{nc}, \tag{EC.96}$$

where we can easily verify that the adjustment term (last four terms on RHS) is negative since $\theta^1_c \le \theta^1_d$, $\theta^2_c \ge \theta^2_d$ and $R_m \ge R_c \ge R_{nc}$. Hence prices decrease with retirement.

Now suppose prices are decreased relative to the case of no retirement in all states in each period from T to $T - t$, and consider period $T - t - 1$. The corresponding comparison is then (suppressing notation)

$$DG > \mathbb{E}[u^i_c] - \mathbb{E}[u^i_d]. \tag{EC.97}$$

By the induction assumption, prices are lower than prices with no retirement in all future periods until the target, and hence (by Lemma EC.1) collusive profits are lower as well. If, in addition, $\mathbb{E}[u^i_d]$ is higher than in the no retirement case, the RHS is decreased and we are done. If $\mathbb{E}[u^i_d]$ is lower, if we can show that $\mathbb{E}[u_d^i(T-t-1)] \geq \mathbb{E}[u_d^i(T-t)]$, we have the same result. To see that this holds, we note from above that the expected profits can be written as

$$\mathbb{E}[u_d^i(T-t-1,\Pi^i)] = \delta[(1-\alpha)\mathbb{E}[u_d^i(T-t,\Pi^i)] + \alpha(\overline{P}(D-k) + \mathbb{E}[u_d^i(T-t,\Pi^i + \overline{P}(D-k))]]. \tag{EC.98}$$

In the corresponding stationary game, $\mathbb{E}[u_d^i(T-t-1,\Pi^i)] = \mathbb{E}[u_d^i(T-t,\Pi^i)] = \mathbb{E}[u_d^i(T-t,\Pi^i + \overline{P}(D-k))]$. Now, when $\mathbb{E}[u_d^i(T-t,\Pi^i)] < \pi_{nc}$, if $\mathbb{E}[u_d^i(T-t,\Pi^i + \overline{P}(D-k))] \geq \mathbb{E}[u_d^i(T-t,\Pi^i)]$, this implies that the expected utility must increase.

To see that $\mathbb{E}[u_d^i(T-t,\Pi^i + \overline{P}(D-k))] \geq \mathbb{E}[u_d^i(T-t,\Pi^i)]$ when $\mathbb{E}[u_d^i(T-t,\Pi^i)] < \pi_{nc}$, we can use a simple induction argument similar to that in Lemma EC.1, but adjusted for the expected value function in the punishment equilibrium in the case $\mathbb{E}[u_d^i(T-t,\Pi^i)] < \pi_{nc}$. In the last period, when the expected value is lower, it will clearly be higher in the next period. For the induction step, if $\mathbb{E}[u_d^i(T-t,x)] \leq \mathbb{E}[u_d^i(T-t,x+q_1)]$,

$$\mathbb{E}[u_d^i(T-t-1,x)] = \delta[(1-\alpha)\mathbb{E}[u_d^i(T-t,x)] + \alpha\mathbb{E}_q[q + \mathbb{E}[u_d^i(T-t,x+q)]]] \tag{EC.99}$$

$$\leq \delta[(1-\alpha)\mathbb{E}[u_d^i(T-t,x+q_1)] + \alpha\mathbb{E}_q[q + \mathbb{E}[u_d^i(T-t,x+q+q_1)]]] \tag{EC.100}$$

$$= \mathbb{E}[u_d^i(T-t-1,x+q_1)]. \tag{EC.101}$$

We then have the entire argument for prices being lower with retirement, compared with the stationary game.

Consider now the uncapacitated case. To see that there will never be collusion when plants missing targets are retired with certainty, consider the last period to reach the target $t = T$. Suppose that $\Delta\Pi^{min} = \min\{\Delta\Pi^i, \Delta\Pi^j\} > 0$, so that both plants are under the target. If $Dp/2 < \Delta\Pi^{min}$ so that neither can reach its target, both will be retired with certainty. Therefore, there is no punishment to create a collusive outcome, and the one-shot equilibrium will always be played. This is true also if just one of the plants cannot reach its target: it has no future and will thus deviate.

