

Electronic Companion for “Risk-Aversion and B2B Contracting under Asymmetric Information: Evidence from Managed Print Services”

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A Proofs

A.1 Proof of Theorem 1.

To prove Theorem 1, we first present and prove two lemmas. Lemma 4 states that the portfolio optimization problem (9) with R contracting groups is equivalent to R individual screening problems. Lemma 5 further simplifies the optimization problem of each contracting group. In the analysis, we use $h(\cdot)$ to denote the probability density function of the Beta distribution $Beta(1, 1/\kappa)$. (Here we assume that no exclusion of printers is allowed. This is consistent with practice where when a MPS provider contracts with an companies company, all the company’s printers are covered.)

Lemma 4. *The provider’s portfolio optimization with all R contracting groups reduces to a set of optimization problems, one per contracting group. The optimal contract design for contracting*

group r ($r = 1, 2, \dots, R$) solves the following optimization:

$$\max_{\{(F_r(\theta), \vec{P}_r(\theta)) : \theta \in [0, 1]\}} \mathbb{E}_{\theta_r} [MV_r(F_r(\theta_r), \vec{P}_r(\theta_r))] \quad (22a)$$

$$s.t. \quad \mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} [U_r(\vec{P}_r(\theta'), F_r(\theta'), \theta)] \leq \mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} [U_r(\vec{P}_r(\theta), F_r(\theta), \theta)] \quad \forall \theta, \theta' \in [0, 1], \quad (22b)$$

$$\mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} [U_r(\vec{P}_r(\theta), F_r(\theta), \theta)] \geq U_{0r} \quad \forall \theta \in [0, 1], \quad (22c)$$

where

$$\begin{aligned} MV_r(F_r(\theta), \vec{P}_r(\theta)) &= N_r \tau_r [(\vec{P}_r(\theta) - \vec{s}_r^0)^T \vec{\Phi} (\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 + F_r(\theta))] \\ &\quad - \lambda N_r \tau_r \left\{ N_r \tau_r \left([\vec{P}_r(\theta) - \vec{s}_r^0]^T \vec{\Phi} \vec{1}_2 \right)^2 \sigma^2 + \tau_r \left(\vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{s}_r^0 \right) \right. \\ &\quad \left. + \left(\vec{P}_r^T(\theta) \vec{V}_r^0 \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{V}_r^s \right) \right\}, \end{aligned} \quad (23)$$

and

$$\vec{s}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{s}_{rj}, \quad \vec{\mu}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{\mu}_{rj}, \quad \vec{V}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{V}_{rj}, \quad \vec{V}_r^s = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{V}_{rj} \vec{s}_{rj}, \quad (24a)$$

$$\vec{V}_{rj} = \text{Var} \left(\vec{Q}_{rjt}(\vec{p}_r, f_r) \middle| \epsilon_r, \vec{\omega}_{rj} \right) = \mathbb{E} \left[\vec{\Phi} \vec{\xi}_{rjt} \vec{\xi}_{rjt}^T \vec{\Phi} \right]. \quad (24b)$$

Proof. In optimization (9), constraints of different contracting groups are independent of each other. Therefore, proving Lemma 4 reduces to proving that the objective function of (9), namely, the expectation $\mathbb{E}_{\Theta}[\cdot]$ over the entire portfolio, can be written as the sum of expectations over each contracting group.

The remainder of the proof establishes this result by first showing that the mean-variance term within the expectation $\mathbb{E}_{\Theta}[\cdot]$ can be written as the sum of R mean-variance terms, each of which is associated with one contracting group.

Start by simplifying the objective function (9). The summation in the variance and expectation

terms satisfies

$$\begin{aligned}
& \sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[Y_{rjt}(\vec{Q}_{rjt}(\vec{p}_r, f_r), \vec{p}_r, f_r) \right] \\
&= \sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left\{ \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{Q}_{rjt}(\vec{P}_r(\theta), F_r(\theta)) + F_r(\theta) - \delta_{rjt} \right\} \\
&= \sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \left[\vec{\Phi} \left(\vec{\Psi}_r \theta + \vec{\mu}_{rj} - \vec{P}_r(\theta) \right) + \vec{\Phi} \left(\epsilon_r \vec{1}_2 + \vec{\omega}_{rj} + \vec{\xi}_{rjt} \right) \right] \tag{25a}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} [F_r(\theta) - \delta_{rjt}], \tag{25b}
\end{aligned}$$

where (25a) follows from (5).

Next consider the variance of (25). Because $\vec{P}_r(\theta) - \vec{s}_{rj}$ and $\vec{\Phi}(\vec{\Psi}_r \theta + \vec{\mu}_{rj} - \vec{P}_r(\theta))$ in (25a), and $F_r(\theta)$ in (25b) are constants, variance of (25) is equivalent to

$$\begin{aligned}
& \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{\Phi} \left(\epsilon_r \vec{1}_2 + \vec{\omega}_{rj} + \vec{\xi}_{rjt} \right) - \sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \delta_{rjt} \right) \\
&= \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{\Phi} \vec{1}_2 \epsilon_r \right) + \tag{26a}
\end{aligned}$$

$$\begin{aligned}
& \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{\Phi} \vec{\omega}_{rj} \right) + \tag{26b}
\end{aligned}$$

$$\begin{aligned}
& \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{\Phi} \vec{\xi}_{rjt} \right) + \tag{26c}
\end{aligned}$$

$$\begin{aligned}
& \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \delta_{rjt} \right), \tag{26d}
\end{aligned}$$

where the decomposition follows from the fact that random variables ϵ , ω , ξ , and δ are orthogonal to each other.

