Electronic Companion

Strategic Release of Information in Platforms: Entry, Competition, and Welfare

Kostas Bimpikis Giacomo Mantegazza *

EC 1 Proofs of results from the main text

Proposition 1 *For every history where only seller i joins the platform, there exist unique prices $\hat{p}^M(\rho)$ and p^M such that it is optimal to set $p_{i,1} = \hat{p}^M(\rho)$ and $p_{i,2} = p^M$. Moreover, $\hat{p}^M > p^M$ if and only if $\rho > 0$.*

Proof. The proof proceeds backwards. At period $t = 2$, it is optimal to set price $p_{i,2}$ such that

$$\max_{p_{i,2}} \quad \rho(1-\alpha)p_{i,2}\overline{F}(p_{i,2}) + (1-\rho)\times 0. \tag{EC.1}$$

Notice that the maximand reproduces equation (3), since $\mathbb{E}_\rho[\mathbf{1}\{\omega = 1\}] = \rho$ and $\mathbb{E}[\{\mathbf{1}\{B_2^* = i\}] = \overline{F}(p_{i,2})$. This is a one-shot monopoly pricing problem, as analyzed in Lariviere (2006). Recalling our notation, $g(p)$ is the generalized failure rate of F and is $h(p)$ the failure rate, for which we have

$$g(p) = ph(p) = p\frac{f(p)}{1-F(p)}.$$

The First Order Condition (FOC) for (EC.1) is

$$\overline{F}(p)[1-g(p)] = 0, \tag{EC.2}$$

and since we assume IFR, [1] a solution to (EC.2) exists, is unique and is interior; we denote it by p^M and write $\pi^M = p^M\overline{F}\left(p^M\right)$.

*Graduate School of Business, Stanford University. Email: {kostasb,giacomom}@stanford.edu

[1] Implicitly we assume $\lim_{p\to a} g(p) < 1$ and $\lim_{p\to b} g(p) > 1$, which rule out distributions such that it would always be optimal to set $p^M = a$ or $p^M = b$. We write IFR for the increasing failure rate property and IGFR for the increasing generalized failure rate property.

Using this result and plugging back into equation (4) for period $t = 1$, seller i's optimal price in this period must solve

$$\max_{p_{i,1}} \quad (1-\alpha)\left[p_{i,1}\overline{F}\left(p_{i,1}\right) + F\left(p_{i,1}\right)\rho\pi^M\right]. \tag{EC.3}$$

The first summand, $p_{i,1}\overline{F}\left(p_{i,1}\right)$, represent the gross expected profit from selling the good to the first customer; with probability $F\left(p_{i,1}\right)$ the first buyer has too low a valuation for the good, and the sale does not happen, so that the unitary good of seller i is still available for purchase, should the second customer arrive.

The FOC associated to (EC.3) can be written as

$$\overline{F}(p)\left[1 - g(p) + \rho\pi^M h(p)\right] = 0. \tag{EC.4}$$

Notice that $h(p) > 0$ for all $p \in (a, b)$, so that

$$\lim_{p\to a}\left[1 - g(p) + \rho\pi^M h(p)\right] = 1 - \lim_{p\to a} g(p) + \rho\pi^M \lim_{p\to a} h(p) > 0,$$

because $\lim_{p\to a} g(p) < 1$. Also

$$\lim_{p\to b}\left[1 - g(p) + \rho\pi^M h(p)\right] = 1 + \lim_{p\to b}\left[\left(\frac{\rho\pi^M}{p} - 1\right) g(p)\right].$$

If $b = \infty$ the limit becomes $1 - \lim_{p\to\infty} g(p)$, which is negative by assumption. If b is finite, then notice that

$$\frac{\rho\pi^M}{b} = \underbrace{\rho\overline{F}\left(p^M\right)}_{<1}\underbrace{\frac{p^M}{b}}_{<1} < 1.$$

Together with $\lim_{p\to b} g(p) > 1$, this implies

$$\lim_{p\to b}\left[1 - g(p) + \rho\pi^M h(p)\right] < 0,$$

which shows that a solution to (EC.4) exists and is in the interior. To show uniqueness of the maximizer, consider the second derivative of the objective function, which is well-defined since f is assumed to be differentiable, given by

$$-2f(p) + f'(p)(\rho\pi^M - p).$$

Using (EC.4), we can rewrite it as

$$-2f(p^*) - f'(p^*)\frac{\overline{F}(p^*)}{f(p^*)},$$

where p^* denotes that it is evaluated at the stationary point. Then the Second Order Condition is

$$\begin{aligned} -2f(p^*) - f'(p^*)\frac{\overline{F}(p^*)}{f(p^*)} < 0 &\iff 2f(p^*) + f'(p^*)\frac{\overline{F}(p^*)}{f(p^*)} > 0 \\ &\iff f(p)^2 + \overline{F}(p)^2 h'(p) > 0. \end{aligned}$$

This condition is satisfied at every stationary point because the distribution is IFR, i.e. $h'(p) \geq 0$. We showed that the objective function in (EC.3) is strictly concave at any stationary point, which we are assured to exist. Since it is a continuously differentiable function (because F necessarily has continuous density), this implies that it cannot have more than one stationary point, which is also the unique global maximizer; denote it by $\hat{p}^M$.

Finally, plugging p^M in (EC.4) and using (EC.2), we have

$$\overline{F}\left(p^M\right)\left[1 - g\left(p^M\right) + \rho\pi^M h\left(p^M\right)\right] = \overline{F}\left(p^M\right)\rho\pi^M h\left(p^M\right) \geq 0,$$

with equality if and only if $\rho = 0$; therefore $\hat{p}^M \geq p^M$. ■

Corollary 1 *The functions W^M and $\hat{V}^M$ are increasing and strictly convex in ρ. Furthermore, $\hat{p}^M$ is increasing in ρ.*

Proof. Setting aside α, which does not affect monotonicity and curvature, we consider the function

$$V^M(\rho) = \hat{p}^M(\rho)\overline{F}\left(\hat{p}^M(\rho)\right) + \rho\pi^M F\left(\hat{p}^M(\rho)\right),$$

where we highlight that $\hat{p}^M$ is a function of ρ, too. The derivative with respect to ρ is

$$\begin{aligned} \frac{\partial V^M}{\partial \rho} &= \frac{\partial \hat{p}^M}{\partial \rho}\left[\overline{F}\left(\hat{p}^M\right) - \hat{p}^M f\left(\hat{p}^M\right) + \rho\pi^M f\left(\hat{p}^M\right)\right] + \pi^M F\left(\hat{p}^M\right) \\ &= \frac{\partial \hat{p}^M}{\partial \rho}\overline{F}\left(\hat{p}^M\right)\left[1 - g\left(\hat{p}^M\right) + \rho\pi^M h\left(\hat{p}^M\right)\right] + \pi^M F\left(\hat{p}^M\right) \\ &= \pi^M F\left(\hat{p}^M\right) > 0, \end{aligned}$$

where the last equality holds because $\hat{p}^M$ must satisfy (EC.4) (equivalently, by the envelope theorem). For the second derivative,

$$\frac{\partial^2 V^M}{\partial \rho^2} = \frac{\partial}{\partial \rho}\left[\pi^M F\left(\hat{p}^M\right)\right] = \pi^M f\left(\hat{p}^M\right)\frac{\partial \hat{p}^M}{\partial \rho} > 0,$$

since, as we will show, $\hat{p}^M$ is strictly increasing in ρ.

Applying the Implicit Function Theorem to (EC.4), one obtains

$$\frac{\partial \hat{p}^M}{\partial \rho} = \frac{-\pi^M h\left(\hat{p}^M\right)}{\rho\pi^M h'\left(\hat{p}^M\right) - g'\left(\hat{p}^M\right)}. \tag{EC.5}$$

While the numerator is negative, we can rewrite the denominator as

$$\begin{aligned}
&\rho\pi^M h'\left(\hat{p}^M\right) - g'\left(\hat{p}^M\right) = \left(\rho\pi^M - \hat{p}^M\right) h'\left(\hat{p}^M\right) - h\left(\hat{p}^M\right) \\
&= -\frac{\overline{F}\left(\hat{p}^M\right)}{f\left(\hat{p}^M\right)} h'\left(\hat{p}^M\right) - h\left(\hat{p}^M\right) < 0,
\end{aligned}$$

because $h(p) > 0$ on all the support. Comparing this with the numerator in (EC.5) we get $\partial\hat{p}^M/\partial\rho > 0$, which justifies the above claim about V^M. ∎

Proposition 2 *At every history when both sellers join the platform, the unique equilibrium prices are given as $p_{i,1} = p_{j,1} = \rho\pi^M$ and*

$$p_{i,2} = \begin{cases} p^M & \textit{if } j \textit{ makes a sale at } t = 1, \\ 0 & \textit{otherwise.} \end{cases}$$

Proof. Recalling the payoff function for seller i from equation (8), notice that when $p_j^1 > p^M$, then also $p_j^1 > \rho\pi^M$ for every $\rho \in [0, 1]$. This means that

$$\rho\pi^M \overline{F}\left(p_j^1\right) < p_j^1 \overline{F}\left(p_j^1\right) < \pi^M,$$

so that the optimal price is $p_i^1 = p^M$ if $p_j^1 > p^M$. On the other hand, when $p_j^1 \leq p^M$, $\Pi^S_{1,i}(p_i^1; \rho)$ is strictly increasing for $p_i^1 < p_j^1$ and constant thereafter; therefore, the optimal action is some $p_i^1 > p_j^1$ whenever $\rho\pi^M > p_j^1$ and some $p_i^1 \geq p_j^1$ when $\rho\pi^M = p_j^1$. Finally, when $\rho\pi^M < p_j^1$, there is no well-defined best reply: for every $p_i^1 < p_j^1$ there exists $\varepsilon > 0$ such that

$$(p_i^1 + \varepsilon)\overline{F}\left(p_i^1 + \varepsilon\right) > p_i^1 \overline{F}\left(p_i^1\right),$$

and $p_i^1 + \varepsilon < p_j^1$, while

$$\Pi_{1,i}^S \left(p_i^1 = p_j^1; \rho\right) < \lim_{p \uparrow p_j^1} p\overline{F}(P).$$

Therefore the best-response correspondence for seller i is

$$BR_i(p_j^1) = \begin{cases} \left(p_j^1, \infty\right), & p_j^1 < \rho\pi^M \\ \left[p_j^1, \infty\right), & p_j^1 = \rho\pi^M \\ \varnothing, & \rho\pi^M < p_j^1 \leq p^M \\ p^M, & p_j^1 > p^M \end{cases}$$

Now, it is easy to see that the only pair (p_i^1, p_j^1) for which the equilibrium condition

$$\begin{cases} \{p_i^1\} \subseteq BR_i(p_j^1) \\ \{p_j^1\} \subseteq BR_j(p_i^1) \end{cases}$$

is satisfied is exactly $(\rho\pi^M, \rho\pi^M)$. ■

Corollary 2 *The functions W^D and $\hat{V}^D$ as defined in (9) and (10) are increasing and concave in belief ρ.*

Proof. Let us consider only the function $V^D(\rho) = \rho\pi^M \overline{F}\left(\rho\pi^M\right)$. Then we have

$$\begin{aligned} &\frac{\partial V^D}{\partial \rho} = \pi^M \left[\overline{F}\left(\rho\pi^M\right) - \rho\pi^M f\left(\rho\pi^M\right)\right] \geq 0 \\ &\iff \overline{F}\left(\rho\pi^M\right) \geq \rho\pi^M f\left(\rho\pi^M\right) \iff 1 \geq g\left(\rho\pi^M\right). \end{aligned}$$

But remember $\rho\pi^M = \rho p^M \overline{F}(p^M) < p^M$ and by IGFR property it must be $1 = g(p^M) \geq g\left(\rho\pi^M\right)$; therefore V^D is increasing in ρ.

Moreover, it can be checked that

$$\frac{\partial^2 V^D}{\partial \rho^2} = -\left(\pi^M\right)^2 \left[f\left(\rho\pi^M\right)\left(1 - g\left(\rho\pi^M\right)\right) + \overline{F}\left(\rho\pi^M\right) g'\left(\rho\pi^M\right)\right] \leq 0,$$

again by IGFR and what we proved above. ■

Proposition 3 *Let ρ denote the sellers' common posterior belief about demand after they obtain the platform's message. Then, the following hold along the equilibrium path:*

(i) Suppose that $c > W^M(0)$. Then, there exists $\rho^M(c,\alpha)$ such that if $\rho \geq \rho^M(c,\alpha)$ a single seller joins the platform and, otherwise, there is no entry. In addition, when $c \geq W^M(1)$, we have $\rho^M(c,\alpha) \geq 1$, i.e., entry is too costly for sellers irrespective of their beliefs about demand.

(ii) Suppose that $W^D(1) \leq c \leq W^M(0)$. Then, a single seller joins irrespective of the platform's information disclosure policy.

(iii) Suppose that $c < W^D(1)$. Then, there exists $\rho^D(c,\alpha)$ such that if $\rho > \rho^D(c,\alpha)$ both sellers join the platform and, otherwise, there is a single entrant.

Proof. As further discussed in Appendix A, under Assumption 2 the platform's expected profits are always strictly larger with just one entrant than when both sellers join. This is formalized in the following lemma.

Lemma 1 *Suppose F satisfies these conditions simultaneously:*

$$\overline{F}\left(\pi^M\right)\left(1-g(\pi^M)\right) \geq \frac{F(p^M)}{2}$$
$$2\overline{F}\left(\pi^M\right) \leq 1+F(p^M),$$

Then

$$2\rho^D\pi^M\overline{F}\left(\rho^D\pi^M\right) \leq \hat{p}^M\left(\rho^D\right)\overline{F}\left(\hat{p}^M\left(\rho^D\right)\right)+\rho^D\pi^M F\left(\hat{p}^M\left(\rho^D\right)\right).$$

for all $\rho \in [0,1]$

For $c > W^M(1)$, since $W^M(1)$ is the maximal amount seller i can expect to earn under any market condition, there will be no entrance; this is equivalent to $\rho^M(c,\alpha) > 1$, with ρ^M defined as below.

For $W^M(0) < c \leq W^M(1)$, since the function W^M is strictly increasing in the belief, there exists a unique solution in $[0,1]$ to the equation $W^M(\rho) = c$; denote it with $\rho^M(c,\alpha)$. When $\rho < \rho^M$, then $W^D(\rho) < W^M(\rho) < c$; hence, no seller has incentive to enter. If instead $\rho > \rho^M$, then $W^M(\rho) > c > W^D(\rho)$. Suppose seller i decides to enter: it follows that seller j is better off by staying out. Vice versa, if j joins, then i should stay out to maximize profits. Therefore, at equilibrium only one seller joins, and the other stays out. Finally, suppose $\rho = \rho^M$, so that $W^M(\rho) = c > W^D(\rho)$: if seller j stays out, seller i is indifferent between joining and not joining. Since the platform's expected payoff is higher when i enters than when she does not, the Sender-preferred equilibrium requires that i enters at belief ρ^M.

Take now $W^D(1) \leq c \leq W^M(0)$. For every $\rho \in (0,1)$, $W^M(\rho) > c > W^D(\rho)$, and therefore the same argument of before applies. For $\rho = 0$, if $c < W^M(0)$, once again we are in the same case as above; if $c = W^M(0)$, then at indifference the equilibrium definition requires one seller to join. Finally, for $\rho = 1$, if $c > W^D(1)$, then entry is never profitable for both sellers, and hence just one joins; if instead $c = W^D(1)$, if seller i joins, then seller j is indifferent between entering and not. Since the platform is better off with one seller, our equilibrium requires that j decides to stay out.

