Appendix

We start with some technical lemmas that are useful for establishing the results. Then we present the proofs to our results. For ease of notation, we denote $M'(t) = \frac{\partial M(t)}{\partial t}$.

Lemma 7. $\bar{A} = sA_H - yA_L + (u_3 y - u_2 s)Z > 0$, $D = u_1 s^2 - (u_2 + u_4)sy + u_3 y^2 > 0$.

Proof. According to Inequality (3) and (4), $\bar{A} = sA_H - yA_L + (u_3 y - u_2 s)Z = y(A_H - A_L) + (u_3 y - u_2 s)Z + (s-y)A_H > y(A_H - A_L) + (s-y)A_L + (u_3 y - u_2 s)Z > y(A_H - A_L) + (s-y)u_3 Z + (u_3 y - u_2 s)Z = y(A_H - A_L) + s(u_3 - u_2)Z > 0$. $D = u_1 s^2 - (u_2 + u_4)sy + u_3 y^2 = s(u_1 s - u_2 y) + y(u_3 y - u_4 s) > y(u_1 s - u_2 y + u_3 y - u_4 s) = y[(u_1 - u_4)s + y(u_3 - u_2)] > 0$. $\square$

Lemma 8. *$M(t)$ increases in t and h, and $2M(t) \geq tM'(t)$.*

Proof. $M'(t) = \int_0^t [2h^2 t + 2h(1-h)k] f(k)dk > 0$, and $\frac{\partial M(t)}{\partial h} = \int_0^t 2(t-k)[ht + (1-h)k] f(k)dk > 0$. $2M(t) \geq \int_0^t \left[2h^2 t^2 + 4h(1-h)tk + 2(1-h)^2 k^2\right] f(k)dk > tM'(t)$. $\square$

Proof of Proposition 1. $\frac{\partial G(t)}{\partial t} = \frac{-\bar{A}DM'(t)}{[1+DM(t)]^2} < 0$, $\frac{\partial P_H^{(t)}}{\partial t} = \frac{-(u_1 s - u_2 y)\bar{A}M'(t)}{[1+DM(t)]^2} < 0$, $\frac{\partial P_L^{(t)}}{\partial t} = \frac{B\bar{A}M'(t)}{[1+DM(t)]^2} > 0$. $\square$

Lemma 9. $D[1 - M(0)] < 1$.

Proof. Because we assume that for all k, $Z - (\beta - \xi)k^{(t)} x^{(t)}(k^{(t)}) > 0$, therefore for all $t \geq 0$, we have $Z > yG(t)$, which is $Z > \frac{y\bar{A}}{1+DM(t)} \Leftrightarrow 1 + DM(t) > \frac{y\bar{A}}{Z}$. This implies that $1 + DM(0) > \frac{y\bar{A}}{Z}$ because $M(t)$ increases in t. And because $y\bar{A} > DZ \Leftrightarrow y[sA_H - yA_L + (u_3 y - u_2 s)Z] > Z[s(u_1 s - u_4 y) + y(u_3 y - u_2 s)] \Leftrightarrow s[yA_H - u_1 sZ] > y[yA_L - u_4 yZ]$, which holds by (4). Therefore we have proved that $1 + DM(0) > \frac{y\bar{A}}{Z} > D$, which implies that $D[1 - M(0)] < 1$. $\square$

Lemma 10. *There exists $\varepsilon > 0$, such that $W^{(\varepsilon)} > W^{(0)}$.*

Proof. We prove by contradiction. Suppose for all $\varepsilon > 0$, such that $W^{(\varepsilon)} < W^{(0)}$. With $W^{(t)}$ given in (10) and $P_L^{(t)} > P_L^{(0)}$ for $t > 0$ (by Lemma 2), we must have for all $\varepsilon > 0$, $G^2(\varepsilon)M(\varepsilon) < G^2(0)M(0)$. This inequality can be transformed as $1 < D\sqrt{M(\varepsilon) \cdot M(0)}$. Because $D[1 - M(0)] < 1$ by Lemma 9, the inequality $1 < D\sqrt{M(\varepsilon) \cdot M(0)}$ implies that $1 - M(0) < \sqrt{M(\varepsilon) \cdot M(0)}$, which is $\sqrt{M(\varepsilon) \cdot M(0)} + M(0) > 1$ for all $\varepsilon > 0$. However, since $M(0) < \frac{1}{2}$, there exists a small enough $\varepsilon > 0$, such that $M(\varepsilon) < \frac{1}{2}$, and thus $\sqrt{M(\varepsilon) \cdot M(0)} +$

$M(0) < \frac{1}{2} + \frac{1}{2} = 1$, which is contradictory to the conclusion that $\sqrt{M(\varepsilon)\cdot M(0)} + M(0) > 1$ for all $\varepsilon > 0$. Therefore there exists $\varepsilon > 0$, such that $W^{(\varepsilon)} > W^{(0)}$. □

Lemma 11. *$t^2 G(t)$ increases in t.*

Proof. $\frac{\partial t^2 G(t)}{\partial t} = \frac{\bar{A}t[2+2DM(t)-tDM'(t)]}{[1+DM(t)]^2} > 0$, which holds by Lemma 8. □

Lemma 12. *For farmer k, where $k \geq t^{(d)}$, her payoff $\pi^{(t)}(k^{(t)})$ decreases in t for $t \leq k$.*

Proof. For any t_1, t_2, where $t_1 < t_2 \leq k$, if the shared level increases from t_1 to t_2, the change to a farmer k's payoff is given in (11) with w replaced by t_2, and t replaced by t_1 respectively, and

$$\pi^{(t_2)}(k) - \pi^{(t_1)}(k) = (P_L^{(t_2)} - P_L^{(t_1)})\left\{Z - \frac{k^2 D(G(t_2)+G(t_1))}{2B}\right\} < (P_L^{(t_2)} - P_L^{(t_1)})[Z - \frac{(t^{(d)})^2 DG(t^{(d)})}{B}] = 0,$$

where the inequality holds because $P_L^{(t_2)} > P_L^{(t_1)}$ and $\forall t \leq k$, $k^2 G(t) \geq (t^{(d)})^2 G(t^{(d)})$. This is because if $t < t^{(d)}$, then $k^2 G(t) > k^2 G(t^{(d)}) \geq (t^{(d)})^2 G(t^{(d)})$; otherwise $k^2 G(t) \geq t^2 G(t) \geq (t^{(d)})^2 G(t^{(d)})$ by Lemma 11. □

