Online Appendix

EC.1. Proofs

Proof of Proposition 1: We first calculate the expected duration of a contract in which both agents exert full effort at all times. We solve the problem by backward induction on the state of the game where the states are defined by the number of success for each agent. First, consider the state when both agents have already achieved one success, then the expected arrival time for the second success is given by:

$$\int_0^\infty 2\lambda t e^{-2\lambda t} dt = -te^{-2\lambda t}\Big|_0^\infty + \int_0^\infty e^{-2\lambda t} dt = -\frac{1}{2\lambda} e^{-2\lambda t}\Big|_0^\infty = \frac{1}{2\lambda}.$$

Next, consider the state of the game with a leader (an agent with one success) and a laggard (an agent with no success). Then the expected arrival time for the second success can be expressed as:

$$\int_0^\infty \left[\lambda t + \lambda\left(t + \frac{1}{2\lambda}\right)\right] e^{-2\lambda t} dt = \frac{1}{2\lambda} + \int_0^\infty \frac{1}{2} e^{-2\lambda t} dt = \frac{1}{2\lambda} + \frac{1}{4\lambda} = \frac{3}{4\lambda},$$

where with instantaneous probability λ the leader may obtain the second success at time t or the laggard may hit the first success at time t (proceeding to the above-mentioned state) in which case the expected duration of the contract is $t + 1/(2\lambda)$. Finally, considering the state when neither of the agents has one success, the expected duration of the contract is as follows:

$$\int_0^\infty 2\lambda\left(t + \frac{3}{4\lambda}\right) e^{-2\lambda t} dt = \frac{1}{2\lambda} + \int_0^\infty \frac{3}{2} e^{-2\lambda t} dt = \frac{1}{2\lambda} + \frac{3}{4\lambda} = \frac{5}{4\lambda}. \tag{EC.1}$$

Clearly, given the cost of effort at each instant, the principal has to offer each agent at least $5c/(4\lambda)$ so that each agent's ex-ante expected payoff is non-negative. ■

Proof of Proposition 2: To derive the symmetric pure-strategy Nash equilibrium with full effort, let us fix agent $-i$'s effort $x_{k,l,t}^{-i} = 1$ for all k, l, and t and find conditions under which agent i optimally chooses $x_{k,l,t}^{i} = 1$ for all k, l, and t. For notational simplicity, we drop the superscript i. Consider the state of the game where both agents have already achieved one success, using (3) we can write:

$$V_{1,1,t} = \max_{x_{1,1,\tau}} \int_t^\infty x_{1,1,\tau}(\lambda R - c) e^{-\int_t^\tau \lambda(x_{1,1,s}+1)ds} d\tau.$$

The agent's problem is an infinite horizon problem, so it is stationary. Thus, we can drop the subscript t and write the equivalent Bellman equation for the agent's problem as follows:

$$V_{1,1} = \max_{x_{1,1}} \left\{ x_{1,1}(\lambda R - c)dt + (1 - \lambda x_{1,1} dt - \lambda dt) V_{1,1} \right\},$$

Note that $-cx_{1,1}dt$ denotes the agent's cost of effort within the time interval $(t, t+dt)$, while $x_{1,1}\lambda dt$ denotes the probability that a success arrives within $(t, t+dt)$, in which case the agent receives R. On the other hand, the probability that her opponent achieves the second success in that time interval is λdt, and in this case, the agent receives zero reward. With probability $(1 - \lambda x_{1,1} dt - \lambda dt)$, neither the agent nor her opponent achieves the second success, in which case the contest continues and the agent anticipates to receive a continuation payoff of $V_{1,1}$ due to stationarity. Simplifying the above expression and dividing both sides by dt, we obtain the following Hamilton-Jacobi- Bellman (hereafter HJB) equation for the agent's problem:

$$0 = \max_{x_{1,1}} \left\{ \underbrace{-cx_{1,1}}_{cost} + \underbrace{\lambda x_{1,1}\left(R - V_{1,1}\right)}_{benefit} - \underbrace{\lambda V_{1,1}}_{externality} \right\}. \tag{EC.2}$$

First, second, and third terms reflect the agent's flow cost of effort, her flow benefit from effort, and the externality imposed by her opponent's effort, respectively. Since the HJB in (EC.2) is linear in $x_{1,1}$, it can be concluded that $x_{1,1} = 1$ is optimal if and only if

$$R - V_{1,1} \geq \frac{c}{\lambda}. \tag{EC.3}$$

The above condition implies that each agent finds it optimal to work if the principal rewards the agent with an additional utility of at least c/λ upon the arrival of a success.

Next, consider the state of the game where agent i is the leader with one success and agent $-i$ is the laggard with no success. Bellman and HJB equations for agent i can be expressed as follows:

$$\begin{aligned} V_{1,0} &= \max_{x_{1,0}} \left\{x_{1,0}(\lambda R - c)dt + \lambda V_{1,1}dt + (1 - \lambda x_{1,0}dt - \lambda dt)V_{1,0}\right\} \\ &\Rightarrow 0 = \max_{x_{1,0}} \left\{x_{1,0}\left(\lambda R - c - \lambda V_{1,0}\right) + \lambda\left(V_{1,1} - V_{1,0}\right)\right\}, \end{aligned} \tag{EC.4}$$

The first line admits a similar interpretation as in the previous case, except that if the laggard (agent $-i$) obtains a success, the leader agent i receives a continuation payoff equal to $V_{1,1}$. From (EC.4), we can derive the Incentive Compatibility (hereafter IC) constraint for agent i which tells us that $x_{1,0} = 1$ is incentive compatible if and only if

$$R - V_{1,0} \geq \frac{c}{\lambda}. \tag{EC.5}$$

When agent i is the laggard with no success and agent $-i$ is the leader with one success, we can rewrite agent i's problem in (4) as follows:

$$\begin{aligned} V_{0,1} &= \max_{x_{0,1}} \left\{x_{0,1}(\lambda V_{1,1} - c)dt + (1 - \lambda x_{0,1}dt - \lambda dt)V_{0,1}\right\} \\ &\Rightarrow 0 = \max_{x_{0,1}} \left\{x_{0,1}\left(\lambda V_{1,1} - c - \lambda V_{0,1}\right) - \lambda V_{0,1}\right\}, \end{aligned} \tag{EC.6}$$

which implies that exerting full effort for the laggard is optimal if and only if the following IC constraint holds:

$$V_{1,1} - V_{0,1} \geq \frac{c}{\lambda}. \tag{EC.7}$$

Finally, before the arrival of any success, the continuation payoff of agent i is given by:

$$\begin{aligned} V_{0,0} &= \max_{x_{0,0}} \left\{x_{0,0}(\lambda V_{1,0} - c)dt + \lambda V_{0,1}dt + (1 - \lambda x_{0,0}dt - \lambda dt)V_{0,0}\right\} \\ &\Rightarrow 0 = \max_{x_{0,0}} \left\{x_{0,0}\left(\lambda V_{1,0} - c - \lambda V_{0,0}\right) + \lambda\left(V_{0,1} - V_{0,0}\right)\right\}. \end{aligned} \tag{EC.8}$$

From (EC.8), exerting $x_{0,0}=1$ is incentive compatible for agent i if and only if

$$V_{1,0}-V_{0,0}\geq \frac{c}{\lambda}. \tag{EC.9}$$

We are now ready to show that $R=3c/\lambda$ is the minimum required fixed reward to induce agent i (and by symmetry agent $-i$ as well) to exert full effort at all times, hence achieving the minimum expected lead time $\underline{T}$. From (EC.7), $V_{1,1}\geq c/\lambda$ since $V_{0,1}$ has to be non-negative. Also, from (EC.2), under full effort, one can verify that $V_{1,1}=\frac{1}{2}\left(R-\frac{c}{\lambda}\right)$. Combining these together, we require $\frac{1}{2}\left(R-\frac{c}{\lambda}\right)\geq\frac{c}{\lambda}$, which boils down to $R\geq 3c/\lambda$. Thus, we need $R=3c/\lambda$ at the minimum to ensure that (EC.7) is satisfied, and hence it is incentive compatible for the laggard to exert full effort.

It remains to show that $R=3c/\lambda$ satisfies all IC constraints. It is straightforward to check that (EC.3) is satisfied, that is $R-V_{1,1}=\frac{3c}{\lambda}-\frac{c}{\lambda}>\frac{c}{\lambda}$. Plugging in the value of $V_{1,1}=c/\lambda$ into (EC.6), we find that $V_{0,1}=0$ and so the IC constraint in (EC.7) for the laggard is binding. Similarly, plugging in the value of $V_{1,1}=c/\lambda$ into (EC.4), it can be concluded that $V_{1,0}=3c/(2\lambda)$ and so the IC constraint in (EC.5) for the leader is satisfied since $R-V_{1,0}=\frac{3c}{\lambda}-\frac{3c}{2\lambda}=\frac{3c}{2\lambda}>\frac{c}{\lambda}$. Finally, plugging in the values of $V_{1,0}=3c/(2\lambda)$ and $V_{0,1}=0$ into (EC.8), one can verify that $V_{0,0}=c/(4\lambda)$ and so the IC constraint in (EC.9) for each agent is satisfied as $V_{1,0}-V_{0,0}=\frac{3c}{2\lambda}-\frac{c}{4\lambda}=\frac{5c}{4\lambda}>\frac{c}{\lambda}$. ■

Proof of Proposition 3: Consider a flexible-reward contest with $R_{2,0}=2c/\lambda$ and $R_{2,1}=3c/\lambda$ where the principal commits to disclose any success upon its arrival. Similar to the previous case, we analyze the problem by moving backward on the state of the game where the states are defined by the number of successes of the agents. Let us fix agent $-i$'s effort $x^{-i}_{k,l,t}=1$ for all k, l, and t and find conditions under which agent i optimally chooses $x^{i}_{k,l,t}=1$ for all k, l, and t. Consider the state of the game where both agents have already achieved one success. The Bellman equation and the corresponding HJB for agent i's problem can be expressed as follows:

$$\begin{aligned}V_{1,1}&=\max_{x_{1,1}}\{x_{1,1}(\lambda R_{2,1}-c)dt+(1-\lambda x_{1,1}dt-\lambda dt)V_{1,1}\}\\ \Rightarrow 0&=\max_{x_{1,1}}\left\{x_{1,1}\left(\lambda R_{2,1}-c-\lambda V_{1,1}\right)-\lambda V_{1,1}\right\},\end{aligned} \tag{EC.10}$$

where we use the fact that the winner receives $R_{2,1}$ in this state of the game. From (EC.10), we can derive that $x_{1,1}=1$ is optimal if and only if

$$R_{2,1}-V_{1,1}\geq\frac{c}{\lambda}. \tag{EC.11}$$

Next, consider the state of the game with a leader and a laggard. The Bellman equation and the corresponding HJB for the leader's problem (which we assume to be agent i) can be written as:

$$\begin{aligned}V_{1,0}&=\max_{x_{1,0}}\{x_{1,0}(\lambda R_{2,0}-c)dt+\lambda V_{1,1}dt+(1-\lambda x_{1,0}dt-\lambda dt)V_{1,0}\}\\ \Rightarrow 0&=\max_{x_{1,0}}\left\{x_{1,0}\left(\lambda R_{2,0}-c-\lambda V_{1,0}\right)+\lambda\left(V_{1,1}-V_{1,0}\right)\right\},\end{aligned} \tag{EC.12}$$

where we use the fact that the winner receives $R_{2,0}$ in this state of the game. From (EC.12), we can derive the IC constraint for the leader which tells us that $x_{1,0}=1$ is incentive compatible if and only if

$$R_{2,0}-V_{1,0}\geq \frac{c}{\lambda}. \tag{EC.13}$$

Similarly, we can express the Bellman equation and the corresponding HJB for the laggard's problem (assuming to be agent i) as follows:

$$\begin{aligned} V_{0,1} &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1}-c)dt+(1-\lambda x_{0,1}dt-\lambda dt)V_{0,1}\} \\ \Rightarrow 0 &= \max_{x_{0,1}} \{x_{0,1}\left(\lambda V_{1,1}-c-\lambda V_{0,1}\right)-\lambda V_{0,1}\}, \end{aligned} \tag{EC.14}$$

which implies that exerting full effort for the laggard is optimal if and only if the following IC constraint holds

$$V_{1,1}-V_{0,1}\geq \frac{c}{\lambda}. \tag{EC.15}$$

Finally, before the arrival of any success, the continuation value of each agent is given by:

$$\begin{aligned} V_{0,0} &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0}-c)dt+\lambda V_{0,1}dt+(1-\lambda x_{0,0}dt-\lambda dt)V_{0,0}\} \\ \Rightarrow 0 &= \max_{x_{0,0}} \{x_{0,0}\left(\lambda V_{1,0}-c-\lambda V_{0,0}\right)+\lambda\left(V_{0,1}-V_{0,0}\right)\}. \end{aligned} \tag{EC.16}$$

From (EC.16), exerting $x_{0,0}=1$ is incentive compatible for each agent if and only if

$$V_{1,0}-V_{0,0}\geq \frac{c}{\lambda}. \tag{EC.17}$$

We now verify that the proposed flexible-reward schedule in Proposition 3 satisfies all of the above IC constraints and spends the minimum first-best expected reward. Given (EC.15), we can see that $V_{1,1}=c/\lambda$ is the minimum required continuation payoff to incentivize the laggard to put full effort. From (EC.10), we know that $V_{1,1}=\frac{1}{2}\left(R_{2,1}-\frac{c}{\lambda}\right)$. Thus, the principal has to specify a reward $R_{2,1}=3c/\lambda$ in order to satisfy $V_{1,1}=c/\lambda$. Given these values, it is straightforward to check that the IC constraint in (EC.11) is satisfied, that is $R_{2,1}-V_{1,1}=\frac{3c}{\lambda}-\frac{c}{\lambda}>\frac{c}{\lambda}$. Also, plugging in the value of $V_{1,1}=c/\lambda$ into (EC.14), we obtain that $V_{0,1}=0$ and so the IC constraint for the laggard is binding. Next, from (EC.17), we can conclude that $V_{1,0}=c/\lambda$ is the minimum required continuation payoff to motivate the agents to exert effort. Plugging in this value into (EC.12), $R_{2,0}=2c/\lambda$ is needed to satisfy the HJB. It follows that the IC constraint in (EC.13) is indeed binding for the leader as $R_{2,0}-V_{1,0}=\frac{2c}{\lambda}-\frac{c}{\lambda}=\frac{c}{\lambda}$. Finally, given $V_{1,0}=c/\lambda$ and $V_{0,1}=0$, we conclude by (EC.16) that $V_{0,0}=0$ which shows that the last IC constraint in (EC.17) is also binding, that is $V_{1,0}-V_{0,0}=\frac{c}{\lambda}-0=\frac{c}{\lambda}$. Therefore, full effort is incentive compatible at all times and $\underline{T}$ can be achieved.

To calculate the expected reward of this flexible-reward contest, note that when both agents have already obtained one success, the expected reward of the contest is $R_{2,1}=3c/\lambda$. When there is a leader and a laggard, the expected reward can be computed as follows:

$$\int_t^{\infty}\lambda\left(\frac{2c}{\lambda}+\frac{3c}{\lambda}\right)e^{-2\lambda(\tau-t)}d\tau=\frac{5c}{2\lambda}.$$

To interpret the above equation note that if the leader obtains her second success, the reward is $R_{2,0} = 2c/\lambda$ and if the laggard obtains her first success, the state of the game transitions to the case where both agents have already obtained one success and the reward is adjusted upward to $R_{2,1} = 3c/\lambda$. Finally, the ex-ante expected reward of the contest is given by:

$$\int_0^\infty 2\lambda\left(\frac{5c}{2\lambda}\right)e^{-2\lambda t}dt = \frac{5c}{2\lambda}.\blacksquare$$

Proof of Proposition 4: First, we derive (5). Note that, by Bayes' rule, the probability that agent i assigns at time $t+dt$ to the event that her opponent has succeeded once, given p_t^i, can be expressed as follows:

$$p_{t+dt}^i = \frac{p_t^i(1-x_{1,t}^{-i}\lambda dt) + (1-p_t^i)x_{0,t}^{-i}\lambda dt}{p_t^i(1-x_{1,t}^{-i}\lambda dt) + 1 - p_t^i},$$

where the numerator is the probability that the game has not ended yet given that the opponent is in the second stage, and the denominator is the total probability that the contest has not finished yet. The law of motion can be obtained by subtracting p_t^i from both sides, dividing by dt, and taking the limit as $dt \to 0$.