Now suppose $\Delta\Pi^{min} = \min\{\Delta\Pi^i, \Delta\Pi^j\} > 0$ but that the $Dp < \Delta\Pi^{min} < Dp/2$ so that both can make it by deviating but not by colluding. The plants only have a future if they deviate, so they will. The same is true if just one of the plants can make the target by deviating but not by colluding.

Now suppose $\Delta\Pi^{min} = \min\{\Delta\Pi^i, \Delta\Pi^j\} > 0$ and that $Dp/2 \geq \Delta\Pi^{min}$, so that both can make their targets by colluding. Deviating ensures that the plant will enjoy monopoly profits which are larger than the collusive continuation profits. Therefore the plants will deviate.

Finally, if one of the plants is below the target and the other has reached its target already, the latter one always has an incentive to deviate to enjoy monopoly profits. To conclude, in the last period, there is no collusion. Consequently, there is no punishment for the period $T-1$, and the same reasoning holds. It follows by induction that the plants never collude if at least one of them starts below the target. $\square$

EC.4. Results in Section 4: Diversification

EC.4.1. Proof of Proposition 8

To find single-shot equilibria for the diversified case, let us again divide the demand into regions. We will only consider equilibria where in high demand situations the firms offer their entire capacity with the same price.

Each generating company has capacity k divided into low (0) marginal cost and high (c) marginal cost capacity. let firm F1 have λk baseload capacity and $(1-\lambda)k$ peak capacity, and firm F2 correspondingly $(1-\lambda)k$ baseload capacity and λk peak capacity, with $\lambda \in [0, 1/2]$.

Part (i): $D \leq \lambda k$. First, when $D \leq \lambda k$, a single low MC plant from either firm may alone serve the whole demand. Therefore the firms will undercut each other to bring the price to 0.

Part (ii): $\lambda k < D \leq k$. Second, when we have $\lambda k < D \leq k$, the firms will play mixed strategies as follows. High MC plants will offer their marginal cost c and the low MC plants will mix prices lower or equal to c. This means that the high MC plants effectively place a price cap of c on the mixed strategies played by the low MC plants. Since both firms have total capacity k and can thus serve the entire demand in this region, a single firm does not have an incentive to deviate to higher prices. Clearly there is no pure strategy equilibrium for the low MC plants due to cyclic offering similarly to the setting in the one-shot game. The equilibrium mixed strategies can be derived using arguments similar to the Appendix of **?** (essentially requiring the mixed strategy to make the competitor indifferent over the support) and are as follows.

Denote $G^i(p) = Pr(p^i \leq p)$ the distribution of the mixed strategy of generator i. In the equilibrium we have the firms offering with mixed strategies on $[\underline{p}, c]$ with distributions

$$G^1(p) = \frac{\min\{(1-\lambda)k, D\}(p-\underline{p})}{p(\lambda k + \min\{(1-\lambda)k, D\} - D)} \tag{EC.102}$$

$$G^2(p) = \frac{\lambda k(p-\underline{p})}{p(\lambda k + \min\{(1-\lambda)k, D\} - D)} \tag{EC.103}$$

forming the unique mixed equilibrium, with $\underline{p} = c(D-\lambda k)/\min\{(1-\lambda)k, D\}$. This corresponds to expected profits of

$$\pi^1 = \lambda k \underline{p} \tag{EC.104}$$

$$\pi^2 = \min\{(1-\lambda)k, D\}\underline{p} \tag{EC.105}$$

so the predominantly low cost supplier (F2) makes a larger profit on average, as expected. However, notice that it is also held to its minmax profit $c(D-\lambda k)$ in this equilibrium, making this the lowest expected profit equilibrium it could be held to as a result of a punishment in a repeated game. We note that this is not necessarily the only equilibrium in this demand region (an equilibrium could also involve mixing up to the price cap – similarly to the next paragraph) but this equilibrium can be used to hold the other firm to lower profits than a higher-pricing equilibrium and would thus be used to create a higher punishment.

