Appendix A: Proofs and Counterexamples

As a reminder, we list the key notations used throughout the paper in Table 3.

Table 3: Key notations

c_f and c_v	fixed and variable cost of the utility
$d(\cdot)$	electricity demand of the market
α	authorized return imposed by the regulator
p_b	buyback price paid by the utility to excess energy of adopters
S	a random variable capturing the ratio of generation an adopter to her demand
L	a random variable capturing the levelized cost of solar electricity
ρ and $\bar{\rho} = 1 - \rho$	portions of nonadopters and adopters
p	retail electricity price charged by the utility
η and $\bar{\eta} = 1 - \eta$	portions of adopters installing no storage system and a storage system
$\gamma(\cdot)$ and γ_b	demand adjustment factors of an adopter and of an adopter with a storage system
$\beta_d^{\bar{\rho}}(p_b)$ and $\beta_c^{\bar{\rho}}(p_b)$	demand multiplier and cost multiplier with adoption level $\bar{\rho}$ and buyback price p_b
$\beta^{\bar{\rho}}(p_b) = \beta_c^{\bar{\rho}}(p_b)/\beta_d^{\bar{\rho}}(p_b)$	ratio of cost multiplier to demand multiplier
$\mathcal{R}(p; p_b)$	adoption level response function for a given retail price p and buyback price p_b
$\mathcal{P}(\bar{\rho}; p_b)$	price response function for a given adoption level $\bar{\rho}$ and buyback price p_b
κ	subscription fee of an adopter
$\bar{\lambda}$	portion of fixed utility cost covered by subscription fees

As a summary, we list the parameter values used in numerical illustrations in Table 4. These illustrations are provided for linear demand $d(p) = a - bp$ and uniformly distributed S over $[s_l, s_h]$ and L over $[l_{min}, l_{max}]$.

Table 4: Parameter values

c_v	c_f	a	b	α	p_b	s_l	s_h	l_{min}	l_{max}
4 ¢	2×10^{10} ¢	5×10^9 kWh	6×10^7 (kWh)^2/¢	0.1	11 ¢	0.4	2	4 ¢	20 ¢

Proof Lemma 1 The maximizer p_α maximizes $(p-(1+\alpha)c_v)d(p)$, which is nonnegative for $d(p)$ approaching zero as p approaches infinity. So the maximizer must make $(p-(1+\alpha)c_v)d(p)$ nonnegative, and hence $p_\alpha \geq (1+\alpha)c_v$. Setting the derivative of the objective equal to zero yields the optimality equation $-d'(p)/d(p) = (p-(1+\alpha)c_v)^{-1}$. The left-hand side $-d'(p)/d(p)$ is increasing for a logconcave demand while the right-hand side $(p-(1+\alpha)c_v)^{-1}$ is strictly decreasing in p from ∞ down to 0. So these sides intersect only once and yield a unique price solution $p_\alpha > (1+\alpha)c_v$. This solution p_α cannot be the minimizer of the objective $(p-(1+\alpha)c_v)d(p)$ because it achieves a positive value while $p = (1+\alpha)c_v$ achieves zero. Then p_α is the unique maximizer. These arguments also imply unimodality of the objective $(p-(1+\alpha)c_v)d(p)$.

For $\alpha < \alpha'$, we have the right hand-sides $(p-(1+\alpha)c_v)^{-1} < (p-(1+\alpha')c_v)^{-1}$ while the left-hand side $-d'(p)/d(p)$ is independent of α; the right-hand side shifts to right and the left-hand side remains constant. A larger value of price is required to equate these sides when α is larger, i.e., p_α is increasing. $\square$

Proof of Theorem 1 a) Since $(p-(1+\alpha)c_v)d(p)$ is unimodal in p, $pd(p)-(1+\alpha)(c_v d(p)+c_f) = 0$ has at most two solutions. If it has no solution, $pd(p)-(1+\alpha)(c_v d(p)+c_f) < 0$ for all prices and the utility problem is infeasible. If it has only one solution larger than p_0, we have $p_0 d(p_0)-(1+\alpha)(c_v d(p_0)+c_f) < 0$ as the left-hand side of the inequality is continuous and negative at $p = 0$. If it has two solutions greater than p_0, we again obtain $p_0 d(p_0)-(1+\alpha)(c_v d(p_0)+c_f) < 0$. Similarly, if both solutions are less than p_0, we have $p_0 d(p_0)-(1+\alpha)(c_v d(p_0)+c_f) < 0$. In a binding utility problem, $p_0 d(p_0)-(1+\alpha)(c_v d(p_0)+c_f) \geq 0$ and hence at least one of the solutions of $pd(p)-(1+\alpha)(c_v d(p)+c_f) = 0$ must be less than p_0. Let that unique solution be p^* and note $p^* < p_0$.

Repeating the above argument for $c_v d(p) + c_f$ instead of $(1+\alpha)(c_v d(p) + c_f)$ and using the solvency condition $p_0 d(p_0) - (c_v d(p_0) + c_f) \geq 0$ instead of the binding condition $p_0 d(p_0) - (1+\alpha)(c_v d(p_0) + c_f) \geq 0$, we obtain the single solution $p^{min} < p_0$ of $pd(p) - (c_v d(p) + c_f) = 0$. Since $c_v d(p) + c_f < (1+\alpha)(c_v d(p) + c_f)$, the revenue $pd(p)$ starting at zero with $p = 0$ crosses $c_v d(p) + c_f$ before $(1+\alpha)(c_v d(p) + c_f)$, i.e., $p^{min} < p^*$. Once it crosses $c_v d(p) + c_f$, it remains below $(1+\alpha)(c_v d(p) + c_f)$ until $p = p^*$. So, prices in $[p^{min}, p^*]$ satisfy the constraint $pd(p) \in (c_v d(p) + c_f)[1, 1+\alpha]$ and are feasible. Prices in $[0, p^{min}) \cup (p^*, p_0]$ are infeasible.

By unimodality of the profit $(p - c_v)d(p)$ and $p^* < p_0$, the profit is increasing over $[p^{min}, p^*]$. Moreover, the profit at p^* is larger than the profit at any price in $[p^{min}, p^*)$. Then p^* is the only candidate optimal solution that is less than p_0.

We now prove that no price $p > p_0$ can yield a larger profit than $(p^* - c_v)d(p^*) - c_f$. By the regulation constraint, a particular feasible price p satisfies $(p - c_v)d(p) - c_f \leq \alpha(c_f + c_v d(p))$. Moreover for $p^* < p_0 < p$, we obtain $\alpha(c_f + c_v d(p)) < \alpha(c_f + c_v d(p^*))$ as the demand is decreasing in price. For such a p, we note $(p - c_v)d(p) - c_f \leq \alpha(c_f + c_v d(p)) < \alpha(c_f + c_v d(p^*)) = (p^* - c_v)d(p^*) - c_f$, i.e., the profit at p^* is larger than that at $p > p_0$. Hence, the largest feasible profit is obtained with price p^*. That is, $p^* \leq p_0$ for all c_v and c_f values that make the utility problem feasible.

b) When the authorized return is binding, the optimal price is characterized by $pd(p) = (1+\alpha)(c_f + c_v d(p))$, which can be rewritten as

$$\frac{(1+\alpha)c_f}{d(p)} = p - (1+\alpha)c_v.$$

A drop in the demand from d_0 to d_1 increases the left-hand side of the equality. To maintain the equality and the optimality, the corresponding right-hand side must be higher, which is only possible with a higher price. As a result, lower demand implies a higher optimal price.

c) The proportional drop in the demand does not change the unconstrained optimal price p_0. Hence,

$$\bar{\alpha}(d_0) = \frac{p_0}{c_v + c_f/d_0(p_0)} - 1 \geq \frac{p_0}{c_v + c_f/d_1(p_0)} - 1 = \bar{\alpha}(d_1).$$

If the regulator's authorized return α is such that $\alpha \leq \bar{\alpha}(d_1)$, we use b) to conclude. If $\bar{\alpha}(d_1) < \alpha \leq \bar{\alpha}(d_0)$, the authorized return is binding the problem with higher demand d_0 but not with lower demand d_1. The optimal price in the problem with d_1 is p_0 and is greater than the optimal price in the problem with d_0. If $\alpha > \bar{\alpha}(d_0)$, both problems have the same optimal price p_0. □

Proof of Theorem 2 We show that the event $[\Gamma^m_{m+1} \geq 0]$ implies $[\Gamma^m_n \geq 0]$ for a given consumer. For brevity, $\bar{p}^{m,n}$ denotes the average of $n - m$ prices starting with p^m. $\Gamma^m_{m+1} \geq 0$ if and only if

$$t_L \bar{\Lambda}^{m,m+1} \leq \bar{p}^{m,m+1}(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+.$$

From this and $\bar{\Lambda}^{m,n} \leq \bar{\Lambda}^{m,m+1}$ almost surely for the consumer, we get the first two inequalities below.

$$t_L \bar{\Lambda}^{m,n} \leq t_L \bar{\Lambda}^{m,m+1} \leq \bar{p}^{m,m+1}(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+ \leq \bar{p}^{m,n}(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+.$$

The last equality above is from $\bar{p}^{m,m+1} \leq \bar{p}^{m,n}$, which follows the rational price expectation condition. The string of inequalities above implies $\Gamma^m_n \geq 0$.

$\mathrm{P}(\Gamma^m \geq 0, \Gamma^m_{m+1} \geq 0) = \mathrm{P}(L^m \vee (t_L(L^m - L^{m+1})) \leq p^m(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+)$ follows from the definitions of Γ^m and Γ^m_{m+1}. Since L^m and $\bar{\Lambda}^{m,m+1} = L^m - L^{m+1}$ stochastically decrease over time, so does $L^m \vee (t_L(L^m - L^{m+1}))$. Hence $\mathrm{P}(\Gamma^m \geq 0, \Gamma^m_{m+1} \geq 0)$ increases over time. Similarly, $\mathrm{P}(\Gamma^m \geq 0)$ increases over time. □

Proof of Lemma 2 Since $\hat{\Gamma}^m$ and Γ^m differ by a positive multiplier, $\mathrm{P}(\hat{\Gamma}^m \geq 0) = \mathrm{P}(\Gamma^m \geq 0) = \mathrm{P}(L^m \leq p^m(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+)$. By using algebra, we can show that the event $[\hat{\Gamma}^m_{m+1} \geq 0]$ occurs if $t_L(L^m - L^{m+1}\tilde{d}(p^{m+1})/\tilde{d}(p^m))S \leq p^m(S \wedge 1) + p_b(S-1)^+$. These lead to the probability expression in the lemma statement. $\square$

Proof of Lemma 3 a) The demand $d^{s,\bar{\rho}}(p; p_b)$ is a scaled version of $d(p)$ by $\rho + \bar{\rho}\eta\mathbb{E}[\gamma(p_b) - S]^+ + \bar{\rho}\bar{\eta}\mathbb{E}[\gamma - S]^+$ and it can be obtained by changing the units of energy (e.g., from kWh to MWh) used in $d(p)$. This does not harm the logconcave property and $d^{s,\bar{\rho}}(p)$ remains logconcave.

Let the maximizing price of the unconstrained profit be $p\beta_d^{\bar{\rho}}(p_b)d(p) - (1+\alpha)\beta_c^{\bar{\rho}}(p_b)c_v d(p) = \beta_d^{\bar{\rho}}(p_b)[p - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v]d(p)$ be $p_\alpha^{s,\bar{\rho}}$. This maximization is structurally similar to the maximization of $[p - (1+\alpha)c_v]d(p)$. So we can use Lemma 1 to establish the claims in a).

b) For $p_b \geq c_v$, we obtain $\beta^{\bar{\rho}}(p_b) \geq 1$, which in the derivative of $pd^{s,\bar{\rho}}(p; p_b) - (1+\alpha)c^{s,\bar{\rho}}(p; p_b)$ leads to the inequality below associated with the first order condition.

$$\frac{-d'(p)}{d(p)} = \left(p - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v\right)^{-1} \geq \left(p - (1+\alpha)c_v\right)^{-1}.$$

Recall that $-d'(p)/d(p) = (p - (1+\alpha)c_v)^{-1}$ yields p_α. Since $-d'(p)/d(p)$ is increasing in p by logconcavity, we obtain $p_\alpha^{s,\bar{\rho}} \geq p_\alpha$.

