

Online Appendix

Delegated Concept Testing in New Product Development

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Proof of Proposition 1. (i) Testing concept $i = 1$ gives the firm an expected profit of $(q_1 v_1 - K)/2 - c$, and the firm will hence test this concept if and only if its expected profit contribution is non-negative. Suppose $(q_1 v_1 - K)/2 - c \geq 0$. Because $q_i v_i \geq q_{i+1} v_{i+1}$, the firm will, upon receiving a good signal ($s_1 = g$), select concept $i = 1$ and stop the testing process. In contrast, if it receives a bad signal ($s_1 = b$), it will discard concept $i = 1$, and test concept $i = 2$, but only if $(q_2 v_2 - K)/2 - c > 0$. The result follows by induction for all $i \in \mathcal{N}$.

(ii) With parallel testing and because $q_i v_i \geq q_{i+1} v_{i+1}$, the firm selects concept i for development if and only if $s_i = g$ and $s_j = b$ for all $j < i$. As a result, the expected profit of including concept i in the test set is given by $(q_i v_i - K)/2^i - c$, and the firm will thus test it if and only if $(q_i v_i - K)/2^i - c \geq 0$.

(iii) For $\delta = 1$, it is straightforward to verify that $\Pi_{\text{seq}}^{\text{fb}} \geq \Pi_{\text{par}}^{\text{fb}}$. Now suppose $\delta < 1$. For $c = 0$, we have $\mathcal{I}_{\text{seq}}^{\text{fb}} = \mathcal{I}_{\text{par}}^{\text{fb}} = \mathcal{N}$, and thus $\Pi_{\text{par}}^{\text{fb}} > \Pi_{\text{seq}}^{\text{fb}}$. By continuity of $\Pi_{\text{par}}^{\text{fb}}$ and $\Pi_{\text{seq}}^{\text{fb}}$ in c , there exists $\underline{c} > 0$ such that $\Pi_{\text{par}}^{\text{fb}} > \Pi_{\text{seq}}^{\text{fb}}$ for all $c < \underline{c}$. In contrast, if $c \in [(q_2 v_2 - K)/4, (q_2 v_2 - K)/2]$, then $\mathcal{I}_{\text{par}}^{\text{fb}} = \{1\}$ and $\mathcal{I}_{\text{seq}}^{\text{fb}} \supset \mathcal{I}_{\text{par}}^{\text{fb}}$, which implies $\Pi_{\text{seq}}^{\text{fb}} > \Pi_{\text{par}}^{\text{fb}}$.

Proof of Proposition 2. We begin our proof by formally deriving the firm's optimization problem S. We then solve S to determine the agent's optimal compensation package, the optimal test set, and the firm's expected profits.

The Optimization problem S: Following the revelation principle, we restrict attention to the optimal contract that induces, for each concept $i \in \mathcal{I}_S$, high-effort testing and truthful recommendations. Such a contract must satisfy several incentive compatibility (IC) constraints; that is, it must be optimal for the agent to test each concept $i \in \mathcal{I}_S$ with high effort and to truthfully reveal her signals, assuming the firm will select the first concept for which it receives a good recommendation.

Denote $\hat{\pi}_{i-1}$ the agent's expected continuation utility before testing concept $i \in \mathcal{I}_S$ under the assumption that she will carry out all future concept tests with high effort and truthfully reveal her signals. Consider an arbitrary concept $i \in \mathcal{I}_S$. Given $e_i = h$ and $s_i = g$, the agent will receive an expected utility of $\pi_i^{gg} = q_i u_{ig} + (1 - q_i) u_{ib}$ when making a good recommendation ($r_i = g$), and $\pi_i^{bg} = u_{ia} + (\delta/2)^{n-i} u_{it} + \delta \hat{\pi}_i$ when making a bad recommendation ($r_i = b$). Similarly, given $e_i = h$ and $s_i = b$, she will receive an expected utility of $\pi_i^{gb} = (1 - q_i) u_{ig} + q_i u_{ib}$ when choosing $r_i = g$, and

$\pi_i^{bb} = u_{ia} + (\delta/2)^{n-i}u_{it} + \delta\hat{\pi}_i$ when choosing $r_i = b$. Also, given truth telling, her expected utility from exerting high effort is $\pi_i(h) = (q_i u_{ig} + (1 - q_i)u_{ib})/2 + (u_{ia} + (\delta/2)^{n-i}u_{it} + \delta\hat{\pi}_i)/2 - c$, whereas low effort yields $\pi_i(l) = (u_{ig}/2 + u_{ib}/2)/2 + (u_{ia} + (\delta/2)^{n-i}u_{it} + \delta\hat{\pi}_i)/2$. The IC constraints (2)-(4) follow from requiring that $\pi_i^{gg} \geq \pi_i^{bg}$, $\pi_i^{bb} \geq \pi_i^{gb}$, and $\pi_i(h) \geq \pi_i(l)$ for all $i \in \mathcal{I}_S$. This also leads to $\hat{\pi}_{i-1} = (q_i u_{ig} + (1 - q_i)u_{ib} + u_{ia} + (\delta/2)^{n-i}u_{it} - 2c + \delta\hat{\pi}_i)/2$ for all $i \in \mathcal{I}_S$ and $\hat{\pi}_{|\mathcal{I}_S|} = 0$. Finally, limited liability enforces non-negative compensation, hence implying (6).