Part (iii): $D > k$. Finally, when demand exceeds the total capacity of a single firm, the firms will play mixed strategies up to the price cap $\overline{P}$. Again, no pure strategy equilibria can exist due to cyclical undercutting and price increases; also, the derivation is similar to **?**, so we omit some details. Suppose that the firms offer their entire capacities k at a single price according to $G^i(p)$, with common support in the set $[\underline{p}, \overline{P}]$. Profits for the firms under mixed strategies are

$$\pi^1(p) = G^2(p)\left[(p(D-k) - c\max\{D-k-\lambda k, 0\}\right] + (1-G^2(p))\left[pk - (1-\lambda)ck\right] \quad \text{(EC.106)}$$

$$\pi^2(p) = G^1(p)\left[(p(D-k) - c\max\{D-k-(1-\lambda)k, 0\}\right] + (1-G^2(p))\left[pk - \lambda ck\right] \quad \text{(EC.107)}$$

where in each of the equations, the first term is the case where the firm offers higher than its competitor and vice versa. In the equilibrium, the firms need to be indifferent over their choice of p, so that $\pi^i(p) = \overline{\pi}^i \; \forall\, p$. Then we can solve for

$$G^1(p) = \frac{pk - \lambda ck - \overline{\pi}^2}{p(2k-D) - c\left(\lambda k - \max\{D-k-(1-\lambda)k, 0\}\right)} \quad \text{(EC.108)}$$

$$G^2(p) = \frac{pk - (1-\lambda)ck - \overline{\pi}^1}{p(2k-D) - c\left((1-\lambda)k - \max\{D-k-\lambda k, 0\}\right)}. \quad \text{(EC.109)}$$

Noting that $G^i(\underline{p}) = 0$, we have

$$\overline{\pi}^1 = \underline{p}k - (1-\lambda)ck \quad \text{(EC.110)}$$

$$\overline{\pi}^2 = \underline{p}k - \lambda ck. \quad \text{(EC.111)}$$

To find $\underline{p}$, we need to find the values of the distributions at $\overline{P}$:

$$\lim_{p\uparrow\overline{P}} G^1(p) = \frac{\overline{P}k - \underline{p}k}{\overline{P}(2k-D) - c\left(\lambda k - \max\{D-k-(1-\lambda)k, 0\}\right)} \quad \text{(EC.112)}$$

$$\lim_{p\uparrow\overline{P}} G^2(p) = \frac{\overline{P}k - \underline{p}k}{\overline{P}(2k-D) - c\left((1-\lambda)k - \max\{D-k-\lambda k, 0\}\right)}. \quad \text{(EC.113)}$$

Note that these are not necessarily equal. Therefore, only the larger of them can be equal to one. To find if the the limit is larger for Firm 1, we need to compare the denominators of the two expressions and check if

$$\lambda k - \max\{D-k-(1-\lambda)k, 0\} > (1-\lambda)k - \max\{D-k-\lambda k, 0\}, \quad \text{(EC.114)}$$

which is never true for $\lambda \leq 1/2$. Therefore, we have $G^2(\overline{P}) = 1$ and can solve for

$$\underline{p} = \frac{1}{k}\left(\overline{P}(D-k) - c(\max\{D-k(1+\lambda), 0\} - (1-\lambda)k)\right)\right) \geq c, \quad \text{(EC.115)}$$

which can be plugged in the expected profit equations to get

$$\overline{\pi}^1 = \overline{P}(D-k) - c\max\{D-k(1+\lambda), 0\} \quad \text{(EC.116)}$$

$$\overline{\pi}^2 = \overline{P}(D-k) - c\max\{D-k(1+\lambda), 0\} + ck(1-2\lambda). \quad \text{(EC.117)}$$

Again, the predominantly low marginal cost producer makes a larger profit (with equal profits when $\lambda = 1/2$ as expected). Here, Firm 1 is held to minmax profits. Firm 2 makes larger than minmax profits (here its minmax profits would be equal to $\overline{P}(D-k) - c\max\{D - k(2-\lambda), 0\}$) if $k < D < k(2-\lambda)$. That is, the firm makes larger than minmax profits the more often, the more specialised the firms are. $\square$

EC.4.2. Proof of Proposition 9

Let us derive the deviation gains and punishments for the two asymmetric firms.