To obtain $p_\alpha + (1+\alpha)(\beta^{\bar{\rho}}(p_b) - 1)c_v \geq p_\alpha^{s,\bar{\rho}}$, first let $\bar{p} = p_\alpha + (1+\alpha)(\beta^{\bar{\rho}}(p_b) - 1)c_v$ so that $(\bar{p} - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v)^{-1} = (p_\alpha - (1+\alpha)c_v)^{-1}$. For $p \in [p_\alpha, \bar{p}]$, we have $-d'(p)/d(p)$ increasing from $(p_\alpha - (1+\alpha)c_v)^{-1}$ to higher values, while $(p - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v)^{-1}$ is decreasing from a value higher than $(p_\alpha - (1+\alpha)c_v)^{-1}$ to $(p_\alpha - (1+\alpha)c_v)^{-1}$. So $-d'(p)/d(p)$ intersects $(p - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v)^{-1}$ in $[p_\alpha, \bar{p}]$. This intersection is $p_\alpha^{s,\bar{\rho}}$ and we arrive at $p_\alpha^{s,\bar{\rho}} \leq \bar{p} = p_\alpha + (1+\alpha)(\beta^{\bar{\rho}}(p_b) - 1)c_v$.

c) For the linear demand $d(p) = a - bp$, the unconstrained price maximizer is $p_0^{s,\bar{\rho}} = a/(2b) + \beta^{\bar{\rho}}(p_b)c_v/2$ and $d(p_0^{s,\bar{\rho}}) = a/2 - \beta^{\bar{\rho}}(p_b)bc_v/2$ for $a \geq \beta^{\bar{\rho}}(p_b)bc_v$; for $a < \beta^{\bar{\rho}}(p_b)bc_v$, $p_0^{s,\bar{\rho}} = a/b$, $d(p_0^{s,\bar{\rho}}) = 0$ and $\bar{\alpha}^{s,\bar{\rho}} = 0$, establishing the result. The maximum binding authorized return for $a \geq \beta^{\bar{\rho}}(p_b)bc_v$ is

$$\bar{\alpha}^{s,\bar{\rho}} = \frac{a^2 - \beta^{\bar{\rho}^2}(p_b)b^2c_v^2}{2b\beta^{\bar{\rho}}(p_b)c_v(a - \beta^{\bar{\rho}}(p_b)bc_v) + 4bc_f/\beta_d^{\bar{\rho}}(p_b)} - 1.$$

The inequality $\bar{\alpha}^{s,\bar{\rho}} \leq \bar{\alpha}$ is equivalent to $1 - 2^{-1}(\bar{\alpha}^{s,\bar{\rho}} + 1)^{-1} \leq 1 - 2^{-1}(\bar{\alpha} + 1)^{-1}$, so we prove the latter inequality. Since

$$1 - \frac{1}{2}\frac{1}{\bar{\alpha}^{s,\bar{\rho}} + 1} = 1 - \frac{2c_f b/\beta_d^{\bar{\rho}}(p_b) + \beta^{\bar{\rho}}(p_b)c_v ab - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2}{a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2} = \frac{a^2 - 2c_f b/\beta_d^{\bar{\rho}}(p_b) - \beta^{\bar{\rho}}(p_b)c_v ab}{a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2},$$

we need to prove

$$\frac{a^2 - 2c_f b/\beta_d^{\bar{\rho}}(p_b) - \beta^{\bar{\rho}}(p_b)c_v ab}{a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2} \leq \frac{a^2 - 2c_f b - c_v ab}{a^2 - c_v^2b^2}.$$

Note that $\beta_d^{\bar{\rho}}(p_b) \leq 1 \leq \beta^{\bar{\rho}}(p_b)$, $-2c_f b/\beta_d^{\bar{\rho}}(p_b) \leq -2c_f b$ and $0 \leq a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2 \leq a^2 - c_v^2b^2$. So we prove the stricter inequality

$$\frac{a^2 - 2c_f b - \beta^{\bar{\rho}}(p_b)c_v ab}{a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2} \leq \frac{a^2 - 2c_f b - c_v ab}{a^2 - c_v^2b^2}.$$

Since $\beta^{\bar{\rho}}(p_b) \geq 1$, we have $-2c_f b/(a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2) \leq -2c_f b/(a^2 - c_v^2b^2)$, then it suffices to prove

$$\frac{a^2 - \beta^{\bar{\rho}}(p_b)c_v ab}{a^2 - \beta^{\bar{\rho}^2}(p_b)c_v^2b^2} \leq \frac{a^2 - c_v ab}{a^2 - c_v^2b^2} \quad \text{or} \quad \frac{a/(c_v b) - \beta^{\bar{\rho}}(p_b)}{a/(c_v b) - \beta^{\bar{\rho}^2}(p_b)c_v b/a} \leq \frac{a/(c_v b) - 1}{a/(c_v b) - c_v b/a}.$$

Let $\tilde{a} = a/(c_v b)$, what remains to establish is that $(\tilde{a}-\beta)/(\tilde{a}-\beta^2/\tilde{a})$ is decreasing in β. Taking the derivative of this ratio with respect to β and examining its sign $-(\tilde{a}-\beta^2/\tilde{a}) - (\tilde{a}-\beta)(-2\beta/\tilde{a}) = -\beta^2/\tilde{a} + 2\beta - \tilde{a} = -(\beta-\tilde{a})^2/\tilde{a} \le 0$, the ratio is decreasing in β. This completes the proof for the linear demand.

With $c_f = 0$, $\bar{\alpha}^{s,\bar{\rho}} = p_0^{s,\bar{\rho}}/(\beta^{\bar{\rho}}(p_b)c_v)$ and $\bar{\alpha} = p_0/c_v$. It suffices to show $p_0^{s,\bar{\rho}} \le \beta^{\bar{\rho}}(p_b)p_0$:

$$p_0^{s,\bar{\rho}} \le p_0 + (\beta^{\bar{\rho}}(p_b)-1)c_v \le \beta^{\bar{\rho}}(p_b)p_0,$$

where the first inequality is specializing part c) for $\alpha = 0$ and the second is from $c_v \le p_0$. □

Since p_b is fixed during the game, we use the notation $\gamma = \gamma(p_b)$ until the proof of Lemma 6.

Proof Lemma 4 a) For $\mathrm{P}(S>\gamma) > 0$ and $x \in [0,1]$, we have

$$\beta_g(x;S) \quad = \quad \eta x \int_{\gamma}^{\gamma-1/\eta+1/(\eta x)} (u-\gamma) f_S(u)du + (1-x)\int_{\gamma-1/\eta+1/(\eta x)}^{\infty} f_S(u)du.$$

and

$$\begin{aligned}
\frac{d\beta_g(x;S)}{dx} &= \eta \int_{\gamma}^{\gamma-1/\eta+1/(\eta x)} (u-\gamma) f_S(u)du + \eta x(1/\eta)(-1/x^2)(\gamma-1/\eta+1/(\eta x)-\gamma) f_S(\gamma-1/\eta+1/(\eta x)) \\
&\quad - \int_{\gamma-1/\eta+1/(\eta x)}^{\infty} f_S(u)du + (1-x)(-1/\eta)(-1/x^2) f_S(\gamma-1/\eta+1/(\eta x)) \\
&= \eta \int_{\gamma}^{\gamma-1/\eta+1/(\eta x)} (u-\gamma) f_S(u)du - \int_{\gamma-1/\eta+1/(\eta x)}^{\infty} f_S(u)du.
\end{aligned}$$

The first integral above decrease in x, while the second increases, so their difference decreases and $\beta_g(x;S)$ is concave in x. In particular, $(d/dx)\beta_g(x;S)|_{x=0} = \int_{\gamma}^{\infty}(u-\gamma)f_S(u)du > 0$ and $(d/dx)\beta_g(x;S)|_{x=1} = -\int_{\gamma}^{\infty} f_S(u)du = -\mathrm{P}(S>\gamma) < 0$. The derivative crosses zero once and in the interval $[0,1]$. So $\beta_g(x;S)$ has a unique maximizer $x(S)$ that solves $\eta\int_{\gamma}^{\gamma-1/\eta+1/(\eta x)}(u-\gamma)f_S(u)du = 1 - F_S(\gamma-1/\eta+1/(\eta x))$ and $x(S) \in [0,1]$.

By (2), we have $\mathbb{E}[1-x+\eta x(\gamma-S)]^+ + \mathbb{E}[\bar{\eta}x(\gamma_b-S)]^+ + \beta_g(x;S) = 1-x+\eta x\mathbb{E}[\gamma-S]^+ + \bar{\eta}x\mathbb{E}[\gamma_b-S]^+$ whose right-hand side is linear in x. Taking the second derivative of the equality,

$$\frac{\partial}{\partial x^2}\mathbb{E}[1-x+\eta x(\gamma-S)]^+ + \frac{\partial}{\partial x^2}\mathbb{E}[\bar{\eta}x(\gamma_b-S)]^+ + \frac{\partial}{\partial x^2}\beta_g(x;S) = 0.$$

By concavity of $\beta_g(x;S)$ proved above and multiplying it by $p_b/c_v \ge 1$, we obtain

$$\frac{\partial}{\partial x^2}\mathbb{E}[1-x+\eta x(\gamma-S)]^+ + \frac{\partial}{\partial x^2}\mathbb{E}[\bar{\eta}x(\gamma_b-S)]^+ + \frac{p_b}{c_v}\frac{\partial}{\partial x^2}\beta_g(x;S) \le 0,$$

where the left hand-side is equal to the second derivative of $\beta_h(x;S)$ and yields the concavity of $\beta_h(.)$.

b) In this part $\gamma = 1$ and $\eta = 1$. Then the optimality equation can be reorganized as

$$\begin{aligned}
1 &= \int_1^{1/x} u\frac{f_S(u)}{1-F_S(1)}du = \int_1^{1/x} u f_{S|S\ge1}(u)du = \int_1^{1/x}\int_0^u f_{S|S\ge1}(u)dvdu \\
&= \int_0^1 dv \int_1^{1/x} f_{S|S\ge1}(u)du + \int_1^{1/x}\int_v^{1/x} f_{S|S\ge1}(u)dudv = F_{S|S\ge1}(1/x) + \int_1^{1/x} F_{S|S\ge1}(1/x) - F_{S|S\ge1}(v)dv.
\end{aligned}$$

This condition can be rewritten as

$$1 = \int_1^{1/x} \frac{F_{S|S\ge1}(1/x) - 1 + 1 - F_{S|S\ge1}(v)}{1-F_{S|S\ge1}(1/x)}dv = \int_1^{1/x}\left(\frac{\bar{F}_{S|S\ge1}(v)}{\bar{F}_{S|S\ge1}(1/x)} - 1\right)dv \tag{8}$$

Since S' is greater than S in hazard rate order, we have for $s, r \geq 0$

$$\mathrm{P}(S' > s+r | S' \geq r) \geq \mathrm{P}(S > s+r | S \geq r) \;\Rightarrow\; \mathrm{P}(S' > s+r | S' \geq 1, S' \geq r) \geq \mathrm{P}(S > s+r | S \geq 1, S \geq r)$$

Hence $S'|S' \geq 1$ is greater than $S|S \geq 1$ in hazard rate order and $\bar{F}_{S'|S'\geq 1}(t)/\bar{F}_{S|S\geq 1}(t)$ increases in t. This yields

$$\frac{\bar{F}_{S'|S'\geq 1}(1/x)}{\bar{F}_{S|S\geq 1}(1/x)} \geq \frac{\bar{F}_{S'|S'\geq 1}(v)}{\bar{F}_{S|S\geq 1}(v)} \iff \frac{\bar{F}_{S|S\geq 1}(v)}{\bar{F}_{S|S\geq 1}(1/x)} \geq \frac{\bar{F}_{S'|S'\geq 1}(v)}{\bar{F}_{S'|S'\geq 1}(1/x)}$$

and in turn

$$\int_1^{1/x} \left(\frac{\bar{F}_{S|S\geq 1}(v)}{\bar{F}_{S|S\geq 1}(1/x)} - 1 \right) dv \geq \int_1^{1/x} \left(\frac{\bar{F}_{S'|S'\geq 1}(v)}{\bar{F}_{S'|S'\geq 1}(1/x)} - 1 \right) dv.$$

This inequality, in view of the right-hand side of (8) decreasing in x, implies $x(S) \geq x(S')$.

Now we investigate the magnitude of maximizer $x(S)$. Suppose to the contrary that the maximizer $x(S) < 1/a$. Then for $x \in [x(S), 1/a)$, $1/x > a$ and the derivative of $\beta_g(x; S)$ is

$$\int_1^{1/x} u f_S(u) du - (1 - F_S(1)) > \int_1^{a} u f_S(u) du - (1 - F_S(1)) = \int_1^{a} (u-1) f_S(u) du \geq 0.$$

So $\beta_g(x; S)$ is increasing over $x \in [x(S), 1/a)$ and $x(S) < 1/a$ cannot be the maximizer. We must then have $x(S) \geq 1/a$.

