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EC.1. Proof of Lemma 1

Consider any feasible control $\pi \in \Pi$. Per our notations, π corresponds to the demand rate sequence $\{\boldsymbol{\lambda}^{t,\pi}\}_{t=1}^T$. Since $\{\boldsymbol{\lambda}^{t,\pi}\}_{t=1}^T$ satisfies the constraints in **OPT** almost surely, the sequence $\{\mathbf{E}[\boldsymbol{\lambda}^{t,\pi}]\}$ must satisfy the constraints in **DET**. Moreover, by Assumption A4 and Jensen's inequality,

$$\mathbf{E}\left[\sum_{t=1}^T R^t(\boldsymbol{\lambda}^{t,\pi})\right] = \mathbf{E}\left[\sum_{t=1}^T r^t(\boldsymbol{\lambda}^{t,\pi})\right] \le \sum_{t=1}^T r^t\left(\mathbf{E}\left[\boldsymbol{\lambda}^{t,\pi}\right]\right).$$

Therefore, we conclude that $J^* \le J^D$. ■

EC.2. Proof of Lemma 2

We will prove that Lemma 2 holds under the more general setting in Section 6 with heterogeneous service time and advance reservation. Note that **DET-H** is a concave optimization problem with linear constraints. Therefore, Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient. To prove Lemma 2, we will construct a set of primal and dual variables that satisfy the KKT conditions for the scaled system, where the primal solution is characterized as in Lemma 2.

Consider the non-scaled system. Let $\mu_i^{t,D}$ be the dual variable corresponding to the constraint for resource type i in period t in **DET-H**. By the KKT conditions, the following must hold:

$$\frac{\partial r^t\left(\boldsymbol{\lambda}^{t,D}\right)}{\partial \lambda_k^t} = \sum_{i=1}^I a_{ik}\cdot\left(\sum_{s=t+\ell_k}^{\min\{t+n_k+\ell_k-1,T\}} \mu_i^{s,D}\right), \qquad \forall k, t\le T; \tag{EC.1}$$

$$\sum_{k=1}^K \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} a_{ik}\cdot\lambda_k^{s,D} - C_i \le 0, \qquad \forall i, t\le T; \tag{EC.2}$$

$$\mu_i^{t,D}\cdot\left(\sum_{k=1}^K \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} a_{ik}\cdot\lambda_k^{s,D} - C_i\right) = 0, \qquad \forall i, t\le T; \tag{EC.3}$$

$$\mu_i^{t,D} \ge 0, \qquad \forall i, t\le T. \tag{EC.4}$$

Correspondingly, for the θ-th system, the KKT conditions are given by

$$\frac{\partial r^{t,(\theta)}\left(\boldsymbol{\lambda}^{t}\right)}{\partial \lambda_k^t} = \sum_{i=1}^I a_{ik}\cdot\left(\sum_{s=t+\theta\cdot\ell_k}^{\min\{t+\theta\cdot(n_k+\ell_k)-1,\theta\cdot T\}} \mu_i^{s}\right), \qquad \forall k, t\le \theta\cdot T; \tag{EC.5}$$

$$\sum_{k=1}^K \sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} a_{ik}\cdot\lambda_k^{s} - \theta\cdot C_i \le 0, \qquad \forall i, t\le \theta\cdot T; \tag{EC.6}$$

$$\mu_i^t \left(\sum_{k=1}^{K} \sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} a_{ik} \cdot \lambda_k^s - \theta \cdot C_i \right) = 0, \qquad \forall i, t \le \theta \cdot T; \tag{EC.7}$$

$$\mu_i^t \ge 0, \qquad \forall i, t \le \theta \cdot T. \tag{EC.8}$$

We construct a set of primal and dual variables according to

$$\boldsymbol{\lambda}^{t,(\theta),D} = \boldsymbol{\lambda}^{\lceil \frac{t}{\theta} \rceil, D}, \qquad \mu_i^{t,(\theta),D} = \mu_i^{\lceil \frac{t}{\theta} \rceil, D} \cdot \mathbf{1}\left\{ \frac{t}{\theta} \in \mathbf{N}_+ \right\}, \qquad \forall i, t \le \theta \cdot T. \tag{EC.9}$$

It suffices to show that the variables defined in (EC.9) satisfy conditions (EC.5) - (EC.8). We start with condition (EC.5). Fix t and let $v = t \bmod \theta \in [0, \theta)$. Then, there exists $t' \in \mathbf{N}_+$ such that $t = \theta \cdot t' + v$. The second summation on the RHS of (EC.5) can be written as follows:

$$\begin{aligned} \sum_{s=t+\theta\cdot\ell_k}^{\min\{t+\theta\cdot(n_k+\ell_k)-1, \theta\cdot T\}} \mu_i^{s,(\theta),D} &= \sum_{s=\theta\cdot(t'+\ell_k)+v}^{\min\{\theta\cdot(t'+n_k+\ell_k)+v-1, \theta\cdot T\}} \mu_i^{s,(\theta),D} \\ &= \begin{cases} \sum_{s=t'+\ell_k}^{\min\{t'+n_k+\ell_k-1, T\}} \mu_i^{s,D} & \text{if } v = 0 \\ \sum_{s=t'+\ell_k+1}^{\min\{t'+n_k+\ell_k, T\}} \mu_i^{s,D} & \text{if } v \ge 1 \end{cases} = \sum_{s=\lceil t/\theta \rceil + \ell_k}^{\min\{\lceil t/\theta \rceil + n_k + \ell_k - 1, T\}} \mu_i^{s,D} \end{aligned}$$

where the second equality follows by (EC.9). Then (EC.5) follows directly from (EC.1) and the fact that $r^{t,(\theta)} = r^{\lceil t/\theta \rceil}$ (see the definition in Section 6). This confirms condition (EC.5).

We now show that condition (EC.6) is satisfied. Per our notation above, $t = \theta \cdot t' + v$ where $v = t \bmod \theta \in [0, \theta)$. We can write (i) $t - \theta n_k - \theta \ell_k + 1 = \theta \cdot (t' - n_k - \ell_k) + v + 1$ if $t \ge \theta(n_k + \ell_k)$, and (ii) $t - \theta \ell_k = \theta \cdot (t' - \ell_k) + v$ if $t \ge \theta \ell_k + 1$. For all k such that $t \ge \theta(n_k + \ell_k)$ (equivalently, $t' \ge n_k + \ell_k$), we can write:

$$\begin{aligned} \sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} \lambda_k^{s,(\theta),D} &= \sum_{s=t-\theta n_k - \theta \ell_k + 1}^{t-\theta\cdot\ell_k} \lambda_k^{\lceil \frac{t}{\theta} \rceil, D} \\ &= (\theta - v) \cdot \lambda_k^{t'-n_k-\ell_k+1, D} + \sum_{s=t'-n_k-\ell_k+2}^{t'-\ell_k} \theta \cdot \lambda_k^{s,D} + v \cdot \lambda_k^{t'-\ell_k+1, D} \\ &= (\theta - v) \cdot \left(\sum_{s=t'-n_k-\ell_k+1}^{t'-\ell_k} \lambda_k^{s,D} \right) + v \cdot \left(\sum_{s=t'-n_k-\ell_k+2}^{t'-\ell_k+1} \lambda_k^{s,D} \right). \end{aligned} \tag{EC.10}$$