To derive the symmetric pure-strategy Nash equilibrium with full effort, we shall fix the opponent's effort $x_{k,t}^{-i} = 1$ for all k and t and try to find conditions under which agent i best-responds by choosing $x_{k,t}^i = 1$ for all k, t.

Consider the problem faced by an agent who has not yet achieved a success. Dropping the superscript i in (7) by using the symmetry of agents, the equivalent Bellman equation for the agent's problem is as follows:

$$V_{0,t} = \max_{x_{0,t}}\left\{-cx_{0,t}dt + x_{0,t}\lambda V_{1,t}dt + (1 - x_{0,t}\lambda dt - p_t\lambda dt)V_{0,t+dt}\right\}. \tag{EC.18}$$

Note that $cx_{0,t}dt$ denotes the agent's cost of effort within the time interval $(t, t+dt)$, while $x_{0,t}\lambda dt$ denotes the probability that a success arrives within $(t, t+dt)$, in which case the agent receives a continuation payoff, $V_{1,t}$. On the other hand, the probability that her opponent is in the second stage and achieves a success in this time interval is $p_t\lambda dt$, and in that case, the agent receives a continuation value of zero. With probability $(1 - x_{0,t}\lambda dt - p_t\lambda dt)$, neither the agent achieves a success, nor does her opponent achieve the second success, in which case the contest continues and the agent anticipates to receive her continuation payoff, $V_{0,t+dt}$. Given that we have an infinite horizon dynamic model with no deadline, from (EC.18) one can verify that the continuation payoff solely depends on the probability p_t rather than time itself. Thus, we can define a stationary Bellman function $V_{k,p}$ for $k \in \{0,1\}$ that does not depend on time but depends on the current state of p_t. Let p be a state variable that corresponds to the probability that each agent assigns to the fact that her opponent is in the second stage under no information disclosure. Then, we can express each agent's continuation payoff as $V_{k,p}$. Thus, we can rewrite (EC.18) as follows:

$$V_{0,p} = \max_{x_{0,p}}\left\{-cx_{0,p}dt + x_{0,p}\lambda V_{1,p}dt + (1 - x_{0,p}\lambda dt - p\lambda dt)V_{0,p+dp}\right\}. \tag{EC.19}$$

Using a Taylor expansion (Ito's Lemma), we have

$$V_{0,p+dp} \simeq V_{0,p} + V_{0,p}^{'} dp = V_{0,p} + \lambda(1-p)^2 V_{0,p}^{'} dt,$$

where we have used that $x_{k,t}^{-i} = 1$ and $dp = \lambda(1-p)^2 dt$ according to (5). Substituting this expression into (EC.19), dropping the terms of the order dt^2 (since $dt^2 \simeq 0$), canceling terms and dividing both sides by dt, we obtain the following HJB equation for the agent's problem:

$$0 = \max_{x_{0,p}} \left\{ \underbrace{-cx_{0,p}}_{cost} + \underbrace{x_{0,p}\lambda\left(V_{1,p} - V_{0,p}\right)}_{benefit} - \underbrace{\lambda\left[pV_{0,p} - (1-p)^2 V_{0,p}'\right]}_{externality} \right\}. \tag{EC.20}$$

Note that the first term reflects the agent's flow cost of effort, the second term reflects her flow benefit from effort, and the third term captures the externality imposed by her opponent's effort. Since the HJB in (EC.20) is linear in $x_{0,p}$, we conclude that $x_{0,p} = 1$ is optimal if and only if

$$V_{1,p} - V_{0,p} \geq \frac{c}{\lambda}. \tag{EC.21}$$

The above IC constraint implies that an agent with no success finds it optimal to work if the principal rewards the agent with additional utility of at least c/λ upon the arrival of a success.

Next, consider the problem faced by an agent who has achieved one success as formulated in (6). Since the continuation payoffs of agents depend on the state variable p rather than time, the principal's problem is also stationary (i.e., independent of t) and hence it is optimal for the principal to choose a reward schedule that depends only on p. In other words, an agent who achieves two successes first is rewarded R_p, where p is her belief about her opponent's progress. As a result, after dropping the superscript i in (6) by using the symmetry of agents, the corresponding Bellman equation for the agent's problem is given by:

$$V_{1,p} = \max_{x_{1,p}} \{-cx_{1,p}dt + x_{1,p}\lambda R_p dt + (1 - x_{1,p}\lambda dt - p\lambda dt)V_{1,p+dp}\}$$

which using the previous techniques gives us the following HJB eqution

$$0 = \max_{x_{1,p}} \left\{ \underbrace{-cx_{1,p}}_{cost} + \underbrace{x_{1,p}\lambda\left(R_p - V_{1,p}\right)}_{benefit} - \underbrace{\lambda\left[pV_{1,p} - (1-p)^2 V_{1,p}'\right]}_{externality} \right\}. \tag{EC.22}$$

Since the HJB in (EC.22) is linear in $x_{1,p}$, we conclude that $x_{1,p} = 1$ is optimal if and only if

$$R_p - V_{1,p} \geq \frac{c}{\lambda}. \tag{EC.23}$$

We are now ready to prove that full effort is incentive compatible at all times given the proposed flexible-reward schedule in Proposition 4. First, notice that when we fix the opponent's effort $x_{k,t}^{-i} = 1$ for all t and solve (5) with initial condition $p_0^i = 0$, we obtain $p_t = \lambda t/(1+\lambda t)$ as stated in the proposition. Second, note that if an agent with no success receives a continuation payoff $V_{1,p} = c/\lambda$, $\forall p$, by substituting this value into the integral form of the agent's problem in (7), we obtain $V_{0,p} = 0$. Hence, (EC.21) is always binding. Moreover, if $V_{1,p} = c/\lambda$, the flexible-reward schedule $R_p = (2+p)c/\lambda$ always satisfies (EC.23). Plugging in $R_p = (2+p)c/\lambda$ into (EC.22), it can be verified that $V_{1,p} = c/\lambda$ for all p is a solution. Finally, plugging in $V_{1,p} = c/\lambda$ into (EC.20), one can verify that $V_{0,p} = 0$ for all p is a solution. Therefore, the design is incentive compatible at all times and achieves $\underline{T}$.

Finally, we can verify that this design spends the first-best expected reward. To show this, we compute the expected reward that the principal has to pay under this design. Denote by $R_{k,l}$ the principal's expected payout conditional on the first agent having achieved $k \in \{0,1\}$ successes, and the second agent having achieved $l \in \{0,1\}$ successes. Let us consider the state of the game when both agents have already achieved one success, then the expected payout is given by:

$$R_{1,1,t} = \int_t^\infty 2\lambda \left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) \frac{c}{\lambda} e^{-2\lambda(\tau-t)} d\tau = \frac{3c}{\lambda} - \frac{2c}{\lambda} e^{2(1+\lambda t)} \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx,$$

where the first equality can be interpreted as follows: if any agent obtains the second success during interval $(\tau, \tau + d\tau)$ which happens with probability $2\lambda d\tau$, the principal has to pay $(2 + p_\tau)c/\lambda$ to the winner, provided that none of the agents have already obtained the second success by time τ which is captured by the term $e^{-2\lambda(\tau-t)}$, and the second equality is obtained by change of variables.

Next, consider the state of the game with a leader and a laggard. Then the expected payout can be computed as follows:

$$\begin{aligned}
R_{1,0,t} &= \int_t^\infty \lambda \left[\left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) \frac{c}{\lambda} + R_{1,1,\tau}\right] e^{-2\lambda(\tau-t)} d\tau \\
&= \int_t^\infty \left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) c e^{-2\lambda(\tau-t)} d\tau + \int_t^\infty \lambda R_{1,1,\tau} e^{-2\lambda(\tau-t)} d\tau \\
&= \frac{1}{2} R_{1,1,t} + \int_t^\infty \lambda \left[\frac{3c}{\lambda} - \frac{2c}{\lambda} e^{2(1+\lambda\tau)} \int_{2(1+\lambda\tau)}^\infty \frac{e^{-x}}{x} dx\right] e^{-2\lambda(\tau-t)} d\tau \\
&= \frac{3c}{\lambda} - \frac{c}{\lambda} e^{2(1+\lambda t)} \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx - 2c e^{2(1+\lambda t)} \int_t^\infty \int_{2(1+\lambda\tau)}^\infty \frac{e^{-x}}{x} dx d\tau \\
&= \frac{2c}{\lambda} + \frac{c}{\lambda} e^{2(1+\lambda t)} (1 + 2\lambda t) \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx.
\end{aligned}$$

Finally, starting from time zero, the expected reward of the contest is given by:

$$\begin{aligned}
R_{0,0,0} &= \int_0^\infty 2\lambda R_{1,0}(t) e^{-2\lambda t} dt \\
&= \int_0^\infty \left[4c e^{-2\lambda t} + 2c e^2 (1 + 2\lambda t) \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx\right] dt \\
&= \frac{2c}{\lambda} + \frac{c}{2\lambda} = \frac{5c}{2\lambda}. \blacksquare
\end{aligned}$$

Proof of Theorem 1: We first prove that under any information disclosure policy, there exists a flexible-reward contest that attains the absolute minimum expected lead time at the first-best cost if $\overline{R} \geq \frac{3c}{\lambda}$. To see this, note that the principal can achieve this goal by organizing a flexible-reward contest similar to the one in Proposition 3 by committing to pay the winner $R_{2,0} = 2c/\lambda$ when one agent achieves the second success before the other agent obtaining any success and $R_{2,1} = 3c/\lambda$ if the second success is obtained when both agents have already succeeded once. Since the principal has commitment power and observes successes, conditioning the reward schedule on the state of the contest in which it ends under any information disclosure policy is feasible. To verify that this reward schedule induces both agents to spend full effort at all times under any information disclosure policy, we fix agent $-i$'s effort to 1 at all times, and prove that agent i best-responds by playing the same strategy. Notice that an agent i with one success holding a belief p about her rival's partial progress finds it optimal to spend full effort if and only if $[pR_{2,1}+(1-p)R_{2,0}]-[pV_{1,1}+(1-p)V_{1,0}]\geq c/\lambda$ (similar to condition (EC.23)), where $V_{1,1}=\frac{1}{2}(R_{2,1}-\frac{c}{\lambda})$ and $V_{1,0}=\frac{1}{2}(R_{2,0}-\frac{c}{\lambda})+\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})$ if she spends full effort in equilibrium. Under this reward schedule, we obtain $V_{1,1}=V_{1,0}=\frac{c}{\lambda}$ and hence the incentive compatibility condition for this agent is indeed slack, implying that full effort is incentive compatible at all times. Next, consider an agent i with no success holding a belief p about her rival's progress. Spending full effort for agent i is optimal if and only if $V_{1,p}-V_{0,p}\geq c/\lambda$ (similar to condition (EC.21)), where $V_{1,p}=pV_{1,1}+(1-p)V_{1,0}=\frac{c}{\lambda}$ under this flexible-reward schedule. In addition, $V_{0,p}=pV_{0,1}+(1-p)V_{0,0}=0$ given that $V_{0,1}=\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})-\frac{c}{2\lambda}=0$ and $V_{0,0}=\frac{1}{4}(R_{2,0}-\frac{c}{\lambda})+\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})-\frac{3c}{4\lambda}=0$ under full effort provision and this reward schedule. Thus, the incentive compatibility condition for this agent is binding. Putting these together, in equilibrium, both agents exert full effort at all times which minimizes the contest's expected lead time. Moreover, this contest spends the first-best expected reward as the principal pays $2c/\lambda$ or $3c/\lambda$ each with probability $1/2$ in this design.

Next, we prove that given an information disclosure policy, if there exists a first-best (flexible-reward) contest that attains the absolute minimum expected lead time $\underline{T}$ at the minimum cost $\underline{R}$, we must have that $\overline{R}\geq\frac{3c}{\lambda}$. Suppose not, that is $\overline{R}<\frac{3c}{\lambda}$, and there exists a lead-time minimizing first-best contest which we denote by $\mathbb{C}$. Start with the observation that agents should always exert full effort in $\mathbb{C}$ because otherwise it is not possible to achieve the absolute minimum expected lead time $\underline{T}$. We first claim that if a contest achieves the absolute minimum expected lead time $\underline{T}$ by paying the first-best reward, it must be the case that, in equilibrium, each agent's ex-ante expected payoff is zero. To see this, note that the sum of agents' surplus in every first-best contest is

$$V^{i}_{0,0,0}+V^{-i}_{0,0,0}=\mathbb{E}\left[R_T.\mathbb{1}_{\{i\ or\ -i\ wins\}}\right]-2c\underline{T}$$
$$\Leftrightarrow\mathbb{E}\left[R_T.\mathbb{1}_{\{i\ or\ -i\ wins\}}\right]=V^{i}_{0,0,0}+V^{-i}_{0,0,0}+2c\underline{T},\qquad\text{(EC.24)}$$

where T is the random termination time of the contest. Notice that the left-hand side in (EC.24) admits its minimum value $\underline{R}$ (i.e., the first-best expected reward) if and only if the right-hand side admits its lower bound which implies that each agent, in contest $\mathbb{C}$, must earn zero ex-ante expected utility. Following this observation, consider an agent i with no success holding a belief p_t at time $t>0$ about her rival's partial progress. We claim that $0<p_t<1$. If not, then this means that agent i receives full information about her rival's partial progress at time t. But then if the opponent obtains a success by t, agent i quits immediately, contradicting first-best assumption, because her continuation payoff under the full effort provision in the first-best contest would be

$$V_{0,1,t} = \int_t^{\infty} (\lambda V_{1,1,\tau} - c) e^{-2\lambda(\tau - t)} d\tau < 0. \tag{EC.25}$$

The inequality above follows because $\overline{R} < \frac{3c}{\lambda}$ and hence $V_{1,1,\tau} = \frac{1}{2}\mathbb{E}[R_T] - \frac{c}{2\lambda} < \frac{1}{2} \times \frac{3c}{\lambda} - \frac{c}{2\lambda} = \frac{c}{\lambda}$ owing to the fact that each agent wins the expected reward with equal probability and the second success is arrived after $\frac{1}{2\lambda}$ periods of time, on average, which costs each agent $\frac{c}{2\lambda}$.

Let $V_{0,t}$ be agent i's continuation payoff under contest $\mathbb{C}$ at time t (where she holds a belief p_t about her rival's progress). We next prove that $V_{0,t} = 0$ for all $t > 0$. Recall from (EC.24) that $V_{0,0,0} = 0$. If $V_{0,t} > 0$ at some t, there is a profitable deviation for agent i where she exerts no effort until time t and then starts exerting full effort and receives a strictly positive expected utility than the equilibrium under $\mathbb{C}$, which is a contradiction. Similarly, we can argue that in contest $\mathbb{C}$ and at any t, $V_{1,t} = \frac{c}{\lambda}$. To see this note that $\frac{c}{\lambda}$ is the minimum necessary continuation payoff to induce an agent with no success to work. Now suppose there is an interval $(t', t' + dt)$ during which $\mathbb{E}[V_{1,t}] > \frac{c}{\lambda}$. Then, agent i with no success can again deviate and earn strictly positive surplus by exerting full effort only during this interval and shirking at all other times (if the agent succeeds, she earns $\lambda\mathbb{E}[V_{1,t}] - c > 0$). Hence, a contradiction. Thus, in contest $\mathbb{C}$, we have $V_{0,t} = 0$ and $V_{1,t} = \frac{c}{\lambda}$ for all t.

Finally, under full effort provision in contest $\mathbb{C}$ we can write

$$V_{0,t} = (\lambda V_{1,t} - c)dt + \lambda(p_t \times 0 + (1 - p_t) \times V_{0,1,t})dt + (1 - 2\lambda dt)V_{0,t+dt}.$$

To understand the above expected continuation payoff for an agent i with no success, note that if agent i exerts full effort during $(t, t + dt)$, she receives $(\lambda V_{1,t} - c)dt$ and if her opponent exerts full effort and obtains a success, the contest ends if this is her second success or agent i receives $V_{0,1,t}$ if this is her first success. Otherwise, the contest continues. Plugging in $V_{0,t} = V_{0,t+dt} = 0$ and $V_{1,t} = \frac{c}{\lambda}$ in the above equation, we obtain $V_{0,1,t} = 0$ which cannot be true by (EC.25). Thus, we have a contradiction, and a first-best contest $\mathbb{C}$ cannot exist if $\overline{R} < \frac{3c}{\lambda}$. ■

Proof of Proposition 5: We prove the proposition in multiple steps.