For uniformly distributed S over $[0, a]$, the maximizer solves $\int_1^{1/x} u(1/a)du = (a-1)/a$ or $(1/x^2 - 1)/2 = a-1$, which leads to the maximizer $x(U[0.a]) = (2a-1)^{-1/2}$. For exponentially distributed S with parameter λ, the maximizer over $[0,1]$ solves $\int_1^{1/x} u\lambda \exp(-\lambda u)du = \exp(-\lambda)$. With some algebra, this condition becomes $1 + \lambda/x = e^{\lambda(1/x-1)}$. □

Proof of Lemma 5 We first rewrite (5) as

$$\big(\beta_d^{\bar{\rho}} p - \beta_c^{\bar{\rho}}(1+\alpha)c_v\big) d(p) = (1+\alpha)c_f. \tag{9}$$

As $\beta_d^{\bar{\rho}} \leq \beta_c^{\bar{\rho}}$, this equality can be solved only by $p \geq (1+\alpha)c_v$.

a) From (2), we have $\mathbb{E}(\rho + \eta\bar{\rho}\gamma - \eta\bar{\rho}S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}(\gamma_b - S))^+ + \beta_g(\bar{\rho}; S) = \rho + \eta\bar{\rho}\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}\mathbb{E}(\gamma_b - S)^+$ for $\bar{\rho}, \eta \in [0,1]$. Applying this twice for $\bar{\rho}_l \leq \bar{\rho}_h \leq x(S)$, we get $\rho_l + \eta\bar{\rho}_l\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_l\mathbb{E}(\gamma_b - S)^+ = \mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+ + \beta_g(\bar{\rho}_l; S)$ and $\rho_h + \eta\bar{\rho}_h\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_h\mathbb{E}(\gamma_b - S)^+ = \mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ + \beta_g(\bar{\rho}_h; S)$. By using $(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) = [\rho_h + \eta\bar{\rho}_h\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_h\mathbb{E}(\gamma_b - S)^+] - [\rho_l + \eta\bar{\rho}_l\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_l\mathbb{E}(\gamma_b - S)^+]$, we obtain

$$\begin{aligned} &(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) \\ &\quad = [\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ - \mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+] + [\mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ - \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+] \\ &\qquad + \beta_g(\bar{\rho}_h; S) - \beta_g(\bar{\rho}_l; S). \end{aligned} \tag{10}$$

From Lemma 4, since $\beta_g(\bar{\rho}; S)$ is a concave function maximized at $x(S)$, we have $\beta_g(\bar{\rho}_h; S) - \beta_g(\bar{\rho}_l; S) \geq 0$. Using the last inequality with $p_b \geq c_v$ in (10) yields $(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) \leq [\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ - \mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+] + [\mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ - \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+] + [\beta_g(\bar{\rho}_h; S) - \beta_g(\bar{\rho}_l; S)]p_b/c_v$. Negating the last inequality gives $-(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) \geq -[\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ - \mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+] - [\mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ - \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+] - [\beta_g(\bar{\rho}_h; S) - \beta_g(\bar{\rho}_l; S)]p_b/c_v$. We preserve this inequality by further multiplying it side-by-side with $p \geq (1+\alpha)c_v$ as the left-hand side is nonnegative, i.e., $-(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) = (\bar{\rho}_h - \bar{\rho}_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+) \geq 0$ on account of our assumption $\mathbb{E}(\gamma - S)^+ \vee \mathbb{E}(\gamma_b - S)^+ \leq 1$. Hence, $-(\rho_h - \rho_l)(1 - \eta\mathbb{E}(\gamma - S)^+ - \bar{\eta}\mathbb{E}(\gamma_b - S)^+)p \geq -([\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ - \mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+] + [\mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ - \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+])(1+\alpha)c_v - [\beta_g(\bar{\rho}_h; S) - \beta_g(\bar{\rho}_l; S)](p_b/c_v)(1+\alpha)c_v$, or rearranging for $\bar{\rho}_h \geq \bar{\rho}_l$

$$\begin{aligned}&[\rho_h + \eta\bar{\rho}_h\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_h\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ + \beta_g(\bar{\rho}_h; S)p_b/c_v](1+\alpha)c_v \\ &\leq [\rho_l + \eta\bar{\rho}_l\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_l\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+ + \beta_g(\bar{\rho}_l; S)p_b/c_v](1+\alpha)c_v.\end{aligned} \tag{11}$$

With the optimal price $\mathcal{P}(\bar{\rho}_l)$, from (9). we have

$$\begin{aligned}&\Big([\rho_l + \eta\bar{\rho}_l\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_l\mathbb{E}(\gamma_b - S)^+]p \\ &-[\mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+ + \beta_g(\bar{\rho}_l; S)p_b/c_v](1+\alpha)c_v\Big)d(p) \\ &< (1+\alpha)c_f \quad \text{for } (1+\alpha)c_v \leq p < \mathcal{P}(\bar{\rho}_l).\end{aligned}$$

By (11),

$$\begin{aligned}&\Big([\rho_h + \eta\bar{\rho}_h\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_h\mathbb{E}(\gamma_b - S)^+]p \\ &-[\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ + \beta_g(\bar{\rho}_h; S)p_b/c_v](1+\alpha)c_v\Big)d(p) \\ &< (1+\alpha)c_f \quad \text{for } (1+\alpha)c_v \leq p < \mathcal{P}(\bar{\rho}_l).\end{aligned}$$

The left-hand side of this inequality is (9) written with $\bar{\rho} = \bar{\rho}_h$. The optimality equation for $\mathcal{P}(\bar{\rho}_h)$ cannot be satisfied by $p < \mathcal{P}(\bar{\rho}_l)$, so $\mathcal{P}(\bar{\rho}_h) \geq \mathcal{P}(\bar{\rho}_l)$.

b) We first prove the following claim: If $x(S) \leq \bar{\rho}_l < \bar{\rho}_m < \bar{\rho}_h \leq 1$, $\bar{\rho}_m - \bar{\rho}_l = \bar{\rho}_h - \bar{\rho}_m$ and $\mathcal{P}(\bar{\rho}_l) \geq \mathcal{P}(\bar{\rho}_m)$, then $\mathcal{P}(\bar{\rho}_m) \geq \mathcal{P}(\bar{\rho}_h)$.

If (11) were to hold strictly for $\bar{\rho}_l < \bar{\rho}_m$, it would result in $\mathcal{P}(\bar{\rho}_l) < \mathcal{P}(\bar{\rho}_m)$, which is the opposite of the hypothesis $\mathcal{P}(\bar{\rho}_l) \geq \mathcal{P}(\bar{\rho}_m)$ of the claim. This argument by contradiction indicates that (11) is reversed for $x(S) < \bar{\rho}_l \leq \bar{\rho}_m$:

$$\begin{aligned}&[\rho_l + \eta\bar{\rho}_l\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_l\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_l + \eta\bar{\rho}_l\gamma - \eta\bar{\rho}_l S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_l(\gamma_b - S))^+ + \beta_g(\bar{\rho}_l; S)p_b/c_v](1+\alpha)c_v \\ &\leq [\rho_m + \eta\bar{\rho}_m\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_m\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_m + \eta\bar{\rho}_m\gamma - \eta\bar{\rho}_m S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_m(\gamma_b - S))^+ + \beta_g(\bar{\rho}_l; S)p_b/c_v](1+\alpha)c_v.\end{aligned} \tag{12}$$

Collecting multipliers of p and c_v, we arrive at

$$[\rho_l - \rho_m + (\bar{\rho}_l - \bar{\rho}_m)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)]p \leq (\beta_h(\bar{\rho}_l; S) - \beta_h(\bar{\rho}_m; S))(1+\alpha)c_v.$$

From $\bar{\rho}_m - \bar{\rho}_l = \bar{\rho}_h - \bar{\rho}_m$, we get $\rho_m - \rho_h + (\bar{\rho}_m - \bar{\rho}_h)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+) = \rho_l - \rho_m + (\bar{\rho}_l - \bar{\rho}_m)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)$ and then

$$\begin{aligned}&[\rho_m - \rho_h + (\bar{\rho}_m - \bar{\rho}_h)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)]p = [\rho_l - \rho_m + (\bar{\rho}_l - \bar{\rho}_m)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)]p \\ &\leq (\beta_h(\bar{\rho}_l; S) - \beta_h(\bar{\rho}_m; S))(1+\alpha)c_v \leq (\beta_h(\bar{\rho}_m; S) - \beta_h(\bar{\rho}_h; S))(1+\alpha)c_v,\end{aligned}$$

where the last inequality is from the concavity of β_h in Lemma 4 and $\bar{\rho}_m - \bar{\rho}_l = \bar{\rho}_h - \bar{\rho}_m$. Writing the last inequality explicitly $[\rho_m - \rho_h + (\bar{\rho}_m - \bar{\rho}_h)(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)]p \leq (\mathbb{E}(\rho_m + \bar{\rho}_m\gamma - \bar{\rho}_m S)^+ -$

$\mathbb{E}(\rho_h + \bar{\rho}_h\gamma - \bar{\rho}_h S)^+ + (\mathbb{E}(\bar{\eta}\bar{\rho}_m(\gamma_b - S)^+ - \mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S)^+)) + [\beta_g(\bar{\rho}_m; S) - \beta_g(\bar{\rho}_h; S)]p_b/c_v)(1+\alpha)c_v$ and reorganizing it, we obtain a counterpart of (12) for $\bar{\rho}_m \leq \bar{\rho}_h$:

$$\begin{aligned}&[\rho_m + \eta\bar{\rho}_m\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_m\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_m + \eta\bar{\rho}_m\gamma - \eta\bar{\rho}_m S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_m(\gamma_b - S))^+ + \beta_g(\bar{\rho}_m; S)p_b/c_v](1+\alpha)c_v \\ &\leq [\rho_h + \eta\bar{\rho}_h\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}_h\mathbb{E}(\gamma_b - S)^+]p - [\mathbb{E}(\rho_h + \eta\bar{\rho}_h\gamma - \eta\bar{\rho}_h S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}_h(\gamma_b - S))^+ + \beta_g(\bar{\rho}_h; S)p_b/c_v](1+\alpha)c_v.\end{aligned} \tag{13}$$

This leads to $\mathcal{P}(\bar{\rho}_m) \geq \mathcal{P}(\bar{\rho}_h)$ for $\bar{\rho}_h > \bar{\rho}_m$. That is, starting with $\mathcal{P}(\bar{\rho}_l) \geq \mathcal{P}(\bar{\rho}_m)$, we have obtained $\mathcal{P}(\bar{\rho}_m) \geq \mathcal{P}(\bar{\rho}_h)$ and proved the claim. The claim allows us to extend the decreasing property to $[\bar{\rho}_b, 1]$. $\square$

Counterexample in Support of Lemma 5b) It can be shown that Lemma 5a) does not hold for $x(S) \leq \bar{\rho}_0 < \bar{\rho}_1$, i.e., $\mathcal{P}(\bar{\rho}_0) \geq \mathcal{P}(\bar{\rho}_1)$. Let $\gamma(p_b) = 1$ and $S \sim U[0, a]$ with $a = 5$ and so $x(S) = x(U[0, a]) = 1/3$ by Lemma 4. For a $\bar{\rho}$, the values of $\mathbb{E}(1 - S)^+$, $\mathbb{E}(1 - \bar{\rho}S)^+$ and $\mathbb{E}(\bar{\rho}(S-1)^+ \wedge \rho)$ can be calculated as follows.

$$\mathbb{E}(1 - S)^+ = \int_0^1 \frac{1}{5}(1 - u)du = \frac{1}{5}(u - \frac{u^2}{2})\bigg|_0^1 = \frac{1}{5}(1 - \frac{1}{2}) = 1/10,$$

$$\mathbb{E}(1 - \bar{\rho}S)^+ = \int_0^{1/\bar{\rho}} \frac{1}{5}(1 - \bar{\rho}u)du = \frac{1}{5}(u - \bar{\rho}\frac{u^2}{2})\bigg|_0^{1/\bar{\rho}} = \frac{1}{5}(\frac{1}{\bar{\rho}} - \frac{1}{2\bar{\rho}}) = 1/(10\bar{\rho}),$$

and from (2), we have $\mathbb{E}(\bar{\rho}(S-1)^+ \wedge \rho) = -1/(10\bar{\rho}) + \rho + \bar{\rho}/10$. For a general $\bar{\rho} \geq 1/3$, we have $\beta_d^{\bar{\rho}} = \rho + \bar{\rho}/10$ and $\beta_c^{\bar{\rho}}(p_b) = 1/(10\bar{\rho}) + (-1/(10\bar{\rho}) + \rho + \bar{\rho}/10)p_b/c_v$, and specialize (5) for linear demand $d(p) = 800 - 12p$

$$(\rho + \bar{\rho}/10)p/(1+\alpha) - c_v/(10\bar{\rho}) + \big(-1/(10\bar{\rho}) + \rho + \bar{\rho}/10\big)p_b = c_f/(800 - 12p).$$

With parameter values $c_v = 5$, $c_f = 1000$, $p_b = 35$ and $\alpha = 0.05$, the above equation has solutions $(\bar{\rho}_0, p_0) = (0.75, 31.478)$ and $(\bar{\rho}_1, p_1) = (0.95, 30.627)$. That is, $\mathcal{P}(\bar{\rho}_0) = p_0 = 31.478 > 30.627 = p_1 = \mathcal{P}(\bar{\rho}_1)$ for $\bar{\rho}_0 = 0.75 < 0.95 = \bar{\rho}_1$. $\diamond$

Proof of Theorem 3 Consider the explicit version of the price optimality equation (9)

$$\begin{aligned}&\big[\rho + \bar{\rho}(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}(\mathbb{E}(\gamma_b - S)^+))\big]\mathcal{P}(\bar{\rho})d(\mathcal{P}(\bar{\rho})) \\ &-(1+\alpha)\Big[\mathbb{E}(\rho + \eta\bar{\rho}\gamma - \eta\bar{\rho}S)^+ + \mathbb{E}(\bar{\eta}\bar{\rho}(\gamma_b - S)^+) + \mathbb{E}(\eta\bar{\rho}(S - \gamma)^+ \wedge \rho)\frac{p_b}{c_v}\Big]c_v d(\mathcal{P}(\bar{\rho})) = (1+\alpha)c_f.\end{aligned} \tag{14}$$

Specializing this for $\bar{\rho} = 1$ yields $d(\mathcal{P}(1))(\mathcal{P}(1) - (1+\alpha)c_v) = (1+\alpha)c_f/(\eta\mathbb{E}(\gamma - S)^+ + \bar{\eta}\mathbb{E}(\gamma_b - S)^+)$ and then $\mathcal{P}(1) \geq \mathcal{P}(0) = p^*$ by Theorem 1 as $\mathbb{E}(\gamma - S)^+ \vee \mathbb{E}(\gamma_b - S)^+ \leq 1$.

a) $\mathcal{R}(\mathcal{P}(0)) = 0$ implies that $\bar{\rho}^0 = 0$ is a fixed point, the market remains at this point ($\bar{\rho}^e = 0, p^e = \mathcal{P}(0)$).

