Additional Proofs and Further Details

Appendix EC.1: Explicit lower bound for $f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1}}$

Previously, we introduced the discrete Bass model (26) on a one-sided line. We can similarly define *one-sided diffusion in D dimensions*, where peers effect can only be exerted in e.g., the negative direction of each coordinate. The discrete Bass model on the one-sided D-dimensional Cartesian network is given by

$$X_{\mathbf{j}}(0)=0, \qquad \mathbf{j}\in\mathbb{Z}^d, \tag{EC.1a}$$

and for any $\mathbf{j}\in\mathbb{Z}^d$, as $\Delta t\to 0$,

$$\mathbb{P}(X_{\mathbf{j}}(t+\Delta t)=1\,|\,\mathbf{X}(t))=\begin{cases}\left(p+\frac{q}{D}N_{\mathbf{j}}^{\text{1-sided}}(t)\right)\Delta t, & \text{if } X_{\mathbf{j}}(t)=0,\\ 1, & \text{if } X_{\mathbf{j}}(t)=1,\end{cases} \tag{EC.1b}$$

where

$$N_{\mathbf{j}}^{\text{1-sided}}(t):=\sum_{i=1}^{D}X_{\mathbf{j}+\hat{\mathbf{e}}_i}(t) \tag{EC.1c}$$

is the number of adopters connected to $\mathbf{j}$ in the one-sided case.

In 1D we saw that $f_{\text{bdry}}^{\mathbb{Z}^+}(t;p,q)=f^{\text{1D}}\left(t;p,\frac{q}{2}\right)$, see (6). There is no similar explicit expression for $f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1}}$. In particular, we have

Lemma EC.1. *Let $f^{\text{D},\,\text{1-sided}}(t;p,q)$ denote the adoption level in the discrete Bass model* (EC.1) *on the one-sided D-dimensional Cartesian grid. Then*

$$f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1}}>f^{\text{D},\,\text{1-sided}}\left(t;p,\frac{q}{2}\right). \tag{EC.2}$$

Proof. Start from the discrete Bass model (12) on $\mathbb{Z}^+\times\mathbb{Z}^{D-1}$. If we delete all the directional edges $(\mathbf{j})\longrightarrow(\mathbf{j}+\hat{\mathbf{e}}_i)$, where $i=1,\dots,D$, we obtain the one-sided D-dimensional discrete Bass model (EC.1), with q replaced by $\frac{q}{2}$. Since the deleted edges are influential to the boundary nodes $j_1\equiv 1$, then by the *dominance principle* (Fibich et al. 2019),

$$f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1}}(t;p,q)>f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1},\,\text{1-sided}}\left(t;p,\frac{q}{2}\right).$$

If we now extend the one-sided network from $\mathbb{Z}^+\times\mathbb{Z}^{D-1}$ to $\mathbb{Z}^D$, the added edges are noninfluential to the nodes $j_1\equiv 1$. Therefore,

$$f_{\text{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1},\,\text{1-sided}}\left(t;p,\frac{q}{2}\right)=f_{(j_1=1,\mathbf{j}_{-1})}^{\mathbb{Z}^D,\,\text{1-sided}}\left(t;p,\frac{q}{2}\right)=f^{\text{D},\,\text{1-sided}}\left(t;p,\frac{q}{2}\right),$$

where the last equality follows from translation invariance. Hence, the result follows. □

In order to express inequality (EC.2) using the two-sided Bass model on $\mathbb{Z}^D$, let us recall

CONJECTURE EC.1 **(Fibich et al. (2019))**. *The expected adoption level in the discrete Bass models* (EC.1) *and* (1) *on homogeneous D-dimensional one-sided and two-sided Cartesian networks on $\mathbb{Z}^D$, respectively, are identical, i.e.,* $f^{\mathrm{D},1-\mathrm{sided}}(t;p,q) = f^{\mathrm{D}}(t;p,q)$.