Step 1: *We first verify that the strategy of the agents in the proposition forms a symmetric equilibrium.*

To check this, we fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy. First, using condition (EC.23), it is easy to verify that exerting full effort is incentive compatible for an agent with one success for all p since $V_{1,p}$ can not exceed $\overline{R} - c/\lambda$. Given that agent $-i$ exerts effort $x_{0,t}^{-i} = p_r$ for $t \geq t_r$, and $x_{1,t}^{-i} = 1$, by (5) we obtain $\dot{p}_t^i = 0$. As a result, $p_t^i = p_r$ for $t \geq t_r$. Following this observation, note that if agent i with no success, holding a belief p_r, receives a continuation payoff $V_{1,p_r} = c/\lambda$, by substituting this value into the integral form of the agent's problem in (7), we get $V_{0,p_r} = 0$. Hence, the incentive compatibility condition in (EC.21) is binding for all $t \geq t_r$ implying that agent i is indifferent between any level of effort and so exerting $x_{0,p_r} = p_r$ is optimal. Plugging in $\overline{R} = (2+p_r)c/\lambda$ into (EC.22), it can be verified that $V_{1,p} = c/\lambda$ is a solution for all $t \geq t_r$ where $p_t = p_r$. Finally, to prove that exerting full effort is optimal for agent i with no success for all $p < p_r$, we move backward from time t_r associated with belief p_r and prove that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen by the following analysis:

$$V_{1,p} - V_{0,p} =$$
$$-cdt + \lambda\overline{R}dt + (1-\lambda dt - p\lambda dt)V_{1,p+dp} + cdt - \lambda V_{1,p}dt - (1-\lambda dt - p\lambda dt)V_{0,p+dp} \geq$$
$$\lambda\overline{R}dt + (1-\lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V_{1,p}dt \geq \frac{c}{\lambda},$$

where the last inequality results from the fact that

$$V_{1,p} = \int_t^\infty (\lambda\overline{R} - c)e^{-\int_t^\tau \lambda(1+p_s)ds}d\tau \leq \int_t^\infty (\lambda\overline{R} - c)e^{-\lambda(1+p)(\tau-t)}d\tau$$
$$= \frac{\lambda\overline{R} - c}{\lambda(1+p)} \leq \overline{R} - (1+p)\frac{c}{\lambda},$$

where the first line results from the fact that p_t is weakly increasing and the second line holds if and only if $(2+p)c/\lambda \leq \overline{R}$ which is satisfied for $p \leq p_r$. This verifies the equilibrium. Next, we prove the uniqueness of the symmetric equilibrium.

Step 2: *Let p_r solve $(2+p_r)c/\lambda = \overline{R}$. Under no information disclosure, there is no symmetric equilibrium in which an agent with no success exerts full effort at some $p > p_r$.*

First note that $x_{0,p} < 1$ for some p. This is because as p approaches 1, $V_{1,p}$ approaches $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda) < c/\lambda$ given the budget constraint, where we use the fact that an agent with one success exerts full effort at all times. Then, suppose t is the first time at which the belief of an agent with no success reaches its maximum level (p_{max}) in a symmetric equilibrium, and $p_{max} > p_r$. Let us focus on a region where the belief is strictly increasing and reaches p_{max} for the first time. If the agent exerts full effort at p_{max}, by (5) p strictly increases which is a contradiction. Therefore, we must have $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} \leq c/\lambda$. This condition implies that exerting zero effort is optimal at belief p_{max}. Then, we consider the following two cases:

- $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} = c/\lambda$: To find the agent's optimal effort at belief $p_{max} - dp$, given that $x_{0,p_{max}} = 0$, we write the following:

$$V_{1,p_{max}} - V_{0,p_{max}} =$$
$$(\lambda\overline{R} - c)dt + (1-\lambda dt - p_{max}\lambda dt)V_{1,p_{max}+dp} - (1 - p_{max}\lambda dt)V_{0,p_{max}+dp} =$$
$$V_{1,p_{max}+dp} - V_{0,p_{max}+dp} + \left[\lambda\overline{R} - c - \lambda V_{1,p_{max}+dp} - p_{max}\lambda\left(V_{1,p_{max}+dp} - V_{0,p_{max}+dp}\right)\right]dt$$
$$= \frac{c}{\lambda} + (\lambda\overline{R} - c - \lambda V_{1,p_{max}+dp} - p_{max}c)dt < \frac{c}{\lambda},$$

where the last inequality results from the fact that $V_{1,p_{max}+dp} \geq c/\lambda$ and $\overline{R} = (2+p_r)c/\lambda < (2+p_{max})c/\lambda$. Thus, an agent with no success exerts zero effort at belief $p_{max} - dp$ and by (5) p decreases which is a contradiction.

- $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} < c/\lambda$: From the previous case, we know that an agent with no success must exert full effort at belief $p_{max} - dp$ which requires $V_{1,p_{max}} - V_{0,p_{max}} \geq c/\lambda$. By continuity of $V_{1,p} - V_{0,p}$, we conclude that $V_{1,p_{max}} - V_{0,p_{max}} = c/\lambda$. Doing the same analysis as before, we find that the agent finds it optimal to put zero effort at belief $p_{max} - 2dp$ which violates the assumption that the agent's belief is strictly increasing in this region.

Step 3: *Let p_r solve $(2+p_r)c/\lambda = \overline{R}$. Under no information disclosure, there is no symmetric equilibrium in which an agent with no success does not exert full effort at some $p < p_r$.*

Suppose τ is the first time that the agent with no success does not exert full effort. Let $t > \tau$ be the first time at which the belief of an agent with no success in the equilibrium reaches its minimum level (p_{min}) and $p_{min} < p_r$. Let us focus on a region where the agent's belief is strictly decreasing and reaches p_{min} for the first time. If the agent exerts zero effort at belief p_{min}, by (5) p strictly decreases which is a contradiction. Therefore, we must have $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} \geq c/\lambda$. This condition implies that exerting full effort is optimal at belief p_{min}. Then we consider two cases:

- $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} = c/\lambda$: To find the agent's optimal effort at belief $p_{min} - dp$, given that $x_{0,p_{min}} = 1$, we can write the following:

$$
\begin{gathered}
V_{1,p_{min}} - V_{0,p_{min}} = \\
(\lambda\overline{R} - c)dt + (1 - \lambda dt - p_{min}\lambda dt)V_{1,p_{min}+dp} - (\lambda V_{1,p_{min}+dp} - c)dt - (1 - \lambda dt - p_{min}\lambda dt)V_{0,p_{min}+dp} = \\
V_{1,p_{min}+dp} - V_{0,p_{min}+dp} + \left[\lambda\overline{R} - \lambda V_{1,p_{min}+dp} - (1+p_{min})\lambda\left(V_{1,p_{min}+dp} - V_{0,p_{min}+dp}\right)\right] dt = \\
\frac{c}{\lambda} + \left[\lambda\overline{R} - \lambda V_{1,p_{min}+dp} - (1+p_{min})\, c\right] dt > \frac{c}{\lambda},
\end{gathered}
$$

  where the last inequality results from the fact that

$$
\begin{aligned}
V_{1,p} = \int_t^\infty (\lambda\overline{R} - c)e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau \leq \int_t^\infty (\lambda\overline{R} - c)e^{-\lambda(1+p_{min})(\tau - t)} d\tau \\
= \frac{\lambda\overline{R} - c}{\lambda(1+p_{min})} \leq \overline{R} - (1+p_{min})\frac{c}{\lambda},
\end{aligned}
$$

  where the first line results from the fact that p_{min} is the minimum belief in the equilibrium and the second line holds since $(2+p_{min})c/\lambda \leq \overline{R}$. Therefore, the agent exerts full effort at belief $p_{min} - dp$ and by (5) p strictly increases which is a contradiction.

- $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} > c/\lambda$: From the previous case, we know that an agent with no success must exert zero effort at belief $p_{min} - dp$ which requires $V_{1,p_{min}} - V_{0,p_{min}} \leq c/\lambda$. By continuity of $V_{1,p} - V_{0,p}$, we conclude that $V_{1,p_{min}} - V_{0,p_{min}} = c/\lambda$. Doing the same analysis as before, we find that the agent finds it optimal to put full effort at belief $p_{min} - 2dp$ which violates the assumption that the agent's belief is strictly decreasing in this region.

From steps 2 and 3, we conclude that the symmetric equilibrium in the proposition is unique. ■

Proof of Proposition 6: We build on the proofs of Propositions 5 and EC.1. As before, let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify the best response of agent i. Consider the very last instant of the first cycle at which the belief of agent i reaches p_r. The IC condition (EC.21) implies that full effort is optimal for an agent with no success if and only if $V_{1,p} - V_{0,p} \geq c/\lambda$. We can rewrite this condition at time t_r associated with belief p_r as follows:

$$p_r V_{1,1} + (1-p_r)V_{1,quit} - (1-p_r)V_{0,0} \geq c/\lambda. \tag{EC.26}$$

The above condition can be interpreted as follows: if agent i obtains her first success at t_r, her expected continuation payoff is given by $p_r V_{1,1} + (1-p_r)V_{1,quit}$ anticipating that the principal discloses full information at the end of the cycle. Therefore, with probability p_r her opponent has already made partial progress which in that case they keep working until the end and the continuation payoff is $V_{1,1}$, or her opponent quits if she has not obtained any success and the continuation payoff is $V_{1,quit}$. On the other hand, if agent i does not succeed at t_r, she quits if her opponent has progressed to the second stage. Otherwise, the contest and the beliefs reset and a new cycle begins with a continuation payoff of $V_{0,0}$.

Given that an agent with one success always puts full effort until the end, we know $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ and $V_{1,quit} = \overline{R} - c/\lambda$. Moreover, the upper bound for $V_{0,0}$ is given by $V_{0,0}^F$, where F stands for full information, which is the continuation payoff if full information is provided during each cycle. To see this, suppose that full information is provided during each cycle. We consider two cases: i) if agent i obtains the first success, her opponent immediately quits. This leads to a higher continuation payoff than the case of silent period where the opponent keeps working until the end of the cycle; ii) if agent i's opponent obtains the first success, agent i's best response is to quit. However, in a silent period, agent i earns a negative ex-post payoff. Therefore, the upper bound for $V_{0,0}$ is given by $V_{0,0}^F = \frac{1}{2}(\overline{R} - 2c/\lambda)$. Plugging in these values into (EC.26), it is easy to verify that the condition is binding implying that full effort is optimal.

Finally, to prove that exerting full effort is optimal for agent i with no success during each cycle where $p < p_r$, we can show that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen by the following analysis:

$$V_{1,p} - V_{0,p} =$$
$$\int_t^{t_r} (\lambda\overline{R} - c)e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau + [p_r V_{1,1} + (1-p_r)V_{1,quit}]\, e^{-\int_t^{t_r} \lambda(1+p_s)ds}$$
$$- \int_t^{t_r} (\lambda V_{1,\tau} - c)e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau - (1-p_r)V_{0,0}e^{-\int_t^{t_r} \lambda(1+p_s)ds}$$

$$= \int_t^{t_r} \lambda \left(\overline{R} - V_{1,\tau}\right) e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau + [p_r V_{1,1} + (1-p_r) V_{1,quit} - (1-p_r) V_{0,0}] \, e^{-\int_t^{t_r} \lambda(1+p_s)ds} \geq \frac{c}{\lambda},$$

where the last inequality can be verified after plugging in the values of $V_{1,p}$, $V_{1,1}$, $V_{1,quit}$, and $V_{0,0}$ into the above expression, computing the above integral and some tedious algebra. ■

Proof of Proposition 7: We already show that an agent with one success finds it optimal to put full effort if and only if

$$\overline{R} - V_{1,p} \geq \frac{c}{\lambda}, \tag{EC.27}$$

which always holds as $V_{1,p} \leq \overline{R} - c/\lambda$ for all p. Next, consider the continuation payoff of an agent i with no success from any time t (after the silent period) onward as follows:

$$V_{0,t}^i = \max_{x_{0,\tau}^i} \int_t^\infty x_{0,\tau}^i (\lambda V_{1,\tau}^i - c) e^{-\int_t^\tau [x_{0,s}^i \lambda + p_s^i x_{1,s}^{-i} \lambda + p_s^i \gamma] ds} d\tau, \tag{EC.28}$$

where by choosing effort $x_{0,\tau}^i$ during interval $(\tau, \tau + d\tau)$, the agent incurs a cost $c x_{0,\tau}^i d\tau$ and if a success arrives, she enters the second stage and enjoys a continuation payoff of $V_{1,\tau}^i$. If her opponent completes the task during interval $(\tau, \tau + d\tau)$, agent i receives zero reward. Moreover, if the principal discloses partial progress of agent i's opponent, agent i quits and receives zero utility because her continuation payoff upon the arrival of her first success falls below c/λ. To see the evolution of belief in (8) note that, by Bayes' rule, the probability that agent i assigns at time $t + dt$ to the event that her opponent has succeeded once, given p_t^i, can be expressed as follows:

$$p_{t+dt}^i = \frac{p_t^i (1 - x_{1,t}^{-i} \lambda dt - \gamma dt) + (1 - p_t^i) x_{0,t}^{-i} \lambda dt}{p_t^i (1 - x_{1,t}^{-i} \lambda dt - \gamma dt) + 1 - p_t^i},$$

where the numerator is the probability that the game has not ended yet and no information is received given that the opponent is in the second stage, and the denominator is the total probability that the contest has not finished yet and no information is disclosed. The law of motion can be obtained by subtracting p_t^i from both sides, dividing by dt, and taking the limit as $dt \to 0$.

Let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy if $t \geq t_r$. Using p as the state variable, consider the Bellman equation for the maximization problem of agent i with no success as follows:

$$V_{0,p} = \max_{x_{0,p}} \{ -c x_{0,p} dt + x_{0,p} \lambda V_{1,p} dt + [1 - x_{0,p} \lambda dt - p \lambda dt - p \gamma dt] V_{0,p+dp} \}.$$

Using the same techniques as before, we can derive the following HJB equation:

$$0 = \max_{x_{0,p}} \{ -c x_{0,p} + x_{0,p} \lambda (V_{1,p} - V_{0,p}) - p \lambda V_{0,p} - p \gamma V_{0,p} + (1-p)[\lambda - p\lambda - p\gamma] V_{0,p}' \}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if

$$V_{1,p} - V_{0,p} \geq c/\lambda, \tag{EC.29}$$

which is similar to (EC.21). To derive the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success, we can write:

$$V_{1,p} = p V_{1,1} + (1-p) V_{1,0}. \tag{EC.30}$$

where $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and $V_{1,0}$ is the expected continuation payoff if the opponent has not progressed to the second stage. Given the value of $\gamma = \lambda(1-p_r)/p_r$ under PSD, by (8) we obtain $p_t = p_r$ remains constant after the initial silent period. Therefore, at any threshold belief p_r, we have:

$$V_{1,0} = \int_t^\infty \left[(\lambda\overline{R} - c) + \lambda\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + \gamma\left(\overline{R} - \frac{c}{\lambda}\right)\right] e^{-(2\lambda+\gamma)(\tau - t)} d\tau,$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns in expectation $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$, or partial progress may be disclosed, in that case the leader gets $(\overline{R} - c/\lambda)$. Taking the above integral, we obtain:

$$V_{1,0} = \frac{3\lambda + 2\gamma}{2(2\lambda+\gamma)}\left(\overline{R} - \frac{c}{\lambda}\right). \tag{EC.31}$$

Therefore, to verify that (EC.29) holds at any $t \geq t_r$, it is enough to verify this condition at the threshold belief p_r as follows:

$$V_{1,p_r} = p_r V_{1,1} + (1-p_r)V_{1,0} = p_r\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + (1-p_r)\frac{3\lambda+2\gamma}{2(2\lambda+\gamma)}\left(\overline{R} - \frac{c}{\lambda}\right) = \frac{c}{\lambda}, \tag{EC.32}$$

where the last equality results from substituting $\gamma = \lambda(1-p_r)/p_r$ and $\overline{R} = (2+p_r)c/\lambda$. Also using (*EC*.28), we obtain $V_{0,p_r} = 0$ for $t \geq t_r$ and hence spending full effort is incentive compatible for all $t \geq t_r$. It remains to show that full effort is incentive compatible for an agent with no success for $t < t_r$. To prove this, we can move backward in time to show that if an agent with no success finds it optimal to spend full effort at any belief p' where $p \leq p' \leq p_r$, then exerting full effort is optimal at belief $p - dp$. This can be seen from the following:

$$\begin{aligned} V_{1,p} - V_{0,p} &= -cdt + \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)V_{1,p+dp} + cdt - \lambda V_{1,p}dt - (1 - \lambda dt - p\lambda dt)V_{0,p+dp} \\ &\geq \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V_{1,p}dt \geq \frac{c}{\lambda}. \end{aligned}$$