Proof of Lemma 1 and Lemma 2. The two lemmas follow immediately from the discussions in the main text. □

Proof of Proposition 2. We first prove that $W^{(t)}$ is strictly quasiconcave in $M(t)$. Taking derivative,

$$\frac{\partial W^{(t)}}{\partial M(t)} = \frac{\bar{A}}{2[1+DM(t)]^3}\left\{\bar{A} + 2BZ + DM(t)(2BZ - \bar{A})\right\}.$$

$\frac{\bar{A}}{2[1+DM(t)]^3} > 0$ and it strictly decreases in $M(t)$. For $\left\{\bar{A} + 2BZ + DM(t)(2BZ - \bar{A})\right\}$, if $\bar{A} > 2BZ$, then it decreases in $M(t)$; otherwise, it is always positive. Therefore, $W^{(t)}$ is strictly quasiconcave in $M(t)$. Because $M(t)$ monotonically increases in t, $W^{(t)}$ is also strictly quasiconcave in t. By Lemma 10, $t^{(c)} > 0$. Therefore the unique maximizer $t^{(c)}$ is either 1 if $\bar{A} + 2BZ > (\bar{A} - 2BZ)DM(1)$, or otherwise satisfies

$$DM(t) = \frac{\bar{A} + 2BZ}{\bar{A} - 2BZ}. \tag{18}$$

Since $M(t)$ increases in t and h by Lemma 8, and $\frac{\bar{A}+2BZ}{D(\bar{A}-2BZ)} \propto c$, therefore $\frac{\partial t^{(c)}}{\partial h} < 0$, and $\frac{\partial t^{(c)}}{\partial c} > 0$. □

Proof of Lemma 3. First, recall from Lemma 2 that $P_L^{(w)}$ is increasing in w so that $P_L^{(w)} - P_L^{(t)}$ increases in w and is positive for any $w > t$. Second, since $G(w)$ decreases in w, the term $Z - \frac{k^2 D}{2B}\left[G(w) + G(t)\right]$ is increasing in w. These two observations imply that farmer k's best response can be described as follows:

it is optimal for farmer k to share "full knowledge" so that $w^* = k$ if $Z > \frac{k^2 D}{2B}\left[G(k) + G(t)\right]$; and share "no (additional) knowledge" so that $w^* = t$ if $Z < \frac{k^2 D}{2B}\left[G(k) + G(t)\right]$. $\square$

Proof of Proposition 3. We consider the deviation of a farmer k. If $k \geq t^{(d)}$, by Lemma 12, farmer k would not deviate to sharing higher than $t^{(d)}$. Meanwhile her deviation to sharing lower than $t^{(d)}$ is futile. If $k < t^{(d)}$, farmer k's deviation to sharing lower than k is also futile. Note that $t^{(d)}$ satisfies $Z = \frac{t^2 DG(t)}{B}$, which is equivalent to $\frac{t^2 D\bar{A}}{B[1+DM(t)]} = Z$. Because $\frac{\partial t^2 G(t)}{\partial t} > 0$ (by Lemma 11), $M(t)$ increases in h by Lemma 8, and $\frac{t^2 D\bar{A}}{B[1+DM(t)]}$ decreases in c, therefore $\frac{\partial t^{(d)}}{\partial c} > 0$ and $\frac{\partial t^{(d)}}{\partial h} > 0$. $\square$

Proof of Proposition 4. We prove this proposition in two steps.

Step 1: For any farmer k, where $k \in [0, t^{(d)})$, sharing knowledge k is not weakly dominated.

Let t be the highest knowledge shared by others, where $t < k$. By Lemma 11, $BZ = (t^{(d)})^2 DG(t^{(d)}) > k^2 DG(k)$. And $k^2 G(t)$ decreases in t and is equal to $k^2 G(k)$ when $t = k$. Therefore, there must exist $t' < k$, such that when the highest knowledge shared by others is t', the unique best response for farmer k is to share knowledge k (by Lemma 3).

Step 2: For any farmer k, where $k \in [t^{(d)}, 1]$, not sharing is the weakly dominant strategy.

Let t be the highest knowledge shared by others, where $t < k$. Consider a strategy of farmer k is to share knowledge w with $w \in (0, k]$. (i) If $t < w$, by Lemma 12, not sharing is strictly better than sharing w. (ii) If $t \geq w$, not sharing is as good as sharing w. $\square$

Proof of Proposition 5. This proposition follows immediately from equations (14) and (15). $\square$

Proof of Proposition 6. We first prove that when both $t^{(c)}$ and $t^{(d)}$ are interior, $t^{(c)} > t^{(d)}$, and then use this result to prove Proposition 6.