Given high-effort testing and truthful recommendations, the firm will select concept i with probability $P(s_i = g, s_j = b \forall j < i) = 1/2^i$ and receive a discounted expected value of $\delta^{i-1}(q_i v_i - K)$. The agent's compensation tied to concept i is as follow: with probability $P(\Theta_i = G | s_i = g)P(s_i = g, s_j = b \forall j < i) = q_i/2^i$ she will receive $\delta^{i-1}u_{ig}$, with probability $P(\Theta_i = B | s_i = g)P(s_i = g, s_j = b \forall j < i) = (1 - q_i)/2^i$ she will receive $\delta^{i-1}u_{ib}$, with probability $P(s_i = b, s_j = b \forall j < i) = 1/2^i$ she will receive $\delta^{i-1}u_{ia}$, and with probability $P(s_i = b, s_j = b \forall j) = 1/2^n$ she will receive $\delta^{n-1}u_{it}$. Summing over all $i \in \mathcal{I}_S$ yields the firm's expected profits as given in (1).

Solving S: (i) Fix any $\mathcal{I}_S \subseteq \mathcal{N}$. Using the definition of $\hat{\pi}_0$, it is true that $\Pi_S = \sum_{i=1}^n (\delta^{i-1}/2^i)(q_i v_i - K - 2c) - \hat{\pi}_0$. Maximizing Π_S is hence equivalent to minimizing $\hat{\pi}_0$. As a first step, we derive the minimum feasible $\hat{\pi}_{i-1}$ for any given fixed $\hat{\pi}_i$: (a) If $\delta\hat{\pi}_i < 4(1 - q_i)c/(2q_i - 1)$, then it is optimal to set $u_{ig} = 4c/(2q_i - 1)$, $u_{ib} = 0$, and $u_{ia} = 4(1 - q_i)c/(2q_i - 1) - \delta\hat{\pi}_i$, yielding $\hat{\pi}_{i-1} = (3 - 2q_i)c/(2q_i - 1)$; (b) if $4(1 - q_i)c/(2q_i - 1) \leq \delta\hat{\pi}_i \leq 4q_i c/(2q_i - 1)$, then it is optimal to set $u_{ig} = 4c/(2q_i - 1)$, $u_{ib} = 0$, and $u_{ia} = 0$, yielding $\hat{\pi}_{i-1} = c/(2q_i - 1) + \delta\hat{\pi}_i/2$; and (c) if $\delta\hat{\pi}_i > 4q_i c/(2q_i - 1)$, then it is optimal to set $u_{ig} = \delta\hat{\pi}_i/q_i$, $u_{ib} = 0$, and $u_{ia} = 0$, yielding $\hat{\pi}_{i-1} = \delta\hat{\pi}_i - c$. Taken together, cases (a)-(c) imply that $\hat{\pi}_{i-1}$ is non-decreasing in $\hat{\pi}_i$ for all $i \in \mathcal{I}_S$. As a result, minimizing $\hat{\pi}_0$ is equivalent to separately minimizing $\hat{\pi}_i$ for each $i \in \mathcal{I}_S$, starting with $\hat{\pi}_{|\mathcal{I}_S|} = 0$ and using backwards induction. The optimal compensation package thus satisfies $u_{ig}^S = 4c/(2q_i - 1) + [\delta\hat{\pi}_i/q_i - 4c/(2q_i - 1)]^+$, $u_{ib}^S = 0$, $u_{ia}^S = [4(1 - q_i)c/(2q_i - 1) - \delta\hat{\pi}_i]^+$, and $u_{it}^S = 0$ for all $i \in \mathcal{I}_S$.

(ii) We prove $\mathcal{I}_S^* \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$ by contradiction. Suppose that there exists a concept $i \in \mathcal{I}_S^*$ with $q_i v_i - K - 2c < 0$; Proposition 1(i) hence implies that $i \notin \mathcal{I}_{\text{seq}}^{\text{fb}}$. Applying part (i) together with (5) yields $\hat{\pi}_{i-1} = (q_i u_{ig}^S + u_{ia}^S + \delta\hat{\pi}_i)/2 - c$. Now consider the best case scenario that the firm can fully utilize $\hat{\pi}_{i-1}$ to reduce the agent's compensation associated to concept $j = i - 1$ —otherwise the firm would miss out on the full strategic value of $\hat{\pi}_{i-1}$. Then by part (i), $u_{i-1,g}^S = 4c/(2q_{i-1} - 1)$ and $u_{i-1,a}^S = 4(1 - q_{i-1})c/(2q_{i-1} - 1) - \delta\hat{\pi}_{i-1}$, and the compensation u_j^S for all other concepts $j \notin \{i - 1, i\}$ is unaffected. The profit contribution of concepts $i - 1$ and i is thus given by $\Pi_{i-1,i} = \delta^{i-2}(q_{i-1}v_{i-1} - K - 4c/(2q_{i-1} - 1) + \delta\hat{\pi}_{i-1})/2^{i-1} + \delta^{i-1}(q_i v_i - K - q_i u_{ig}^S - u_{ia}^S)/2^i$. Rearranging terms reveals that $\Pi_{i-1,i} = \delta^{i-2}(q_{i-1}v_{i-1} - K - 4c/(2q_{i-1} - 1) + \delta^2\hat{\pi}_i/4)/2^{i-1} + \delta^{i-1}(q_i v_i - K - 2c)/2^i$; and because (a) $q_i v_i - K - 2c < 0$ and (b) if concept i is removed, then prior $\hat{\pi}_i$ now becomes $\hat{\pi}_{i-1}$ and the firm