To show the last inequality, we need to show that

$$V_{1,p<p_r} \leq \overline{R} - (1+p)\frac{c}{\lambda}.$$

We prove this in two steps. First, we prove that

$$\begin{aligned} & V_{1,p<p_r} \leq \frac{1}{1+p}\left(\overline{R} - \frac{c}{\lambda}\right) \\ \Leftrightarrow V_{1,p} = pV_{1,1} + (1-p)V_{1,0} &= p\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r - t)}\right)\left(\overline{R} - \frac{c}{\lambda}\right) \\ &\leq \frac{1}{1+p}\left(\overline{R} - \frac{c}{\lambda}\right) \Leftrightarrow p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r-t)}\right) \leq \frac{1}{1+p}. \end{aligned}$$

To show the last inequality, we can drop the term $e^{-2\lambda(t_r - t)}$ and substitute for γ to see that

$$\begin{aligned} p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r-t)}\right) &\leq p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\lambda(1-p_r)/p_r}{4(2\lambda + \lambda(1-p_r)/p_r)}\right) \\ &\leq p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\lambda(1-p)/p}{4(2\lambda+\lambda(1-p)/p)}\right) = \frac{1}{1+p}. \end{aligned}$$

In the second step, we prove that

$$\frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)\leq \overline{R}-(1+p)\frac{c}{\lambda}$$

which holds if and only if $(2+p)c/\lambda\leq \overline{R}$ which is satisfied for $p\leq p_r$. Therefore, an agent with no success puts full effort until she succeeds, or the contest ends, or partial progress is disclosed. ■

Proof of Proposition 8: We prove the theorem in multiple steps. To gain insights for why our proposed PSD improves upon other canonical disclosure policies, we prove a more general result. Suppose the principal commits to disclose information about any partial progress at constant rate $\lambda(x_0-p_r)/p_r$ after t_r so that in equilibrium an agent with no success reduces her effort to $x_0\geq p_r$ for all $t\geq t_r$. Notice that no information disclosure is a special case with $x_0=p_r$ and $\gamma=0$, and PSD is a special case with $x_0=1$ and $\gamma\equiv\lambda(1-p_r)/p_r$ for all $t\geq t_r$.

Step 1: *We calculate the expected lead time of the contest under PSD.*

Denote by $T_{k,l,t}$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward. Let us consider the state of the game when both agents have already obtained one success. Then the expected arrival time for the second success is given by:

$$T_{1,1,t}=\int_t^{\infty}2\lambda(\tau-t)e^{-2\lambda(\tau-t)}d\tau=\frac{1}{2\lambda}.$$

Here, information disclosure does not affect the outcome since both agents exert full effort until the end. Next, consider the state of the game with a leader (an agent with one success) and a laggard (an agent with no success). Then, the expected lead time of the contest from any time $t\geq t_r$ can be expressed as follows:

$$\begin{aligned}T_{1,0,t\geq t_r}&=\int_t^{\infty}\left[\lambda(\tau-t)+x_0\lambda\left(\tau-t+\frac{1}{2\lambda}\right)+\frac{\lambda(x_0-p_r)}{p_r}\left(\tau-t+T_{1,quit,\tau}\right)\right]e^{-\left(\lambda+x_0\lambda+\frac{\lambda(x_0-p_r)}{p_r}\right)(\tau-t)}d\tau\\&=\frac{2+p_r}{2\lambda\left(1+p_r\right)},\end{aligned}\tag{EC.33}$$

where $T_{1,quit,\tau}$ is the expected arrival time for the second success once the principal discloses that the leader has made partial progress and the laggard quits, namely,

$$T_{1,quit,t}=\int_t^{\infty}\lambda(\tau-t)e^{-\lambda(\tau-t)}d\tau=\frac{1}{\lambda}.$$

(EC.33) can be interpreted as follows: conditional on reaching to any instant τ, the leader exerts full effort and if she succeeds the contest ends at $\tau-t$, or the laggard who is putting x_0 effort may achieve her first success and in that case the contest's expected lead time is $\tau-t+1/(2\lambda)$, or information may be disclosed by the principal and in that case the laggard quits and the contest ends by the leader at $\tau-t+1/\lambda$ in expectation. Interestingly, $T_{1,0,t\geq t_r}$ is independent of x_0. Next, for any $t<t_r$, the expected lead time is given by:

$$T_{1,0,t<t_r}=\int_t^{t_r}\left[\lambda(\tau-t)+\lambda\left(\tau-t+\frac{1}{2\lambda}\right)\right]e^{-2\lambda(\tau-t)}d\tau+\left(t_r-t+\frac{2+p_r}{2\lambda\left(1+p_r\right)}\right)e^{-2\lambda(t_r-t)}$$

$$= \frac{3}{4\lambda} + \frac{1-p_r}{4\lambda(1+p_r)} e^{-2\lambda(t_r - t)}, \tag{EC.34}$$

where we use the fact that no information is disclosed by the principal before t_r. Finally, the ex-ante expected lead time of the contest for any $t \geq t_r$ can be expressed as follows:

$$T_{0,0,t\geq t_r} = \int_t^\infty 2x_0\lambda \left(\tau - t + T_{1,0,\tau\geq t_r}\right) e^{-2x_0\lambda(\tau - t)} d\tau$$

$$= \int_t^\infty 2x_0\lambda \left[\tau - t + \frac{2+p_r}{2\lambda(1+p_r)}\right] e^{-2x_0\lambda(\tau-t)} d\tau = \frac{1+p_r+x_0(2+p_r)}{2x_0\lambda(1+p_r)}, \tag{EC.35}$$

where we use that an agent with no success exerts effort x_0 after t_r, and for any $t < t_r$ is given by:

$$T_{0,0,t<t_r} = \int_t^{t_r} 2\lambda \left(\tau - t + T_{1,0,\tau<t_r}\right) e^{-2\lambda(\tau-t)} d\tau + \left(t_r - t + T_{0,0,t_r}\right) e^{-2\lambda(t_r-t)}$$

$$= \int_t^{t_r} 2\lambda \left[\tau - t + \frac{3}{4\lambda} + \frac{1-p_r}{4\lambda(1+p_r)} e^{-2\lambda(t_r-\tau)}\right] e^{-2\lambda(\tau-t)} d\tau + \left[t_r - t + \frac{1+p_r+x_0(2+p_r)}{2x_0\lambda(1+p_r)}\right] e^{-2\lambda(t_r-t)}$$

$$= \left[\frac{2(1+p_r) - x_0(1+3p_r) + 2x_0\lambda(1-p_r)(t_r-t)}{4x_0\lambda(1+p_r)}\right] e^{-2\lambda(t_r-t)} + \frac{5}{4\lambda}, \tag{EC.36}$$

given that both agents exert full effort before t_r.

Under PSD, we have $x_0 = 1$ after t_r. Also, $p_r = \lambda t_r/(1+\lambda t_r)$. Plugging in these values into (EC.36), the expected lead time of the contest under PSD is given by:

$$T_{0,0,0} = \left[\frac{1-p_r+2\lambda(1-p_r)t_r}{4\lambda(1+p_r)}\right] e^{-2\lambda t_r} + \frac{5}{4\lambda} = \frac{1}{4\lambda} e^{-2\lambda t_r} + \frac{5}{4\lambda}. \tag{EC.37}$$

Step 2: *We prove that PSD dominates no information disclosure.*

This immediately follows from the previous step. We already show that $T_{1,0,t}$ is independent of x_0. This means the expected lead time of the contest from any time t onward once the first success is obtained is the same across any design with constant information disclosure of rate $\lambda(x_0 - p_r)/p_r$ that stimulates constant effort x_0 after t_r in the equilibrium. However, according to (EC.36), $T_{0,0,0}$ is decreasing in x_0 and probabilistic encouragement design ensures that $x_0 = 1$ as long as both agents have zero success which results in the minimum expected lead time within this class of contests. Notice that no information disclosure or any disclosure with a rate lower than γ fails to encourage full effort and hence is dominated by the probabilistic encouragement design. Finally, we can compute the expected lead time of the contest under no information disclosure by plugging in $x_0 = p_r$ into (EC.36).

Step 3: *We prove that PSD dominates full information disclosure.*

This step is easy to verify. Note that under full information, the laggard quits upon the arrival of the first success at any time t. Therefore, the expected lead time in this case is given by:

$$T^F_{0,0,0} = \int_0^\infty 2\lambda \left(t + \frac{1}{\lambda}\right) e^{-2\lambda t} dt = \frac{3}{2\lambda},$$

where F stands for full information. However, under probabilistic encouragement design, the principal delays the stopping time of the laggard by t_r periods of time on average if success arrives after time t_r (given that $\gamma = 1/t_r$) and by $2t_r - t$ periods of time on average if success arrives at any time $t < t_r$. It is easy to see that $T^F_{0,0,0} < (5 + e^{-2\lambda t_r})/(4\lambda)$.

Step 4: *We prove that PSD dominates cyclic information disclosure.*

During the first cycle in a design with cyclic information disclosure, if the first success arrives at time $t < t_r$, both agents put full effort during the cycle and the laggard quits at time t_r at the end of the cycle. Therefore, we can write:

$$T^C_{1,0,t<t_r} = \int_t^{t_r} \left[\lambda(\tau - t) + \lambda\left(\tau - t + \frac{1}{2\lambda}\right)\right] e^{-2\lambda(\tau - t)} d\tau + \left(t_r - t + \frac{1}{\lambda}\right) e^{-2\lambda(t_r - t)} = \frac{3}{4\lambda} + \frac{1}{4\lambda} e^{-2\lambda(t_r - t)},$$

where C stands for cyclic information disclosure. Given this, the ex-ante expected lead time of the contest under cyclic information disclosure is given by:

$$T^C_{0,0,0} = \int_0^{t_r} 2\lambda\left(t + \frac{3}{4\lambda} + \frac{1}{4\lambda} e^{-2\lambda(t_r - t)}\right) e^{-2\lambda t} dt + \left(t_r + T_{0,0,t_r}\right) e^{-2\lambda t_r}$$

$$\Rightarrow T^C_{0,0,0} = \frac{t_r\ e^{-2\lambda t_r}}{2\left(1 - e^{-2\lambda t_r}\right)} + \frac{5}{4\lambda}, \tag{EC.38}$$

where we use the fact that $T^C_{0,0,0} = T^C_{0,0,t_r}$ as the game resets at time t_r. However, under PSD, information is disclosed at least t_r periods on average after the success is arrived. It is easy to check that $T^C_{0,0,0} > \left(5 + e^{-2\lambda t_r}\right)/(4\lambda)$. Thus, PSD dominates cyclic disclosure. ■

Proof of Theorem 2: We first verify the equilibrium under $PCSD$ and then prove that this information disclosure policy minimizes the contest's expected lead time. As before, an agent with one success finds it optimal to spend full effort at all times since $V_{1,p} \leq \overline{R} - c/\lambda$ for all p. Next, consider the continuation payoff of an agent i with no success from any time $t \geq \underline{t}$ (phase 2) onward:

$$V^i_{0,t} = \max_{x^i_{0,\tau}} \int_t^\infty x^i_{0,\tau}(\lambda V^i_{1,\tau} - c) e^{-\int_t^\tau [x^i_{0,s}\lambda + p^i_s x^{-i}_{1,s}\lambda + (1-p^i_s) x^{-i}_{0,s}\lambda\phi_s] ds} d\tau. \tag{EC.39}$$

To see the evolution of belief in (9) note that, by Bayes' rule, the probability that agent i assigns at time $t + dt$ to the event that her rival has succeeded once, given p^i_t, can be expressed as follows:

$$p^i_{t+dt} = \frac{p^i_t(1 - x^{-i}_{1,t}\lambda dt) + (1 - p^i_t) x^{-i}_{0,t}\lambda dt(1 - \phi_t)}{p^i_t(1 - x^{-i}_{1,t}\lambda dt) + (1 - p^i_t)[x^{-i}_{0,t}\lambda dt(1 - \phi_t) + 1 - x^{-i}_{0,t}\lambda dt]},$$

where the numerator is the probability that the contest has not ended yet and no information is disclosed given that the opponent has succeeded once, and the denominator is the total probability that the contest has not finished yet and no information is disclosed. The law of motion can be obtained by subtracting p^i_t from both sides, dividing by dt, and taking the limit as $dt \to 0$.

Let us fix the strategy of agent $-i$ to the proposed one in the equilibrium and verify that agent i best-responds by playing the same strategy. Using p as the state variable, consider the Bellman equation for the maximization problem of agent i with no success as follows:

$$V_{0,p} = \max_{x_{0,p}} \left\{-c x_{0,p} dt + x_{0,p}\lambda V_{1,p} dt + [1 - x_{0,p}\lambda dt - p\lambda dt - (1-p)\lambda\phi_p dt] V_{0,p+dp}\right\}.$$

Using the same techniques as before, we can derive the following HJB equation:

$$0 = \max_{x_{0,p}} \{-c x_{0,p} + x_{0,p}\lambda\left(V_{1,p} - V_{0,p}\right) - p\lambda V_{0,p} - (1-p)\lambda\phi_p V_{0,p} + (1-p)[\lambda - p\lambda - (1-p)\lambda\phi_p] V'_{0,p}\}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if

$$V^\phi_{1,p} - V^\phi_{0,p} \geq c/\lambda, \tag{EC.40}$$

where the superscript ϕ refers to $PCSD$. To derive the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success after $\underline{t}$ (in phase 2), we can write:

$$V_{1,p}^{\phi} = pV_{1,1}^{\phi} + (1-p)V_{1,0}^{\phi}, \tag{EC.41}$$

where $V_{1,1}^{\phi} = \frac{1}{2}(\overline{R} - c/\lambda)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and

$$\begin{aligned} V_{1,0}^{\phi} &= \phi\left(\overline{R} - \frac{c}{\lambda}\right) + (1-\phi)\int_t^{\infty}\left[(\lambda\overline{R} - c) + \lambda\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right)\right]e^{-2\lambda(\tau - t)}d\tau \\ &= \phi\left(\overline{R} - \frac{c}{\lambda}\right) + (1-\phi)\frac{3}{4}\left(\overline{R} - \frac{c}{\lambda}\right), \end{aligned}$$

is the expected continuation payoff if the opponent has not progressed to the second stage given that if the principal immediately discloses the change of state, the rival quits and agent i receives $(\overline{R} - c/\lambda)$, otherwise, during each interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns in expectation $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$. Under the proposed $PCSD$, we have $\phi_p = [\frac{4c/\lambda}{\overline{R} - c/\lambda} - 3 + p]/(1-p)$ for $p \in [\underline{p}, \overline{p})$. Given the assumption that the rival spends full effort in the equilibrium and after substituting for ϕ_p in (9), we obtain (12) for the evolution of p_t. Notice that p_t and ϕ_t are strictly increasing in time for $t \in [\underline{t}, \overline{t})$. From $t \geq \overline{t}$, $\phi_t = 1$ which holds the belief constant at $\overline{p}$ from $\overline{t}$ onward. Given the value of ϕ_p and the above equations, it is straightforward to verify that $V_{1,p}^{\phi} = c/\lambda$ for any $p \geq \underline{p}$. Using (EC.39), $V_{0,p}^{\phi} = 0$, and hence spending full effort is incentive compatible for agent i for all $t \geq \underline{t}$.

Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < \underline{p}$ where $\underline{p} < p_r$, by showing that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq \underline{p}$, then exerting full effort is optimal at belief $p - dp$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. This verifies the equilibrium.