Given both $t^{(d)}$ and $t^{(c)}$ are interior, $t^{(d)}$ is the solution to $BZ = t^2 DG(t)$, and $t^{(c)}$ is the solution to $DM(t) = \frac{\bar{A}+2BZ}{\bar{A}-2BZ}$, which can be transformed as $BZ = \frac{DM(t)-1}{2} G(t)$ When $B = 0$, $t^{(d)} = 0 \leq t^{(c)}$. When $B > 0$, the proof is as follows. Because $t^2 G(t)$ increases in t by Lemma 11, if we can prove that $\frac{2D(t^{(c)})^2}{DM(t^{(c)})-1} > 1$, then we are done. Actually, for any t such that $DM(t) - 1 > 0$, we can show that $1 + 2Dt^2 > DM(t)$. By Lemma 9, we have $1 > D[1 - M(0)]$. If we can prove that $D[1 - M(0)] + 2Dt^2 > DM(t)$, then we are done. Because $D[1 - M(0)] + 2Dt^2 > DM(t) \Leftrightarrow 1 + 2t^2 > M(t) + M(0)$. Let $V(t) = 1 + 2t^2 - M(0) - M(t)$. Since $V(0) = 1 - 2M(0) \geq 0$, and $\frac{\partial V(t)}{\partial t} = 4t - \int_0^t 2h[ht + (1-h)k] f(k) dk > 4t - 2htF(t) > 4t - 2t > 0$, we can conclude that $V(t) \geq 0$ for all $t > 0$. And thus the inequality $1+2t^2 > M(t)+M(0)$ is proved, which leads to $1+2Dt^2 > DM(t)$, and thus $t^{(c)} > t^{(d)}$ when they are both interior. Because $t^{(c)}$ and $t^{(d)}$ are both continuous and increasing in production cost c, with $t^{(c)} > t^{(d)}$ when they are both interior, we can conclude that $t^{(c)} > t^{(d)}$.

Since $W^{(t)}$ is strictly quasiconcave in t with its maximizer being $t^{(c)}$, by Lemma 10, we have $0 \leq t^{(d)} \leq t^{(c)}$, $W^{(0)} \leq W^{(d)} \leq W^{(c)}$. □

Proof of Lemma 4. We prove this lemma in three steps.

Step 1: For any farmer k with $k \in [t^{(d)}, 1]$, no strategy is weakly dominated.

Let t be the highest knowledge shared by others. $\forall t \leq k$, by Lemma 12, farmer k's unique best response is to share t. Therefore no strategy of farmer k is weakly dominated.

Step 2: No Nash equilibrium with shared level lower than $t^{(d)}$.

Suppose there is a Nash equilibrium with shared level t, where $t < t^{(d)}$. For a farmer $t+\varepsilon \in [t, t^{(d)})$, if he deviates to share at $t+\varepsilon$, the change to her payoff, denoted by $\Delta(\varepsilon)$, is:

$$\Delta(\varepsilon) = \left(P_L^{(t+\varepsilon)} - P_L^{(t)}\right)\left\{Z - \frac{D(t+\varepsilon)^2}{2B}\left[G(t+\varepsilon)+G(t)\right]\right\}.$$

By Lemma 2, $P_L^{(t+\varepsilon)} > P_L^{(t)}$, $\forall \varepsilon > 0$. And by Lemma 11, $Z = \frac{D(t^{(d)})^2 G(t^{(d)})}{B} > \frac{Dt^2 G(t)}{B}$ and $(t+\varepsilon)^2[G(t+\varepsilon)+G(t)]$ increases in ε. Thus there must exist $\varepsilon > 0$ such that $t+\varepsilon < t^{(d)}$, and $\Delta(\varepsilon) > 0$, and thus farmer $t+\varepsilon$ benefits from deviation.

Step 3: $\forall t \in [t^{(d)}, 1]$, there exists a Nash equilibrium with shared level t.

We claim what follows is a Nash equilibrium: any farmer with knowledge in $[0,t]$ shares her own knowledge while others share at t. For farmers sharing at t, he will not deviate to sharing higher than t, by Lemma 12. And any farmer's deviation to sharing lower is futile. □

Proof of Lemma 5. **Step 1: For a farmer k, $k \in [t^{(d)}, t^{(c)})$, not sharing is weakly dominant strategy.**

The proof is similar to that for Proposition 4, and thus we omit here.

Step 2: A farmer k with $k \in [t^{(c)}, 1]$, either shares $t^{(c)}$ or not sharing.

For farmer k, not sharing weakly dominates sharing knowledge w as long as $w > 0$ and $w \neq t^{(c)}$. The proof is similar to that for Proposition 4, and thus we omit here. When the highest knowledge shared by others is higher than $t^{(c)}$, farmer k's sharing $t^{(c)}$ is strictly better than not sharing.

Step 3: We examine two cases: (i) $R > \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)})$; (ii) $R < \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)})$.

Case i: $R > \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)})$. Suppose there is an equilibrium with shared level being $t < t^{(c)}$. If $t < t^{(d)}$, then it is contradicted by the proof for Lemma 4. If $t \geq t^{(d)}$, farmer $t^{(c)}$ will deviate to sharing at $t^{(c)}$, since $R > \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)})$.

Case ii: $R < \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)})$. The Nash equilibrium in Proposition 3 remains valid under this case, because any farmer with knowledge in $[t^{(c)}, 1]$ would not deviate to sharing at $t^{(c)}$ as $R < \pi^{(d)}(t^{(c)}) - \pi^{(c)}(t^{(c)}) \leq \pi^{(d)}(k) - \pi^{(c)}(k), \forall k \geq t^{(c)}$, where $\pi^{(d)}(k) - \pi^{(c)}(k)$ is given in (11) with w replaced by d, t replaced by c, and by Lemma 11, $\pi^{(d)}(k) - \pi^{(c)}(k)$ increases in k. However, what follows is also a Nash equilibrium: farmers with knowledge in $[0, t^{(d)})$ share their own knowledge and farmers with knowledge in $[t^{(c)}, 1]$ share at $t^{(c)}$, while others share nothing. $\square$

Proof of Proposition 7. We prove this proposition in five steps.

Step 1: For a farmer k, $k \in [t^{(c)}, 1]$, sharing $t^{(c)}$ weakly dominates sharing w, $\forall w \in (t^{(c)}, k]$.

Suppose the highest knowledge shared by others is t. (i) When $t > w$, sharing $t^{(c)}$ is as good as sharing w. (ii) When $t < w$, farmer k has a higher payoff if sharing at $t^{(c)}$ (by Lemma 12).

Step 2: For a farmer k, $k \in [t^{(c)}, 1]$, sharing at $t^{(c)}$ is not weakly dominated.