Recall that the contracting group-level random effects $\epsilon_r \stackrel{iid}{\sim} N(0, \sigma^2)$. Using $\vec{s}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{s}_{rj}$,

the variance term in (26a) satisfies

$$\begin{aligned} \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{1}_2 \epsilon_r \right) &= \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \tau_r [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{1}_2 \epsilon_r \right) \\ &= \text{Var} \left(\sum_{r=1}^R \tau_r \left[N_r \vec{P}_r(\theta) - \sum_{j=1}^{N_r} \vec{s}_{rj} \right]^T \vec{\Phi} \vec{1}_2 \epsilon_r \right) = \sum_{r=1}^R \left(N_r \tau_r [\vec{P}_r(\theta) - \vec{s}_r^0]^T \vec{\Phi} \vec{1}_2 \right)^2 \sigma^2. \end{aligned} \quad (27)$$

Recall that $\vec{\omega}_{rj} \stackrel{iid}{\sim} N(0, \vec{\Sigma})$. The variance term in (26b) is

$$\begin{aligned} \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{\omega}_{rj} \right) &= \text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \tau_r [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{\omega}_{rj} \right) \\ &= \sum_{r=1}^R \sum_{j=1}^{N_r} \tau_r^2 [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{\Sigma} \vec{\Phi} [\vec{P}_r(\theta) - \vec{s}_{rj}]. \end{aligned} \quad (28)$$

Recall that $\vec{\xi}_{rjt}$ have zero mean and are independent across printers. Using \vec{V}_{rj} in (24b), the variance term in (26c) is

$$\text{Var} \left(\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{\Phi} \vec{\xi}_{rj}(t) \right) = \sum_{r=1}^R \sum_{j=1}^{N_r} \tau_r [\vec{P}_r(\theta) - \vec{s}_{rj}]^T \vec{V}_{rj} [\vec{P}_r(\theta) - \vec{s}_{rj}]. \quad (29)$$

Finally, because (26d) is independent of the contract terms, it can be dropped from the objective function. Therefore, combining (27)–(29) and dropping terms that are independent of the contract, the variance term in the objective function can be simplified as follows

$$\begin{aligned} &\sum_{r=1}^R [N_r \tau_r (\vec{P}_r(\theta) - \vec{s}_r^0)^T \vec{\Phi} \vec{1}_2]^2 \sigma^2 + \\ &\sum_{r=1}^R N_r \tau_r^2 [\vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{s}_r^0] + \end{aligned} \quad (30a)$$

$$\sum_{r=1}^R N_r \tau_r [\vec{P}_r^T(\theta) \vec{V}_r^0 \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{V}_r^s]. \quad (30b)$$

where $\vec{V}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{V}_{rj}$, $\vec{V}_r^s = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{V}_{rj} \vec{s}_{rj}$, (30a) corresponds to (28) and (30b) to (29).

Now consider the expectation term in the objective function (9). Using $\mathbb{E}[\epsilon_r] = 0$, $\mathbb{E}[\vec{\omega}_{rj}] = 0$,

and $\mathbb{E}[\vec{\xi}_{rj}(t)] = 0$, equation (25) yields

$$\begin{aligned} & \mathbb{E} \left[\sum_{r=1}^R \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \left[\vec{P}_r(\theta) - \vec{s}_{rj} \right]^T \vec{\Phi} \left(\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_{rj} \right) + F_r(\theta) \right] \\ &= \sum_{r=1}^R N_r \tau_r \left[(\vec{P}_r(\theta) - \vec{s}_r^0)^T \vec{\Phi} (\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0) + F_r(\theta) \right] + \sum_{r=1}^R \tau_r \left[N_r (\vec{s}_r^0)^T \vec{\Phi} \vec{\mu}_r^0 - \sum_{j=1}^{N_r} \vec{s}_{rj}^T \vec{\Phi} \vec{\mu}_{rj} \right]. \end{aligned} \quad (31)$$

The last term in (31) is independent of the contract and can be dropped.

Therefore, conditional on the value of θ , the mean-variance objective on the overall cumulative cash flow can be written as a summation of R mean-variance terms $MV_r(F_r(\theta), \vec{P}_r(\theta))$ ($r = 1, 2, \dots, R$), where $MV_r(F_r(\theta), \vec{P}_r(\theta))$ is as defined in equation (23).

Because $\theta_r \perp \epsilon_r \perp \vec{\omega}_{rj}$ and θ_r is independent across contracting groups, the expectation $\mathbb{E}_{\Theta}[\cdot]$ can be written as the summation of $\mathbb{E}_{\theta_r}[MV_r(F_r(\theta_r), \vec{P}_r(\theta_r))]$, establishing the lemma. \square

Lemma 5. *The provider's optimization (22) is equivalent to the following problem.*

$$\max_{\{(F_r(\theta), \vec{P}_r(\theta)) : \theta \in [0, 1]\}} \int_0^1 MV_r(F_r(\theta), \vec{P}_r(\theta)) h(\theta) d\theta, \quad (32a)$$

$$s.t. \quad \frac{du_r^{**}(\theta)}{d\theta} = \vec{\Psi}_r^T \vec{\Phi} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0] \quad (32b)$$

$$\vec{\Psi}_r^T \vec{\Phi} \frac{d\vec{P}_r(\theta)}{d\theta} \leq 0, \quad (32c)$$

$$u_r^{**}(0) \geq u_{0r}, \quad (32d)$$

where $MV_r(F_r(\theta), \vec{P}_r(\theta))$ is defined in (23), and

$$u_r^{**}(\theta) = \frac{1}{2} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right]^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right] - F_r(\theta), \quad (33a)$$

$$u_{0r} = \frac{U_{0r}}{N_r \tau_r} - \frac{1}{2} \left(\sigma^2 \vec{1}_2^T \vec{\Phi} \vec{1}_2 + \frac{1}{N_r} \sum_{j=1}^{N_r} \mathbb{E}[\vec{\omega}_{rj}^T \vec{\Phi} \vec{\omega}_{rj}] - \frac{1}{N_r \tau_r} \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \mathbb{E} \left[\vec{\xi}_{rjt}^T \vec{\Phi} \vec{\xi}_{rjt} \right] \right). \quad (33b)$$