When $c < W^D(1)$, then since W^D is strictly increasing there exists a unique solution in $[0, 1]$ to the equation $W^D(\rho) = c$; denote this solution by $\rho^D(c, \alpha)$. Whenever $\rho < \rho^D$, then $W^M(\rho) > c > W^D(\rho)$ and thus only one joins in equilibrium. If $\rho > \rho^D$, then $W^D(\rho) > c$; thus, if seller i joins, also seller j is better off by joining, and notice that seller i does not have incentive to change its behaviour, because by opting out she would earn less than $W^D(\rho)$. Hence, it is equilibrium that both enter. Finally, for $\rho = \rho^D$, if seller i enters then j is indifferent between joining and not; again by the Sender-preferred equilibrium definition, it must be that she stays out. Hence, at belief ρ^D only one seller enters. This concludes the proof. ■

Lemma 1 *Suppose F satisfies these conditions simultaneously:*

$$\overline{F}\left(\pi^M\right)\left(1 - g(\pi^M)\right) \geq \frac{F(p^M)}{2}$$
$$2\overline{F}\left(\pi^M\right) \leq 1 + F(p^M),$$

Then

$$2\rho^D \pi^M \overline{F}\left(\rho^D \pi^M\right) \leq \hat{p}^M\left(\rho^D\right)\overline{F}\left(\hat{p}^M\left(\rho^D\right)\right) + \rho^D \pi^M F\left(\hat{p}^M\left(\rho^D\right)\right).$$

for all $\rho \in [0, 1]$

Proof. In Corollary 1 we proved that $\hat{V}^M(\rho)$ is strictly convex. Therefore, (A.3) is implied by

$$2\rho\pi^M \overline{F}\left(\rho\pi^M\right) \leq \pi^M + \rho\pi^M F(p^M), \tag{EC.6}$$

where the RHS is the first order approximation of $\hat{p}^M(\rho)\overline{F}\left(\hat{p}^M(\rho)\right) + \rho\pi^M F\left(\hat{p}^M(\rho)\right)$ at $\rho = 0$. Notice that (EC.6) is trivially satisfied as a strict inequality at $\rho = 0$. To prove that inequalities (A.1)-(A.2) imply (EC.6), we show that under Assumption 2 the function l, defined by

$$l(\rho) = 1 + \rho F(p^M) - 2\rho\overline{F}\left(\rho\pi^M\right),$$

is decreasing in ρ and satisfies $l(1) \geq 0$. Indeed

$$l(1) = 1 + F(p^M) - 2\overline{F}\left(\pi^M\right) \geq 0$$

is precisely condition (A.2). We can also notice that, as proved in Corollary 2, $2\rho\overline{F}\left(\rho\pi^M\right)$ is concave, so that $l(\rho)$ is convex. Hence, to show that it is decreasing it is sufficient (and necessary) that $\left.\frac{\partial l(\rho)}{\partial \rho}\right|_{\rho=1} \leq 0$. Therefore, since

$$\frac{\partial l(\rho)}{\partial \rho} = F(p^M) - 2\overline{F}\left(\rho\pi^M\right)\left(1 - g\left(\rho\pi^M\right)\right)$$

the condition (A.1) in Lemma 1 implies that also $\left.\frac{\partial l(\rho)}{\partial \rho}\right|_{\rho=1} \leq 0$ is satisfied. ■

Theorem 1 *Suppose that Assumptions 1 and 2 hold. Then, there exists an optimal policy $(\mathcal{D}^*, M^*)$ that involves sending one of two messages, i.e., $M^* = \{Y, N\}$. Moreover, if we let $\mathcal{D}^*(\omega)$ denote the probability that message Y is sent when the state of the world is ω under information disclosure policy $(\mathcal{D}^*, \{Y, N\})$, an optimal policy for the platform takes the following form:*

(i) When $c > W^M(0)$, then

$$\mathcal{D}^*(1) = \begin{cases} 1 & \text{for } \mu < \rho^M(c,\alpha) \\ q_u^M & \text{for } \mu \geq \rho^M(c,\alpha) \end{cases} \qquad \text{and} \qquad \mathcal{D}^*(0) = \begin{cases} q_l^M & \text{for } \mu < \rho^M(c,\alpha) \\ 0 & \text{for } \mu \geq \rho^M(c,\alpha) \end{cases},$$

where the disclosure probabilities are equal to

$$q_l^M = \frac{\mu(1-\rho^M(c,\alpha))}{\rho^M(c,\alpha)(1-\mu)} \quad \text{and} \quad q_u^M = \frac{\mu - \rho^M(c,\alpha)}{\mu(1-\rho^M(c,\alpha))};$$

(ii) When $W^D(1) \leq c \leq W^M(0)$, full-disclosure is optimal, i.e.,

$$\mathcal{D}^*(\omega) = \omega;$$

(iii) Finally, when $c < W^D(1)$,

$$\mathcal{D}^*(1) = \begin{cases} 1 & \text{for } \mu \leq \rho^D(c,\alpha) \\ q_u^D & \text{for } \mu > \rho^D(c,\alpha) \end{cases} \qquad \text{and} \qquad \mathcal{D}^*(0) = \begin{cases} q_l^D & \text{for } \mu \leq \rho^M(c,\alpha) \\ 0 & \text{for } \mu > \rho^D(c,\alpha) \end{cases},$$

where the disclosure probabilities are equal to

$$q_l^D = \frac{\mu(1-\rho^D(c,\alpha))}{\rho^D(c,\alpha)(1-\mu)} \quad \text{and} \quad q_u^D = \frac{\mu - \rho^D(c,\alpha)}{\mu(1-\rho^D(c,\alpha))}.$$

Proof. We first show that the problem of the platform is equivalent to one with just one receiver.

Consider the following game between two players: one, which we call P', is an *alter ego* of the platform, while the other one, which we call S, represents the system of sellers. S's action space is the set $\{00, 01, 11\}$. P' and S engage in a two-stage game in which first P' observes the state of the world and sends a message according to an information disclosure policy; subsequently S takes an action, which affects its own and the payoff of P'. The expected utility of S is given by the following function,[2]

$$u(I, \mu) = \begin{cases} c & I = 00 \\ W^M(\mu) & I = 01 \\ W^D(\mu)\mathbf{1}\{W^D(\mu) \leq c\} + \left[W^M(\mu) + \varepsilon\right]\mathbf{1}\{W^D(\mu) > c\} & I = 11 \end{cases}$$

for some $\varepsilon > 0$. Let us interpret action 00 being *No one joins*, 01 being *Only one joins* and 11 being *Both join*. S's optimal action given belief μ about the state of the world is the same as the entry decision that realizes in our model. The expected utility of P' is instead

$$\hat{u}(I, \mu) = \begin{cases} 0 & I = 00 \\ \hat{V}^M(\mu) & I = 01 \\ \hat{V}^D(\mu) & I = 11 \end{cases}$$

The information design problem of P' is equivalent to that of the platform in the original model, in the sense that every optimal solution for P' is optimal for the platform and vice-versa. This setting satisfies the hypotheses of Anunrojwong, Iyer, and Lingenbrink (2020), so we can use one of their preliminary results and take the message space to be $M = \Delta(\Omega) = \Delta(\{0, 1\}) = [0, 1]$ without loss of generality. The problem can then be stated as

$$\begin{aligned} &\max_{\tau \in \Delta([0,1])} \mathbb{E}_{\rho \sim \tau}\left[\hat{u}(J, \rho)\right] \quad \text{s.t.} \\ &J \in \operatorname*{arg\,max}_{I \in \{00,01,11\}} u(I, \rho) \; \forall \rho \in [0, 1] \\ &\mathbb{E}_\tau[\rho] = \mu \end{aligned}$$

Let us define the function $\hat{V} : [0, 1] \to \mathbb{R}_+$, given by $\hat{V}(\rho) = 0$ for $c > W^M(1)$, and further as

$$\hat{V}(\rho) = \begin{cases} 0 & \text{if } \rho < \rho^M(c, \alpha) \\ \hat{V}^M(\rho) & \text{if } \rho \geq \rho^M(c, \alpha) \end{cases}$$

[2]Expectation is taken with respect to μ.

for $W^M(0) < c \leq W^M(1)$. As $\hat{V}(\rho) = \hat{V}^M(\rho)$ for $W^D(1) \leq c \leq W^M(0)$. Finally, as

$$\hat{V}(\rho) = \begin{cases} \hat{V}^M(\rho) & \text{if } \rho \leq \rho^D(c, \alpha) \\ \hat{V}^D(\rho) & \text{if } \rho > \rho^D(c, \alpha) \end{cases}$$

for $c < W^D(1)$. Since $\hat{V}(\rho) = \hat{u}(J, \rho)$ if $J \in \arg\max_{I \in \{00,01,11\}} u(I, \rho)$ $\forall \rho \in [0, 1]$, the original problem can be further restated as

$$\begin{aligned} &\max_{\tau \in \Delta([0,1])} \mathbb{E}_{\rho \sim \tau} \left[\hat{V}(\rho) \right] \quad \text{s.t.} \\ &\mathbb{E}_\tau[\rho] = \mu \end{aligned}$$

This is now a problem similar to those analyzed by, e.g., Kamenica and Gentzkow (2011) and Aumann and Maschler (1995), and we identify the optimal value function for the platform with the "concavification" of $\hat{V}$, i.e.

$$V^*(\mu) = \sup \left\{ y : \ (y, \mu) \in \mathtt{cov}(\hat{V}) \right\},$$

where $\mathtt{cov}(\hat{V})$ denotes the convex hull of the graph of $\hat{V}$. In other terms, this is the concave closure of $\hat{V}$. We now turn to characterizing V^* in each of the cases we have for $\hat{V}$. To avoid trivialities, suppose $c \leq W^M(1)$.

When $W^M(0) < c \leq W^M(1)$, $\hat{V}$ stays at zero for $\mu < \rho^M$ and then jumps to a positive convex function. Notice that full-disclosure is not optimal, unless $c = W^M(0)$: in fact, the value function corresponding to such policy is equal to $\hat{V}^M(1)\mu$. However, it has

$$\begin{aligned} \hat{V}^M(\mu) &> \hat{V}^M(1) + \alpha \mu \pi^M F\left(\hat{p}^M(1)\right)(\mu - 1) \\ &\geq \hat{V}^M(1)\mu, \end{aligned}$$

where the first equality holds because the RHS is a linear approximation to $\hat{V}^M$ at $\mu = 1$ and the second by algebra. Thus, the concavification of $\hat{V}$ is the piecewise-linear function

$$V^*(\mu) = \begin{cases} \dfrac{1}{\rho^M} \dfrac{\alpha c}{1-\alpha} \mu & \mu < \rho^M \\ \dfrac{\mu - \rho^M}{1 - \rho^M} \left(\hat{V}^M(1) - \dfrac{\alpha c}{1-\alpha} \right) + \dfrac{\alpha c}{1-\alpha} & \mu \geq \rho^M \end{cases}$$

since $\hat{V}^M(\rho^M) = \frac{\alpha c}{1-\alpha}$. Thus, we infer that the optimal strategy for the platform is to alternatively induce beliefs 0 and ρ^M when $\mu < \rho^M$, and beliefs ρ^M and 1 when $\mu \geq \rho^M$. To determine the optimal probability with which each belief is induced, we employ the constraint on τ given by $\mathbb{E}_\tau[\rho] = \mu$: in this case τ should put mass only on two points in $[0, 1]$ and their mean must be μ. Hence: for $\mu < \rho^M$ we need $0 \times (1 - \tau) + \tau \times \rho^M = \mu$ so that $\tau = \frac{\mu}{\rho^M}$. An information disclosure policy that sends message ρ^M with probability q_l^M (and 0 with complementary probability) when $\omega = 0$, and with probability 1 when $\omega = 1$, induces this distribution of posterior beliefs; for $\mu \geq \rho^M$, the constraint requires $\rho^M \times (1-\tau)+1\times\tau = \mu$, which implies $\tau = \frac{\mu-\rho^M}{1-\rho^M}$. A policy sending message ρ^M with probability q_u^M (and 0 with complementary probability) when $\omega = 1$, and always message 0 when $\omega = 0$, induces this distribution.

Suppose now $W^D(1) \leq c \leq W^M(0)$: since $\hat{V}(\mu) = \hat{V}^M(\mu)$ and the latter is strictly convex, the concavification V^* is the straight line joining $(0, \pi^M)$ and $(1, \hat{V}^M(1))$ in the (μ, v) plane. Consequently, one easily reads that the platform optimal strategy is to fully reveal all the information it has. This is achieved, for example, by the policy that sends message 0 when $\omega = 0$ and message 1 when $\omega = 1$.

Finally, assume $c < W^D(1)$. To find the optimal policy for this case we use Assumption 2. Full transparency is not optimal because the value function of this policy would be $\widetilde{V}(\mu) = \alpha\left[\pi^M + \mu\pi^M\left(2\overline{F}(\pi^M) - 1\right)\right]$, and we have for $\mu > 0$

$$\begin{aligned}\hat{V}^M(\mu) &> \alpha\left[\pi^M + \mu\pi^M F(p^M)\right]\\ &\geq \alpha\left[\pi^M + \mu\pi^M\left(2\overline{F}(\pi^M) - 1\right)\right] = \widetilde{V}(\mu),\end{aligned}$$

where the first inequality follows because the RHS is the linear approximation of $\hat{V}^M$ at $\mu = 0$ and the second reduces to the second condition in (11). In addition, and always thanks to conditions (11), $\hat{V}^D$ always lies below a linear approximation to $\hat{V}^M$ at $\mu = 0$: this implies that for any threshold ρ^D the straight line joining $(\rho^D, \hat{V}^M(\rho^D))$ with $(1, 2\pi^M\overline{F}(\pi^M))$, which is the value of a policy that induces beliefs ρ^D and 1, always dominates any other policy that induces different beliefs when $\rho^D < \mu \leq 1$.[3] Hence, also in this case the optimal value function is a piecewise-linear function, given by

$$V^*(\mu) = \begin{cases}\alpha\pi^M + \left(\hat{V}^M(\rho^D) - \alpha\pi^M\right)\dfrac{\mu}{\rho^D} & \mu \leq \rho^D\\ \hat{V}^M(\rho^D) + \left(\alpha 2\pi^M\overline{F}(\pi^M) - \hat{V}^M(\rho^D)\right)\dfrac{\mu-\rho^D}{1-\rho^D} & \mu > \rho^D\end{cases}$$

As before, this value function tells that the optimal policy is to alternatively induce beliefs 0 and ρ^D when $\mu \leq \rho^D$ and beliefs ρ^D and 1 when $\mu > \rho^D$. Following the same reasoning of the case

[3] In particular, the first condition in (11) ensures that it is not optimal to induce beliefs ρ^D and $\eta < 1$ for $\rho^D < \mu \leq \eta$ and do nothing for $\mu > \eta$.

above where $W^M(0) < c \leq W^M(1)$, one can check that an optimal policy is to send message ρ^D with probability 1 when $\omega = 1$ and with probability q_l^D (and 0 with complementary probability) when $\omega = 0$, if $\mu \leq \rho^D$; and to send message 1 with probability q_u^D (and ρ^D with complementary probability) when $\omega = 1$ and ρ^D with probability 1 when $\omega = 0$, if $\mu > \rho^D$.

Finally, let us observe that the policy we have described always sends one of two possible messages for every combination of c and α. Therefore, the message space can be restricted to $\{Y, N\}$, so that we recover the policy given in the statement of the proposition. ■

Corollary 3 *The following hold true:*

(i) The optimal information disclosure policy always yields strictly higher profits for the platform than no-disclosure. In addition, it strictly outperforms full-disclosure too, unless $W^D(1) \leq c \leq W^M(0)$.

(ii) No-disclosure yields strictly higher profits than full-disclosure if (a) $c > W^M(0)$ and $\mu \geq \rho^M$, or (b) $c < W^D(1)$ and $\mu \leq \rho^D$.

Proof. Since $\mu = 0$ and $\mu = 1$ are absorbing cases in which the disclosure problem is trivial (in fact, the prior belief cannot be modified), without loss of generality assume $\mu \neq 0, 1$.

Notice that the first part of the corollary follows directly from Theorem 1. For the second part, suppose $W^M(0) < c \leq W^M(1)$ and $\mu < \rho^M$: the no-disclosure policy gives 0 profit in this case, while full-disclosure yields $\mu\hat{V}^M(1) > 0$, so full-disclosure strictly dominates no-disclosure.
If $\mu > \rho^M$, then no-disclosure strictly dominates full-disclosure if and only if

$$\mu\hat{V}^M(1) < \hat{V}^M(\mu) \iff \frac{\hat{V}^M(1)}{1} < \frac{\hat{V}^M(\mu)}{\mu}.$$

$\frac{\hat{V}^M(\mu)}{\mu}$ is a strictly decreasing function of μ, since

$$\frac{\partial}{\partial\mu}\left(\frac{\hat{V}^M(\mu)}{\mu}\right) = \alpha\frac{\mu F\left(\hat{p}^M(\mu)\right)\pi^M - \hat{V}^M(\mu)}{\mu^2} < 0,$$

where the inequality follows by equation (3.1), and thus claim (a) holds.