For any knowledge level $w < t^{(c)}$, consider the following scenario. Suppose all other farmers with knowledge higher than t share at t, where $t \in (w, t^{(c)})$. If farmer k shares at w, then he gets no reward, and her payoff from the market is $\pi^{(t)}(k)$. If he shares $t^{(c)}$, her gets a reward R and her payoff from the market is $\pi^{(c)}(k)$. Since $R + \pi^{(c)}(k) - \pi^{(t)}(k)$ is continuous and increases in t, and is strictly positive when $t = t^{(c)}$, there must exist a $t' \in (w, t^{(c)}]$, such that $R + \pi^{(c)}(k) > \pi^{(t')}(k)$. That is, in this case, farmer k's sharing at $t^{(c)}$ is strictly better than sharing at w.

Step 3: For a farmer k with $k \in [0, t^{(c)}]$, sharing at k is not weakly dominated.

The proof is similar to Step 2 and thus we omit here.

Step 4: No pure-strategy Nash equilibrium with shared level lower than $t^{(c)}$.

Let t be the shared level in some equilibrium, where $t < t^{(c)}$, then every farmer with knowledge in $[t, 1]$ must share at t with an expected reward of $\frac{R(1-F(t^{(c)}))}{1-F(t)} < R$. However, there exists a farmer $t + \varepsilon$ with $\varepsilon > 0$ will deviate to sharing at $t + \varepsilon$. By doing so, this farmer gets reward R and her loss from the market, if any, is $\pi^{(t)}(t+\varepsilon) - \pi^{(t+\varepsilon)}(t+\varepsilon)$, which is continuous in ε and is equal to 0 when $\varepsilon = 0$. Given $R - \frac{R(1-F(t^{(c)}))}{1-F(t)}$ is fixed, the statement above is valid.

Step 5: There exists a pure-strategy Nash equilibrium of shared level $t^{(c)}$.

What follows is a Nash equilibrium: for farmer k, if $k \le t^{(c)}$, he shares knowledge k; otherwise, he shares $t^{(c)}$. Since sharing higher than $t^{(c)}$ is weakly dominated, and unilateral deviation to sharing lower is futile. Hence it is a Nash equilibrium when farmers only use admissible strategies. ☐

Proof of Proposition 8. For any two values h_1, h_2, where $h_1 < h_2$, we have $t_{h_1}^{(d)} \le t_{h_2}^{(d)} \le t_{h_2}^{(c)}$ (by Propositions 3 and 6). Therefore, $M(t_{h_1}^{(d)}, h_1) \le M(t_{h_2}^{(d)}, h_2) \le M(t_{h_2}^{(c)}, h_2)$, where we write $M(t,h)$ to stress its dependence on h. Because $M(t_{h_2}^{(c)}, h_2)$ is the unique maximizer to the strictly quasiconcave function $W^{(t)}$, farmer welfare $W^{(d)}$ increases in h. By Lemma 8, $Q_H^{(d)} = \frac{s\bar{A}M(t,h)}{1+DM(t,h)}$ increases in $M(t,h)$. ☐

Proof of Lemma 6. Recall that for this case, we have $h_1 < h_2$ and $t_{h_1}^{(d)} < t_{h_2}^{(d)}$. For a farmer k, we examine three cases: (i) $k \in [0, t_{h_1}^{(d)}]$; (ii) $k \in (t_{h_1}^{(d)}, t_{h_2}^{(d)})$; and (iii) $k \in [t_{h_2}^{(d)}, 1]$. And we write $M(t,h)$, $Q_H^{(d)}(h)$, $k_h^{(t)}$, $G_h(t)$ and $\pi^{(d)}(k_{h_2}^{(d)}, h)$ to stress their dependences on h. It is important to note that $P_L^{(d)}(h_2) > P_L^{(d)}(h_1)$ by Proposition 8, and $G_{h_2}(t_{h_2}^{(d)}) < G_{h_1}(t_{h_1}^{(d)})$ because $G_h(t)$ decreases in $M(t,h)$, while the latter increases in t and h.

Case i: $k \in [0, t_{h_1}^{(d)}]$**,**

$$\begin{aligned}\pi^{(d)}(k_{h_2}^{(d)}, h_2) - \pi^{(d)}(k_{h_1}^{(d)}, h_1) &= \frac{(k_{h_2}^{(d)})^2}{2} G_{h_2}^2(t_{h_2}^{(d)}) + Z\big[P_L^{(d)}(h_2) - P_L^{(d)}(h_1)\big] - \frac{(k_{h_1}^{(d)})^2}{2} G_{h_1}^2(t_{h_1}^{(d)}) \\ &> \big[P_L^{(d)}(h_2) - P_L^{(d)}(h_1)\big]\Big\{Z - \frac{(k_{h_1}^{(d)})^2}{2}\frac{D[G_{h_2}(t_{h_2}^{(d)}) + G_{h_1}(t_{h_1}^{(d)})]}{B}\Big\} > 0,\end{aligned}$$

which holds because $\frac{D(k_{h_1}^{(d)})^2 G_{h_2}(t_{h_2}^{(d)})}{B} < \frac{D(k_{h_1}^{(d)})^2 G_{h_1}(t_{h_1}^{(d)})}{B} < \frac{D(t_{h_1}^{(d)})^2 G_{h_1}(t_{h_1}^{(d)})}{B} = Z$.

Case ii: $k \in [t_{h_2}^{(d)}, 1]$**,**

Because $t_{h_1}^{(d)} < t_{h_2}^{(d)}$, following the proof for Lemma 12, we have $\pi^{(d)}(k_{h_2}^{(d)}, h_2) - \pi^{(d)}(k_{h_1}^{(d)}, h_1) < 0$ for $k > t_{h_2}^{(d)}$.