Proof. First consider the IC constraint (22b). From equation (7),

$$\begin{aligned}
U_r \left(\vec{P}_r(\theta), F_r(\theta), \theta \right) &= \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} U_{rjt} \left(\vec{P}_r(\theta), F_r(\theta), \theta \right) \\
&= \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \frac{1}{2} \left(\vec{\Psi}_r \theta + \vec{\mu}_{rj} - \vec{P}_r(\theta) + \epsilon_r \vec{1}_2 + \vec{\omega}_{rj} + \vec{\xi}_{rjt} \right)^T \vec{\Phi} \\
&\quad \times \left(\vec{\Psi}_r \theta + \vec{\mu}_{rj} - \vec{P}_r(\theta) + \epsilon_r \vec{1}_2 + \vec{\omega}_{rj} + \vec{\xi}_{rjt} \right) - F_r(\theta).
\end{aligned}$$

Given that $\mathbb{E}[\epsilon_r] = 0$, $\mathbb{E}[\vec{\omega}_{rj}] = 0$, $\mathbb{E}[\vec{\xi}_{rj}(t)] = 0$, and $\epsilon_r \perp \vec{\omega}_{rj} \perp \vec{\xi}_{rj}(t)$, and using $\vec{\mu}_r^0 = \frac{1}{N_r} \sum_{j=1}^{N_r} \vec{\mu}_{rj}$, $\mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} [U_r(\vec{P}(\theta), F(\theta), \theta)]$ satisfies

$$\begin{aligned}
&\mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} \left[U_r \left(\vec{P}_r(\theta), F_r(\theta), \theta \right) \right] \\
&= N_r \tau_r \left[\frac{1}{2} \left(\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right)^T \vec{\Phi} \left(\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right) - F_r(\theta) \right]
\end{aligned} \tag{34a}$$

$$+ \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \frac{1}{2} \mathbb{E} \left[\sigma^2 \vec{1}_2^T \vec{\Phi} \vec{1}_2 + \vec{\omega}_{rj}^T \vec{\Phi} \vec{\omega}_{rj} + \vec{\xi}_{rjt}^T \vec{\Phi} \vec{\xi}_{rjt} \right]. \tag{34b}$$

Because (34b) is invariant in θ , the IC constraint (22b) reduces to

$$\begin{aligned}
&\frac{1}{2} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta') + \vec{\mu}_r^0 \right]^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta') + \vec{\mu}_r^0 \right] - F_r(\theta') \\
&\leq \frac{1}{2} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right]^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right] - F_r(\theta)
\end{aligned} \tag{35}$$

That is, the contract designed for contracting group r of type- θ then solves the following problem:

$$\theta = \arg \max_{\theta'} \frac{1}{2} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta') + \vec{\mu}_r^0 \right]^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta') + \vec{\mu}_r^0 \right] - F_r(\theta'). \tag{36}$$

The first- and second-order local conditions for the contract $(f_r(\theta), \vec{p}_r(\theta))$ to be incentive compatible are, respectively:

$$\left(\frac{d\vec{P}_r(\theta)}{d\theta} \right)^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right] + \frac{dF_r(\theta)}{d\theta} = 0, \tag{37a}$$

$$\vec{\Psi}_r^T \vec{\Phi} \frac{d\vec{P}_r(\theta)}{d\theta} \leq 0. \tag{37b}$$

Because the Spence-Mirrlees condition is satisfied by the utility associated with each contracting group, the local first and second order conditions are necessary and sufficient for incentive compatibility (Laffont and Martimort 2001).

Let

$$u_r^{**}(\theta) = \frac{1}{2} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right]^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right] - F_r(\theta). \quad (38)$$

Then the first-order IC constraint (37a) is equivalent to the following constraint on $u_r^{**}(\theta)$:

$$\frac{du_r^{**}(\theta)}{d\theta} = \vec{\Psi}^T \vec{\Phi} \left[\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right]. \quad (39)$$

Finally consider the participation constraint (22c). Combining (34) and (38) yields

$$u_r^{**}(\theta) \geq u_{0r} \quad \forall \theta \in [0, 1], \quad (40)$$

where

$$u_{0r} = \frac{U_{0r}}{N_r \tau_r} - \frac{1}{2} \left(\sigma^2 \vec{1}_2^T \vec{\Phi} \vec{1}_2 + \frac{1}{N_r} \sum_{j=1}^{N_r} \mathbb{E}[\vec{\omega}_{rj}^T \vec{\Phi} \vec{\omega}_{rj}] - \frac{1}{N_r \tau_r} \sum_{j=1}^{N_r} \sum_{t=1}^{\tau_r} \mathbb{E} \left[\vec{\xi}_{rjt}^T \vec{\Phi} \vec{\xi}_{rjt} \right] \right). \quad (41)$$

Furthermore, because $\vec{\Phi} [\vec{\Psi}_r \theta - \vec{p}_r + \vec{\mu}_r^0]$ is the expected monthly volume (equation (5)), which is positive by definition, and $\vec{\Psi}_r > 0$, $du_r^{**}(\theta)/d\theta > 0$ in (39). Therefore, (40) reduces to a constraint on the lowest type $\theta = 0$:

$$u_r^{**}(0) \geq u_{0r}, \quad (42)$$

completing the proof. □

Proof of Theorem 1. From Lemma 5, optimization (22) for contracting group r is equivalent to optimization (32), which we shall solve below. We first ignore constraint (32c).