Therefore, we have

COROLLARY EC.1. *Let Conjecture EC.1 hold. Then* $f_{\mathrm{bdry}}^{\mathbb{Z}^+\times\mathbb{Z}^{D-1}} > f^{\mathrm{D}}\left(t;p,\frac{q}{2}\right)$.

Appendix EC.2: Global effect of an internal boundary

The global effect of internal boundaries is also $O(\frac{1}{M})$. To see that, we compute the difference h_M between the expected adoption level $f^{[1,\ldots,2M]}$ on a two-sided line of length $2M$ (Figure EC.1A), and the expected adoption level $f^{[1,\ldots,M\,|\,M+1,\ldots,2M]}$ on a two-sided line of length $2M$ that has an internal boundary between nodes M and $M+1$ (Figure EC.1B). Obviously, $f^{[1,\ldots,M\,|\,M+1,\ldots,2M]} \equiv f^{[1,\ldots,M]}$.

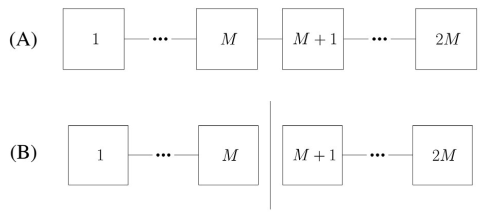

Figure EC.1 Line of length $2M$ without (A) and with (B) an internal boundary.

LEMMA EC.2. *Let*

$$h_M(t;p,q) := f^{[1,\ldots,2M]}(t;p,q) - f^{[1,\ldots,M\,|\,M+1,\ldots,2M]}(t;p,q) \tag{EC.3}$$

denote the effect of an internal boundary on the aggregate adoption on a two-sided line of length $2M$. *Then*

$$h_M(t;p,q) \sim \frac{\psi(t;p,q)}{M}, \qquad M\to\infty, \tag{EC.4}$$

where ψ *is given by* (11).

Proof. Since $f^{[1,\ldots,2M]} - f^{[1,\ldots,M]} = \left(f^{1\mathrm{D}} - f^{[1,\ldots,M]}\right) - \left(f^{1\mathrm{D}} - f^{[1,\ldots,2M]}\right)$, the result follows from Lemma 3. □

Figure EC.2A confirms numerically that the effect of an internal boundary decreases with M, and Figure EC.2B confirms that $Mh_M \to \psi$ as $M\to\infty$, where ψ is independent of M, see (EC.4).

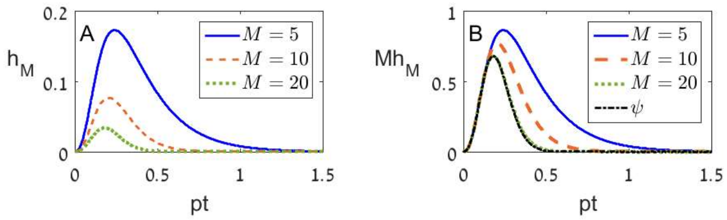

Figure EC.2 (A) Effect of an internal boundary on the adoption level, see (EC.3), as a function of time, for various values of M. Here $\frac{q}{p} = 100$. (B) Same as (A) with Mh_M as a function of time. The line for $M = 20$ is already indistinguishable from ψ, see (11).

Appendix EC.3: Further details on the empirical analysis

The primary raw data for the empirical analysis consists of administrative data on residential rooftop solar installations acquired under a non-disclosure agreement from the Connecticut Green Bank. These include the date of the installation and address of the installation, along with several other variables we do not use in this study. The data include nearly all solar installations in Connecticut through the end of 2019. These data are supplemented with U.S. Census Bureau data at the Census block group level (a unit that contains between 600 and 3,000 people) from the 2015-2018 American Community Survey. For our study we use median household income, median number of rooms in owner-occupied households, median home values, population, and census block area.