When $W^D(1) \leq c \leq W^M(0)$ full-disclosure yields larger profits, because $\hat{V}^M$ is strictly convex. So, assume $c < W^D(1)$: full-disclosure gives $\mu\hat{V}^D(1) + (1-\mu)\hat{V}^M(0)$. If $\mu \leq \rho^D$, no-disclosure earns $\hat{V}^M(\mu)$. As in the proof of Theorem 1, conditions (11) imply that the linear approximation of $\hat{V}^M$ at $\mu = 0$ is always strictly larger than the value of full-disclosure. Recalling the convexity of $\hat{V}^M$ yields the result. Finally, for $\mu > \rho^D$, full-disclosure yields strictly larger profits if the linear approximation to $\hat{V}^D$ at $\mu = 1$ is always strictly smaller than the value of full-disclosure. Notice the two are equal at $\mu = 1$ by construction, so it is enough (by concavity of $\hat{V}^D$) to show that $\frac{\partial \hat{V}^D}{\partial \mu} > \hat{V}^D(1) - \hat{V}^M(0)$ at $\mu = 1$. From Corollary 2, we have

$$\frac{\partial \hat{V}^D(1)}{\partial \mu} = 2\pi^M \left[\overline{F}(\pi^M) - \pi^M f(\pi^M)\right] > \pi^M \left[2\overline{F}(\pi^M) - 1\right]$$
$$\iff 1 > 2\overline{F}\left(\pi^M\right) g(\pi^M).$$

By the first inequality in (11) we have $2\overline{F}\left(\pi^M\right) - F\left(p^M\right) \geq 2\overline{F}\left(\pi^M\right) g(\pi^M)$, and the LHS is smaller than 1 by the second inequality in (11), so also (b) holds. ■

Theorem 2 *Suppose that $c < W^D(1)$. Then, if $\mu \leq \rho^D(c, \alpha)$, social welfare, consumer surplus and profits for the sellers increase under the optimal policy. On the other hand, if $\mu > \rho^D(c, \alpha)$, then profits for sellers increase; however, both consumer surplus and aggregate welfare decrease.*

Proof. Suppose first that $\mu \leq \rho^D$: under the optimal policy, if $\omega = 1$ message Y is always sent, which induces belief ρ^D; if $\omega = 0$, message Y is sent with probability q_l^D, and message N with complementary probability, so that after N belief 0 obtains . The variation in social welfare is then

$$\begin{aligned}
\Delta SW = \mu \Bigg[&\int_{\hat{p}^M(\rho^D)}^{\infty} v dF + F\left(\hat{p}^M(\rho^D)\right) \int_{p^M}^{\infty} v dF + c - \int_{\hat{p}^M(\mu)}^{\infty} v dF - F\left(\hat{p}^M(\mu)\right) \int_{p^M}^{\infty} v dF - c \Bigg] \\
&+ (1-\mu) \left[q_l^D \left(\int_{\hat{p}^M(\rho^D)}^{\infty} v dF + c \right) + \left(1 - q_l^D\right) \left(\int_{\hat{p}^M(0)}^{\infty} v dF + c \right) - \int_{\hat{p}^M(\mu)}^{\infty} v dF - c \right] \\
&= \tau_l^D SW^M(\rho^D) + (1 - \tau_l^D) SW^M(0) - SW^M(\mu),
\end{aligned}$$

recalling Definition 2 and that $\tau_l^M = \frac{\mu}{\rho^D}$ from the proof of Theorem 1. Since we know that $\mu = \tau_l^D \rho^D + (1 - \tau_l^D) \times 0$, whether social welfare increases or not depends on whether the function $SW^M(\mu)$ is convex or concave. Moreover, it is easy to see that in this case $CS^M(\mu) = SW^M(\mu) - V^M(\mu) - c$, where $V^M = \hat{V}^M + W^M$, and therefore

$$\begin{aligned}
&\Delta CS = \tau_l^D CS^M(\rho^D) + \left(1 - \tau_l^D\right) CS^M(0) - CS^M(\mu) \\
&= \tau_l^D \left[SW^M(\rho^D) - V^M(\rho^D) - c\right] + (1 - \tau_l^D) \left[SW^M(0) - V^M(0) - c\right] - SW^M(\mu) + V^M(\mu) + c.
\end{aligned}$$

Since V^M is convex, if $CS^M(\mu)$ is convex also $SW^M(\mu)$ is: under the uniform assumption one obtains

$$CS(\mu) = \frac{1}{2}\left(\frac{1}{2} + \frac{\mu}{8}\right)^2 + \frac{\mu}{8}\left(\frac{\mu}{8} - \frac{1}{2}\right),$$

which is strictly convex. Hence both social welfare and consumer surplus increase by adopting the optimal policy. Finally, since also W^M is convex, the profit of the seller that decides to join increases; since the expected profit of the other seller remains unchanged, information disclosure also increases aggregate sellers' profits in this case.

Consider now the case $\mu > \rho^D$. We begin with sellers' profits: after message Y both sellers join, while after N, which induces ρ^D, only one enters; accordingly the variation in aggregate profits is

$$\begin{aligned}\Delta \sum_{i=1}^{2} \Pi_{0,i}^S =& \mu \Bigg[q_u^D (1-\alpha) 2\pi^M \overline{F}(\pi^M) + \left(1-q_u^D\right)(1-\alpha)\left(\hat{p}^M(\rho^D)\overline{F}\left(\hat{p}^M(\rho^D)\right) + F\left(\hat{p}^M(\rho^D)\right)\pi^M + c\right) \\ & -(1-\alpha)\overline{F}\left(\mu\pi^M\right)\pi^M(1+\mu)\Bigg] + (1-\mu)\Bigg[(1-\alpha)\hat{p}^M(\rho^D)\overline{F}\left(\hat{p}^M(\rho^D)\right) + c - (1-\alpha)\mu\pi^M\overline{F}(\mu\pi^M)\Bigg] \\ =& (1-\alpha)\Big[\tau_u^D 2\pi^M\overline{F}(\pi^M) + (1-\tau_u^D)\left(\hat{p}^M(\rho^D)\overline{F}\left(\hat{p}^M(\rho^D)\right) + F\left(\hat{p}^M(\rho^D)\right)\pi^M\right) \\ & - 2\mu\pi^M\overline{F}(\mu\pi^M)\Big] + c(1-\tau_u^D) \\ =& \frac{1-\alpha}{\alpha}\left[\tau_u^D\hat{V}^D(1) + (1-\tau_u^D)\hat{V}^M(\rho^D) - \hat{V}^D(\mu)\right] + c(1-\tau_u^D) \\ =& \frac{1-\alpha}{\alpha}\left[V^*(\mu) - \hat{V}^D(\mu)\right] + c(1-\tau_u^D) \geq 0,\end{aligned}$$

where $\tau_u^D = \frac{\mu-\rho^D}{1-\rho^D}$ and the last inequality follows by definition of V^*. The inequality is strict for all $\mu \neq \rho^D, 1$. Let us now consider consumer surplus, whose variation is

$$\Delta CS = \tau_u^D CS^D(1) - (1-\tau_u^D)CS^M(\rho^D) - CS^D(\mu).$$

We cannot immediately deduce the sign of the change in consumer surplus, because with the entry of a second seller the curvature changes. Using the uniformity hypothesis we have that $\rho^D = 2\left(1-\sqrt{1-\dfrac{4c}{1-\alpha}}\right)$, $W^D(1) = \dfrac{3}{16}(1-\alpha)$ so that

$$\Delta CS = \frac{\tau_u^D}{2} + (1-\tau_u^D)\left[\frac{1}{2}\left(\frac{1}{2}+\frac{\rho^D}{8}\right)^2 + \frac{\rho^D}{8}\left(\frac{\rho^D}{8}-\frac{1}{2}\right)\right] - \frac{\mu^2}{8} + \frac{\mu}{8} - \frac{1}{2} < 0,$$

for all $c < \dfrac{3}{16}(1-\alpha)$, $\alpha \in [0,1)$ and $\mu > \rho^D$. Finally, the variation in social welfare is

$$\begin{aligned}\Delta SW =& \tau_u^D SW^D(1) - (1-\tau_u^D) SW^M(\rho^D) - SW^D(\mu)\\ =& \tau_u^D \left(\frac{7}{8}\right) + (1-\tau_u^D)\left(\frac{5}{128}(\rho^D)^2 + \frac{\rho^D}{8} + \frac{3}{8} + c\right) - \frac{3}{8}\mu - \frac{1}{2}\\ =& (1-\mu)\left[\frac{27}{128} + \frac{c}{1-\rho^D} - \frac{5\rho^D}{128} - \frac{43}{128(1-\rho^D)}\right].\end{aligned}$$

Further algebra shows that $\Delta SW(\mu) < 0$ always. ■

Theorem 3 *When $c > W^M(0)$, both social welfare and consumer surplus increase under the optimal policy. In addition, when $\mu \geq \rho^M(c,\alpha)$, the profits for sellers increase, as well (otherwise, they remain unchanged).*

Proof. Notice first that the claim for the case $\mu \geq \rho^M$ follows the same argument based on convexity of $CS^M(\cdot)$ that we employed for the proof of Theorem 2.

Assume then that $\mu < \rho^M$: when $\omega = 1$ the platform sends message Y and belief ρ^M is induced, whereby one seller joins the market; when $\omega = 0$, with probability q_l^M message Y is sent and N otherwise, so that after N the induced belief is 0 and no seller enters. Under this the variation in social welfare can be written as

$$\begin{aligned}\Delta SW =& \mu\left[\int_{\hat{p}^M(\rho^M)}^{\infty} v dF + F\left(\hat{p}^M(\rho^M)\right)\int_{p^M}^{\infty} v dF + c - 2c\right]\\ &+ (1-\mu)\left[q_l^M\left(\int_{\hat{p}^M(\rho^M)}^{\infty} v dF + c - 2c\right) + (1-q_l^M)(2c-2c)\right]\\ =& \tau_l^M\left[\int_{\hat{p}^M(\rho^M)}^{\infty} v dF + \rho^M F\left(\hat{p}^M(\rho^M)\right)\int_{p^M}^{\infty} v dF - c\right]\\ =& \tau_l^M\left[\underbrace{\int_{\hat{p}^M(\rho^M)}^{\infty}\left(v - \hat{p}^M(\rho^M)\right) dF + \rho^M F\left(\hat{p}^M(\rho^M)\right)\int_{p^M}^{\infty}(v-p^M) dF}_{\text{consumer surplus}} + \hat{V}^M(\rho^M)\right] \geq 0,\end{aligned}$$

where $\tau_l^M = \frac{\mu}{\rho^M}$ from the proof of Theorem 1. Therefore, social welfare always increases.

Let us now consider aggregate sellers' profits: the variation is

$$\begin{aligned}\Delta\sum_{i=1}^{2}\Pi_{0,i} =&\mu\left[(1-\alpha)\hat{p}^M\left(\rho^M\right)\overline{F}\left(\hat{p}^M(\rho^M)\right)+(1-\alpha)F\left(\hat{p}^M(\rho^M)\right)\pi^M+c-2c\right]\\ &+(1-\mu)\left[q_l^M\left((1-\alpha)\hat{p}^M\left(\rho^M\right)\overline{F}\left(\hat{p}^M(\rho^M)\right)+c-2c\right)+\left(1-q_l^M\right)(2c-2c)\right]\\ =&\tau_l^M\left[W^M\left(\rho^M\right)-c\right]=0,\end{aligned}$$

where the last equality follows by definition of ρ^M. So aggregate sellers' profits remain the same for all prior beliefs. Finally, notice that without disclosure no seller would join, and therefore consumer surplus would be zero; under the optimal policy one seller enters and, since $\hat{p}^M(\rho^M)$ is smaller than the upper bound of the support of F, there is a non null probability that a sale will realize in the first period, so that consumer surplus under optimal disclosure is positive. Hence, the optimal policy always increases consumer surplus. We then conclude that the optimal policy increases all our metrics of welfare; notice that this conclusion holds even without the uniformity assumption. ■

EC 2 Noisy Signals

The setting we consider assumes that the platform is able to directly observe *in advance* the realized state of the world, and then condition the messages it sends to sellers on this observation. Our interpretation of this assumption is that the platform can generate a "perfectly" informative signal about the state of the demand, i.e., one that is it always correct, and then condition its messaging policy on it. In this appendix, we extend our model to allow for "noisy" signals and explore also how the accuracy of the platform's signal affects the policy it chooses to implement.

EC 2.1 Model

We consider a model akin to that of Section 2, but more general as far as the platform's signal is concerned: as before, there are two sellers who evaluate joining the platform to sell the single unit of a homogeneous good they are endowed with, and an uncertain number of buyers that have random valuations for the good. All assumptions set forth in Sections 2 and 3 are taken to hold also in the current setting, and the sequence of play after the platform's message is the same as well.

Let there be a set of states of the world $\Omega = \{0, 1\}$, with $\omega = 1$ denoting that the demand is high, i.e., that a second customer will look for the good on the platform in $t = 2$. The commonly shared prior probability on Ω is that $\mathbb{P}(\omega = 1) = \mu$. The platform cannot directly observe the realized ω; rather, it observes a partially accurate signal $\varphi \in \mathscr{F}$ about the state of the world and conditions its messaging policy on it. Formally, define the random variable φ taking values in $\{0, 1\}$ and such that

$$\varphi \mid \omega = \begin{cases} \omega & \text{with probability } a \\ 1 - \omega & \text{with probability } 1 - a \end{cases}$$

In words, conditional on the realization of the state of the world, φ is equal to the state of the world only with probability a (i.e., the signal is "accurate" only with some probability, which is taken to be an exogenous parameter and reflects the platform's ability to forecast its demand). Throughout this section we assume that $a \geq \frac{1}{2}$, with $a = \frac{1}{2}$ denoting a situation in which signals are not informative at all about the true state of the world, and $a = 1$ describing perfect ones; thus, this enriched model encompasses the one described in Section 2. Importantly, the accuracy level is common knowledge among the players of the game. The main difference we introduce is in the definition of *information disclosure* policies available to the platform, which can now only be conditioned on the signal. Formally, we denote an information disclosure policy as the mapping between the space of φ and the space of distributions over messages,

$$\mathcal{D} : \varphi \mapsto q(m \mid \varphi) \in \Delta(M).$$

The platform commits to a disclosure policy before observing the realization of the signal, and sellers are assumed to know that the messages are sent conditional on the signal and not the true demand.

Equilibrium The payoff functions of the agents participating in the game and the sequence of play are the same as in Section 2, and therefore we can apply the same equilibrium given in Definition 1, with one minor modification. In fact, for any history h_t with $t \geq 0$, the platform's belief is the posterior belief determined by Bayes' rule after observing the realization of the signal, while for sellers it is the posterior obtained by Bayes' rule after observing the message sent by the platform, but not the signal. In other words, φ is the platform's private information.

Similarly to before, the message sent by the platform to sellers induces the sellers to hold a new belief about the state of the demand, based on which they decide whether to join the platform. Hence, all results from Section 3 apply also in this context, and in particular those relating to the shape of the function $\hat{V}(\rho)$ that gives the expected profit of the platform at belief ρ. Therefore, we only need to characterize the optimal disclosure policy of the platform to be able to compare this setting with the one analyzed before.

EC 2.2 Optimal Information Disclosure

Suppose the platform adopts some information disclosure policy with message space M. Since the sellers know that each message $m \in M$ is sent to them according to the signal obtained by the platform, which is inaccurate in general, when they form their posterior belief they take into account this additional layer of uncertainty. As a result, not all beliefs in $[0, 1]$ can be induced, as the following lemma shows.

Lemma 1. *Take any information disclosure policy $(\mathcal{D}, M)$ and suppose the prior belief of demand being high is μ. Then the maximal posterior belief that can be induced is*

$$\mu_{\max}(\mu, a) = \frac{a\mu}{1 - \mu + a\,(2\mu - 1)}, \tag{EC.7}$$

and the minimal is

$$\mu_{\min}(\mu, a) = \frac{\mu(1 - a)}{a\,(1 - 2\mu) + \mu}. \tag{EC.8}$$

When $a = \frac{1}{2}$, i.e., the signal is not informative about the state of the world, Lemma 1 clarifies that the platform can never induce a posterior belief different from the prior, because the sellers know that any message based on uninformative signals cannot provide more information about the state of the world than what they already have; symmetrically, when $a = 1$ and foresight is perfect, $\mu_{\max} = 1$ and $\mu_{\min} = 0$, so that any belief can be reached as a posterior; finally, for information that is partially accurate, $\mu_{\min} > 0$ and $\mu_{\max} < 1$. Thus, Lemma 1 restricts the range of distributions over posteriors that the platform can induce, and their support: in particular, for any given prior μ the support of any distribution τ over posteriors must be a subset of the interval $[\mu_{\min}, \mu_{\max}]$, that depends on the value of the prior.

The restriction on the range of posterior beliefs brought by imperfect signals makes the analysis of the optimal disclosure policy more challenging, because the standard theory of information design does not allow for the possibility of imperfect observation.[4] Nevertheless, we can prove a characterization of the optimal disclosure policy similar to that of Kamenica and Gentzkow (2011), but specific to our setting.

