Electronic Companion

Appendix A: Proof of Results in Section 4

A.1. Proof of Lemma 1

The Revelation Principle (Myerson 1982) implies that there exists a direct revelation mechanism, which is optimal for the benchmark problem. The optimal direct revelation mechanism, which we refer to as Mechanism D, can be characterized by a set of functions denoted as $\left(Q_n^D(\hat{\boldsymbol{t}}), \tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}), Z_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}), M_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}), Y_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}), U_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) : n=1,2,\ldots,N\right)$ and implemented by the following steps:

(i) The buyer announces the sequence of events of Mechanism D as well as the associated functions (namely, $Q_n^D(\hat{\boldsymbol{t}})$, $\tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$, $Z_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$, $M_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$, $Y_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$, and $U_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$).

(ii) The buyer solicits the suppliers' types and announces the reported type vector $\hat{\boldsymbol{t}}$.

(iii) Supplier n is asked to exert an input effort $Q_n^D(\hat{\boldsymbol{t}})$. Then, the vector of random yields $\boldsymbol{\theta}$ is realized.[6]

(iv) Supplier n is also asked to purchase $\tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ units from the reliable supply sources of component n, and deliver $Z_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ units to the buyer. Thus, Supplier n's leftover quantity, denoted as $\tilde{U}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ is determined by:

$$\tilde{U}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) = \theta_n Q_n^D(\hat{\boldsymbol{t}}) + \tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) - Z_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}). \tag{A.1}$$

The buyer then pays $M_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ to Supplier n.

(v) The buyer procures an additional quantity $Y_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ of component n from the corresponding reliable supply source and sells $U_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})$ units for salvage value. Therefore, the quantity of component n available for assembling the final product equals $\left(Z_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) + Y_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) - U_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})\right)$.

Given that all suppliers other than Supplier n report their types $\boldsymbol{t_{-n}}$ truthfully in step (ii), we define $P_n^D\left(\hat{t}_n \middle| \boldsymbol{t},\boldsymbol{\theta}\right)$ as the profit of Supplier n, who reports his type t_n as $\hat{t}_n$. From the definition of Mechanism D, we have:

$$P_n^D\left(\hat{t}_n \middle| \boldsymbol{t},\boldsymbol{\theta}\right) = M_n^D(\hat{t}_n,\boldsymbol{t_{-n}},\boldsymbol{\theta}) + v_n\tilde{U}_n^D(\hat{t}_n,\boldsymbol{t_{-n}},\boldsymbol{\theta}) - p_n\tilde{Y}_n^D(\hat{t}_n,\boldsymbol{t_{-n}},\boldsymbol{\theta}) - t_n h_n\left(Q_n^D(\hat{t}_n,\boldsymbol{t_{-n}})\right).$$

[6] Mechanism D can enforce suppliers to take the prescribed actions in steps (iii) and (iv) by imposing large penalties for deviations, as the suppliers' actions in these steps are observable to the buyer in the benchmark problem. The same holds for Mechanism A, which will be defined below.

The equilibrium outcome under Mechanism D is as follows: For any realization of $\boldsymbol{t}$ and $\boldsymbol{\theta}$, Supplier n reports his type truthfully ($\hat{t}_n = t_n$), exerts an input effort of $Q_n^D(\boldsymbol{t})$, purchases $\tilde{Y}_n^D(\boldsymbol{t},\boldsymbol{\theta})$ from the reliable supply source, delivers $Z_n^D(\boldsymbol{t},\boldsymbol{\theta})$ to the buyer, and receives a payment of $M_n^D(\boldsymbol{t},\boldsymbol{\theta})$. Thus,

$$\mathbb{E}_{(\boldsymbol{t},\boldsymbol{\theta})}\left[P_n^D\left(t_n\middle|\boldsymbol{t},\boldsymbol{\theta}\right)\right] \geq \mathbb{E}_{(\boldsymbol{t},\boldsymbol{\theta})}\left[P_n^D\left(\hat{t}_n\middle|\boldsymbol{t},\boldsymbol{\theta}\right)\right], \qquad \forall t_n, \hat{t}_n \in \mathcal{T}_n. \tag{A.2}$$

Moreover, for every $\boldsymbol{t}$ and $\boldsymbol{\theta}$, the buyer's profit in the equilibrium under Mechanism D equals:

$$r\left(\min_{n=1,2,\ldots,N}\left(Z_n^D(\boldsymbol{t},\boldsymbol{\theta}) + Y_n^D(\boldsymbol{t},\boldsymbol{\theta}) - U_n^D(\boldsymbol{t},\boldsymbol{\theta})\right)\right) + \sum_{n=1}^{N}\left(v_n U_n^D(\boldsymbol{t},\boldsymbol{\theta}) - p_n Y_n^D(\boldsymbol{t},\boldsymbol{\theta}) - M_n^D(\boldsymbol{t},\boldsymbol{\theta})\right). \tag{A.3}$$

We now define an alternative mechanism, namely, Mechanism A. The implementation steps of Mechanism A are identical to those of Mechanism D, except for the following modifications:

- In step (iv), the buyer asks Supplier n to purchase no quantity from the reliable supply source of component n and to deliver exactly the production quantity $\theta_n Q_n^D(\hat{\boldsymbol{t}})$, ensuring zero leftover quantity. The payment to Supplier n is $\left(M_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) - p_n\tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) + v_n\tilde{U}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})\right)$.
- In step (v), the buyer purchases $\left(\tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) + Y_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})\right)$ units of component n from the corresponding reliable supply source and sells $\left(\tilde{U}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta}) + U_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})\right)$ units for salvage value.

Mechanism A is feasible because all supplier actions, namely, the choices of input efforts, production quantities, purchasing quantities from the reliable supply sources, and delivered quantities are observable to the buyer in the benchmark scenario. Furthermore, Mechanism A satisfies the two properties in Lemma 1.

Assume that all suppliers other than Supplier n report their types truthfully under Mechanism A. It is straightforward to verify that, for any realization of $\boldsymbol{t}$ and $\boldsymbol{\theta}$, Supplier n's profit from reporting his type as $\hat{t}_n$ equals $P_n^D\left(\hat{t}_n\middle|\boldsymbol{t},\boldsymbol{\theta}\right)$, which is the same as his profit under Mechanism D. Inequality (A.2) implies that Supplier n finds it optimal to report his type truthfully. Therefore, in the equilibrium under Mechanism A, all suppliers report their types truthfully.

Consequently, the buyer's profit under Mechanism A is as follows:

$$\underbrace{r\left(\min_{n=1,\ldots,N}\left(\theta_n Q_n^D(\boldsymbol{t}) + \tilde{Y}_n^D(\boldsymbol{t},\boldsymbol{\theta}) + Y_n^D(\boldsymbol{t},\boldsymbol{\theta}) - U_n^D(\boldsymbol{t},\boldsymbol{\theta}) - \tilde{U}_n^D(\boldsymbol{t},\boldsymbol{\theta})\right)\right)}_{\text{Revenue from assembled product}} - \underbrace{\sum_{n=1}^{N} v_n\left(\tilde{Y}_n^D(\boldsymbol{t},\boldsymbol{\theta}) + Y_n^D(\boldsymbol{t},\boldsymbol{\theta})\right)}_{\text{Cost from reliable supply sources}}$$

$$+\underbrace{\sum_{n=1}^{N} p_n\big(\tilde{U}_n^D(\boldsymbol{t},\boldsymbol{\theta})+U_n^D(\boldsymbol{t},\boldsymbol{\theta})\big)}_{\text{Salvage value}}-\underbrace{\sum_{n=1}^{N}\big(M_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})-p_n\tilde{Y}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})+v_n\tilde{U}_n^D(\hat{\boldsymbol{t}},\boldsymbol{\theta})\big)}_{\text{Payment to suppliers}}$$

$$=r\Big(\min_{n=1,2,\ldots,N}\big(Z_n^D(\boldsymbol{t},\boldsymbol{\theta})+Y_n^D(\boldsymbol{t},\boldsymbol{\theta})-U_n^D(\boldsymbol{t},\boldsymbol{\theta})\big)\Big)+\sum_{n=1}^{N}\big(v_nU_n^D(\boldsymbol{t},\boldsymbol{\theta})-p_nY_n^D(\boldsymbol{t},\boldsymbol{\theta})-M_n^D(\boldsymbol{t},\boldsymbol{\theta})\big),$$

(using (A.1))

which is the same as (A.3). This proves that Mechanism A, which satisfies the two properties in Lemma 1, is also optimal. ■

A.2. Proof of Lemma 2

For any given $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})$, the buyer's revenue from assembled product, plus the salvage values, and minus the purchasing costs incurred from the reliable supply sources equals $\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})$ (defined in Lemma 2), where $\boldsymbol{x}$ is the vector of production quantities of the suppliers, $\boldsymbol{y}$ is the vector of quantities the buyer purchases from the reliable supply sources, and $\boldsymbol{u}$ is the vector of leftover quantities. Thus, for any given $\boldsymbol{x}$, the buyer maximize $\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})$ by choosing $\boldsymbol{y}$ and $\boldsymbol{u}$.

We now show that the solution to the maximization problem exists. To this end, we prove that fixing $\boldsymbol{x}$, function $\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})$ is a continuous function of $(\boldsymbol{y},\boldsymbol{u})$, and the feasible region is bounded:

Recall that in Lemma 2, we define function $\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})$ as follows:

$$\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})=r\left(\min_{n\in\mathcal{N}}\{x_n+y_n-u_n\}\right)+\sum_{n=1}^{N}(v_nu_n-p_ny_n).$$

For any given $\boldsymbol{x}$, this is obviously a continuous function of $(\boldsymbol{y},\boldsymbol{u})$. Moreover, since $\lim_{\zeta\to+\infty} r'(\zeta)=0$, we have that $\forall\epsilon>0$, $\exists\bar{\zeta}(\epsilon)<+\infty$, such that $0\le r'(\zeta)<\epsilon$, $\forall\zeta\ge\bar{\zeta}(\epsilon)$. Therefore, for all $\zeta\ge\bar{\zeta}(\epsilon)$:

$$\frac{\partial}{\partial\zeta_n}r(\min\{\zeta_1,\ldots,\zeta_N\})\Big|_{\zeta_n=\zeta}<\epsilon,\ \forall n=1,2,\ldots,N.$$

By taking $\epsilon=\min\{p_1,\ldots,p_N\}$, we have that for any given $\boldsymbol{x}$ and for all $\zeta\ge\bar{\zeta}(\epsilon)$, the following inequality holds:

$$\frac{\partial}{\partial y_n}\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})\Big|_{y_n=\zeta}<0,\ \forall n=1,2,\ldots,N,\text{ and }\mathbf{0}\le\boldsymbol{u}\le\boldsymbol{x}.$$

Therefore, the feasible region of $(\boldsymbol{y},\boldsymbol{u})$ is bounded by $0\le y_n\le\bar{\zeta}(\epsilon)$ and $0\le u_n\le x_n$ for all n. Thus, for any given $\boldsymbol{x}\ge\mathbf{0}$, the solution to the maximization problem exists. In Lemma 2, we denote the solution as

$$(\boldsymbol{Y}^*(\boldsymbol{x}),\boldsymbol{U}^*(\boldsymbol{x}))\in\arg\max_{\substack{\boldsymbol{y}\ge\mathbf{0}\\0\le\boldsymbol{u}\le\boldsymbol{x}}}\ \hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u}).$$

The monotonicity of $r^*(\boldsymbol{x})$ is straightforward. For any given $\tilde{\boldsymbol{x}}\ge\boldsymbol{x}$, we have

$$r^*(\tilde{\boldsymbol{x}})\ge\hat{r}(\tilde{\boldsymbol{x}},\boldsymbol{Y}^*(\boldsymbol{x}),\boldsymbol{U}^*(\boldsymbol{x}))\ge\hat{r}(\boldsymbol{x},\boldsymbol{Y}^*(\boldsymbol{x}),\boldsymbol{U}^*(\boldsymbol{x}))=r^*(\boldsymbol{x}).$$

We now prove the concavity of $r^*(\boldsymbol{x})$. Let $\hat{u}_n=-u_n$ and define

$$\tilde{r}(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{u}})=\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u})=r\left(\min_{n\in\mathcal{N}}\{x_n+y_n+\hat{u}_n\}\right)-\sum_{n=1}^{N}\left(v_n\hat{u}_n+p_ny_n\right).$$

Since $r(\cdot)$ and $\min\{\cdot\}$ are both increasing and concave, $\tilde{r}(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{u}})$ is a concave function. Therefore, for any $\epsilon>0$ and $\boldsymbol{x}^{(\boldsymbol{i})}\ge\mathbf{0}$, where $i\in\{1,2\}$, there exists $\left(\boldsymbol{y}^{(\boldsymbol{i})},\hat{\boldsymbol{u}}^{(\boldsymbol{i})}\right)$, where $\boldsymbol{y}^{(\boldsymbol{i})}\ge\mathbf{0}$ and $-\boldsymbol{x}^{(\boldsymbol{i})}\le\hat{\boldsymbol{u}}^{(\boldsymbol{i})}\le\mathbf{0}$, such that $r^*(\boldsymbol{x}^{(\boldsymbol{i})})-\epsilon<\tilde{r}(\boldsymbol{x}^{(\boldsymbol{i})},\boldsymbol{y}^{(\boldsymbol{i})},\hat{\boldsymbol{u}}^{(\boldsymbol{i})})$. Therefore, for any $\beta\in[0,1]$, the following inequality holds:

$$\begin{aligned}&\beta r^*(\boldsymbol{x}^{(\mathbf{1})})+(1-\beta)r^*(\boldsymbol{x}^{(\mathbf{2})})-\epsilon<\beta\tilde{r}\left(\boldsymbol{x}^{(\mathbf{1})},\boldsymbol{y}^{(\mathbf{1})},\hat{\boldsymbol{u}}^{(\mathbf{1})}\right)+(1-\beta)\tilde{r}\left(\boldsymbol{x}^{(\mathbf{2})},\boldsymbol{y}^{(\mathbf{2})},\hat{\boldsymbol{u}}^{(\mathbf{2})}\right)\\&\le\tilde{r}\left(\beta\boldsymbol{x}^{(\mathbf{1})}+(1-\beta)\boldsymbol{x}^{(\mathbf{2})},\beta\boldsymbol{y}^{(\mathbf{1})}+(1-\beta)\boldsymbol{y}^{(\mathbf{2})},\beta\hat{\boldsymbol{u}}^{(\mathbf{1})}+(1-\beta)\hat{\boldsymbol{u}}^{(\mathbf{2})}\right)\le r^*(\beta\boldsymbol{x}^{(\mathbf{1})}+(1-\beta)\boldsymbol{x}^{(\mathbf{2})}).\end{aligned}$$

The second inequality follows from the concavity of $\tilde{r}(\cdot,\cdot,\cdot)$, and the third inequality is obtained by the definition of $r^*(\cdot)$. Thus, $r^*(\boldsymbol{x})$ is concave for all $\boldsymbol{x}\ge\mathbf{0}$.

Next, we prove that $r^*(\cdot)$ is a supermodular function. Since $r(\cdot)$ is an increasing function, we have that for all non-negative $x_n^{(i)}$, $y_n^{(i)}$, and $\hat{u}_n^{(i)}$, where $n=1,2,\ldots,N$ and $i\in\{1,2\}$, the following inequality holds:

$$\begin{aligned}&r\left(\min_{n\in\mathcal{N}}\{x_n^{(1)}+y_n^{(1)}+\hat{u}_n^{(1)}\}\right)+r\left(\min_{n\in\mathcal{N}}\{x_n^{(2)}+y_n^{(2)}+\hat{u}_n^{(2)}\}\right)\\&\le r\left(\min_{n\in\mathcal{N}}\{\min\{x_n^{(1)}+y_n^{(1)}+\hat{u}_n^{(1)},x_n^{(2)}+y_n^{(2)}+\hat{u}_n^{(2)}\}\}\right)\\&\quad+r\left(\min_{n\in\mathcal{N}}\{\max\{x_n^{(1)}+y_n^{(1)}+\hat{u}_n^{(1)},x_n^{(2)}+y_n^{(2)}+\hat{u}_n^{(2)}\}\}\right).\end{aligned}\tag{A.4}$$

Therefore, $r(\min_{n\in\mathcal{N}}\{y_n+x_n+\hat{u}_n\})$ is a supermodular function of $(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{u}})$, and thus $\tilde{r}(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{u}})$ is also supermodular. Furthermore, supermodularity is preserved by the pointwise maximization operation as follows: Let $i \in \{1,2\}$. For any $\boldsymbol{x^{(i)}} \geq \mathbf{0}$, there exist $\left(\boldsymbol{y^{(i)}}, \hat{\boldsymbol{u}}^{(\boldsymbol{i})}\right)$, where $\boldsymbol{y^{(i)}} \geq \mathbf{0}$ and $-\boldsymbol{x^{(i)}} \leq \hat{\boldsymbol{u}}^{(\boldsymbol{i})} \leq \mathbf{0}$, such that

$$\begin{aligned} r^*(\boldsymbol{x^{(1)}}) + r^*(\boldsymbol{x^{(2)}}) &= \tilde{r}\left(\boldsymbol{x^{(1)}}, \boldsymbol{y^{(1)}}, \hat{\boldsymbol{u}}^{(1)}\right) + \tilde{r}\left(\boldsymbol{x^{(2)}}, \boldsymbol{y^{(2)}}, \hat{\boldsymbol{u}}^{(2)}\right) \\ &\leq \tilde{r}\left(\boldsymbol{x^{(1)}} \vee \boldsymbol{x^{(2)}}, \boldsymbol{y^{(1)}} \vee \boldsymbol{y^{(2)}}, \hat{\boldsymbol{u}}^{(1)} \vee \hat{\boldsymbol{u}}^{(2)}\right) + \tilde{r}\left(\boldsymbol{x^{(1)}} \wedge \boldsymbol{x^{(2)}}, \boldsymbol{y^{(1)}} \wedge \boldsymbol{y^{(2)}}, \hat{\boldsymbol{u}}^{(1)} \wedge \hat{\boldsymbol{u}}^{(2)}\right) \\ &\leq r^*\left(\boldsymbol{x^{(1)}} \vee \boldsymbol{x^{(2)}}\right) + r^*\left(\boldsymbol{x^{(1)}} \wedge \boldsymbol{x^{(2)}}\right). \end{aligned} \tag{A.5}$$

Therefore, $r^*(\boldsymbol{x})$ is supermodular. ■

A.3. Proof of Proposition 1

Under a direct revelation mechanism, $\left\{(\boldsymbol{Q}(\hat{\boldsymbol{t}}), \boldsymbol{M}(\hat{\boldsymbol{t}})) : \hat{\boldsymbol{t}} \in \boldsymbol{\mathcal{T}}\right\}$, Supplier n's profit depends on $\hat{\boldsymbol{t}}$, which is the vector of the suppliers' reported types. Given that all suppliers other than Supplier n report their type truthfully (i.e., $\hat{\boldsymbol{t}}_{-\boldsymbol{n}} = \boldsymbol{t}_{-\boldsymbol{n}}$), we denote the optimal profit of Supplier n, whose cost type is t_n, as:

$$P_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}}) := \max_{\hat{t}_n \in \mathcal{T}_n} \left\{M_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}}) - c_n(Q_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}}), t_n)\right\} = \max_{\hat{t}_n \in \mathcal{T}_n} \left\{M_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}}) - t_n h_n(Q_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}}))\right\}.$$

Note that $P_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}})$ is the pointwise maximum of a set of affine functions and is thus a convex function of t_n. Therefore, for any given $\boldsymbol{t}_{-\boldsymbol{n}} \in \boldsymbol{\mathcal{T}}_{-\boldsymbol{n}}$, function $P_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}})$ is also uniformly continuous and almost everywhere differentiable on $t_n \in \mathcal{T}_n$.