Let the participation constraint of the lowest type be binding

$$u_r^{**}(0) = u_{0r}. \quad (43)$$

Then by equation (32b), we have

$$u_r^{**}(\theta) = u_{0r} + \vec{\Psi}_r^T \vec{\Phi} \int_0^\theta [z \vec{\Psi}_r - \vec{P}_r(z) + \vec{\mu}_r^0] dz. \quad (44)$$

Using the definition of $u_r^{**}(\theta)$, equation (44) yields

$$F_r(\theta) = \frac{1}{2} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0]^T \vec{\Phi} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0] - u_{0r} - \vec{\Psi}_r^T \vec{\Phi} \int_0^\theta [z \vec{\Psi}_r - \vec{P}_r(z) + \vec{\mu}_r^0] dz. \quad (45)$$

Then in the objective function of (32), the integral over $f_r(\theta)$ becomes

$$\begin{aligned} \int_0^1 F_r(\theta) h(\theta) d\theta &= \int_0^1 \frac{1}{2} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0]^T \vec{\Phi} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0] h(\theta) d\theta \\ &\quad - \vec{\Psi}_r^T \vec{\Phi} \int_0^1 \left[\int_0^\theta [z \vec{\Psi}_r - \vec{P}_r(z) + \vec{\mu}_r^0] dz \right] h(\theta) d\theta - u_{0r}. \end{aligned} \quad (46)$$

Using integration by parts, the double integral on the right side of equation (46) becomes

$$\begin{aligned} &\int_0^1 \int_0^\theta [z \vec{\Psi}_r - \vec{P}_r(z) + \vec{\mu}_r^0] h(\theta) dz d\theta \\ &= H(\theta) \int_0^\theta [z \vec{\Psi}_r - \vec{P}_r(z) + \vec{\mu}_r^0] dz \Big|_0^1 - \int_0^1 H(\theta) [\theta \vec{\Psi}_r - \vec{P}_r(\theta) + \vec{\mu}_r^0] d\theta \\ &= \int_0^1 \bar{H}(\theta) [\theta \vec{\Psi}_r - \vec{P}_r(\theta) + \vec{\mu}_r^0] d\theta, \end{aligned}$$

where $H(\theta)$ is the cumulative distribution function of θ , $\bar{H}(\theta) = 1 - H(\theta)$. Thus, the integral over $f_r(\theta)$ becomes

$$\int_0^1 \left(\frac{1}{2} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0] - \frac{\bar{H}(\theta)}{h(\theta)} \vec{\Psi}_r \right)^T \vec{\Phi} [\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0] h(\theta) d\theta - u_{0r}. \quad (47)$$

Plugging (47) into the first term of $MV_r(F_r(\theta), \vec{P}_r(\theta))$ in equation (23) yields

$$\begin{aligned} &\int_0^1 N_r \tau_r \left[\left(\vec{P}_r(\theta) - \vec{s}_r^0 \right)^T \vec{\Phi} \left(\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0 \right) + F_r(\theta) \right] h(\theta) d\theta \\ &= N_r \tau_r \int_0^1 \left(\frac{\vec{P}_r(\theta) + \theta \vec{\Psi}_r + \vec{\mu}_r^0}{2} - \vec{s}_r^0 - \frac{\bar{H}(\theta)}{h(\theta)} \vec{\Psi}_r \right)^T \vec{\Phi} [\theta \vec{\Psi}_r - \vec{P}_r(\theta) + \vec{\mu}_r^0] h(\theta) d\theta - N_r \tau_r u_{0r}. \end{aligned} \quad (48)$$

Combining (48) with the rest three terms of $MV_r(F_r(\theta), \vec{P}_r(\theta))$ in (23), the integral in (32a) is

$$\int_0^1 MV_r(F_r(\theta), \vec{P}_r(\theta)) h(\theta) d\theta = N_r \tau_r \int_0^1 \left(\frac{\vec{P}_r(\theta) + \theta \vec{\Psi}_r + \vec{\mu}_r^0}{2} - \vec{s}_r^0 - \frac{\bar{H}(\theta)}{h(\theta)} \vec{\Psi}_r \right)^T \vec{\Phi} [\theta \vec{\Psi}_r - \vec{P}_r(\theta) + \vec{\mu}_r^0] h(\theta) d\theta \quad (49a)$$

$$- N_r \tau_r u_{0r} - \lambda N_r^2 \tau_r^2 \sigma^2 \int_0^1 \left([\vec{P}_r(\theta) - \vec{s}_r^0]^T \vec{\Phi} \vec{1}_2 \right)^2 h(\theta) d\theta \quad (49b)$$

$$- \lambda N_r \tau_r^2 \int_0^1 \left(\vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{s}_r^0 \right) h(\theta) d\theta \quad (49c)$$

$$- \lambda N_r \tau_r \int_0^1 \left(\vec{P}_r^T(\theta) \vec{V}_r^0 \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{V}_r^s \right) h(\theta) d\theta. \quad (49d)$$

Therefore, finding the function $\vec{P}_r(\theta)$ that optimizes equation (49) is equivalent to the following pointwise maximization

$$\max_{\vec{p}_r} \left(\frac{\vec{p}_r + \theta \vec{\Psi}_r + \vec{\mu}_r^0}{2} - \vec{s}_r^0 - \frac{\bar{H}(\theta)}{h(\theta)} \vec{\Psi}_r \right)^T \vec{\Phi} [\theta \vec{\Psi}_r - \vec{p}_r + \vec{\mu}_r^0] \quad (50a)$$

$$- \lambda N_r \tau_r \sigma^2 \left([\vec{p}_r - \vec{s}_r^0]^T \vec{\Phi} \vec{1}_2 \right)^2 - \lambda \tau_r \left(\vec{p}_r^T \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{p}_r - 2 \vec{p}_r^T \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{s}_r^0 \right) \quad (50b)$$

$$- \lambda \left(\vec{p}_r^T \vec{V}_r^0 \vec{p}_r - 2 \vec{p}_r^T \vec{V}_r^s \right). \quad (50c)$$

Maximization (50) is a concave function of \vec{p}_r . First-order condition yields

$$\vec{P}_r^*(\theta) = \vec{s}_r^0 + \left(2\lambda \vec{\Gamma}_r + \vec{\Phi} \right)^{-1} \left[\vec{\Phi} \frac{\bar{H}(\theta)}{h(\theta)} \vec{\Psi}_r + 2\lambda (\vec{V}_r^s - \vec{V}_r^0 \vec{s}_r^0) \right], \quad (51)$$

where $\vec{\Gamma}_r$ is given in Theorem 1.