Theorem 1 (Optimal policy characterization)**.** *Let there be given a prior μ and accuracy level a. The platform's optimal profit $V^*(\mu)$ can always be achieved with $|M| = 2$. Moreover,*

$$V^*(\mu) = \sup\left\{z : (\mu, z) \in \boldsymbol{cov}_{ltd}\left(\hat{V};\mu\right)\right\},$$

where $\boldsymbol{cov}_{ltd}\left(\hat{V};\mu\right) = \boldsymbol{cov}\left\{(\rho, v) \in \Gamma\left(\hat{V}\right) : \rho \in [\mu_{\min}(\mu, a),\, \mu_{\max}(\mu, a)]\right\}$; $\boldsymbol{cov}$ denotes the convex hull of a set and $\Gamma(\cdot)$ is the graph of a function.

More intuitively, Theorem 1 first makes clear that a binary message space is sufficient to achieve the optimal value, even in this more complicated setting; secondly, it characterizes the optimal profit of the platform in terms of what we call *sliding* concavification of $\hat{V}$. Essentially, the sliding concavification of $\hat{V}$ assigns to μ the value that one would obtain by computing the concavification of $\hat{V}$ at μ, with domain restricted to $[\mu_{\min}(\mu, a),\, \mu_{\max}(\mu, a)]$. Further applying this machinery, we obtain the following corollary that details the optimal information disclosure policy.

Corollary 1 (Platform's optimal policy)**.** *There exists an optimal policy $(\mathcal{D}^*, M^*)$ with $M^* = \{Y, N\}$. Assume that $a > \frac{1}{2}$ and let $q(m \mid \varphi)$ denote the probability that message m is sent after signal φ (we suppress the dependence on μ for readability). Then an optimal policy for the platform takes the following form:*

[4] There is little work that has analyzed the case of a privately informed sender, e.g. Hedlund (2017), whose framework does not apply to our setting.

(i) When $c > W^M(0)$, then

$$(q(Y \mid 1), q(Y \mid 0)) = \begin{cases} (1,0) & \text{for } \mu \text{ s.t. } \mu_{\max} \leq \rho^M(c,\alpha) \\ \left(q^1_{\mu_{\min},\rho^M}, q^0_{\mu_{\min},\rho^M}\right) & \text{for } \mu < \rho^M(c,\alpha) \text{ s.t. } \mu_{\min} < \rho^M < \mu_{\max} \\ (1,1) & \text{for } \mu = \rho^M(c,\alpha) \\ \left(q^1_{\rho^M,\mu_{\max}}, q^0_{\rho^M,\mu_{\max}}\right) & \text{for } \mu > \rho^M(c,\alpha) \text{ s.t. } \mu_{\min} < \rho^M < \mu_{\max} \\ (1,0) & \text{for } \mu \text{ s.t. } \mu_{\min} \geq \rho^M(c,\alpha) \end{cases}$$

(ii) When $W^D(1) \leq c \leq W^M(0)$, then

$$(q(Y \mid 1), q(Y \mid 0)) = \left(q^1_{\mu_{\min},\mu_{\max}}, q^0_{\mu_{\min},\mu_{\max}}\right)$$

(iii) Finally, when $c < W^D(1)$

$$(q(Y \mid 1), q(Y \mid 0)) = \begin{cases} (1,0) & \text{for } \mu \text{ s.t. } \mu_{\max} \leq \rho^D(c,\alpha) \\ \left(q^1_{\mu_{\min},\rho^D}, q^0_{\mu_{\min},\rho^D}\right) & \text{for } \mu < \rho^D(c,\alpha) \text{ s.t. } \mu_{\min} < \rho^D < \mu_{\max} \\ (1,1) & \text{for } \mu = \rho^D(c,\alpha) \\ \left(q^1_{\rho^D,\mu_{\max}}, q^0_{\rho^M,\mu_{\max}}\right) & \text{for } \mu > \rho^D(c,\alpha) \text{ s.t. } \mu_{\min} \leq \rho^D < \mu_{\max} \\ (1,1) & \text{for } \mu \text{ s.t. } \mu_{\min} > \rho^D(c,\alpha) \end{cases}$$

In the above, $\left(q^1_{x,y}, q^0_{x,y}\right)$ for $x < \mu < y$ are the unique solutions to the system

$$\begin{cases} \dfrac{\left[q^1_{x,y} a + q^0_{x,y}(1-a)\right]\mu}{\left[q^1_{x,y} a + q^0_{x,y}(1-a)\right]\mu + \left[q^1_{x,y}(1-a) + q^0_{x,y} a\right](1-\mu)} = y \\ \\ \dfrac{\left[\left(1-q^1_{x,y}\right)a + \left(1-q^0_{x,y}\right)(1-a)\right]\mu}{\left[\left(1-q^1_{x,y}\right)a + \left(1-q^0_{x,y}\right)(1-a)\right]\mu + \left[\left(1-q^1_{x,y}\right)(1-a) + \left(1-q^0_{x,y}\right)a\right](1-\mu)} = x \end{cases}$$

It is worthwhile to compare the policy of Corollary 1 with that of Theorem 1 in Section 4. With direct observability of the state of demand the platform is able to induce the competitive scenario it prefers, by discouraging or enticing entry. In the current setting this is no longer possible, because not all posterior beliefs can be obtained; as a result, taking as given the accuracy of the signals, there only exists a restricted range of prior beliefs for which the platform can alter the level competition compared to what would result without information disclosure. In greater detail, this occurs when the outside option is large ($c > W^M(0)$) and the prior belief is such that

$\mu_{\min} < \mu < \rho^M < \mu_{\max}$; and when the cost of entry is low ($c < W^D(1)$) and the prior belief is such that $\mu_{\min} < \rho^D < \mu < \mu_{\max}$. Intuitively, these are the cases where the range of feasible posterior beliefs includes the entry threshold ρ^M or ρ^D. However, in all these cases the platform's optimal information disclosure induces the type of competition that results from optimal disclosure in the case of perfectly informative signals: one of the sellers is nudged to join when none of them would not, and is discouraged from entry when both would join. Conveniently, in the limit where $a \to 1$, we recover exactly the policy from Theorem 1. Finally, notice that when the value of the outside option is low and the prior is such that $\mu_{\min} > \rho^D$, the platform's optimal policy is to send an uninformative message, which is never optimal under perfect foresight of the state of demand.

Platform's value for more accurate signals The discussion above stressed that the level of accuracy of the platform's signals is an important driver in its ability to induce a wider range of posterior beliefs, and thereby achieve higher profits. While it is clear that producing uninformative signals ($a = \frac{1}{2}$) is equivalent to not engaging in information disclosure at all, and that perfect foresight ($a = 1$) makes it possible to achieve the first-best of Section 4, it is also interesting to assess the marginal return from improving accuracy. In fact, this is the relevant metric for platforms to ascertain the return on investments in analytic capabilities able to generate the signals. Figure 1 depicts by how much would the platform's profits increase if it had a-precise signals instead of uninformative ones (or, equivalently, if it received no signal at all) , and we consider three values of accuracy ranging from $a = 0.6$ to $a = 0.9$. We concentrate on the case of low outside option because of its greater relevance for applications. We make a number of observations from the figure, which we can reconcile with the theory from Section 4.

First, for those prior beliefs for which information disclosure does not modify the number of entrants, we see quite a small gain from increasing the precision of the signal: this again points to the relevance of information disclosure first, and foremost, as a tool to modify the degree of competition on the markets. This intuition is further reinforced if one considers those prior beliefs for which information disclosure actually prevents entry of one of the sellers, at which the increase in profits that occurs can be as high as 50%.

Second, we observe that, contrary to Figure 4, the increase in profits presents discontinuities in the beliefs, and that these discontinuities occur at different beliefs for different accuracy levels. The jumps happen at those beliefs for which $\mu_{\min} = \rho^D$: intuitively, for all those priors for which the minimum attainable posterior is larger than the threshold ρ^D, the platform can no longer prevent entry using information disclosure, and therefore needs to settle for its second-best (which is not to disclose anything). This provides a new insight: increasing the precision of signals is particularly profitable for platforms that would otherwise be unable to persuade some of the sellers not to join.

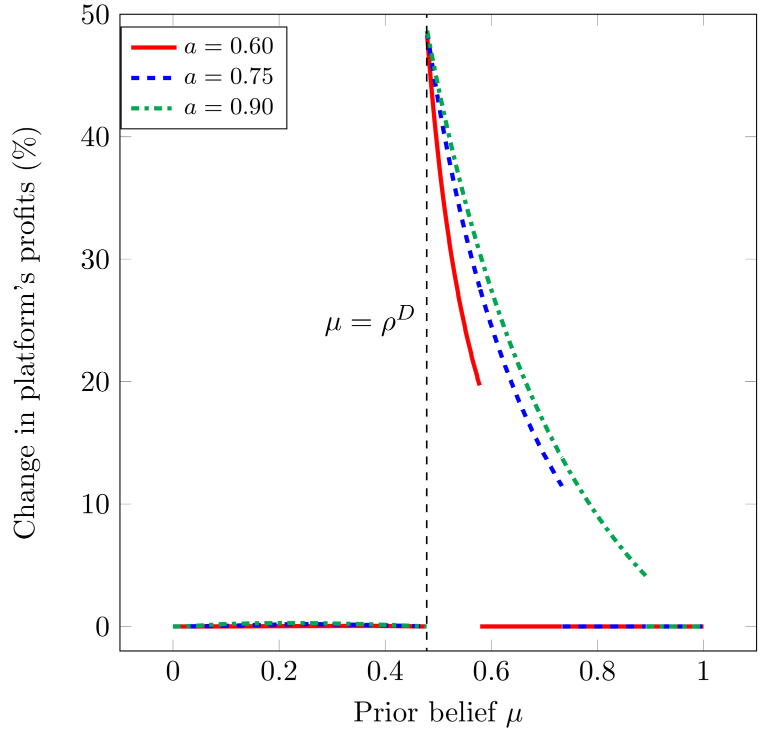


Figure 1: Percentage *increase* in platform's optimal profits from having a-precise signals instead of completely uninformative ones ($a = .5$). Results obtained with $v \sim U[0, 1]$, $\alpha = 5\%$ and $c = 0.1$.

Finally, if we consider those prior beliefs for which increasing the precision of signals does not fundamentally alter the disclosure policy, we note that there are decreasing marginal returns from accuracy. This last intuition is made formal in the following lemma (for which we assume that the buyers' willingness to pay is uniformly distributed, for the sake of simplicity).

Lemma 2. *Suppose $c < W^D(1)$ and fix: a prior μ, and accuracy level $\overline{a}$ such that there exists $\epsilon > 0$ for which $\mu_{\min}(\mu, a) < \rho^D < \mu < \mu_{\max}$ holds for every $a \in (\overline{a} - \epsilon, \overline{a} + \epsilon)$. Let $V^*(a; \mu)$ denote the optimal platform's profit at prior μ when the accuracy level is a. Then*

$$\frac{\partial^2}{\partial a^2} V^* (\overline{a}; \mu) < 0,$$

i.e., the optimal profit is (locally) concave in the accuracy of the signals.

EC 2.3 Proofs EC 2

Lemma 1

Proof. Take a disclosure policy $(\mathcal{D}, M)$ with some arbitrary message space M, and without loss of generality assume $|M| \geq 2$; if not, it is clear that the only posterior consistent with Bayes' rule is the prior μ. Denote by $\mathscr{R}(\mathcal{D}, M)$ the range of posterior beliefs that can be achieved with this policy.

Take $m \in M$ and write $q(m \mid \varphi)$ for the probability that message m is sent conditional on the realized signal being φ; also without loss of generality, assume $q(m \mid \varphi) > 0$ for at least one realization of the signal, since otherwise the message has probability zero of being sent and the posterior belief cannot be determined. The sellers' posterior belief that $\{\omega = 1\}$ is then[5]

$$\begin{aligned}&\mathbb{P}\left(\omega = 1 \mid m\right)\\ &\quad = \frac{\left[q(m \mid \varphi = 1)a + q(m \mid \varphi = 0)(1-a)\right]\mu}{\left[q(m \mid \varphi = 1)a + q(m \mid \varphi = 0)(1-a)\right]\mu + \left[q(m \mid \varphi = 1)(1-a) + q(m \mid \varphi = 0)a\right](1-\mu)}.\end{aligned}$$

The posterior probability as a function of $\left(q(m \mid \varphi)\right)_{\varphi=0,1}$ is both quasi-convex and quasi-concave, because sub-(super-)level sets are half-spaces. In particular, it can be verified that it is maximized at $q(m \mid \varphi = 0) = 0$ and $q(m \mid \varphi = 1) \in (0, 1]$, and it achieves a value of

$$\frac{a\mu}{1 - \mu + a\left(2\mu - 1\right)}.$$

Similarly, the minimum is

$$\frac{\mu(1-a)}{a\left(1 - 2\mu\right) + \mu},$$

and is achieved at $q(m \mid \varphi = 0) \in (0, 1]$ and $q(m \mid \varphi = 1) = 0$. Finally, since $\mathbb{P}(\omega = 1 \mid m)$ is a continuous function, every posterior in $[\mu_{\min}, \mu_{\max}]$ can be achieved. This shows that $\mathscr{R}(\mathcal{D}, M) \subseteq [\mu_{\min}, \mu_{\max}]$ for any disclosure policy.

Take now a policy with $|M| = 2$, $M = \{m_1, m_2\}$. One immediately verifies that $\mathbb{P}(\omega = 1 \mid m_2)$ is minimized for $q(m_1 \mid \varphi = 1) = 1$ and $q(m_1 \mid \varphi = 0) \in [0, 1)$, and that the minimum is exactly $\mu_{\min}$. Likewise, $\mathbb{P}(\omega = 1 \mid m_1)$ attains its maximum of $\mu_{\max}$ at $q(m_1 \mid \varphi = 1) \in (0, 1]$ and $q(m_1 \mid \varphi = 0) = 0$. It follows that at $q(m_1 \mid \varphi = 1) = 1$ and $q(m_1 \mid \varphi = 0) = 0$ the posterior after m_1 is maximized and that after m_2 is minimized. Hence, for any policy with $|M| = 2$, $\mathscr{R}(\mathcal{D}, M) = [\mu_{\min}, \mu_{\max}]$, which also proves that for any policy with arbitrary message space the range of feasible beliefs must equal the interval $[\mu_{\min}, \mu_{\max}]$. ■

Theorem 1

Proof. Let $\mu \mid m$ be shorthand for $\mathbb{P}(\omega = 1 \mid m)$. The platform's optimization problem can be

[5] Implicitly assuming that $\mu \neq 0, 1$. These prior beliefs cannot be modified by any form of persuasion.

stated as:

$$\max_{M,\,(q(m|\varphi=1),\,q(m|\varphi=0))_{m\in M}} \mu\left[a\sum_{m\in M} q(m\mid\varphi=1)\hat{V}(\mu\mid m)+(1-a)\sum_{m\in M} q(m\mid\varphi=0)\hat{V}(\mu\mid m)\right]$$
$$+(1-\mu)\left[a\sum_{m\in M} q(m\mid\varphi=0)\hat{V}(\mu\mid m)+(1-a)\sum_{m\in M} q(m\mid\varphi=1)\hat{V}(\mu\mid m)\right]$$

$$\text{s.t.}\ \sum_{m\in M} q(m\mid\varphi=i)=1\quad\forall i=0,1$$
$$q(m\mid\varphi=i)\geq 0\quad\forall m\in M,\,\forall i=0,1$$

Since the objective is upper semi-continuous, there exists an optimal value $V^*(\mu)$ which is attained in the feasible region. Then there exist M^* and $(q^*(m\mid\varphi=i))_{i\in\{0,1\},m\in M}$ such that

$$V^*(\mu)=\mu\left[a\sum_{m\in M} q^*(m\mid\varphi=1)\hat{V}(\mu\mid m)+(1-a)\sum_{m\in M} q^*(m\mid\varphi=0)\hat{V}(\mu\mid m)\right]$$
$$+(1-\mu)\left[a\sum_{m\in M} q^*(m\mid\varphi=0)\hat{V}(\mu\mid m)+(1-a)\sum_{m\in M} q^*(m\mid\varphi=1)\hat{V}(\mu\mid m)\right].$$

Denote by τ^* the distribution over posteriors induced by this policy. This distribution is Bayes plausible and such that $V^*(\mu)=\mathbb{E}_{\rho\sim\tau^*}\left[\hat{V}(\rho)\right]$. Moreover, by Lemma 1, $V^*(\mu)\in \texttt{cov}\hat{V}\left([\mu_{\min},\mu_{\max}]\right)$, the convex hull of the image of $[\mu_{\min},\mu_{\max}]$ through $\hat{V}$. Hence, $(\mu,V^*(\mu))\in \texttt{cov}\left(hyp\left(\hat{V}\right)|_{[\mu_{\min},\mu_{\max}]}\right)$, the convex hull of the hypograph of $\hat{V}$ restricted to the interval of feasible posteriors. This is a connected subset of $\mathbb{R}^2$, and therefore by the Fenchel-Bunt theorem there exists $\overline{\tau}$ that satisfies the following: (i) $(\mu,V^*(\mu))=\mathbb{E}_{\rho\sim\overline{\tau}}\left[(\rho,z(\rho))\right]$, with $(\rho,z(\rho))\in\texttt{cov}\left(hyp\left(\hat{V}\right)|_{[\mu_{\min},\mu_{\max}]}\right)$ so that $\overline{\tau}$ is Bayes plausible too; (ii) $supp(\overline{\tau})\in[\mu_{\min},\mu_{\max}]$ and $|supp(\overline{\tau})|\leq 2$. We now state a Lemma from the appendix of a working version of Kamenica and Gentzkow (2011), specialized to our setting, whose proof we also report for ease of reference.[6]

Lemma 3 (Kamenica and Gentzkow (2009))**.** *Given μ and $S\subset hyp(\hat{V}|_{[\mu_{\min},\mu_{\max}]})$, if $(\mu,V^*(\mu))$ is in the convex hull of S, it is also in the convex hull of the intersection of S and graph of $\hat{V}$ restricted to the feasible set of posteriors.*

Proof. We restrict to the case where $S=\{(\rho_1,z_1),(\rho_2,z_2)\}$ and suppose $(\mu,V^*(\mu))=\gamma(\rho_1,z_1)+(1-\gamma)(\rho_2,z_2)$ for some $\gamma\in[0,1]$. Towards a contradiction, assume $z_1<\hat{V}(\rho_1)$. Then we have

[6] This paper is available at https://www.wallis.rochester.edu/assets/pdf/wallisseminarseries/bayesianPersuasion.pdf

$V^*(\mu) = \gamma z_1 + (1-\gamma)z_2 < \gamma \hat{V}(\rho_1) + (1-\gamma)z_2$. But then $V^*(\mu)$ cannot be the optimal value because, as we will prove shortly, any Bayes plausible distribution with binary support in $[\mu_{\min}, \mu_{\max}]$ can be obtained by a disclosure policy. This is a contradiction and therefore $z_i = \hat{V}(\rho_i)$ for $i = 1, 2$. ■

It then follows that $V^*(\mu) = \mathbb{E}_{\rho\sim\overline{\tau}}\left[\hat{V}(\rho)\right]$, which proves that a distribution over posteriors with at most binary support is sufficient to achieve the optimum.