For all k such that $\theta \ell_k + 1 \le t \le \theta(n_k + \ell_k) - 1$ (equivalently, $\ell_k \le t' \le n_k + \ell_k - 1$), we can write:

$$\sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} \lambda_k^{s,(\theta),D}$$

$$
\begin{aligned}
&= \sum_{s=1}^{t-\theta\cdot\ell_k} \lambda_k^{\lceil \frac{t}{\theta} \rceil, D} = \sum_{s=1}^{t'-\ell_k} \theta \cdot \lambda_k^{s,D} + v \cdot \lambda_k^{t'_k-\ell_k+1,D} \\
&= (\theta - v) \cdot \left(\sum_{s=1}^{t'-\ell_k} \lambda_k^{s,D} \right) + v \cdot \left(\sum_{s=1}^{t'-\ell_k+1} \lambda_k^{s,D} \right) \\
&= (\theta - v) \cdot \left(\sum_{s=(t'-n_k-\ell_k+1)^+}^{t'-\ell_k} \lambda_k^{s,D} \right) + v \cdot \left(\sum_{s=(t'-n_k-\ell_k+2)^+}^{t'-\ell_k+1} \lambda_k^{s,D} \right). \qquad \text{(EC.11)}
\end{aligned}
$$

For all k such that $t \le \theta\ell_k$, none of them have consumed any resource yet by time t. Therefore, by (EC.10) and (EC.11), the total consumption of resource type i under $\lambda^{(\theta),D}$ can be written as:

$$
\begin{aligned}
&\sum_{k=1}^{K} a_{ik} \cdot \left(\sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} \lambda_k^{s,(\theta),D} \right) \\
&\quad = (\theta - v) \cdot \sum_{k=1}^{K} a_{ik} \cdot \left(\sum_{s=(t'-n_k-\ell_k+1)^+}^{(t'-\ell_k)^+} \lambda_k^{s,D} \right) + v \cdot \sum_{k=1}^{K} a_{ik} \cdot \left(\sum_{s=(t'-n_k-\ell_k+2)^+}^{(t'-\ell_k+1)^+} \lambda_k^{s,D} \right) \\
&\quad \le (\theta - v) \cdot C_i + v \cdot C_i = C_i
\end{aligned}
$$

where the inequality follows from (EC.2). This confirms condition (EC.6).

We now prove condition (EC.7). Note that (EC.7) holds automatically for any period t that is not an integer multiple of θ. If $t = \theta \cdot t'$ for some $t' \in \mathbf{N}_+$, we have:

$$
\begin{aligned}
&\mu_i^{t,(\theta),D} \cdot \left(\sum_{k=1}^{K} \sum_{s=(t-\theta\cdot(n_k+\ell_k)+1)^+}^{(t-\theta\cdot\ell_k)^+} a_{ik} \cdot \lambda_k^{s,(\theta),D} - \theta \cdot C_i \right) \\
&= \mu_i^{t',D} \cdot \left(\sum_{k=1}^{K} \sum_{u=(t'-(n_k+\ell_k)+1)^+}^{(t'-\ell_k)^+} \sum_{s=(u-1)\theta+1}^{u\theta} a_{ik} \cdot \lambda_k^{\lceil \frac{s}{\theta} \rceil, D} - \theta \cdot C_i \right) \\
&= \theta \cdot \mu_i^{t',D} \cdot \left(\sum_{k=1}^{K} \sum_{u=(t'-(n_k+\ell_k)+1)^+}^{(t'-\ell_k)^+} a_{ik} \cdot \lambda_k^{u,D} - C_i \right) = 0
\end{aligned}
$$

where the last equality holds by (EC.3). Lastly, condition (EC.8) is satisfied automatically. ■

EC.3. Proof of Theorem 1

Let $\{\boldsymbol{p}^t\}$ denote the price sequences computed under DPC(ϵ). The proof of Theorem 1 is divided into two steps. In the first step, we define an event where we can explicitly characterize $\boldsymbol{p}^t$ and then show that it happens with a high probability; in the second step, we bound the total revenue losses under DPC(ϵ) by bounding the losses on and off the high-probability event separately. Let $\boldsymbol{\Delta}^t(\boldsymbol{p}^t) = \boldsymbol{D}^t(\boldsymbol{p}^t) - \boldsymbol{\lambda}^t(\boldsymbol{p}^t)$ (i.e., $\boldsymbol{\Delta}^t(\boldsymbol{p}^t)$ is the error from the expected demand in period t under price $\boldsymbol{p}^t$). For notational brevity, we will simply write $\boldsymbol{\lambda}^t = \boldsymbol{\lambda}^t(\boldsymbol{p}^t)$ and $\boldsymbol{\Delta}^t = \boldsymbol{\Delta}^t(\boldsymbol{p}^t)$.

Step 1

For some positive δ whose exact value is to be determined later, define a sequence of events $\{\mathcal{A}_j^b(\epsilon,\delta)\}$ as follows:

$$\mathcal{A}_j^b(\epsilon,\delta) \;=\; \left\{\max_{t\le bn}\left|\sum_{s=(b-1)n+1}^{t}\Delta_j^s\right| \;<\; \delta\right\} \qquad \text{for all } j\le[1,J] \text{ and } b\in\left[1,\frac{T}{n}\right]. \tag{EC.12}$$

Intuitively, on the set $\mathcal{A}_j^b$, the magnitude of cumulative demand errors of service type j within service cycle b is uniformly bounded by δ. We further define $\mathcal{G}(\epsilon,\delta) := \cap_{b,j}\mathcal{A}_j^b(\epsilon,\delta)$. Per our discussions at the beginning of this section, $\mathcal{G}(\epsilon,\delta)$ is our high-probability event. We formalize it in the following lemma.

Lemma EC.1. *Suppose that $\epsilon \le \min\{\varphi_L,\varphi_U\}\cdot\underline{n}$ and $\delta = (\underline{n}-1)(\epsilon-1)/(3\underline{n})$. Then, on $\mathcal{G}(\epsilon,\delta)$, the following two conditions hold for all $t\le T$:*

(i) $\boldsymbol{C}^t \succeq \boldsymbol{e}$; and

(ii) for all j, $\lambda_j^{t,D} > 0$ implies $\lambda_j^{t,D} - \epsilon/\underline{n} \in (0,\lambda_U)$.