Next, we prove that this design minimizes the contest's expected lead time. Note that the expected lead time is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Fix an arbitrary contest and observe that upon the arrival of the first success at time t associated with belief p, we can write $V_{1,p} = pV_{1,1} + (1-p)V_{1,0,p}$ where $V_{1,p}$ is the expected continuation payoff of an agent who just succeeded and $V_{1,1} = \frac{1}{2}(\overline{R} - \frac{c}{\lambda})$ (is a constant) since both agents spend full effort after achieving the first success under any design. We claim that $V_{1,0,p} = (\lambda\overline{R} - c)T_{1,0,p}$ where $V_{1,0,p}$ is the expected continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage and $T_{1,0,p}$ is the expected time between the end of the contest and the arrival of the first success at t (associated with belief p). To prove this claim, we can write

$$V_{1,0,p} = (\overline{R} - \frac{c}{\lambda}) - \int_t^\infty x_{0,\tau}\lambda\frac{1}{2}(\overline{R} - \frac{c}{\lambda})e^{-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau$$
$$= (\lambda\overline{R} - c)[\frac{1}{\lambda} - \int_t^\infty x_{0,\tau}\lambda\frac{1}{2\lambda}e^{-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau] = (\lambda\overline{R} - c)T_{1,0,p}. \quad \text{(EC.42)}$$

To understand the first equality above, note that $(\overline{R} - \frac{c}{\lambda})$ is the continuation payoff of an agent with one success in the absence of any opponent. In the presence of an opponent and during any interval $(\tau, \tau + d\tau)$, if the leader succeeds, she loses none of this continuation payoff, but if her opponent succeeds (for any effort level of an agent with no success in the equilibrium), the leader loses $\frac{1}{2}(\overline{R} - \frac{c}{\lambda})$ as she needs to compete with her rival in the second stage (recall that $V_{1,1} = \frac{1}{2}(\overline{R} - \frac{c}{\lambda})$). The second equality follows by factoring out the term $(\lambda\overline{R} - c)$. The third equality results from the definition of $T_{1,0,p}$ where the expected duration of a contest after the arrival of the first success with only one agent is given by $\frac{1}{\lambda}$. In the presence of a laggard and during any interval $(\tau, \tau + d\tau)$, if the leader succeeds, the expected duration does not change, but if the laggard succeeds, the expected duration reduces by $\frac{1}{2\lambda}$ owing to the fact that two agents are working full time until the task is complete which takes $\frac{1}{2\lambda}$ on average (a reduction of $\frac{1}{2\lambda}$ compared to $\frac{1}{\lambda}$). Thus, we prove our claim.

Following the above arguments, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. We next show this result.

First, notice that when $\overline{R} \leq \frac{7c}{3\lambda}$, phase 1 does not exist under $PCSD$ (i.e., $\underline{t} = \underline{p} = 0$). The principal chooses ϕ_t such that $V_{1,p} = c/\lambda$ for all p which in turn keeps $V_{0,p} = 0$ for all p. Indeed, this is the minimum necessary continuation payoff $V_{1,p}$ at each instant to incentivize an agent with no success to work; otherwise, the contest does not proceed to state $\{1,0\}$. This proves our claim in this case.

Second, suppose $\overline{R} > \frac{7c}{3\lambda}$. First, consider an agent with no success holding a belief $p > \overline{p} = \frac{2(\overline{R}-2c/\lambda)}{\overline{R}-c/\lambda}$ ($\overline{p}$ is defined in the theorem). The optimal action for this agent is to quit because even if she succeeds, her payoff is not sufficient to compensate her for her cost of effort as indicated below:

$$V_{1,p} = pV_{1,1} + (1-p)V_{1,0} \leq p\frac{1}{2}(\overline{R} - \frac{c}{\lambda}) + (1-p)(\overline{R} - \frac{c}{\lambda}) < \frac{c}{\lambda},$$

where the first inequality results from the facts that an agent with one success always spends full effort (therefore, $V_{1,1} = \frac{1}{2}(\overline{R} - \frac{c}{\lambda})$) and the maximum value of $V_{1,0}$ is obtained if we assume that the rival immediately quits (therefore, $V_{1,0} = \overline{R} - \frac{c}{\lambda}$). Second, consider an agent with no success holding a belief $\underline{p} \leq p \leq \overline{p}$. Under $PCSD$ and for all such p, $V_{1,p} = c/\lambda$ which is the bare minimum continuation payoff to incentivize any effort in the first stage. Thus, $PCSD$ minimizes $V_{1,p}$ in this region for all p. Finally, consider an agent with no success holding a belief $p < \underline{p}$. Under $PCSD$ and for all such p, $V_{1,p}$ is strictly greater than c/λ but it is the minimum continuation payoff possible as, under $PCSD$, this agent puts full effort until the end and the principal will never disclose this partial progress to her rival, inducing the rival to keep spending full effort until the contest ends. Thus, $PCSD$ minimizes $V_{1,p}$ in this region, too. Putting these together, $PCSD$ grants the minimum surplus to an agent who obtains a success when $\overline{R} > \frac{7c}{3\lambda}$ (and when $\overline{R} \leq \frac{7c}{3\lambda}$).

In conclusion, $PCSD$ minimizes the expected lead time of the contest after the arrival of the first success. Since it achieves the same goal before the arrival of the first success, this policy minimizes the project expected lead time. ■

EC.2. Additional Results and Robustness Checks

PROPOSITION EC.1. *When the principal is budget-constrained and commits to full information disclosure, there exists a unique symmetric equilibrium in which both agents exert full effort until the first success arrives. After that, the laggard quits and the leader puts full effort until the end.*

Proof of Proposition EC.1: Incentive compatibility conditions (EC.3) and (EC.5) show that an agent with one success finds it optimal to put full effort until the end. Given this observation, we have $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda) < c/\lambda$. Immediately, from IC condition (EC.7), it can be concluded that the laggard quits. Using this observation, we obtain $V_{1,0} = \overline{R} - c/\lambda$ and $V_{0,0} = \frac{1}{2}(\overline{R} - 2c/\lambda) \geq 0$. Therefore, IC condition (EC.9) is satisfied as $V_{1,0} - V_{0,0} = \frac{1}{2}\overline{R} > c/\lambda$. ■

PROPOSITION EC.2. *When the principal is budget-constrained, and commits to $PCSD$ with a flexible reward according to $R_{2,1} = \overline{R}$ and $R_{2,0,t} = \frac{7c/\lambda - \overline{R} - p_t(\overline{R} + c/\lambda)}{2(1-p_t)}$ if $t < \underline{t}$, otherwise $R_{2,0,t} = \overline{R}$ if $t \geq \underline{t}$, where t is the time at which the first success is obtained, $p_t = \lambda t/(1+\lambda t)$, and $\underline{t}$ is defined in Theorem 2, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.*

The amount of cost savings (CS) relative to the optimal $PCSD$ contest with a fixed reward is

$$CS = \int_0^{\underline{t}} (2\lambda I_t) e^{-2\lambda t} dt, \tag{EC.43}$$

where

$$I_t = \int_t^{\infty} \lambda(\overline{R} - R_{2,0,t}) e^{-2\lambda(\tau - t)} d\tau = \frac{1}{2}(\overline{R} - R_{2,0,t}). \tag{EC.44}$$

Proof of Proposition EC.2: In the proof of Theorem 2, we already show that $V^{\phi}_{1,p} = c/\lambda$ for $t \geq \underline{t}$ where $R_{2,1} = R_{2,0} = \overline{R}$ which makes $V^{\phi}_{0,p} = 0$ for $t \geq \underline{t}$ and hence exerting full effort is incentive compatible. For $t < \underline{t}$ and with the proposed flexible reward in the Proposition, one can verify that

$$\begin{aligned} V^{\phi}_{1,p} &= pV^{\phi}_{1,1} + (1-p)V^{\phi}_{1,0} \\ &= p\frac{1}{2}(\overline{R} - \frac{c}{\lambda}) + (1-p)\frac{1}{2}[R_{2,0,t} - \frac{c}{\lambda} + \frac{1}{2}(\overline{R} - \frac{c}{\lambda})] = \frac{c}{\lambda}, \end{aligned}$$

where t is the time at which the first success is obtained and p is the associated belief with that time. Therefore, (EC.40) is binding implying that an agent with no success finds it optimal to exert full effort. In addition, the reward structure is such that the incentive compatibility condition is slack for an agent with one success. Indeed, this design leaves no surplus for the agents while inducing them to exert the same level of effort under $PCSD$.

To understand expressions (EC.43) and (EC.44) note that the principal can save money by paying a lower reward $R_{2,0,t}$ compared to $\overline{R}$ if the first success arrives at $t < \underline{t}$, and the leader obtains her second success before the laggard obtains any success. The expressions measure this value. ■

EC.2.1. Splitting the Reward between Stages

In this section, we consider and analyze the possibility of splitting the reward between stages for a budget-constrained principal ($2c/\lambda < \overline{R} < 3c/\lambda$) to see if any improvement in lead-time minimization can be obtained. It is worth noting that for a case with no budget restriction, an interim reward will not be useful because giving a single final reward already achieves the first best.

Because any intermediate reward given publically leads to full information disclosure, which we show to be suboptimal since the laggard quits immediately, we shall focus on the case where the interim reward is given privately. Our analysis indicates that Theorem 2 can be easily extended to settings where the rewards for the two stages are separated, like in Bimpikis et al. (2019). Specifically, we prove that our probabilistic change-of-state information disclosure policy ($PCSD$) remains optimal when the principal specifies $\alpha\overline{R}$ for the first agent who completes stage one and $(1-\alpha)\overline{R}$ for the first agent who completes the task (i.e., both stages). As we shall see in the proof of Theorem EC.1, any feasible splitting must satisfy $0 \le \alpha \le (\overline{R} - c/\lambda)/\overline{R}$, because otherwise, the project cannot be completed.

Theorem EC.1. *When the principal is budget-constrained and commits to splitting the reward between stages according to the above rule for a given α, the following probabilistic change-of-state disclosure policy ($PCSD$) minimizes the expected lead-time of the contest:*

(Phase 1) The principal discloses no information to the agents up to time $\underline{t} = \frac{\underline{p}}{\lambda(1-\underline{p})}$ where

$$\underline{p} = \begin{cases} 0 & \text{if } \dfrac{2c}{\lambda} < \overline{R} \le \dfrac{7c}{(3+\alpha)\lambda}, \\ \\ \dfrac{(3+\alpha)\overline{R} - 7c/\lambda}{(1+3\alpha)\overline{R} - c/\lambda} & \text{if } \dfrac{7c}{(3+\alpha)\lambda} < \overline{R} < \dfrac{3c}{\lambda}. \end{cases} \tag{EC.45}$$

(Phase 2) At each instant $(t+dt)$ after $\underline{t}$, the principal discloses partial progress with probability

$$\phi_t^* = \begin{cases} \dfrac{\frac{c}{\lambda}(7-p_t) - \overline{R}[3+\alpha(1-3p_t) - p_t]}{(1-p_t)[(1-\alpha)\overline{R} - \frac{c}{\lambda}]} & \text{if } \underline{t} \le t < \overline{t}, \\ \\ 1 & \text{if } t \ge \overline{t}, \end{cases} \tag{EC.46}$$

if it arrived during interval $(t, t+dt)$ *where* p_t *is the unique solution to the ordinary differential equation (ODE)*

$$\dot{p}_t = \lambda(1-p_t)^2(1-\phi_t^*), \tag{EC.47}$$

with boundary conditions $p_{\underline{t}} = \underline{p}$ *and* $p_{\overline{t}} = \overline{p} \equiv \frac{2(\overline{R}-2c/\lambda)}{(1+\alpha)\overline{R}-c/\lambda}$.

(Equilibrium) Under PCSD, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.

Proof of Theorem EC.1: Fix a development contest with an intermediate reward $\alpha\overline{R}$ for the first agent to complete stage one and a final reward $(1-\alpha)\overline{R}$ for the first agent to complete both stages. Note as before that the IC condition for an agent with one success to spend strictly positive effort can be expressed as $(1-\alpha)\overline{R} - V_{1,p} \geq c/\lambda$ which always holds as long as $(1-\alpha)\overline{R} \geq c/\lambda$. If, on the other hand, $(1-\alpha)\overline{R} < c/\lambda$, an agent with one success has no incentives to work and hence the project will never be completed. Therefore, we must have $\alpha \leq (\overline{R}-c/\lambda)/\overline{R}$ which induces an agent with one success to keep exerting full effort until the project is complete. We now verify the equilibrium under $PCSD$ by fixing the strategy of agent $-i$ to the proposed one in the equilibrium and showing that agent i best-responds by playing the same strategy. Consider the continuation payoff of an agent i with no success from any time $t \geq \underline{t}$ (phase 2) onward. Using a similar approach as in the proof of Theorem 2, the IC constraint for this agent implies that $x_{0,p} = 1$, if and only if:

$$V_{1,p}^{\phi} - V_{0,p}^{\phi} = pV_{1,1}^{\phi} + (1-p)(\alpha\overline{R} + V_{1,0}^{\phi}) - V_{0,p}^{\phi} \geq c/\lambda, \tag{EC.48}$$

where the first equality is obtained by expanding, $V_{1,p}^{\phi}$, the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success after $\underline{t}$ (in phase 2). In particular, $V_{1,1}^{\phi} = \frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda]$ is the expected continuation payoff if the opponent has already progressed to the second stage, and

$$\begin{aligned} V_{1,0}^{\phi} &= \phi[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-\phi)\int_t^{\infty}\left[\lambda(1-\alpha)\overline{R} - c + \lambda\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]\right]e^{-2\lambda(\tau-t)}d\tau \\ &= \phi[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-\phi)\frac{3}{4}[(1-\alpha)\overline{R} - \frac{c}{\lambda}], \end{aligned}$$

is the expected continuation payoff if the opponent has not progressed to the second stage given that if the principal immediately discloses the change of state, the rival quits (since $\frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda] < c/\lambda$, $\forall\alpha$) and agent i receives $[(1-\alpha)\overline{R} - c/\lambda]$, otherwise, during each interval $(\tau, \tau+d\tau)$, the leader puts full effort and earns in expectation $[\lambda(1-\alpha)\overline{R} - c]d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda]$. The evolution of p_t in (EC.47) is obtained from (9) given the assumption that the rival spends full effort in the equilibrium. Notice that p_t and ϕ_t^* are strictly increasing in time for $t \in [\underline{t}, \overline{t})$. From $t \geq \overline{t}$, $\phi_t^* = 1$ which holds the belief constant at $\overline{p}$ from $\overline{t}$ onward. Given the value of ϕ_p^* and the above equations, it is straightforward to verify that $pV_{1,1}^{\phi} + (1-p)(\alpha\overline{R} + V_{1,0}^{\phi}) = c/\lambda$ for any $p \geq \underline{p}$. This makes $V_{0,p}^{\phi} = 0$, and by (EC.48) spending full effort is incentive compatible for agent i for all $t \geq \underline{t}$. Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < \underline{p}$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. This verifies the equilibrium.

Next, we prove that this design minimizes the contest's expected lead time using a similar approach as in the proof of Theorem 2. Note that the expected lead time is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Observe that upon the arrival of the first success at time t associated with belief p, the expected continuation payoff of an agent who just succeeded is $V_{1,p} = pV_{1,1} + (1-p)[\alpha\overline{R} + V_{1,0,p}]$ where $V_{1,1} = \frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]$ (is a constant) since both agents spend full effort after achieving the first success under any design. We can also show that $V_{1,0,p} = [\lambda(1-\alpha)\overline{R} - c]T_{1,0,p}$ where $V_{1,0,p}$ is the expected continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage and $T_{1,0,p}$ is the expected time between the end of the contest and the arrival of the first success at t (associated with belief p). To see this, it is enough to note that

$$
\begin{aligned}
V_{1,0,p} &= [(1-\alpha)\overline{R} - \frac{c}{\lambda}] - \int_t^{\infty} x_{0,\tau}\lambda\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]e^{-\int_t^{\tau}\lambda(1+x_{0,s})ds}d\tau \\
&= [\lambda(1-\alpha)\overline{R} - c][\frac{1}{\lambda} - \int_t^{\infty} x_{0,\tau}\lambda\frac{1}{2\lambda}e^{-\int_t^{\tau}\lambda(1+x_{0,s})ds}d\tau] = [\lambda(1-\alpha)\overline{R} - c]T_{1,0,p}. \qquad \text{(EC.49)}
\end{aligned}
$$

Thus, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. The proof follows a similar argument as in the proof of Theorem 2. First, notice that when $\overline{R} \leq \frac{7c}{(3+\alpha)\lambda}$, phase 1 does not exist under $PCSD$ (i.e., $\underline{t} = \underline{p} = 0$). The principal chooses ϕ_t such that $V_{1,p} = c/\lambda$ for all p which in turn keeps $V_{0,p} = 0$ for all p, and this is the minimum necessary continuation payoff $V_{1,p}$ at each instant to incentivize an agent with no success to work; otherwise, the contest does not proceed to state $\{1,0\}$. This proves our claim in this case.