We now prove that a mechanism is incentive compatible *if and only if* the following two conditions are satisfied simultaneously: (i) The prescribed input effort $Q_n(t_n, \boldsymbol{t}_{-\boldsymbol{n}})$ is decreasing in t_n. (ii) The payment function $M_n(\cdot)$ satisfies

$$M_n(t_n, \boldsymbol{t}_{-\boldsymbol{n}}) = P_n(\bar{t}_n|\boldsymbol{t}_{-\boldsymbol{n}}) + t_n h_n(Q_n(t_n, \boldsymbol{t}_{-\boldsymbol{n}})) + \int_{s_n=t_n}^{\bar{t}_n} h_n(Q_n(s_n, \boldsymbol{t}_{-\boldsymbol{n}}))d\, s_n. \tag{A.6}$$

We now proceed to establish this claim.

First we show the *only if* condition. The IC constraint is equivalent to

$$\begin{gathered} M_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}}) - \hat{t}_n h_n(Q_n(\hat{t}_n, \boldsymbol{t}_{-\boldsymbol{n}})) \geq [M_n(\boldsymbol{t}) - t_n h_n(Q_n(\boldsymbol{t}))] + t_n h_n(Q_n(\boldsymbol{t})) - \hat{t}_n h_n(Q_n(\boldsymbol{t})), \\ -(t_n - \hat{t}_n) h_n(Q_n(\boldsymbol{t})) \geq P_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}}) - P_n(\hat{t}_n|\boldsymbol{t}_{-\boldsymbol{n}}), \quad \forall \hat{t}_n \text{ and } \boldsymbol{t}. \end{gathered} \tag{A.7}$$

Fix arbitrary $t_n \in \mathcal{T}_n$, where $P_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}})$ is differentiable. By taking $\hat{t}_n \downarrow t_n$, we have

$$\frac{dP_n(t_n|\boldsymbol{t}_{-\boldsymbol{n}})}{d\, t_n} \geq -h_n(Q_n(t_n, \boldsymbol{t}_{-\boldsymbol{n}})).$$

Analogously, by taking $\hat{t}_n \uparrow t_n$, we have

$$\frac{dP_n(t_n|\boldsymbol{t_{-n}})}{d\,t_n} \leq -h_n(Q_n(t_n,\boldsymbol{t_{-n}})), \text{ and thus } \frac{dP_n(t_n|\boldsymbol{t_{-n}})}{d\,t_n} = -h_n(Q_n(t_n,\boldsymbol{t_{-n}})).$$

Since $P_n(\cdot)$ is convex and $h_n(\cdot)$ is strictly increasing, $Q_n(t_n,\boldsymbol{t_{-n}})$ is decreasing in t_n. Since $h_n(q_n)$ is non-negative for all q_n, we have $P_n(t_n|\boldsymbol{t_{-n}})$ is decreasing in t_n. Also, $P_n(t_n|\boldsymbol{t_{-n}})$ is almost everywhere differentiable, and thus under an incentive compatible mechanism, the payment function $M_n(\cdot)$ is defined by the input effort function $Q_n(\cdot)$ as shown in equation (A.6).

For the *if* condition, we note that if $Q_n(t_n,\boldsymbol{t_{-n}})$ is decreasing in t_n, and (A.6) is satisfied, then (A.7) holds, and thus the mechanism satisfies the IC constraint.

As formulated in $\mathbb{P}^{OE}$, the buyer's problem is:

$$\max_{\boldsymbol{M}(\cdot),\boldsymbol{Q}(\cdot)} \int_{\boldsymbol{\mathcal{T}}} \left[\mathbb{E}_{\boldsymbol{\theta}}\left[r^*(\theta_1 Q_1(\boldsymbol{t}),\ldots,\theta_N Q_N(\boldsymbol{t}))\right] - \sum_{n=1}^{N} M_n(\boldsymbol{t})\right] \boldsymbol{dF}(\boldsymbol{t}), \tag{A.8}$$

subject to the IR and IC constraints. Recall that in Proposition 1, we denote the solution to this problem by $(\boldsymbol{M}^*(\boldsymbol{t}),\boldsymbol{Q}^*(\boldsymbol{t}))$. Since $P_n(t_n|\boldsymbol{t_{-n}})$ is decreasing in t_n, the IR constraint can be replaced by the one with the largest cost type $\bar{t}_n$ as follows:

$$P_n(\bar{t}_n|\boldsymbol{t_{-n}}) = M_n(\bar{t}_n,\boldsymbol{t_{-n}}) - t_n h_n(Q_n(\bar{t}_n,\boldsymbol{t_{-n}})) \geq 0, \;\; \forall n, \text{ and } \boldsymbol{t_{-n}}, \tag{A.9}$$

Furthermore, the IC constraint can be replaced by (A.6), and the condition that $Q_n(t_n,\boldsymbol{t_{-n}})$ is decreasing in t_n on $\mathcal{T}_n$. Substituting (A.6) into the objective function of (A.8), we have

$$\begin{aligned}&\int_{\boldsymbol{\mathcal{T}}} \left[\mathbb{E}_{\boldsymbol{\theta}}\left[r^*(\theta_1 Q_1(\boldsymbol{t}),\ldots,\theta_N Q_N(\boldsymbol{t}))\right] - \sum_{n=1}^{N} P_n(\bar{t}_n|\boldsymbol{t_{-n}}) - \sum_{n=1}^{N} t_n h_n(Q_n(\boldsymbol{t}))\right] \boldsymbol{f}(\boldsymbol{t}) d\boldsymbol{t} \\ &- \int_{\boldsymbol{\mathcal{T}}} \sum_{n=1}^{N} \int_{s_n=t_n}^{\bar{t}_n} h_n(Q_n(s_n,\boldsymbol{t_{-n}})) ds_n \boldsymbol{f}(\boldsymbol{t}) d\boldsymbol{t}.\end{aligned} \tag{A.10}$$

Interchanging the order of integration for each n in the last term, we have

$$\begin{aligned}&\int_{\boldsymbol{\mathcal{T}_{-n}}} \int_{s_n=\underline{t}_n}^{\bar{t}_n} h_n(Q_n(s_n,\boldsymbol{t_{-n}})) \int_{t_n=\underline{t}_n}^{s_n} f_n(t_n) dt_n ds_n \boldsymbol{f_{-n}}(\boldsymbol{t_{-n}}) d\boldsymbol{t_{-n}} \\ &= \int_{\boldsymbol{\mathcal{T}_{-n}}} \int_{t_n=\underline{t}_n}^{\bar{t}_n} h_n(Q_n(\boldsymbol{t})) F_n(t_n) dt_n \boldsymbol{f_{-n}}(\boldsymbol{t_{-n}}) d\boldsymbol{t_{-n}} = \int_{\boldsymbol{\mathcal{T}}} h_n(Q_n(\boldsymbol{t})) \frac{F_n(t_n)}{f_n(t_n)} \boldsymbol{f}(\boldsymbol{t}) d\boldsymbol{t}.\end{aligned}$$

Therefore, (A.10) equals

$$\begin{aligned}&\int_{\boldsymbol{\mathcal{T}}} \hat{R}(\boldsymbol{Q}(\boldsymbol{t}),\boldsymbol{t}) \boldsymbol{f}(\boldsymbol{t}) d\boldsymbol{t} - \int_{\boldsymbol{\mathcal{T}}} \sum_{n=1}^{N} P_n(\bar{t}_n|\boldsymbol{t_{-n}}) \boldsymbol{f}(\boldsymbol{t}) d\boldsymbol{t}, \\ &\text{where } \hat{R}(\boldsymbol{q},\boldsymbol{t}) = \mathbb{E}_{\boldsymbol{\theta}}\left[r^*(\theta_1 q_1,\ldots,\theta_N q_N)\right] - \sum_{n=1}^{N} h_n(q_n)\psi_n(t_n) \text{ and } \psi_n(t_n) = t_n + \frac{F_n(t_n)}{f_n(t_n)}.\end{aligned} \tag{A.11}$$

To minimize the latter integral in (A.11), we minimize $P_n(\bar{t}_n|\boldsymbol{t_{-n}})$ under the constraint of (A.9). The solution to this minimization problem is as follows:

$$P_n(\bar{t}_n|\boldsymbol{t_{-n}}) = M_n(\bar{t}_n, \boldsymbol{t_{-n}}) - t_n h_n(Q_n(\bar{t}_n, \boldsymbol{t_{-n}})) = 0, \quad \forall n, \text{ and } \boldsymbol{t_{-n}}.$$

From (A.6), the solution to problem (A.8), which we denote by $(\boldsymbol{M}^*(\boldsymbol{t}), \boldsymbol{Q}^*(\boldsymbol{t}))$, satisfies the following equality:

$$M_n^*(\boldsymbol{t}) = t_n h_n(Q_n^*(\boldsymbol{t})) + \int_{s_n=t_n}^{\bar{t}_n} h_n(Q_n^*(s_n, \boldsymbol{t_{-n}}))\, ds_n, \quad n = 1, 2, \ldots, N,$$

which is part of the result in Proposition 1.

We now establish the function of the suppliers' input efforts $\boldsymbol{Q^*}(\boldsymbol{t})$ which solves (A.8). If we temporarily neglect the monotonicity constraint of $Q_n(t_n, \boldsymbol{t_{-n}})$, then to maximize the first integral in (A.11), we can simply maximize the integrand for every $\boldsymbol{t}$. That is,

$$\boldsymbol{Q}(\boldsymbol{t}) \in \arg\max_{\boldsymbol{q} \geq \boldsymbol{0}} \ \hat{R}(\boldsymbol{q}, \boldsymbol{t}), \quad \forall \boldsymbol{t}. \tag{A.12}$$

Therefore, if we can find a choice of $\boldsymbol{Q}(\boldsymbol{t})$ for every $\boldsymbol{t}$ that is decreasing in t_n and satisfies (A.12), then this choice is optimal for $\mathbb{P}^{OE}$.

To achieve such a result, we first show that $\arg\max_{\boldsymbol{q} \geq \boldsymbol{0}} \hat{R}(\boldsymbol{q}, \boldsymbol{t})$ is bounded for all $\boldsymbol{t}$: (i) $\hat{R}(\boldsymbol{q}, \boldsymbol{t})$ is continuous. (ii) Since $\lim_{z \to +\infty} r'(z) = 0$, and $\underline{t}_n > 0$, for some sufficiently large $\bar{q} > 0$ and $\boldsymbol{\mathcal{Q}} := [0, \bar{q}]^N$, we have

$$\arg\max_{\boldsymbol{q} \geq \boldsymbol{0}} \hat{R}(\boldsymbol{q}, \boldsymbol{t}) \in \boldsymbol{\mathcal{Q}}, \ \forall \boldsymbol{t}.$$

It is easy to see that $\boldsymbol{\mathcal{Q}}$ is a bounded lattice. In what follows, we will prove that the function $\boldsymbol{Q^*}(\boldsymbol{t})$ defined in Proposition 1 is decreasing in $\boldsymbol{t}$ and satisfies (A.12).

As shown in Lemma 2, $r^*(\boldsymbol{z})$ is a supermodular function. Thus, for any $\boldsymbol{\theta}$, $r^*(\theta_1 q_1, \ldots, \theta_N q_N)$ is a supermodular function of $\boldsymbol{q} \in \boldsymbol{\mathcal{Q}} \subset \mathbb{R}_+^N$. Since the expectation operator preserves supermodularity, $\mathbb{E}_{\boldsymbol{\theta}}\big[r^*(\theta_1 q_1, \ldots, \theta_N q_N)\big]$ is also a supermodular function of $\boldsymbol{q}$. Also, $\sum_{n=1}^N h_n(q_n)\psi_n(t_n)$ is supermodular, and consequently, $\hat{R}(\boldsymbol{q}, \boldsymbol{t})$ is a supermodular function of $(\boldsymbol{q}, \boldsymbol{t}) \in \boldsymbol{\mathcal{Q}} \times \boldsymbol{\mathcal{T}}$. Additionally, for all $\boldsymbol{t}$ and $\tilde{\boldsymbol{t}}$ in $\boldsymbol{\mathcal{T}}$, with $\boldsymbol{t} \leq \tilde{\boldsymbol{t}}$, the following quantity

$$\hat{R}(\boldsymbol{q}, \tilde{\boldsymbol{t}}) - \hat{R}(\boldsymbol{q}, \boldsymbol{t}) = \sum_{n=1}^N h_n(q_n)[\psi_n(t_n) - \psi_n(\tilde{t_n})],$$

is decreasing in $\boldsymbol{q}$. Therefore, $\hat{R}(\boldsymbol{q}, \boldsymbol{t})$ has decreasing differences in $(\boldsymbol{q}, \boldsymbol{t})$ on $\boldsymbol{\mathcal{Q}} \times \boldsymbol{\mathcal{T}}$.

We have verified the conditions for the following Lemma A.1 (Theorem 2.8.1 of Topkis 2011):

LEMMA A.1. *If $\mathcal{Q}$ is a lattice, $\mathcal{T}$ is a partially ordered set, $\hat{R}(\boldsymbol{q},\boldsymbol{t})$ is supermodular in $\boldsymbol{q}$ on $\mathcal{Q}$ for each $\boldsymbol{t}$ in $\mathcal{T}$, and $\hat{R}(\boldsymbol{q},\boldsymbol{t})$ has decreasing differences in $(\boldsymbol{q},\boldsymbol{t})$ on $\mathcal{Q}\times\mathcal{T}$, then* $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})$ *is decreasing in the strong set order in $\boldsymbol{t}$ on $\mathcal{T}$.*

Strong set order (Topkis 2011): For two sets S and S', we say that S is greater than or equal to S' in the strong set order if for any $s\in S$ and $s'\in S'$, $s\wedge s'\in S'$ and $s\vee s'\in S$.

From Lemma A.1, we have that for all $t_n,\tilde{t}_n\in\mathcal{T}_n$ with $t_n<\tilde{t}_n$, the set $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},t_n,\boldsymbol{t_{-n}})$ is greater than or equal to $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\tilde{t_n},\boldsymbol{t_{-n}})$ in the strong set order for all $\boldsymbol{t_{-n}}$. Thus, for all $\tilde{\boldsymbol{q}}\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\tilde{t}_n,\boldsymbol{t_{-n}})$ and $\boldsymbol{q}\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},t_n,\boldsymbol{t_{-n}})$, we have:

$$\tilde{\boldsymbol{q}}\wedge\boldsymbol{q}\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\tilde{t}_n,\boldsymbol{t_{-n}}),\text{ and }\tilde{\boldsymbol{q}}\vee\boldsymbol{q}\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},t_n,\boldsymbol{t_{-n}}).\tag{A.13}$$

Next, we prove that $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})$ is a *compact sublattice* for all $\boldsymbol{t}$. Picking arbitrary $\hat{\boldsymbol{q}}$ and $\tilde{\boldsymbol{q}}$ from $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})$, the supermodularity of $\hat{R}(\boldsymbol{q},\boldsymbol{t})$ implies

$$0\leq\hat{R}(\hat{\boldsymbol{q}},\boldsymbol{t})-\hat{R}(\hat{\boldsymbol{q}}\wedge\tilde{\boldsymbol{q}},\boldsymbol{t})\leq\hat{R}(\hat{\boldsymbol{q}}\vee\tilde{\boldsymbol{q}},\boldsymbol{t})-\hat{R}(\tilde{\boldsymbol{q}},\boldsymbol{t})\leq 0.$$

Therefore, both $\hat{R}(\hat{\boldsymbol{q}}\wedge\tilde{\boldsymbol{q}},\boldsymbol{t})$ and $\hat{R}(\hat{\boldsymbol{q}}\vee\tilde{\boldsymbol{q}},\boldsymbol{t})$ belong to the set $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})$, which is thus a sublattice. Moreover, since for all $\boldsymbol{t}$, we have $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})\subset\mathcal{Q}$, which is bounded, and $\hat{R}(\boldsymbol{q},\boldsymbol{t})$ is continuous, we have that $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\boldsymbol{t})$ is a compact set. For any *compact sublattice*, there exists exactly one least element (Corollary 2.3.2 of Topkis 2011). Let

$$\boldsymbol{Q}^*(\boldsymbol{t})=\bigwedge\left\{\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\ \hat{R}(\boldsymbol{q},\boldsymbol{t})\right\}$$

be the *least element* of the argument of the maximum, for each $\boldsymbol{t}$. Precisely, we define

$$Q_n^*(\boldsymbol{t})=\min\left\{\tilde{q_n}\middle|\tilde{\boldsymbol{q}}\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\ \hat{R}(\boldsymbol{q},\boldsymbol{t})\right\},\ n=1,2,\ldots,N.$$

Since $\boldsymbol{Q}^*(\boldsymbol{t})\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\ \hat{R}(\boldsymbol{q},\boldsymbol{t})$ for all $\boldsymbol{t}$, from (A.13), we have

$$\boldsymbol{Q}^*(t_n,\boldsymbol{t_{-n}})\wedge\boldsymbol{Q}^*(\tilde{t_n},\boldsymbol{t_{-n}})\in\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\hat{R}(\boldsymbol{q},\tilde{t_n},\boldsymbol{t_{-n}}),\ \forall n,\ t_n<\tilde{t_n},\text{ and }\boldsymbol{t_{-n}}.$$

Since $\boldsymbol{Q}^*(\tilde{t_n},\boldsymbol{t_{-n}})$ is the *least element* of $\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}}\ \hat{R}(\boldsymbol{q},\tilde{t_n},\boldsymbol{t_{-n}})$, we have

$$\boldsymbol{Q}^*(t_n,\boldsymbol{t_{-n}})\geq\boldsymbol{Q}^*(t_n,\boldsymbol{t_{-n}})\wedge\boldsymbol{Q}^*(\tilde{t_n},\boldsymbol{t_{-n}})=\boldsymbol{Q}^*(\tilde{t_n},\boldsymbol{t_{-n}}).$$

Therefore, $Q_n^*(t_n,\boldsymbol{t_{-n}})$ is decreasing in t_n for any given $\boldsymbol{t_{-n}}$ and satisfies (A.12). Consequently, the mechanism $(\boldsymbol{Q}^*(\hat{\boldsymbol{t}}),\boldsymbol{M}^*(\hat{\boldsymbol{t}}))$ in Proposition 1 is optimal for problem $\mathbb{P}^{OE}$. ■

Appendix B: Proof of Results in Section 5

B.1. Proof of Proposition 2

We use backward induction and establish the PBE in the following four induction steps.