avoids one round of profit discounting, it is true that the firm can increase profits by removing concept i from \mathcal{I}_S^* . As a result, $i \notin \mathcal{I}_S^*$ and the claim follows.

Consider the following setting with two concepts: $v_1 = 5.95$, $q_1 = 1$, $v_2 = 10$, $q_2 = 0.6$, $c = 0.02$, $K = 5$, and $\delta = 1$. It is optimal to test concept $i = 1$ first and $i = 2$ second, although $q_1 v_1 < q_2 v_2$. Finally, if the concepts in \mathcal{I}_S can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1}$, $q_i \geq q_{i+1}$ and $(1 - q_i)4c/(2q_i - 1) \leq \delta \hat{\pi}_i \leq 4q_i c/(2q_i - 1)$ for all $i \in \mathcal{I}_S \setminus \{|\mathcal{I}_S|\}$, then cases (a) and (b) in part (i) imply that $q_i u_{ig} + (1 - q_i)u_{ib} + u_{ia} < q_{i+1} u_{i+1g} + (1 - q_{i+1})u_{i+1b} + u_{i+1a}$ for all $i \in \mathcal{I}_S \setminus \{|\mathcal{I}_S|\}$. By (1) and the assumption that $q_i v_i \geq q_{i+1} v_{i+1}$ it follows readily that it is optimal to test the concepts in increasing order of i .

(iii) This result follows directly from inserting the optimal compensation package as specified in Proposition 2(i) in (1) and rearranging terms.

Proof of Proposition 3. The derivation of optimization problem M is equivalent to the derivation of optimization problem S as detailed in the proof of Proposition 2. In particular, for each $i \in \mathcal{I}_M$, $\pi_i^{gg} = q_i u_{ig} + (1 - q_i)u_{ib}$, $\pi_i^{bg} = \pi_i^{bb} = u_{ia} + P(s_j = b \forall j > i) \delta^{n-i} u_{it}$, $\pi_i^{gb} = (1 - q_i)u_{ig} + q_i u_{ib}$, $\pi_i(h) = (q_i u_{ig} + (1 - q_i)u_{ib})/2 + (u_{ia} + P(s_j = b \forall j > i) \delta^{n-i} u_{it})/2 - c$, and $\pi_i(l) = (u_{ig}/2 + u_{ib}/2)/2 + (u_{ia} + P(s_j = b \forall j > i) \delta^{n-i} u_{it})/2$. Since for concept $i = n$, with $n = |\mathcal{I}_M|$, u_{na} and u_{nt} are paid at the same time, we only require $u_{na} + u_{nt} \geq 0$ to ensure non-negative wages.

(i) Given the structure of M, maximizing the firm's expected profit is equivalent to minimizing each agent's compensation package individually. For every $i \in \mathcal{I}_M$, note that (10) implies that $u_{ig} > u_{ib}$, which allows us to rewrite (8) and (9) as $q_i u_{ig} + (1 - q_i)u_{ib} \geq u_{ia} + 2^{i-n} \delta^{n-i} u_{it} \geq (1 - q_i)u_{ig} + q_i u_{ib}$. It follows readily that $u_{ia} + 2^{i-n} \delta^{n-i} u_{it} = (1 - q_i)u_{ig} + q_i u_{ib}$, and u_{ig} and u_{ib} should be chosen as low as possible. By (10) and (11), these minimal payments are $u_{ig} = 4c/(2q_i - 1)$ and $u_{ib} = 0$. Moreover, the firm is indifferent between paying u_{ia} or u_{it} ; without loss of optimality, we can hence set $u_{ia} = 4(1 - q_i)c/(2q_i - 1)$ and $u_{it} = 0$. Finally, $u_{ia}/u_{ig} = 1 - q_i < 1/2$ because $q_i > 1/2$.

(ii)-(iii) Given the optimal compensation package derived in part (i), the firm's expected profit reads $\Pi_M = \sum_{i \in \mathcal{I}_M} (\delta^{i-1}/2^i)(q_i v_i - K - 4c/(2q_i - 1))$ —revealing that each concept i 's net profit contribution is independent of the number and identity of the other concepts to be tested. As a result, it is optimal to test all concepts $i \in \mathcal{N}$ for which $q_i v_i - K - 4c/(2q_i - 1) \geq 0$. Thus, $\mathcal{I}_M^* = \{i \in \mathcal{N} \mid q_i v_i - K - 4c/(2q_i - 1) \geq 0\}$, and by Proposition 1(i) it follows that $\mathcal{I}_M^* \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$. Furthermore, because $(\delta^{i-1}/2^i)$ is decreasing in i , the firm maximizes Π_M by testing the concepts in \mathcal{I}_M^* in decreasing order of $q_i v_i - 4c/(2q_i - 1)$.