Part (i): Deviation gains. Recall that F1 has λk low MC capacity and $(1-\lambda)k$ high MC capacity, while F2 has $(1-\lambda)k$ low MC capacity and λk high MC capacity, with the fraction $\lambda \le 1/2$. We assume further that in the collusive equilibrium the generators are offering their entire capacities at the same price and are allocated production according to their (equal) capacities.

To find deviation gains, consider current period profits for a firm when colluding or when deviating by undercutting. If $D \le k$, the firm can capture the entire demand, while for larger D it can only serve its capacity k. For F1, these profits are

$$\pi^1 = \begin{cases} D\overline{P}/2 - c(D/2 - \lambda k)^+ & \text{if collude} \\ D\overline{P} - c(D - \lambda k)^+ & \text{if deviate} \end{cases} \qquad (D \le k) \tag{EC.118}$$

$$\pi^1 = \begin{cases} D\overline{P}/2 - c(D/2 - \lambda k)^+ & \text{if collude} \\ k\overline{P} - c(1-\lambda)k & \text{if deviate} \end{cases} \qquad (D > k). \tag{EC.119}$$

For F2, the corresponding profits are

$$\pi^2 = \begin{cases} D\overline{P}/2 - c(D/2 - (1-\lambda)k)^+ & \text{if collude} \\ D\overline{P} - c(D - (1-\lambda)k)^+ & \text{if deviate} \end{cases} \qquad (D \le k) \tag{EC.120}$$

$$\pi^2 = \begin{cases} D\overline{P}/2 - c(D/2 - (1-\lambda)k)^+ & \text{if collude} \\ k\overline{P} - c\lambda k & \text{if deviate} \end{cases} \qquad (D > k). \tag{EC.121}$$

The deviation gains can be found as the difference of the deviation and collusive profits. We need to consider the different cases the $(\cdot)^+$-functions with respect to the parameters D and λ.

Consider first the low demand case $D \le k$. For F1 there are two parameter thresholds for the $(\cdot)^+$-functions: $\lambda = D/2k$ and $\lambda = D/k$. For F2 the thresholds are $\lambda = (k-D)/k$ and $\lambda = (k - D/2)/k$. Therefore we need to find the deviation gains in the different intervals

$$F^1 : \lambda \in [0 \ldots \frac{D}{2k} \ldots \frac{D}{k} \ldots \frac{1}{2}] \tag{EC.122}$$

$$F^2 : \lambda \in [0 \ldots \frac{k-D}{k} \ldots \frac{k - \frac{D}{2}}{k} \ldots \frac{1}{2}]. \tag{EC.123}$$

Comparing the thresholds for the two firms, we have the following:

$$\frac{k-D}{k} > \frac{D}{2k} \iff D < \frac{2k}{3} \tag{EC.124}$$
$$\frac{k-D}{k} > \frac{D}{k} \iff D < \frac{k}{2} \tag{EC.125}$$
$$\frac{k-\frac{D}{2}}{k} > \frac{D}{2k} \iff D < k \tag{EC.126}$$
$$\frac{k-\frac{D}{2}}{k} > \frac{D}{k} \iff D < \frac{2k}{3}. \tag{EC.127}$$

Using these, we have three demand intervals: $D \in [0 \ldots k/2 \ldots 2k/3 \ldots k]$. For each of these intervals, λ also has different intervals:

$$1^\circ \quad D \in [0, \frac{k}{2}]: \quad \lambda \in [0 \ldots \frac{D}{2k} \ldots \frac{D}{k} \ldots \frac{1}{2}] \tag{EC.128}$$
$$2^\circ \quad D \in [\frac{k}{2}, \frac{2k}{3}]: \quad \lambda \in [0 \ldots \frac{D}{2k} \ldots \frac{k-D}{k} \ldots \frac{1}{2}] \tag{EC.129}$$
$$3^\circ \quad D \in [\frac{2k}{3}, k]: \quad \lambda \in [0 \underbrace{\ldots}_{(i)} \frac{k-D}{k} \underbrace{\ldots}_{(ii)} \frac{D}{2k} \underbrace{\ldots}_{(iii)} \frac{1}{2}]. \tag{EC.130}$$