Proof. Condition (ii) immediately follows from the definition of φ_L and φ_U in Section 3. We prove condition (i) by induction. If $t=1$, by definition, $C^1 = \boldsymbol{C} \succeq \boldsymbol{e}$; this confirms our base case. For $t>1$, assume that conditions (i) and (ii) are satisfied for all $s\le t-1$. Note that we only need to argue for the case where $t\ge n+1$, since $C_i^{n+1}\ge 1$ automatically implies that $C_i^s > 1$ for all $s\le n$. By condition (ii) and the inductive assumption, we can write: $\lambda_j^s = \lambda_j^{s,D} - (\epsilon/\underline{n})\cdot\mathbf{1}\{\lambda_j^{s,D} > 0\}$ for all $s\le t-1$. Let $S_i^t := \{j : \sum_{s=t-n+1}^{t-1}\lambda_j^{s,D} > 0, a_{ij}=1\}$. Note that, by definition of DPC, $j\notin S_i^t$ implies $\sum_{s=t-n+1}^{t-1} a_{ij}\cdot D_j^s(\boldsymbol{p}^s) = 0$; in other word, the set S_i^t is the collection of service types that may consume strictly positive amount of resource i at the beginning of period t. If $S_i^t=\emptyset$, then $C_i^t = C_i > 1$. If, however, $S_i^t\ne\emptyset$, on the set $\mathcal{G}(\epsilon,\delta)$, we have:

$$C_i^t = C_i - \sum_{s=t-n+1}^{t-1}\sum_{j=1}^{J} a_{ij}\cdot D_j^s(\boldsymbol{p}^s)$$

$$
\begin{aligned}
&= C_i - \sum_{s=t-n+1}^{t-1} \sum_{j \in S_i^t} a_{ij} \cdot \left(\lambda_j^{s,D} - \frac{\epsilon}{\underline{n}} \cdot \mathbf{1}\{\lambda_j^{s,D} > 0\} + \Delta_j^s \right) \\
&= C_i - \sum_{s=t-n+1}^{t-1} \sum_{j \in S_i^t} a_{ij} \cdot \lambda_j^{s,D} + \sum_{j \in S_i^t} a_{ij} \cdot \left(\frac{\epsilon}{\underline{n}} \cdot \sum_{s=t-n+1}^{t-1} \mathbf{1}\{\lambda_j^{s,D} > 0\} - \sum_{s=t-n+1}^{t-1} \Delta_j^s \right) \\
&\geq 0 + \sum_{j \in S_i^t} a_{ij} \cdot \left(\frac{\underline{n}-1}{\underline{n}} \cdot \epsilon - 3 \cdot \delta \right) > 0. \qquad \text{(EC.13)}
\end{aligned}
$$

where the first inequality follows since $\sum_{s=t-n+1}^{t-1} \sum_{j=1}^{J} a_{ij} \cdot \lambda_j^{s,D} \leq C_i$. As for the last inequality, note that, (i) $\underline{n} \leq \sum_{s=t-n+1}^{t-1} \mathbf{1}\{\lambda_j^{t,D} > 0\} + 1$ for $j \in S_i^t$ (by definition); and (ii) for any pair (t_1, t_2) with $t_2 = t_1 + n - 1 \leq T$, $t_1 \in [(b-1)n+1, bn]$ and $t_2 \in [bn+1, (b+1)n]$ for some $b \leq T/n - 1$, we have:

$$
\left| \sum_{s=t_1}^{t_2} \Delta_j^s \right| \leq \left| \sum_{s=t_1}^{bn} \Delta_j^s \right| + \left| \sum_{s=bn+1}^{t_2} \Delta_j^s \right| \leq \left| \sum_{s=(b-1)n+1}^{bn} \Delta_j^s \right| + \left| \sum_{s=(b-1)n+1}^{t_1} \Delta_j^s \right| + \left| \sum_{s=bn+1}^{t_2} \Delta_j^s \right| \leq 2\delta + \delta = 3\delta,
$$

where the last inequality follows from the definition of $\mathcal{A}_j^b(\epsilon, \delta)$ in (EC.12). This verifies condition (i) for period t and concludes the inductive argument. ■

By definition of DPC, a direct consequence of Lemma EC.1 is that, on $\mathcal{G}(\epsilon, \delta)$, we have:

$$
\lambda_j^t \; = \; \lambda_j^{t,D} - \frac{\epsilon}{\underline{n}} \cdot \mathbf{1}\left\{\lambda_j^{t,D} > 0\right\}. \qquad \text{(EC.14)}
$$

Let $\bar{\mathcal{A}}$ denote the complement of $\mathcal{A}$. We now state and proof a lemma that bounds $\mathbf{P}\left(\bar{\mathcal{G}}(\epsilon, \delta)\right)$ from above.

Lemma EC.2. *For any $\gamma \in (0, 1]$, the following holds:*

$$
\mathbf{P}\left(\bar{\mathcal{G}}(\epsilon, \delta)\right) \leq \frac{2JT}{n} \exp\left\{ \min\left\{ \max_i C_i, n \right\} \cdot \gamma^2 - \delta \cdot \gamma \right\}
$$

Proof. We first compute a bound for $\mathbf{P}(\bar{\mathcal{A}}_j^b(\epsilon, \delta))$. Fix b and j. Note that, for an arbitrary $\gamma > 0$, we can bound:

$$
\begin{aligned}
\mathbf{P}\left(\bar{\mathcal{A}}_j^b(\epsilon, \delta)\right) &\leq \frac{\mathbf{E}\left[\exp\left\{\gamma \left| \sum_{s=(b-1)n+1}^{bn} \Delta_j^s \right| \right\}\right]}{\exp\{\gamma\delta\}} \\
&\leq \frac{\mathbf{E}\left[\exp\left\{\gamma \sum_{s=(b-1)n+1}^{bn} \Delta_j^s \right\}\right] + \mathbf{E}\left[\exp\left\{-\gamma \sum_{s=(b-1)n+1}^{bn} \Delta_j^s \right\}\right]}{\exp\{\gamma\delta\}},
\end{aligned}
$$

where the first inequality follows from a sub-Martingale inequality (see Williams 1991) and the last inequality holds because $e^{|x|} \leq e^x + e^{-x}$ for all x. Since $D_j^t(\boldsymbol{\lambda}^t)$ is a Bernoulli random variable with success probability λ_j^t, by the Moment Generating Function of Bernoulli random variable,

$$\begin{aligned}\mathbf{E}\left[\exp\left\{\gamma \sum_{s=(b-1)n+1}^{bn} \Delta_j^s\right\}\right] &= \prod_{s=(b-1)n+1}^{bn} \mathbf{E}\left[\exp\left\{\gamma \cdot \Delta_j^s\right\}\right] \\ &= \prod_{s=(b-1)n+1}^{bn} \left[e^{\gamma} \cdot \lambda_j^s + 1 - \lambda_j^s\right] \cdot e^{-\gamma \lambda_j^s} \\ &\leq \prod_{s=(k-1)n+1}^{kn} \exp\left\{(e^{\gamma} - 1 - \gamma)\lambda_j^s\right\}.\end{aligned}$$

Now, for all $|x| \leq 1$, it holds that $e^x - 1 - x \leq x^2$. Moreover, for any i such that $a_{ij} = 1$ (such i always exists since we assume that $\sum_i a_{ij} > 0$),

$$\sum_{s=(k-1)n+1}^{kn} \lambda_j^s = \frac{1}{a_{ij}} \cdot \left(\sum_{s=(k-1)n+1}^{kn} a_{ij} \cdot \lambda_j^s\right) \leq \frac{1}{a_{ij}} \cdot \left(\sum_{s=(k-1)n+1}^{kn} \sum_v a_{iv} \cdot \lambda_v^s\right) \leq \min\{C_i, n\}$$

where the last inequality holds since there is at most one arrival each period. Since $\min\{C_i, n\} \leq \min\{\max_i C_i, n\}$, we can bound:

$$\mathbf{E}\left[\exp\left\{\gamma \sum_{s=(b-1)n+1}^{bn} \Delta_j^s\right\}\right] \leq \exp\left\{\min\left\{\max_i C_i, n\right\} \cdot \gamma^2\right\} \quad \text{for all } \gamma \in (0,1]. \tag{EC.15}$$