Second, suppose $\overline{R} > \frac{7c}{(3+\alpha)\lambda}$. Consider an agent with no success holding a belief $p > \overline{p} = \frac{2(\overline{R} - 2c/\lambda)}{(1+\alpha)\overline{R} - c/\lambda}$ ($\overline{p}$ is defined in the theorem). The optimal action for this agent is to quit because even if she succeeds, her payoff is not sufficient to compensate her for her cost of effort as indicated below:

$$
V_{1,p} = pV_{1,1} + (1-p)[\alpha\overline{R} + V_{1,0}] \leq p\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-p)[\alpha\overline{R} + (1-\alpha)\overline{R} - \frac{c}{\lambda}] < \frac{c}{\lambda},
$$

where the first inequality results from the facts that an agent with one success always spends full effort (therefore, $V_{1,1} = \frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]$) and the maximum value of $V_{1,0}$ is obtained if we assume that the rival immediately quits (therefore, $V_{1,0} = (1-\alpha)\overline{R} - \frac{c}{\lambda}$). Second, consider an agent with no success holding a belief $\underline{p} \leq p \leq \overline{p}$. Under $PCSD$ and for all such p, $V_{1,p} = c/\lambda$ which is the bare minimum continuation payoff to incentivize any effort in the first stage. Thus, $PCSD$ minimizes $V_{1,p}$ in this region for all p. Finally, consider an agent with no success holding a belief $p < \underline{p}$. Under $PCSD$ and for all such p, $V_{1,p}$ is strictly greater than c/λ but it is the minimum continuation payoff possible as, under $PCSD$, this agent puts full effort until the end and the principal will never disclose this partial progress to her rival, inducing the rival to keep spending full effort until the contest ends. Thus, $PCSD$ minimizes $V_{1,p}$ in this region, too. Putting these together, $PCSD$ grants the minimum surplus to an agent who obtains a success when $\overline{R} > \frac{7c}{(3+\alpha)\lambda}$ (and when $\overline{R} \leq \frac{7c}{(3+\alpha)\lambda}$).

In conclusion, $PCSD$ minimizes the expected lead time of the contest after the arrival of the first success. Since it achieves the same goal before the arrival of the first success, this policy remains optimal and minimizes the project expected lead time even when the principal splits the reward between stages. ■

Theorem EC.1 shows the robustness of our finding to the case with an interim reward. Yet, one might question whether splitting rewards is in fact desirable. We answer this question numerically. Specifically, we consider 200 instances of $\overline{R}$ in the region $(2c/\lambda, 3c/\lambda)$, and show that in *all of these instances*, the optimal lead-time minimizing contest is a $PCSD$ design that allocates the entire budget to the final stage/reward (*i.e.*, $\alpha = 0$). The results are illustrated in Figure EC.1.

The intuition for the suboptimality of interim rewards is as follows. Recall that, in our development context, the key challenge for a budget-constrained principal is to encourage an agent who is failing to obtain any success over time to keep exerting effort. To mitigate this discouragement, the principal employs monetary and non-monetary (information design) incentives to minimize the project lead time. However, assigning a portion of the reward to the intermediate stage only amplifies the early contest incentives, specifically benefiting an agent who assigns a high chance to achieving the first success. Yet, such an agent already has sufficient incentives in the absence of any interim reward. However, an agent who fails to achieve a success for a while has incentive issues because she believes her opponent is likely to have progressed to the second stage. For such an agent, an interim reward has no value because she assigns low probability to getting the interim reward. On the contrary, interim reward reduces incentives for such an agent by diverting some of the attainable final reward to an unattainable interim reward. To mitigate the loss of incentives, the principal needs to disclose more information to persuade such an agent, which is suboptimal. Therefore, giving interim rewards is not desirable in development contests.

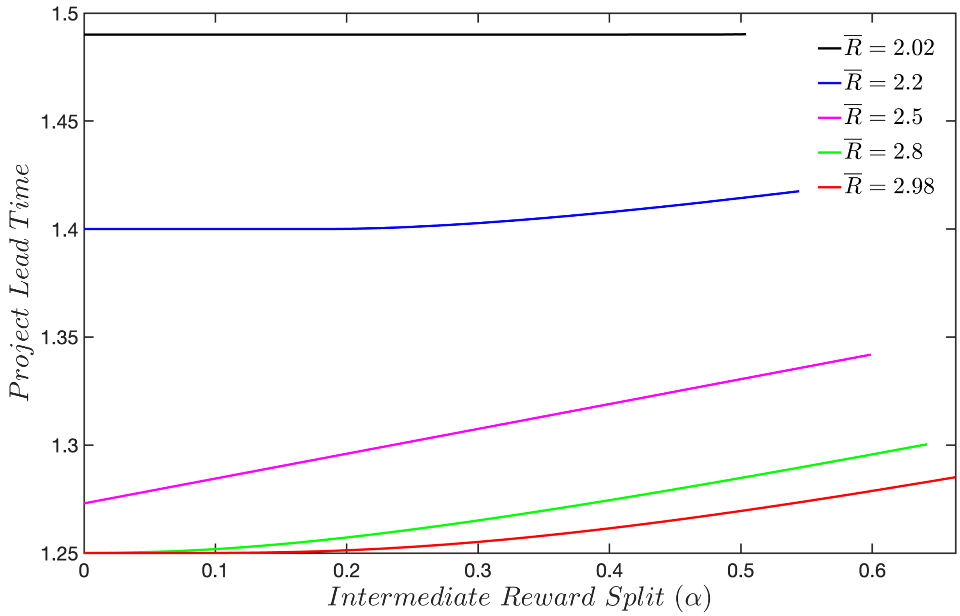

Figure EC.1 No splitting of reward ($\alpha = 0$) combined with probabilistic change-of-state disclosure ($PCSD$) minimizes lead time. Setting: $c = \lambda = 1$.

EC.2.2. Probabilistic vs Deterministic Delay

In this section, we will compare deterministic and probabilistic delay in sharing partial progress.

Consider a case where the principal commits to disclose any partial progress with t_d periods of delay. This mechanism leads to an initial silent period of length t_d during which agents' beliefs drift upward to p_d according to (5). During interval $(t_d, t_d + dt)$, if the principal announces partial progress, an agent with no success (i.e., the laggard) quits. Otherwise, each agent's belief remains constant at p_d as she realizes that no progress has been made during the initial dt period of the contest, akin to a contest that starts at dt instead of time 0. Intuitively, the principal wishes to extend this delay as long as possible to keep the laggard working. Thus, we consider the following deterministic design policy in which an agent with no success spends full effort until information is disclosed by the principal, derive the unique symmetric equilibrium under this policy, and compare the contest's expected lead time under this policy with that under PSD.

Proposition EC.3. *Suppose that the principal is budget-constrained. In the deterministic delay design, the principal commits to disclose partial progress after a delay of length t_d given by*

$$(1 + e^{-2\lambda t_d})(\overline{R} - c/\lambda) = 2(1 + \lambda t_d)(3c/\lambda - \overline{R}). \tag{EC.50}$$

Furthermore, under the deterministic delay design:

(i) An agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress.

An agent who has achieved one success exerts full effort until the end.

(ii) Delay length $t_d < t_r = \frac{p_r}{\lambda(1-p_r)}$ *where* $p_r = \frac{\lambda \overline{R}}{c} - 2$.

(iii) The expected lead time of the contest is given by $(5+e^{-2\lambda t_d})/(4\lambda)$*, which is strictly larger than the expected lead time under PSD.*

Proof of Proposition EC.3: Part (i). As before, we fix the strategy of agent $-i$ to the proposed one in the theorem and verify that agent i best-responds by playing the same strategy. Condition (EC.21) implies that full effort is optimal for an agent with no success if and only if $V_{1,p} - V_{0,p} \geq c/\lambda$. We use this condition to pin down t_d. Consider the very last instant during the initial silent period of length t_d at which an agent with no success finds it optimal to work. Then, we must have:

$$V^d_{1,p_d} = p_d V^d_{1,1} + (1-p_d)V^d_{1,0} = \frac{c}{\lambda}, \tag{EC.51}$$

where $p_d = \lambda t_d/(1+\lambda t_d)$ is the belief of agent i at time t_d anticipating the equilibrium behavior of agent $-i$. We use the superscript d to refer to a design with delay. Given that the principal discloses any progress with a delay t_d, agent i's belief remains constant for $t \geq t_d$. Therefore, at any threshold belief $p = p_d$, we can write:

$$V^d_{1,0} = \int_t^{t+t_d} \left[(\lambda \overline{R} - c) + \lambda \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right)\right] e^{-2\lambda(\tau - t)} d\tau + e^{-2\lambda t_d}\left(\overline{R} - \frac{c}{\lambda}\right),$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) and the continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$. If neither the leader achieves the second success, nor does the laggard achieve her first success, the principal discloses progress t_d periods of time after its arrival and in that case the laggard quits and the leader gets $(\overline{R} - c/\lambda)$. Taking the above integral, we obtain:

$$V^d_{1,0} = \left(\frac{3}{4} + \frac{1}{4} e^{-2\lambda t_d}\right)\left(\overline{R} - \frac{c}{\lambda}\right). \tag{EC.52}$$

Substituting the above value and $V^d_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ into (EC.51) and simplifying the equation, we find that t_d must solve (EC.50). Finally, to prove that exerting full effort is optimal for agent i with no success for all $p < p_d$, we move backward from time t_d associated with belief p_d and prove that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_d$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen from the following:

$$\begin{aligned} V^d_{1,p} - V^d_{0,p} &= -cdt + \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)V^d_{1,p+dp} + cdt - \lambda V^d_{1,p}dt - (1 - \lambda dt - p\lambda dt)V^d_{0,p+dp} \\ &\geq \lambda \overline{R}dt + (1 - \lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V^d_{1,p}dt \geq \frac{c}{\lambda}. \end{aligned}$$

To show the last inequality, we need to show that

$$V^d_{1,p} \leq \overline{R} - (1+p)\frac{c}{\lambda}.$$

We prove this in two steps. First, we prove that

$$V_{1,p}^d \leq \frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)$$
$$\Leftrightarrow V_{1,p}^d = pV_{1,1}^d + (1-p)V_{1,0}^d = p\frac{1}{2}\left(\overline{R}-\frac{c}{\lambda}\right) + (1-p)\left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_d}\right)\left(\overline{R}-\frac{c}{\lambda}\right) \leq \frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)$$
$$\Leftrightarrow p\frac{1}{2} + (1-p)\left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_d}\right) \leq \frac{1}{1+p} \Leftrightarrow 1 + e^{-2\lambda t_d} \leq \frac{2}{1+p}.$$

To show the last inequality, we use that $p = \lambda t/(1+\lambda t)$ and $t \leq t_d$. Thus, it is enough to show that

$$1 + e^{-2\lambda t_d} \leq 1 + e^{-2\lambda t} \leq \frac{2(1+\lambda t)}{1+2\lambda t} \Leftrightarrow 1 - (1+2\lambda t)e^{-2\lambda t} \geq 0,$$

and the last inequality holds given that the left-hand-side is increasing in t and at $t = 0$, it is binding. In the second step, we prove that

$$\frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right) \leq \overline{R} - (1+p)\frac{c}{\lambda}$$

which holds if and only if $(2+p)c/\lambda \leq \overline{R}$ which is satisfied for $p \leq p_d$. This verifies the equilibrium.

Part (ii). To prove that $t_d < t_r$, first notice that the left-hand-side of (EC.51) is strictly decreasing in t_d. To see this, note that

$$\frac{\partial V_{1,p_d}^d}{\partial t_d} = \frac{\partial p_d}{\partial t_d}V_{1,1}^d - \frac{\partial p_d}{\partial t_d}V_{1,0}^d + (1-p_d)\frac{\partial V_{1,0}^d}{\partial t_d} < 0,$$

where the above inequality holds since $\partial p_d/\partial t_d > 0$, $V_{1,1}^d < V_{1,0}^d$ and $\partial V_{1,0}^d/\partial t_d < 0$ according to (EC.52). Following this observation, suppose that $t_d = t_r$. Then, we can show that $V_{1,p_r}^d < c/\lambda$ implying that $t_d < t_r$. To see this, recall from (EC.32) that under PSD we have:

$$V_{1,p_r}^r = p_r V_{1,1}^r + (1-p_r)V_{1,0}^r = \frac{c}{\lambda},$$

where superscript r refers to the PSD. Therefore, to prove that $V_{1,p_r}^d < c/\lambda$, it is enough to show that

$$V_{1,0}^d < V_{1,0}^r \Leftrightarrow \left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_r}\right)\left(\overline{R}-\frac{c}{\lambda}\right) < \frac{3\lambda+2\gamma}{2(2\lambda+\gamma)}\left(\overline{R}-\frac{c}{\lambda}\right).$$

Using the fact that $\gamma = 1/t_r$ and further simplifying the above inequality, we need to show:

$$1 - (1+2\lambda t_r)e^{-2\lambda t_r} > 0,$$

which always holds for $t_r > 0$. This completes the proof of part (ii).

Part (iii). We first calculate the expected lead time of the contest when the principal discloses partial progress with a delay of length t_d. Denote by $T_{k,l,t}^d$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward under a design with delay. Consider any time t when the first success arrives. Then, the expected lead time of the contest from t can be expressed as follows:

$$T_{1,0,t}^d = \int_t^{t+t_d}\left[\lambda(\tau-t) + \lambda\left(\tau - t + \frac{1}{2\lambda}\right)\right]e^{-2\lambda(\tau-t)}d\tau + \left(t_d+\frac{1}{\lambda}\right)e^{-2\lambda t_d} = \frac{3}{4\lambda} + \frac{1}{4\lambda}e^{-2\lambda t_d}, \tag{EC.53}$$

where we use the fact that the laggard quits once the principal discloses progress t_d periods after its arrival. Using the above expression, the ex-ante expected lead time of the contest is:

$$T^d_{0,0,0} = \int_0^\infty 2\lambda \left(t + \frac{3}{4\lambda} + \frac{1}{4\lambda}e^{-2\lambda t_d}\right) e^{-2\lambda t} dt = \frac{1}{4\lambda} e^{-2\lambda t_d} + \frac{5}{4\lambda}. \tag{EC.54}$$

Recall that the expected lead time of the contest under PSD is given by $\frac{1}{4\lambda}e^{-2\lambda t_r} + \frac{5}{4\lambda}$. Part (iii) of the theorem follows from Part (ii) where we show that $t_d < t_r$. ■

EC.2.3. Discounting

In this section, we extend Proposition 1, Proposition 3 which also extends Theorem 1, and Theorem 2 to the case where the principal and the agents discount future payoffs at rate $r > 0$. Generalizations of other results follow in a similar fashion and are available upon request from the authors.

EC.2.3.1. First-Best Contract with Observable Effort and Discounting

PROPOSITION EC.4. *There exists an individually rational "first-best" contract that achieves the minimum expected lead time* $\underline{T}$ *with the minimum required compensation of* $\frac{2c}{2\lambda+r} + \frac{4\lambda c(3\lambda+r)}{(2\lambda+r)^3}$ *to agents.*

Proof of Proposition EC.4: Each agent incurs a flow cost of c while working during the contract. Given this, consider the state when both agents have already achieved one success. Then, each agent's expected cost in such a contract from any time t is given by:

$$\int_t^\infty 2\lambda \left(\int_t^\tau ce^{-r(s-t)} ds\right) e^{-2\lambda(\tau-t)} d\tau = \frac{c}{2\lambda + r}.$$

Next, consider the state of the game with a leader and a laggard. Then, each agent's expected cost from any time t can be expressed as follows:

$$\int_t^\infty \left[\lambda \int_t^\tau ce^{-r(s-t)} ds + \lambda \left(\int_t^\tau ce^{-r(s-t)} ds + \frac{c}{2\lambda + r} e^{-r(\tau - t)}\right)\right] e^{-2\lambda(\tau-t)} d\tau = \frac{c(3\lambda + r)}{(2\lambda + r)^2}.$$

Finally, each agent's ex-ante expected cost is given by:

$$\int_0^\infty 2\lambda \left(\int_0^t ce^{-rs} ds + \frac{c(3\lambda + r)}{(2\lambda + r)^2} e^{-rt}\right) e^{-2\lambda t} dt = \frac{c}{2\lambda + r} + \frac{2\lambda c(3\lambda + r)}{(2\lambda + r)^3}.$$

Multiplying the above value by 2 gives us the first-best cost of the principal. ■

EC.2.3.2. Full Information Disclosure with Flexible Reward and Discounting

PROPOSITION EC.5. *Under full information disclosure, a flexible-reward contest with* $R_{2,0} = \frac{c(2\lambda+r)}{\lambda^2}$ *and* $R_{2,1} = \frac{c(3\lambda+r)}{\lambda^2}$ *achieves the minimum expected lead time* $\underline{T}$ *with the first-best expected reward of* $\frac{2c}{2\lambda+r} + \frac{4\lambda c(3\lambda+r)}{(2\lambda+r)^3}$.