We can define the inverse map $\mathcal{R}^{-1}(\cdot)$ uniquely as $\mathcal{R}^{-1}(\bar{\rho}) = \min\{x : \mathcal{R}(x) \geq \bar{\rho}\}$. Then because of increasing $\mathcal{R}^{-1}(\cdot)$, we have $p \geq \mathcal{R}^{-1}(\bar{\rho})$ if and only if $\mathcal{R}(p) \geq \bar{\rho}$. Let $p_{max} = \mathcal{R}^{-1}(1)$ be the smallest market price that converts all consumers to adopters. We allow for p_{max} to approach infinity if $\mathcal{R}(\bar{\rho}) < 1$ for all $\bar{\rho}$. For $p \geq p_{max}$, we have $\mathcal{R}(p) = 1$. Let p_{min} be the largest market price that keeps all consumers as nonadopters. Note $\mathcal{R}(\cdot) : [p_{min}, p_{max}] \to [0, 1]$ and $\mathcal{R}^{-1}(\cdot) : [0, 1] \to [p_{min}, p_{max}]$.

$\mathcal{R}(\cdot)$ and $\mathcal{P}(\cdot)$ are continuous. Continuity of $\mathcal{R}$ is directly from its definition. Continuity of $\mathcal{R}^{-1}$ is from the last paragraph and strictly increasing $\mathcal{R}$. Continuity of $\mathcal{P}$ is due to first the continuity of $\beta_d^{\bar{\rho}}$ and $\beta_c^{\bar{\rho}}(p_b)$ in $\bar{\rho}$, and then to the continuity of all terms in the optimal price equation.

If $\mathcal{P}(1) \geq p_{max}$, then $\mathcal{R}(\mathcal{P}(1)) \geq \mathcal{R}(p_{max}) = 1$ and $(1, p_{max})$ is a fixed point, where all consumers are adopters. If $\mathcal{P}(1) < p_{max}$, we consider the given condition $\mathcal{R}(\mathcal{P}(0)) > 0$. We must have $\mathcal{P}(0) > \mathcal{R}^{-1}(0)$;

otherwise $\mathcal{R}(\mathcal{P}(0)) \leq 0$ yields the contradiction $\mathcal{R}(\mathcal{P}(0)) = 0$. We have $\mathcal{P}(0) > \mathcal{R}^{-1}(0)$, $\mathcal{P}(1) < p_{max} = \mathcal{R}^{-1}(1)$ and continuity of $\mathcal{P}(\bar{\rho})$ and $\mathcal{R}^{-1}(\bar{\rho})$ for $\bar{\rho} \in (0,1)$. By the intermediate value theorem, $\mathcal{P}(\bar{\rho})$ and $\mathcal{R}^{-1}(\bar{\rho})$ cross each other at least once over $(0,1)$. This completes the proof of existence for a fixed point $\mathcal{R}(\mathcal{P}(\bar{\rho})) = \bar{\rho}$ and we can always pick the smallest as $\bar{\rho}^e$ when multiple fixed points exist. Since we start with $\bar{\rho}^0 = 0$, the iteration converges to the smallest fixed point $(\bar{\rho}^e, \mathcal{P}(\bar{\rho}^e))$. From Corollary 1, we obtain $p^* \leq p^e \leq p_0$, which combined with the unimodality of profit and $\beta^{\bar{\rho}} \geq 1$ give the profit bounds. By $\beta_d^{\bar{\rho}} \leq 1$ moreover, $\beta_d^{\bar{\rho}}(p^e - \beta^{\bar{\rho}} c_v)d(p^e) \leq (p_0 - \beta^{\bar{\rho}} c_v)d(p_0)$ and $c_f/\beta_d^{\bar{\rho}} \geq c_f$.

b) From Lemma 5, $\mathcal{P}$ is increasing and then decreasing. Recall that $\bar{\rho}_s$ is the adoption level at which $\mathcal{P}$ switches from increasing to decreasing with the understanding that $\bar{\rho}_s = 1$ when $\mathcal{P}$ remains to be increasing.

If $\mathcal{P}$ is decreasing at $\bar{\rho}^e$, we have $\bar{\rho}^e > \bar{\rho}_s \geq x(S)$ such that $\mathcal{P}(\bar{\rho})$ is increasing over $[0, \bar{\rho}_s]$ and decreasing over $[\bar{\rho}_s, \bar{\rho}^e]$. Then $(\bar{\rho}^n, p^n)$ are increasing in n as long as $\bar{\rho}^n \leq \bar{\rho}_s$, in particular for $\bar{\rho}^n \leq x(S)$. Once $\bar{\rho}^n > \bar{\rho}_s$, $\bar{\rho}^n$ or p^n can increase or decrease.

If $\mathcal{P}$ is increasing at $\bar{\rho}^e$, we can argue as above that both adoption level and price increase in n towards the fixed point $(\bar{\rho}^e, \mathcal{P}(\bar{\rho}^e))$.

c) $\mathrm{P}(S \leq \gamma) = 1$ in condition i) implies $\beta_g(x;S) = 0$, $\beta^{\bar{\rho}} = 1$ and $\beta_d^{\bar{\rho}} = 1 - \bar{\rho} + \eta(\gamma - \mathbb{E}[S])\bar{\rho} + \bar{\eta}\mathbb{E}[\gamma_b - S]^+\bar{\rho}$, which decreases linearly in $\bar{\rho}$. This decrease pulls $c_f/\beta_d^{\bar{\rho}}$ up in (5) and in turn increases prices by Theorem 1. Then, $\mathcal{P}(\bar{\rho})$ is increasing in $\bar{\rho} \in [0,1]$. So prices and adoption levels increase towards the equilibrium.

Under condition ii), $p_b = c_v$ so

$$\begin{aligned}
d^{s,\bar{\rho}}(p;c_v) &= [\rho + \eta\bar{\rho}\mathbb{E}(\gamma - S)^+ + \bar{\eta}\bar{\rho}\mathbb{E}(\gamma_b - S)^+]d(p) = \beta_d^{\bar{\rho}} d(p); \\
c^{s,\bar{\rho}}(p;c_v) &= c_f + \left[\mathbb{E}(\rho + \eta\bar{\rho}(\gamma - S))^+ + \bar{\eta}\bar{\rho}\mathbb{E}(\gamma_b - S)^+ + \mathbb{E}(\eta\bar{\rho}(S-\gamma)^+ \wedge \rho)\right] c_v d(p) = c_f + c_v d^{s,\bar{\rho}}(p;c_v),
\end{aligned}$$

where the last equality is due to first (2) and then to the equality of the terms in the square brackets. Then $\beta^{\bar{\rho}} = 1$ and $\beta_d^{\bar{\rho}} = 1 - \bar{\rho} + \eta\mathbb{E}(\gamma - S)^+\bar{\rho} + \bar{\eta}\mathbb{E}(\gamma_b - S)^+\bar{\rho}$, which decreases linearly in $\bar{\rho}$. The rest of the argument is identical to the argument above for the condition i).

In the remainder we consider condition iii). $\mathcal{P}(\cdot)$ is increasing over $[0,1]$ if it is increasing around $\bar{\rho} = 1$ by Lemma 5b. We use the elasticity of demand at the price $\mathcal{P}(\bar{\rho} = 1)$ to obtain this increasing property at $\bar{\rho} = 1$. Taking derivative of both sides of (14) with respect to $\bar{\rho}$ and inserting $\gamma = \eta = 1$ lead to

$$\begin{aligned}
0 &= -(1 - \mathbb{E}(1-S)^+)\mathcal{P}(\bar{\rho})d(\mathcal{P}(\bar{\rho})) + \left[1 - \bar{\rho}(1 - \mathbb{E}(1-S)^+)\right]\left[\frac{\partial \mathcal{P}(\bar{\rho})}{\partial \bar{\rho}} d(\mathcal{P}(\bar{\rho})) + \mathcal{P}(\bar{\rho})\frac{\partial d(p)}{\partial p}\Big|_{p=\mathcal{P}(\bar{\rho})} \frac{\partial \mathcal{P}(\bar{\rho})}{\partial \bar{\rho}}\right] \\
&\quad - (1+\alpha)\frac{\partial}{\partial \bar{\rho}}\left[\mathbb{E}(1 - \bar{\rho}S)^+ + \mathbb{E}(\bar{\rho}(S-1)^+ \wedge \rho)\frac{p_b}{c_v}\right] c_v d(\mathcal{P}(\bar{\rho})) \\
&\quad - (1+\alpha)\left[\mathbb{E}(1 - \bar{\rho}S)^+ + \mathbb{E}(\eta\bar{\rho}(S-1)^+ \wedge \rho)\frac{p_b}{c_v}\right] c_v \frac{\partial d(p)}{\partial p}\Big|_{p=\mathcal{P}(\bar{\rho})} \frac{\partial \mathcal{P}(\bar{\rho})}{\partial \bar{\rho}}\Bigg].
\end{aligned}$$

First note

$$\begin{aligned}
&\frac{\partial}{\partial \bar{\rho}}\left[\mathbb{E}(1 - \bar{\rho}S)^+ + \mathbb{E}(\bar{\rho}(S-1)^+ \wedge \rho)\frac{p_b}{c_v}\right] \\
&= \frac{\partial}{\partial \bar{\rho}}\mathbb{E}\mathbb{1}_{S \leq 1/\bar{\rho}}(1 - \bar{\rho}S) + \frac{p_b}{c_v}\frac{\partial}{\partial \bar{\rho}}\bar{\rho}\mathbb{E}\mathbb{1}_{S \geq 1}\mathbb{1}_{S \leq 1/\bar{\rho}}(S-1) + \frac{p_b}{c_v}\frac{\partial}{\partial \bar{\rho}}(1 - \bar{\rho})\mathbb{E}\mathbb{1}_{S \geq 1/\bar{\rho}} \\
&= -\mathbb{E}\mathbb{1}_{S \leq 1/\bar{\rho}}S + \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{1 \leq S \leq 1/\bar{\rho}}(S-1) - \frac{p_b}{c_v}\bar{\rho}(1/\bar{\rho}^2)(1/\bar{\rho} - 1)f_S(1/\bar{\rho}) - \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{S \geq 1/\bar{\rho}} + \frac{p_b}{c_v}(1 - \bar{\rho})(1/\bar{\rho}^2)f_S(1/\bar{\rho}) \\
&= -\mathbb{E}\mathbb{1}_{S \leq 1/\bar{\rho}}S + \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{1 \leq S \leq 1/\bar{\rho}}(S-1) - \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{S \geq 1/\bar{\rho}}.
\end{aligned}$$

We can insert the last equality in the derivative of (14) and set $\bar{\rho} = 1$.

$$0 = -(1 - \mathbb{E}(1-S)^+)\mathcal{P}(1)d(\mathcal{P}(1)) + \mathbb{E}(1-S)^+\frac{\partial \mathcal{P}(\bar{\rho})}{\partial \bar{\rho}}\left[d(\mathcal{P}(1)) + \mathcal{P}(1)\frac{\partial d(p)}{\partial p}\Big|_{p=\mathcal{P}(1)}\right]$$

$$+ (1+\alpha)\Big[\mathbb{E}\mathbb{1}_{S\le 1}S + \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{S\ge 1}\Big]c_v d(\mathcal{P}(1)) - (1+\alpha)\Big[\mathbb{E}(1-S)^+\Big]c_v \frac{\partial d(p)}{\partial p}\mid_{p=\mathcal{P}(1)} \frac{\partial \mathcal{P}(\bar\rho)}{\partial \bar\rho}\Big].$$

The derivative of (14) at $\bar\rho = 1$ can be rewritten as

$$d(\mathcal{P}(1))\ LHS = \mathbb{E}(1-S)^+ \frac{\partial \mathcal{P}(\bar\rho)}{\partial \bar\rho}\ RHS$$

where

$$\begin{aligned} LHS &= [1-\mathbb{E}(1-S)^+]\mathcal{P}(1) - (1+\alpha)\Big[\mathbb{E}\mathbb{1}_{S\le 1}S + \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{S\ge 1}\Big]c_v. \\ RHS &= d(\mathcal{P}(1)) + \frac{\partial d(p)}{\partial p}\mid_{p=\mathcal{P}(1)} (\mathcal{P}(1) - (1+\alpha)c_v) \\ &= \frac{d(\mathcal{P}(1))}{\mathcal{P}(1)}\left(\mathcal{P}(1) + \frac{d'(\mathcal{P}(1))}{d(\mathcal{P}(1))/\mathcal{P}(1)}(\mathcal{P}(1) - (1+\alpha)c_v)\right) \\ &= \frac{d(\mathcal{P}(1))}{\mathcal{P}(1)}\big(\mathcal{P}(1) - \epsilon_d(\mathcal{P}(1))(\mathcal{P}(1) - (1+\alpha)c_v)\big). \end{aligned}$$

The elasticity at $\mathcal{P}(1)$ is denoted by $\epsilon_d(\mathcal{P}(1))$. The price is increasing in $\bar\rho$ at $\bar\rho = 1$ if $LHS, RHS \ge 0$.