Case iii: $k \in [t_{h_1}^{(d)}, t_{h_2}^{(d)}]$**,**

$$\pi^{(d)}(k_{h_2}^{(d)}, h_2) - \pi^{(d)}(k_{h_1}^{(d)}, h_1) = \frac{(k_{h_2}^{(d)})^2}{2} G_{h_2}^2(t_{h_2}^{(d)}) + Z\big[P_L^{(d)}(h_2) - P_L^{(d)}(h_1)\big] - \frac{k^2}{2} G_{h_1}^2(t_{h_1}^{(d)}),$$

where $\Delta = \pi^{(d)}(k_{h_2}^{(d)}, h_2) - \pi^{(d)}(k_{h_1}^{(d)}, h_1)$ is a concave function of k, since $k_{h_2}^{(d)} = k + h_2[t_{h_2}^{(d)} - k]^+$, and

$$\frac{\partial^2 \Delta}{\partial k^2} = (1-h_2)^2 G_{h_2}^2(t_{h_2}^{(d)}) - G_{h_1}^2(t_{h_1}^{(d)}) < 0.$$

Because Δ is continuous and positive when $k = t^{(d)}_{h_1}$ and is negative when $k = t^{(d)}_{h_2}$, there must exist $\theta_h \in (t^{(d)}_{h_1}, t^{(d)}_{h_2})$ such that farmer k's payoff is increased if and only if $k < \theta_h$. □

Proof of Proposition 9. To avoid confusion, we might as well write $W^{(t)}(\tau)$ as $W(t,\tau)$. Moreover, let $t^{(c)}_{\tau}$ be the efficient shared level at price sensitivity τ. In this case $W^{(c)}(\tau) - W^{(0)}(\tau) = W(t^{(c)}_{\tau},\tau) - W(0,\tau)$. If $\frac{\partial W^2(t,\tau)}{\partial\tau\partial t} < 0$, then we have $\frac{\partial W(t,\tau)}{\partial\tau} - \frac{\partial W(0,\tau)}{\partial\tau} < 0$, and thus given any $\tau_1 > \tau_2$, we have $W(t^{(c)}_{\tau_1},\tau_1) - W(0,\tau_1) < W(t^{(c)}_{\tau_1},\tau_2) - W(0,\tau_2) < W(t^{(c)}_{\tau_2},\tau_2) - W(0,\tau_2)$. In conclude, if $\frac{\partial W^2(t,\tau)}{\partial\tau\partial t} < 0$, then we have the platform impact $W(t^{(c)}_{\tau},\tau) - W(0,\tau)$ decreases in τ.

To prove that $\frac{\partial W^2(t,\tau)}{\partial\tau\partial t} < 0$, we might as well prove $\frac{\partial W^2[M(t),\tau]}{\partial\tau\partial M(t)} = \frac{\partial W^2[M(t),\tau]}{\partial M(t)\partial\tau} < 0$, because $W^2(t,\tau)$ is a continuous function of $M(t)$ and τ. For convenience of notation, we shall simply write $G(t,\tau)$ as G, and $M(t)$ as M, $P^{(t)}_L = P_L$, in the following.

Since $W[M,\tau] = \frac{MG^2}{2} + ZP_L = \frac{MG^2}{2} + Z[A_L - \tau u_3 Z + \tau BMG]$, taking derivative of $W[M,\tau]$ with regard to M, we have

$$
\begin{aligned}
\frac{\partial W[M,\tau]}{\partial M} &= \frac{1}{2}[G^2 + 2MG\frac{\partial G}{\partial M}] + \tau ZB[M\frac{\partial G}{\partial M} + G],\\
\frac{\partial^2 W[M,\tau]}{\partial M\partial\tau} &= G\frac{\partial G}{\partial\tau} + M\Big[\frac{\partial G}{\partial M}\frac{\partial G}{\partial\tau} + G\frac{\partial^2 G}{\partial M\partial\tau}\Big] + BZ\left\{G + M\frac{\partial G}{\partial M} + \tau M\frac{\partial^2 G}{\partial M\partial\tau} + \tau\frac{\partial G}{\partial\tau}\right\},
\end{aligned}
\tag{19}
$$

where

$$
\begin{aligned}
\frac{\partial G}{\partial M} &= \frac{-\tau D(\tilde{A} + \tau VZ)}{(1+\tau DM)^2} = \frac{-\tau DG^2}{(\tilde{A} + \tau VZ)} < 0, \qquad \frac{\partial G}{\partial\tau} = \frac{VZ - DM\tilde{A}}{(1+\tau DM)^2} = \frac{(G-\tilde{A})G}{\tau(\tilde{A}+\tau VZ)} < 0,\\
\frac{\partial^2 G}{\partial M\partial\tau} &= \frac{-D[\tilde{A}(1-\tau DM) + 2\tau VZ)]}{(1+\tau DM)^3}, \qquad \tilde{A} = sA_H - yA_L, \qquad V = u_3 y - u_2 s.
\end{aligned}
$$

Incorporating $\frac{\partial G}{\partial M}$, $\frac{\partial G}{\partial\tau}$, and $\frac{\partial^2 G}{\partial M\partial\tau}$ into (19), we have

$$
\frac{\partial^2 W[M,\tau]}{\partial M\partial\tau} = \frac{G[DM\tilde{A}(\tau DM - 2) + VZ(1-2\tau DM)]}{(1+\tau DM)^3} + \frac{BZ[2\tau VZ + (1-\tau DM)\tilde{A}]}{(1+\tau DM)^3}.
$$

When $u_4 > u_2$, we have $B = (u_3 y - u_4 s) < (u_3 y - u_2 s) = V$. Therefore if $2\tau VZ + (1-\tau DM)\tilde{A} > 0$ for all τ and $M(t)$, that is $2VZ + [1 - DM(1)]\tilde{A} > 0$, then we have