Let $n = |\mathcal{I}_S^*|$, and suppose that for all $i \in \mathcal{I}_S^*$, $q_n v_n - K - 4c/(2q_n - 1) \geq 0$, $q_n v_n \leq q_i v_i$, and $q_n \leq q_i$. It follows immediately that $q_i v_i - K - 4c/(2q_i - 1) \geq q_n v_n - K - 4c/(2q_n - 1) \geq 0$ for all \mathcal{I}_S^* , and thus, $\mathcal{I}_S^* \subseteq \mathcal{I}_M^*$. Finally, consider the following setting with three concepts: $v_1 = 10$, $q_1 = 0.9$, $v_2 = 9.5$, $q_2 = 0.6$, $v_3 = 9$, $q_3 = 0.59$, $c = 0.02$, $K = 5$, and $\delta = 1$. Then, $q_3 v_3 - K - 4c/(2q_3 - 1) < 0$, and also, $\mathcal{I}_M^* = \{1, 2\} \subset \mathcal{I}_S^* = \{1, 2, 3\}$.

Proof of Proposition 4. We begin with a formal derivation of the optimization problem P (analogously to the proof of Proposition 2). For any fixed $\mathcal{I}_P \subseteq \mathcal{N}$, the firm will select, after having received all recommendations, concept $i \in \mathcal{I}_P$ if and only if $r_i = g$ and $q_i(v_i - u_{ig}) - (1 - q_i)u_{ib} - \sum_{k \neq i} u_{ka}$ is largest among all concepts with a good recommendation (Note that if the firm receives only bad recommendations, then it will select none of the concepts). Accordingly, constraint (16) orders all concepts $i \in \mathcal{I}_P$ according to their relative attractiveness to the firm; represented by the index j in $y_i^{(j)}$.

Next, we derive the IC constraints for a concept $i \in \mathcal{I}_P$ that is the j th most attractive alternative (i.e., $y_i^{(j)} = 1$). Given $e_i = h$ and $s_i = g$, agent i will receive an expected utility of $\pi_{ij}^{gg} = (q_i u_{ig} + (1 - q_i)u_{ib})/2^{j-1} + (1 - 1/2^{j-1})u_{ia}$ when making a good recommendation ($r_i = g$), and $\pi_{ij}^{bg} = u_{ia} + u_{it}/2^{n-1}$ when making a bad recommendation ($r_i = b$). Similarly, given $e_i = h$ and $s_i = b$, she will receive an expected utility of $\pi_{ij}^{gb} = ((1 - q_i)u_{ig} + q_i u_{ib})/2^{j-1} + (1 - 1/2^{j-1})u_{ia}$ when choosing $r_i = g$, and $\pi_{ij}^{bb} = u_{ia} + u_{it}/2^{n-1}$ when choosing $r_i = b$. Also, given truth telling, her expected utility from exerting high effort is $\pi_{ij}(h) = (q_i u_{ig} + (1 - q_i)u_{ib})/2^j + (u_{ia} + (1/2)^{n-j}u_{it})/2^j + (1 - 1/2^{j-1})u_{ia} - c$, whereas low effort yields $\pi_{ij}(l) = (u_{ig}/2 + u_{ib}/2)/2^j + (u_{ia} + (1/2)^{n-j}u_{it})/2^j + (1 - 1/2^{j-1})u_{ia}$. The IC constraints (13)-(15) follow from requiring that $\pi_{ij}^{gg} \geq \pi_{ij}^{bg}$, $\pi_{ij}^{bb} \geq \pi_{ij}^{gb}$, and $\pi_{ij}(h) \geq \pi_{ij}(l)$ for all $i, j \in \mathcal{I}_P$, while adhering to the order of attractiveness as defined by $y_i^{(j)}$. Finally, limited liability enforces non-negative compensation, hence implying (18), and (17) follows from the fact that each concept $i \in \mathcal{I}_P$ must be assigned to one and only one attractiveness rank.

Given $y_i^{(j)} = 1$, the firm develops concept i with probability $1/2^j$ and receives an expected value of $q_i v_i - K$. Agent i 's compensation is then u_{ig} with probability $q_i/2^j$, u_{ib} with probability $(1 - q_i)/2^j$, u_{ia} with probability $1 - 1/2^j$, and u_{it} with probability $1/2^n$. Summing over $i, j \in \mathcal{I}_P$ yields (12).