This gives us nine intervals in total where to derive the deviation gains for each of the two firms. These are presented in Table EC.1. It is easy to see that the deviation gain is always (weakly) larger for F2.

Table EC.1 Deviation gains for the low demand case.

		F1	**F2**
1°	**(i)**	$D(\overline{P}-c)/2$	$D\overline{P}/2$
	(ii)	$D\overline{P}/2 - c(D-\lambda k)$	$D\overline{P}/2$
	(iii)	$D\overline{P}/2$	$D\overline{P}/2$
2°	**(i)**	$D(\overline{P}-c)/2$	$D\overline{P}/2$
	(ii)	$D\overline{P}/2 - c(D-\lambda k)$	$D\overline{P}/2$
	(iii)	$D\overline{P}/2 - c(D-\lambda k)$	$D\overline{P}/2 - c(D-(1-\lambda)k)$
3°	**(i)**	$D(\overline{P}-c)/2$	$D\overline{P}/2$
	(ii)	$D(\overline{P}-c)/2$	$D\overline{P}/2 - c(D-(1-\lambda)k)$
	(iii)	$D\overline{P}/2 - c(D-\lambda k)$	$D\overline{P}/2 - c(D-(1-\lambda)k)$

When $D > k$, from the profits above we can find the threshold $D = 2(1-\lambda)k$ for F2. For F1 the $(\cdot)^+$-functions are always positive. Therefore we have the following deviation gains (Table EC.2). Again, the deviation gain is (weakly) larger for F2. Together, these results suggest that, all else equal, firm 2 would be more likely to deviate from implicit collusion. However, punishments for the firms differ as well.

The deviation gains can further be used to derive the relationship between collusive thresholds $\underline{D}^*$ and $\overline{D}^*$. This is done by equating the deviation gains. For F2, we have the following cases:

Table EC.2 Deviation gains for the high demand case.

	F1	**F2**
1° $D \le 2(1-\lambda)k$:	$(\overline{P}-c)(k-D/2)$	$(k-D/2)\overline{P}-ck\lambda$
2° $D > 2(1-\lambda)k$:	$(\overline{P}-c)(k-D/2)$	$(\overline{P}-c)(k-D/2)$

1. $\underline{D}^*\overline{P}/2 = (k-\overline{D}^*/2)\overline{P} - ck\lambda \iff \overline{D}^* = 2k - \frac{2c\lambda}{\overline{P}}k - \underline{D}^*$
2. $\underline{D}^*\overline{P}/2 = (k-\overline{D}^*/2)(\overline{P}-c) \iff \overline{D}^* = 2k - \underline{D}^*\overline{P}/(\overline{P}-c)$
3. $\underline{D}^*\overline{P}/2 - c(\underline{D}^* - (1-\lambda)k) = (k-\overline{D}^*/2)\overline{P} - ck\lambda \iff \overline{D}^* = 2k - \underline{D}^* - 2(k-\underline{D}^*)c/\overline{P}$
4. $\underline{D}^*\overline{P}/2 - c(\underline{D}^* - (1-\lambda)k) = (k-\overline{D}^*/2)(\overline{P}-c) \iff \overline{D}^* = 2k - \underline{D}^*\overline{P}/(\overline{P}-c) + (\underline{D}^* - (1-\lambda)k)2c/(\overline{P}-c)$.