Step one: We examine the suppliers' strategies after realizing their production quantities $\boldsymbol{x}$. In this step, Supplier n's decisions include the purchase quantity $\tilde{y}_n$ from the reliable supply source and the delivered quantity z_n. The leftover quantity $\tilde{u}_n$ is determined by x_n, z_n and $\tilde{y}_n$ with $\tilde{u}_n = x_n + \tilde{y}_n - z_n$. To be concise, we define $\hat{W}_n^S(x_n, o_n, \hat{t}_n)$ as part of the payment function $W_n^S(x_n, z_n, o_n, \hat{t}_n)$ as follows:

$$\hat{W}_n^S(x_n, o_n, \hat{t}_n) = \begin{cases} \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \overline{\beta}_n(o_n)x_n, & \text{if } 0 \le x_n < \alpha_{n,1}o_n, \\ \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n + \underline{\beta}_n(o_n)x_n, & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \alpha_{n,1}o_n \le x_n \le o_n, & \\ \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n + \underline{\beta}_n(o_n)o_n, & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } x_n \ge o_n. & \end{cases}$$

Therefore, we have

$$W_n^S(x_n, z_n, o_n, \hat{t}_n) = \hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n\left(x_n - z_n\right)^+. \tag{B.1}$$

Disregarding the sunk costs incurred before this step, Supplier n aims to maximize his monetary income, which includes the payment from the buyer and the salvage value from the leftover component (if any), minus the purchasing cost incurred from the reliable supply source, as follows:

$$\max_{z_n, \tilde{u}_n, \tilde{y}_n} W_n^S(x_n, z_n, o_n, \hat{t}_n) + v_n\tilde{u}_n - p_n\tilde{y}_n. \tag{B.2}$$

Given $\tilde{u}_n = x_n + \tilde{y}_n - z_n$ and (B.1), this problem is equivalent to

$$\max_{z_n, \tilde{y}_n}\left\{-v_n\left(x_n - z_n\right)^+ + v_n\left(x_n + \tilde{y}_n - z_n\right) - p_n\tilde{y}_n\right\}.$$

The set of solutions to this problem is $\{(z_n, \tilde{y}_n) : 0 \le z_n \le x_n, \text{ and } \tilde{y}_n = 0\}$. Thus, the set of solutions to (B.2) is $\{(z_n, \tilde{u}_n, \tilde{y}_n) : z_n = x_n - \tilde{u}_n, 0 \le \tilde{u}_n \le x_n, \text{ and } \tilde{y}_n = 0\}$.

We now consider the following set of choices of Supplier n: Supplier n purchases zero quantity from the reliable supply source (i.e., $\tilde{y}_n = 0$), delivers a quantity of $z_n = x_n - U_n^*(\boldsymbol{x})$ to the buyer, and sell the leftover quantity of $\tilde{u}_n = U_n^*(\boldsymbol{x})$ to the salvage market. This set of quantities satisfies

$$\left(x_n - U_n^*(\boldsymbol{x}), U_n^*(\boldsymbol{x}), 0\right) \in \left\{(z_n, \tilde{u}_n, \tilde{y}_n) : z_n = x_n - \tilde{u}_n, 0 \le \tilde{u}_n \le x_n, \text{ and } \tilde{y}_n = 0\right\}. \tag{B.3}$$

and is thus a solution to (B.2). Therefore, for any given $\boldsymbol{x}$ produced by the suppliers, it is a dominant strategy for Supplier n to choose $\tilde{y}_n = 0$, $z_n = x_n - U_n^*(\boldsymbol{x})$, and $\tilde{u}_n = U_n^*(\boldsymbol{x})$. By following this dominant strategy, Supplier n delivers a quantity of $(x_n - U_n^*(\boldsymbol{x}))$ of his component to the buyer, receives a payment of $\left(\hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n U_n^*(\boldsymbol{x})\right)$ from the buyer, and has a salvage value of $v_n U_n^*(\boldsymbol{x})$ for his leftover components. As a result, Supplier n's monetary income equals $\hat{W}_n^S(x_n, o_n, \hat{t}_n)$. Moreover, step (vii) of the SC mechanism shows that the buyer subsequently purchases a quantity of $y_n = Y_n^*(\boldsymbol{x})$ of component n from the reliable supply source herself. Consequently, the buyer's revenue from the final product minus her total cost in this step – consisting of payments to the suppliers and purchasing costs incurred from the reliable supply sources – equals:

$$\begin{aligned} & r\left(\min_{n\in\mathcal{N}}\{x_n + Y_n^*(\boldsymbol{x}) - U_n^*(\boldsymbol{x})\}\right) - \sum_{n=1}^{N}\left(\hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n U_n^*(\boldsymbol{x})\right) - \sum_{n=1}^{N} p_n Y_n^*(\boldsymbol{x}) \\ & = r^*(\boldsymbol{x}) - \sum_{n=1}^{N} \hat{W}_n^S(x_n, o_n, \hat{t}_n). \end{aligned} \tag{B.4}$$

This equality is implied by the definition of $Y_n^*(\boldsymbol{x})$, $U_n^*(\boldsymbol{x})$, and $r^*(\boldsymbol{x})$ in Lemma 2.

Step two: We now examine the suppliers' choices of their input efforts $\boldsymbol{q}$ in response to the buyer's choice of $\boldsymbol{o}$. We show that for any type $\hat{t}_n$ Supplier n reveals in the screening and any o_n chosen by the buyer, Supplier n has a dominant strategy to choose an input effort equal to the buyer's desired quantity o_n; i.e., choose $q_n = o_n$. We now proceed to establish this claim.

Since $\underline{\beta}_n(o_n) = \frac{t_n h_n'(o_n)}{\sum_{i=1}^I \alpha_{n,i}\lambda_{n,i}}$, and $h_n(\cdot)$ is convex, we have that for all $q_n > o_n$,

$$\begin{aligned} & \frac{t_n\left[h_n(q_n) - h_n(o_n)\right]}{q_n - o_n} \geq \underline{\beta}_n(o_n)\sum_{i=1}^{I}\alpha_{n,i}\lambda_{n,i}, \text{ and thus} \\ & o_n\underline{\beta}_n(o_n)\sum_{i=1}^{I}\alpha_{n,i}\lambda_{n,i} - t_n h_n(o_n) \geq q_n\underline{\beta}_n(o_n)\sum_{i=1}^{I}\alpha_{n,i}\lambda_{n,i} - t_n h_n(q_n), \quad \forall t_n \in \mathcal{T}_n. \end{aligned} \tag{B.5}$$

Add $\mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n\sum_{i=1}^I\lambda_{n,i}$ to both sides:

$$\begin{aligned} & \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n\sum_{i=1}^{I}\lambda_{n,i} \\ & \quad + o_n\underline{\beta}_n(o_n)\sum_{i=1}^{I}\alpha_{n,i}\lambda_{n,i} - t_n h_n(o_n) \end{aligned}$$

$$
\begin{aligned}
&= \mathbb{E}_{\theta_n}\left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n)\right] - t_n h_n(o_n) \qquad \text{(from the definition of } \hat{W}_n^S(x_n, o_n, \hat{t}_n))\\
&\geq \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1} o_n \sum_{i=1}^{I} \lambda_{n,i}\\
&\quad + \underline{\beta}_n(o_n) q_n \sum_{i=1}^{I} \alpha_{n,i}\lambda_{n,i} - t_n h_n(q_n) \qquad \text{(since (B.5))}\\
&\geq \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1} o_n \sum_{i=1}^{I} \lambda_{n,i}\\
&\quad + \underline{\beta}_n(o_n) \sum_{i=1}^{I} \left(\max\{\alpha_{n,i} q_n, o_n\} \cdot \lambda_{n,i}\right) - t_n h_n(q_n)\\
&= \mathbb{E}_{\theta_n}\left[\hat{W}_n^S(\theta_n q_n, o_n, \hat{t}_n)\right] - t_n h_n(q_n), \quad \forall \hat{t}_n \in \mathcal{T}_n. \quad \text{(from the definition of } \hat{W}_n^S(x_n, o_n, \hat{t}_n))
\end{aligned}
\tag{B.6}
$$

That is, choosing any input effort of $q_n > o_n$ is less profitable than choosing $q_n = o_n$. Furthermore, since $\overline{\beta}_n(o_n) = \frac{\bar{t}_n h_n'(o_n)}{\alpha_{n,1}\lambda_{n,1}}$, and $h_n(\cdot)$ is convex, we have the following inequality for all $q_n < o_n$:

$$
\overline{\beta}_n(o_n)\alpha_{n,1}\lambda_{n,1} \geq \frac{t_n\left[h_n(o_n) - h_n(q_n)\right]}{o_n - q_n}, \text{ and therefore}
$$

$$
o_n\overline{\beta}_n(o_n)\alpha_{n,1}\lambda_{n,1} - t_n h_n(o_n) \geq q_n\overline{\beta}_n(o_n)\alpha_{n,1}\lambda_{n,1} - t_n h_n(q_n)\ \forall t_n \in \mathcal{T}_n.
$$

By adding $\sum_{i=2}^{I} \lambda_{n,i}\hat{W}_n^S(\alpha_{n,i}o_n, o_n, \hat{t}_n) + (\lambda_{n,0} + \lambda_{n,1})(\mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n))$ to both sides, we have the following inequality:

$$
\begin{aligned}
&\sum_{i=0}^{I} \lambda_{n,i}\hat{W}_n^S(\alpha_{n,i}o_n, o_n, \hat{t}_n) - t_n h_n(o_n) = \mathbb{E}_{\theta_n}\left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n)\right] - t_n h_n(o_n)\\
&\geq \sum_{i=2}^{I} \lambda_{n,i}\hat{W}_n^S(\alpha_{n,i}o_n, o_n, \hat{t}_n) + \sum_{j=0}^{1} \lambda_{n,j}\hat{W}_n^S(\alpha_{n,j}q_n, o_n, \hat{t}_n) - t_n h_n(q_n)\\
&\geq \sum_{i=0}^{I} \lambda_{n,i}\hat{W}_n^S(\alpha_{n,i}q_n, o_n, \hat{t}_n) - t_n h_n(q_n) \qquad \text{(since } W_n^S(x_n, o_n, \hat{t}_n) \text{ is increasing in } x_n)\\
&= \mathbb{E}_{\theta_n}\left[\hat{W}_n^S(\theta_n q_n, o_n, \hat{t}_n)\right] - t_n h_n(q_n), \qquad \forall \hat{t}_n \in \mathcal{T}_n.
\end{aligned}
\tag{B.7}
$$

This inequality proves that Supplier n's profit, by choosing any input effort $q_n < o_n$, is less than or equal to that by choosing o_n. Therefore, it is a dominant strategy for Supplier n to choose the input effort q_n equal to the desired quantity o_n. By choosing $q_n = o_n$, Supplier n's monetary income minus the non-refundable deposit, equals

$$\begin{aligned}\mathbb{E}_{\theta_n}\left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n)\right] &= \sum_{i=0}^{I} \lambda_{n,i} \hat{W}_n^S(\alpha_{n,i} o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n) \\ &= \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) - \mathbb{E}_{\boldsymbol{t_{-n}}}\left[h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\psi_n(\hat{t}_n)\right].\end{aligned} \tag{B.8}$$

Furthermore, given that Supplier n always exerts the input effort of $q_n = o_n$ for any o_n chosen by the buyer, the production quantities satisfies $\boldsymbol{x} = (\theta_n o_n : n = 1, 2, \ldots, N)$. The buyer's profit equals the summation of her revenue from final product and the non-refundable deposits paid by the suppliers, minus her payment to the suppliers, and minus her purchasing costs incurred from the reliable supply sources. From (B.4), for any given $\boldsymbol{o}$ and $\hat{\boldsymbol{t}}$, the buyer's total profit equals

$$\begin{aligned}& r\left(\min_{n\in\mathcal{N}}\{x_n + Y_n^*(\boldsymbol{x}) - U_n^*(\boldsymbol{x})\}\right) + \sum_{n=1}^{N} D_n^S(o_n, \hat{t}_n) - \sum_{n=1}^{N}\left(\hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n U_n^*(\boldsymbol{x})\right) - p_n Y_n^*(\boldsymbol{x}) \\ &= r^*(\boldsymbol{x}) - \sum_{n=1}^{N} \hat{W}_n^S(x_n, o_n, \hat{t}_n) + \sum_{n=1}^{N} D_n^S(o_n, \hat{t}_n) \\ &= r^*(\theta_1 o_1, \theta_2 o_2, \ldots, \theta_N o_N) - \sum_{n=1}^{N} \hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) + \sum_{n=1}^{N} D_n^S(o_n, \hat{t}_n) \qquad \text{(since } x_n = \theta_n o_n\text{)}.\end{aligned}$$

Step three: In this step, we show that given the suppliers' dominant strategies for choosing their input efforts ($q_n = o_n$ for all n), an optimal vector of desired quantities for the buyer is $\boldsymbol{o} = \boldsymbol{Q}^*(\hat{\boldsymbol{t}})$, where $\hat{\boldsymbol{t}}$ is the vector of types the suppliers reveal in the screening. We now establish this claim.

The buyer's problem is to choose a vector of desired quantities $\boldsymbol{o}$, which maximizes the buyer's expected total profit. That is,

$$\max_{\boldsymbol{o}\geq\boldsymbol{0}} \mathbb{E}_{\boldsymbol{\theta}}\left[r^*(\theta_1 o_1, \ldots, \theta_N o_N) - \sum_{n=1}^{N}\left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n)\right]\right].$$

The latter part is a summation of (B.8), for all $n = 1, 2, \ldots, N$. Since in (B.8), $M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})$ and $h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\psi_n(\hat{t}_n)$ are independent of o_n, the buyer's problem is equivalent to

$$\max_{\boldsymbol{o}\geq\boldsymbol{0}} \mathbb{E}_{\boldsymbol{\theta}}\left[r^*(\theta_1 o_1, \ldots, \theta_N o_N)\right] - \sum_{n=1}^{N} h_n(o_n)\psi_n(\hat{t}_n).$$

The following vector of desired quantities is a solution of this problem:

$$\boldsymbol{o} = \boldsymbol{Q}^*(\hat{\boldsymbol{t}}) = \bigwedge\left\{\arg\max_{\boldsymbol{q}\geq\boldsymbol{0}} \mathbb{E}_{\boldsymbol{\theta}}[r^*(\theta_1 q_1, \ldots, \theta_N q_N)] - \sum_{n=1}^{N} h_n(q_n)\psi_n(\hat{t}_n)\right\}. \tag{B.9}$$

Therefore, the buyer has a dominant strategy to choose the desired quantity of $o_n = Q_n^*(\hat{t})$, for all $n = 1, 2, \ldots, N$.

Step four: We prove that the suppliers reveal their types truthfully in the PBE; i.e., Supplier n chooses contract $\big(D_n^S(o_n, t_n), W_n^S(x_n, o_n, t_n)\big)$ from his menu, for every $n = 1, 2, \ldots, N$. We now establish this claim.