$LHS \ge 0$ if $\mathcal{P}(1) \ge (1+\alpha)p_b$, because

$$[1-\mathbb{E}(1-S)^+]\mathcal{P}(1) - (1+\alpha)\Big[\mathbb{E}\mathbb{1}_{S\le 1}S + \frac{p_b}{c_v}\mathbb{E}\mathbb{1}_{S\ge 1}\Big]c_v \ge \mathbb{E}(S\wedge 1)\mathcal{P}(1) - (1+\alpha)\Big[\frac{p_b}{c_v}\mathbb{E}(S\wedge 1)\Big]c_v.$$

□

Note that $\gamma(p_b) = \eta = 1$ in the price mechanism section, whose proofs are below.

Proof of Lemma 6 Since $p_b' < p_b$, we have $\mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b'/c_v \le \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b/c_v$ for each $\bar\rho$. By adding $\mathbb{E}(1-\bar\rho S)^+$ to both sides of inequality and multiplying by $-(1+\alpha)c_v$, we have $-[\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b'/c_v](1+\alpha)c_v \ge -[\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b/c_v](1+\alpha)c_v$, which leads to

$$\begin{aligned} &[\rho + \bar\rho\mathbb{E}(1-S)^+]p - [\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b/c_v](1+\alpha)c_v \\ &\quad \le [\rho + \bar\rho\mathbb{E}(1-S)^+]p - [\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b'/c_v](1+\alpha)c_v. \end{aligned} \tag{15}$$

With the optimal price $\mathcal{P}(\bar\rho; p_b')$ of the solar model, we have

$$\Big([\rho+\bar\rho\mathbb{E}(1-S)^+]p - [\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b'/c_v](1+\alpha)c_v\Big)d(p) < (1+\alpha)c_f \quad \text{for } (1+\alpha)c_v \le p < \mathcal{P}(\bar\rho, p_b').$$

By (15),

$$\Big([\rho+\bar\rho\mathbb{E}(1-S)^+]p - [\mathbb{E}(1-\bar\rho S)^+ + \mathbb{E}(\bar\rho(S-1)^+\wedge\rho)p_b/c_v](1+\alpha)c_v\Big)d(p) < (1+\alpha)c_f \quad \text{for } (1+\alpha)c_v \le p < \mathcal{P}(\bar\rho, p_b').$$

The above inequality becomes the price optimality equation if it is turned to equality. The optimality equation for $\mathcal{P}(\bar\rho, p_b)$ cannot be satisfied by $p < \mathcal{P}(\bar\rho, p_b')$, so $\mathcal{P}(\bar\rho, p_b) \ge \mathcal{P}(\bar\rho, p_b')$. Hence, $\mathcal{P}(\bar\rho; p_b)$ is increasing in p_b for every $\bar\rho$.

$\mathcal{R}(p; p_b)$ is increasing in p_b by our construction. Since $\mathcal{P}(\bar\rho; p_b)$ and $\mathcal{R}(p; p_b)$ increase in p_b so does $\mathcal{P}(\mathcal{R}(\bar\rho; p_b); p_b)$ for every $\bar\rho$. □

Proof of Theorem 4 $\mathcal{P}(\cdot; p_b)$ is continuous in p_b because it solves for the price p in $pd(p) - c_f/\beta_d^{\bar\rho} = (1+\alpha)c_v d(p)\beta\bar\rho(p_b)$ where $\beta\bar\rho(p_b)$ is continuous in p_b. $\mathcal{R}(\cdot; p_b)$ is also continuous in p_b. Combining these, $\mathcal{P}\circ\mathcal{R}(p; p_b)$ is continuous in p_b for each p. By this continuity, $\mathcal{P}\circ\mathcal{R}(\hat p^t; p_b^{min}) \le \hat p^t \le \mathcal{P}\circ\mathcal{R}(\hat p^t; p_b^{max})$ and the intermediate value theorem, there exists a buyback price p_b^t in $[p_b^{min}, p_b^{max}]$ such that $\mathcal{P} \circ \mathcal{R}(\hat{p}^t; p_b^t) = \hat{p}^t$. Uniqueness of p_b^t follows from Lemma 6; in particular, $\mathcal{P} \circ \mathcal{R}(\cdot; p_b)$ is increasing in p_b. □

Proof of Lemma 7 a) Increase of the optimal regulated price with respect to p_b is analogous to that in Lemma 6. So we focus on its change with respect to λ. The optimal regulated price solves $(p - (1+\alpha)c_v\beta^{\bar{\rho}}(p_b))d(p) = (1+\alpha)\lambda c_f/\beta_d^{\bar{\rho}}$. The left-hand side is unimodal and increasing in p for feasible p, see also Figure 2. When λ increases, the optimal price equality can be maintained only by increasing p. So, $\mathcal{P}(\cdot; p_b, \lambda)$ increases in λ. To show that $\mathcal{R}(\cdot; p_b, \lambda)$ is increasing in p_b and λ, we examine the right hand-side of (7). This right-hand side increases in both p_b and λ, and so does $\mathcal{R}(\cdot; p_b, \lambda)$.

b) We first provide a proof for a general p_b^{min}. We consider increasing buyback price and indirectly allocated fixed charge ratio from $(p_b^{min}, 0)$ to (p_b, λ) on the straight line piece represented as $(p_b^{min}, 0) + \lambda(p_b - p_b^{min}, 1)$ for $\lambda \in [0,1]$. Projecting $\mathcal{P} \circ \mathcal{R}(p^n; p_b, \lambda)$ on this line piece, we have the price parameterized by only λ: $\mathcal{P} \circ \mathcal{R}(p^n; (p_b^{min}, 0) + \lambda(p_b - p_b^{min}, 1))$. From $\mathcal{P} \circ \mathcal{R}(p^n; (p_b^{min}, 0)) \le p^n \le p^{n+1} \le \mathcal{P} \circ \mathcal{R}(p^n; (p_b^{min}, 0) + (p_b - p_b^{min}, 1))$ and the intermediate value theorem, we know the existence of λ^t such that $p^n = \mathcal{P} \circ \mathcal{R}(p^n; (p_b^{min}, 0) + \lambda^t(p_b - p_b^{min}, 1))$ and the buyback price $p_b^{min} + \lambda^t(p_b - p_b^{min})$ stops the spiral. If $p_b \ge c_v$, the condition in the lemma is trivially satisfied with $p_b^{min} = c_v$ as $\mathcal{P} \circ \mathcal{R}(p^n; (c_v, 0)) = (1+\alpha)c_v \le p^n$. □

Proof of Theorem 5 We first show how the price p^* is achieved. We already know $\beta^{\bar{\rho}}(p_b = c_v) = \beta_c^{\bar{\rho}}(p_b = c_v)/\beta_d^{\bar{\rho}}(p_b = c_v) = 1$. Since λ is set equal to $1 - \bar{\rho}\mathbb{E}(S \wedge 1)$, we have $\beta_d^{\bar{\rho}}(c_v) = \lambda$. With $\beta^{\bar{\rho}}(c_v) = 1$ and $\beta_d^{\bar{\rho}}(c_v) = \lambda$, the optimality equation (6) becomes $pd(p) = (1+\alpha)(c_v d(p) + c_f)$, which is identical to the optimality equation in the base model. Hence, $p_b = c_v$ and $\lambda = 1 - \bar{\rho}\mathbb{E}(S \wedge 1)$ yield the optimal price of p^*. The spiral then remains at $\bar{\rho}$ and the cost recovery condition becomes

$$\max_p \big(\beta_d^{\bar{\rho}^n}(c_v)p - \beta_c^{\bar{\rho}^n}(c_v)c_v\big)d(p) = \max_p \big(\lambda p - \lambda c_v\big)d(p) \ge \lambda c_f.$$

This condition is the same as the solvency condition $\max_p (p - c_v)d(p) \ge c_f$ in the base model.

What remains is the specification of $\bar{\rho}$. For the market price of p^* and the buyback price of c_v, setting $\lambda = 1 - \bar{\rho}\mathbb{E}(S \wedge 1)$ yields $\mathcal{R}(p^*; c_v, 1 - \bar{\rho}\mathbb{E}(S \wedge 1))$, i.e.,

$$\mathcal{R}(p^*; c_v, 1 - \bar{\rho}\mathbb{E}(S \wedge 1)) \;=\; \mathrm{P}\left(L \le p^* - (p^* - c_v)\left(1 - \frac{1}{S}\right)^+ - \frac{(1+\alpha)c_f}{d(p^*)}\frac{\mathbb{E}(S \wedge 1)}{S}\right).$$

From $\bar{\lambda} = \bar{\rho}\mathbb{E}(S \wedge 1)$, we obtain

$$\kappa = \frac{\bar{\lambda}}{\bar{\rho}}\frac{(1+\alpha)c_f}{N} = \mathbb{E}(S \wedge 1)\frac{(1+\alpha)c_f}{N}.$$

□

Proof of Lemma 8 For a general ratio, the formula for root follows from the associated quadratic equation given above the lemma. The smaller root of quadratic equation as $bp^2 - \big[a + (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v b\big]p + \big[a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v + (1+\alpha)c_f/\beta_d^{\bar{\rho}}\big] = 0$ is

$$p \;=\; a/(2b) + (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/2 - \sqrt{\Big(a/(2b) + (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/2\Big)^2 - a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/b - (1+\alpha)c_f/\beta_d^{\bar{\rho}}b},$$

which is nonnegative if real.

We need to show that the discriminant $\big(a/(2b) + (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/2\big)^2 - a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/b - (1+\alpha)c_f/\beta_d^{\bar{\rho}}b \ge 0$. This can be done by writing the return bound as

$$\alpha + 1 \le \bar{\alpha}^{s,\bar{\rho}} + 1 = \frac{a^2 - \beta^{\bar{\rho}^2}(p_b)b^2c_v^2}{2b\beta^{\bar{\rho}}(p_b)c_v(a - b\beta^{\bar{\rho}}(p_b)c_v) + 4bc_f/\beta_d^{\bar{\rho}}},$$

which leads to $(1+\alpha)[2b\beta^{\bar{\rho}}(p_b)c_v(a-b\beta^{\bar{\rho}}(p_b)c_v)+4bc_f/\beta^{\bar{\rho}}_d] \leq (a^2-\beta^{\bar{\rho}^2}(p_b)b^2c_v^2)$. Dividing both sides by $4b^2$ gives $(1+\alpha)[a\beta^{\bar{\rho}}(p_b)c_v/(2b)-\beta^{\bar{\rho}^2}(p_b)c_v^2/2+c_f/(\beta^{\bar{\rho}}_d b)] \leq (a/(2b))^2-\beta^{\bar{\rho}^2}(p_b)(c_v/2)^2$, or equivalently

$$(a/(2b))^2-\beta^{\bar{\rho}^2}(p_b)(c_v/2)^2-(1+\alpha)a\beta^{\bar{\rho}}(p_b)c_v/(2b)+(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v^2/2-(1+\alpha)c_f/(\beta^{\bar{\rho}}_d b) \geq 0.$$

By adding $(1+\alpha)^2\beta^{\bar{\rho}^2}(p_b)c_v^2/4$ and $a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/(2b)$ to the left-hand side and deducting the same,

$$\begin{aligned}(1+\alpha)^2\beta^{\bar{\rho}^2}(p_b)c_v^2/4-(1+\alpha)^2\beta^{\bar{\rho}^2}(p_b)c_v^2/4+a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/(2b)-a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/(2b)+(a/(2b))^2\\ -\beta^{\bar{\rho}^2}(p_b)(c_v/2)^2-(1+\alpha)a\beta^{\bar{\rho}}(p_b)c_v/(2b)+(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v^2/2-(1+\alpha)c_f/(\beta^{\bar{\rho}}_d b) \geq 0.\end{aligned}$$

Reorganizing terms gives us $(a/(2b)+(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/2)^2-a(1+\alpha)\beta^{\bar{\rho}}(p_b)c_v/b-(1+\alpha)c_f/(b\beta^{\bar{\rho}}_d) \geq \beta^{\bar{\rho}^2}(p_b)c_v^2\alpha^2/4 \geq 0$. Hence, the discriminant is nonnegative and the roots are real.