When θ follows beta distribution $Beta(1, 1/\kappa)$, $\bar{H}(\theta)/h(\theta) = (1 - \theta)\kappa$. It is easy to check that $\vec{P}_r^*(\theta)$ satisfies the earlier ignored constraint (32c). Therefore, $\vec{P}_r^*(\theta)$ in equation (51) is the equilibrium variable price.

The equilibrium fixed price $F_r^*(\theta)$ follows by combining equations (51) and (45). \square

A.2 Proofs of statements in §5

Proof of Lemma 1. From equation (11), the observed variable prices \vec{p}_r^* satisfies

$$\vec{p}_r^* = \vec{P}_r^*(\theta_r) = \vec{s}_r^0 + \left(2\lambda \vec{\Gamma}_r + \vec{\Phi} \right)^{-1} \left[\vec{\Phi} (1 - \theta_r) \kappa \vec{\Psi}_r + 2\lambda \left(\vec{V}_r^s - \vec{V}_r^0 \vec{s}_r^0 \right) \right]. \quad (52)$$

This gives

$$\vec{\Psi}_r(1 - \theta_r) = \frac{1}{\kappa} \vec{\Phi} \left(2\lambda \vec{\Gamma}_r + \vec{\Phi} \right) (\vec{p}_r^* - \vec{s}_r^0) - \frac{2\lambda}{\kappa} \vec{\Phi} \left(\vec{V}_r^s - \vec{V}_r^0 \vec{s}_r^0 \right),$$

which yields equation (15). \square

Proof of Lemma 2. Substituting equations (15), (16) and (19) into equation (17) yields

$$\vec{q}_{rj}^* = \vec{\Phi} \vec{\Psi}_r - \frac{1}{\kappa} \left(2\lambda \vec{\Gamma}_r + \vec{\Phi} \right) (\vec{p}_r^* - \vec{s}_r^0) + \frac{2\lambda}{\kappa} \left(\vec{V}_r^s - \vec{V}_r^0 \vec{s}_r^0 \right) - \vec{\Phi} \vec{p}_r^* + \vec{\Phi} \vec{1}_2 \vec{\gamma}^T \vec{Z}_{rj} + \vec{\Phi} \vec{1}_2 \epsilon_r + \vec{\Phi} \vec{\omega}_{rj}. \quad (53)$$

In equation (53), term $\vec{\Gamma}_r$ is defined in equation (13b), which is repeated below for convenience:

$$\vec{\Gamma}_r = \vec{V}_r^0 + N_r \tau_r \sigma^2 \vec{\Phi} \vec{1}_2 \vec{1}_2^T \vec{\Phi} + \tau_r \vec{\Phi} \vec{\Sigma} \vec{\Phi}.$$

Substituting equation (3b) for $\vec{\Phi}$ and recalling that $\vec{\Sigma}$ is the diagonal covariance matrix of shock $\vec{\omega}_{rj}$ with diagonals σ_b^2 and σ_c^2 , we can write $\vec{\Gamma}_r$ as follows

$$\vec{\Gamma}_r = \vec{V}_r^0 + N_r \tau_r \begin{pmatrix} g_1 & g_2 \\ g_2 & g_3 \end{pmatrix} + \tau_r \begin{pmatrix} h_1 & h_2 \\ h_2 & h_3 \end{pmatrix} \quad (54)$$

where

$$\vec{V}_r^0 = \begin{pmatrix} V_{rb}^0 & V_{rbc}^0 \\ V_{rbc}^0 & V_{rc}^0 \end{pmatrix} \quad (55)$$

is the observed average covariance matrix of the monthly print volume from all devices in contracting group r (equation (13a)), and g_i and h_i ($i = 1, 2, 3$) are nonlinear functions of model parameters:

$$g_1 = \sigma^2 (\phi_b + \phi_{bc})^2, \quad (56a)$$

$$g_2 = \sigma^2 (\phi_b + \phi_{bc})(\phi_c + \phi_{bc}), \quad (56b)$$

$$g_3 = \sigma^2 (\phi_c + \phi_{bc})^2, \quad (56c)$$

$$h_1 = \phi_b^2 \sigma_b^2 + \phi_{bc}^2 \sigma_c^2, \quad (56d)$$

$$h_2 = \phi_{bc} (\phi_b \sigma_b^2 + \phi_c \sigma_c^2), \quad (56e)$$

$$h_3 = \phi_{bc}^2 \sigma_b^2 + \phi_c^2 \sigma_c^2. \quad (56f)$$

Substituting equation (54) into equation (53) and writing all vectors in terms of their BW and color components, we obtain the following two nonlinear random-effects models on the observed BW and color monthly print volume, q_{rjb}^* and q_{rjc}^* . The model on the average BW volume q_{rjb}^* is

$$q_{rjb}^* = G^1(\vec{K}_{rj}; \vec{\Lambda}) + u_r^1 + e_{rj}^1 \quad (57)$$

where errors u_r^1 and e_{rj}^1 are as defined in the lemma and $G^1(\vec{K}_{rj}; \vec{\Lambda})$ is