Given that the buyer and the suppliers will follow their dominant strategies established in the previous three steps, and that all suppliers other than Supplier n reveal their types truthfully, we examine the expected profit of Supplier n, who reveals his type as $\hat{t}_n$. From (B.8), Supplier n's monetary income minus the non-refundable deposit paid by Supplier n, in expectation over $\boldsymbol{t_{-n}}$ and $\boldsymbol{\theta}$, equals:

$$\mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}}) + h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\psi_n(\hat{t}_n) - h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\psi_n(\hat{t}_n)\right] = \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right]. \tag{B.10}$$

Thus, Supplier n's profit, in expectation over $\boldsymbol{t_{-n}}$ and $\boldsymbol{\theta}$, equals

$$\mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}}) - t_n h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\right]. \tag{B.11}$$

Recall that in Proposition 1, we establish the OE* mechanism characterized by function $\boldsymbol{Q}^*(\cdot)$ and $\boldsymbol{M}^*(\cdot)$. This mechanism satisfies the IC constraint, and therefore, function $\boldsymbol{Q}^*(\cdot)$ and $\boldsymbol{M}^*(\cdot)$ satisfy the following inequality:

$$\begin{aligned} M_n^*(\boldsymbol{t}) - t_n h_n(Q_n^*(\boldsymbol{t})) &\geq M_n^*(\hat{t}_n, \boldsymbol{t_{-n}}) - t_n h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}})), \text{ and thus} \\ \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\boldsymbol{t}) - t_n h_n(Q_n^*(\boldsymbol{t}))\right] &\geq \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}}) - t_n h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\right], \ \forall \hat{t}_n \in \mathcal{T}_n \text{ and } \boldsymbol{t} \in \boldsymbol{\mathcal{T}} \end{aligned} \tag{B.12}$$

Analogously, since the OE* mechanism satisfies the IR constraint, we have that function $\boldsymbol{Q}^*(\cdot)$ and $\boldsymbol{M}^*(\cdot)$ satisfy the following inequality for all $t_n \in \mathcal{T}_n$,:

$$\mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\boldsymbol{t}) - t_n h_n(Q_n^*(\boldsymbol{t}))\right] \geq 0. \tag{B.13}$$

Consequently, Supplier n willingly participates in the SC mechanism and reveals his type truthfully. The result follows. ■

B.2. Proof of Theorem 1

We now consider the profit of each participant in this PBE under the SC mechanism. From (B.11), in expectation over $\boldsymbol{t_{-n}}$ and θ_n, Supplier n's profit under the SC mechanism equals $\mathbb{E}_{\boldsymbol{t}_{-n}}\left[M_n^*(\boldsymbol{t}) - t_n h_n(Q_n^*(\boldsymbol{t}))\right]$, which is the same as that under OE*. Furthermore, by plugging $o_n = Q_n^*(\boldsymbol{t})$ and $\hat{t}_n = t_n$ into (B.9), we have that the buyer's expected profit in the PBE equals

$$\mathbb{E}_{(\boldsymbol{\theta},\boldsymbol{t})}\left[r^*\left(\theta_1 Q_1^*(\boldsymbol{t}), \theta_2 Q_2^*(\boldsymbol{t}), \ldots, \theta_N Q_N^*(\boldsymbol{t})\right) - \sum_{n=1}^{N} \hat{W}_n^S(\theta_n Q_n^*(\boldsymbol{t}), Q_n^*(\boldsymbol{t}), t_n) + \sum_{n=1}^{N} D_n^S(Q_n^*(\boldsymbol{t}), t_n)\right]$$
$$= \mathbb{E}_{(\boldsymbol{\theta},\boldsymbol{t})}\left[r^*\left(\theta_1 Q_1^*(\boldsymbol{t}), \theta_2 Q_2^*(\boldsymbol{t}), \ldots, \theta_N Q_N^*(\boldsymbol{t})\right) - \sum_{n=1}^{N} h_n(Q_n^*(\boldsymbol{t}))\psi_n(t_n)\right]$$

which is also the same as that under the OE* mechanism. ■

Appendix C: Limitations of "Rigid" Mechanisms: An Example

We refer to a mechanism as "rigid" if suppliers receive payment only upon delivering the exact ordered amount. The following example illustrates that rigid mechanisms may fail to achieve the optimal profit for the buyer in settings when suppliers face supply uncertainty.

Example 1: Consider a special case of our model, where a buyer procures a product from an unreliable supplier. The buyer's revenue from selling the product is $r(z) = 2\sqrt{z}$, where z denotes the product's quantity. The supplier's production cost is $c(q) = q$, which is linear and deterministic with respect to the input effort q. The production quantity x equals the input effort q with probability of 0.5 and equals $q/2$ otherwise, which represents a supply disruption. We assume that both the buyer and the supplier can purchase supplemental product from an alternative, reliable supply source at a unit price of $p = 2$. Consistent with our model, the input effort q remains unobservable to the buyer, whereas the production quantity x is observable.

- In the optimal mechanism for this special case, the buyer suggests that the supplier exert an input effort $q = \frac{3+2\sqrt{2}}{8} \approx 0.73$ and deliver the production quantity to the buyer. The buyer agrees to pay the supplier $\frac{4}{3}q$ if he delivers q units of product and pay $\frac{2}{3}q$ if the delivery quantity is $\frac{1}{2}q$. The supplier finds it optimal to follow this suggestion. The buyer's expected profit is $\frac{3+2\sqrt{2}}{8} \approx 0.73$.
- On the other hand, the optimal rigid mechanism specifies that the supplier should deliver $\frac{4}{9}$ units of the product, and the corresponding payment is $\frac{2}{3}$. The supplier's optimal strategy involves an input effort $q = \frac{4}{9}$. If the produced quantity equals $\frac{1}{2}q = \frac{2}{9}$, the supplier willingly purchases $\frac{2}{9}$ units from the alternative supply source and delivers the entire amount of product on hand. In this case, the buyer's expected profit equals $\frac{2}{3} \approx 0.67$, which is less than 0.73.

Example 1 demonstrates that no rigid mechanism can achieve the optimal profit for the buyer when suppliers face supply uncertainty. To optimize the buyer's profit, it is necessary to use a quantity-flexible mechanism such as our SC mechanism.

C.1. Proof of Results in Example 1

In this proof, we use the same notation as in the base model. We eliminate the supplier's index n since there is only one supplier in Example 1. Additionally, we eliminate the supplier's type t since there is only one possible type.

Part 1: The optimal mechanism for the buyer.

Let us denote the mechanism design problem in Example 1 as $\mathbb{P}_1$. The setting is a special case of our base model discussed in Section 3. Therefore, the analysis and results of our base model (Section 5) are applicable. Here, we briefly outline the key steps establishing the optimal mechanism:

We first consider a benchmark problem where the buyer directly observes the supplier's input effort q and purchasing quantity from the reliable supply source $\tilde{y}$. Let z^T be the quantity that satisfies $r'(z^T) = p$, where $r(z) = 2\sqrt{z}$ is the function of the buyer's revenue defined in Example 1. Here, we have $z^T = \frac{1}{4}$. Using backward induction, Lemma 2 in Section 4 implies that the buyer purchases $z^T - x$ units of product from the reliable supply source if the production quantity x falls below z^T. Otherwise, if the production quantity x exceeds z^T, then the buyer does not purchase any quantity from the reliable supply source. Thus, the two output quantities in Proposition 1 in Section 4 are as follows:

$$Q^* = \arg\max_{\boldsymbol{q} \geq \mathbf{0}} \left\{ \mathbb{E}_{\boldsymbol{\theta}} \left[r^*(\theta q) \right] - q \right\} = \frac{3 + 2\sqrt{2}}{8} \, ,$$

$$\text{and} \;\; M^* = Q^* = \frac{3 + 2\sqrt{2}}{8} \, .$$

Consequently, the optimal mechanism for the benchmark problem is as follows: The buyer pays the supplier an amount of M^* if and only if the supplier exerts an input effort of Q^* and delivers the entire production quantity. Under this mechanism, the supplier finds it optimal to exert the prescribed input effort Q^* and deliver the entire production quantity (either Q^* or $\frac{Q^*}{2}$) to the buyer. Also, the buyer does not purchase any quantity from the reliable supply source since even if a disruption occurs, the supplier produces and delivers $\frac{Q^*}{2}$, which is greater than z^T. This optimal mechanism for the benchmark problem establishes an upper bound (namely, $\frac{3+2\sqrt{2}}{8}$) on the buyer's expected profit in problem $\mathbb{P}_1$.

We now consider the mechanisms for problem $\mathbb{P}_1$. In Section 5, we have established an optimal and practical *screening mechanism* (SC) for problem $\mathbb{P}$ where the buyer's profit achieves the upper bound. Since $\mathbb{P}_1$ is a special case of $\mathbb{P}$, the corresponding SC mechanism is also optimal for $\mathbb{P}_1$. Moreover, in Example 1, we propose an alternative and simpler mechanism for $\mathbb{P}_1$. It is easy to verify that the buyer's expected profit under that mechanism also achieves the upper bound of the buyer's profit $\frac{3+2\sqrt{2}}{8}$, thereby establishing the optimality of the mechanism.

Part 2: The optimal rigid mechanism.

We now examine the optimal rigid mechanism. A rigid mechanism, denoted as (z,w), consists of an order quantity denoted by z and a corresponding payment denoted by w. The supplier receives a payment of w if he delivers exactly z units to the buyer, and zero payment otherwise.

The supplier accepts a typical rigid mechanism if and only if the following *participation constraint* is satisfied: The supplier's expected profit is non-negative by following his optimal strategy. It is easy to verify that to deliver z units, the supplier incurs a cost of at least z. Therefore, a necessary condition for participation is $w \geq z$. We now consider the following three cases regarding the supplier's input effort q, for a given contract (z,w) which satisfies the participation constraint:

- **Case 1:** *The supplier chooses an input effort* $q = z$*:*
  If disruption occurs, and the production quantity x equals $\frac{q}{2}$, then the supplier willingly purchases $\tilde{y} = \frac{q}{2}$ units of the product from the reliable supply sources to fulfill the order, since $w \geq z$. Thus, the supplier's expected profit equals $w - q - \frac{p\tilde{y}}{2} = w - \frac{3z}{2}$, since $p = 2$.
- **Case 2:** *The supplier chooses an input effort* $0 < q < z$*:*
  We consider the following four sub-cases regarding the supplier's other decisions:
  - *Sub-case 2.1:* The supplier fulfills the order by delivering z units of product to the buyer, regardless of whether disruption occurs ($\theta = \frac{1}{2}$) or not ($\theta = 1$). Therefore, the expected payment to the supplier equals w and the supplier's expected cost equals $q + \frac{p(z-q)}{2} + \frac{p}{2}\left(z - \frac{q}{2}\right)$. Consequently, the supplier's expected profit is $w - 2z + \frac{q}{2} < w - \frac{3z}{2}$.
  - *Sub-case 2.2:* The supplier fulfills the order by delivering z units of product to the buyer when no disruption occurs ($\theta = 1$), but chooses not to fulfill the order in the event of a disruption ($\theta = \frac{1}{2}$). In this case, the expected payment to the supplier equals $\frac{w}{2}$ and the supplier's expected cost equals $q + \frac{p(z-q)}{2}$, where $p = 2$. Consequently, the supplier's expected profit is $\frac{w}{2} - z \leq w - \frac{3z}{2}$, since $w \geq z$.

  - *Sub-case 2.3:* The supplier chooses not to fulfill the order when no disruption occurs ($\theta = 1$), but fulfills the order in the event of a disruption ($\theta = \frac{1}{2}$). Thus, the expected payment to the supplier equals $\frac{w}{2}$ and the supplier's expected cost equals $q + \frac{p}{2}(z - \frac{q}{2})$, where $p = 2$. Consequently, the supplier's expected profit is $\frac{w}{2} - z - \frac{q}{2} < w - \frac{3z}{2}$, since $w \geq z$ and $q > 0$.
  - *Sub-case 2.4:* The supplier does not fulfill the order, regardless of whether disruption occurs ($\theta = \frac{1}{2}$) or not ($\theta = 1$). The supplier's profit is negative in this case, since the supplier incurs a positive production cost, but receives zero payment. Thus, in this case, it is better for the supplier not to participate.

  Therefore, choosing any input effort $0 < q < z$ is less profitable than choosing $q = z$, for any contract (w, z) which satisfies the participation constraint.

- **Case 3:** *The supplier chooses an input effort $q > z$:*
  Since choosing any $q > 2z$ is obviously less profitable than choosing $q = 2z$, without loss of generality, we focus on the choice of input effort q where $z < q \leq 2z$. When no disruption occurs ($\theta = 1$), the production quantity x equals $q > z$. The supplier thus willingly delivers the production quantity to the buyer and receives a payment of w. When disruption occurs ($\theta = \frac{1}{2}$), we consider the following two cases:
  - *Sub-case 3.1:* The supplier fulfills the order when disruption occurs ($x = \frac{q}{2} \leq z$). To fulfill the order, the purchase quantity from the reliable supply source is $z - \frac{q}{2}$. Therefore, the supplier's expected profit equals $w - q - \frac{p}{2}(z - \frac{q}{2}) = w - z - \frac{q}{2} < w - \frac{3z}{2}$.
  - *Sub-case 3.2:* The supplier chooses not to fulfill the order when disruption occurs. Thus, the expected payment to the supplier is $\frac{w}{2}$ and the production cost equals q. Consequently, the supplier's expected profit is $\frac{w}{2} - q \leq w - q - \frac{z}{2} < w - \frac{3z}{2}$, since $w \geq z$ and $q > z$.

  As a result, choosing any input effort $q > z$ is less profitable than choosing $q = z$.

Therefore, for any given rigid contract (z, w) that satisfies the participation constraint, the supplier has a dominant strategy to choose an input effort of $q = z$, purchases $\frac{z}{2}$ units of product from the reliable supply source if disruption occurs, and delivers the ordered amount of z to the buyer. The supplier earns a corresponding profit of $w - \frac{3z}{2}$. Furthermore, the participation constraint requires that the supplier's optimal profit is non-negative with $w - \frac{3z}{2} \geq 0$. To search for an optimal rigid mechanism for the buyer, we can, without loss of generality, restrict attention to the set of mechanisms where the supplier earns zero profit; i.e., $w = \frac{3z}{2}$.

Recall that $z^T = \frac{1}{4}$ is the quantity that satisfies $r'(z^T) = p$. Given the supplier's dominant strategy above, the buyer's purchase quantity from the reliable supply source is solely determined by the ordered amount z as follows: the buyer will purchase $z^T - z$ units of product from the reliable supply source, if the delivered quantity z falls below z^T. Otherwise, if the delivered quantity z exceeds z^T, then the buyer will not purchase any quantity from the reliable supply source. Therefore, the buyer's expected profit, denoted as $\tilde{R}(z)$, is determined by the order quantity z as follows:

$$\tilde{R}(z) = \begin{cases} r(z) - w = 2\sqrt{z} - \frac{3z}{2}, & \text{if } z \geq \frac{1}{4}, \\ r(z^T) - w - p\left(z^T - z\right) = \frac{z+1}{2}, & \text{if } z < \frac{1}{4}. \end{cases} \qquad (\text{since } w = \frac{3z}{2})$$

The maximum of this function equals $\tilde{R}(z = \frac{4}{9}) = \frac{2}{3}$, which is less than the buyer's optimal profit $\frac{3+2\sqrt{2}}{8}$ under the optimal mechanism. ∎

Appendix D: Proof of Results in Section 7

D.1. Proof of Proposition 3

Similar to the notations used in Section 3 and Section 4, let y_n denote the quantity of component n that the buyer purchases from the corresponding reliable supply source. Additionally, we denote Supplier n's leftover quantity by $\tilde{u}_n$. For the benchmark problem $\mathbb{P}^{OES}$, the buyer also has access to the salvage market, and we denote her leftover quantity of component n by u_n.

Similar to Lemma 1 in Section 4, Lemma D.1 below demonstrates that to search for an optimal mechanism for the benchmark problem $\mathbb{P}^{OES}$, we can restrict attention to the direct revelation mechanisms where Supplier n neither purchases any quantity from the reliable supply source nor sells any quantity for salvage value.

LEMMA D.1. *There exists an optimal mechanism for the benchmark scenario with the following two properties for every $n = 1, 2, \ldots, N$: (1) Supplier n does not purchase any component from the reliable supply source, and (2) Supplier n delivers the entire production quantity to the buyer. In other words, under this optimal mechanism, for any realization of the vector of production-cost types $\mathbf{t}$ and the vector of random yields $\boldsymbol{\theta}$, we have $\tilde{u}_n = \tilde{y}_n = 0$ and $z_n = x_n = \theta_n q_n$.*

Proof of Lemma D.1:

The Revelation Principle (Myerson 1982) implies that there exists a direct revelation mechanism, which is optimal for the benchmark problem; we refer the optimal direct revelation mechanism as Mechanism D^S. In such a direct revelation mechanism, for every $n = 1, 2, \ldots, N$, Supplier n makes four distinct decisions, namely, the reported type $\hat{t}_n$, input effort q_n, purchase quantity from the reliable supply source $\tilde{y}_n$, and the delivered quantity z_n. Since p_n and v_n are stochastic and realized after Supplier n produces their components in this extension, the purchasing quantities from reliable supply sources and the leftover quantities may depend on the realizations of $\boldsymbol{p}$ and $\boldsymbol{v}$, where $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$ and $\boldsymbol{v} = (v_1, v_2, \ldots, v_N)$.

Therefore, Mechanism D^S can be characterized by a set of functions denoted as $\left(Q_n^D(\hat{\boldsymbol{t}}), \tilde{Y}_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}), Z_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}), M_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}) : n = 1, 2, \ldots, N\right)$. The implementation of Mechanism D^S is similar to that of a direct revelation mechanism for problem $\mathbb{P}^{OE}$ proposed in Section 4. In addition to the sequence of events of a direct revelation mechanism for problem $\mathbb{P}^{OE}$, under Mechanism D^S, the buyer asks Supplier n to purchase a quantity of $\tilde{Y}_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$ from the reliable supply source and delivers $Z_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$ units of component n. Here, we omit the other detailed implementations steps. The equilibrium outcome under Mechanism D^S is as follows: For any realization of $\boldsymbol{t}$, $\boldsymbol{\theta}$, $\boldsymbol{p}$, and $\boldsymbol{v}$, Supplier n reports his type truthfully ($\hat{t}_n = t_n$), exerts an input effort of $Q_n^D(\boldsymbol{t})$, purchases $\tilde{Y}_n^D(\boldsymbol{t}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$ units from the reliable supply sources, delivers a quantity of $Z_n^D(\boldsymbol{t}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$ to the buyer, and receives a payment of $M_n^D(\boldsymbol{t}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$.