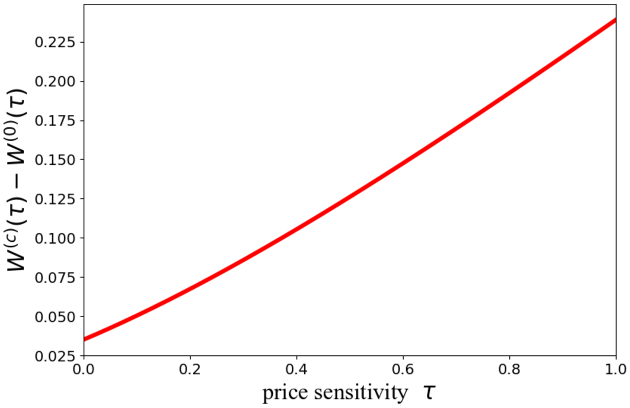
$$
\begin{aligned}
\frac{\partial W^2[M,\tau]}{\partial M\partial\tau} &< \frac{G[DM\tilde{A}(\tau DM - 2) + VZ(1-2\tau DM)]}{(1+\tau DM)^3} + \frac{VZ[2\tau VZ + (1-\tau DM)\tilde{A}]}{(1+\tau DM)^3}\\
&= \frac{(DM\tilde{A} - VZ)[(\tau DM - 2)\tilde{A} - 3\tau VZ]}{(1+\tau DM)^4}.
\end{aligned}
$$

Therefore, $\frac{\partial^2 W[M,\tau]}{\partial M\partial\tau} < 0$ if we have $u_4 > u_2$, $2VZ + (1 - DM(1))\tilde{A} > 0$, and $DM(0)\tilde{A} - VZ > 0$.

To summarize, when $u_4 > u_2$ and $\frac{DM(1)-1}{2} < \frac{(u_3 y - u_2 s)Z}{sA_H - yA_L} < DM(0)$, the platform impact $W^{(c)}(\tau) - W^{(0)}(\tau)$ decreases in the price sensitivity τ.

It is also worth noting that, when the sufficient conditions are not satisfied, it is possible that $W^{(c)}(\tau) - W^{(0)}(\tau)$ is not decreasing in τ. For example, when $u_1 = 0.45$, $u_2 = 0.14$, $u_3 = 0.44$, $u_4 = 0.28$, $A_H = 1.71$, $A_L = 1.21$, $Z = 2.7$, $s = 0.74$, $y = 0.73$, $h = 0.8$, and farmers' knowledge $k \sim U[0,1]$, we have $\frac{(u_3 y - u_2 s)Z}{sA_H - yA_L} = 1.54 > DM(0) = 0.08$. In this case, the platform contribution, $W^{(c)}(\tau) - W^{(0)}(\tau)$, increases in the price sensitivity τ, which is illustrated in Figure 9. □

Figure 9: Example When the Platform Impact, $W^{(c)}(\tau) - W^{(0)}(\tau)$, Increases in the Price Sensitivity τ



Proof of Proposition 10. The proof for quasi-concavity is similar to that for Proposition 2, we thus omit here. We shall only prove the sensitivity of the platform contribution to the platform size.

Because $W^{(0)}$ is a constant, therefore we only need to prove that $W^{(c)}(\theta) = W(t_\theta^{(c)}, \theta)$ increases in the platform size θ. Actually, if we can prove that for all t, $W(t,\theta)$ increases in θ, then we have $W(t_{\theta_1}^{(c)}, \theta_1) < W(t_{\theta_1}^{(c)}, \theta_2) < W(t_{\theta_2}^{(c)}, \theta_2)$, for any $0 \le \theta_1 < \theta_2 \le 1$.

Recall that from (17) that $W(t,\theta) = \frac{\theta M(t)}{2} G^2(t,\theta) + \theta Z P_L^{(t)}(\theta) + (1-\theta)\Big[Q_H^{(0)} P_H^{(t)}(\theta) + Q_L^{(0)} P_L^{(t)}(\theta) - \frac{M(0)G^2(0)}{2}\Big] = \frac{\theta M(t)}{2} G^2(t,\theta) + \frac{1-\theta}{2} M(0)G(0,0)[2G(t,\theta) - G(0,0)] + ZP_L^{(t)}$. Take derivative to $W(t,\theta)$ with regard to θ, we have

$$\frac{\partial W(t,\theta)}{\partial \theta} = \frac{M(t)}{2} G(t,\theta)^2 + \theta M(t) G(t,\theta) \frac{\partial G(t,\theta)}{\partial \theta} - \frac{M(0)G(0,0)[2G(t,\theta) - G(0,0)]}{2} + (1-\theta) M(0) G(0,0) \frac{\partial G(t,\theta)}{\partial \theta} + Z \frac{\partial P_L^{(t)}}{\partial \theta},$$

where $\frac{\partial G(t,\theta)}{\partial \theta} = \frac{D[M(0)-M(t)]G(0,0)}{(1+\theta DM(t))^2} < 0$, $\frac{\partial P_L^{(t)}}{\partial \theta} = B[M(t)G(t,\theta) + \theta M(t) \frac{\partial G(t,\theta)}{\partial \theta} - M(0)G(0,0)]$.

Incorporating $\frac{\partial G(t,\theta)}{\partial\theta}$ and $\frac{\partial P_L^{(t)}}{\partial\theta}$ into $\frac{\partial W(t,\theta)}{\partial\theta}$, we get

$$\frac{\partial W(t,\theta)}{\partial\theta}=\frac{G(0,0)[M(t)-M(0)]}{2(1+\theta DM(t))^3}\Big\{\big[1+2(2\theta-1)DM(0)-\theta DM(t)+\theta^3D^3M(0)M^2(t)+\theta D^2M(0)M(t)(3\theta-2)\big]G(0,0)+2BZ(1+\theta DM(t))\Big\}.$$

Therefore if $4M(0)>M(1)$ and $\{1-2DM(0)[1+DM(1)]\}G(0,0)+2BZ>0$, we have $W(t,\theta)$ increases in θ, and thus the platform impact increase with the platform size. $\square$

Appendix B: General Learning Mode

In the original model, we assume that farmers only learn from the highest level of knowledge shared among farmers. This assumption is admittedly strong, although it is reasonable in certain real world applications. In this section, we drop this assumption and consider a more general learning mode. The basic idea for this general learning mode is that the more knowledge sharing among farmers, the more productive farmers will be after learning from each other. To this end, we define an order function for us to compare different sharing outcomes. Farmers' overall productivity is summarized by the second moment of cumulative distribution function of the post-learning knowledge of farmers. With this general learning mode as well as some other facilitating assumptions, we find that farmers' voluntary sharing is not adequate in maximizing their total welfare. In the following, we shall first declare some notation for sharing outcome, based on which, we then define our order function. Lastly, we shall introduce our general learning mode as well as its implications.