The regulated utility breaks even when it makes zero profit at the optimal price $p_0^{s,\bar{\rho}}$, i.e., $[p_0^{s,\bar{\rho}}\beta^{\bar{\rho}}_d-c_v\beta^{\bar{\rho}}_c(p_b)]d(p_0^{s,\bar{\rho}})=c_f$. Dividing both sides by $\beta^{\bar{\rho}}_d$ gives $[p_0^{s,\bar{\rho}}-c_v\beta^{\bar{\rho}}(p_b)]d(p_0^{s,\bar{\rho}})=c_f/\beta^{\bar{\rho}}_d$. By substituting $p_0^{s,\bar{\rho}}=a/(2b)+\beta^{\bar{\rho}}(p_b)c_v/2$ and $d(p_0^{s,\bar{\rho}})=a/2-\beta^{\bar{\rho}}(p_b)bc_v/2$, we obtain

$$(a/(2b)-\beta^{\bar{\rho}}(p_b)c_v/2)(a/2-\beta^{\bar{\rho}}(p_b)bc_v/2)=c_f/\beta^{\bar{\rho}}_d.$$

Solving for c_f yields the break-even fixed cost expression in the lemma statement.

For a small ratio, $\beta^{\bar{\rho}}(p_b)=1$ and $\beta^{\bar{\rho}}_d=(1-\bar{\rho}\bar{s})$. Then

$$p \;=\; a/(2b)+(1+\alpha)c_v/2-\sqrt{\Big(a/(2b)+(1+\alpha)c_v/2\Big)^2-(1+\alpha)ac_v/b-(1+\alpha)c_f/(b(1-\bar{\rho}\bar{s}))}.$$

This is nonnegative, independent of p_b and increases in $\bar{\rho}$. ☐

Appendix B: Supplementary Material

Random Demand

We can adapt our model to a situation where demand is random by introducing the multiplicative noise ξ whose expected value is 1. Then, we must consider the expected profits and the expected revenues in the utility problem. For multiplicative and independent ξ, we can separate its expectation from other expectations. For example, revenue random variable is $p\xi d(p)$ and expected revenue is $pd(p)$. The utility problem becomes

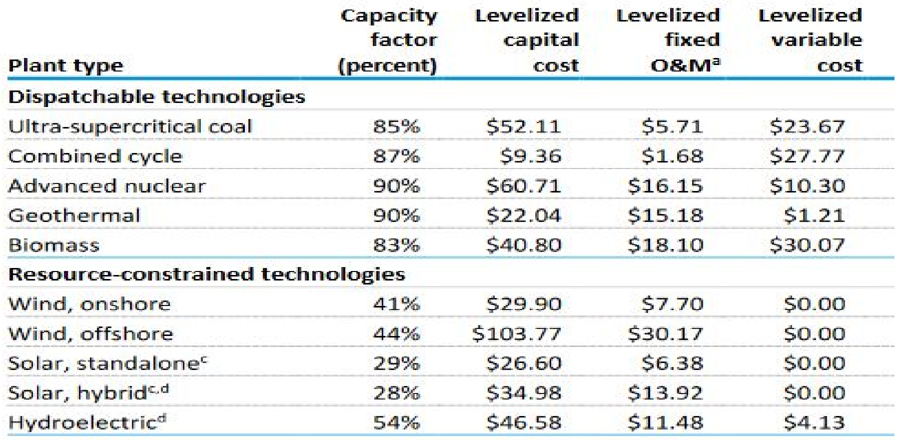
$$\max_{p\geq 0} \left\{\mathbb{E}p\xi d(p) - \mathbb{E}(c_v\xi d(p) + c_f) : \ \mathbb{E}p\xi d(p) \in \mathbb{E}(c_v\xi d(p) + c_f)[1, 1+\alpha]\right\},$$

which is equivalent to (1). This equivalence breaks down if the objective or the constraint is modified to incorporate risk sensitivity, e.g., through a chance constraint for the allowed revenue.

In the utility problem with adopters, the random utility demand is equal to $\beta_d^{\bar{\rho}}(p_b)\xi d(p)$ and the random utility cost is $\beta_c^{\bar{\rho}}(p_b)c_v\xi d(p) + c_f$. Then, the expected utility demand and generation and buyback-related cost in the solar model are $d^{s,\bar{\rho}}(p;p_b) = \beta_d^{\bar{\rho}}(p_b)d(p)$ and $c^{s,\bar{\rho}}(p;p_b) = \beta_c^{\bar{\rho}}(p_b)\ c_v d(p) + c_f$. All of our results holds under the random demand setting.

Buyback Price Exceeds Variable Cost

We first establish the value of variable cost c_v by using the data in Figure 9. Numbers in the figure are from Table 1b in the EIA report titled "Levelized Costs of New Generation Resources in the Annual Energy Outlook 2022"[1]. The highest variable cost there is 3 ¢/kWh for Biomass technology, which generates a small portion of energy in most markets. Taking other technologies into account, we can safely claim that the variable cost is 3 or below. Then we check the value of wholesale electricity price. The lowest zonal average wholesale price in Ercot market –our motivating case– in 2021 is in the West region and it is \$35.51/MWh on page xi of a Potomac Economics Report titled "2021 State of the Market Report for the Ercot Electricity Markets"[2], or 3.5 ¢/kWh. Hence, the highest variable cost is less than the lowest wholesale price, or the wholesale price exceeds the variable cost.



Plant type	Capacity factor (percent)	Levelized capital cost	Levelized fixed O&M^{a}	Levelized variable cost
Dispatchable technologies				
Ultra-supercritical coal	85%	\$52.11	\$5.71	\$23.67
Combined cycle	87%	\$9.36	\$1.68	\$27.77
Advanced nuclear	90%	\$60.71	\$16.15	\$10.30
Geothermal	90%	\$22.04	\$15.18	\$1.21
Biomass	83%	\$40.80	\$18.10	\$30.07
Resource-constrained technologies				
Wind, onshore	41%	\$29.90	\$7.70	\$0.00
Wind, offshore	44%	\$103.77	\$30.17	\$0.00
Solar, standalone^{c}	29%	\$26.60	\$6.38	\$0.00
Solar, hybrid^{c,d}	28%	\$34.98	\$13.92	\$0.00
Hydroelectric^{d}	54%	\$46.58	\$11.48	\$4.13

Figure 9: The largest variable cost of \$30.07/MWh corresponds to 3 cents per kWh.

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[1] Available at https://www.eia.gov/outlooks/aeo/pdf/electricity_generation.pdf.

[2] Available at https://www.potomaceconomics.com/wp-content/uploads/2022/05/2021-State-of-the-Market-Report.pdf

As mentioned in Callander et al. (2018), buyback prices are often set equal to the retail market price or the wholesale market price. This is validated by Pickerel (2022)[3] and the Ercot buyback prices listed at the Quickelectricity website[4]. This website also lists buyback prices other than the retail or wholesale market price, an example is Rhythm energy in Figure 10. The buyback price 10.2 ¢/kWh of Rhythm energy is well above the wholesale price of 3.5 ¢/kWh. So, the buyback price is not below the wholesale price.

Rhythm Energy	Solar Buyback 12	10.2¢ / kWh	21.1¢ / kWh	Available for solar power systems below 20 kW. No solar buyback limit, and unused credits are rolled over to the next month.

Figure 10: Rhythm energy pays buyback price of 10.2 cents per kWh and charges 21.1 cents per kWh.

Combining the conclusions of the last two paragraphs based on empirical data, we conclude that the buyback price exceeds the variable cost, i.e., $p_b \geq c_v$.

The inequality of $p_b \geq c_v$ is not surprising as the regulators suggest competitive buyback prices to promote homeowners' solar generation. If the inequality fails with a hypothetically low buyback price, this low buyback price –rather than promoting such solar generation– turns the homeowners to cheap energy suppliers for the utility. This hypothetical scenario is not the intention of regulators who hence suggest and insist on $p_b \geq c_v$. With this deduction through contradiction, we also arrive at $p_b \geq c_v$.

Additional Justification for the Game

For the one-shot version of (5), we write β functions only in terms of prices. With $\beta_d(p;p_b) = 1 - \mathcal{R}(p;p_b)\mathbb{E}(S \wedge 1)$, the demand becomes $d^s(p;p_b) = \beta_d(p;p_b)d(p)$. Note that $d^s(p;p_b) \leq d(p)$ and decreases in p_b. That is, a high buyback price increases the adoption level and reduces the demand. The cost becomes

$$c^s(p;p_b) = c_v\beta_c(p;p_b)d(p) + c_f,$$

where $\beta_c(p;p_b) = 1 - \mathcal{R}(p;p_b)\mathbb{E}(S \wedge 1) + \mathbb{E}(\mathcal{R}(p;p_b)(S-1)^+ \wedge (1 - \mathcal{R}(p;p_b)))p_b/c_v$ and $\beta_c(p;c_v) = \beta_d(p;c_v)$. Letting $\beta(p;p_b) = \beta_c(p;p_b)/\beta_d(p;p_b)$, we have $\beta(p;p_b) \geq 1$ for $p_b \geq c_v$. The solar model still applies but with the demand $d^s(p;p_b)$ and cost $c^s(p;p_b)$.

If the profit $(p\beta_d(p;p_b) - c_v\beta_c(p;p_b))d(p)$ with no fixed cost is unimodal in the price, we can proceed as in Theorem 1 to obtain the (one-shot) optimal price equation

$$pd(p) = (1+\alpha)(c_v\beta(p;p_b)d(p) + c_f/\beta_d(p;p_b)). \tag{16}$$

In a particular implementation, the unimodality of profit can be checked by plotting it for given parameters. If the unimodality holds, this paragraph specifies the market price. If the unimodality fails, we can instead consider the game of §2.2.2.

Recall that the game starts with $\bar{\rho}^0 = 0$ and proceeds as $p^n = \mathcal{P}(\bar{\rho}^n;p_b)$ and $\bar{\rho}^n = \mathcal{R}(p^{n-1};p_b)$. It then indirectly describes the retail price of (16) because of the next result. This description further justifies the game motivated independently in the main body.

Lemma 9. *A price p^e that solves $\mathcal{P}(\mathcal{R}(p^e;p_b);p_b) = p^e$ also solves (16).*

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[3] UPDATE: Which states offer net metering?. Available at https://www.solarpowerworldonline.com/2020/03/which-states-offer-net-metering/

[4] https://quickelectricity.com/2018-solar-panel-incentives-texas-net-metering-buyback-programs

Proof Let $\bar{\rho}^e = \mathcal{R}(p^e;p_b)$ for the given p^e. Since $p^e = \mathcal{P}(\bar{\rho}^e;p_b)$, p^e satisfies $p^e d(p^e) = (1+\alpha)$ $(\beta^{\bar{\rho}^e}(p_b)c_v d(p^e) + c_f/\beta_d^{\bar{\rho}^e})$. In view of (16), it suffices to show two equalities $\beta_d(p^e;p_b) = \beta_d^{\bar{\rho}^e}$ and $\beta(p^e;p_b) = \beta^{\bar{\rho}^e}(p_b) = \beta_c^{\bar{\rho}^e}(p_b)/\beta_d^{\bar{\rho}^e}$. For the first equality, we proceed as $\beta_d(p^e;p_b) = 1 - \mathcal{R}(p^e;p_b)\mathbb{E}(S \wedge 1) = 1 - \bar{\rho}^e\mathbb{E}(S \wedge 1) = \beta_d^{\bar{\rho}^e}$. Using $\beta_d(p^e;p_b) = \beta_d^{\bar{\rho}^e}$ to obtain the second equality $\beta(p^e;p_b) = \beta_c^{\bar{\rho}^e}(p_b)/\beta_d^{\bar{\rho}^e}$, we note $\beta(p^e;p_b) = \beta_c(p^e;p_b)/\beta_d(p^e;p_b) = \beta_c(p^e;p_b)/\beta_d^{\bar{\rho}^e}$. Hence, we need to show $\beta_c(p^e;p_b) = \beta_c^{\bar{\rho}^e}(p_b)$. This follows from $\beta_c(p^e;p_b) = 1 - \mathcal{R}(p^e;p_b)\mathbb{E}(S \wedge 1) + (\mathcal{R}(p^e;p_b)\mathbb{E}(S-1)^+ \wedge (1 - \mathcal{R}(p^e;p_b)))p_b/c_v = 1 - \bar{\rho}^e\mathbb{E}(S \wedge 1) + (\bar{\rho}^e\mathbb{E}(S-1)^+ \wedge \rho^e)p_b/c_v = \beta_c^{\bar{\rho}^e}(p_b)$. ☐