(i) Assume \mathcal{I}_P to be an informative test set. We solve P by first ignoring (16), and then showing that the obtained solution is also feasible—and thus optimal—when including (16). Without (16), the structure of P reveals that maximizing the firm's expected profit is equivalent to separately minimizing agent i 's compensation whenever $y_i^{(j)} = 1$, for all $i \in \mathcal{I}_P$. For each $i \in \mathcal{I}_P$ and assuming $y_i^{(j)} = 1$, (15) implies that $u_{ig} > u_{ib}$, which allows us to rewrite (13) and (14) as $q_i u_{ig} + (1 - q_i)u_{ib} \geq u_{ia} + 2^{j-n}u_{it} \geq (1 - q_i)u_{ig} + q_i u_{ib}$. It follows that agent i 's compensation is minimized when $u_{ia} + 2^{j-n}u_{it} = (1 - q_i)u_{ig} + q_i u_{ib}$, and u_{ig} and u_{ib} are chosen as low as possible. By (15) and (18), these minimal payments are $u_{ig} = 2^{j+1}c/(2q_i - 1)$ and $u_{ib} = 0$. Moreover, (12) reveals that the firm prefers paying u_{it} over u_{ia} ; therefore $u_{ia} = 0$ and $u_{it} = 2^{n+1}(1 - q_i)c/(2q_i - 1)$. Inserting these payments in (12) and using (17) gives $\Pi_P = \sum_{j=1}^n \sum_{i=1}^n y_i^{(j)}(q_i v_i/2^j) - \sum_{i=1}^n 2c/(2q_i - 1) - \sum_{j=1}^n K/2^j$. Given that \mathcal{I}_P is an informative test set, we have $q_i v_i \geq q_{i+1} v_{i+1}$, and hence, in optimum, $y_i^{(i)} = 1$ for all $i \in \mathcal{I}_P$, and $y_i^{(j)} = 0$ for all $j \neq i$. Furthermore, this solution satisfies (17) and is thus feasible. Finally, it remains to show that the solution also satisfies (16). However, this is obvious because

we can rewrite (16) as follow: $q_i v_i - (2^{i+1} q_i c / (2q_i - 1)) \geq q_{i+1} v_{i+1} - (2^{i+2} q_{i+1} c / (2q_{i+1} - 1))$, and this is true for any informative test set.

(ii) For all $i, j \in \mathcal{I}_P^*$ such that $y_i^{(j)} = 1$, we know from constraints (13)-(15) that $u_{ig} \geq 2^{j+1} c / (2q_i - 1)$ and $u_{ia} + 2^{j-n} u_{it} \geq (1 - q_i) u_{ig}$. As a result, we can bound concept i 's net profit contribution as follow: $\Pi_i^{(j)} \leq (q_i v_i - K - q_i u_{ig} - u_{ia} - 2^{j-n} u_{it}) / 2^j \leq (q_i v_i - K - u_{ig}) / 2^j \leq (q_i v_i - K) / 2^j - 2c / (2q_i - 1)$. It follows that a necessary condition for $i \in \mathcal{I}_P^*$ is that $(q_i v_i - K) / 2^j - 2c / (2q_i - 1) \geq 0$ for some $j \in \mathcal{I}_P^*$. It is evident that this can only be true if $q_i v_i - K - 4c / (2q_i - 1) \geq 0$, which together with Proposition 3(ii) concludes that $\mathcal{I}_P^* \subseteq \mathcal{I}_M^*$.

To prove that there exist cases in which $\mathcal{I}_P^* \cap \mathcal{I}_{\text{par}}^{\text{fb}} = \emptyset$, consider the following setting with three concepts: $v_1 = 100$, $q_1 = 0.55$, $v_2 = 85$, $q_2 = 0.64$, $v_3 = 53$, $q_3 = 1$, $c = 0.6$, and $K = 50$. In this case, $\mathcal{I}_P^* = \{3\}$, whereas $\mathcal{I}_{\text{par}}^{\text{fb}} = \{1, 2\}$.

(iii) For informative test sets this result follows directly from part (i). For test sets that are not informative, Π_P is an upper bound on expected firm profits because the compensation package defined in part (i) solves a relaxed version of optimization problem P without constraint (16).

Proof of Proposition 5. (i) Suppose $\delta < 1$. For $c = 0$, Propositions 2-4 imply that $\mathcal{I}_P^* = \mathcal{I}_M^* = \mathcal{I}_S^* = \mathcal{N}$ and that the concepts are tested in decreasing order of $q_i v_i$. It follows readily that $\Pi_P^* > \Pi_M^* = \Pi_S^*$. Thus, by continuity of the expected profits in c , there exists $\underline{c} > 0$ such that $\Pi_P^* > \max\{\Pi_M^*, \Pi_S^*\}$ for all $c < \underline{c}$. Now suppose $\delta = 1$. By Proposition 3(iii) and 4(iii), we have $\Pi_P^* = \Pi_P(\mathcal{I}_P^*) \leq \Pi_M(\mathcal{I}_P^*) \leq \Pi_M(\mathcal{I}_M^*) = \Pi_M^*$, and the result follows immediately.