Similarly, for F1, we have

1. $\underline{D}^*\overline{P}/2 = (k-\overline{D}^*/2)(\overline{P}-c) \iff \overline{D}^* = 2k - \frac{\overline{P}}{\overline{P}-c}\underline{D}^*$
2. $\underline{D}^*/2(\overline{P}-c) = (k-\overline{D}^*/2)(\overline{P}-c) \iff \overline{D}^* = 2k - \underline{D}^*$
3. $\underline{D}^*\overline{P}/2 - c(\underline{D}^* - \lambda k) = (k-\overline{D}^*/2)(\overline{P}-c) \iff \overline{D}^* = 2k - \frac{\overline{P}}{\overline{P}-c}\underline{D}^* + \frac{2c}{\overline{P}-c}(\underline{D}^* - \lambda k)$.

To find the prices $P(D)$ that the firms would be able to sustain, we again note that the deviation gain must remain constant between the thresholds. Using the deviation gains, we can derive the prices for the two firms:

$$\text{F1}: P(D) = (\overline{P}-c)\frac{\underline{D}^*/2}{k-D/2} + c \tag{EC.131}$$

$$\text{F2}: P(D) = (\overline{P}-c)\frac{\underline{D}^*/2}{k-D/2} + c \quad \text{if } D, \overline{D}^* \ge 2k(1-\lambda) \tag{EC.132}$$

$$\text{F2}: P(D) = \overline{P}\frac{\underline{D}^*/2}{k-D/2} + c - c\frac{k\lambda}{k-D/2} \quad \text{if } D < 2k(1-\lambda), \quad \overline{D}^* \ge 2k(1-\lambda) \tag{EC.133}$$

$$\text{F2}: P(D) = \overline{P}\frac{\underline{D}^*/2}{k-D/2} \quad \text{if } D, \overline{D}^* < 2k(1-\lambda). \tag{EC.134}$$

In the first case of F2, the prices are the same for both firms. In the third case, the price of F2 is lower. In the second case, F2 is lower if $\lambda < 1 - \frac{\underline{D}^*}{2k}$, but since $\overline{D}^* \ge 2k(1-\lambda)$, this is never true (strictly). Hence F1's price is at least as low in the second case. Therefore it is not clear which of the firms can sustain a higher price at a given demand with the same parameters. We note that the lower of these prices does not in fact depend on λ (except through $\underline{D}^*$). Moreover, as λ decreases, holding other things constant, the third case becomes more prevalent, with lower prices more often.

The mixed strategy thresholds $\{\hat{D}, \check{D}\}$ can be derived similarly to Proposition 1.

Furthermore, there may be another range $\{\dot{D}, \ddot{D}\}$ in which mixed strategies are preferred when $\frac{k}{2} < D \le k$. This happens when the mixed strategy profits in this region exceed those with the collusive prices. The mixed strategy profits are

$$\pi^1 = \frac{c\lambda k(D-\lambda k)}{\min\{(1-\lambda)k, D\}} \tag{EC.135}$$

$$\pi^2 = c(D-\lambda k). \tag{EC.136}$$

These never exceed the monopoly profits $D\overline{P}/2$ but may exceed $\underline{D}^*\overline{P}/2$ when the threshold is low, for example for F2:

$$c(D-\lambda k) > \underline{D}^*\overline{P}/2 \iff D > \underline{D}^*\frac{\overline{P}}{2c} + \lambda k. \tag{EC.137}$$

Note that all else equal, this happens less often with higher λ.

Part (ii): Punishments. The punishments for the two firms depend on the difference of the collusive profits and the one-shot equilibrium profits that the firms would gain after deviation. Recall that the punishment can be written as

$$M = \frac{\gamma}{1-\gamma}\int_0^{\overline{D}} (\pi_c(D) - \pi_o(D))\, dF(D). \tag{EC.138}$$

We have already specified π_o in the one-shot equilibrium results of Proposition 8. It remains to write down the collusive profits π_c and combine these two.