Note that similar arguments can also be applied to $\mathbf{E}\left[\exp\left\{-\gamma \sum_{s=(b-1)n+1}^{bn} \Delta_j^s\right\}\right]$. Putting all things together, for all b, j and $\gamma \in (0,1]$, we have:

$$\mathbf{P}(\bar{\mathcal{A}}_j^b(\epsilon, \delta)) \leq 2 \cdot \exp\left\{\min\left\{\max_i C_i, n\right\} \cdot \gamma^2 - \delta \cdot \gamma\right\}.$$

Then the proof is concluded by the sub-additive property of probability. ■

Step 2

Let $r^u = \max_t \max_{\boldsymbol{\lambda}^t \in \Omega_\lambda} r^t(\boldsymbol{\lambda}^t)$. By Lemma EC.1, we can bound the expected loss of DPC(ϵ) as follows.

$$J^D - \mathbf{E}[R^{DPC(\epsilon)}]$$

$$
\begin{aligned}
&= \sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^{t,D}) - \mathbf{E}\left[\sum_{t=1}^{T} r^t(\boldsymbol{p}^t)\right] \\
&= \sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^{t,D}) - \mathbf{E}\left[\sum_{t=1}^{T} r^t(\boldsymbol{p}^t)\middle| \mathcal{G}(\epsilon,\delta)\right] \mathbf{P}\left(\mathcal{G}(\epsilon,\delta)\right) - \mathbf{E}\left[\sum_{t=1}^{T} r^t(\boldsymbol{p}^t)\middle| \bar{\mathcal{G}}(\epsilon,\delta)\right] \mathbf{P}\left(\bar{\mathcal{G}}(\epsilon,\delta)\right) \\
&= \sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^{t,D}) - \mathbf{E}\left[\sum_{t=1}^{T} r^t\left(\hat{p}^t\right)\right] - 2\cdot\mathbf{E}\left[\sum_{t=1}^{T} r^t(\boldsymbol{p}^t)\middle| \bar{\mathcal{G}}(\epsilon,\delta)\right] \mathbf{P}\left(\bar{\mathcal{G}}(\epsilon,\delta)\right) \\
&\le \sum_{t=1}^{T}\left[r^t(\boldsymbol{\lambda}^{t,D}) - r^t\left(\lambda_1^{t,D} - \frac{\epsilon}{n}\cdot\mathbf{1}\{\lambda_1^{t,D}>0\},\ldots,\lambda_J^{t,D} - \frac{\epsilon}{n}\cdot\mathbf{1}\{\lambda_J^{t,D}>0\}\right)\right] \\
&\qquad + \frac{4r^u JT}{n}\cdot\exp\left\{\min\left\{\max_i C_i, n\right\}\cdot\gamma^2 - \delta\cdot\gamma\right\} \\
&\le T\cdot\left(\frac{\Psi J\epsilon}{n} + \frac{4r^u JT}{n}\exp\left\{\min\left\{\max_i C_i, n\right\}\cdot\gamma^2 - \delta\cdot\gamma\right\}\right)
\end{aligned}
$$

where the first inequality holds for arbitrary $\gamma\in(0,1]$ by Lemma EC.2, the second inequality holds by Assumption A5. Taking $\delta=(\underline{n}-1)(\epsilon-1)/(3\underline{n})$ and $\gamma=(\epsilon-1)/(6\cdot\min\{\max_i C_i,n\})$ yields:

$$
\frac{J^D - \mathbf{E}[R^{DPC(\epsilon)}]}{T} \le M_1\cdot\left[\frac{\epsilon}{\underline{n}} + \frac{T}{n}\ \exp\left\{-\frac{(\epsilon-1)^2}{36\cdot\min\{\max_i C_i, n\}}\right\}\right]
$$

for some $M_1>0$ independent of T, C, n, and $\epsilon\in(1,\min\{\varphi_L,\varphi_U\}\underline{n}]$. (It is required that $1<\epsilon\le 6\cdot\min\{\max_i C_i,n\}+1$ to ensure that $\gamma\in(0,1]$.) This completes the proof. ■

EC.4. Proof of Theorem 3

Let $\{\boldsymbol{p}^t\}$ and $\{\boldsymbol{\lambda}^t\}$ be the price and demand rate sequences under DPC-B. We will bound the loss of DPC-B. The proof for each case follows a similar two-step argument as in the proof of Theorem 1: We first define a high-probability event on which we can explicitly characterize $\boldsymbol{p}^t$, and then we compute a bound for the revenue loss under DPC-B(m,ϵ).

Step 1

For some positive $\delta_k^b = o(m_k)$ whose exact value is to be determined later, define a sequence of events $\{\mathcal{A}_k^b(\epsilon_k,m_k,\delta_k)\}$ as follows:

$$
\mathcal{A}_k^b(\epsilon_k,m_k,\delta_k) \ = \ \left\{\max_{t\le bm_k}\left|\sum_{s=(b-1)m_k+1}^{t}\Delta_k^s\right| \ < \ \delta_k^b\right\} \quad \text{for all } b \text{ and } k. \tag{EC.16}
$$

We further define $\mathcal{G}(\epsilon,m,\delta)=\cap_{b=1}^{B_k}\cap_{j=1}^{J}\cap_{k=1}^{K}\mathcal{A}_k^b(\epsilon_k,m_k,\delta_k^b)$. Similar with the proof of Theorem 1, $\mathcal{G}(\epsilon,m,\delta)$ is our high-probability event. We state a lemma.

Lemma EC.3. *Suppose that* $\epsilon_k \le \underline{n}_k \min\left\{1, \frac{1+4m_k\cdot\min\{\varphi_L,\varphi_U\}}{\underline{n}_k+4m_k}\right\}$, $m_k \le \underline{n}_k$, *and*

$$\delta_k^b = \begin{cases} \frac{\underline{n}-1}{\underline{n}}\cdot\frac{\min_k \epsilon_k - 1}{4K} & \text{if } \min\{t : t\in\mathcal{T}_k^b\} \le n_K + \ell_K + 1 \\ \frac{\underline{n}-1}{\underline{n}}\cdot\frac{\epsilon_k-1}{4} & \text{otherwise} \end{cases} \tag{EC.17}$$

Then, on $\mathcal{G}(\epsilon, m, \delta)$, *the following two conditions hold for all* $t \le T$:

(i) $\boldsymbol{C}^t \succeq \boldsymbol{e}$; *and*

(ii) for all k, $\lambda_k^{t,D} \in (0,\lambda_U)$ *implies* $\lambda_k^{t,D} - \epsilon_k/\underline{n}_k - (\sum_{s\in\mathcal{T}_k^{\beta_k(t)-1}}\Delta_k^s)/m_k \in (0,\lambda_U)$.