We now show that any Bayes plausible distribution τ with $supp(\tau) \in [\mu_{\min}, \mu_{\max}]$ and $|supp(\tau)| \leq 2$ can be obtained from some disclosure policy with $|M| = 2$. First, notice that in the case $supp(\tau) = \{\mu\}$ the claim is trivially true. Suppose then $supp(\tau) = \{\rho_1, \rho_2\}$, and without loss of generality $\rho_1 < \mu < \rho_2$ (since otherwise τ cannot be Bayes plausible). Let $M = \{m_1, m_2\}$. The linear system of equations

$$\begin{cases} q(m_1 \mid \varphi = 1)a + q(m_1 \mid \varphi = 0)(1-a) \\ \quad = \dfrac{\rho_1}{\mu}\left\{\left[q(m_1 \mid \varphi = 1)a + q(m_1 \mid \varphi = 0)(1-a)\right]\mu + \left[q(m_1 \mid \varphi = 0)a + q(m_1 \mid \varphi = 1)(1-a)\right](1-\mu)\right\} \\ \\ q(m_2 \mid \varphi = 1)a + q(m_2 \mid \varphi = 0)(1-a) \\ \quad = \dfrac{\rho_1}{\mu}\left\{\left[q(m_2 \mid \varphi = 1)a + q(m_2 \mid \varphi = 0)(1-a)\right]\mu + \left[q(m_2 \mid \varphi = 0)a + q(m_2 \mid \varphi = 1)(1-a)\right](1-\mu)\right\} \\ \\ q(m_1 \mid \varphi = i) + q(m_2 \mid \varphi = i) = 1 \quad \forall i = 0, 1 \\ q(m \mid \varphi = i) \geq 0 \quad \forall m \in M, \forall i = 0, 1 \end{cases}$$

in the unknowns $q(m \mid \varphi = i)$ always has a solution, and therefore defines a Bayes plausible distribution with support $\{\rho_1, \rho_2\}$. However, there exists only one such distribution, so it must be that τ equals the distribution over posterior induced by the solutions to the system. In conclusion then, every Bayes plausible distribution over posteriors with binary support included in $[\mu_{\min}, \mu_{\max}]$ can be obtained by some disclosure policy with binary message space.

Given the equivalence between Bayes plausible distributions and disclosure policy we proved before, we can restate the platform's optimization problem as

$$\begin{aligned} &\max_{\tau\in\Delta([0,1])} \mathbb{E}_{\rho\sim\tau}\left[\hat{V}(\rho)\right] \\ &\text{s.t.} \quad \mathbb{E}_\tau[\rho] = \mu \\ &\qquad supp(\tau) = \{\mu_1, \mu_2\} \subset [\mu_{\min}, \mu_{\max}] \end{aligned}$$

Define now the convex hull of the graph of $\hat{V}$ restricted to the domain $[\mu_{\min}, \mu_{\max}]$ at μ as

$$\texttt{cov}_{\texttt{ltd}}\left(\hat{V};\mu\right) = \texttt{cov}\left\{(\rho, v) \in \Gamma\left(\hat{V}\right) :\ \rho \in [\mu_{\min}(\mu, a),\ \mu_{\max}(\mu, a)]\right\}$$

and notice that from all the previous parts of the proofs it follows that

$$V^*(\mu) = \sup\left\{z :\ (\mu, z) \in \texttt{cov}_{\texttt{ltd}}\left(\hat{V};\mu\right)\right\}$$

which finally proves the last claim and concludes the proof. ■

Corollary 1

Proof. The proof of the statement follows the same logic of the proof of Theorem 1 in Section 4, and entails verifying that the proposed policy gives the same value as the sliding concavification. As before, assume $c \leq W^M(1)$, since otherwise the platform's profit is always zero.

Suppose first that $c > W^M(0)$. When the prior is such that $\mu_{\max} < \rho^M$, any policy is optimal because irrespective of persuasion no seller join. If instead $\mu_{\max} = \rho^M$, it is optimal that the largest of the posteriors induced be ρ^M, since otherwise there would be no entry; let μ_2 be the smallest posterior belief induced: the value of such policy is

$$\frac{\mu - \mu_2}{\mu_{\max} - \mu_2}\hat{V}(\rho^M),$$

which is decreasing in μ_2, and therefore it is optimal to induce $\mu_{\min}$ as smallest posterior beliefs. One then verifies that the policy proposed induces exactly these two posteriors. When $\mu < \rho^M$ but $\mu_{\min} < \rho^M < \mu_{\max}$, notice that for any posteriors μ_1 and μ_2 it must be $\mu_2 < \mu$ and $\mu_1 > \rho^M > \mu$ (since otherwise the profits would be zero). Thus the value of this policy is

$$\frac{\mu - \mu_2}{\mu_1 - \mu_2}\hat{V}(\mu_1),$$

which is decreasing in μ_2, so that it is optimal to induce $\mu_{\min}$; moreover, this value is decreasing in μ_1 as long as $\mu_1 \geq \rho^M$, because

$$(\mu_{\min} - \mu_1)\frac{\partial \hat{V}(\mu_1)}{\partial \mu_1} + \hat{V}(\mu_1) < 0,$$

and therefore it is optimal to set $\mu_1 = \rho^M$. When $\mu = \rho^M$, it cannot be optimal to set the lowest posterior strictly below ρ^M, because otherwise (by the same reasoning as before) the optimum would be to set the largest posterior equal to ρ^M, which does not satisfy Bayes plausibility; hence, it is optimal to leave the prior unchanged, which is achieved by a policy that sends the same message with probability one irrespective of the signal realized. Finally, for $\mu > \rho^M$ the platform maximizes its profits by inducing the beliefs μ_1 and μ_2 such that $|\mu_1 - \mu_2|$ is maximal and there is entry at both: this follows from convexity of $\hat{V}$. Notice that the same reasoning carries out for $W^D(1) \leq c \leq W^M(0)$.

Suppose now that $c < W^D(1)$. As long as the prior is such that $\mu_{\max} \leq \rho^D$, convexity of $\hat{V}$ gives that the policy is the same as for $W^D(1) \leq c \leq W^M(0)$. For $\mu < \rho^D$ but $\mu_{\min} < \rho^D < \mu_{\max}$, it cannot be optimal to induce a posterior larger than ρ^D: this follows from Assumption 2 as in the proof of Theorem 1, where we ruled out full disclosure policies (which necessarily dominate any policy that induces posterior belief larger than ρ^D in this case). By convexity then, it is optimal to induce beliefs $\mu_{\min}$ and ρ^D. Always owing to Assumption 2, for $\mu = \rho^D$ it is optimal to leave the posterior unchanged. When $\mu > \rho^D$ but $\mu_{\min} \leq \rho^D$, it is clearly never optimal to induce a posterior less than ρ^D, which implies that the lower posterior belief must equal ρ^D; then Assumption 2 implies that the optimal upper posterior must be equal to $\mu_{\max}$. Finally, when $\mu_{\min} > \rho^D$, concavity of $\hat{V}$ yields that it is optimal to leave the prior belief unchanged. ■

Lemma 2

Proof. Under the hypotheses of the Lemma, the optimal disclosure policy is to induce belief $\mu_{\max}$ with probability $\frac{\mu-\rho^D}{\mu_{\max}-\rho^D}$ and ρ^D with complementary probability. Therefore,

$$V^*(\overline{a};\mu) = \frac{\mu - \rho^D}{\mu_{\max} - \rho^D}\hat{V}(\mu_{\max}) + \frac{\mu_{\max} - \mu}{\mu_{\max} - \rho^D}\hat{V}(\rho^D).$$

Recall that when the customers' willingness to pay is uniformly distributed it has

$$\rho^D = 2\left(1 - \sqrt{1 - \frac{4c}{1-\alpha}}\right)$$

and

$$\hat{V}(\rho^D) = \alpha\left(\frac{1}{2} + \frac{\rho^D}{8}\right)^2$$
$$\hat{V}(\mu_{\max}) = \alpha\frac{\mu_{\max}}{8}(4 - \mu_{\max})$$

Moreover, it is assumed that there exists $\epsilon > 0$ for which $\mu_{\min}(\mu, a) < \rho^D < \mu < \mu_{\max}$ holds for every $a \in (\overline{a} - \epsilon, \overline{a} + \epsilon)$, which implies that the function $V^*(a;\mu)$ is twice differentiable in a neighbourhood of $\overline{a}$. But then it is a matter of algebra to show that

$$\frac{\partial^2 V^*(\overline{a};\mu)}{\partial a^2} = \alpha \frac{\partial^2}{\partial a^2}\left[\frac{\mu-\rho^D}{\mu_{\max}-\rho^D}\frac{\mu_{\max}}{8}(4-\mu_{\max}) + \frac{\mu_{\max}-\mu}{\mu_{\max}-\rho^D}\left(\frac{1}{2}+\frac{\rho^D}{8}\right)^2\right] < 0$$

■

EC 3 Revenue Share

In the description of our model we argued that the platform may find it impossible to tailor the revenue share it retains to different market conditions because of practical constraints; this justified taking α exogenous. In this appendix we explore the outcomes that would obtain if the platform could optimize also this quantity. In fact, a potential threat to our main results is, that leaving the platform the ability to choose α could lead to a "balancing" between the negative effects of optimal information disclosure and the positive effect of a low enough revenue share.

Suppose that the same model of Section 2 holds,[7] with the only difference that the platform chooses both the share of revenue it wants to retain and the information disclosure policy; sellers have the same action space as before. For the sake of tractability, we also assume that the buyers' willingness to pay is uniformly distributed on the unit interval. Formally, since for fixed α and $(\mathcal{D}, M)$ the game is the same as before and the sellers' optimal strategies do not change, we can write the platform's problem as

$$\max_{\alpha\in[0,1],\,(\mathcal{D},M)} \mathbb{E}_{\rho\sim\tau}\left[\hat{V}(\rho)\right],$$

where $\hat{V}(\cdot)$ is the function giving the expected profit of the platform when the induced belief is ρ; the expectation is taken with respect to the distribution τ over beliefs induced by the mechanism $(\mathcal{D}, M)$.[8] Notice that the optimal information disclosure policy derived in Section 4 is parametrized by $\alpha \in [0, 1]$, and so for each α we can identify the optimal $(\mathcal{D}, M)$; thus, we can write

$$\max_{\alpha\in[0,1],\,(\mathcal{D},M)} \mathbb{E}\left[\hat{V}(\rho)\right] = \max_{\alpha\in[0,1]} \max_{(\mathcal{D},M)} \mathbb{E}\left[\hat{V}(\rho)\right] = \max_{\alpha\in[0,1]} V^*(\mu),$$

where V^* is the concavification of $\hat{V}$, i.e. the optimal expected value of the policy to the platform given c and α. Without loss of generality we will assume $c \leq W^M(1)$; if it were larger, no one would ever join and the problem would be vacuous.

Notice that the platform faces a basic trade-off in this optimization problem. Since α concurs to determine how many sellers will join the market in equilibrium, as the opportunity cost decreases the platform should be able to retain an always larger revenue share, for fixed number of entrants. At the same time, we established that it is always more profitable to have a single seller on the market rather than two. The choice of the revenue share must then trade-off the incentive to appropriate a larger share of each transaction and the effects on competition and volumes of a larger α. The following proposition shows how this tension is resolved.

[7] We also revert to assuming that signals about demand are perfectly informative.

[8] A more formal definition can also be found in the proof of Theorem 1 in Appendix EC 1.

Proposition 1. *The optimal revenue share α^* takes value in the set*

$$\left\{1-\frac{64}{25}c,\, 1-\frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2},\, 1-4c,\, \overline{\alpha}\right\},$$

where $\overline{\alpha}$ solves

$$4\sqrt{\frac{c}{1-\alpha}}(1-\mu)(2-\alpha)=5(\mu+4)(1-\alpha).$$

The proof provides a more extensive description of the (rather involved) cases where each of these values are optimal. Notice, however, that the case where $\alpha^*=\overline{\alpha}$ is a "residual" one, in the sense that if c and μ were randomly and independently chosen from their respective ranges, the probability that this case would obtain is close to 0. Therefore, in what follows we will concentrate on the other, more relevant, instances. We highlight two main facts that follow from Proposition 1: (i) if α^* is chosen, there always occurs at most one entry; (ii) the optimal revenue share is decreasing in the opportunity cost. Hence, our previous intuition about the relation between α^* and the opportunity cost is confirmed.

The exact value of α^* depends both on the value of the opportunity cost and on the prior belief, and therefore requires the same precise knowledge of the market conditions of the optimal information disclosure policy of Section 4. However, optimally choosing the revenue share makes the disclosure policy straightforward to implement, because it degenerates to either full-disclosure or no-disclosure; Table 1 details this. The intuition is that, once it is established that it is optimal for the platform to have just one seller, the optimal α is the one that selects the best threshold belief $\rho^M(c,\alpha)$. This is because in the optimal disclosure with just one entrant, ρ^M is one of the induced beliefs. When the opportunity cost is large, it is better for the platform to retain a share that sets $\rho^M=1$, which lets one seller join only if two customers come, rather than trying to lower ρ^M so much that there would always be entry: the intuition is that to have the latter the revenue share would need to become too small. As the value of the outside option decreases, α is chosen (first) for ρ^M to exactly match the prior, and then to have $\rho^M=0$. In both cases one seller always joins, and the information disclosure policy moves from no-disclosure to full-disclosure. The simplicity of the ensuing information disclosure policy is a salient feature of this model, and points towards a direction we have hinted at in Section 2: adjusting the revenue share can be (partially) substituted with optimal information disclosure. When α is taken as given, the disclosure policy (i.e. the probability with which the true state is revealed) is made contingent on the market primitives; however, if the revenue share is allowed to be adjusted, then its optimal value subsumes also the effects of strategic information disclosure, so that the latter becomes straightforward.

The consequences on welfare of giving the platform an additional profit lever depend on which setting is taken as a benchmark. In fact, we briefly note that when α^* is chosen, the hypotheses of

Optimal α	**Induced policy**	**Entry**
$1-\dfrac{64}{25}c$	Full disclosure	$\rho^M=1 \Rightarrow$ One seller iff $\omega=1$
$1-\dfrac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$	Uninformative	$\rho^M=\mu \Rightarrow$ One seller always in
$1-4c$	Full disclosure	$\rho^M=0 \Rightarrow$ One seller always in

Table 1: Optimal information disclosure when α^* is chosen.