$$\begin{aligned} G^1(\vec{K}_{rj}; \vec{\Lambda}) &= (\phi_b \psi_b + \phi_{bc} \psi_c) - \left(\frac{2\lambda}{\kappa}\right) K_r^1 - \left(\frac{2\lambda}{\kappa} g_1\right) K_r^2 - \left(\frac{2\lambda}{\kappa} h_1\right) K_r^3 - \left(\frac{2\lambda}{\kappa} g_2\right) K_r^4 \\ &\quad - \left(\frac{2\lambda}{\kappa} h_2\right) K_r^5 + (\phi_b + \phi_{bc}) \vec{\beta}^T \vec{K}_r^6 - \left(\frac{\phi_b}{\kappa}\right) K_r^7 - \left(\frac{\phi_{bc}}{\kappa}\right) K_r^8 \\ &\quad + \left(\frac{2\lambda}{\kappa}\right) K_r^9 - (\phi_b) K_r^{10} - (\phi_{bc}) K_r^{11} + (\phi_b + \phi_{bc}) (\vec{\gamma}^T) \vec{K}_{rj}^{12}, \end{aligned} \quad (58)$$

The model on the average color volume q_{rjc}^* is

$$q_{rjc}^* = G^2(\vec{K}_{rj}; \vec{\Lambda}) + u_r^2 + e_{rj}^2, \quad (59)$$

where errors u_r^2 and e_{rj}^2 are defined in the lemma and $G^2(\vec{Y}; \vec{\Lambda})$ is

$$\begin{aligned} G^2(\vec{K}_{rj}; \vec{\Lambda}) &= (\phi_{bc} \psi_b + \phi_c \psi_c) - \left(\frac{2\lambda}{\kappa}\right) K_r^{13} - \left(\frac{2\lambda}{\kappa} g_2\right) K_r^2 - \left(\frac{2\lambda}{\kappa} h_2\right) K_r^3 - \left(\frac{2\lambda}{\kappa} g_3\right) K_r^4 \\ &\quad - \left(\frac{2\lambda}{\kappa} h_3\right) K_r^5 + (\phi_c + \phi_{bc}) \vec{\beta}^T \vec{K}_r^6 - \left(\frac{\phi_{bc}}{\kappa}\right) K_r^7 - \left(\frac{\phi_c}{\kappa}\right) K_r^8 \\ &\quad + \left(\frac{2\lambda}{\kappa}\right) K_r^{14} - (\phi_{bc}) K_r^{10} - (\phi_c) K_r^{11} + (\phi_b + \phi_{bc}) (\vec{\gamma}^T) \vec{K}_{rj}^{12}. \end{aligned} \quad (60)$$

In equations (58) and (60), Y^i ($i = 1, \dots, 14$) are explanatory variables defined as follows

$$K_r^1 = V_{rb}^0 (p_{rb}^* - s_{rb}^0) + V_{rbc}^0 (p_{rc}^* - s_{rc}^0), \quad K_r^2 = N_r \tau_r (p_{rb}^* - s_{rb}^0), \quad K_r^3 = \tau_r (p_{rb}^* - s_{rb}^0), \quad (61a)$$

$$K_r^4 = N_r \tau_r (p_{rc}^* - s_{rc}^0), \quad K_r^5 = \tau_r (p_{rc}^* - s_{rc}^0), \quad \vec{K}_r^6 = \vec{W}_r, \quad K_r^7 = p_{rb}^* - s_{rb}^0, \quad (61b)$$

$$K_r^8 = p_{rc}^* - s_{rc}^0, \quad K_r^9 = V_{rb}^s - V_{rb}^0 s_{rb}^0 - V_{rbc}^0 s_{rc}^0, \quad K_r^{10} = p_{rb}^*, \quad K_r^{11} = p_{rc}^*, \quad (61c)$$

$$\vec{K}_{rj}^{12} = \vec{Z}_{rj}, \quad K_r^{13} = V_{rbc}^0 (p_{rb}^* - s_{rb}^0) + V_{rc}^0 (p_{rc}^* - s_{rc}^0), \quad K_r^{14} = V_{rc}^s - V_{rbc}^0 s_{rb}^0 - V_{rc}^0 s_{rc}^0. \quad (61d)$$

By definition, $VP_{rj}^* = p_{rb}^* q_{rjb}^* + p_{rc}^* q_{rjc}^*$. Thus, using models (57) and (59), the econometric model with the variable payment as the response variable is:

$$VP_{rj}^* = G^3(\vec{K}_{rj}; \vec{\Lambda}) + u_r^3 + e_{rj}^3, \quad (62)$$

where u_r^3 and e_{rj}^3 are defined in the lemma and $G^3(\vec{K}_{rj}; \vec{\Lambda})$ is the following nonlinear function of model parameters

$$G^3(\vec{K}_{rj}; \vec{\Lambda}) = p_{rb}^* G^1(\vec{K}_{rj}; \vec{\Lambda}) + p_{rc}^* G^2(\vec{K}_{rj}; \vec{\Lambda}). \quad (63)$$

□

A.3 Proof of Lemma 3

Proof of Lemma 3. When the provider knows the type of each contracting group, it maximizes the mean-variance objective $MV_r(F(\theta), \vec{P}(\theta))$ defined in equation (23), which is repeated below for the reader's convenience:

$$\begin{aligned} MV_r(F_r(\theta), \vec{P}_r(\theta)) &= N_r \tau_r [(\vec{P}_r(\theta) - \vec{s}_r^0)^T \vec{\Phi} (\vec{\Psi}_r \theta - \vec{P}_r(\theta) + \vec{\mu}_r^0) + F_r(\theta)] \\ &\quad - \lambda N_r \tau_r \left\{ N_r \tau_r \left(\left[\vec{P}_r(\theta) - \vec{s}_r^0 \right]^T \vec{\Phi} \vec{1}_2 \right)^2 \sigma^2 + \tau_r \left(\vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{\Phi} \vec{\Sigma} \vec{\Phi} \vec{s}_r^0 \right) \right. \\ &\quad \left. + \left(\vec{P}_r^T(\theta) \vec{V}_r^0 \vec{P}_r(\theta) - 2 \vec{P}_r^T(\theta) \vec{V}_r^s \right) \right\}. \end{aligned}$$