Proof. Firstly, on the set $\mathcal{G}(\epsilon,m,\delta)$, it is straightforward to check that $\left|\epsilon_k/\underline{n}_k + \sum_{s\in\mathcal{T}_k^b}\Delta_k^s/m_k\right| \le \epsilon_k/\underline{n}_k + \delta_k^b/m_k$ for all b and k. This, together with the definition of φ_L and φ_U, verifies condition (ii). We now verify condition (i) by induction. The base case (i.e., for $t=1$) is trivially satisfied. For $t>1$, assume that conditions (i) and (ii) are satisfied for all $s \le t-1$. Let $S_i^t := \{k : \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+}\lambda_k^{s,D} > 0, a_{ik}=1\}$ be the collection of service types that will consume a unit of resource i in period t. (It is straightforward to check that $\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} a_{ik}D_k^s(\boldsymbol{p}^s) = 0$ if $k\notin S_i^t$.) Now, if $S_i^t=\emptyset$, then $C_i^t = C_i - \sum_{k=1}^K\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} a_{ik}D_k^s(\boldsymbol{p}^s) = C_i \ge 1$. If $S_i^t \cap \{v : t\ge n_v+\ell_v\} = \emptyset$, then $n_k+\ell_k-1\ge t$ for all $k\in S_i^t$. But, this means that the consumption of resource k in period $t-1$ can always be bounded from above by the consumption of resource k in period t as follows:

$$\begin{aligned}\sum_{k=1}^K a_{ik}\cdot\left(\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} D_k^s(\boldsymbol{p}^s)\right) &= \sum_{k\in S_i^t} a_{ik}\cdot\left(\sum_{s=1}^{(t-\ell_k-1)^+} D_k^s(\boldsymbol{p}^s)\right) \le \sum_{k\in S_i^t} a_{ik}\cdot\left(\sum_{s=1}^{(t-\ell_k)^+} D_k^s(\boldsymbol{p}^s)\right) \\ &= \sum_{k\in S_i^t} a_{ik}\cdot\left(\sum_{s=(t+1-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} D_k^s(\boldsymbol{p}^s)\right) \le \sum_{k=1}^K a_{ik}\cdot\left(\sum_{s=(t+1-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} D_k^s(\boldsymbol{p}^s)\right),\end{aligned}$$

where both the first and the second equalities follow because $n_k+\ell_k-1\ge t$. Thus, the capacity constraint at period t directly implies the capacity constraint at period t. In other words, $C_i^{t+1}\ge 1$ directly implies $C_i^t \ge 1$. Therefore it suffices to check the case where $S_i^t\cap\{v : t\ge n_v+\ell_v\}\neq\emptyset$. Now suppose that there exists two constants $b_k \ge b_k' \ge 0$ such that $(t-\ell_k-1)^+\in\mathcal{T}_k^{b_k}$ and $(t-n_k-\ell_k+1)^+\in\mathcal{T}_k^{b_k'}$. Note that $\Delta_k^t = \Delta_k^t\cdot\mathbf{1}\{\lambda_k^{t,D}>0\}$ since $\Delta_k^t=0$ whenever $\lambda_k^{t,D}=0$. By condition (ii) and the inductive assumption, we have $\lambda_k^t = \lambda_k^{t,D} - (\epsilon_k/\underline{n}_k + \sum_{s\in\mathcal{T}_k^{\beta_k(t)-1}}\Delta_k^s)\cdot\mathbf{1}\{\lambda_k^{t,D}>0\}$. Putting these together, the total resource consumption of resource i at the beginning of period t can be characterized explicitly as follows:

$$\begin{aligned}
C_i^t &= C_i - \sum_{k=1}^{K} a_{ik}\cdot\left[\sum_{\substack{s\geq(t-n_k-\ell_k)^+\\ s\in\mathcal{T}_k^{b'_k}}} D_k^s(\boldsymbol{p}^s) + \sum_{b=b'_k+1}^{b_k-1}\sum_{s\in\mathcal{T}_k^b} D_k^s(\boldsymbol{p}^s) + \sum_{\substack{s\leq(t-\ell_k-1)^+\\ s\in\mathcal{T}_k^{b_k}}} D_k^s(\boldsymbol{p}^s)\right]\\
&= C_i - \sum_{k\in S_i^t}\sum_{\substack{s\geq(t-n_k-\ell_k+1)^+\\ s\in\mathcal{T}_k^{b'_k}}} a_{ik}\cdot\left[\lambda_k^{s,D} - \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k}\sum_{v\in\mathcal{T}_k^{b'_k-1}}\Delta_k^v - \Delta_k^s\right)\cdot\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right]\\
&\quad - \sum_{k\in S_i^t}\sum_{b=b'_k+1}^{b_k-1}\sum_{s\in\mathcal{T}_k^b} a_{ik}\cdot\left[\lambda_k^{s,D} - \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k}\sum_{v\in\mathcal{T}_k^{b-1}}\Delta_k^v - \Delta_k^s\right)\cdot\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right]\\
&\quad - \sum_{k\in S_i^t}\sum_{\substack{s\leq(t-\ell_k-1)^+\\ s\in\mathcal{T}_k^{b_k}}} a_{ik}\cdot\left[\lambda_k^{s,D} - \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k}\sum_{v\in\mathcal{T}_k^{b_k-1}}\Delta_k^v - \Delta_k^s\right)\cdot\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right]\\
&= C_i - \sum_{k\in S_i^t}\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} a_{ik}\cdot\lambda_k^{s,D} + \sum_{k\in S_i^t} a_{ik}\cdot\frac{\epsilon_k}{\underline{n}_k}\cdot\left(\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+}\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right)\\
&\quad - \sum_{k\in S_i^t} a_{ik}\cdot\underbrace{\left(\sum_{s\in\mathcal{T}_k^{b'_k-1}}\Delta_k^s\right)\cdot\left(\sum_{v\geq(t-n_k-\ell_k+1)^+,\,v\in\mathcal{T}_k^{b'_k}}\frac{\mathbf{1}\left\{\lambda_k^{v,D}>0\right\}}{m_k}\right)}_{\text{Error A}}\\
&\quad + \sum_{k\in S_i^t} a_{ik}\cdot\underbrace{\left(\sum_{s<(t-n_k-\ell_k+1)^+,\,s\in\mathcal{T}_k^{b'_k}}\Delta_k^s\cdot\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right)}_{\text{Error B}}\\
&\quad + \sum_{k\in S_i^t} a_{ik}\cdot\underbrace{\sum_{s\in\mathcal{T}_k^{b_k-1}}\Delta_k^s\cdot\left(1-\frac{\sum_{v\leq(t-\ell_k-1)^+,\,v\in\mathcal{T}_k^{b_k}}\mathbf{1}\left\{\lambda_k^{v,D}>0\right\}}{m_k}\right)}_{\text{Error C}}\\
&\quad + \sum_{k\in S_i^t} a_{ik}\cdot\underbrace{\left(\sum_{s\leq(t-\ell_k-1)^+,\,s\in\mathcal{T}_k^{b_k}}\Delta_k^s\cdot\mathbf{1}\left\{\lambda_k^{s,D}>0\right\}\right)}_{\text{Error D}}
\end{aligned}$$

where the term "Error A" represents redundant error-correction terms induced over periods $\{s : s\geq(t-n_k-\ell_k+1)^+, s\in\mathcal{T}_k^{b'_k}\}$; the term "Error B" represents over-correction of error in $\mathcal{T}_k^{b'_k}$ due to incomplete $\mathcal{T}_k^{b'_k}$; the term "Error C" represents under-correction of error in $\mathcal{T}_k^{b_k-1}$ due to incomplete $\mathcal{T}_k^{b_k}$; the term "Error D" represents error terms in $\mathcal{T}_k^{b_k}$. On $\mathcal{G}(\epsilon, m, \delta)$, it is not difficult to see that

$$|\text{Error A }| \leq \delta_k^{b'_k-1}, \quad |\text{Error B}| \leq \delta_k^{b'_k}, \quad |\text{Error C}| \leq \delta_k^{b_k-1}, \quad |\text{Error D}| \leq \delta_k^{b_k} \tag{EC.18}$$