First, let us look at the some notation for sharing outcomes. Let I denote the cumulative distribution function of the knowledge shared by farmers. Here I can be interpreted as a short form for "imparted knowledge". Please note the difference between I and F, where F is the cumulative distribution function of the endowed knowledge levels among farmers. For a farmer with endowed knowledge k, if she shares knowledge at level t, where $t\in[0,k]$, then she accounts for an instance of k in F, and an instance of t in I. We use two illustrating examples to explain further the constitution of I. The first example is that all of the farmers share knowledge at level 0. In this case, $I(x)=\text{Prob}(t\leq x)=0$ if $x<0$, and $I(x)=1$, if $x\geq 0$. Next, we take the sharing equilibrium in Proposition 3 as our second example. In this equilibrium, farmers with knowledge lower than $t^{(d)}$ share their own knowledge, while farmers with knowledge higher than $t^{(d)}$ do not share, that is sharing at level 0. In this case, the cumulative distribution function for the shared knowledge among farmers is as follows, where F, again, is the cumulative distribution function for farmers' endowed knowledge.

$$I(x) = \begin{cases} 0, \quad \text{if } x < 0, \\ 1 - F(t^{(d)}), \quad \text{if } x = 0, \\ F(x) + 1 - F(t^{(d)}), \quad \text{if } x \in (0, t^{(d)}], \\ 1, \quad \text{if } x > t^{(d)}. \end{cases}$$

We can see that F is commonly known among farmers ex-ante, while I depends on the sharing outcome.

Next, We define order function O_I, where $O_I : R \to R^2$ to compare different sharing outcomes I. O_I is a key probability index for I. For $x \in R$, $O_I(x) = [a, b]$, where a is the entry that record the aggregate density of values (i.e., the shared knowledge levels) that is larger than x, that is, $a = \text{Prob}(X > x) = 1 - I(x)$. b, with default value being 0, is the entry that record the discrete number of values that are larger than x when $a = 0$. For example, for the voluntary sharing outcome in Proposition 3, denoted by I_1,

$$O_{I_1}(x) = \begin{cases} [1, 0], \quad \text{if } x < 0, \\ [F(t^{(d)}), 0], \quad \text{if } x = 0, \\ [F(t^{(d)}) - F(x), 0], \quad \text{if } x \in (0, t^{(d)}], \\ [0, 0], \quad \text{if } x > t^{(d)}. \end{cases}$$

If, in addition to the sharing outcome in Proposition 3, a farmer with knowledge 1 shares knowledge at level 1, then the first entry of $O_{I_2}(x)$ remains the same, while the second entry will be a little different, where I_2 denotes the new sharing outcome. Specifically,

$$O_{I_2}(x) = \begin{cases} [1, 0], \quad \text{if } x < 0, \\ [F(t^{(d)}), 0], \quad \text{if } x = 0, \\ [F(t^{(d)}) - F(x), 0], \quad \text{if } x \in (0, t^{(d)}], \\ [0, 1], \quad \text{if } x \in (t^{(d)}, 1), \\ [0, 0], \quad \text{if } x \geq 1. \end{cases}$$

Note that, by the definition of the order function O_I, besides aggregate density, we also concern about discrete values of sharing, because a high level of knowledge shared could potentially enhance the overall productivity among farmers, as in the real-world applications of WeFarm and SCN.

Given this ordering function O_I, we are able to compare different sharing outcomes I.

Definition 1. *For all x, if $O_{I_i}(x) \geq O_{I_j}(x)$, then I_i dominates I_j.*

Please be noted that since O_I is two-dimensional, it is possible for us to have two sharing outcomes I_i and I_j, and we cannot compare them.

One can see that if a sharing outcome is of higher order than another, then in general it means that there is more sharing in the former case. We assume that more knowledge sharing leads to a higher aggregate productivity among farmers, where farmers' aggregate productivity is defined as the second moment of the distribution of their post-learning knowledge. Let F^I be the cumulative distribution of farmers' post-learning knowledge given the sharing outcome is I. With a little abuse to the notation, farmers' aggregate productivity is denoted by $M(F^I)$, where $M(F^I) = \int_0^1 (k^I)^2 f(k)dk$, and k^I is the post-learning knowledge of farmer k under the sharing outcome I and a general learning mode. Our key assumption of the learning outcome can be stated as follows.

Assumption 1. *If I_i dominates I_j by Definition 1, then $M(F^{I_i}) \geq M(F^{I_j})$.*

Along with Assumption 1, we further make some other assumptions.

Assumption 2. *If farmer k learns from a knowledge of level t, then his post-learning knowledge level is denoted by k^t, where $k^{(t)} = k + h[t-k]^+$.*

We assume the same linear learning function as defined in (6). Note that although the learning function remains the same, however, in this extension, we do not have specific restriction on which level to learn for a farmer k. Instead, we only have Assumption 1 on the learning outcome.