(ii) By Proposition 4(iii), $\Pi_P^* \leq \sum_{i \in \mathcal{I}_P^*} ((q_i v_i - K) / 2^i - 2c / (2q_i - 1))$. Now suppose that the firm fixes the identity and ordering of the concepts in \mathcal{I}_P^* , but uses a sequential testing strategy with multiple agents. Then: $\Pi_M(\mathcal{I}_P^*) = \sum_{i \in \mathcal{I}_P^*} \delta^{i-1} (q_i v_i - K - 4c / (2q_i - 1)) / 2^i$. By comparison, $\Pi_M(\mathcal{I}_P^*) > \Pi_P^*$ if $c > \bar{c} \equiv \sum_{i \in \mathcal{I}_P^*} ((1 - \delta^{i-1}) (q_i v_i - K) / 2^i) / \sum_{i \in \mathcal{I}_P^*} ((2(1 - (\delta/2)^{i-1}) / (2q_i - 1)))$, and by optimality, it follows that $\Pi_P^* < \Pi_M(\mathcal{I}_P^*) \leq \Pi_M^* \leq \max\{\Pi_M^*, \Pi_S^*\}$ if $c > \bar{c}$.

(iii) With $|\mathcal{I}_S^*| = 2$, Propositions 2(iii) and 3(iii) imply that $\Pi_S^* = (q_1 v_1 - K - \delta \hat{\pi}_1) / 2 + \delta (q_2 v_2 - K - 4c / (2q_2 - 1)) / 4$, with $\hat{\pi}_1 = (3 - 2q_2) c / (2q_2 - 1)$, and $\Pi_M(\mathcal{I}_S^*) = (q_1 v_1 - K - 4c / (2q_1 - 1)) / 2 + \delta (q_2 v_2 - K - 4c / (2q_2 - 1)) / 4$. It follows that $\Pi_M(\mathcal{I}_S^*) > \Pi_S^*$ if $q_2 < 1/2 + \delta (2q_1 - 1) / (4 + \delta (2q_1 - 1)) \leq 7/10$, and the result follows from the fact that $\Pi_M^* \geq \Pi_M(\mathcal{I}_S^*)$.

(iv) By Proposition 2(iii) and 3(iii), we have $\Pi_M^* = \sum_{i \in \mathcal{I}_M^*} (\delta^{i-1} / 2^i) (q_i v_i - K - 4c / (2q_i - 1))$ and $\Pi_S(\mathcal{I}_M^*) = \sum_{i \in \mathcal{I}_M^*} \delta^{i-1} (q_i v_i - K - \max\{4q_i c / (2q_i - 1), \delta \hat{\pi}_i, 4c / (2q_i - 1) - \delta \hat{\pi}_i\}) / 2^i \leq \Pi_S^*$, and clearly, $\Pi_S(\mathcal{I}_M^*) \geq \Pi_M^*$ if $\delta \hat{\pi}_i \leq 4q_i c / (2q_i - 1)$ for all $i \in \mathcal{I}_M^*$. We now show that this condition holds if $q_{i+1} \geq \underline{q}_i$ for all $i \in \mathcal{I}_M^*$.

According to (5), $\delta \hat{\pi}_n = 0$ and $\delta \hat{\pi}_{i-1}$ increases in $\delta \hat{\pi}_i$ for all $i \in \mathcal{I}_M^*$. Moreover, if $\delta \hat{\pi}_i = 4q_i c / (2q_i - 1)$, then $\delta \hat{\pi}_{i-1} = \delta (4q_i c / (2q_i - 1) - c)$. Thus, by induction, if $\delta \hat{\pi}_i \leq 4q_i c / (2q_i - 1)$, then $\delta \hat{\pi}_{i-1} \leq$

$\delta(4q_i c / (2q_i - 1) - c)$, and $\delta(4q_i c / (2q_i - 1) - c) \leq 4q_{i-1} c / (2q_{i-1} - 1)$ if $q_i \geq \underline{q}_{i-1}$. Finally, it is easy to show that $\underline{q}_i \leq q_i$ and $\underline{q}_i \leq 5/6$.

(v) Assume $q_i = q$ for all $i \in \mathcal{N}$. Then part (a) is a direct application of Proposition 5(iv). We proceed to prove part (b): For brevity, let $|\mathcal{I}_M^*| = n_M$, $|\mathcal{I}_P^*| = n_P$, $|\mathcal{I}_{\text{seq}}^{\text{fb}}| = n_{\text{seq}}$, and $|\mathcal{I}_{\text{par}}^{\text{fb}}| = n_{\text{par}}$. Given $q_i = q$ for all $i \in \mathcal{N}$, it is always optimal to test the concepts in decreasing order of v_i when testing them sequentially, and under parallel testing concept i 's attractiveness will decrease in v_i . It follows from Propositions 3(ii) and 4(ii) that $\mathcal{I}_P^* \subseteq \mathcal{I}_{\text{par}}^{\text{fb}}$, $\mathcal{I}_M^* \subseteq \mathcal{I}_{\text{seq}}^{\text{fb}}$, and consequently, $n_P \leq n_{\text{par}}$, $n_M \leq n_{\text{seq}}$. Next, without loss of generality, we relabel the concepts such that $v_i \geq v_{i+1}$ for all $i \in \mathcal{N}$. Given these preliminaries, we will prove the result by showing that for any $n_P \geq 0$, $\Pi_{\text{seq}}^{\text{fb}} \geq \Pi_{\text{par}}^{\text{fb}}$ implies $\Pi_M^* \geq \Pi_P^*$, and the result then follows by applying part (a).