Theorem 3 are satisfied; in turn, these imply that letting the platform apply the optimal information disclosure weakly increases consumer surplus, compared to the case where nothing is disclosed, but the same α^* is employed.[9] However, a more interesting benchmark is the setting in which the the platform sets neither the revenue share nor information disclosure, i.e. takes some α as given and reveals nothing. In this case, letting the platform use the additional lever makes buyers considerably worse off.

Theorem 2. *Let c, α and μ be given such that, with no-disclosure, at least one seller would join the platform. If $c>\frac{1}{36}$, or $c\leq\frac{1}{36}$ and $\mu\notin\left(\dfrac{1-36c}{14c+1},\overline{\mu}\right)$, then consumer surplus decreases when the platform optimally chooses both the revenue share and the disclosure policy.*

The belief $\overline{\mu}$ is defined in the proof of Proposition 1, and the the interval $\left(\dfrac{1-36c}{14c+1},\overline{\mu}\right)$ is the range of values of the prior for which, if the opportunity cost is very small, the optimal revenue share is $\overline{\alpha}$. Since no analytic results can be obtained for this case, one should interpret Theorem 2 as sufficient condition; indeed, numerical examples suggest that the claim also holds for most of the cases where $\alpha^*=\overline{\alpha}$.[10] The intuition behind the stark result is that when the platform is choosing α optimally, it makes sure that just one seller joins the market (and so makes buyers worse off compared to all cases where two would enter) and that she has all the information needed to set the highest price possible. As an illustrative example, consider what happens when $c>\dfrac{1}{4}$: in this case $\alpha^*=1-\frac{64}{25}c$ and the policy is of full disclosure, with a seller joining only when two customers are expected; under this, the first customer is either faced with the highest possible monopoly price or does not find a seller at all.

Overall, this section improved our understanding of the platform's market making abilities. Firstly, we can conclude that relaxing the assumption that α is exogenous does not alter, but strengthens, our insights from the previous sections: the platform can potentially severely harm the

[9] Notice that when α^* induces a no-disclosure policy, there is actually no change in consumer surplus.

[10] In any case, as remarked before, the range of values for which the theorem does not apply is very small.

buyers, and letting it choose the optimal revenue share only improves its ability to extract surplus to increase its profits. The fundamental mechanism through which the platform achieves this is its ability to modify the sellers' incentives to join the market. By selecting the most profitable competitive structure, the optimal revenue share basically incorporates all the gains that would come from optimal information disclosure, which is the reason why the optimal policy is so simple under α^*. In turn, this highlights that optimally choosing the revenue share and the information disclosure are, in a sense, substitutes. This reinforces our intuition that, provided α cannot be adapted to ever changing market conditions, information disclosure can be used in spite, thus constituting a flexible tool for managing market thickness.

EC 3.1 Proofs EC 3

Proposition 1

Proof. Before proceeding, let us specify the functional form many quantities of interest take in the uniform case. It has:

$$\rho^M(c,\alpha) = 4\left(2\sqrt{\frac{c}{1-\alpha}} - 1\right) \qquad \rho^D(c,\alpha) = 2\left(1 - \sqrt{1 - \frac{4c}{1-\alpha}}\right).$$

One can also check that $W^M(1) = (1-\alpha)\frac{25}{64}$, $W^M(0) = (1-\alpha)\frac{1}{4}$ and $W^D(1) = (1-\alpha)\frac{3}{16}$. Since α cannot be larger than 1, without loss of generality $c \le \frac{25}{64}$. Thus, we have the following cases for V^*: when $\alpha > 1 - \frac{64}{25}c$, $V^*(\mu) = 0$ for all beliefs and we call it (for short) case (a); when $\alpha \in \left(1-4c, 1-\frac{64}{25}c\right] \cap [0,1]$, then

$$V^*(\mu) = \begin{cases} \dfrac{\alpha c}{1-\alpha}\dfrac{\mu}{\rho^M} & \text{if } \mu < \rho^M \\ \dfrac{\mu-\rho^M}{1-\rho^M}\left(\dfrac{25}{64}\alpha - \dfrac{\alpha c}{1-\alpha}\right) + \dfrac{\alpha c}{1-\alpha} & \text{if } \mu \ge \rho^M \end{cases}$$

and we denote it case (b); when $\alpha \in \left(1-\frac{16}{3}c, 1-4c\right] \cap [0,1]$, it obtains

$$V^*(\mu) = \frac{\alpha}{4} + \frac{9}{64}\alpha\mu,$$

termed case (c); finally, when $\alpha \in \left[0, 1-\frac{16}{3}c\right] \cap [0,1]$, we have

$$V^*(\mu) = \begin{cases} \dfrac{\alpha}{4} + \alpha\left[\left(\dfrac{1}{2} + \dfrac{\rho^D}{8}\right)^2 - \dfrac{1}{4}\right]\dfrac{\mu}{\rho^D} & \text{if } \mu \le \rho^D \\ \alpha\left(\dfrac{1}{2} + \dfrac{\rho^D}{8}\right)^2 + \alpha\left[\dfrac{3}{8} - \left(\dfrac{1}{2} + \dfrac{\rho^D}{8}\right)^2\right]\dfrac{\mu-\rho^D}{1-\rho^D} & \text{if } \mu > \rho^D \end{cases}$$

which is denoted as case (d). When any of the intersections above is empty, V^* is not defined on that interval. Notice also that V^* as just defined is also a continuous function of α, so a maximum in $[0,1]$ exists.

Note that not all of the cases are feasible for every c. Indeed, when $\frac{1}{4} < c \leq \frac{25}{64}$, only case (b) is (partially) feasible: since $1-4c<0$, α ranges between 0 and $1-\frac{64}{25}c$; consequently, the smallest ρ^M that can be achieved is $8\sqrt{c}-4>0$. When $\frac{3}{16} < c \leq \frac{1}{4}$ case (b) is feasible, and case (c) is partially feasible. Finally, for $c \leq \frac{3}{16}$ both (b) and (c) are entirely feasible and (d) is partially feasible (it becomes entirely feasible only when $c=0$).

Let us notice that when $c = \frac{25}{64}$ then the only possible choice is $\alpha = 0$ and therefore the problem is trivial. Without loss of generality, assume then $c < \frac{25}{64}$. We first show that, if feasible, it is never optimal for the platform to set α so low that case (d) would obtain. In fact, suppose $c \leq \frac{3}{16}$ and denote by $\alpha(c)$ some $\alpha \in \left(1-\frac{16}{3}c, 1-4c\right]$ and by $\alpha(d)$ some $\alpha \in \left[0, 1-\frac{16}{3}c\right]$. For fixed c, $\alpha(d)$ determines a threshold ρ^D. The expected profit for the platform without information disclosure under case (c) is

$$\hat{V}_{(c)}(\mu) = \alpha(c)\left[\hat{p}^M(\mu)\overline{F}\left(\hat{p}^M(\mu)\right) + \mu\pi^M F\left(\hat{p}^M(\mu)\right)\right],$$

and for case (d) with $\mu \leq \rho^D$ it has

$$\hat{V}_{(d)}(\mu) = \alpha(d)\left[\hat{p}^M(\mu)\overline{F}\left(\hat{p}^M(\mu)\right) + \mu\pi^M F\left(\hat{p}^M(\mu)\right)\right].$$

Since $\alpha(c) > \alpha(d)$, $\hat{V}_{(c)}(\mu) > \hat{V}_{(c)}(\mu)$ for $\mu \leq \rho^D$. Moreover, when $\mu > \rho^D$ so that two sellers join, we know that the platform gets strictly higher payoff when just one enters; therefore:

$$\alpha(d)\left[\hat{p}^M(\mu)\overline{F}\left(\hat{p}^M(\mu)\right) + \mu\pi^M F\left(\hat{p}^M(\mu)\right)\right] > \alpha(d)2\mu\pi^M\overline{F}\left(\mu\pi^M\right) = \hat{V}_{(d)}(\mu),$$

and hence $\hat{V}_{(d)}(\mu) < \hat{V}_{(c)}(\mu)$ for $\mu > \rho^D$ as well. Recalling the definition of V^*, it follows $V^*_{(c)} \geq V^*_{(d)}$ for every $\alpha(c)$, $\alpha(d)$, $c \leq \frac{3}{16}$ and μ. Finally, with $c > \frac{3}{16}$ case (d) is not feasible and therefore we conclude that $\alpha \in \left[0, 1-\frac{16}{3}c\right]$ can never be optimal. We will now analyze each remaining "piece" of V^*, and then derive the optimal α. This is accomplished by first looking for the local maximum of each bit, and then using monotonicity to identify the global maximum.

Consider now the function

$$gb_l : \alpha \longmapsto \frac{\alpha c}{1-\alpha}\frac{\mu}{\rho^M}.$$

Since $c < \frac{25}{64}$, $\alpha \neq 1$ always. Moreover, this is the value of the revenue share α when $\mu < \rho^M$; using the definition of ρ^M one obtains

$$\mu < \rho^M \Longleftrightarrow \alpha > 1 - \frac{c}{\left(\frac{1}{2} + \frac{\mu}{8}\right)^2}.$$

For all μ in the unit interval we have

$$1 - 4c \leq 1 - \frac{c}{\left(\frac{1}{2} + \frac{\mu}{8}\right)^2},$$

and therefore $\rho^M \neq 0$ always, so that the function is always well defined. It can be checked that its first derivative is positive if and only if $c > \frac{1-\alpha}{(2-\alpha)^2}$, where the RHS reaches a maximum of $\frac{1}{4}$ at $\alpha = 0$. Hence for $c > \frac{1}{4}$ the function is strictly increasing on all its domain and its maximum is attained at $\alpha = 1 - \frac{64}{25}c$. Moreover, for $c \leq \frac{1}{4}$ the function is convex for $\alpha > 1 - 4c$, and consequently the maximum is found at either extreme of the domain. It has

$$gb_l\left(1 - \frac{c}{\left(\frac{1}{2} + \frac{\mu}{8}\right)^2}\right) \leq gb_l\left(1 - \frac{64}{25}c\right),$$

if and only if $c \leq \frac{15}{64}$, or $c > \frac{15}{64}$ and $\mu \geq 16(1 - 4c)$.

We now study the function

$$gb_u : \alpha \longmapsto \frac{\mu - \rho^M}{1 - \rho^M}\left(\frac{25}{64}\alpha + \frac{\alpha c}{1 - \alpha}\right) + \frac{\alpha c}{1 - \alpha}.$$

It can be checked that it is strictly concave for all $\mu \neq 1$. Since this is the value of α when $\mu \geq \rho^M$, its domain is the interval (provided c is small enough)

$$\left(1 - 4c, 1 - \frac{c}{\left(\frac{1}{2} + \frac{\mu}{8}\right)^2}\right].$$

The FOC for it to be maximized is

$$4\sqrt{\frac{c}{1 - \alpha}}(1 - \mu)(2 - \alpha) = 5(\mu + 4)(1 - \alpha). \tag{EC 3.1}$$

The first derivative evaluated at $\alpha = 1 - \frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$ is positive for all μ when $c > \frac{1}{36}$; when $c \leq \frac{1}{36}$ there exists $\overline{\mu}$ such that gb_u' is positive if $\mu \geq \overline{\mu}$. Such $\overline{\mu}$ is the unique root in $[0, 1]$ of the polynomial

$$\mu^3 + 7\mu^2 + (8 + 64c)\mu + 576c - 16,$$

with variable μ and parameter c. It follows that $gb'_u(1-4c) > 0$ for $c > \frac{1}{36}$, and when $c \leq \frac{1}{36}$ we have $gb'_u(1-4c) > 0$ if $\mu < \frac{1-36c}{14c+1} < \overline{\mu}$. Therefore, we have three possible cases: (i) when $gb'_u(1-4c) \leq 0$ the maximum is achieved at $\alpha = 1-4c$; (ii) if $gb'_u(1-4c) > 0$ and

$$gb'_u\left(1 - \frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}\right) < 0,$$

then the function is maximized at the solution of (EC 3.1); finally (iii), if

$$gb'_u\left(1 - \frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}\right) \geq 0,$$

the maximum is found at $\alpha = 1 - \frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$.

Finally, let us consider

$$gc : \alpha \longmapsto \alpha\left(\frac{1}{4} + \frac{9}{64}\mu\right),$$

which is maximized at $\alpha = 1-4c$, since it is linear in α.

Recall that the objective function we want to maximize is continuous in α. Putting together all previous observations, we arrive at the following cases.

1. $\frac{1}{4} < c \leq \frac{25}{64}$: only case (b) is feasible, and therefore from our discussion it must be that the platform wants $\rho^M > \mu$, with $\alpha^* = 1 - \dfrac{64}{25}c$.
2. $\frac{15}{64} < c \leq \frac{1}{4}$: only case (b) is feasible, but compared to the previous point gb_l is no longer monotone, while gb_u is increasing. Thus it has:
   (i) $\mu > 16(1-4c)$: $\alpha^* = 1 - \dfrac{64}{25}c$, because gb_L is convex and the maximum is attained at the right end of the domain.
   (ii) $\mu = 16(1-4c)$: $\alpha^* \in \left\{1 - \dfrac{64}{25}c, 1 - \dfrac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}\right\}$, because gb_l attains the same value at both ends of the domain.
   (iii) $\mu < 16(1-4c)$: $\alpha^* = 1 - \dfrac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$, because gb_l is convex and the maximum is attained at the left end of the domain.
3. $\dfrac{1}{36} < c \leq \dfrac{15}{64}$: gb_u is increasing and gb_l is always smaller than gb_u, so $\alpha^* = 1 - \dfrac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$.
4. $0 < c \leq \dfrac{1}{36}$: where the maximum is attained depends on the monotonicity of gb_u. From before, it has:

(i) $\mu \geq \overline{\mu}$: gb_u is increasing, and thus $\alpha^* = 1 - \dfrac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$.

(ii) $\dfrac{1-36c}{14c+1} < \mu < \overline{\mu}$: gb_u admits an interior maximum, attained at the α^* solving equation (EC 3.1).

(iii) $\mu \leq \dfrac{1-36c}{14c+1}$: gb_u is decreasing and therefore $\alpha^* = 1-4c$.

This concludes the proof. ■

Theorem 2

Proof. We will start by recollecting from the previous proofs the value of expected consumer welfare when no optimization takes place; denote by α an exogenously given revenue share. If no seller joins, which happens for $\alpha > 1 - \frac{64}{25}c$, or $1-4c < \alpha \leq 1 - \frac{64}{25}c$ and $\mu < \rho^M(c,\alpha)$, the consumer surplus is 0. When only one seller enters, which occurs if $1-4c < \alpha \leq 1-\frac{64}{25}c$ and $\mu \geq \rho^M(c,\alpha)$, or $1-\frac{16}{3}c < c \leq 1-4c$, or $c \leq 1-\frac{16}{3}c$ and $\mu \geq \rho^D(c,\alpha)$, the expected consumer surplus is

$$\int_{\frac{1}{2}+\frac{\mu}{8}}^{1}\left(v-\frac{1}{2}-\frac{\mu}{8}\right)dv + \mu\left(\frac{1}{2}+\frac{\mu}{8}\right)\int_{\frac{1}{2}}^{1}\left(v-\frac{1}{2}\right)dv = \frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 + \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right).$$

Finally, both sellers join only when $c \leq 1 - \dfrac{16}{3}c$ and $\mu > \rho^D(c,\alpha)$; the consumer surplus is given by

$$\int_{\frac{\mu}{4}}^{\infty}\left(v-\frac{\mu}{4}\right)dv + \mu\left(\left(1-\frac{\mu}{4}\right)\int_{\frac{1}{2}}^{\infty}\left(v-\frac{1}{2}\right)dv + \frac{\mu}{4}\int_{0}^{\infty} v dv\right) = \frac{\mu^2}{8} - \frac{\mu}{8} + \frac{1}{2}.$$

Let us first suppose that $\alpha^* = 1 - \frac{64}{25}c$, so that the induced policy is full-disclosure and in particular $\rho^M(c,\alpha^*) = 1$; this implies that one seller will enter when $\omega = 1$, and she is certain that a second customer comes, while no one joins when $\omega = 0$. Hence the expected consumer surplus induced by this policy is

$$\mu\left[\int_{\frac{5}{8}}^{1}\left(v-\frac{5}{8}\right)dv + \frac{5}{8}\int_{\frac{1}{2}}^{1}\left(v-\frac{1}{2}\right)dv\right] + (1-\mu)\times 0 = \frac{19}{128}\mu.$$

Algebra shows that

$$\frac{19}{128}\mu - \frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 - \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) \leq 0,$$

and

$$\frac{19}{128}\mu - \frac{\mu^2}{8} + \frac{\mu}{8} - \frac{1}{2} \leq 0,$$

for all $\mu \in [0,1]$.