Shape of β Functions

For clear plots, we set $\gamma(p_b) = 1$. The demand multiplier $\beta_d^{\bar{\rho}} = 1 - \bar{\rho}\mathbb{E}[S \wedge 1]$ decreases linearly in $\bar{\rho}$. The cost multiplier $\beta_c^{\bar{\rho}} = 1 - \bar{\rho}\mathbb{E}[S \wedge 1] + \mathbb{E}[\bar{\rho}(S-1)^+ \wedge \rho]p_b/c_v = \beta_d^{\bar{\rho}} + \mathbb{E}[\bar{\rho}(S-1)^+ \wedge \rho]p_b/c_v$ coincides with the demand multiplier $\beta_d^{\bar{\rho}}$ at $\bar{\rho} \in \{0,1\}$. For $\bar{\rho} \in (0,1)$, the cost multiplier is larger, i.e., $\beta_c^{\bar{\rho}} \geq \beta_d^{\bar{\rho}}$. As a minimum of two linear functions in $\bar{\rho}$, the function $[\bar{\rho}(S-1)^+] \wedge [1-\bar{\rho}]$ is concave in $\bar{\rho}$ for each realization of S. This concavity remains intact under expectation with respect to S, multiplication by the positive term p_b/c_v and addition of $\beta_d^{\bar{\rho}}$, hence the cost multiplier $\beta_c^{\bar{\rho}}$ is concave in $\bar{\rho}$. The ratio $\beta^{\bar{\rho}} = \beta_c^{\bar{\rho}}/\beta_d^{\bar{\rho}}$ is 1 at $\bar{\rho} \in \{0,1\}$ and no less otherwise. We have $\beta_d^{\bar{\rho}} \leq \beta_c^{\bar{\rho}} \leq \beta^{\bar{\rho}}$, see Figure 11. This figure is drawn for $p_b = 11$¢, $c_v = 4$¢ and S is uniformly distributed over $[0.4, 2]$ on the left and over $[0.4, 4]$ on the right.

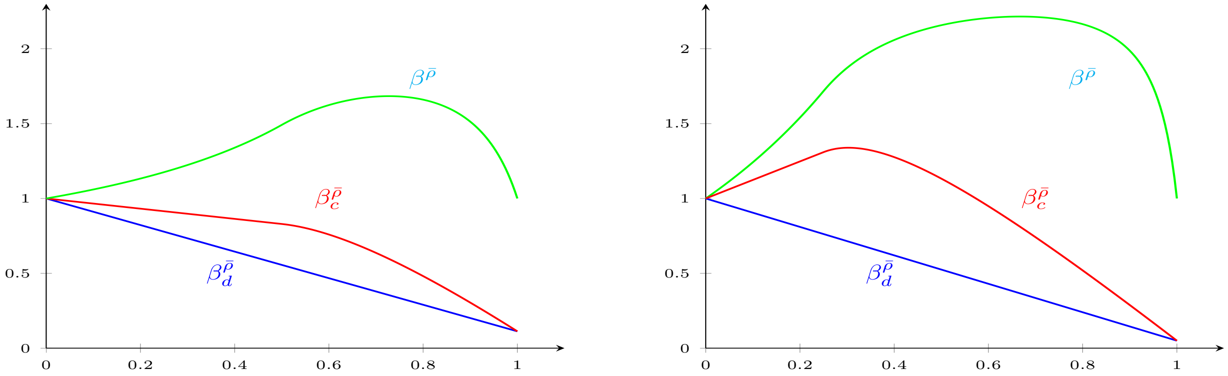


Figure 11: β Functions. S is uniform over $[0.4, 2]$ on the left and over $[0.4, 4]$ on the right.

$\beta^{\bar{\rho}}$ must decrease down to 1, while $\beta_d^{\bar{\rho}}$ is also decreasing, as $\bar{\rho}$ approaches 1. If the decrease in $\beta^{\bar{\rho}}$ dominates that in $\beta_d^{\bar{\rho}}$, the price solution of (5) decreases as $\bar{\rho}$ increases towards 1. This explains the dropping price for $\bar{\rho} > x(S)$ in Figure 5.

Justification for Using the Limit of Levelized Cost in the Game

We obtain in §2.2.1 that forward-looking consumers considering the relative surplus switch with probability

$$\mathrm{P}\left(\left(t_L(L^m - L^{m+1})\right) \vee L^m \leq p(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+\right).$$

This probability can be denoted by $\mathcal{R}_m(p)$. Because of stochastically decreasing L^m and $L^m - L^{m+1}$, we obtain $\mathcal{R}_m(p) \leq \mathcal{R}_{m+1}(p)$. The limit of the levelized cost as $m \to \infty$ is L and accordingly adoption level responses $\{\mathcal{R}_m(p)\}$ converge to $\mathcal{R}(p) = \mathrm{P}\left(L \leq p(1 \wedge S^{-1}) + p_b(1 - S^{-1})^+\right)$, i.e., $\mathcal{R}_m(p) \uparrow \mathcal{R}(p)$ or $\mathcal{R}_m^{-1}(\bar{\rho}) \downarrow \mathcal{R}^{-1}(\bar{\rho})$; see Figure 12.

The adoption level response of consumers shift over periods until this response converges to $\mathcal{R}$. An equilibrium can be defined by $\mathcal{R}_m(\mathcal{P}(\bar{\rho}_m)) = \bar{\rho}_m$ with $\mathcal{R}_m \neq \mathcal{R}$, but the market is unlikely to stay at this equilibrium $(\bar{\rho}_m, p_m = \mathcal{P}(\bar{\rho}_m))$ after period $m+1$ when the consumers shift their responses to $\mathcal{R}_{m+1}$ and

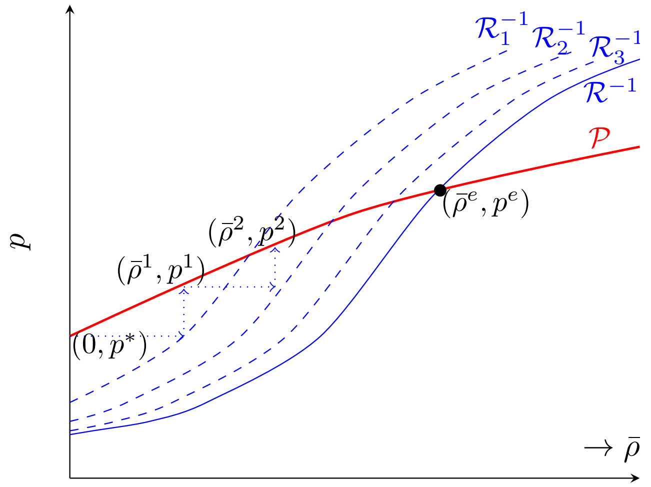


Figure 12: Shifting adoption level responses towards $\mathcal{R}$ yield equilibrium $(\bar\rho^e, p^e)$.

then to $\mathcal{R}_{m+2}$ and so on. Only if $\mathcal{R}_n(p_m) = \mathcal{R}_m(p_m)$ for $n \geq m$, then $(\bar\rho_m, p_m)$ remains as an equilibrium. But then $(\bar\rho_m, p_m)$ also solves $\mathcal{R}(\mathcal{P}(\bar\rho)) = \bar\rho$. Hence, we can set the equilibrium condition directly as $\mathcal{R}(\mathcal{P}(\bar\rho)) = \bar\rho$ and use only the adoption level response $\mathcal{R}$ in the analysis of the game between consumers and the utility.

Obtaining $\beta_d^{\bar\rho}$ and $\beta_c^{\bar\rho}(p_b)$ with a General Ratio and Linear Demand

For the ratio distributed uniformly over $[s_l, s_h]$ and $s_h > 1$, we obtain the following.

$$\begin{aligned}
\mathbb{E}(S \wedge 1) &= 1 - \mathbb{E}(1-S)^+ = 1 - \frac{1}{s_h - s_l}\int_{s_l}^{1}(1-s)ds = 1 - \frac{(1-s_l)^2}{2(s_h-s_l)}. \\
\mathbb{E}(1-\bar\rho S)^+ &= \frac{1}{s_h - s_l}\int_{s_l}^{s_h \wedge \bar\rho^{-1}}(1-\bar\rho s)ds = \frac{1}{s_h - s_l}\Big((s_h \wedge \bar\rho^{-1}) - s_l - \frac{\bar\rho}{2}((s_h \wedge \bar\rho^{-1})^2 - s_l^2)\Big) \\
&= \frac{(s_h \wedge \bar\rho^{-1}) - s_l}{s_h - s_l}\Big(1 - \frac{\bar\rho}{2}((s_h \wedge \bar\rho^{-1}) + s_l)\Big) = \frac{(s_h \wedge \bar\rho^{-1}) - s_l}{2(s_h - s_l)}\Big(2 - (s_h\bar\rho \wedge 1) - s_l\bar\rho\Big). \\
\mathbb{E}(\bar\rho(S-1)^+ \wedge (1-\bar\rho)) &= \frac{1}{s_h - s_l}\int_{1}^{s_h}\bar\rho(s-1) \wedge (1-\bar\rho)ds \\
&= \frac{\bar\rho}{s_h - s_l}\int_{1}^{s_h \wedge \bar\rho^{-1}}(s-1)ds + \frac{1-\bar\rho}{s_h - s_l}\int_{1 \vee \bar\rho^{-1}}^{s_h} ds \\
&= \frac{\bar\rho}{2(s_h - s_l)}\Big((s_h \wedge \bar\rho^{-1})^2 - 2(s_h \wedge \bar\rho^{-1}) + 1\Big) + \frac{1-\bar\rho}{s_h - s_l}(s_h - \bar\rho^{-1})^+ \\
&= \frac{1}{2(s_h - s_l)}\Big(\bar\rho((s_h \wedge \bar\rho^{-1}) - 1)^2 + 2(1-\bar\rho)(s_h - (s_h \wedge \bar\rho^{-1}))\Big).
\end{aligned}$$

These are inserted into $\beta_d^{\bar\rho}$ and $\beta_c^{\bar\rho}(p_b)$ expressions for computations. Using $\beta_d^{\bar\rho} = 1 - \bar\rho\mathbb{E}(S \wedge 1)$ and $\beta_c^{\bar\rho}(p_b) = \mathbb{E}(1-\bar\rho S)^+ + (\bar\rho\mathbb{E}(S-1)^+ \wedge \rho)p_b/c_v$,

$$\begin{aligned}
\beta_d^{\bar\rho} &= 1 - \left(1 - \frac{(1-s_l)^2}{2(s_h-s_l)}\right)\bar\rho, \\
\beta_c^{\bar\rho}(p_b) &= \frac{(s_h \wedge \bar\rho^{-1}) - s_l}{2(s_h - s_l)}\Big(2 - (s_h\bar\rho \wedge 1) - s_l\bar\rho\Big) \\
&\quad + \frac{1}{2(s_h - s_l)}\Big(\bar\rho((s_h \wedge \bar\rho^{-1}) - 1)^2 + 2(1-\bar\rho)(s_h - (s_h \wedge \bar\rho^{-1}))\Big)\frac{p_b}{c_v}.
\end{aligned}$$

Solar Model with Buyback Price at Variable Cost: $p_b = c_v$

With $p_b = c_v$, a constant in this section, we use the notation $\gamma = \gamma(c_v)$ and $\bar{\rho}(p) = \mathcal{R}(p; c_v)$. The adoption level $\bar{\rho}(p)$ is assumed to be convex. If $\mathrm{P}(S \le \gamma(p_b)) = 1$, the buyback price becomes irrelevant and the results of this section directly apply.

The expected utility demand and cost expressions for the solar model are

$$\begin{aligned} d^s(p) &= \beta_d(p) d(p); \\ c^s(p) &= c_f + \big(\mathbb{E}(\rho(p) + \bar{\rho}(p)(\gamma - S))^+ + \mathbb{E}(\bar{\rho}(p)(S-\gamma)^+ \wedge \rho(p))\big)\, c_v d(p) = c_f + c_v d^s(p), \end{aligned}$$

where the demand multiplier by Sell column of Table 2 and (2) is $\beta_d(p) = \rho(p) + \bar{\rho}(p)\mathbb{E}(\gamma - S)^+ = \mathbb{E}(\rho(p) + \bar{\rho}(p)\gamma - \bar{\rho}(p)S)^+ + \mathbb{E}(\bar{\rho}(p)(S-\gamma)^+ \wedge \rho(p)) \le 1$. The multiplier drops in p towards $\mathbb{E}(\gamma - S)^+$. $S = 0$ or $\bar{\rho} = 0$ implies $\beta_d = 1$, i.e., the revenue and cost in the solar model reduce to those in the base model.