Because the company has no private information, the provider maximizes $MV_r(F(\theta), \vec{P}(\theta))$ of contracting group r ($r = 1, 2, \dots, R$) subject to the participation constraint: $\mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)} [U_r(\vec{P}_r(\theta), F_r(\theta), \theta)] \geq U_{0r}$, where $\mathbb{E}_{(\vec{\xi}, \vec{\omega}, \epsilon)}$ is the expected utility over the contracting horizon, and U_{0r} is the reservation utility of contracting group r as defined in Lemma 4.

From equation (34) and using $u_r^{**}(\theta)$ and u_{0r} defined in equations (38) and (41), the (IR) constraint can be rewritten as $u_r^{**}(\theta) \geq u_{0r}$. Thus, the provider's contract design problem under symmetric information is to maximize $MV_r(F(\theta), \vec{P}(\theta))$ under the (IR) constraint $u_r^{**}(\theta) \geq u_{0r}$. At the optimal solution, the (IR) constraint is binding and thus

$$F_r(\theta) = \frac{1}{2} [\vec{\Psi}_r \theta + \vec{\mu}_r^0 - \vec{P}_r(\theta)]^T \vec{\Phi} [\vec{\Psi}_r \theta + \vec{\mu}_r^0 - \vec{P}_r(\theta)] - u_{0r}. \quad (64)$$

Substituting equation (64) into the mean-variance objective yields a concave function in the variable price $\vec{P}_r(\theta)$. Thus, the first-order condition yields that the optimal variable prices satisfy equation (20). From equations (64) and (12), the optimal fixed price satisfies (21), completing the proof. \square

B Computational details of estimating the econometric model parameters

We used Matlab for the maximum likelihood estimation (MLE) of the BW model, the Color model, and the Payment model in Lemma 2. For each model, the built-in unconstrained optimization functions *fminsearch* and *fminunc* with quasi-Newton algorithm is used (we apply logistic transformation to $\kappa \in [0, 1]$ and logarithmic transformations to λ , ϕ_b , ϕ_c , ψ_b , ψ_c , σ^2 , σ_b^2 and σ_c^2 so that all parameters being estimated are defined on $(-\infty, +\infty)$). The tolerance levels are 10^{-8} for both the objective function value and the variables. The maximum iteration is set to be 5000 and the maximum number of function evaluations is set to be 80000.

In solving the optimization problem, we randomly generate 200 initial points for the solver. Thus, the solver solves the same optimization problem 200 times from 200 initial points using two algorithms *fminsearch* and *fminunc*. We then use the output that generates the highest log-likelihood value as the point estimates. This way we can reduce the possibility of being trapped in a local optimum. For a given set of initial points, solving the optimization takes about 20 seconds

using *fminunc* and about 3 mins using *fminsearch* on a High Performance Computing node with 12 Intel processing cores and 48GB of memory.

Before working with real data, we carry out simulation and confirm that the best output can recover the true parameter values used to generate the data. Furthermore, when varying the random seeds and ranges for generating the initial points, the optimization procedure provides very similar outputs, thus providing computational evidence for the numerical stability of our solution.

The confidence intervals are computed using parametric bootstrapping. Take the BW model as an example. Let $\hat{\Lambda}$ denote the estimated value of the parameters from the BW model. With $\hat{\Lambda}$, we can compute $G^1(\vec{K}_{rj}; \hat{\Lambda})$ and fully characterize the distributions of u_r^1 and e_{rj}^1 in Lemma 2. We obtain 44 samples from the estimated distribution of u_r^1 as the group-level errors, and obtain 1021 samples from the estimated distribution of e_{rj}^2 as the printer-level errors. Adding these errors to the mean $G^1(\vec{K}_{rj}; \hat{\Lambda})$, we obtain a bootstrap data set. We carry out the same procedure as explained above on this bootstrap data set to get a set of parameter estimates. We generate a total of 300 bootstrap data sets, and obtain 300 estimates of the parameters. We use the standard deviation of these 300 estimates and the estimate from the actual data set to construct the confidence intervals. (For more on parametric bootstrapping, see e.g., Ch. 6.5 in Efron and Tibshirani 1994).

C Alternative evidence of risk-aversion and out-of-sample tests

C.1 Alternative evidence of provider's risk-aversion

Table 2 shows that the RA assumption holds under different model specifications. However, given the complexity of the nonlinear random-effects models in Lemma 2, concerns on the robustness of the result may still arise. In this subsection, we show that our result is robust by testing the following hypothesis using a reduced-form approach.

Hypothesis 1.

(H_0) *The provider is risk-neutral.*

(H_1) *The provider is risk-averse.*

From Theorem 1, the contract prices depend on the covariance matrix of the monthly print

volume, \vec{V}_{rj} (equation (10)), only when the provider is risk-averse. Consequently, the company's print volume and monthly payment depend on \vec{V}_{rj} only when the provider is risk-averse. This means that under the null hypothesis (H_0), the observed average BW print volume q_{rjb}^* , average color print volume q_{rjc}^* , and the average variable payment VP_{rj}^* , would not depend on the covariance matrix V_{rj} . Under the alternative hypothesis (H_1), however, q_{rjb}^* , q_{rjc}^* , and VP_{rj}^* would depend on V_{rj} . This observation allows us to test Hypothesis 1 by doing likelihood ratio tests, as explained below.