Proof of Proposition EC.5: Consider a flexible-reward contest with $R_{2,0} = \frac{c(2\lambda+r)}{\lambda^2}$ and $R_{2,1} = \frac{c(3\lambda+r)}{\lambda^2}$ where the principal commits to disclose any success upon its arrival. Let us fix agent $-i$'s effort $x^{-i}_{k,l,t} = 1$ for all k, l, and t and find conditions under which agent i optimally chooses $x^i_{k,l,t} = 1$ for all k, l, and t. Consider the state of the game where both agents have already achieved one success. The Bellman equation and the corresponding HJB for agent i's problem can be expressed as follows:

$$V_{1,1} = \max_{x_{1,1}} \{x_{1,1}(\lambda R_{2,1} - c)dt + (1 - \lambda x_{1,1}dt - \lambda dt - rdt)V_{1,1}\}$$
$$\Rightarrow 0 = \max_{x_{1,1}} \{x_{1,1}(\lambda R_{2,1} - c - \lambda V_{1,1}) - (\lambda + r)V_{1,1}\}, \quad \text{(EC.55)}$$

where we use the fact that the winner receives $R_{2,1}$ in this state of the game. From (EC.55), we can derive that $x_{1,1} = 1$ is optimal if and only if $R_{2,1} - V_{1,1} \geq c/\lambda$. Next, consider the state of the game with a leader and a laggard. The Bellman equation and the corresponding HJB for the leader's problem (which we assume to be agent i) can be written as:

$$V_{1,0} = \max_{x_{1,0}} \{x_{1,0}(\lambda R_{2,0} - c)dt + \lambda V_{1,1}dt + (1 - \lambda x_{1,0}dt - \lambda dt - rdt)V_{1,0}\}$$
$$\Rightarrow 0 = \max_{x_{1,0}} \{x_{1,0}(\lambda R_{2,0} - c - \lambda V_{1,0}) + \lambda (V_{1,1} - V_{1,0}) - rV_{1,0}\}, \quad \text{(EC.56)}$$

where we use the fact that the winner receives $R_{2,0}$ in this state of the game. From (EC.56), we can derive the IC constraint for the leader which tells us that $x_{1,0} = 1$ is incentive compatible if and only if $R_{2,0} - V_{1,0} \geq c/\lambda$. Similarly, we can express the Bellman equation and the corresponding HJB for the laggard's problem (assuming to be agent i) as follows:

$$V_{0,1} = \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c)dt + (1 - \lambda x_{0,1}dt - \lambda dt - rdt)V_{0,1}\}$$
$$\Rightarrow 0 = \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c - \lambda V_{0,1}) - (\lambda + r)V_{0,1}\}, \quad \text{(EC.57)}$$

which implies that exerting full effort for the laggard is optimal if and only if $V_{1,1} - V_{0,1} \geq c/\lambda$. Finally, before the arrival of any success, the continuation value of each agent is given by:

$$V_{0,0} = \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c)dt + \lambda V_{0,1}dt + (1 - \lambda x_{0,0}dt - \lambda dt - rdt)V_{0,0}\}$$
$$\Rightarrow 0 = \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c - \lambda V_{0,0}) + \lambda (V_{0,1} - V_{0,0}) - rV_{0,0}\}. \quad \text{(EC.58)}$$

From (EC.58), exerting $x_{0,0} = 1$ is incentive compatible for each agent if and only if $V_{1,0} - V_{0,0} \geq \dfrac{c}{\lambda}$.

We now verify that the proposed flexible-reward schedule in Proposition EC.5 satisfies all of the above IC constraints and spends the minimum first-best expected reward. We know that $V_{1,1} = c/\lambda$ is the minimum required continuation payoff to incentivize the laggard to put full effort. From (EC.55), we know that $V_{1,1} = \frac{\lambda R_{2,1} - c}{2\lambda + r}$. Thus, the principal has to specify a reward $R_{2,1} = \frac{c(3\lambda + r)}{\lambda^2}$ in order to satisfy $V_{1,1} = c/\lambda$. Given these values, the IC constraint $R_{2,1} - V_{1,1} \geq c/\lambda$ is satisfied. Also, plugging in the value of $V_{1,1} = c/\lambda$ into (EC.57), we obtain that $V_{0,1} = 0$ and so the IC constraint for the laggard is binding. Next, we know that $V_{1,0} = c/\lambda$ is the minimum required continuation payoff to motivate the agents to exert effort from the beginning. Plugging in this value into (EC.56), $R_{2,0} = \frac{c(2\lambda + r)}{\lambda^2}$ is needed to satisfy the HJB. It follows that the IC constraint for the leader is satisfied as $R_{2,0} - V_{1,0} \geq \frac{c}{\lambda}$. Finally, given $V_{1,0} = c/\lambda$ and $V_{0,1} = 0$, we conclude by (EC.58) that $V_{0,0} = 0$ which shows that the last IC constraint $V_{1,0} - V_{0,0} = \frac{c}{\lambda} - 0 = \frac{c}{\lambda}$ is binding. Therefore, full effort is incentive compatible at all times which means this design achieves the minimum expected lead time $\underline{T}$.

To calculate the expected reward of the contest with flexible reward, note that when both agents have already obtained one success, the expected reward of the contest with discounting is given by

$$\int_t^{\infty} 2\lambda \left[\frac{c(3\lambda+r)}{\lambda^2} e^{-r(\tau-t)}\right] e^{-2\lambda(\tau-t)} d\tau = \frac{2c(3\lambda+r)}{\lambda(2\lambda+r)}.$$

When there is a leader and a laggard, the expected reward can be computed as follows:

$$\int_t^{\infty} \lambda \left[\frac{c(2\lambda+r)}{\lambda^2} + \frac{2c(3\lambda+r)}{\lambda(2\lambda+r)}\right] e^{-(2\lambda+r)(\tau-t)} d\tau = \frac{c}{\lambda} + \frac{2c(3\lambda+r)}{(2\lambda+r)^2}.$$

Finally, the ex-ante expected reward of the contest is given by:

$$\int_0^{\infty} 2\lambda \left[\frac{c}{\lambda} + \frac{2c(3\lambda+r)}{(2\lambda+r)^2}\right] e^{-(2\lambda+r)t} dt = \frac{2c}{2\lambda+r} + \frac{4\lambda c(3\lambda+r)}{(2\lambda+r)^3},$$

which is the first-best expected reward. ■

EC.2.3.3. Optimal Lead-Time Minimizing Development Contests with Discounting

THEOREM EC.2. *The following probabilistic change-of-state disclosure design, which we call PCSD, minimizes the expected lead-time of the contest when the principal is budget-constrained: (Phase 1) The principal discloses no information to the agents up to time* $\underline{t} = \frac{\underline{p}}{\lambda(1-\underline{p})}$ *where*

$$\underline{p} = \begin{cases} 0 & \text{if } \frac{c(2\lambda+r)}{\lambda^2} < \overline{R} \le \frac{c(2\lambda+r)}{\lambda^2} + \frac{c}{3\lambda+r}, \\ \frac{3\lambda+r}{\lambda} - \frac{(2\lambda+r)^2 c}{\lambda^2(\lambda\overline{R}-c)} & \text{if } \frac{c(2\lambda+r)}{\lambda^2} + \frac{c}{3\lambda+r} < \overline{R} < \frac{c(3\lambda+r)}{\lambda^2}. \end{cases} \tag{EC.59}$$

(Phase 2) At each instant $(t+dt)$ *after* $\underline{t}$*, the principal discloses partial progress with probability*

$$\phi_t^* = \begin{cases} \dfrac{\frac{c}{\lambda} - p_t \frac{\lambda\overline{R}-c}{2\lambda+r} - (1-p_t)\frac{\lambda\overline{R}-c+\lambda\frac{\lambda\overline{R}-c}{2\lambda+r}}{2\lambda+r}}{(1-p_t)[\frac{\lambda\overline{R}-c}{\lambda+r} - \frac{\lambda\overline{R}-c+\lambda\frac{\lambda\overline{R}-c}{2\lambda+r}}{2\lambda+r}]} & \text{if } \underline{t} \le t < \overline{t}, \\ 1 & \text{if } t \ge \overline{t}, \end{cases} \tag{EC.60}$$

if it arrived during interval $(t, t+dt)$ *where* p_t *is the unique solution to the ordinary differential equation (ODE)*

$$\dot{p}_t = \lambda(1-p_t)^2(1-\phi_t^*), \tag{EC.61}$$

with boundary conditions $p_{\underline{t}} = \underline{p}$ *and* $p_{\overline{t}} = \overline{p} \equiv \frac{(2\lambda+r)[\lambda(\lambda\overline{R}-c)-(\lambda+r)c]}{\lambda^2(\lambda\overline{R}-c)}$.

(Equilibrium) Under PCSD, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.

Proof of Theorem EC.2: Verifying the equilibrium under *PCSD* is straightforward and follows the steps provided in the proof of Theorem 2. Hence, we omit this part. Here, we prove that this design minimizes the contest's expected lead time. Note that the expected lead time of the contest is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Fix an arbitrary contest and observe that upon the arrival of the first success at time t associated with belief p, we can write $V_{1,p} = pV_{1,1} + (1-p)V_{1,0,p}$ where $V_{1,p}$ is the expected (discounted) continuation payoff of an agent who just succeeded, $V_{1,1} = \frac{\lambda\overline{R}-c}{2\lambda+r}$ (is a constant) since both agents spend full effort after achieving the first success under any design, and $V_{1,0,p}$ is the expected (discounted) continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage. We claim that $V_{1,0,p} = (\lambda\overline{R}-c)T_{1,0,p}$ where $T_{1,0,p}$ is the expected (discounted) time between the end of the contest and the arrival of the first success at t (associated with belief p). To better understand the principal's objective function in the case of discounting, assume that the principal incurs a flow cost of 1 as long as the contest is running. The principal aims to minimize this cost which is equivalent to the lead-time minimization objective in the original model. The difference here is that this flow cost is discounted over time at rate r. To prove our claim, we can write

$$\begin{aligned} V_{1,0,p} &= \frac{\lambda\overline{R}-c}{\lambda+r} - \int_t^\infty x_{0,\tau}\lambda\left(\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}\right)(\lambda\overline{R}-c)e^{-r(\tau-t)-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau \\ &= (\lambda\overline{R}-c)\left[\frac{1}{\lambda+r} - \int_t^\infty x_{0,\tau}\lambda\left(\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}\right)e^{-r(\tau-t)-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau\right] = (\lambda\overline{R}-c)T_{1,0,p}. \end{aligned} \quad \text{(EC.62)}$$

To understand the first equality above, note that $\frac{\lambda\overline{R}-c}{\lambda+r}$ is the (discounted) continuation payoff of an agent with one success in the absence of any opponent. In the presence of an opponent and during any interval $(\tau, \tau+d\tau)$, if the leader succeeds, she loses none of this continuation payoff, but if her opponent succeeds (for any effort level of an agent with no success in the equilibrium), the leader loses $\frac{\lambda\overline{R}-c}{\lambda+r} - \frac{\lambda\overline{R}-c}{2\lambda+r}$ as she needs to compete with her rival in the second stage (recall that $V_{1,1} = \frac{\lambda\overline{R}-c}{2\lambda+r}$). The second equality follows by factoring out the term $(\lambda\overline{R}-c)$. The third equality results from the definition of $T_{1,0,p}$ where the expected discounted cost of running the contest after the arrival of the first success with only one agent is given by $\frac{1}{\lambda+r}$. In the presence of a laggard and during any interval $(\tau, \tau+d\tau)$, if the leader succeeds, the expected discounted cost does not change, but if the laggard succeeds, the expected discounted cost reduces by $\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}$ owing to the fact that two agents are working full time until the task is complete. Thus, we prove our claim.

Following the above arguments, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. It can be shown that $PCSD$ indeed minimizes $V_{1,p}$ for all p by following the same steps as in the proof of Theorem 2. This completes the proof. ■

EC.2.4. Different Poisson Arrival Rates for Different Stages

In this extension, we consider a case where the hazard rate of success in stage 1 is λ_1 and in stage 2 is λ_2. Proposition 3 can be extended to accommodate different hazard rates in a straightforward manner to show that a flexible-reward contest with $R_{2,0} = c/\lambda_1 + c/\lambda_2$ and $R_{2,1} = 2c/\lambda_1 + c/\lambda_2$ induces both agents to exert full effort at all times. This contest ends with probability 1/2 before the arrival of any success for the laggard in which case the principal spends $R_{2,0} = c/\lambda_1 + c/\lambda_2$; and with probability 1/2, the contest ends after the arrival of the first success for the laggard (the state when both agents have obtained one success) in which case the principal pays $R_{2,1} = 2c/\lambda_1 + c/\lambda_2$. Thus, the expected reward of the contest equals the first-best expected reward $3c/(2\lambda_1) + c/\lambda_2$. Similarly, the result in Proposition 4 can be extended to this setting if the principal gradually increases the reward schedule over time according to $R_t = (1+p_t)c/\lambda_1 + c/\lambda_2$ where the equilibrium belief of each agent about the partial progress of her opponent, p_t, is given by

$$\frac{\lambda_1 \left[e^{(\lambda_1-\lambda_2)t} - 1\right]}{\lambda_1 e^{(\lambda_1-\lambda_2)t} - \lambda_2} = p_t. \tag{EC.63}$$

Therefore, a principal with sufficient funds, with access to $2c/\lambda_1 + c/\lambda_2$, can find an appropriate flexible-reward schedule for any information disclosure policy that attains the absolute minimum expected lead time at the minimal cost of incentives. We next introduce the updated PSD in a case with different Poisson arrival rates (other results can be generalized similarly).

Definition EC.1. The "*probabilistic state disclosure design*" prescribes no information to the agents up to time t_r that solves

$$\frac{\lambda_1 \left[e^{(\lambda_1-\lambda_2)t_r} - 1\right]}{\lambda_1 e^{(\lambda_1-\lambda_2)t_r} - \lambda_2} = p_r, \tag{EC.64}$$

where p_r solves

$$(1+p_r)c/\lambda_1 + c/\lambda_2 = \overline{R}. \tag{EC.65}$$

After that it discloses any partial progress with rate $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$.

The following proposition describes the equilibrium under this design which is identical to the one in Proposition 7.