Through the specialization of the authorized return regulation in (1) to the solar model,

$$\text{Utility problem with adopters and } p_b = c_v\text{:} \quad \max_{p \ge 0} \; \big\{ p d^s(p) - c^s(p) : \; p d^s(p) \in c^s(p)[1, 1+\alpha] \big\}. \tag{17}$$

In line with the base model prices, we let p^s_α be the maximizer of $p d^s(p) - (1+\alpha)c^s(p)$. We investigate the solar model to find the optimal price but first need to identify binding authorized returns. The binding value of α satisfies $p^s_0 d^s(p^s_0) \ge (1+\alpha)c^s(p^s_0)$, which can be expressed as $\alpha \le \bar{\alpha}^s$.

$$\text{Highest binding return:} \;\; \bar{\alpha}^s = \frac{p^s_0}{c_v + c_f/(\beta_d(p_0) d(p^s_0))} - 1.$$

Lemma 10. a) *The demand $d^s(p)$ inherits logconcavity from $d(p)$. For each α, the objective $p d^s(p) - (1+\alpha)c^s(p)$ is unimodal, its maximizer p^s_α is unique, $p^s_\alpha \ge (1+\alpha)c_v$ and increasing in α. In particular, $p^s_{-1} \ge 0$, $p^s_0 \ge c_v$ and $p^s_{-1} \le p^s_0$.*
b) *Unregulated prices decrease with the presence of adopters: $p^s_\alpha \le p_\alpha$.*
c) *No variable cost or no fixed cost implies $\bar{\alpha}^s \le \bar{\alpha}$.*

Part a) of Lemma 10 is analogous to that of Lemma 3. Part b) is intuitively from the faster dropping solar model demand $\beta_d(p)d(p)$ in price p than the base model demand $d(p)$. We remark that $p^s_\alpha \le p_\alpha$ in the case of $p_b = c_v$ of Lemma 10 and $p^{s,\bar{\rho}}_\alpha \ge p_\alpha$ in the case of the game in Lemma 3 are not inconsistent. Note the unconstrained maximizers

$$p^s_\alpha = \arg\max_p \; [p - (1+\alpha)c_v]\beta_d(p)d(p) \;\; \text{and} \;\; p^{s,\bar{\rho}}_\alpha = \arg\max_p \; [p - (1+\alpha)\beta^{\bar{\rho}}(p_b)c_v]d(p).$$

Presence of adopters reduces the demand as $\beta_d(p)d(p) \le d(p)$ in the case of $p_b = c_v$, whereas it increases the cost as $\beta^{\bar{\rho}}(p_b)c_v \ge c_v$ in the game. The reduction in demand leads to lower price than p_α; the increase in cost leads to higher price than p_α.

Part c) relates the binding authorized returns by $\bar{\alpha}^s \le \bar{\alpha}$ for $c_v = 0$ or $c_f = 0$. The case of no variable cost is relevant for utilities using renewable resources (e.g., solar and wind) for generating electricity because their fuel cost, the largest driver of the variable cost, is zero. On the other hand, utilities that buy energy to sell to consumers have little fixed costs. When $\bar{\alpha}^s \le \bar{\alpha}$, the regulator needs to be stricter in the solar model to sway the utility away from its unregulated (unconstrained) solution. If a regulator applies the same authorized return α for $\bar{\alpha}^s \le \alpha \le \bar{\alpha}$, its return regulation becomes irrelevant in the solar model despite being relevant in the base model. This is a consequence of dropping utility returns with adopters; the drop can be so large and the unregulated return can already be so small that the regulator cannot justifiably ask the utility to further reduce its return. This can potentially make the regulator irrelevant in the presence of large number of adopters.

The base and solar models are similar. The solar model (17) has the logconcave demand $d^s(p)$ by Lemma 10a) and the same structure as the base model (1). Hence, Theorem 1 applies to the solar model and yields the next result involving the inflated fixed cost $c_f/\beta_d(p) \geq c_f$.

Corollary 3. *a) The unique optimal price $p^{s,*}$ of a regulated utility, with the buyback price c_v and the binding authorized return α, is the lowest price p that solves*

$$pd(p) = (1+\alpha)(c_v d(p) + c_f/\beta_d(p)). \tag{18}$$

b) Also $p^{s,*} \geq p^*$ when both solar and base models have the same binding authorized return.

In Corollary 3, we assume the same binding authorized returns in the base and solar models to compare prices. This assumption is supported by the dependence of returns more on macroeconomic conditions than electricity generation technology.

The term $c_f/\beta_d(p)$ in (18) represents the higher amount of fixed cost allocated on the remaining consumers in the solar model with respect to the base model. The utility is bound to increase its regulated price to compensate for this extra cost. Two drivers of the price increase are a binding authorized return and a positive fixed cost. Without a fixed cost, the regulated price is the same in the base and solar models. Conversely, the unregulated price, even with a fixed cost, is lower in the solar model by Lemma 10b).

A high regulated price motivates a large number of consumers to become adopters and this interaction is captured by increasing $\bar{\rho}(p)$ in p. Then higher prices induce more adopters, who in turn pull the prices even higher because of (18) and $\beta_d(p) = 1-\bar{\rho}(p)(1-\mathbb{E}(\gamma - S)^+)$ decreasing in p. Such an interplay between prices and adoption levels can happen over time in stages rather than at once. Each stage then ends with a higher price and adoption level pair than the pair it starts with. This death spiral is detrimental to the utility for leaving it with a financially precarious low demand.

Proof of Lemma 10 a) We have $d^s(p) = \beta_d(p)d(p)$. Product of logconcave functions is logconcave, so it is sufficient to argue that $\beta_d(p) = \rho(p) + \bar{\rho}(p)\mathbb{E}(\gamma - S)^+$ is logconcave. Writing $\beta_d(p) = 1 - \bar{\rho}(p)(1 - \mathbb{E}(\gamma - S)^+)$, the logconcavity of $\beta_d(p)$ follows from the concavity of $-\bar{\rho}(p)$ and preservation of concavity under a logarithmic transformation.

The maximizer p^s_α maximizes $[p - (1+\alpha)c_v]d^s(p)$. This is structurally similar to the maximization of $[p - (1+\alpha)c_v]d(p)$. So we use the steps in the proof of Lemma 1 to establish the claims in b).

b) The price p_α maximizes $(p - (1+\alpha)c_v)d(p)$. Since this objective is unimodal, the derivative of objective is positive at low prices: $d(p) + (p - (1+\alpha)c_v)d'(p) > 0$ for $p < p_\alpha$. The derivative of solar objective $(p - (1+\alpha)c_v)\beta_d(p)d(p)$ is $(p - (1+\alpha)c_v)d(p)\beta'_d(p) + [d(p) + (p - (1+\alpha)c_v)d'(p)]\beta_d(p)$. Setting this equal to zero, we obtain

$$(p - (1+\alpha)c_v)d(p)(-\beta'_d(p))/\beta_d(p) = d(p) + (p - (1+\alpha)c_v)d'(p).$$

The left-hand side of the optimality equation is always nonnegative, the right-hand side is nonnegative only if $p \leq p_\alpha$. The equality can be achieved only with $p \leq p_\alpha$, so $p^s_\alpha \leq p_\alpha$.

c) When $c_v = 0$, p^s_0 maximizes $p\beta_d(p)d(p)$. Thus, $p^s_0\beta_d(p^s_0)d(p^s_0) \leq p^s_0 d(p^s_0) \leq p_0 d(p_0)$, where the first inequality from $\beta_d(p_0) \leq 1$ and the second is from the optimality of p_0. Hence, we obtain

$$\bar{\alpha}^s = \frac{p^s_0\beta_d(p^s_0)d(p^s_0)}{c_f} - 1 \leq \frac{p_0 d(p_0)}{c_f} - 1 = \bar{\alpha}.$$

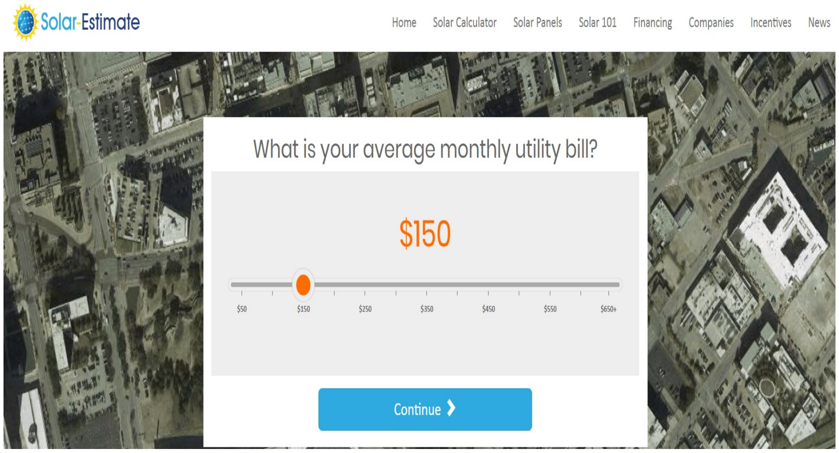
When $c_f = 0$, from part c), $\bar{\alpha}^s = p^s_0/c_v - 1 \leq p_0/c_v - 1 = \bar{\alpha}$. □

Proof of Corollary 3 a) This part follows from the discussion immediately before the theorem.

b) With the optimal price p^* of the base model, we have $\Big(p-(1+\alpha)c_v\Big)d(p) < (1+\alpha)c_f$ for $(1+\alpha)c_v \leq p < p^*$. Since $\beta_d(p) \leq 1$, we get $(p-(1+\alpha)c_v)d(p) < (1+\alpha)c_f/\beta_d(p)$. In view of (18), the optimality equation for $p^{s,*}$ cannot be satisfied by $p < p^*$, so $p^{s,*} \geq p^*$. □

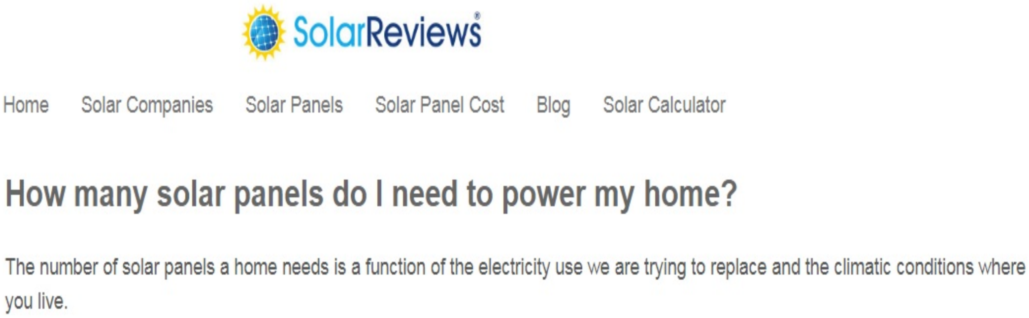
More Real-Life Examples for Linking Generation Capacity to Demand

Figure 13 has SolarEstimate, SolarReviews and EnergySage examples in which the generation capacity is determined by starting from the demand.



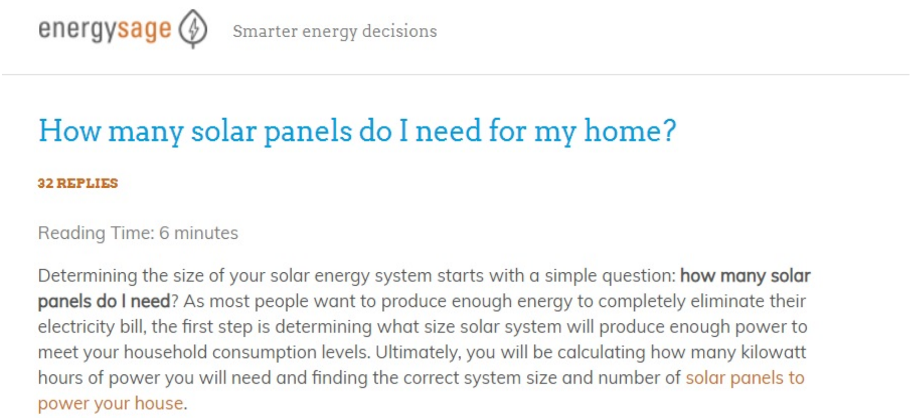
What is your average monthly utility bill?

$150



How many solar panels do I need to power my home?

The number of solar panels a home needs is a function of the electricity use we are trying to replace and the climatic conditions where you live.



How many solar panels do I need for my home?

32 REPLIES

Reading Time: 6 minutes

Determining the size of your solar energy system starts with a simple question: **how many solar panels do I need**? As most people want to produce enough energy to completely eliminate their electricity bill, the first step is determining what size solar system will produce enough power to meet your household consumption levels. Ultimately, you will be calculating how many kilowatt hours of power you will need and finding the correct system size and number of solar panels to power your house.

Figure 13: An adopter's average monthly bill (demand) is an important and usually the first-considered variable to determine the generation capacity. Source: https://www.solar-estimate.org, https://www.solarreviews.com and https://news.energysage.com.