We geocoded each of the addresses in the residential solar installation data using the Google Maps geocoding API (see `https://developers.google.com/maps/documentation/geocoding/overview`). This gives a latitude and longitude value for each solar installation. These are then input into ArcMap along with the date of the installation. We next identify each of the municipalities that had a Solarize campaign. Table EC.1 shows the start and end dates of the 51 municipality-level campaigns included in this analysis. See (Gillingham and Bollinger 2021) for a treatment effects analysis of the effectiveness of the campaigns at spurring further adoption of solar. As mentioned in the main text, we focus on examining the diffusion process after these campaigns because they provide a substantial exogenous increase in solar installations in one area, so we can follow how word-of-mouth and peer effects influence diffusion elsewhere. By focusing on municipalities with a reasonable number of solar installations, we can also improve our signal-to-noise ratio.

We use a Python script run in ArcMap to create buffer zones around each of the Solarize municipality boundaries. These are created within the Solarize municipality itself because we are analyzing the effect of external boundaries. We drop any boundaries where the adjacent municipality is also a Solarize municipality. Then we map the geocoded solar installations to the buffer zones. We next calculate the sum of the installations in each buffer zone in the year following the Solarize campaigns.

For the Census data, we perform a join in ArcMap to overlay the U.S. Census block groups geographic areas with the buffers we created. Some block groups may not be entirely within the buffer zone. Thus, we calculate the percentage of the buffer zone covered by each block group and take a weighted average for each Census variable. For example, if there are two block group geographic areas in a buffer zone and 60% of the block group is covered by one block group and 40% another, we would take the weighted average based on these percentages. We create the dependent variable for our analysis by dividing the sum of the installations in each buffer zone by the number of owner-occupied households that we calculate are in each buffer zone.

The data for this analysis are at the municipality-buffer zone type level, where a buffer zone type refers to either being in the boundary buffer zone or the inner core zone. Table EC.2 shows the summary statistics for the cross-sectional data with 152 observations, where an observation is a municipality i zone category $j \in \{\text{boundary buffer}, \text{inner core}\}$. Thus, the analysis includes a total of 76 municipalities. As mentioned above, we excluded the borders where the municipality on the other side of the border is a Solarize municipality (i.e., we examine the border on the other side of the town from the border with the Solarize municipality).

References

Fibich G, Levin T, Yakir O (2019) Boundary effects in the discrete Bass model. *SIAM J. Appl. Math.* 79:914–937.

Gillingham K, Bollinger B (2021) Social learning and solar photovoltaic adoption. *Management Science* 67:7091–7112.