First, suppose $n_P = 0$. The claim is trivially satisfied because $\Pi_P^* = 0 \leq \Pi_M^*$. Second, suppose $n_P = 1$. By Proposition 3(iii) and 4(iii), we have $\Pi_P^* = (qv_1 - K)/2 - 2c/(2q - 1) = \Pi_M(n = 1) \leq \Pi_M(n_M) = \Pi_M^*$, where the inequality follows from the optimality of n_M . Last, suppose $n_P \geq 2$ and define $\Delta\Pi(x, y) = \Pi(x) - \Pi(y)$. Using this notation, we can rewrite the firm's first-best expected profits as follow: $\Pi_{\text{seq}}^{\text{fb}} = \Pi_M^* + \Delta\Pi_M(n_{\text{seq}}, n_M) + \sum_{i=1}^{n_{\text{seq}}} (c(\delta/2)^{i-1}(3 - 2q)/(2q - 1))$, and $\Pi_{\text{par}}^{\text{fb}} = \Pi_P^* + \Delta\Pi_{\text{par}}^{\text{fb}}(n_{\text{par}}, n_P) + \sum_{i=1}^{n_P} (c(3 - 2q)/(2q - 1))$. Hence, $\Pi_{\text{seq}}^{\text{fb}} \geq \Pi_{\text{par}}^{\text{fb}}$ if and only if $\Pi_M^* \geq \Pi_P^* + \Delta\Pi_{\text{par}}^{\text{fb}}(n_{\text{par}}, n_P) - \Delta\Pi_M(n_{\text{seq}}, n_M) + c(3 - 2q)/(2q - 1)(n_P - (1 - (\delta/2)^{n_{\text{seq}}})/(1 - (\delta/2)))$. Moreover, by optimality of n_{par} and n_M , we have $\Delta\Pi_{\text{par}}^{\text{fb}}(n_{\text{par}}, n_P) \geq 0$ and $\Delta\Pi_M(n_{\text{seq}}, n_M) \leq 0$, and also $n_P - (1 - (\delta/2)^{n_{\text{seq}}})/(1 - (\delta/2)) \geq n_P - 2 \geq 0$ because $n_P \geq 2$ by assumption. As a result, $\Pi_M^* \geq \Pi_P^*$.

Finally, we prove by example that the converse statement is not always true. Consider the following setting with four concepts: $v_1 = 10$, $v_2 = 8$, $v_3 = 6$, $v_4 = 4$, $q = 1$, $c = 0.2$, $K = 2$, and $\delta = 0.8$. Then $\Pi_P^* = 3.6 < 3.82 = \Pi_M^*$, but $\Pi_{\text{par}}^{\text{fb}} = 4.2 > 4.14 = \Pi_{\text{seq}}^{\text{fb}}$.

Proof of Proposition 6. For ease of notation, relabel the concepts in \mathcal{I}_S^* and \mathcal{I}_M^* such that it is optimal for the firm to test the concepts in increasing order of index i . Similarly, relabel the concepts in \mathcal{I}_P^* such that concept i is the firm's i th most preferred concept. Then, for any $\mathcal{I} \in \{\mathcal{I}_S^*, \mathcal{I}_M^*, \mathcal{I}_P^*\}$, we have $\rho(\mathcal{I}) = \sum_{i \in \mathcal{I}} q_i / 2^i$.

(i) If $q_i = q$ for all $i \in \mathcal{N}$, then, under sequential testing, the firm always tests concepts in decreasing order of v_i . By Proposition 2 and 3, it follows immediately that $\mathcal{I}_S^* \supseteq \mathcal{I}_M^*$. Together with Proposition 4(ii), this implies that $\mathcal{I}_S^* \supseteq \mathcal{I}_M^* \supseteq \mathcal{I}_P^*$, and hence $\rho(\mathcal{I}_S^*) \geq \rho(\mathcal{I}_M^*) \geq \rho(\mathcal{I}_P^*)$.

(ii) Suppose that $|\mathcal{N}| = 2$ and that $\mathcal{I}_M^* = \{1, 2\}$. By Proposition 2(ii)-(iii), if $q_1 > \hat{q}_1$, then $\mathcal{I}_S^* = \{1\}$, and it follows readily that $\rho(\mathcal{I}_M^*) = q_1/2 + q_2/4 > q_1/2 = \rho(\mathcal{I}_S^*)$.

(iii) We prove the results by example. (a) Consider the following setting with two concepts: $v_1 = 29.8$, $v_2 = 40$, $q_1 = 1$, $q_2 = 0.75$, $c = 0.1$, $K = 10$, and $\delta = 1$. Then $\mathcal{I}_M^* = \mathcal{I}_S^* = \{1, 2\}$ and $\mathcal{I}_{\text{seq}}^{\text{fb}} = \{2, 1\}$, which yields $\rho(\mathcal{I}_M^*) = \rho(\mathcal{I}_S^*) = 11/16 > 5/8 = \rho(\mathcal{I}_{\text{seq}}^{\text{fb}})$. (b) We refer to the example in the proof of Proposition 4(ii), which establishes that $\mathcal{I}_P^* = \{3\}$, but $\mathcal{I}_{\text{par}}^{\text{fb}} = \{1, 2\}$, yielding $\rho(\mathcal{I}_P^*) = 1/2 > 87/200 = \rho(\mathcal{I}_{\text{par}}^{\text{fb}})$.