Consider first the collusive profits for small demand realisations $D \le k$. The collusive profits for F1 depend on how much of its production share it has to cover with high MC capacity. They can be written as

$$\pi_c^1 = \begin{cases} D\overline{P}/2 & \text{if } D \le \underline{D}^* \\ \underline{D}^*\overline{P}/2 - c(D/2-\lambda k)^+ & \text{if } D > \underline{D}^* \end{cases} \qquad (\underline{D}^* \le 2\lambda k) \tag{EC.139}$$

$$\pi_c^1 = \begin{cases} D\overline{P}/2 - c(D/2-\lambda k)^+ & \text{if } D \le \underline{D}^* \\ \underline{D}^*\overline{P}/2 - c(D/2-\lambda k) & \text{if } D > \underline{D}^* \end{cases} \qquad (\underline{D}^* > 2\lambda k). \tag{EC.140}$$

The corresponding profits for F2 are simple since the firm has enough low MC capacity to always cover half of the demand in this region.

$$\pi_c^2 = \begin{cases} D\overline{P}/2 & \text{if } D \le \underline{D}^* \\ \underline{D}^*\overline{P}/2 & \text{if } D > \underline{D}^* \end{cases} \qquad (\underline{D}^* \le k) \tag{EC.141}$$

When $D > k$, we have

$$\pi_c^1 = \begin{cases} DP(D)/2 - c(D/2-\lambda k) & \text{if } D \le \overline{D}^* \\ D\overline{P}/2 - c(D/2-\lambda k) & \text{if } D > \overline{D}^* \end{cases} \tag{EC.142}$$

$$\pi_c^2 = \begin{cases} DP(D)/2 - c(D/2-(1-\lambda)k)^+ & \text{if } D \le \overline{D}^* \\ D\overline{P}/2 - c(D/2-(1-\lambda)k)^+ & \text{if } D > \overline{D}^* \end{cases}, \tag{EC.143}$$

where the maximum sustainable price $P(D)$ can be derived in each case similarly to the symmetric case of Proposition 1.

Taking the difference of the collusive and one-shot profits, we can derive the punishments for the firms. Again, whenever the difference would be negative, the firms would prefer to play mixed strategies of the one-shot equilibrium.

For F1, the difference is

$$\pi_c^1 - \pi_o^1 = \begin{cases} D\overline{P}/2 - c(D/2-\lambda k)^+ - c(D-\lambda k)^+ \lambda k/\min\{(1-\lambda)k, D\} & \text{if } D \le \underline{D}^* \\ \underline{D}^*\overline{P}/2 - c(D/2-\lambda k)^+ - c(D-\lambda k)^+ \lambda k/\min\{(1-\lambda)k, D\} & \text{if } \underline{D}^* < D \le k \\ DP(D)/2 - c(D/2-\lambda k) - \overline{P}(D-k) + c(D-k(1+\lambda))^+ & \text{if } k < D \le \overline{D}^* \\ D\overline{P}/2 - c(D/2-\lambda k) - \overline{P}(D-k) + c(D-k(1+\lambda))^+ & \text{if } D \ge \overline{D}^*. \end{cases} \tag{EC.144}$$

For F2, we have the difference

$$\pi_c^2 - \pi_o^2 = \begin{cases} D\overline{P}/2 - c(D-\lambda k)^+ \quad \text{if } D \le \underline{D}^* \\ \underline{D}^*\overline{P}/2 - c(D-\lambda k)^+ \quad \text{if } \underline{D}^* < D \le k \\ DP(D)/2 - c(D/2-(1-\lambda)k)^+ - \overline{P}(D-k) + c(D-k(1+\lambda))^+ - (1-2\lambda)ck & \text{if } k < D \le \overline{D}^* \\ D\overline{P}/2 - c(D/2-(1-\lambda)k)^+ - \overline{P}(D-k) + c(D-k(1+\lambda))^+ - (1-2\lambda)ck & \text{if } D \ge \overline{D}^*. \end{cases} \tag{EC.145}$$