By the capacity constraint at period $t-1$, we have

$$C_i - \sum_{k \in S_i^t} \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} a_{ik} \cdot \lambda_k^{s,D} \geq 0 \tag{EC.19}$$

Moreover, since $S_i^t \cap \{v : t \geq n_v + \ell_v\} \neq \emptyset$, we can bound:

$$\begin{aligned}
&\sum_{k \in S_i^t} a_{ik} \cdot \frac{\epsilon_k}{\underline{n}_k} \cdot \left(\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} \mathbf{1}\left\{\lambda_k^{s,D} > 0\right\} \right) \\
\geq & \sum_{k \in S_i^t \cap \{v : t \geq n_v + \ell_v\}} a_{ik} \cdot \frac{\epsilon_k}{\underline{n}_k} \cdot \left(\sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k-1)^+} \mathbf{1}\left\{\lambda_k^{s,D} > 0\right\} \right) \\
\geq & \sum_{k \in S_i^t \cap \{v : t \geq n_v + \ell_v\}} a_{ik} \cdot \frac{\epsilon_k}{\underline{n}_k} \cdot \underline{n}_k \\
= & \sum_{k \in S_i^t \cap \{v : t \geq n_v + \ell_v\}} a_{ik} \cdot \epsilon_k .
\end{aligned} \tag{EC.20}$$

Combining all the bounds above (i.e., EC.18 - EC.20) yields:

$$C_i^t \geq \sum_{k \in S_i^t} a_{ik} \cdot \left(\epsilon_k \cdot \mathbf{1}\{t \geq n_k + \ell_k\} - \delta_k^{b'_k-1} - \delta_k^{b'_k} - \delta_k^{b_k-1} - \delta_k^{b_k} \right) \tag{EC.21}$$

for all i and t with $S_i^t \cap \{v : t \geq n_v + \ell_v\} \neq \emptyset$. Lastly, substituting the definition of δ_k^b in (EC.17) into (EC.21) verifies condition (i) and concludes our inductive argument. ■

By definition of $\boldsymbol{p}^t$ under DPC-B, a consequence of Lemma EC.1 is that, on $\mathcal{G}(\epsilon, \delta)$, we have:

$$\lambda_k^t \;=\; \lambda_k^{t,D} - \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k} \cdot \sum_{s \in \mathcal{T}_k^{\beta_k(t)-1}} \Delta_k^s \right) \mathbf{1}\left\{\lambda_k^{t,D} > 0\right\}. \tag{EC.22}$$

Note that (EC.21) is the analogue of (EC.13) in the proof of Theorem 1. The term δ_k in (EC.13) represents a bound on cumulative errors over n periods whereas the term δ_k^b in (EC.21) represents a bound on cumulative errors during m_k periods. As m_k can be different from n (or, more precisely, n_k), the magnitude of δ_k^b in (EC.21) can be optimally tuned whereas the magnitude of δ_k in (EC.13) is determined by system parameters. This highlights the potential room for improvement due to batch adjustments.

The following lemma is analogous to Lemma EC.2.

LEMMA EC.4. *Assume that δ_k is chosen as in (EC.17). For all $\gamma_k \in (0,1]$, there exists some :*

$$\mathbf{P}\left(\bar{\mathcal{G}}(\epsilon,m,\delta)\right) \le \sum_{k=1}^{K} \frac{4T}{m_k} \exp\left\{\min\left\{\max_i C_i, m_k\right\}\cdot\gamma_k^2 - \frac{\min_k \epsilon_k - 1}{8K}\cdot\gamma_k\right\}.$$

Proof. Analogous to the proof of Lemma EC.2, it can be shown that $\mathbf{P}\left(\bar{\mathcal{A}}_k^b(\epsilon_k,m_k,\delta_k)\right) \le 2\cdot \exp\{\min\{\max_i C_i, m_k\}\cdot\gamma_k^2 - \delta_k^b\gamma_k\}$ for all b, k and $\gamma_k \in (0,1]$. Note that there are at most $(n_K + \ell_K + 1)/m_k$ batches before period $n_K+\ell_K+1$, and the maximum number of batches in partition $\mathcal{P}_k$ satisfies $B_k \le T/m_k$. By the sub-additive property of probability and the fact that $(\underline{n}-1)/\underline{n} \ge 1/2$,

$$\begin{aligned}
\mathbf{P}\left(\bar{\mathcal{G}}(\epsilon,m,\delta)\right) &\le \sum_{k=1}^{K} \frac{2(n_K+\ell_K+1)}{m_k} \exp\left\{\min\left\{\max_i C_i, m_k\right\}\cdot\gamma_k^2 - \frac{\min_k \epsilon_k - 1}{8K}\cdot\gamma_k\right\} \\
&\quad + \sum_{k=1}^{K} \frac{2T}{m_k} \exp\left\{\min\left\{\max_i C_i, m_k\right\}\cdot\gamma_k^2 - \frac{\epsilon_k - 1}{8}\cdot\gamma_k\right\} \\
&\le \sum_{k=1}^{K} \frac{4T}{m_k} \exp\left\{\min\left\{\max_i C_i, m_k\right\}\cdot\gamma_k^2 - \frac{\min_k \epsilon_k - 1}{8K}\cdot\gamma_k\right\}. \quad \blacksquare
\end{aligned} \tag{EC.23}$$

<u>**Step 2**</u>

We now bound the average loss of DPC-B(m,ϵ). Similar to Step 2 in the proof of Theorem 1, we can bound the revenue loss as follows:

$$\begin{aligned}
&J^D - \mathbf{E}[R^{DPC-B(m,\epsilon)}] \\
&= \sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^{t,D}) - \mathbf{E}\left[\sum_{t=1}^{T} r^t\left(\hat{p}^t\right)\right] - 2\cdot\mathbf{E}\left[\sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^t)\middle|\bar{\mathcal{G}}(\epsilon,m,\delta)\right]\mathbf{P}\left(\bar{\mathcal{G}}(\epsilon,m,\delta)\right)
\end{aligned} \tag{EC.24}$$