Proof of Proposition 7. Fix \mathcal{N} and let \mathcal{I}_S^* and u_S be given as in Proposition 2. We say that \mathcal{I}_S^* and u_S are renegotiation-proof if there exists no $i \in \mathcal{I}_S^*$ such that the firm, after testing concept i , can offer the agent a new test set \mathcal{I}'_S and a new compensation scheme u'_S that simultaneously increases the firm's expected profits and the agent's expected continuation utility.

(a) If $|\mathcal{N}| = 2$, we have $|\mathcal{I}_S^*| \leq 2$. Obviously, if $|\mathcal{I}_S^*| = 1$, then the testing process ends after the first concept test and there is by definition no opportunity for renegotiation. Now assume $|\mathcal{I}_S^*| = 2$; here the firm could renegotiate after testing concept $i = 1$. However, by comparing the initially optimal compensation for concept $i = 2$ (as given by Proposition 2(i)) with the optimal compensation scheme for testing a single concept (cf. Proposition 3(i)), we can immediately conclude that the firm cannot increase its own expected profits by offering a new compensation scheme. Also, it would never be mutually beneficial to not test concept $i = 2$ if, initially, $2 \in \mathcal{I}_S^*$.

(b) If $q_i = q$ for all $i \in \mathcal{N}$, then it is always optimal to test concepts in decreasing order of v_i . Furthermore, the agent's expected continuation utility is independent of the exact testing order because, for all $i \in \mathcal{I}_S^*$, $\hat{\pi}_i$ is independent of v_i , but only depends on c and q . As a result, the firm can never increase its expected profits by changing the testing order, which implies that \mathcal{I}_S^* and u_S are renegotiation-proof.

(c) A necessary condition for renegotiation to be feasible is that after the test of some concept $i \in \mathcal{I}_S^*$, the firm must find it beneficial to change either the sequence or the identity of the remaining concepts to be tested (otherwise, the firm could only increase expected profits by paying the agent less, which the agent would not accept). However, by Proposition 2(ii), if the concepts in \mathcal{N} can be ordered such that $q_i v_i \geq q_{i+1} v_{i+1}$ and $q_i \geq q_{i+1}$, and for all $i \in \mathcal{I}_S^* \setminus \{n\}$, with $n = |\mathcal{I}_S^*|$, we have $4(1 - q_i)c/(2q_i - 1) \leq \delta \hat{\pi}_i \leq 4q_i c/(2q_i - 1)$, then for any $i \in \mathcal{I}_S^*$, the firm can never benefit from changing the remaining test set or testing order.

Proof of Proposition 8. (a) Assume $\delta = 0$. If, for any concept $i \in \mathcal{I}_S^*$, the firm receives a recommendation of feasibility (i.e., $m_i = g$), then the firm immediately selects concept i for development because, given that $\delta = 0$, any postponement of the selection decision reduces the firm's expected profits. By continuity of the expected profits in δ , there exists $\underline{\delta} > 0$ such that for all $\delta < \underline{\delta}$, it is optimal to stop testing after the first recommendation of feasibility.

(b) Assume $c = 0$. In this case, by Proposition 2, $u^S = 0$ and it is thus optimal to test concepts in decreasing order of $q_i v_i$. That is, concepts are tested in decreasing order of their attractiveness, and it is hence optimal to stop testing after the first recommendation of feasibility. By continuity of the expected profits in c , there exists $\check{c} > 0$ such that the same conclusion holds for all $c < \check{c}$. Next, assume $c > 0$. It is obvious that there exists a $\hat{c} > \check{c}$ such that for all $c > \hat{c}$, testing becomes so expensive that it is never economically rational to continue testing (and to incur additional testing costs) after a recommendation of feasibility.

(c) If $q_i = q$ for all $i \in \mathcal{I}_S^*$, then it is always optimal to test concepts in decreasing order of v_i . Hence it is optimal to stop testing after the first recommendation of feasibility because any further testing cannot reveal a more attractive concept.

(d) Assume $v_i \geq v_{i+1}$ and $q_i \geq q_{i+1}$ for all $i \in \mathcal{I}_S^* \setminus \{n\}$, with $n = |\mathcal{I}_S^*|$. Then, $q_i v_i \geq q_{i+1} v_{i+1}$, and it follows that the firm prefers to select concept i over $i+1$, unless the induced agency costs associated with the selection of concept i are overly large. Yet, if $i \in \mathcal{I}_S^*$ then those agency costs are bounded from above; if they were not, the firm would forgo testing concept i and hence $i \notin \mathcal{I}_S^*$. Now, given that, for all $i \in \mathcal{I}_S^*$, $q_i v_i \geq q_{i+1} v_{i+1}$ and agency costs are bounded from above, there exists some $\Delta_{qv} \geq 0$ such that the firm prefers to halt the testing process after the first recommendation of feasibility if $q_i v_i - q_{i+1} v_{i+1} > \Delta_{qv}$ for all $i \in \mathcal{I}_S^* \setminus \{n\}$.