Based on these, we can compare which of the firms has the larger punishment, all else equal. This is straightforward so we omit most of the details. As an example, let us check whether $\pi_c^1 - \pi_o^1 \ge \pi_c^2 - \pi_o^2$ for the case where $D \ge \overline{D}^*$ (the case $k < D \le \overline{D}^*$ is similar):

$$D\overline{P}/2 - c(D/2-\lambda k) - \overline{P}(D-k) + c(D-k(1+\lambda))^+ \tag{EC.146}$$

$$\ge D\overline{P}/2 - c(D/2-(1-\lambda)k)^+ - \overline{P}(D-k) + c(D-k(1+\lambda))^+ - (1-2\lambda)ck \tag{EC.147}$$

$$\iff -c(D/2-\lambda k) \ge -c(D/2-(1-\lambda)k)^+ - (1-2\lambda)ck \tag{EC.148}$$

Suppose that the $(\cdot)^+$-function is zero, that is, $D \le 2k(1-\lambda)$. Note that we also have $D \ge k$. Then the inequality holds if

$$D/2 - \lambda k \le (1-2\lambda)k \tag{EC.149}$$

$$\iff D \le 2k(1-\lambda), \tag{EC.150}$$

that is, always when the $(\cdot)^+$-function is zero. On the other hand, when this function is positive, the two differences are equal.

Consider next the case where $\underline{D}^* < D \le k$ and $D > \lambda k$ and suppose further that $D > 2\lambda k$:

$$\underline{D}^*\overline{P}/2 - c(D/2-\lambda k)^+ - c(D-\lambda k)\frac{\lambda}{1-\lambda} \ge \underline{D}^*\overline{P}/2 - c(D-\lambda k) \tag{EC.151}$$

$$\iff -c(D/2-\lambda k) - c(D-\lambda k)\frac{\lambda}{1-\lambda} \ge -c(D-\lambda k) \tag{EC.152}$$

$$\iff D(3\lambda-1) \le 2\lambda^2 k \tag{EC.153}$$

$$\iff D \le \frac{2\lambda^2}{3\lambda-1}k \quad (\lambda > \frac{1}{3}). \tag{EC.154}$$

Here, the inequality is reversed if $\lambda < \frac{1}{3}$ and clearly holds then. The multiplier for k on the RHS is always larger than one when $\lambda \in (\frac{1}{3}, 1/2]$ so the inequality always holds.

Going through all the possible cases in a similar way, in fact we find that

$$\pi_c^1 - \pi_o^1 \geq \pi_c^2 - \pi_o^2 \quad \forall D, \tag{EC.155}$$

that is, the difference between the collusive and one-shot profits is at least as large (and sometimes strictly larger) for F1 for any demand realisation (given the same parameters and thresholds). Since the demand realisations are the same for both firms, we can thus conclude that, all else equal, the punishment for F1 from deviating is larger than that for F2. This suggests that F2 would be more likely to deviate than F1. We note that lower punishment also means using mixed strategies on a wider range of demand realisations.

Part (iii): Combining the results. We know from above that, all parameters and thresholds equal, the predominantly low marginal cost generator F2 is more likely to deviate. Further, we know that the prices and thresholds are in fact equal for the two firms in the collusive equilibrium. Therefore, we can focus on the incentives of F2. Increasing the diversification parameter λ affects collusion in several ways. Here we will show that if a collusive equilibrium exists for λ_1, an equilibrium with the same or more collusive thresholds and prices will exist for a $\lambda_2 > \lambda_1$.

Let us assume that there exists an equilibrium with λ with collusive thresholds and prices shared by the two generators. First, we note that the punishment for F2 is non-decreasing in λ. This can be seen from equation (EC.145). This also means that the area where mixed strategies are played decreases when λ increases. Similarly, we can see from Tables EC.1 and EC.2 that the deviation gain for F2 is non-increasing in λ. Moreover, it can be seen from the derivation of collusive thresholds above that the area of demand for which the firms perfectly collude is non-decreasing in the diversification parameter λ. Finally, above we noted that the maximum price that can be sustained in the equilibrium, p_c, is non-decreasing in λ.

Combining these effects, if a collusive equilibrium exists for λ_1, an equilibrium with the same or more collusive thresholds and prices will exist for a $\lambda_2 > \lambda_1$. That is, with higher λ_2, both the firms can still support collusion with the original equilibrium parameters and thresholds. $\square$