We now define an alternative mechanism, namely, Mechanism A^S as follows: First, the buyer solicits suppliers' types. Then, the buyer asks Supplier n to exert an input effort of $Q_n^D(\hat{\boldsymbol{t}})$, purchase zero units from the reliable supply source, and deliver exactly the production quantity of $\theta_n Q_n^D(\hat{\boldsymbol{t}})$ to the buyer. If Supplier n follows the requirements, the buyer pays Supplier n an amount of

$$M_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}) - p_n \tilde{Y}_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}) + v_n \left(\theta_n Q_n^D(\hat{\boldsymbol{t}}) + \tilde{Y}_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v}) - Z_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})\right).$$

Otherwise, Supplier n need to pay a penalty of $x_n v_n$ to the buyer, where x_n is the quantity produced by Supplier n. The buyer then purchases $\tilde{Y}_n^D(\hat{\boldsymbol{t}}, \boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})$ units of component n from the reliable supply source herself. Mechanism A^S is feasible because all the suppliers' actions, namely, the choices of input efforts, production quantities, purchasing quantities from the reliable supply sources, and delivered quantities are observable to the buyer in the benchmark scenario. Furthermore, Mechanism A^S satisfies the two properties in Lemma 1.

Is is straightforward to verify that, in the equilibrium under Mechanism A^S, Supplier n reports his type truthfully and exerts the prescribed effort $Q_n^D(\hat{\boldsymbol{t}})$. Also, for all realization of v_t, Supplier n delivers the entire produced quantity to the buyer, because the penalty $x_n v_n$ for deviation is sufficiently large.

Therefore, for any given $\boldsymbol{t}$ and $\boldsymbol{\theta}$, under both Mechanism D^S and A, the buyer incurs a total cost of $\sum_{n=1}^N M_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})$ and has $Z_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})$ units of component n on hand, and Supplier n earns a profit of

$$M_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})-p_n\tilde{Y}_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})+v_n\left(\theta_n Q_n^D(\boldsymbol{t})+\tilde{Y}_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})-Z_n^D(\boldsymbol{t},\boldsymbol{\theta},\boldsymbol{p},\boldsymbol{v})\right)-t_nh_n\left(Q_n^D(\boldsymbol{t})\right).$$

In other words, the equilibrium outcomes under these two mechanisms are exactly the same for the buyer and each supplier. Thus, Mechanism A^S, which satisfies the two properties in Lemma D.1, is also optimal for $\mathbb{P}^{OES}$. □

We now search for an optimal direct revelation mechanism that satisfies the two properties outlined in Lemma D.1. The sequence of events under a direct revelation mechanism for problem $\mathbb{P}^{OES}$ is identical to that of problem $\mathbb{P}^{OE}$ in Section 4, except for the modification introduced in Section 7.1: In step (i), the buyer still pays Supplier n an amount of $M_n(\hat{\boldsymbol{t}})$ if the supplier exerts the prescribed input effort $Q_n(\hat{\boldsymbol{t}})$ and delivers exactly the quantity he produces. However, if Supplier n deviates, he now pay a penalty of $x_n v_n$ to the buyer, where x_n is the quantity produced.

We also need the following modifications for the formulations of quantities: Recall that in the formulation of problem $\mathbb{P}^{OE}$ in Section 4, function $Y_n(\boldsymbol{x})$ denotes the additional quantity of component n that the buyer procures from the corresponding reliable supply source, and $U_n(\boldsymbol{x})$ denotes the quantity of component n the buyer decides to salvage, where $\boldsymbol{x}=(x_1,x_2,\dots,x_N)=(\theta_1q_1,\theta_2q_2,\dots,\theta_Nq_N)$ is the vector of quantities produced and delivered by the suppliers. For problem $\mathbb{P}^{OES}$, the unit costs $\boldsymbol{p}$ from the reliable supply sources and salvage values $\boldsymbol{v}$ of leftover components are both stochastic, and the buyer's decisions can depend on the realizations of these two vectors. Therefore, under a direct revelation mechanism, we now denote the procurement quantity of component n from the reliable supply source as function $Y_n(\boldsymbol{x},\boldsymbol{p},\boldsymbol{v})$ and the leftover quantity of component n that the buyer sells for salvage value $U_n(\boldsymbol{x},\boldsymbol{p},\boldsymbol{v})$. An optimal set of these two functions are as follows:

LEMMA D.2. *For any given vectors $\boldsymbol{x}$, $\boldsymbol{p}$, and $\boldsymbol{v}$, the buyer's optimal choice of (a) purchasing quantities from the reliable supply sources and (b) salvage quantities, denoted by*

$$(\boldsymbol{Y}^*(\boldsymbol{x},\boldsymbol{p},\boldsymbol{v}),\boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{p},\boldsymbol{v}))\in\operatorname*{arg\,max}_{\substack{\boldsymbol{y}\geq\boldsymbol{0}\\0\leq\boldsymbol{u}\leq\boldsymbol{x}}}\hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u},\boldsymbol{p},\boldsymbol{v})\ \textit{ exists,}$$

$$\textit{where }\ \hat{r}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u},\boldsymbol{p},\boldsymbol{v}):=r\left(\min_{n\in\mathcal{N}}\{x_n+y_n-u_n\}\right)+\sum_{n=1}^N(v_nu_n-p_ny_n).$$

Furthermore, the pointwise maximum of this objective function, defined as,

$$r^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}) := \max_{\substack{\boldsymbol{y} \geq \boldsymbol{0} \\ 0 \leq \boldsymbol{u} \leq \boldsymbol{x}}} \hat{r}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{p}, \boldsymbol{v}),$$

is an increasing, concave, and supermodular function of $(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v})$.

Proof of Lemma D.2:

Using the proof of Lemma 2 in Appendix A.2 and Assumption 7, it is straightforward to prove that $(\boldsymbol{Y}^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}), \boldsymbol{U}^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}))$, as defined in Lemma D.2, exits and is optimal for the buyer, for any given $\boldsymbol{x}$, $\boldsymbol{p}$, and $\boldsymbol{v}$. Denote $\boldsymbol{w} = (\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v})$. In what follows, we prove the monotonicity, concavity, and supermodularity of function $r^*(\boldsymbol{w})$ sequentially:

For any given $\tilde{\boldsymbol{w}} = (\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}) \geq \boldsymbol{w} = (\tilde{\boldsymbol{x}}, -\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{v}})$, we have

$$r^*(\tilde{\boldsymbol{w}}) \geq \hat{r}(\tilde{\boldsymbol{x}}, \boldsymbol{Y}^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}), \boldsymbol{U}^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}), \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{v}}) \geq \hat{r}(\boldsymbol{x}, \boldsymbol{Y}^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}), \boldsymbol{U}^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}), \boldsymbol{p}, \boldsymbol{v}) = r^*(\boldsymbol{w}),$$

which proves that $r^*(\tilde{\boldsymbol{w}})$ is increasing.

We now prove the concavity of $r^*(\boldsymbol{w})$. Let $\hat{u}_n = -u_n$ and $\hat{\boldsymbol{u}} = (\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_N)$. We define

$$\tilde{r}(\boldsymbol{x}, \boldsymbol{y}, \hat{\boldsymbol{u}}, \boldsymbol{p}, \boldsymbol{v}) = \hat{r}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{p}, \boldsymbol{v}) = r\left(\min_{n \in \mathcal{N}} \{x_n + y_n + \hat{u}_n\}\right) - \sum_{n=1}^{N} (v_n \hat{u}_n + p_n y_n).$$

Since $r(\cdot)$ and $\min\{\cdot\}$ are both increasing and concave, $\tilde{r}(\boldsymbol{x}, \boldsymbol{y}, \hat{\boldsymbol{u}}, \boldsymbol{p}, \boldsymbol{v})$ is a concave function. Therefore, for any $\epsilon > 0$ and $\boldsymbol{w}^{(i)} = (\boldsymbol{x}^{(i)}, -\boldsymbol{p}^{(i)}, \boldsymbol{v}^{(i)})$, where $i \in \{1, 2\}$, there exists $\left(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{u}}^{(i)}\right)$, where $\boldsymbol{y}^{(i)} \geq \boldsymbol{0}$ and $-\boldsymbol{x}^{(i)} \leq \hat{\boldsymbol{u}}^{(i)} \leq \boldsymbol{0}$, such that $r^*(\boldsymbol{w}^{(i)}) - \epsilon < \tilde{r}(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)}, \hat{\boldsymbol{u}}^{(i)}, \boldsymbol{p}^{(i)}, \boldsymbol{v}^{(i)})$. Thus, for any $\beta \in [0, 1]$, the following inequality holds:

$$\begin{aligned}
&\beta r^*(\boldsymbol{w}^{(1)}) + (1-\beta) r^*(\boldsymbol{w}^{(2)}) - \epsilon \\
&< \beta \tilde{r}\left(\boldsymbol{x}^{(1)}, \boldsymbol{y}^{(1)}, \hat{\boldsymbol{u}}^{(1)}, \boldsymbol{p}^{(1)}, \boldsymbol{v}^{(1)}\right) + (1-\beta) \tilde{r}\left(\boldsymbol{x}^{(2)}, \boldsymbol{y}^{(2)}, \hat{\boldsymbol{u}}^{(2)}, \boldsymbol{p}^{(2)}, \boldsymbol{v}^{(2)}\right) \\
&\leq \tilde{r}\left(\beta \boldsymbol{x}^{(1)} + (1-\beta)\boldsymbol{x}^{(2)}, \beta \boldsymbol{y}^{(1)} + (1-\beta)\boldsymbol{y}^{(2)}, \beta \hat{\boldsymbol{u}}^{(1)} + (1-\beta)\hat{\boldsymbol{u}}^{(2)}, \beta \boldsymbol{p}^{(1)} + (1-\beta)\boldsymbol{p}^{(2)}, \beta \boldsymbol{v}^{(1)} + (1-\beta)\boldsymbol{v}^{(2)}\right) \\
&\leq r^*(\beta \boldsymbol{w}^{(1)} + (1-\beta)\boldsymbol{w}^{(2)}).
\end{aligned}$$

The second inequality follows from the concavity of $\tilde{r}(\cdot)$, and the third inequality is obtained by the definition of $r^*(\cdot)$. Thus, $r^*(\boldsymbol{w})$ is concave, where $\boldsymbol{w} = (\boldsymbol{x}, -\boldsymbol{q}, \boldsymbol{v})$.

Next, we prove that $r^*(\boldsymbol{w})$ is a supermodular function. In Appendix A.2, inequality (A.4) has proved that $r(\min_{n \in \mathcal{N}}\{y_n + x_n + \hat{u}_n\})$ is a supermodular function of $(\boldsymbol{x}, \boldsymbol{y}, \hat{\boldsymbol{u}})$, and thus $\tilde{r}(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{u}},\boldsymbol{p},\boldsymbol{v})$ is also supermodular. Since supermodularity is preserved by the pointwise maximization operation (see, e.g., inequality (A.5) in Appendix A.2), $r^*(\boldsymbol{w})$ is supermodular. □

When choosing the purchasing quantities from the reliable supply sources $\boldsymbol{y}$ and the leftover quantities $\boldsymbol{u}$, the buyer's problem is a deterministic optimization problem that only depends on the vector of quantities $\boldsymbol{x}$ delivered by the suppliers, the vector of unit purchasing costs $\boldsymbol{p}$ from the reliable supply sources, and the vector of salvage values $\boldsymbol{v}$. Given the solution to this problem established in Lemma D.2, $r^*(\boldsymbol{x},-\boldsymbol{q},\boldsymbol{v})$ is the buyer's corresponding revenue from the assembled product, plus the salvage value from the leftover quantities, and minus the costs incurred in purchasing the supplemental components from the reliable supply sources. In other words, from the buyer's perspective, $r^*(\boldsymbol{x},-\boldsymbol{q},\boldsymbol{v})$ is the value associated with the vector $\boldsymbol{x}$ of delivered components, the vector $\boldsymbol{q}$ of purchasing costs from the reliable supply sources, and the vector $\boldsymbol{v}$ of salvage values in problem $\mathbb{P}^{OES}$.

Using Lemma D.2, a simplified version of problem $\mathbb{P}^{OES}$ can be formulated as follows:

$$\max_{\boldsymbol{M}(\cdot),\boldsymbol{Q}(\cdot)} \int_{\boldsymbol{\mathcal{T}}} \mathbb{E}_{(\boldsymbol{\theta},\boldsymbol{q},\boldsymbol{v})}\left[r^*\big(\theta_1 Q_1(\boldsymbol{t}),\ldots,\theta_N Q_N(\boldsymbol{t}),-\boldsymbol{q},\boldsymbol{v}\big)-\sum_{n=1}^{N} M_n(\boldsymbol{t})\right]\boldsymbol{dF}(\boldsymbol{t}),$$

$$\text{subject to}\ \ \mathbb{E}_{\boldsymbol{\theta}}\left[M_n(\boldsymbol{t})-t_n\tilde{h}_n(Q_n(\boldsymbol{t}),\theta_n)\right]\geq \mathbb{E}_{\boldsymbol{\theta}}\left[M_n(\hat{t}_n,\boldsymbol{t_{-n}})-t_n\tilde{h}_n(Q_n(\hat{t}_n,\boldsymbol{t_{-n}}),\theta_n)\right],\quad \forall \boldsymbol{t},\hat{t}_n, \tag{IC}$$

$$\text{and}\ \ \mathbb{E}_{\boldsymbol{\theta}}\left[M_n(\boldsymbol{t})-t_n\tilde{h}_n(Q_n(\boldsymbol{t}),\theta_n)\right]\geq 0,\quad \forall \boldsymbol{t}. \tag{IR}$$

Since $\boldsymbol{t}$ is independent of $\boldsymbol{\theta}$,$\boldsymbol{q}$, and $\boldsymbol{v}$ (Assumption 5) and $\mathbb{E}_{\boldsymbol{\theta}}\left[\tilde{h}_n(q_n,\theta_n)\right]=\mathbb{E}_{\theta_n}\left[\tilde{h}_n(q_n,\theta_n)\right]=h_n(q_n)$, this problem is identical to the *simplified* formulation of problem $\mathbb{P}^{OE}$ on page 13, except that the function $r^*(\cdot)$ – which represents the value associated with the delivered components – additionally depends on $\boldsymbol{p}$ and $\boldsymbol{v}$ in problem $\mathbb{P}^{OES}$.

It is straightforward to verify that the statement and proof of Proposition 1, with corresponding modifications to the $r^*(\cdot)$ function, remain applicable in problem $\mathbb{P}^{OES}$: The proof of Proposition 1 in Appendix A.3 relies on three properties of the function $r^*(\theta_1 Q_1(\boldsymbol{t}),\ldots,\theta_N Q_N(\boldsymbol{t}))$, namely monotonicity, concavity, and supermodularity. For problem $\mathbb{P}^{OES}$, the counterpart $r^*\left(\theta_1 Q_1(\boldsymbol{t}),\ldots,\theta_N Q_N(\boldsymbol{t}),-\boldsymbol{q},\boldsymbol{v}\right)$ also has these three properties, as established in Lemma D.2. This completes the proof of Proposition 3. ■

Up to this point, we have derived the optimal mechanism for the benchmark problem $\mathbb{P}^{OES}$, which provides an upper bound on the buyer's profit in our original problem $\mathbb{P}^{S}$. In what follows, we proceed to demonstrate that the SCS mechanism we propose for $\mathbb{P}^{S}$ achieves this upper bound.

D.2. Proof of Theorem 2

It is noteworthy that the statement of Theorem 2 essentially combines the statements of Proposition 2 and Theorem 1, with minor notational adjustments. Following a similar approach to the proof of Proposition 2 in Appendix B.1, we use backward induction to establish the PBE under the SCS mechanism in four induction steps:

Step one: We examine the suppliers' strategies after realizing their production quantities $\boldsymbol{x}$, purchasing costs from the reliable supply sources $\boldsymbol{p}$, and salvage values $\boldsymbol{v}$. In this step, Supplier n's decisions include the purchase quantity $\tilde{y}_n$ from the reliable supply source and the delivered quantity z_n. The leftover quantity $\tilde{u}_n$ is determined by x_n, z_n and $\tilde{y}_n$ with $\tilde{u}_n = x_n + \tilde{y}_n - z_n$. To be concise, we define $\hat{W}_n^S(x_n, o_n, \hat{t}_n)$ as part of the payment function $W_n^S(x_n, z_n, o_n, \hat{t}_n, v_t)$ as follows:

$$\hat{W}_n^S(x_n, o_n, \hat{t}_n) := \begin{cases} \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \overline{\beta}_n(o_n)x_n, & \text{if } 0 \le x_n < \alpha_{n,1}o_n, \\ \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n + \underline{\beta}_n(o_n)x_n, & \\ & \text{if } \alpha_{n,1}o_n \le x_n \le o_n, \\ \mathbb{E}_{\boldsymbol{t_{-n}}}\left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})\right] + h_n(o_n)\psi_n(\hat{t}_n) + \left(\overline{\beta}_n(o_n) - \underline{\beta}_n(o_n)\right)\alpha_{n,1}o_n + \underline{\beta}_n(o_n)o_n, & \\ & \text{if } x_n \ge o_n. \end{cases}$$

Therefore, we have

$$W_n^S(x_n, z_n, o_n, \hat{t}_n, v_t) = \hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n\left(x_n - z_n\right)^+. \tag{D.1}$$

Disregarding the sunk costs incurred before this step, Supplier n aims to maximize his monetary income, which includes the payment from the buyer and the salvage value from the leftover component (if any), minus the purchasing cost incurred from the reliable supply source. Therefore, for any given p_n and v_n, Supplier n's problem in this step is as follows:

$$\max_{z_n, \tilde{u}_n, \tilde{y}_n} W_n^S(x_n, z_n, o_n, \hat{t}_n, v_t) + v_n\tilde{u}_n - p_n\tilde{y}_n. \tag{D.2}$$

Given $\tilde{u}_n = x_n + \tilde{y}_n - z_n$ and (D.1), this problem is

$$\max_{z_n, \tilde{y}_n}\left\{-v_n\left(x_n - z_n\right)^+ + v_n\left(x_n + \tilde{y}_n - z_n\right) - p_n\tilde{y}_n\right\}.$$

The set of solutions to this problem is $\{(z_n, \tilde{y}_n) : 0 \le z_n \le x_n, \text{ and } \tilde{y}_n = 0\}$. Thus, the set of solutions to (D.2) is $\{(z_n, \tilde{u}_n, \tilde{y}_n) : z_n = x_n - \tilde{u}_n,\ 0 \le \tilde{u}_n \le x_n, \text{ and } \tilde{y}_n = 0\}$.