Use y_{rj}^i as the generic notation for the response variable in the model, where $i \in \{B, C, P\}$ indicates whether y_{rj}^i equals the average BW print volume q_{rjb}^* , the average color print volume q_{rjc}^* , or the average variable payment VP_{rj}^* . Then the null hypothesis (H_0) predicts the following linear random-effects model, henceforth referred to as model (RN $_i$):

$$(RN_i) \quad y_{rj}^i = \alpha_0^i + \alpha_1^i f_r^* + \alpha_2^i p_{rb}^* + \alpha_3^i p_{rc}^* + (\vec{\alpha}_4^i)^T \vec{W}_r + \alpha_5^i s_{rjb} + \alpha_6^i s_{rjc} + \alpha_7^i Z_{rj} + o_r^i + z_{rj}^i, \quad (65)$$

where $(f_r^*, p_{rb}^*, p_{rc}^*)$ are the contract prices, (s_{rjb}, s_{rjc}) are the variable service costs, \vec{W}_r and Z_{rj} are demographic characteristics, and o_r^i and z_{rj}^i are hierarchical errors.

Hypothesis (H_1) predicts the following model, henceforth referred to as model (RA $_i$):

$$(RA_i) \quad y_{rj}^i = \alpha_0^i + \alpha_1^i f_r^* + \alpha_2^i p_{rb}^* + \alpha_3^i p_{rc}^* + (\vec{\alpha}_4^i)^T \vec{W}_r + \alpha_5^i s_{rjb} + \alpha_6^i s_{rjc} + \alpha_7^i Z_{rj} \\ + (\alpha_8^i V_{rjb} + \alpha_9^i V_{rjc} + \alpha_{10}^i V_{rjbc}) + o_r^i + z_{rj}^i, \quad (66)$$

where $(V_{rjb}, V_{rjc}, V_{rjbc})$ are the variance and covariance of the monthly print volume (equation (10)).

Equations (RN $_i$) and (RA $_i$) are nested random-effects models. Thus, the task of testing Hypothesis 1 reduces to a likelihood ratio test between (RN $_i$) and (RA $_i$). If (H_0) holds, then (RA $_i$) would not provide a better fit to the data than (RN $_i$), and the coefficients of the variance terms V_{rjb} , V_{rjc} and V_{rjbc} would not be significant. The top panel of Table 3 presents the results when testing (RN $_i$) and (RA $_i$) models using the 1,021 printers with cost data. We observe that, for all three choices of response variable, the RN assumption is rejected and the variance terms V_{rjc} , V_{rjb} , and V_{rjbc} are statistically significant.

Finally, to be able to use all 3,065 printers in the test, we remove the variable service costs (s_{rjb}, s_{rjc}) from the list of explanatory variables on the right sides of models (RN $_i$) and (RA $_i$).

Re-running the estimation and the likelihood ratio tests generates results in the bottom panel of Table 3. We report that these expanded tests still reject the null hypothesis that the provider is risk-neutral, with statistically significant variance terms V_{rjc} , V_{rjc} , and V_{rjbc} .

Table 3: Reduced-form analysis results

L.R.T. stands for "likelihood ratio test."

Response	p -values of L.R.T.	t -values of coefficients		
		V_{rjb}	V_{rjc}	V_{rjbc}
<i>1021 printers with cost data</i>				
q_{rjb}^*	< 2.2E-16	15	2	2
q_{rjc}^*	< 2.2E-16	-0.5	18	4
VP_{rj}^*	< 2.2E-16	3	17	0.3
<i>All 3075 printers</i>				
q_{rjb}^*	< 2.2E-16	33	3	5
q_{rjc}^*	< 2.2E-16	0.5	42	4
VP_{rj}^*	< 2.2E-16	6	27	4

C.2 Out-of-sample tests

In this subsection, we perform out-of-sample tests to check whether the contract prices and print volumes predicted by our model agree with the observations. Due to the hierarchical structure of our model, we do the tests on the contracting group level. Specifically, we split the printers within a contracting group into two sub-groups: in-sample (90% of the printers) and out-of-sample (10% of the printers). By our assumption, printers in these two sub-groups have the same private type θ and thus the same group-level WTP $\vec{\zeta}$. Hence, we can use the in-sample printers to find the true average group-level WTP $\vec{\Psi}_r\theta_r$ (Lemma 1), and then use these estimated WTPs to predict the contract prices (Theorem 1) and average print volumes (equation (5)) for the out-of-sample printers. If our contracting model is a good approximation of practice, then our WTP estimate, $\vec{\Psi}_r\theta_r$, should be close to the true baseline WTP of the test printers. Further, the predicted contract prices and volumes should be able to explain the observations from the out-of-sample printers.

Next we describe how we implement this test. There are 44 contracting groups in our data set and, among the 44 contracting groups, 18 groups contain at least ten printers. To have adequate data for the in-sample estimation, we only use these 18 groups for out-of-sample tests. Within

each contracting group, we randomly select 90% of the printers as the in-sample printers, and the remaining 10% as the test printers. We pool the observations from all in-sample printers together, estimate the model parameters, and use Lemma 1 to find the average group-level WTPs $\vec{\Psi}_r \theta_r$ of each contracting group. By Lemma 1, we need to know $\vec{\Psi}_r$ to estimate $\vec{\Psi}_r \theta_r$. Thus, we only use the Payment model in Lemma 2 for the in-sample estimation.

We use Theorem 1 with the estimated WTP $\vec{\Psi}_r \theta_r$ to predict the contracts and print volumes for the out-of-sample printers. Comparing the predicted variable prices with the true contract prices, we report that the R-squares are 0.33 and 0.21 for the BW and color variable prices, and is 0.51 for the fixed price. Comparing the predicted average print volumes with the observations, we report that the R-squares are 0.11 and 0.24 for the BW and color volumes, respectively.

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