Proposition EC.6. *When the principal is budget-constrained, and commits to probabilistic state disclosure design, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.*

Proof of Proposition EC.6: Let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy. We know that an agent with one success finds it optimal to put full effort if and only if $\overline{R} - V_{1,p} \geq c/\lambda_2$ which always holds as long as $\overline{R} \geq c/\lambda_2$ since $V_{1,p} \leq \overline{R} - c/\lambda_2$. Next, notice that the belief of an agent i with no success about the partial progress of her opponent evolves according to:

$$dp_t^i = (1 - p_t^i)(\lambda_1 - p_t^i \lambda_2 - p_t^i \gamma_t)dt. \tag{EC.66}$$

Using p as the state variable and applying the same techniques as before, we can derive the following HJB equation for the maximization problem of agent i:

$$0 = \max_{x_{0,p}} \left\{ -cx_{0,p} + x_{0,p}\lambda_1 \left(V_{1,p} - V_{0,p}\right) - p\lambda_2 V_{0,p} - p\gamma_p V_{0,p} + (1-p)(\lambda_1 - p\lambda_2 - p\gamma_p)V'_{0,p} \right\}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if $V_{1,p} - V_{0,p} \geq c/\lambda_1$. The expected continuation payoff of agent i, holding a belief p, upon the arrival of the first success is $V_{1,p} = pV_{1,1} + (1-p)V_{1,0}$, where $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda_2)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and $V_{1,0}$ is the expected continuation payoff if the opponent has not progressed to the second stage. Given the probabilistic rate of information disclosure $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$, by (EC.66) we obtain $p_t = p_r$ remains constant for $t \geq t_r$. Therefore, at any threshold belief $p = p_r$, we have:

$$V_{1,0} = \int_t^\infty \left[\left(\lambda_2 \overline{R} - c\right) + \lambda_1 \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda_2}\right) + \gamma_r \left(\overline{R} - \frac{c}{\lambda_2}\right) \right] e^{-(\lambda_1 + \lambda_2 + \gamma_r)(\tau - t)} d\tau,$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns $(\lambda_2 \overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) and the continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda_2)$, or partial progress may be disclosed and in that case the leader gets $(\overline{R} - c/\lambda_2)$. Taking the above integral, we obtain:

$$V_{1,0} = \frac{\lambda_1 + 2\lambda_2 + 2\gamma_r}{2(\lambda_1 + \lambda_2 + \gamma_r)} \left(\overline{R} - \frac{c}{\lambda_2}\right). \tag{EC.67}$$

Using this value, we can write

$$V_{1,p_r} = p_r V_{1,1} + (1 - p_r)V_{1,0} = p_r \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda_2}\right) + (1 - p_r)\frac{\lambda_1 + 2\lambda_2 + 2\gamma_r}{2(\lambda_1 + \lambda_2 + \gamma_r)}\left(\overline{R} - \frac{c}{\lambda_2}\right) = \frac{c}{\lambda_1}, \tag{EC.68}$$

where the last equality results from substituting $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$ and $\overline{R} = (1 + p_r)c/\lambda_1 + c/\lambda_2$. Hence, $V_{1,p_r} = c/\lambda_1$ for $t \geq t_r$. This implies $V_{0,p_r} = 0$ for $t \geq t_r$ and hence the IC constraint for an agent with no success is binding. Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < p_r$, by showing that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$, then exerting full effort is optimal at belief $p - dp$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. ■

We conclude this extension by showing that our proposed probabilistic design dominates the two extremes of information disclosure for any pair of λ_1 and λ_2. Generalization of other results follow in a similar fashion and are available upon request from the authors.

THEOREM EC.3. *The expected lead time under PSD is given by (EC.72) which is strictly lower than the expected lead times under no and full information disclosure policies.*

Proof of Theorem EC.3: We prove the theorem in multiple steps. As before, we prove the result for a more general class of contests in which the principal commits to disclose information about any partial progress at constant rate $(\lambda_1 x_0 - p_r\lambda_2)/p_r$ after t_r so that in equilibrium an agent with no success reduces her effort to $x_0 \geq p_r$ for all $t \geq t_r$. It is straightforward to show that no information disclosure is a special case with $x_0 = p_r\lambda_2/\lambda_1$ and $\gamma_t = 0$, and probabilistic state disclosure design is a special case with $x_0 = 1$ and $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$ for all $t \geq t_r$.

Step 1: We first calculate the expected lead time of the contest under PSD. Denote by $T_{k,l,t}$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward. Let us consider the state of the game when both agents have already obtained one success. Then the expected arrival time for the second success is given by:

$$T_{1,1,t} = \int_t^\infty 2\lambda_2(\tau - t)e^{-2\lambda_2(\tau-t)}d\tau = \frac{1}{2\lambda_2}.$$

Here, information disclosure does not affect the outcome since both agents exert full effort until the end. Next, consider the state of the game with a leader and a laggard. Then, the expected lead time of the contest from any time $t \geq t_r$ can be expressed as follows:

$$T_{1,0,t\geq t_r} = \int_t^\infty \left[\lambda_2(\tau - t) + x_0\lambda_1\left(\tau - t + \frac{1}{2\lambda_2}\right) + \frac{\lambda_1 x_0 - p_r\lambda_2}{p_r}\left(\tau - t + T_{1,quit,\tau}\right)\right] \times e^{-\left(\lambda_2 + x_0\lambda_1 + \frac{\lambda_1 x_0 - p_r\lambda_2}{p_r}\right)(\tau-t)}d\tau = \frac{2+p_r}{2\lambda_2(1+p_r)}, \quad \text{(EC.69)}$$

where $T_{1,quit,\tau}$ is the expected arrival time for the second success once the principal discloses that the leader has made partial progress and the laggard quits, namely,

$$T_{1,quit,t} = \int_t^\infty \lambda_2(\tau - t)e^{-\lambda_2(\tau-t)}d\tau = \frac{1}{\lambda_2}.$$

Equation (EC.69) can be interpreted as follows. Conditional on reaching to any instant τ, the leader exerts full effort and if she succeeds the contest ends at $\tau - t$, or the laggard who is putting x_0 effort may achieve her first success and in that case the contest's expected lead time is $\tau - t + 1/(2\lambda_2)$, or information may be disclosed by the principal and in that case the laggard quits and the contest ends by the leader at $\tau - t + 1/\lambda_2$ in expectation. Interestingly, $T_{1,0,t\geq t_r}$ is independent of x_0. Next, for any $t < t_r$, the expected lead time is given by:

$$T_{1,0,t<t_r} = \int_t^{t_r} \left[\lambda_2(\tau - t) + \lambda_1\left(\tau - t + \frac{1}{2\lambda_2}\right)\right] e^{-(\lambda_1+\lambda_2)(\tau-t)}d\tau + \left(t_r - t + \frac{2+p_r}{2\lambda_2(1+p_r)}\right)e^{-(\lambda_1+\lambda_2)(t_r-t)}$$
$$= \frac{\lambda_1 + 2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1 - p_r\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)(1+p_r)}e^{-(\lambda_1+\lambda_2)(t_r-t)}, \quad \text{(EC.70)}$$

where we use the fact that no information is disclosed by the principal before t_r. Finally, the ex-ante expected lead time of the contest for any $t \geq t_r$ can be expressed as follows:

$$T_{0,0,t\geq t_r} = \int_t^\infty 2x_0\lambda_1\left(\tau - t + T_{1,0,\tau\geq t_r}\right)e^{-2x_0\lambda_1(\tau-t)}d\tau$$
$$= \int_t^\infty 2x_0\lambda_1\left[\tau - t + \frac{2+p_r}{2\lambda_2(1+p_r)}\right]e^{-2x_0\lambda_1(\tau-t)}d\tau = \frac{\lambda_2(1+p_r) + x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}, \quad \text{(EC.71)}$$

where we use that an agent with no success exerts effort x_0 after t_r, and for any $t<t_r$ is given by:

$$
\begin{aligned}
T_{0,0,t<t_r} &= \int_t^{t_r} 2\lambda_1 \left(\tau - t + T_{1,0,\tau<t_r}\right) e^{-2\lambda_1(\tau-t)} d\tau + \left(t_r - t + T_{0,0,t_r}\right) e^{-2\lambda_1(t_r-t)} \\
&= \int_t^{t_r} 2\lambda_1 \left[\tau - t + \frac{\lambda_1+2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1 - p_r\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)(1+p_r)} e^{-(\lambda_1+\lambda_2)(t_r-\tau)}\right] e^{-2\lambda_1(\tau-t)} d\tau \\
&\quad + \left[t_r - t + \frac{\lambda_2(1+p_r)+x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}\right] e^{-2\lambda_1(t_r-t)} \\
&= \left[\frac{-1}{2\lambda_1} - \frac{\lambda_1+2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1(\lambda_1-p_r\lambda_2)}{(\lambda_2-\lambda_1)\lambda_2(\lambda_1+\lambda_2)(1+p_r)} + \frac{\lambda_2(1+p_r)+x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}\right] e^{-2\lambda_1(t_r-t)} \\
&\quad - \frac{\lambda_1(\lambda_1-p_r\lambda_2)}{(\lambda_2-\lambda_1)\lambda_2(\lambda_1+\lambda_2)(1+p_r)} e^{-(\lambda_1+\lambda_2)(t_r-t)} + \frac{\lambda_1^2+3\lambda_1\lambda_2+\lambda_2^2}{2\lambda_1\lambda_2(\lambda_1+\lambda_2)}, \qquad \text{(EC.72)}
\end{aligned}
$$

given that both agents exert full effort before t_r.

Under PSD, we have $x_0=1$ after t_r. Also, p_r is given by (EC.63). Plugging in these values into (EC.72) gives us the expected lead time of the contest under PSD.

Step 2: We next prove that PSD dominates no information disclosure. This result follows from the previous step. We already show that $T_{1,0,t}$ is independent of x_0. This means the expected lead time of the contest from any time t onward once the first success is obtained is the same across any design with constant information disclosure of rate $(\lambda_1 x_0 - p_r\lambda_2)/p_r$ that stimulates constant effort x_0 after t_r in the equilibrium. However, according to (EC.72), $T_{0,0,0}$ is decreasing in x_0 and PSD ensures that $x_0=1$ as long as both agents have zero success, which results in the minimum expected lead time within this class of contests. Notice that no information disclosure or any disclosure with a rate lower than γ_r fails to encourage full effort and hence is dominated by PSD. Finally, we can compute the expected lead time of the contest under no information disclosure by plugging in $x_0 = p_r\lambda_2/\lambda_1$ into (EC.72).

Step 3: We finally prove that PSD dominates full information disclosure. Note that under full information, the laggard quits upon the arrival of the first success at any time t. However, under PSD, the principal delays the stopping time of the laggard by $1/\gamma_r$ periods of time on average if success arrives after time t_r and by $(t_r - t + 1/\gamma_r)$ periods of time on average if success arrives at any time $t<t_r$. The result directly follows from the fact that the laggard works for a longer duration under probabilistic design. ■

EC.2.5. Optimal Flexible-Reward Contest with n Agents

In this section, we show that a flexible-reward schedule is even more beneficial when more agents participate in the contest. We start our analysis by first characterizing the optimal flexible-reward schedule under full information disclosure that induces n agents to exert full effort to complete a two-stage task. Then, we compare our proposed optimal design with the optimal fixed-reward contest. The following remark along with Figure EC.2 formally highlight that the benefit of flexible rewards is increasing with the number of agents.

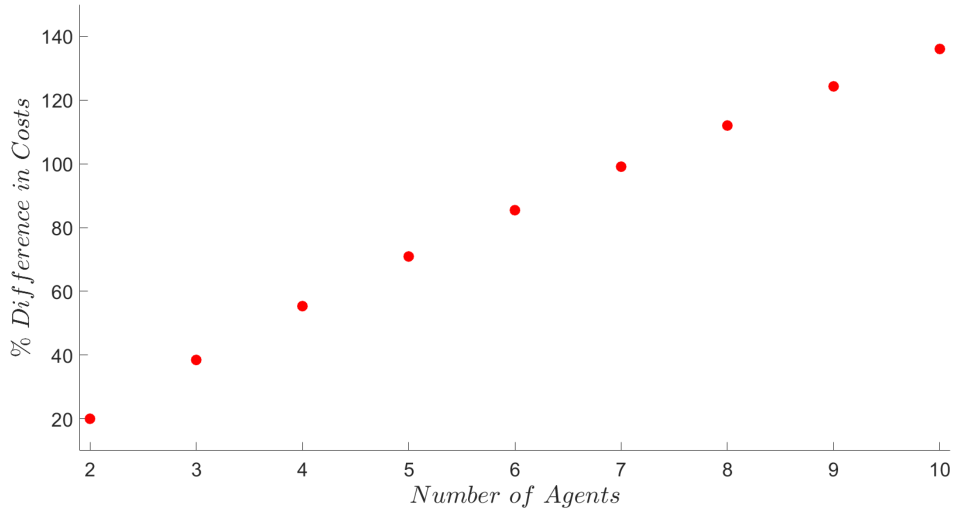

Figure EC.2 Percentage difference between the average rewards under fixed-reward and flexible-reward schedules.

REMARK EC.1. In a two-stage development contest with n agents, the principal can implement an optimal flexible-reward schedule under full information disclosure that achieves the absolute minimum expected lead time by paying the absolute minimum expected reward. The cost savings relative to the optimal fixed-reward contest is increasing in the number of agents.

Proof of Remark EC.1: Consider a flexible-reward contest under full information disclosure in which the principal offers a guaranteed reward of $2c/\lambda$ with the option to increase the reward by c/λ per each additional agent progressing to the second stage. This means if exactly m out of n agents have already achieved one success, the reward for the winner will be $(m+1)c/\lambda$. The optimality of this contest can be seen from the fact that agents' and principal's combined surplus is the same across any design that achieves the absolute minimum expected lead time $\underline{T}$ by inducing full effort at all times. Therefore, the principal's surplus is maximized when the agents' surplus is minimized. The proposed flexible-reward schedule in this case minimizes the agents' surplus by keeping the continuation payoff of an agent with no success equal to zero (her outside option) and the continuation payoff of an agent with one success equal to c/λ which is the bare minimum utility to incentivize first-stage effort. Therefore, this design maximizes the principal's surplus and hence is optimal.

To see why this design keeps the continuation payoff of an agent with one success equal to c/λ, notice that if all agents have already achieved one success, the continuation payoff of each agent under full effort is equal to

$$V_{\underbrace{1,1,\dots,1}_{n}} = \int_0^\infty [\lambda(n+1)\frac{c}{\lambda} - c]e^{-n\lambda t}dt = \frac{c}{\lambda}.$$

Now, suppose this holds for the case when exactly $m+1$ agents have already achieved one success. By induction, when m agents have already achieved one success, we can show that the continuation payoff of each agent with one success is c/λ as follows

$$V_{\underbrace{1,1,...,1}_{m},\underbrace{0,0,...,0}_{n-m}} = \int_0^\infty [\lambda(m+1)\frac{c}{\lambda} - c + (n-m)\lambda\frac{c}{\lambda}]e^{-n\lambda t}dt = \frac{c}{\lambda}.$$

Given the above result, it immediately follows that this design keeps the continuation payoff of an agent with no success equal to zero.

Next, we calculate the expected first-best reward of the contest in this case when n agents exert full effort at all times to complete a two-stage task. Denote by $R_{n,s}$ the principal's expected payout when exactly s agents have not achieved any success. Let us consider the state when all agents have achieved one success (i.e., $s=0$). Then the expected reward of the contest is given by $R_{n,0}=(n+1)c/\lambda$. Now, let us guess that the expected reward is given by

$$R_{n,s} = \frac{c}{n\lambda}\sum_{j=0}^{s}\frac{(n-j)(n-j+1)(s!)}{n^{(s-j)}(j!)}.$$

Notice that the above equation holds for $s=0$. Towards proving the result by induction on s, we can express the expected reward of the contest when exactly $s+1$ agents are in the first stage as follows

$$R_{n,s+1} = \int_0^\infty \left[(n-s-1)\lambda(n-s)\frac{c}{\lambda} + (s+1)\lambda R_{n,s}\right]e^{-n\lambda t}dt = \frac{c}{n\lambda}\sum_{j=0}^{s+1}\frac{(n-j)(n-j+1)[(s+1)!]}{n^{(s+1-j)}(j!)},$$

where the first equality results from the observation that if any one of the $n-s-1$ agents in the second stage obtains another success, the contest ends and the reward is $(n-s)c/\lambda$, and if any one of the $s+1$ agents in the first stage succeeds, the expected payout is $R_{n,s}$, and the second equality follows by substituting $R_{n,s}$ and collecting terms. Thus, our guess is verified.

Finally, considering the state when neither of the agents has one success, the expected first-best reward of the contest is equal to:

$$R_{n,n} = \frac{c}{n\lambda}\sum_{j=0}^{n}\frac{(n-j)(n-j+1)(n!)}{n^{(n-j)}(j!)}. \tag{EC.73}$$

Now, consider a fixed-reward contest that induces all agents to exert full effort at all times. Such a contest requires the minimum reward of $(n+1)c/\lambda$ to give sufficient incentives for working to an agent with no success while all the other agents have progressed to the second stage. This can be seen by the fact that this reward makes $V_{\underbrace{1,1,...,1}_{n}} = c/\lambda$ which is the minimum continuation payoff to stimulate effort.

Finally, the fixed-reward contest spends $[[(n+1)c/\lambda]/R_{n,n}-1]\times 100\%$ more than the optimal flexible-reward contest which one can (tediously) verify that this amount is increasing in n. ∎