We now consider the following set of choices of Supplier n: For any given realization of $\boldsymbol{p}$ and $\boldsymbol{v}$, Supplier n purchases zero quantity from the reliable supply source (i.e., $\tilde{y}_n = 0$), delivers a quantity of $z_n = x_n - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$ to the buyer, and sell the leftover quantity of $\tilde{u}_n = U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$ to the salvage market[7]. This set of quantities satisfies

$$(x_n - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}),\ U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}),\ 0) \in \{(z_n, \tilde{u}_n, \tilde{y}_n) : z_n = x_n - \tilde{u}_n, 0 \le \tilde{u}_n \le x_n, \text{ and } \tilde{y}_n = 0\},$$

and is thus a solution to problem (D.2). Therefore, for any given $\boldsymbol{x}$, $\boldsymbol{p}$, and $\boldsymbol{v}$, it is a dominant strategy for Supplier n to choose $\tilde{y}_n = 0$, $z_n = x_n - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$, and $\tilde{u}_n = U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$. By following this dominant strategy, Supplier n delivers a quantity of $(x_n - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}))$ of his component to the buyer, receives a payment of $\left(\hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})\right)$ from the buyer, and has a salvage value of $v_n U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$ for his leftover components. As a result, Supplier n's monetary income equals $\hat{W}_n^S(x_n, o_n, \hat{t}_n)$. Moreover, under the SCS mechanism, the buyer subsequently purchases a quantity of $y_n = Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$ of component n from the reliable supply source herself. Consequently, for any given vectors $\boldsymbol{x}$, $\boldsymbol{p}$, and $\boldsymbol{v}$, the buyer's revenue from the final product minus her expected cost in this step – consisting of payments to the suppliers and purchasing costs incurred from the reliable supply sources – equals:

$$\begin{aligned}
&r(\min_{n\in\mathcal{N}}\{x_n + Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}) - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})\}) - \sum_{n=1}^{N}(\hat{W}_n^S(x_n, o_n, \hat{t}_n) - v_n U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})) - \sum_{n=1}^{N} p_n Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}) \\
&= r(\min_{n\in\mathcal{N}}\{x_n + Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}) - U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})\}) + \sum_{n=1}^{N}(v_n U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v}) - p_n Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})) - \sum_{n=1}^{N} \hat{W}_n^S(x_n, o_n, \hat{t}_n) \\
&= r^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}) - \sum_{n=1}^{N} \hat{W}_n^S(x_n, o_n, \hat{t}_n).
\end{aligned}$$

The second equality is implied by the definition of $Y_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$, $U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$, and $r^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v})$ in Lemma D.2. This completes Step one.

Step two: The proof of this induction step is the same as *Step two* in Appendix B.1. As a result, for any type $\hat{t}_n$ Supplier n reveals in the screening and any o_n chosen by the buyer, Supplier n

[7] Function $U_n^*(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$ is defined in Lemma D.2 above.

has a dominant strategy to choose an input effort equal to the buyer's desired quantity o_n; i.e., choose $q_n = o_n$. Consequently, for any given $\boldsymbol{o}$ and $\hat{\boldsymbol{t}}$, the buyer's total profit equals:

$$\begin{aligned}
&r^*(\boldsymbol{x}, -\boldsymbol{p}, \boldsymbol{v}) - \sum_{n=1}^{N} \hat{W}_n^S(x_n, o_n, \hat{t}_n) + \sum_{n=1}^{N} D_n^S(o_n, \hat{t}_n) \\
&= r^*(\theta_1 o_1, \theta_2 o_2, \ldots, \theta_N o_N, -\boldsymbol{p}, \boldsymbol{v}) - \sum_{n=1}^{N} \hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) + \sum_{n=1}^{N} D_n^S(o_n, \hat{t}_n) \quad \text{(since } x_n = \theta_n q_n = \theta_n o_n\text{)}.
\end{aligned} \tag{D.3}$$

Step three: In this step, we show that given the suppliers' dominant strategies for choosing their input efforts ($q_n = o_n$ for all n), an optimal vector of desired quantities for the buyer is $\boldsymbol{o} = \boldsymbol{Q}^*(\hat{\boldsymbol{t}})$, where $\hat{\boldsymbol{t}}$ is the vector of types the suppliers reveal in the screening. We now establish this claim.

The buyer's problem is to choose a vector of desired quantities $\boldsymbol{o}$, which maximizes the buyer's total profit obtained in (D.3) in expectation over $(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{v})$. The buyer's problem is:

$$\max_{\boldsymbol{o} \geq \boldsymbol{0}} \mathbb{E}_{(\boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})} \left[r^*(\theta_1 o_1, \theta_2 o_2, \ldots, \theta_N o_N, -\boldsymbol{p}, \boldsymbol{v}) - \sum_{n=1}^{N} \left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n) \right] \right], \tag{D.4}$$

where $\mathbb{E}_{(\boldsymbol{\theta}, \boldsymbol{p}, \boldsymbol{v})} \left[\hat{W}_n^S(\theta_n o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n) \right]$

$$\begin{aligned}
&= \sum_{i=0}^{I} \lambda_{n,i} \hat{W}_n^S(\alpha_{n,i} o_n, o_n, \hat{t}_n) - D_n^S(o_n, \hat{t}_n) \\
&= \mathbb{E}_{\boldsymbol{t_{-n}}} \left[M_n^*(\hat{t}_n, \boldsymbol{t_{-n}}) \right] + h_n(o_n) \psi_n(\hat{t}_n) - \mathbb{E}_{\boldsymbol{t_{-n}}} \left[h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}})) \psi_n(\hat{t}_n) \right].
\end{aligned}$$

Since, $M_n^*(\hat{t}_n, \boldsymbol{t_{-n}})$ and $h_n(Q_n^*(\hat{t}_n, \boldsymbol{t_{-n}}))\psi_n(\hat{t}_n)$ are independent of o_n, the buyer's problem (D.4) is equivalent to

$$\max_{\boldsymbol{o} \geq \boldsymbol{0}} \mathbb{E}_{(\boldsymbol{\theta}, \boldsymbol{q}, \boldsymbol{v})} \left[r^*(\theta_1 o_1, \ldots, \theta_N o_N, -\boldsymbol{q}, \boldsymbol{v}) \right] - \sum_{n=1}^{N} h_n(o_n) \psi_n(\hat{t}_n).$$

Obviously, the following vector of desired quantities is a solution of this problem:

$$\boldsymbol{o} = \boldsymbol{Q}^*(\hat{\boldsymbol{t}}) = \bigwedge \left\{ \arg\max_{\boldsymbol{q} \geq \boldsymbol{0}} \mathbb{E}_{(\boldsymbol{\theta}, \boldsymbol{q}, \boldsymbol{v})} [r^*(\theta_1 q_1, \ldots, \theta_N q_N, -\boldsymbol{q}, \boldsymbol{v})] - \sum_{n=1}^{N} h_n(q_n) \psi_n(\hat{t}_n) \right\}.$$

Therefore, the buyer has a dominant strategy to choose the desired quantity of $o_n = Q_n^*(\hat{t})$, for all $n = 1, 2, \ldots, N$.

Step four is the same as that in Appendix B.1, establishing the PBE under the SCS mechanism.

Following the same argument as in the proof of Theorem 1 (Appendix B.2), the buyer's profit in the PBE under the SCS mechanism achieves its upper bound, as established by the OES* mechanism in the benchmark $\mathbb{P}^{OES}$. Thus, the SCS mechanism is optimal for problem $\mathbb{P}^S$. ■

D.3. Proof of Theorem 3

It is straightforward that the buyer's optimal profit increases with more information. Therefore,

$$\mathcal{P}^{S1} \geq \mathcal{P}^{S2} \geq \mathcal{P}^{S4}, \text{ and } \mathcal{P}^{S1} \geq \mathcal{P}^{S3} \geq \mathcal{P}^{S4}.$$

Additionally, we denote the difference between the high unit production cost and the low unit cost as as $\Delta_c = c_h - c_l$, and the difference between the the high disruption risk and the low risk as $\Delta_p = p_H - p_L$. We now analyze the profit of the four settings sequentially.

Proof of S1: Both the supplier's disruption risk and effort are observable to the buyer in $S1$. The revelation principle (Myerson 1981) implies that, to search for an optimal mechanism, we can restrict attention to the class of direct revelation mechanisms under which the payment is a function of the supplier's input effort. Specifically, a direct revelation mechanism is defined collectively by a set $\{(q_{ij}, m_{ij}) | i = l, h, \text{ and } j = L, H\}$ and can be implemented as follows:

(i) Given the observed disruption risk p_j, where $j \in \{L, H\}$, the buyer specifies to the supplier an input effort q_{ij} and a monetary payment m_{ij} for each cost type $i \in \{l, h\}$. The buyer commits to pay the supplier an amount of m_{ij} if and only if that supplier exerts the specified input effort q_{ij} and delivers exactly the quantity he produces.

(ii) For any realization of the production cost c_i, the supplier finds it optimal to exert the corresponding input effort q_{ij} and to deliver the entire produced amount (if any) to the buyer.

(iii) The buyer pays m_{ij} to the supplier and sells the delivered quantity (if any) for revenue.

Over the set of such mechanisms, the buyer's problem $\mathbb{P}^{S1}$ is to maximize her expected profit, taken over the distributions of both disruption risk type and cost type. Since the buyer's revenue $r(z)$ is zero in the event of a disruption, the formulation of problem $\mathbb{P}^{S1}$ is as follows:

$$\max_{q_{ij}, m_{ij}} \sum_{j=L,H} \sum_{i=l,h} \left(\overline{p}_j \cdot r(q_{ij}) - m_{ij}\right) t_i^c \cdot t_j^d, \quad \text{where } \overline{p}_j = 1 - p_j, \tag{$\mathbb{P}^{S1}$}$$

subject to,

$$m_{ij} - c_i \cdot q_{ij} \geq m_{\hat{i}j} - c_i \cdot q_{\hat{i}j}, \quad \text{for all } i, \hat{i} \in \{l, h\} \text{ and } i \neq \hat{i}, \tag{$IC1$}$$

$$m_{ij} - c_i \cdot q_{ij} \geq 0, \quad \text{for all } i \in \{l, h\}. \tag{$IR1$}$$

We now solve this optimization problem. We denote $P_{ij}^{S1} = m_{ij} - c_i \cdot q_{ij}$ as the supplier's profit when the cost is i, the disruption risk is j, and the input effort is q_{ij}. For any given j, it is straightforward to verify that the $IC1$ constraint holds if and only if $q_{lj} \geq q_{hj}$ and the following inequality is satisfied:

$$\begin{aligned} & m_{hj} - c_h \cdot q_{hj} + \Delta_c \cdot q_{hj} \leq \ m_{lj} - c_l \cdot q_{lj} \ \leq m_{hj} - c_h \cdot q_{hj} + \Delta_c \cdot q_{lj}, \\ \text{that is,} \quad & P_{hj}^{S1} + \Delta_c \cdot q_{hj} \leq \ P_{lj}^{S1} \ \leq P_{hj}^{S1} + \Delta_c \cdot q_{lj}. \end{aligned} \tag{D.5}$$

Therefore, to solve

$$\max_{P_{ij}^{S1}} \sum_{j=L,H} \sum_{i=l,h} \left(\overline{p}_j \cdot r(q_{ij}) - P_{ij}^{S1} - c_i \cdot q_{ij}\right) t_i^c \cdot t_j^d,$$

we use $P_{lj}^{S1} = P_{hj}^{S1} + \Delta_c \cdot q_{hj}$ (from D.5) and $P_{hj}^{S1} = 0$ (from $IR1$). Thus, the following mechanism, denoted as $S1^*$, is optimal for $\mathbb{P}^{S1}$: For both $j \in \{L, H\}$,

$$\begin{aligned} q_{hj}^{S1} =& \arg\max_{q\geq 0} \left\{\overline{p}_j \cdot r(q) - \left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right) \cdot q\right\} = r^{-1}\left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right), \\ q_{lj}^{S1} =& \arg\max_{q\geq 0} \left\{\overline{p}_j \cdot r(q) - c_l \cdot q\right\} = r^{-1}(c_l), \\ m_{lj}^{S1} =& c_l \cdot q_{lj}^{S1} + \Delta_c \cdot q_{hj}^{S1}, \\ \text{and} \ \ m_{hj}^{S1} =& c_h \cdot q_{hj}^{S1}. \end{aligned} \tag{$S1^*$}$$

Note that the constraint $q_{lj} \geq q_{hj}$ is also satisfied. Consequently, the buyer's optimal profit under this mechanism is:

$$\begin{aligned} \mathcal{P}^{S1} = & \ t_l^c \cdot t_L^d \cdot \underbrace{\max_{q_{lL}} \left\{\overline{p}_L \cdot r(q_{lL}) - c_l \cdot q_{lL}\right\}}_{\text{Term 1}} + t_h^c \cdot t_L^d \cdot \underbrace{\max_{q_{hL}} \left\{\overline{p}_L \cdot r(q_{hL}) - \left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right) \cdot q_{hL}\right\}}_{\text{Term 2}} \\ & + t_l^c \cdot t_H^d \cdot \underbrace{\max_{q_{lH}} \left\{\overline{p}_H \cdot r(q_{lH}) - c_l \cdot q_{lH}\right\}}_{\text{Term 3}} + t_h^c \cdot t_H^d \cdot \underbrace{\max_{q_{hH}} \left\{\overline{p}_H \cdot r(q_{hH}) - \left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right) \cdot q_{hH}\right\}}_{\text{Term 4}}. \end{aligned} \tag{D.6}$$

For both setting $S2$ and $S3$, the buyer's profit is smaller than or equal to $\mathcal{P}^{S1}$ since the buyer has less information. For each of these two settings, we will directly propose a *direct revelation* mechanism under which the buyer's profit is equal to $\mathcal{P}^{S1}$.

Proof of S2: This setting differs from $S1$ in that the supplier's input effort q is unobservable to the buyer. For each type ij of the supplier, where $i \in \{l, h\}$ and $j \in \{L, H\}$, our mechanism for $S2$, which we denote as $S2^*$, is characterized by two quantities: an order quantity q_{ij}^{S2} and a corresponding payment w_{ij}^{S2}. The implementation steps of this mechanism are similar to those in $S1$, but the buyer pays w_{ij}^{S2} to the supplier only if the supplier *delivers* exactly q_{ij}^{S2}. The values of quantities are as follows:

$$\begin{aligned}
q_{hj}^{S2} &= q_{hj}^{S1} \;=\; r^{-1}\left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right), \\
q_{lj}^{S2} &= q_{lj}^{S1} \;=\; r^{-1}\left(c_l\right), \\
w_{lj}^{S2} &= \frac{m_{lj}^{S1}}{\overline{p}_j} \;=\; \frac{c_l \cdot q_{lj}^{S1} + \Delta_c \cdot q_{hj}^{S1}}{\overline{p}_j}, \\
\text{and}\;\; w_{hj}^{S2} &= \frac{m_{hj}^{S1}}{\overline{p}_j} \;=\; \frac{c_h \cdot q_{hj}^{S1}}{\overline{p}_j}.
\end{aligned} \tag{$S2^*$}$$

We now verify that the buyer's profit under the $S2^*$ mechanism is the same as that under the $S1^*$ mechanism in setting $S1$.

Fix the supplier's disruption risk p_j, where $j \in \{L, H\}$. If the supplier realizes a cost of c_i and exerts an input effort $q_{\hat{i}j}^{S2} = q_{\hat{i}j}^{S1}$ (where $i, \hat{i} \in \{l, h\}$), then his expected profit is:

$$\overline{p}_j \cdot w_{\hat{i}j}^{S2} + p_j \cdot 0 - c_i \cdot q_{\hat{i}j}^{S2} = m_{\hat{i}j}^{S1} - c_i \cdot q_{\hat{i}j}^{S1},$$

which equals his profit from choosing the same input effort under the $S1^*$ mechanism. Since the $S1^*$ mechanism (in particular, the quantities q_{ij}^{S1} and m_{ij}^{S1}) satisfies the $IC1$ and $IR1$ constraints, the supplier has a dominant strategy to choose q_{ij}^{S2} under the $S2^*$ mechanism. Additionally, the payment equals m_{ij}^{S1} in expectation. That is, both the supplier's (dominant) actions and expected payment are the same under $S1^*$ and $S2^*$. Therefore, the $S2^*$ mechanism is optimal in setting $S2$, as the buyer's profit achieves its upper bound $\mathcal{P}^{S1}$.

Proof of S3: Setting $S3$ differs from $S1$ in that the disruption risk p_j is privately known by the supplier. Our mechanism in setting $S3$ is denoted as $S3^*$. For each combination of ij, where $i \in \{l, h\}$ and $j \in \{L, H\}$, the $S3^*$ mechanism is characterized by three quantities: (1) an order quantity q_{ij}^{S3}, (2) a payment for the supplier's effort m_{ij}^{S3}, and (3) an additional payment for the supplier's delivery w_{ij}^{S3}. The implementation steps of this mechanism are as follows:

(i) For each combination of ij, the buyer specifies an input effort q_{ij}^{S3}, a payment for the supplier's effort m_{ij}^{S3}, and an additional payment for the supplier's delivery w_{ij}^{S3}. The buyer commits to pay the supplier m_{ij}^{S3} if the specified input effort q_{ij}^{S3} is exerted. Additionally, the buyer commits to pay w_{ij}^{S3} if no disruption occurs and the full amount q_{ij}^{S3} is delivered.

(ii) For any realization of the supplier's production cost and disruption risk, the supplier chooses his input effort, realizes the production quantity, and decides the quantity to deliver.

(iii) The buyer pays m_{ij}^{S3} to the supplier if the supplier exerts input effort q_{ij}^{S3} for some ij. Additionally, the buyer pays w_{ij}^{S3} if the supplier delivers q_{ij}^{S3} without disruption. The buyer then sells the delivered quantity (if any) for revenue.