The second expectation after the last equality above can be bounded by $r^u T\cdot\mathbf{P}(\bar{\mathcal{G}}(\epsilon,m,\delta))$ where $r^u = \max_t \max_{\boldsymbol{\lambda}_t\in\Omega_\lambda} r_t(\boldsymbol{\lambda}_t)$. For brevity of notation, define $\eta_e^t = (\mathbf{1}\left\{\lambda_1^{t,D} > 0\right\}, \ldots, \mathbf{1}\left\{\lambda_K^{t,D} > 0\right\})$, $\eta_\epsilon^t = (\epsilon_1/n_1, \ldots, \epsilon_K/n_k)$ and $\eta_\delta^t = \left(\sum_{s\in\mathcal{T}_1^{\beta_1(t)-1}} \Delta_1^s/m_1, \ldots, \sum_{s\in\mathcal{T}_J^{\beta_J(t)-1}} \Delta_K^s/m_K\right)$. Let $u\odot v$ be the

Hadamard product of vectors u and v. Then by (EC.22), we can write $\boldsymbol{\lambda}^t = \boldsymbol{\lambda}^{t,D} - \eta_\epsilon^t \odot \eta_e^t - \eta_\delta^t \odot \eta_e^t$. We can then bound the term inside the first expectation in (EC.24) as follows.

$$\begin{aligned} r^t(\boldsymbol{\lambda}^t) &= r^t\left(\boldsymbol{\lambda}^{t,D} - \eta_\epsilon^t \odot \eta_e^t - \eta_\delta^t \odot \eta_e^t\right) \\ &\geq r^t\left(\boldsymbol{\lambda}^{t,D}\right) - \left[\nabla r^t\left(\boldsymbol{\lambda}^{t,D}\right)\right]^\top \left(\eta_\epsilon^t \odot \eta_e^t\right) - \left[\nabla r^t\left(\boldsymbol{\lambda}^{t,D}\right)\right]^\top \left(\eta_\delta^t \odot \eta_e^t\right) - \Psi \cdot \left|\left|\eta_\epsilon^t \odot \eta_e^t + \eta_\delta^t \odot \eta_e^t\right|\right|_2^2 \\ &\geq r^t\left(\boldsymbol{\lambda}^{t,D}\right) - \Psi \sum_{k=1}^K \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{2\epsilon_k^2}{\underline{n}_k^2} + \frac{2}{m_k}\right) - \left[\nabla r^t\left(\boldsymbol{\lambda}^{t,D}\right)\right]^\top \left(\eta_\delta^t \odot \eta_e^t\right) \end{aligned}$$

where the first inequality follows from Taylor's expansion and Assumption A5, the second inequality follows from triangular inequality, the fact that $\mathbf{E}\left[\left(\sum_{s\in\mathcal{T}_k^b} \Delta_k^s\right)^2\right] \leq m_k$ and Assumption A5. Moreover, since $\{\Delta_k^s\}$ are independent zero-mean random variables, $\mathbf{E}\left(\left[\nabla r^t\left(\boldsymbol{\lambda}^{t,D}\right)\right]^\top \left(\eta_\delta^t \odot \eta_e^t\right)\right) = 0$. Putting the bounds together, for all $r \in [0,1]$, we have:

$$\begin{aligned} &\frac{J^D - \mathbf{E}[R^{DPC-B(m,\epsilon)}]}{T} \\ &\leq \frac{1}{T} \cdot \left[T\Psi \sum_{k=1}^K \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{2\epsilon_k^2}{\underline{n}_k^2} + \frac{2}{m_k}\right) + r^u T \cdot P(\bar{\mathcal{G}}(\epsilon, m, \delta))\right] \\ &\leq \sum_{k=1}^K \frac{\Psi\epsilon_k}{\underline{n}_k} + \frac{2\Psi\epsilon_k^2}{\underline{n}_k^2} + \frac{2\Psi}{m_k} + \frac{2r^u T}{m_k} \exp\left\{\min\left\{\max_i C_i, m_k\right\} \cdot \gamma_k^2 - \frac{\min_k \epsilon_k - 1}{8K} \cdot \gamma_k\right\}. \end{aligned}$$

Taking $\gamma_k = [\min_k \epsilon_k - 1]/(16K \cdot \min\{\max_i C_i, m_k\})$ and noting that $\epsilon_k/\underline{n}_k < 1$ yields:

$$\mathcal{L}(\text{DPC-B}(m,\epsilon)) \leq M_3 \cdot \sum_{k=1}^K \left[\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k} + \frac{T}{m_k} \cdot \exp\left\{-\frac{(\min_k \epsilon_k - 1)^2}{256K^2 \cdot \min\{\max_i C_i, m_k\}}\right\}\right]$$

for some $M_3 > 0$ independent of T, C, n_k, $m_k \leq \underline{n}_k$, and

$$\epsilon_k \in \left(1, \min\left\{\underline{n}_k \cdot \min\left\{1, \frac{1 + 4m_k \cdot \min\{\varphi_L, \varphi_U\}}{4m_k + \underline{n}_k}\right\}, 1 + 16\min\{\max_i C_i, m_k\}\right\}\right].$$

Note that $1 < \epsilon_k < 1 + 16\min\{\max_i C_i, m_k\}$ ensures that $\gamma_k \in (0,1)$. ■

EC.5. Appendix to Numerical Experiment

In this section, we first report numerical details to the experiment in Section 7. To get better understandings of the performances of the proposed heuristics, we further run a second set of numerical experiment to compare DPC-B with two benchmark controls in a simpler setting. To achieve good estimates of performances, we simulate all heuristic controls with 200 Monte Carlo runs; this ensures that the coefficients of variations are all within 1%. All the experiments are implemented on a Windows desktop with Intel i5-7300HQ CPU and 8 GB RAM.

EC.5.1. Additional Details to the Numerical Experiment in Section 7

Table EC.1 reports the numerical results for the performance of DPC(0), DPC(0.2), and DPC-B(1,0.3). Note that, instead of reporting the average loss metric, we report the loss (i.e., $J^{D,(\theta)} - J^{\pi}$, in the column titled "Loss") and the percentage loss (i.e., $(J^{D,(\theta)} - J^{\pi,(\theta)})/J^{D,(\theta)}$, in the column titled "Loss %"; this quantity is on the same order of average loss) compared to $J^{D,(\theta)}$ in order to have a more direct sense of the performance. We also report the runtime of a single Monte Carlo run for different controls in millisecond (in the column titled "Runtime"). As expected, DPC-B dominates DPC, which in turn dominates DPC without the buffer. Moreover, although DPC-B requires significantly more computation time compared to DPC, it is still very fast given the scale of the problem instances.

θ	DPC(0)			DPC(0.2)			DPC-B(1,0.3)		
	Loss	Loss %	Runtime	Loss	Loss %	Runtime	Loss	Loss %	Runtime
1000	5363895	2.499	9.6	2659600	1.239	9.7	1782187	0.830	203.4
2000	7902362	1.841	18.6	3253920	0.758	19.0	2752227	0.641	411.5
3000	9502699	1.476	26.4	4117705	0.640	26.8	3287743	0.511	668.3
4000	10308384	1.201	22.2	5240517	0.610	23.1	3827128	0.446	559.0
5000	12283973	1.145	37.8	5871840	0.547	39.6	3998207	0.373	866.5
6000	13056547	1.014	28.2	5182266	0.402	28.7	3828433	0.297	618.4
7000	13906379	0.926	37.8	4774585	0.318	40.4	2879453	0.192	834.7
8000	14712064	0.857	47.4	4836605	0.282	51.9	1806326	0.105	1202.2

Table EC.1 Performance of different controls with varying θ

Table EC.2 summarizes the performance of DPC and DPC-B under different choices of buffer sizes and batch sizes.