Assumption 3. *Let $S = \{(a_i)_{i\in[0,1]}\}$ be the set of strategy profile of farmers, where $a_i \in [0,i]$ is the sharing strategy of a farmer i. Let $S_{-i} = \{(a_{-i})\}$ be the set of strategy profile of farmers except farmer i, where a_{-i} denotes the sharing strategies of farmers except farmer i. Then for any sharing strategy $a_i = w$, with $w \in (0,i]$, of any i, there exists $a_{-i} \in S_{-i}$, such that $I_{(w,a_{-i})}$ dominates $I_{(0,a_{-i})}$ and $M(F^{I_{(w,a_{-i})}}) > M(F^{I_{(0,a_{-i})}})$, where $I_{(w,a_{-i})}$ (or $I_{(0,a_{-i})}$) is the sharing outcome when farmer i shares w (or 0) while others adopt a_{-i}.*

Assumption 3 means that there must exists a sharing scenario such that a farmer's sharing strategy will make a difference to M. One special scenario in mind is that except a farmer k, $k \in [0,1]$, all other farmers are sharing at 0. If farmer k shares at w, $w \in (0,k]$, then it is reasonable that quite a lot of farmers are learning from w with the implementation of P2P platform. And thus farmer k increases M by increasing her sharing level from 0 to w.

Assumption 4. *Given other farmers' sharing decision, a farmer's sharing decision does not impact her own post-learning knowledge.*

Assumption 4 essentially means that sharing one's own knowledge will not change her own post-learning knowledge, if other farmers sharing decisions remain unchanged.

Assumption 5. $(t^{(c)})^2DG(1) \geq BZ$*, where* $G(1) = \frac{\bar{A}}{1+DM(1)}$*, and* $M(1) = \int_0^1 (k^{(1)})^2 f(k)dk$*, which is the second moment of farmers' post-learning knowledge distribution when all of the farmers learn from knowledge* 1*. The definitions of B, Z, and D is provided in Table 1.*

Assumption 5 means that the efficient knowledge-shared level, $t^{(c)}$, is high enough. In other words, the sharing cost c is large enough and the learning efficiency h is low enough. This is reasonable for smallholders whose production cost is considerably high and the learning efficiency is low.

One can check that our original model, learning from the best, satisfies assumptions 1, 2, 3, and 4. We do not require Assumption 5 in our original model. But here, we make this assumption to make our analysis tractable.

Proposition 11. *When farmers only adopt admissible strategies, with assumptions 1, 2, 3, 4, and 5 in a decentralized system, farmers' voluntary sharing is inadequate in maximizing their total welfare.*

According to Proposition 11, we need to implement central intervention to promote more sharing among farmers. Because with reward mechanisms, the highest shared knowledge level is always selected out, and thus farmers know which is the highest level. In this case, the assumption of learning from the highest knowledge shared is indeed reasonable. Given farmers learn from the highest level of knowledge shared under reward mechanisms, all of the conclusions in Section 5 continue to hold.

Proof to Proposition 11. From the proof for Proposition 2, we can see that farmer welfare W is quasi-concave in M, the second moment of post-learning knowledge distribution. (In the following, we shall generally use M to denote the second moment of the distribution of farmers' knowledge.) $M(t^{(c)})$ is the unique maximizer for W, where $M(t^{(c)})$ is the second moment of knowledge distribution when all of the farmers learn from knowledge $t^{(c)}$. To prove Proposition 11, we only need to prove that in a decentralized system, $M(F^I) \leq M(t^{(c)})$, where F^I is the post-learning knowledge distribution in equilibrium. This inequality means that under this general learning function, voluntary sharing among farmers is still not enough to maximize farmer welfare.

To prove that $M(F^I) \leq M(t^{(c)})$, we only need to prove that for a farmer k, where $k \geq t^{(c)}$, sharing at 0 weakly dominates sharing at w, where $w \geq t^{(c)}$. Given k and w,where $k \geq w \geq t^{(c)}$, the proof can be divided into three steps: (i) scenarios when M is decreased if farmer k shares at 0, rather than sharing at w (ii) scenarios when M remains the same whether farmer k shares at 0 or w; (iii) scenarios when M is increased if farmer k shares at 0, rather than sharing at w.

(i) M is decreased if farmer k shares at 0, rather than sharing at w. In this case, we let $I_1 = I_{(0,a_{-k})}$, and $I_2 = I_{(w,a_{-k})}$, then I_2 dominates I_1, and $M(F^{I_1}) < M(F^{I_2})$ by assumption. Note that by Assumption 4, farmer k's post-learning knowledge is the same whether she shares at 0 or w, as long as others' decisions remain the same. We let $\tilde{k}$ be farmer k's post-learning knowledge, and by Assumption 2, $\tilde{k} \geq k \geq t^{(c)}$. Then the difference in farmer k's payoff between these two sharing strategies is denoted by Δ, where $\Delta = [P_L(M(F^{I_1})) - P_L(M(F^{I_2}))]\{Z - \frac{D\tilde{k}^2[G(M(F^{I_1}))+G(M(F^{I_2}))]}{2B}\}$. Because $M(F^{I_1}) < M(F^{I_2})$, $P_L(M(F^{I_1})) - P_L(M(F^{I_2})) < 0$, as $P_L(M)$ increases in M. Moreover, by Assumption 5, $(t^{(c)})^2 G(1) D \geq BZ$, therefore, $\tilde{k}^2 G(M(F^{I_1})) > \tilde{k}^2 G(M(F^{I_2})) > \tilde{k}^2 G(1) > (t^{(c)})^2 G(1)$, since $G(M)$ decreases in M. Therefore $Z - \frac{D\tilde{k}^2[G(M(F^{I_1}))+G(M(F^{I_2}))]}{2B} < 0$, and $\Delta > 0$. That is, to farmer k, sharing at 0 is strictly better than sharing at w in this scenario.

(ii) M is the same whether a farmer k shares at 0, or sharing at w. Following the above analysis, $\Delta = 0$.

(iii) M is increased if a farmer k shares at 0, rather than sharing at w. This is not possible, since $I_{(0,a_{-k})} \leq I_{(w,a_{-k})}$.

With these three steps, we prove that for a farmer k, with $k \geq t^{(c)}$, sharing at 0 weakly dominates sharing at w, where $w \in [t^{(c)}, k]$. Therefore the highest knowledge shared among farmers is less than $t^{(c)}$. And by Assumption 2, $M(F^I) \leq M(t^{(c)})$ in equilibrium. Therefore under a general sharing mode, farmers' sharing is not adequate in maximizing farmer welfare. $\square$