The quantities under this mechanism are defined as follows:

$$
\begin{aligned}
q_{hj}^{S3} &= q_{hj}^{S1} = r^{-1}\left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c}\right), \\
q_{lj}^{S3} &= q_{lj}^{S1} = r^{-1}(c_l), \\
m_{lH}^{S3} &= m_{lH}^{S1} = c_l \cdot q_{lH}^{S1} + \Delta_c \cdot q_{hH}^{S1}, \\
m_{lL}^{S3} &= m_{lL}^{S1} - \frac{\overline{p}_L \cdot \Delta_C \cdot \left(q_{hL}^{S1} - q_{hH}^{S1}\right)}{\Delta_p} = c_l \cdot q_{lL}^{S1} + \Delta_c \cdot q_{hL}^{S1} - \frac{\overline{p}_L \cdot \Delta_C \cdot \left(q_{hL}^{S1} - q_{hH}^{S1}\right)}{\Delta_p}, \\
m_{hj}^{S3} &= m_{hj}^{S1} = c_h \cdot q_{hj}^{S1}, \\
w_{hH}^{S3} &= w_{hL}^{S3} = w_{lH}^{S3} = 0, \\
\text{and } w_{lL}^{S3} &= \frac{\Delta_C \cdot \left(q_{hL}^{S1} - q_{hH}^{S1}\right)}{\Delta_p}. \qquad (S3^*)
\end{aligned}
$$

That is, the buyer's payment is determined solely by the supplier's input effort when the supplier reveals his type as hH, lH, or hL in the screening (i.e., when the supplier exerts an input effort of q_{hH}^{S3}, q_{lH}^{S3}, or q_{hL}^{S3}). However, if the supplier reveals his type as lL (i.e., exerts an input effort of q_{lL}^{S3}), then his payment is uncertain. The supplier receives m_{lL}^{S3} if a disruption occurs but receives $(m_{lL}^{S3} + w_{lL}^{S3})$ if no disruption occurs.

To demonstrate the performance of the mechanism, we first establish the supplier's strategy in response to the $S3^*$ mechanism. Recall that in setting $S1$, the disruption risk is public information, so in the incentive compatibility constraints, we only need to compare the two types with the same disruption risk; see $IC1$. In contrast, when both the production cost and disruption risk are private information, the incentive compatibility constraints assure that revealing his type truthfully is better than pretending to be any of the other three types. Precisely in setting $S3$, we need:

$$m_{ij} + \overline{p}_j \cdot w_{ij} - c_i \cdot q_{ij} \geq m_{\hat{i}\hat{j}} + \overline{p}_j \cdot w_{\hat{i}\hat{j}} - c_i \cdot q_{\hat{i}\hat{j}}, \quad \text{for all } i, \hat{i} \in \{l,h\},\ j, \hat{j} \in \{L,H\}. \qquad (IC3)$$

Additionally, a direct revelation mechanism must satisfy the IR constraints:

$$m_{ij} + \overline{p}_j \cdot w_{ij} - c_i \cdot q_{ij} \geq 0, \quad \text{for all } i, \hat{i} \in \{l,h\}, \text{ and } j, \hat{j} \in \{L,H\}. \qquad (IR3)$$

We now verify that the $S3^*$ mechanism satisfies these constraints. If the supplier, whose type is ij, chooses the prescribed input effort q_{ij}^{S3} under the $S3^*$ mechanism, his expected profit equals:

$$m_{ij}^{S3} + \overline{p}_j \cdot w_{ij}^{S3} - c_i \cdot q_{ij}^{S3} = m_{ij}^{S1} - c_i \cdot q_{ij}^{S1}. \qquad \text{(D.7)}$$

In contrast, if the supplier chooses $q_{\hat{i}j}^{S3}$, where $\hat{i} \neq i$, then his expected profit equals

$$m_{\hat{i}j}^{S3} + \overline{p}_j \cdot w_{\hat{i}j}^{S3} - c_i \cdot q_{\hat{i}j}^{S3} = m_{\hat{i}j}^{S1} - c_i \cdot q_{\hat{i}j}^{S1}. \qquad \text{(D.8)}$$

Since $\left\{ \left(q_{ij}^{S1}, m_{ij}^{S1}\right) \middle| i = l,h, \text{ and } j = L,H \right\}$ satisfies the $IR1$ and $IC1$ constraints, the agent's profit in (D.7) is non-negative and greater than or equal to that in (D.8). Thus, the $IR3$ constraints are satisfied, and the supplier prefers to choose q_{ij} rather than $q_{\hat{i}j}$.

To compare the supplier's profit between choosing q_{ij}^{S3} and $q_{i\hat{j}}^{S3}$, we consider the following cases:

- Case 1: The supplier's production cost is c_h and the disruption risk is p_j, where $j \in L,H$. Regardless of whether he exerts an effort of q_{hj}^{S3} or $q_{h\hat{j}}^{S3}$, where $\hat{j} \neq j$, he receives a payment that precisely covers his production cost, with $m_{hj}^{S3} = m_{hj}^{S1} = c_h \cdot q_{hj}^{S1}$, resulting in 0 profit. Therefore, the supplier is indifferent between these two choices.
- Case 2: The supplier's production cost is c_l and disruption risk is p_H. If the supplier distorts his disruption risk and chooses q_{lL}^{S3}, then he has a relatively low probability $\overline{p}_H$ to earn w_{lL}^{S3}, which is the payment for delivering q_{lL}^{S3} without disruption. Specifically, his expected profit is:

$$\begin{aligned}
& m_{lL}^{S3} + \overline{p}_H \cdot w_{lL}^{S3} - c_l \cdot q_{lL}^{S3} \\
&= c_l \cdot q_{lL}^{S1} + \Delta_c \cdot q_{hL}^{S1} - \frac{\overline{p}_L \cdot \Delta_C \cdot \left(q_{hL}^{S1} - q_{hH}^{S1}\right)}{\Delta_p} + \overline{p}_H \cdot \frac{\Delta_C \cdot \left(q_{hL}^{S1} - q_{hH}^{S1}\right)}{\Delta_p} - c_l \cdot q_{lL}^{S1} \\
&= \Delta_c \cdot q_{hH}^{S1} \quad = m_{lH}^{S3} + \overline{p}_H \cdot w_{lH}^{S3} - c_l \cdot q_{lH}^{S3},
\end{aligned}$$

  which is the same as the profit from choosing q_{lH}^{S3}.

- Case 3: The supplier's production cost is c_l and disruption risk is p_L. If the supplier distorts his disruption risk and chooses q_{lH}^{S3}, then the supplier's profit is:

$$m_{lH}^{S3} + \overline{p}_L \cdot w_{lH}^{S3} - c_l \cdot q_{lH}^{S3} \;=\; \Delta_c \cdot q_{hH}^{S1} \;<\; \Delta_c \cdot q_{hL}^{S1} \;=\; m_{lL}^{S3} + \overline{p}_L \cdot w_{lL}^{S3} - c_l \cdot q_{lL}^{S3}.$$

  That is, the supplier gets fully reimbursed for his production cost, but earns less information rent as compared to choosing q_{lL}^{S3}.

Together, these three cases prove that the supplier prefers q_{ij} rather than $q_{i\hat{j}}^{S3}$.

Similar to our analysis in (D.7) and (D.8), we can prove that for the supplier whose type is ij, choosing $q_{i\hat{j}}^{S3}$ is better than or equal to choosing $q_{\hat{i}\hat{j}}^{S3}$. As demonstrated above, the supplier prefers q_{ij}^{S3} over $q_{i\hat{j}}^{S3}$. Thus, the supplier prefers q_{ij}^{S3} over $q_{\hat{i}\hat{j}}^{S3}$, for all $\hat{i} \neq i$ and $\hat{j} \neq j$, completing the proof for the $IC3$ constraints.

Therefore, for any type ij, the supplier has a dominant strategy to exert the input effort $q_{ij}^{S3} = q_{ij}^{S1}$ corresponding to his actual type, and the payment, in expectation, equals m_{ij}^{S1} (see (D.7)). Consequently, the buyer's profit under the $S3^*$ mechanism is the same as that under the $S1^*$ mechanism with $\mathcal{P}^{S3} = \mathcal{P}^{S1}$.

Proof of S4: Our goal is to establish an *upper bound* on the buyer's optimal profit $\mathcal{P}^{S4}$ in $S4$, where both the disruption risk and the supplier's effort are unobservable to the buyer. The revelation principle (Myerson 1981) implies that, to search for an optimal mechanism, we can restrict attention to the class of direct revelation mechanisms. A direct revelation mechanism in $S4$ can be defined by a set $\{(q_{ij}, d_{ij}, w_{ij}) | i = l, h, \text{ and } j = L, H\}$ and implemented as follows:

(i) For every ij, the buyer specifies an order quantity q_{ij}, a non-refundable deposit d_{ij}, and a payment w_{ij}^{S3}. The buyer commits to pay the supplier w_{ij} if and only if the supplier reports ij in Step (ii), pays deposit d_{ij} to the buyer, and delivers exactly q_{ij} units of product.

(ii) The supplier is asked to report his type ij and pays the corresponding non-refundable deposit d_{ij} to the buyer.

(iii) The supplier finds it optimal to choose an input effort equal to the order quantity q_{ij}, and deliver the entire produced amount (if any) to the buyer.

(iv) The buyer pays w_{ij} to the supplier if the supplier delivers the order quantity q_{ij}. Otherwise, the payment is 0. The buyer then sells the delivered amount (if any) for revenue.

Over the set of such mechanisms, the buyer's problem, denoted as $\mathbb{P}^{S4}$, is to maximize her expected profit, taken over the distributions of both disruption risk type and cost type. Since the buyer's revenue $r(z)$ is zero in the event of a disruption, the formulation of problem $\mathbb{P}^{S4}$ is:

$$\max_{q_{ij},d_{ij},w_{ij}} \sum_{i=l,h} \sum_{j=L,H} \left(\overline{p}_j \cdot r(q_{ij}) + d_{ij} - \overline{p}_j \cdot w_{ij}\right) t^d_j \cdot t^c_i, \quad \text{where } \overline{p}_j = 1 - p_j, \tag{$\mathbb{P}^{S4}$}$$

subject to,

$$\overline{p}_j \cdot w_{ij} - d_{ij} - c_i \cdot q_{ij} \geq \overline{p}_j \cdot w_{\hat{i}\hat{j}} - d_{\hat{i}\hat{j}} - c_i \cdot q_{\hat{i}\hat{j}}, \quad \text{for all } i, \hat{i} \in \{l,h\},\ j, \hat{j} \in \{L,H\},\ (i,j) \neq (\hat{i},\hat{j}) \tag{$IC4$}$$

$$\overline{p}_j \cdot w_{ij} - d_{ij} - c_i \cdot q_{ij} \geq 0, \quad \text{for all } i, \in \{l,h\} \text{ and } j \in \{L,H\}, \tag{$IR4$}$$

$$\overline{p}_j \cdot w_{ij} - d_{ij} - c_i \cdot q_{ij} \geq -d_{\hat{i}\hat{j}}, \quad \text{for all } i, \hat{i} \in \{l,h\} \text{ and } j, \hat{j} \in \{L,H\}. \tag{EE}$$

The *exerting effort* constraint EE ensures that the supplier prefers to report his type truthfully and exert the prescribed effort, rather than reporting $\hat{i}\hat{j}$ and exerting zero effort for any $\hat{i}\hat{j}$. Lemma D.3 below simplifies the EE constraint.

LEMMA D.3. *To search for an optimal solution to $\mathbb{P}^{S4}$, we can, without loss of generality, substitute the EE constraint with the condition that the deposits are non-negative; i.e.,*

$$d_{ij} \geq 0, \quad \textit{for all } i \in \{l,h\} \textit{ and } j \in \{L,H\}. \tag{EE^*}$$

Proof of Lemma D.3:

For any given direct revelation mechanism $\{(q_{ij}, d_{ij}, w_{ij}) | i = l, h, \text{ and } j = L, H\}$ which satisfies $IC4$, $IR4$, and EE, if $\min\{d_{ij}\} = a < 0$, then we consider another mechanism denoted as $\{(q_{ij}, \tilde{d}_{ij}, w_{ij}) | i = l, h, \text{ and } j = L, H\}$, where $\tilde{d}_{ij} = d_{ij} - a \geq 0$ for all i and j. Since the original mechanism $\{(q_{ij}, d_{ij}, w_{ij})\}$ satisfies the EE constraint, we have the following inequality:

$$\begin{aligned}
&\overline{p}_j \cdot w_{ij} - d_{ij} - c_i \cdot q_{ij} \geq -a > 0 \\
&\overline{p}_j \cdot w_{ij} - \tilde{d}_{ij} - c_i \cdot q_{ij} \geq -a + d_{ij} - \tilde{d}_{ij} = 0, \quad \text{for all } i \text{ and } j.
\end{aligned}$$

Thus, the mechanism $\{(q_{ij}, \tilde{d}_{ij}, w_{ij})\}$ satisfies the $IR4$ constraint. Moreover, it is straightforward to verify that $\{(q_{ij}, \tilde{d}_{ij}, w_{ij})\}$ also satisfies $IC4$ (respectively, EE) because the original mechanism $\{(q_{ij}, d_{ij}, w_{ij})\}$ satisfies $IC4$ (respectively, EE). □

Therefore, the buyer's optimal profit $\mathcal{P}^{S4}$ is the optimized objective function value of problem $\mathbb{P}^{S4}$ with EE substituted by EE^*. This problem consists of twelve $IC4$ constraints for different pairs of ij and $\hat{i}\hat{j}$, four $IR4$ constraints for different values of ij, and four EE^* constraints. In what follows, we will derive a set of *necessary conditions* of these constraints.

Denote $P_{ij} = \overline{p}_j \cdot w_{ij} - d_{ij} - c_i \cdot q_{ij}$. When the supplier's type is hH, the $IR4$ constraint implies that:

$$P_{hH} = \overline{p}_H \cdot w_{hH} - d_{hH} - c_h \cdot q_{hH} \geq 0. \tag{D.9}$$

Combining with the EE^* constraint, we also obtain:

$$\begin{aligned} \overline{p}_H \cdot w_{hH} - c_h \cdot q_{hH} &\geq d_{hH} \geq 0, \\ \text{and thus} \quad w_{hH} &\geq \frac{c_h \cdot q_{hH}}{\overline{p}_H}. \end{aligned} \tag{D.10}$$

From $IC4$, we have the following three streams of inequalities:

$$\begin{aligned} P_{hL} &\geq \overline{p}_H \cdot w_{hH} - d_{hH} - c_h \cdot q_{hH} + (\overline{p}_L \cdot w_{hH} - \overline{p}_H \cdot w_{hH}) \\ &= P_{hL} + \Delta_p \cdot w_{hH} \\ &\geq \frac{\Delta_p \cdot c_h \cdot q_{hH}}{\overline{p}_H}, \qquad \text{(from (D.9) and (D.10))} \end{aligned} \tag{D.11}$$

$$\begin{aligned} P_{lH} &\geq \overline{p}_H \cdot w_{hH} - d_{hH} - c_h \cdot q_{hH} + (c_h \cdot q_{hH} - c_l \cdot q_{hH}) \\ &= \Delta_c \cdot q_{hH}, \qquad \text{(from (D.9))} \end{aligned} \tag{D.12}$$

$$\begin{aligned} P_{lL} &\geq \overline{p}_L \cdot w_{hL} - d_{hL} - c_h \cdot q_{hL} + (c_h \cdot q_{hL} - c_l \cdot q_{hL}) \\ &= P_{hL} + \Delta_c \cdot q_{hL} \\ &\geq \frac{\Delta_p \cdot c_h \cdot q_{hH}}{\overline{p}_H} + \Delta_c \cdot q_{hL}. \qquad \text{(from (D.11))} \end{aligned} \tag{D.13}$$

If we only consider constraints (D.11), (D.12), and (D.13), the maximization problem of $\mathbb{P}^{S4}$ is:

$$\begin{aligned} &\max_{q_{ij}, P_{ij}} \left\{ \sum_{i=l,h} \sum_{j=L,H} \left(\overline{p}_j \cdot r(q_{ij}) - P_{ij} - c_i \cdot q_{ij}\right) t_j^d \cdot t_i^c \right\} \\ &\leq \max_{q_{ij}} \left\{ t_l^c \cdot t_L^d \cdot \left(\overline{p}_L \cdot r(q_{hH}) - \frac{\Delta_p \cdot c_h \cdot q_{hH}}{\overline{p}_H} - \Delta_c \cdot q_{hL} - c_l \cdot q_{lL} \right) \right. \end{aligned}$$

$$
\begin{aligned}
&+ t_h^c \cdot t_L^d \cdot \left(\overline{p}_L \cdot r(q_{hL}) - \frac{\Delta_p \cdot c_h \cdot q_{hH}}{\overline{p}_H} - c_h \cdot q_{hL} \right) \\
&+ t_l^c \cdot t_H^d \cdot (\overline{p}_H \cdot r(q_{lH}) - \Delta_c \cdot q_{hH} - c_l \cdot q_{lH}) \\
&+ t_h^c \cdot t_H^d \cdot (\overline{p}_H \cdot r(q_{hH}) - c_h \cdot q_{hH}) \Bigg\} \\
= & t_l^c \cdot t_L^d \cdot \underbrace{\max_{q_{lL}} \{\overline{p}_L \cdot r(q_{lL}) - c_l \cdot q_{lL}\}}_{\text{Term 1}} \\
&+ t_h^c \cdot t_L^d \cdot \underbrace{\max_{q_{hL}} \left\{ \overline{p}_L \cdot r(q_{hL}) - \left(c_h + \frac{t_l^c \cdot \Delta_c \cdot q_{hL}}{t_h^c} \right) \cdot q_{hL} \right\}}_{\text{Term 2}} \\
&+ t_l^c \cdot t_H^d \cdot \underbrace{\max_{q_{lH}} \{\overline{p}_H \cdot r(q_{lH}) - c_l \cdot q_{lH}\}}_{\text{Term 3}} \\
&+ t_h^c \cdot t_H^d \cdot \underbrace{\max_{q_{hH}} \left\{ \overline{p}_H \cdot r(q_{hH}) - \left(c_h + \frac{t_l^c \cdot t_L^d \cdot \Delta_p \cdot c_h}{t_h^c \cdot t_H^d \cdot \overline{p}_H} + \frac{t_L^d \cdot \Delta_p \cdot c_h}{t_H^d \cdot \overline{p}_H} + \frac{t_l^c \cdot \Delta_c}{t_h^c} \right) \cdot q_{hH} \right\}}_{\text{Term 4}}.
\end{aligned} \tag{D.14}
$$

We denote the right-hand side of (D.14) as $\overline{\mathcal{P}}^{S4}$. Since we have only considered a set of *necessary conditions* of the constraints in problem $\mathbb{P}^{S4}$, $\overline{\mathcal{P}}^{S4}$ serves as an upper bound for $\mathcal{P}^{S4}$, which represents the optimized objective function value of problem $\mathbb{P}^{S4}$.