Suppose now $\alpha^* = 1 - \frac{c}{\left(\frac{1}{2}+\frac{\mu}{8}\right)^2}$, which implies $\rho^M(c, \alpha^*) = \mu$ and therefore no-disclosure. Under this policy, one seller always enters, with a posterior belief equal to her prior. Thus consumer surplus is equal to

$$\frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 + \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) \geq 0,$$

and once again one verifies that

$$\frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 + \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 - \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) = 0,$$

and

$$\frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 + \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) - \frac{\mu^2}{8} + \frac{\mu}{8} - \frac{1}{2} \leq 0,$$

for all $\mu \in [0, 1]$.

Finally, assume $\alpha^* = 1 - 4c$, so that $\rho^M(c, \alpha^*) = 0$ and the policy is again of full-disclosure. One seller always joins, which implies that the consumer surplus is

$$\mu\left[\int_{\frac{5}{8}}^{1}\left(v-\frac{5}{8}\right)dv + \frac{5}{8}\int_{\frac{1}{2}}^{1}\left(v-\frac{1}{2}\right)dv\right] + (1-\mu)\int_{\frac{1}{2}}^{1}\left(v-\frac{1}{2}\right)dv = \frac{3}{128}\mu + \frac{1}{8}.$$

Further algebra gives that

$$\frac{3}{128}\mu + \frac{1}{8} - \frac{1}{2}\left(\frac{1}{2}+\frac{\mu}{8}\right)^2 - \frac{\mu}{8}\left(\frac{\mu}{8}-\frac{1}{2}\right) \leq 0,$$

and

$$\frac{3}{128}\mu + \frac{1}{8} - \frac{\mu^2}{8} + \frac{\mu}{8} - \frac{1}{2} \leq 0,$$

for all $\mu \in [0, 1]$, which proves the claim. ■

EC 4 Market with Many Sellers

The model employed until Section 5 is stylized, but thanks to its simplicity we could neatly identify the essential aspects of our problem. We now seek to understand whether the insights developed also hold in a richer setting. To this end, we develop a model that allows for an arbitrary number of sellers and buyers to join the platform: our analysis will show that the previous intuitions carry over. However, this comes at the cost of a more contrived model, which will require some additional assumptions to be made tractable. We now describe its details, and postpone the discussion of the differences with the basic model to the end of the section.

EC 4.1 Model and equilibrium

The game we consider is dynamic, with three time periods, $t \in \{-1, 0, 1\}$: in period $t = -1$ the platform commits to the information disclosure policy; in $t = 0$, it observes the realization of the state of demand and sends a message to a countably infinite pool $\mathcal{S}$ of potentially differentiated sellers, who, in turn, make an entry decision. Finally, in $t = 1$, those of them who decided to join compete on prices. If sellers do not join the platform, they have an outside option valued at c. Analogously to the basic model, the platform generates its profits by retaining a fixed share of the value of the sales made by each participating seller. As in Section 2, we denote by h_t a generic history of the game in period t, with the convention that $h_{-1} = \varnothing$.

Buyers As before, we abstract away the behaviour of buyers, and take as primitive the demand function that arises from their (implicit) optimal decision. We use a demand system employed in Bimpikis, Crapis, and Tahbaz-Salehi (2019), which is in turn micro-founded in Myatt and Wallace (2015); this results from a continuum of buyer . When a subset $S \subset \mathcal{S}$ of sellers decide to join the platform, and each posts price $(p_j)_{j \in S}$, seller i faces a demand Q_i for her product given by

$$Q_i = \max\{0, \theta + (\phi - 1)P - \phi p_i\}, \tag{EC 4.1}$$

where $\theta > 0$, $P = \dfrac{1}{|S|}\sum_{j \in S} p_j$ is the average price posted by all participating sellers and $\phi > 1$. This demand system allows us to generally and economically model a market where many sellers sell (possibly) differentiated products. The parameter ϕ captures the degree of substitutability between goods supplied by different sellers: higher values of ϕ indicate more homogeneous markets. To model the idea that, when the number of participants in the market increases, the level of differentiation decreases[11], we assume that ϕ depends on the number $|S| = N$ of sellers that decide

[11] Instances of models where this phenomenon occurs are the classic Hotelling (linear city) or Salop (1979) competition games.

to join the market, so that $\phi = d(N) = N^{\varepsilon}$, $\varepsilon \geq 2$.

As discussed later, we include uncertainty about demand by assuming that θ is a random variable. A large realization of θ models an instance of "high" demand, while with a small realization we instantiate low demand in the market.

Sellers There is a countably infinite set of potential entrants, denoted by $\mathcal{S} = \{1, 2, \ldots\}$. We define the sellers' actions and payoffs backwards as in Section 2, with p_i the price posted by seller i and $E_i \in \{0, 1\}$ denoting her entry decision. Starting from $t = 1$, at any history h_1 such that subset $S \subset \mathcal{S}$ of seller decided to join the platform, for any $i \in S$ the payoff is

$$\Pi_{1,i}^{S}\left(\sigma_i^S; h_1\right) = (1-\alpha)\mathbb{E}_{\rho}\left[\mathbb{E}_{p_i \sim \sigma_i^S}\left[\max\left\{0, \theta + (\phi - 1)P - \phi p_i\right\} p_i \mid \theta\right]\right],$$

where ρ is the posterior belief at h_1. It is assumed that sellers get to observe how many of their competitors joined the platform before deciding on prices. Moreover, we suppose that each seller has an infinite inventory of her product, so that she can always entirely satisfy demand Q_i. Based on this continuation payoff, one defines also the utility of seller $i \in \mathcal{S}$ at time $t = 0$.

$$\Pi_{0,i}^{S}\left(\sigma_i^S; h_0\right) = \mathbb{E}_{E_i \sim \sigma_i^S}\left[\mathbf{1}\{E_i = 1\}\Pi_{1,i}^{S}\left(\sigma_i^S; \langle h_0, E_i = 1, E_{-i}\rangle\right) + c\mathbf{1}\{E_i = 0\}\right].$$

The entry decision is taken after having observed the message sent by the platform, which affects the expectation about θ and therefore influences $\Pi_{1,i}^{S}$.

Platform and information structure Demand is unknown at period $t = -1$, and this is modeled by assuming that θ in equation (EC 4.1) is a random variable, taking values in $\Theta = \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L$. We assume $\theta_L > \frac{\theta_H}{2}$, i.e. that the realizations of demand are sufficiently similar; this is just to simplify exposition. The commonly shared prior probability that $\{\theta = \theta_H\}$ is denoted by μ as before. Consistently with Section 2, an information disclosure policy is a pair $(\mathcal{D}, M)$ such that

$$\mathcal{D} : \Theta \to \Delta(M).$$

Since the platform only moves at period $t = -1$, its payoff is[12]

$$\Pi^{P}\left((\mathcal{D}, M); h_{-1}\right) = \frac{\alpha}{1-\alpha}\mathbb{E}_{\theta \sim \mu}\left[\mathbb{E}_{m \sim \mathcal{D}(\theta)}\left[\sum_{i \in \mathcal{S}} \mathbf{1}\{E_i = 1\}\Pi_{0,i}^{S}\left(\sigma_i^S; \langle h_{-1}, (\mathcal{D}, M), m\rangle\right) \mid \theta\right]\right].$$

[12]Similarly to before, we can interpret this as saying that the platform receives perfectly informative signals about θ.

Under the rules of the game described in the previous paragraphs, the series above converges for all $c > 0$, since only finitely many of the sellers will join in equilibrium.

There are some important differences between this model and the one presented in Section 2. Besides the number of agents taking part in the transactions, these concern the timing of the game and the representation of demand; the two issues will be dealt together. Notice that simply expanding the number of buyers and sellers in the basic model, while leaving unchanged its dynamic pricing, gives rise to a much more difficult game, where existence and payoff equivalence of equilibria cannot be assured.[13] Therefore, in order to increase the number of market participants, and still retain a tractable model, we need to compress the time dimension. This also necessarily implies the different way in which we capture variability in demand.

EC 4.1.1 Equilibrium definition

The equilibrium concept we consider is completely analogous to that of the basic model: Sender-preferred perfect Bayesian equilibrium in pure strategies. Hence, the formal definition is very similar to that in Section 2.1.

Definition 1. *A collection of strategy-belief pairs $(\sigma_k, \gamma_k)_{k\in\mathcal{S}\cup\{P\}}$ is a Sender-preferred PBE if the following conditions are satisfied:*

(1) For every seller $i \in \mathcal{S}$, every time period $t \in \{0, 1\}$, and every history h_t, we have

$$\Pi^S_{t,i}\left(\sigma^S_i; h_t\right) \geq \Pi^S_{t,i}\left(\sigma'^S_i; h^t\right),$$

where σ'^S_i denotes a feasible strategy for seller i. Moreover, $\sigma^S_i(h_1) = \sigma^S_j(h_1)$ for all i, j such that $E_i = E_j = 1$ at $t = 0$

(2) For every agent k, $\gamma_k \mid \varnothing = \mu$, i.e., both sellers and the platform share a common prior μ. Moreover,

(i) For $i \in S$, $\gamma_i \mid h$ is determined by Bayes' rule after history h.

(ii) For the platform, $\gamma_P \mid h = \delta_{\{\theta\}}$ for all $h \neq \varnothing$, where θ is the realization of the state of the world.

(3) For fixed $(\mathcal{D}, M)$, whenever there exist multiple assessments such that all of the previous conditions hold, then $(\sigma_i, \gamma_i)_{i\in\mathcal{S}}$ yields the highest payoff for the platform.

[13] E.g. Martínez-De-Albéniz and Talluri (2011) conclude that with two sellers endowed with finite inventory, and facing multiple customers, there exists no equilibrium whenever buyers have random willingness to pay.

(4) Finally, $(\mathcal{D}, M)$ is the information disclosure policy that maximizes the platform's profits

$$\Pi^P\left((\mathcal{D}, M); h_{-1}\right) \geq \Pi^P\left((\mathcal{D}, M)'; h_{-1}\right),$$

assuming that sellers follow the strategies prescribed by the equilibrium.

The only substantial difference with the equilibrium in Section 2.1 is that here we impose that all entrants post the same price. This is required for simplicity reasons, and it allows to concentrate on the effects that information disclosure has on competition, since it eliminates possible sources of heterogeneity. Furthermore, it facilitates the comparison with the outcomes of the basic model.

EC 4.2 Equilibrium analysis

The identification of equilibrium decisions proceeds backwards, and therefore we begin with equilibrium play at period $t = 1$.

Proposition 2 (Pricing game)**.** *At every history h_1 when a subset S of sellers decide to join, and the belief is ρ, the only symmetric equilibrium in pure strategies gives*

$$p^* = \frac{\mathbb{E}_\rho[\theta] N}{N[d(N) + 1] + 1 - d(N)}.$$

Each seller expects to earn

$$W_i(\rho, N, \alpha) = (1 - \alpha)\left(\mathbb{E}_\rho[\theta]\right)^2 \frac{N\left[d(N)(N-1) + 1\right]}{\left[d(N)(N-1) + N + 1\right]^2},$$

and the platform's expected profits are

$$\hat{V}(\rho, N, \alpha) = \alpha\left(\mathbb{E}_\rho[\theta]\right)^2 \frac{N^2\left[d(N)(N-1) + 1\right]}{\left[d(N)(N-1) + N + 1\right]^2},$$

where $|S| = N$.

Further manipulations show the following facts: (i) holding ρ and α fixed, seller i's profits are strictly decreasing in N, and so are equilibrium prices; instead, (ii) $W_i(\rho, N, \alpha)$ increases in the belief for fixed N. We also note that $\hat{V}$ is convex in the belief ρ.

As an ideal counterpart of Proposition 3, the following result describes equilibrium entry decisions in this game.

Proposition 3 (Entry equilibrium)**.** *Given outside option value c and platform's commission α,* *define* $N_{max}(c,\alpha)$ *and* $N_{min}(c,\alpha)$ *as*

$$N_{max}(c,\alpha) = \max\{N: \ W_i(1,N,\alpha) \geq c\},$$
$$N_{min}(c,\alpha) = \max\{N: \ W_i(0,N,\alpha) \geq c\}.$$

For each $k \in \{N_{min}(c,\alpha)+1,\ldots,N_{max}(c,\alpha)\}$ *define the threshold beliefs* ρ_k *such that*

$$W_i(\rho_k,k,\alpha) = c.$$

Then at any history h_0 *such that the posterior belief is* ρ*:*

(i) if $\rho \leq \rho_{N_{min}+1}$*,* $N_{min}(c,\alpha)$ *sellers join;*

(ii) if $\rho_k < \rho \leq \rho_{k+1}$*,* k *sellers join (with* $k \in \{N_{min}(c,\alpha)+1,\ldots,N_{max}(c,\alpha)\}$*);*

(iii) if $\rho > \rho_{N_{max}}$*,* $N_{max}(c,\alpha)$ *sellers join.*

Proposition 3 shows that the pattern of entry at equilibrium in this model is not substantially different from that of the setting of Section 2: for a fixed value for the outside option and fees, as the belief about demand being high increases more sellers join. The main difference with Proposition 3 is that in this case there are infinitely many potential sellers, and therefore more than one of them may enter as the belief increases. The result is that the unit interval is partitioned in sub-intervals $(\rho_k,\rho_{k+1}]$, and for posterior beliefs within each interval, k sellers join the market. Finally, $N_{min}(c,\alpha)$ and $N_{max}(c,\alpha)$ describe the minimum and maximum number of entrants, respectively, that the value of the outside option and α allow in the market.

EC 4.3 Information disclosure

The information design problem of the platform exhibits now many more degrees of freedom, and the optimal information disclosure policy cannot be determined analytically for every combination of the parameters. The reason for the increased difficulty of the problem resides with the more complex nature of the entry pattern. In fact, in general it has $N_{max}(c,\alpha) - N_{min}(c,\alpha) > 1$, which implies that in principle all possible combinations of threshold beliefs ρ^k as potential posteriors to be induced should be evaluated. However, it is still possible to obtain a basic prediction of our basic model: optimal disclosure harms consumers, when taking as benchmark a policy of no-disclosure.

Theorem 3. *Suppose* $N_{min}(c,\alpha) < N_{max}(c,\alpha)$*. There exists* $\underline{N}$ *such that if* $N_{min}(c,\alpha) \geq \underline{N} \geq 2$ *and* $\hat{V}(1,N_{max}(c,\alpha)) < \hat{V}\left(\rho^{N_{max}},N_{max}(c,\alpha)-1\right)$*, then consumer surplus decreases under the optimal information disclosure policy for at least all* $\mu \geq \rho^{N_{max}}$*.*

It should be noted that the reduction in the welfare of consumers is caused by the same mechanism at work in the basic model. For $\mu \geq \rho^{N_{max}}$ the platform's optimal policy will always be one that induces beliefs 1 or $\rho^{N_{max}}$: when the latter obtains, the number of sellers in the market decreases compared to the benchmark, and this drives prices up and volumes down. Thus, we recover another base prediction of our simpler model, that the platform alters the competitive structure to increase its profits, and this damages consumers.

Theorem 3 provides a sufficient condition for buyers to be harmed by the disclosure from the platform, but it is not necessary. As such, it gives a lower bound on the instances where consumer surplus decreases. Indeed, it is not difficult to find examples where the reduction is consumer surplus is much more widespread; however, this can only be ascertained by first determining the optimal information disclosure policy, which in turn requires fixing some values for the parameters. Next we explore a numerical example where consumer surplus decreases for almost all prior beliefs.

EC 4.3.1 Numerical example

The example described here is intended to show that the implications of employing optimal information disclosure can be extended beyond the lower bound presented in Theorem 3. At the same time, the choice of parameters instantiates a fairly "standard" setting, whose main takeaways can be recovered also with other parameter values.