EC.5.2. Comparing DPC-B with Benchmark Controls

In the second set of experiment, we assume a simpler setting with a single type of demand and resource. Demand function is stationary over time, and is also exponentially decreasing in price,

θ	DPC(0.1)	DPC(0.2)	DPC(0.3)	DPC-B(1,0.1)	DPC-B(1,0.3)	DPC-B(1,0.5)
1000	1.384	1.239	1.468	1.256	0.830	0.718
2000	1.251	0.758	1.162	0.930	0.641	0.660
3000	0.961	0.640	0.916	0.633	0.511	0.592
4000	0.749	0.610	0.897	0.561	0.446	0.506
5000	0.695	0.547	0.849	0.520	0.373	0.460
6000	0.637	0.402	0.680	0.489	0.297	0.301
7000	0.610	0.318	0.543	0.311	0.192	0.216
8000	0.549	0.282	0.458	0.261	0.105	0.100
θ	DPC-B(0.5,0.1)	DPC-B(0.5,0.3)	DPC-B(0.5,0.5)	DPC-B(2,0.1)	DPC-B(2,0.3)	DPC-B(2,0.5)
1000	0.969	0.826	1.206	1.648	1.078	0.751
2000	0.794	0.708	1.044	1.066	0.849	0.724
3000	0.537	0.609	0.831	0.772	0.695	0.623
4000	0.467	0.519	0.654	0.707	0.554	0.582
5000	0.392	0.459	0.552	0.667	0.524	0.492
6000	0.324	0.309	0.452	0.540	0.389	0.359
7000	0.275	0.218	0.311	0.381	0.275	0.274
8000	0.226	0.164	0.231	0.300	0.160	0.148

Table EC.2 Percentage loss of different controls under different choices of control parameters

i.e., we use $\lambda^t(p^t) \equiv \lambda(p^t) = \exp(a - b \cdot p^t)$ with $a = 0.8$ and $b = 0.01$. (The subscript k is dropped here as well in the remaining part of Experiment 2, since there is only one type of demand.) The other problem parameters are set as $n = 1$, $C = 0.7$ and $T = 5$. The resulting deterministic problem has a stationary optimal solution $\lambda_t^D \equiv \lambda^D = 0.7$, with optimal objective value $J^D = 404.85$. Other than DPC-B (we test DPC-B(1,0.4), where we set $m^0 = 1$ by default and ϵ^0 after optimizing the buffer size), we also test two benchmark controls defined as follows:

- **LRC(m)**: This is a simple adaptation of the Linear Rate Control (LRC) proposed in Jasin (2014), whose average loss in the canonical PRM setting is of order $\theta^{-1} \cdot \log \theta$. LRC($m$) slices the selling horizon into batches in the same way as DPC-B. Denoting the sequence of batches as $\mathcal{T}^b$ (we neglect the subscript k since there is only one service type), LRC(m) sets price as follows: at the beginning of period t, if $t \in \mathcal{T}^b$, it first computes $\hat{\boldsymbol{p}}^t$ according to

$$\lambda^t(\hat{p}^t) \;=\; \textsc{Proj}_{[0,\lambda_U]}\left[\lambda^{t,D} - \frac{1}{m}\sum_{s\in\mathcal{T}^b, s<t} \frac{\Delta^s}{\tau^b_{\max} - s + 1}\right]$$

where $\tau^b_{\max} = \max\{t : t \in \mathcal{T}^b\}$. If $C^t \geq 1$, set $p^t = \hat{p}^t$; otherwise, set $p^t = \bar{p}$. In a nutshell, LRC(m) treats different batches as if they are independent, and blindly applies the original LRC in Jasin (2014) within each batch.

- **Erl-Approx**: This heuristic control uses the Erlang-B Formula to compute the blocking probability, which in turn is used to compute the optimal *static* price. Since the exact value of Erlang-B Formula is hard to compute for large scale system, we use the approximation method in Adelman (2008) (see Section 5 in Adelman 2008 for an example on how the approximation is used to optimize price).

We report the losses and percentage losses for all three controls in Table EC.3. For LRC(m), we only report the results for LRC($n^{(\theta)}$) since, from our experiments, setting $m = n^{(\theta)}$ consistently gives the best numerical performance. For Erl-Approx, we also report the optimal demand computed using the approximation in Adelman (2008) in the column titled "Opt. Demand". The relatively poor performance of LRC($n^{(\theta)}$) suggests that a heuristic control that performs well in the canonical PRM setting may not perform equally well in the reusable RM setting (indeed, our results also suggest that the theoretical average loss of LRC in the reusable RM setting is much worse than $\theta^{-1} \cdot \log \theta$). The performance of Erl-Approx is better than LRC($n^{(\theta)}$) but still worse than DPC-B(1,0.4).

EC.6. Example with Zero Optimal Demand

We give a simple example where some of the optimal demand can indeed be zero. Consider the setting with single type of demand and resource. Demand is linearly decreasing in price, i.e. $\lambda^t(p^t) = a^t - b^t \cdot p^t$ (we drop the subscript k since there is only one product). The problem parameters are

theta	DPC-B (1.0.4)		LRC-n		Erl-Approx		
	Loss	Loss %	Loss	Loss %	Loss	Loss %	Opt. Demand
1000	291944	0.721	1609959	3.977	274956	0.679	0.678
2000	336595	0.416	2351197	2.904	380655	0.470	0.684
3000	390181	0.321	3032371	2.497	469264	0.386	0.687
4000	429517	0.265	3508643	2.167	548728	0.339	0.688
5000	469688	0.232	3933702	1.943	602442	0.298	0.689
6000	560774	0.231	4362388	1.796	637323	0.262	0.690
7000	536484	0.189	4618052	1.630	713853	0.252	0.690
8000	569789	0.176	5186992	1.602	781171	0.241	0.691

Table EC.3 Performance of different controls with varying θ

set to be $T = 3$, $n = 2$, $C = 0.5$, $a = [3, 1, 3]$, and $b = [1, 1, 1]$. It is straightforward to check that the optimal demand is $\lambda^{1,D} = \lambda^{3,D} = 0.5$ and $\lambda^{2,D} = 0$. This is reasonable, since $(r^1)'(\lambda^1)|_{\lambda^1=C} = (r^3)'(\lambda^3)|_{\lambda^3=C} = 2 > 1 = (r^2)'(\lambda^2)|_{\lambda^2=0}$ and therefore all capacity is optimally allocated to satisfy demands in periods 1 and 3.

References

Adelman, D. 2008. A simple algebraic approximation to the erlang loss system. *Oper. Res. Lett.* **36**(4) 484–491.

Jasin, S. 2014. Reoptimization and self-adjusting price control for network revenue management. *Oper. Res.* **62**(5) 1168–1178.

Williams, D. 1991. *Probability with martingales*. Cambridge university press.