Table EC.1 Detailed Timeline of Solarize Campaigns

	Start Date	End Date
Round 1		
Durham	Sept 5, 2012	Jan 14, 2013
Westport	Aug 22, 2012	Jan 14, 2013
Portland	Sept 4, 2012	Jan 14, 2013
Fairfield	Aug 28, 2012	Jan 14, 2013
Round 2		
Bridgeport	Mar 26, 2013	July 31, 2013
Coventry	Mar 30, 2013	July 31, 2013
Canton	Mar 19, 2013	July 31, 2013
Mansfield	Mar 11, 2013	July 31, 2013
Windham	Mar 11, 2013	July 31, 2013
Round 3		
Easton	Sept 22, 2013	Feb 9, 2014
Redding	Sept 22, 2013	Feb 9, 2014
Trumbull	Sept 22, 2013	Feb 9, 2014
Ashford	Sept 24, 2013	Feb 11, 2014
Chaplin	Sept 24, 2013	Feb 11, 2014
Hampton	Sept 24, 2013	Feb 11, 2014
Pomfret	Sept 24, 2013	Feb 11, 2014
Greenwich	Oct 2, 2013	Feb 18, 2014
Newtown	Sept 24, 2013	Feb 28, 2014
Manchester	Oct 3, 2013	Feb 28, 2014
West Hartford	Sept 30, 2013	Feb 18, 2014
West Haven	Nov 13, 2013	Apr 8, 2013
Hamden	Nov 18, 2013	Feb 11, 2014
Easton	Sept 22, 2013	Feb 9, 2014
Trumbull	Sept 22, 2013	Feb 9, 2014
Round 4		
Tolland	Apr 23, 2014	Sept 16, 2014
Torrington	Apr 24, 2014	Sept 16, 2014
Simsbury	Apr 29, 2014	Sept 23, 2014
Essex	Apr 29, 2014	Sept 23, 2014
Montville	May 1, 2014	Sept 23, 2014
Brookfield	May 6, 2014	Sept 30, 2014
Bloomfield	May 6, 2014	Sept 30, 2014
Farmington	May 14, 2014	Oct 7, 2014
Haddam	May 15, 2014	Oct 7, 2014
Killingworth	May 15, 2014	Oct 7, 2014
East Lyme	May 22, 2014	Oct 14, 2014
Weston	June 24, 2014	Nov 14, 2014
Round 5		
Avon	Nov 20, 2014	Apr 10, 2015
Griswold	Dec 8, 2014	Apr 28, 2015
Milford	Dec 3, 2014	Apr 23, 2015
Southbury	Nov 19, 2014	Apr 9, 2015
Old Lyme	Dec 4, 2014	Apr 24, 2015
Lyme	Nov 18, 2014	Apr 8, 2015
South Windsor	Nov 10, 2014	Mar 31, 2015
Woodstock	Dec 3, 2014	Apr 23, 2015
Burlington	Nov 19, 2014	Apr 9, 2015
East Granby	Dec 2, 2014	Apr 22, 2015
Suffield	Dec 2, 2014	Apr 22, 2015
Windsor	Dec 2, 2014	Apr 22, 2015
Windsor Locks	Dec 2, 2014	Apr 22, 2015
New Canaan	Dec 2, 2014	Apr 22, 2015
New Hartford	Nov 17, 2014	Apr 7, 2015

Table EC.2 Summary statistics for boundary analysis

	Mean	S.D.	Min.	Max.
Percent installed (%)	1.17	1.04	0.00	7.66
Median household income ($)	98,897	31,411	56,288	241,800
Median number of rooms in homes	6.46	0.836	4.36	9.00
Median home values ($)	374,874	192,696	175,877	998,264
Density (owner-occupied houses/area)	83.7	84.0	7.55	467.4

Notes: There are 152 observations for all of the variables (no missing values), where an observation is a municipality x zone pair. Data are from one year after the period listed for each of the Solarize campaigns in Table EC.1. All dollars are nominal dollars.

Table EC.3 Evidence of Boundary Effects

Boundary Buffer	-0.405	-0.416	-0.413
	(0.166)	(0.156)	(0.107)
Control: Density		-0.002	-0.001
		(0.001)	(0.002)
Control: Income		-5.52e-06	-0.00001
		(4.81e-06)	(9.81e-06)
Control: Rooms		0.483	0.865
		(0.141)	(0.467)
Control: Home Value		-1.81e-06	9.88e-07
		(6.97e-07)	5.40e-06
Constant	1.372	-0.326	-2.60
	(0.141)	(0.723)	(2.40)
Municipality Fixed Effects	No	No	Yes
R-squared	0.038	0.185	0.825
N	152	152	152

Notes: Dependent variable is market share of owner-occupied homes that have installations one year after the Solarize campaigns (mean=0.1). An observation is a municipality x zone pair. Robust standard errors in parentheses.

Table EC.4 Robustness Check: Empirical Results Using Non-Solarize Towns

Boundary Buffer	-0.084	-0.084	-0.084
	(0.524)	(0.228)	(0.040)
Constant	0.862	-15.70	23.57
	(0.359)	(4.42)	(2.22)
Control Variables	No	Yes	Yes
Municipality Fixed Effects	No	No	Yes
R-squared	0.002	0.874	0.997
N	14	14	14

Notes: Dependent variable is market share of owner-occupied homes that have installations one year after the Solarize campaigns (mean=0.1). An observation is a municipality x zone pair. The municipalities included are: Bristol, Hamden, Milford, New Haven, Stonington, Stratford, and Waterbury, and the analysis is performed during the period of Round 3, when none of these municipalities had received a Solarize campaign. Robust standard errors in parentheses.