We now compare $\overline{\mathcal{P}}^{S4}$ with $\mathcal{P}^{S1}$, which represents the buyer's optimal profit in setting $S1$ (see equation (D.6)). Note that Terms 1, 2, and 3 in inequality (D.14) are respectively identical to Terms 1, 2, and 3 in equation (D.6). Moreover, in Assumption 9, we have assumed that the buyer's marginal revenue at 0 is sufficiently large under the following condition:

$$
r'(0) > \frac{1}{\overline{p}_H} \cdot \left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c} \right).
$$

Therefore, Term 4 in (D.6) is *strictly* greater than Term 4 in (D.14) as follows:

$$
\begin{aligned}
&\max_{q_{hH}} \left\{ \overline{p}_H \cdot r(q_{hH}) - \left(c_h + \frac{\Delta_c \cdot t_l^c}{t_h^c} \right) \cdot q_{hH} \right\} \\
&> \max_{q_{hH}} \left\{ \overline{p}_H \cdot r(q_{hH}) - \left(c_h + \frac{t_l^c \cdot t_L^d \cdot \Delta_p \cdot c_h}{t_h^c \cdot t_H^d \cdot \overline{p}_H} + \frac{t_L^d \cdot \Delta_p \cdot c_h}{t_H^d \cdot \overline{p}_H} + \frac{t_l^c \cdot \Delta_c}{t_h^c} \right) \cdot q_{hH} \right\} \geq 0.
\end{aligned}
$$

Since the weights of Term 1, 2, 3, and 4 are strictly positive and identical in both (D.6) and (D.14), we conclude that $\mathcal{P}^{S1} > \overline{\mathcal{P}}^{S4} \geq \mathcal{P}^{S4}$, thereby completing the proof of Theorem 3. ■

Appendix E: Supplemental Details for Section 6

E.1. Definition and Analysis of the m-TPT Mechanism

As assumed in Section 6, when using the m-TPT mechanism, the buyer ignores the distribution of the random yield θ_n and instead takes each supplier's *expected cost per unit* as the supplier's actual cost per unit without supply uncertainty. In other words, the buyer incorrectly conjectures that Supplier n's production cost per unit is $\tilde{t}_n = t_n/\Theta$, where $\Theta = \mathbb{E}[\theta_n] = \lambda + (1-\lambda)\cdot\alpha$. Therefore, from the buyer's perspective, Supplier n's private information becomes his production cost per unit $\tilde{t}_n$ which is uniformly distributed on $[\frac{1}{\Theta}, \frac{2}{\Theta}]$. We denote the cdf. (resp., pdf.) of $\tilde{t}_n$ as $\tilde{F}(\tilde{t}_n) = \Theta\tilde{t}_n - 1$ (resp., $\tilde{f}(\tilde{t}_n) = \Theta$), where $\tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}]$.

The m-TPT mechanism can be implemented in the following steps:

(i) The buyer offers each supplier a menu of contracts

$$\left\{ \left(W^T(\tilde{t}_n), F^T(\tilde{t}_n)\right) : \tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \right\}.$$

Each contract $\left(W^T(\tilde{t}_n), F^T(\tilde{t}_n)\right)$ (henceforth, contract $\tilde{t}_n$ for brevity) consists of an upfront payment $F^T(\tilde{t}_n)$ and a per-unit payment $W^T(\tilde{t}_n)$; these functions are defined below after the implementation steps.

(ii) Each supplier is asked to choose one contract (i.e., choose a $\tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}]$) from the menu. Alternatively, the supplier can choose not to participate in the mechanism, earning no profit. We say Supplier n chooses contract $\tilde{t}_n = 0$ when the supplier does not participate.

(iii) If both suppliers participate, then the buyer pays the upfront payments $F^T(\tilde{t}_n)$ to Supplier n and places an order of $Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$ units from both suppliers, where $m \neq n$. Each participating supplier *must* deliver the ordered amount to the buyer (same below).

(iv) If Supplier n participates while Supplier m does not, the buyer orders $Q^T\left(\tilde{t}_n, 0\right)$ units from Supplier n and purchases the same amount of component m from the reliable supply source.

(v) If both suppliers do not participate, the buyer chooses a quantity of the assembled product sourced solely from the reliable supply sources. To maximize her profit $r(z) - 2pz$, the buyer finds it optimal to choose $z = (\frac{5}{2p})^2$.

Denote the derivative of the buyer's revenue function as $\dot{r}(z) := \frac{\mathrm{d}r(z)}{\mathrm{d}z}$. We now precisely define the functions under the m-TPT mechanism:

$$W^T\left(\tilde{t}_n\right) = \tilde{t}_n + \frac{\tilde{F}(\tilde{t}_n)}{\tilde{f}(\tilde{t}_n)} = \frac{2\Theta\cdot\tilde{t}_n - 1}{\Theta},$$

$$F^T\left(\tilde{t}_n\right) = \left(\tilde{t}_n - W^T\left(\tilde{t}_n\right)\right) \cdot \int_{\frac{1}{\Theta}}^{\frac{2}{\Theta}} Q^T\left(\tilde{t}_n, \tilde{t}_m\right) \,\mathrm{d}\tilde{F}(\tilde{t}_m) + \int_{\frac{1}{\Theta}}^{\frac{2}{\Theta}} \int_{\tilde{t}_n}^{\frac{2}{\Theta}} Q^T\left(s_n, \tilde{t}_m\right) \,\mathrm{d}s_n \,\mathrm{d}\tilde{F}(\tilde{t}_m),$$

$$Q^T\left(\tilde{t}_n, \tilde{t}_m\right) = \begin{cases} \dot{r}^{-1}\left(W^T\left(\tilde{t}_n\right) + W^T\left(\tilde{t}_m\right)\right) = \left(\frac{5\Theta}{2\Theta\cdot\left(\tilde{t}_n+\tilde{t}_m\right)-2}\right)^2, & \text{if } \tilde{t}_m \in [\frac{1}{\Theta}, \frac{2}{\Theta}], \\ \dot{r}^{-1}\left(W^T\left(\tilde{t}_n\right) + p\right) = \left(\frac{5\Theta}{2\Theta\cdot\tilde{t}_n+\Theta\cdot p-1}\right)^2, & \text{if } \tilde{t}_m = 0. \end{cases} \tag{E.1}$$

Therefore, the monetary payment to Supplier n, which we denote as $M^T\left(\tilde{t}_n, \tilde{t}_m\right)$, is:

$$M^T\left(\tilde{t}_n, \tilde{t}_m\right) = \begin{cases} F^T\left(\tilde{t}_n\right) + Q^T\left(\tilde{t}_n, \tilde{t}_m\right) \cdot W^T\left(\tilde{t}_n\right), & \text{if } \tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}], \\ 0, & \text{if } \tilde{t}_n = 0. \end{cases} \tag{E.2}$$

While in the setting of Hu and Qi (2018) without supply uncertainty, the m-TPT mechanism is optimal and satisfies the individual rationality (IR) and incentive compatibility (IC) constraints, the IC and IR constraints may not be satisfied in our setting with supply uncertainty. Specifically, for some realizations of the production type, Supplier n may prefer not to participate in the m-TPT mechanism rather than choosing any contract within the menu. Therefore, we have added Steps (iii) and (iv) to the implementation steps above as a reasonable response to supplier non-participation. Furthermore, under the m-TPT mechanism, suppliers do not necessarily find it optimal to choose the contract corresponding to their actual types. Without loss of generality, we consider that the two rational and symmetric suppliers form a symmetric Bayesian Nash equilibrium (BNE).

To establish the equilibrium, we need each supplier's profit in response to the choices of contracts. Fix contracts $\hat{t}_n$ and $\hat{t}_m$ chosen by the suppliers, where $\hat{t}_n, \hat{t}_m \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}$. Supplier n decides on his input effort $q_n \geq 0$, which incurs a cost of $t_n q_n = \Theta\tilde{t}_n q_n$. Recall that the produced amount equals q_n with probability λ and $\alpha \cdot q_n$ otherwise. If the produced amount is less than the ordered amount, Supplier n must purchase the shortage from the reliable supply source. Therefore, Supplier n, whose actual type is $\tilde{t}_n$, earns an optimal profit as follows[8]:

$$P_n^*\left(\tilde{t}_n, \hat{t}_n, \hat{t}_m\right) = \mathbb{1}[\hat{t}_n \neq 0] \cdot \max_{q_n \geq 0} \left\{ F^T\left(\hat{t}_n\right) + Q^T\left(\hat{t}_n, \hat{t}_m\right) W^T\left(\hat{t}_n\right) - \Theta\tilde{t}_n q_n - \lambda p \cdot \left(Q^T\left(\hat{t}_n, \hat{t}_m\right) - q_n\right)^+ - (1-\lambda)p \cdot \left(Q^T\left(\hat{t}_n, \hat{t}_m\right) - \alpha q_n\right)^+ \right\}.$$

Define function $S(\cdot) : [\frac{1}{\Theta}, \frac{2}{\Theta}] \to [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}$ as a symmetric equilibrium strategy, where for any given realization of type $\tilde{t}_n$, Supplier n chooses contract $S(\tilde{t}_n)$ from his menu. Also, $S(\tilde{t}_n) = 0$ indicates that Supplier n does not participate by following strategy $S(\cdot)$. We simulate discrete values

[8] Let $\mathbb{1}[\hat{t}_n \neq 0] = 1$ if $\hat{t}_n \neq 0$ and 0 otherwise (i.e., when Supplier n does not participate with $\hat{t}_n = 0$).

of $\tilde{t}_n$ distributed over $[\frac{1}{\Theta}, \frac{2}{\Theta}]$ and iteratively search for a strategy $S(\cdot)$ that satisfies the equilibrium condition below:

$$\mathbb{E}_{\tilde{t}_m}\left[P_n^*\left(\tilde{t}_n, S(\tilde{t}_n), S(\tilde{t}_m)\right)\right] \geq \mathbb{E}_{\tilde{t}_m}\left[P_n^*\left(\tilde{t}_n, \hat{t}_n, S(\tilde{t}_m)\right)\right], \text{ for all } \tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \text{ and } \hat{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}.$$

Figure E.1 showcases the equilibrium reporting strategy $S(\tilde{t}_n)$ for two set of parameters in our test-bed: $(\lambda, \alpha) = (0.75, 0.75)$ and $(0.5, 0.5)$. For some values of $\tilde{t}_n$ within its domain, Supplier n chooses a contract corresponding to a higher cost type or opts not to participate in the mechanism. Consequently, the buyer's profit is suboptimal under the m-TPT mechanism.

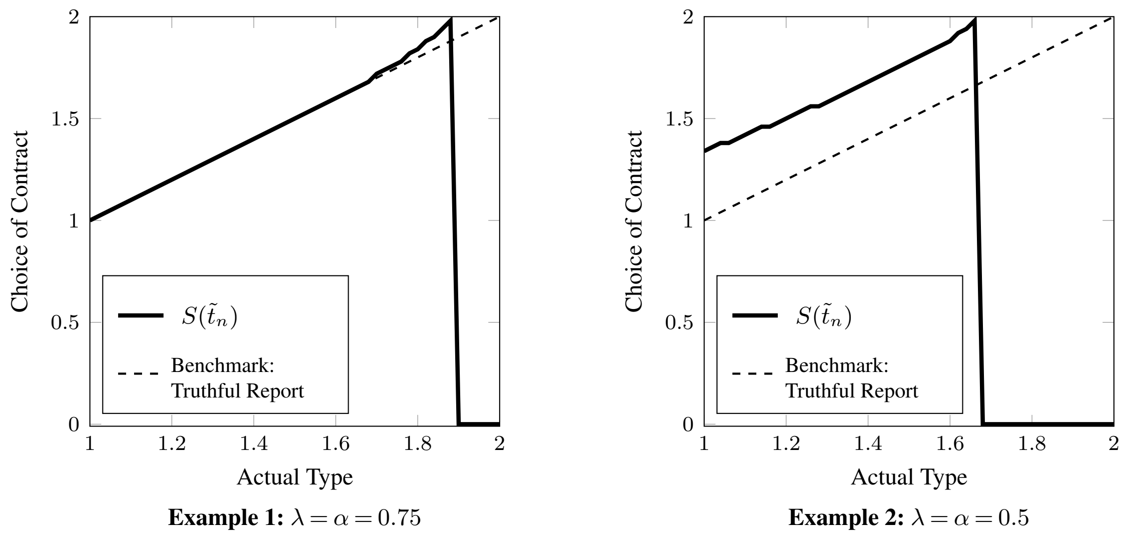

Figure E.1 An illustration of the BNE strategy $S(\tilde{t}_n)$ for different (λ, α).

Given the suppliers' equilibrium strategy $S(\cdot)$ for choosing contracts under the m-TPT mechanism, we have the buyer's corresponding profit $\mathcal{P}^T(\lambda, \alpha)$. Consequently, we have the results illustrated in Figure 1.

E.2. Definition and Analysis of the RF Mechanism

The RF mechanism can be implemented in the following steps:

(i) For every $n, m \in \{1, 2\}$ with $n \neq m$, the buyer offers Supplier n a menu of contracts:

$$\left\{\left(Q^T\left(\tilde{t}_n, \tilde{t}_m\right), M^T\left(\tilde{t}_n, \tilde{t}_m\right)\right) : \tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}]\right\}.$$

Each contract $\left(Q^T\left(\tilde{t}_n, \tilde{t}_m\right), M^T\left(\tilde{t}_n, \tilde{t}_m\right)\right)$ (henceforth, contract $\tilde{t}_n$ for brevity) consists of a desired quantity $Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$ of component n and a payment $M^T\left(\tilde{t}_n, \tilde{t}_m\right)$. These functions depend on $\tilde{t}_m$, which is chosen by the other supplier in step (ii) below, and are precisely defined in equation E.1 and E.2 in Appendix E.1.

(ii) Supplier n is asked to choose one contract (i.e., choose a $\tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}]$) from the menu. Alternatively, the supplier can choose not to participate in the mechanism, earning no profit. We denote non-participation as $\tilde{t}_n = 0$.

(iii) For every $n = 1, 2$, if Supplier n participates, then the buyer pays $M^T\left(\tilde{t}_n, \tilde{t}_m\right)/Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$ for every unit of component n produced by Supplier n, up to the buyer's desired quantity $Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$.

(iv) After receiving deliveries (if any) from the suppliers and making the corresponding payments, the buyer decides her purchase quantities from the reliable supply sources to maximize her revenue from the assembled product minus the associated purchase costs.

Similar to the m-TPT mechanism, the RF mechanism does not necessarily satisfy the incentive compatibility (IC) and individual rationality (IR) constraints, leading to the following responses of the suppliers: (a) Supplier n may prefer to exert an effort different from the desired quantity $Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$ specified by the buyer. (b) During contract selection, the two suppliers form a Bayesian Nash equilibrium (BNE), in which they may find it optimal to misreport their types or choose not to participate.

For any given $\tilde{t}_n, \tilde{t}_m \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}$, the unit payment $M^T\left(\tilde{t}_n, \tilde{t}_m\right)/Q^T\left(\tilde{t}_n, \tilde{t}_m\right)$ is smaller than p in our numerical setting. Therefore, the suppliers never purchase any quantities from the reliable supply sources but willingly delivers precisely his produced quantity (if any) to the buyer under the RF mechanism. Consequently, Supplier n, whose actual type is $\tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}]$, earns an optimal profit as follows:

$$P_n^R\left(\tilde{t}_n, \hat{t}_n, \hat{t}_m\right) = \mathbb{1}[\hat{t}_n \neq 0] \cdot \max_{q_n \geq 0} \left\{ -\Theta \tilde{t}_n q_n + \lambda \cdot M^T\left(\tilde{t}_n, \tilde{t}_m\right) \cdot \min\left\{ \frac{q_n}{Q^T\left(\tilde{t}_n, \tilde{t}_m\right)},\ 1 \right\} + (1-\lambda) \cdot M^T\left(\tilde{t}_n, \tilde{t}_m\right) \cdot \min\left\{ \frac{\alpha q_n}{Q^T\left(\tilde{t}_n, \tilde{t}_m\right)},\ 1 \right\} \right\}.$$

Define function $S^R(\cdot) : [\frac{1}{\Theta}, \frac{2}{\Theta}] \to [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}$ as a symmetric equilibrium strategy of choosing contract from the menu. That is, Supplier n chooses contract $S^R(\tilde{t}_n)$ from his menu in the BNE, when his actual type is $\tilde{t}_n$. We search for a strategy $S^R(\cdot)$ that satisfies the equilibrium condition:

$$\mathbb{E}_{\tilde{t}_m}\left[P_n^R\left(\tilde{t}_n, S^R(\tilde{t}_n), S^R(\tilde{t}_m)\right)\right] \geq \mathbb{E}_{\tilde{t}_m}\left[P_n^R\left(\tilde{t}_n, \hat{t}_n, S^R(\tilde{t}_m)\right)\right],\ \forall \tilde{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \text{ and } \hat{t}_n \in [\frac{1}{\Theta}, \frac{2}{\Theta}] \cup \{0\}.$$

Using the suppliers' equilibrium strategy $S^R(\cdot)$ for choosing contracts under the RF mechanism, we have the buyer's corresponding profit $\mathcal{P}^R(\lambda, \alpha)$. Consequently, we have the performance gap between the RF mechanism and our SC mechanism illustrated in Figure 3.