We fix the following values for the primitives: $\varepsilon = 2$, $\theta_H = 10$, $\theta_L = 5$, $c = 2.9$ and $\alpha = 5\%$. These imply that $N_{min} = 2$ and $N_{max} = 6$. The optimal information disclosure that ensues takes the following form.[14]

Optimal policy An optimal policy has message space $M = \{Y, N\}$. Write $\mathcal{D}(\theta)$ for the probability that message Y is sent when the state of the world is θ. Then,

$$\mathcal{D}(\theta_H) = \begin{cases} 1 & \text{for } \mu \leq \rho^{N_{min}+1} \\ q_k^H & \text{for } \rho^k < \mu \leq \rho^{k+1},\ k \in \{N_{min}+1, \ldots, N_{max}\} \\ \dfrac{\mu - \rho^{N_{max}}}{\mu\,(1 - \rho^{N_{max}})} & \text{for } \mu > \rho^{N_{max}} \end{cases}$$

[14] A formal justification of why the policy takes this form can be found at the end of the proofs section of this appendix.

and

$$\mathcal{D}(\theta_L) = \begin{cases} \dfrac{\mu\left(1-\rho^{N_{min}+1}\right)}{\rho^{N_{min}+1}(1-\mu)} & \text{for } \mu \leq \rho^{N_{min}+1} \\ q_k^L & \text{for } \rho^k < \mu \leq \rho^{k+1},\ k \in \{N_{min}+1, \ldots, N_{max}\} \\ 0 & \text{for } \mu > \rho^{N_{max}} \end{cases}$$

where q_k^L and q_k^H are the unique solutions to the systems

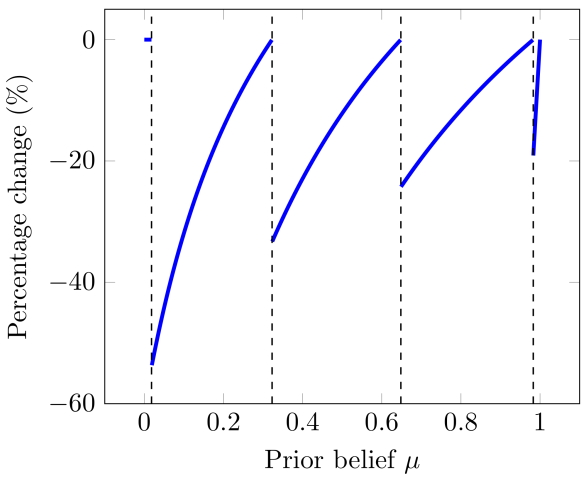
$$\begin{cases} \dfrac{q_k^H \mu}{q_k^H \mu + q_k^L(1-\mu)} = \rho^{k+1} \\ \dfrac{\left(1-q_k^H\right)\mu}{\left(1-q_k^H\right)\mu + \left(1-q_k^L\right)(1-\mu)} = \rho^k \end{cases}$$

for $k \in \{N_{min}+1, \ldots, N_{max}\}$.

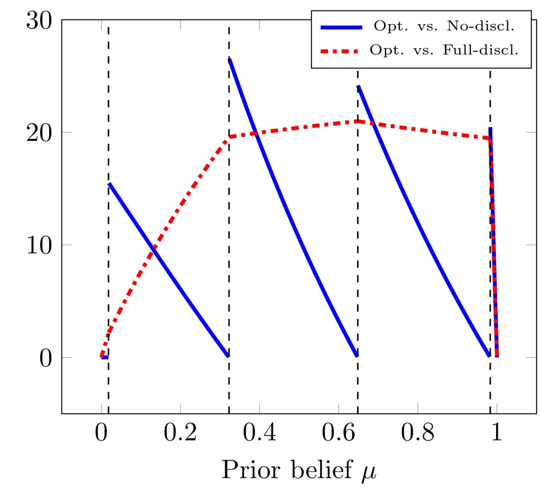
A verbal description of the policy clarifies that the optimal behaviour for the platform is to identify the threshold beliefs which are immediately larger and smaller than the prior μ, and to obfuscate information to induce either of these two as posterior beliefs. Since at the lower belief the incentives to entry are different, the outcome of the policy is to sometimes inducing one of the sellers that would have joined to stay out of the market. When the prior belief is below the threshold that would let $N_{min}+1$ sellers join, the possible posterior beliefs are 0 and $\rho^{N_{min}+1}$, from which follows that the number of participating sellers is not affected. Drawing a comparison with the optimal policy in Section 4, the case of $\mu \leq \rho^{N_{min}+1}$ closely mirrors the case where the value of the outside option is low, i.e., $c < W^D(1)$, with $\mu \leq \rho^D$. On the other hand, the outcomes of the policy when $\mu > \rho^{N_{min}+1}$ can be likened what occurs in our baseline model with low value of outside option and the prior exceeds the threshold ρ^D.

Figure 2 represents the welfare effects of employing the optimal information disclosure policy. From Figure 2a one observes that the optimal policy can lower consumer surplus by as much as 50%. At the same time, it is worth noting the similarity between Figure 2a and the left panel of Figure 5, which strengthens our previous parallelism with the baseline model. The increase in the platform's profits is large as well, as shown in Figure 2b, which also includes a comparison with the full-disclosure policy. The platform earns its additional profits at the expense of buyers: it exploits the fact that it can inflate prices by alternatively increasing the belief held by sellers or by restricting the number of entrants. The comparison with full-disclosure serves to confirm our intuition from Section 4, that fully revealing information may yield even worse outcomes than no-disclosure. Finally, we also obtain that social welfare is lower when the platform employs the optimal information disclosure policy, as given in Figure 2d.

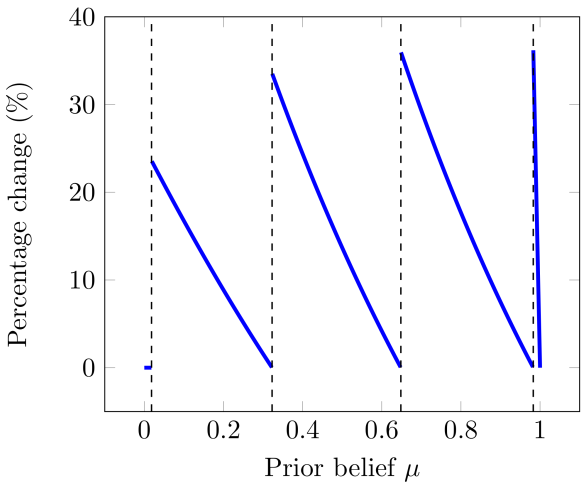
In general, the results of this numerical exercise showed that it possible to retrieve our predictions



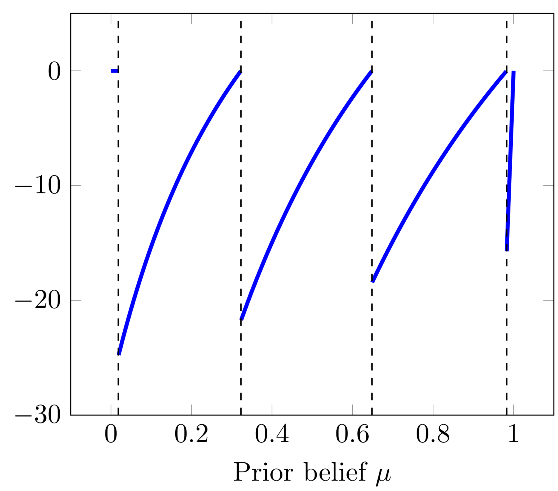
(a) Consumer surplus



(b) Platform's profits



(c) Aggregate profits for sellers



(d) Social welfare

Figure 2: Percentage change due to the platform employing the optimal disclosure policy instead of no-disclosure. Additionally, 2b illustrates the increase in profits with respect to full-disclosure policy. Dashed lines are drawn at the threshold beliefs ρ^3, ρ^4, ρ^5 and ρ^6. Benchmark profits and social welfare are derived with a total number of sellers equal to 10.

also from a more realistic model, that allows for additional customers and sellers. Results of the same type can be obtained by choosing different values for the parameters and selecting different functional forms for $\phi = d(N)$.

EC 4.4 Proofs EC 4

Proposition 2

Proof. Suppose first $N = 1$, in which case the demand faced by the single entrant reduces to $Q = \theta - p_i$, since $P = p_i$. The seller wishes to maximize expected profit and therefore the equilibrium price must solve

$$\max_p (1-\alpha)\mathbb{E}_\rho[p(\theta - p)].$$

Taking the FOC gives that $p^* = \frac{\mathbb{E}_\rho[\theta]}{2}$ and, since we are assuming $\theta_L > \frac{\theta_H}{2}$, the Seller faces positive demand even when the realized intercept is θ_L. Hence the expected profit for Seller i is $\pi_i(\rho, 1, \alpha) = \frac{(\mathbb{E}_\rho[\theta])^2}{4}$.

Suppose now $N > 1$, so that the optimization problem of seller i becomes

$$\max_{p_i} \quad (1-\alpha)\mathbb{E}_\rho \left[p_i \left(\theta + (d(N)-1)\frac{1}{N}\sum_{j=1}^{N} p_j - d(N)p_i \right) \right]. \tag{EC 4.2}$$

Taking the FOC gives

$$\mathbb{E}_\rho[\theta] - 2p_i \frac{(N-1)d(N)+1}{N} + \frac{d(N)-1}{N}\sum_{j\neq i} p_j = 0.$$

Imposing $p_j = p_i$ for every i, j and solving the equation yields

$$p_i^* = \frac{\mathbb{E}_\rho[\theta]N}{N(d(N)+1)+1-d(N)}.$$

At this point, simple substitution of p_i^* into (EC 4.2) gives the expected profit of a single seller. Finally, we can obtain the platform's profit by summing over the N expected profits of the sellers multiplied by the revenue share of the platform. ■

Proposition 3

Proof. Under our assumptions, $W_i(\rho, N, \alpha) \to 0$ for all ρ as $N \to \infty$. Also recall that W_i is increasing in ρ, which implies that the maximum possible profit for a seller obtains when $\rho = 1$ and $N = 1$, while the minimum is zero. Without loss of generality, assume $W_i(1, 1, \alpha) > c$, since otherwise there is never entry. Then $N_{max}(c, \alpha)$ and $N_{min}(c, \alpha)$ are well-defined as

$$N_{max}(c, \alpha) = \max \{N : \ W_i(1, N, \alpha) \geq c\},$$
$$N_{min}(c, \alpha) = \max \{N : \ W_i(0, N, \alpha) \geq c\}.$$

For each $k \in \{N_{min}(c, \alpha) + 1, \ldots, N_{max}(c, \alpha)\}$ define beliefs ρ_k such that

$$W_i(\rho_k, k, \alpha) = c.$$

Under our assumption, each of these equations has a unique solution: it represents the minimum belief such that k sellers would join the platform. In fact, take posterior belief ρ such that $\rho_k < \rho < \rho_{k+1}$: if $k-1$ sellers joined, then one of the sellers that stays out would have incentive to deviate and enter, because $W_i(\rho, k, \alpha) > c$; similarly, if $k+1$ joined then one of the seller in the market would deviate and stay out, because $W_i(\rho, k+1, \alpha) < c$. Hence, exactly k seller join at equilibrium. To establish entry for $\rho = \rho_k$, we evaluate $\hat{V}(\rho_k, k, \alpha)$ and $\hat{V}(\rho_k, k-1, \alpha)$. It has

$$\hat{V}(\rho_k, k-1, \alpha) \geq \hat{V}(\rho_k, k, \alpha \iff \frac{(k-1)^2 \left[d(k-1)(k-2)+1\right]}{\left[d(k)(k-2)+k\right]^2} \geq \frac{k^2 \left[d(k)(k-1)+1\right]}{\left[d(k)(k-1)+k+1\right]^2}, \tag{EC 4.3}$$

where we remind that $d(k) = k^{\varepsilon}$ for some $\varepsilon \geq 2$. Additional algebra shows that (EC 4.3) is always satisfied for every $k > 2$. Hence, we conclude that for $k = 1, 2$, exactly k sellers join at ρ_k, while for $k \geq 3$, $k-1$ sellers join at ρ_k. ■

Theorem 3

Proof. We start by noting that for any optimal policy, the only posterior beliefs that are induced can be $\rho = 0$, $\rho = 1$, or the threshold beliefs ρ^k, for some $k \in \left\{\rho^{N_{min}+1}, \ldots, \rho^{N_{max}}\right\}$ (and possibly all of them). This is because the function $\hat{V}$ is locally increasing and convex in the belief, which by the usual concavification argument implies that it is always possible to improve over a policy that induces posterior $\rho \neq 0$, 1, ρ^k. Observe now that at belief ρ^k exactly $k-1$ sellers join by the Sender-preferred condition of equilibrium. Therefore, at ρ^k the platform's profits reach a local maximum, and then jump downwards. We have

$$\hat{V}(\rho^k, k-1, \alpha) = \alpha \mathbb{E}_{\rho^k}[\theta]^2 \frac{k-1}{h(k-1)} = c\frac{\alpha}{1-\alpha}(k-1)\frac{h(k)}{h(k-1)},$$

where $h(k) = \dfrac{\left[d(k)(k-1)+k+1\right]^2}{k\left[d(k)(k-1)+1\right]}$. It can be shown that $(k-1)\dfrac{h(k)}{h(k-1)}$ is eventually increasing in the number of entrants; denote by $\underline{N}$ the number, which depends on ε only, such that for all $k \geq \underline{N}$, $(k-1)\dfrac{h(k)}{h(k-1)}$ is increasing. It follows that, under our hypotheses, there is a sequence of increasing local maxima at ρ^k, with $\hat{V}\left(\rho^{N_{max}}, N_{max}-1, \alpha\right)$ being also the global maximum of $\hat{V}$ in $[0,1]$. Hence, it must be that at an optimal disclosure policy, for prior $\mu \in \left(\rho^{N_{max}}, 1\right]$, the platform mixes between beliefs $\rho^{N_{max}}$ and 1.

Consider now expected consumer surplus: since at equilibrium all sellers quote the same price, the expected consumer surplus of the customers acquiring the product from one seller amounts to $\frac{1}{2}\mathbb{E}_\rho\left[(\theta-p^*)^2\right]$. Hence, when there are k of them on the platform, the total expected consumer surplus is

$$CS(\rho,k) = \frac{k}{2}\mathbb{E}_\rho\left[\left(\theta - \frac{\mathbb{E}_\rho[\theta]k}{d(k)(k-1)+k+1}\right)^2\right].$$

All else equal, increasing the number of entrants increases consumer welfare, both because the price paid decreases and because a larger demand can be satisfied. Furthermore, algebra proves that $CS(\rho,k)$ is concave in ρ for k fixed, and increasing. Hence, expected consumer surplus is an increasing, piecewise concave function over $[0,1]$, with discontinuities at each threshold belief ρ^k. In particular, it has

$$CS(\rho^{N_{max}}, N_{max}-1) < \lim_{\rho \downarrow \rho^{N_{max}}} CS(\rho, N_{max}) \leq CS(1, N_{max}).$$

Since at any optimal disclosure policy the platform mixes between beliefs $\rho^{N_{max}}$ and 1, under the optimal policy the expected consumer surplus is

$$\tau CS\left(1, N_{max}\right) + (1-\tau)CS\left(\rho^{N_{max}}, N_{max}-1\right),$$

with τ determined by the optimal policy. Concavity of consumer surplus and the previous inequality then yield that under the optimal policy consumer welfare must be lower than under the no-disclosure benchmark. ■

Derivation of the optimal policy in the numerical example As in the proof of Theorem 1, we want to identify the concavification V^* of $\hat{V}$ and then deduce the optimal policy from it. The concavification of $\hat{V}$ can also be defined as the smallest concave function that is everywhere weakly larger than $\hat{V}$.

Notice that the beliefs $\{0, \rho^{N_{min}+1}, \dots, \rho^{N_{max}}, 1\}$ partition the unit interval and, within each cell of the partition, $\hat{V}$ is a convex function of the prior belief μ. Thus, consider the set B given by

$$B = \left\{\left(\rho^k, \hat{V}(\rho^k, k-1, \alpha)\right) : \ k \in \{N_{min}+1, \dots, N_{max}\}\right\} \cup \left\{\left(0, \hat{V}(0, N_{min}, \alpha)\right)\right\} \cup \left\{\left(1, \hat{V}(1, N_{max}, \alpha)\right)\right\}$$

and the piecewise affine function defined as

$$\widetilde{V}(\mu) = \begin{cases} \dfrac{\hat{V}(\rho^{N_{min}+1}, N_{min}, \alpha) - \hat{V}(0, N_{min}, \alpha)}{\rho^{N_{min}+1}}\mu + \hat{V}(0, N_{min}, \alpha) & \mu \leq \rho^{N_{min}+1} \\ \dfrac{\hat{V}(\rho^{k+1}, k, \alpha) - \hat{V}(\rho^{k}, k-1, \alpha)}{\rho^{k+1} - \rho^{k}}\mu + \hat{V}(\rho^{k}, k-1, \alpha) & \rho^{k} < \mu \leq \rho^{k+1} \\ \dfrac{\hat{V}(1, N_{max}, \alpha) - \hat{V}(\rho^{N_{max}}, N_{max}-1, \alpha)}{1 - \rho^{N_{max}}}\mu + \hat{V}(\rho^{N_{max}}, N_{max}-1, \alpha) & \mu > \rho^{N_{max}} \end{cases}$$

$k \in \{N_{min}+1, \ldots, N_{max}\}$, whose graph passes through every point in B. $\widetilde{V}$ is concave because under our choice of parameters the slope of each affine bit decreases as μ moves from 0 to 1; hence, B is a subset of the set of extreme points of the convex hull of the graph of $\hat{V}$. As a consequence, $V^*(\mu) = \hat{V}(\mu, N(\mu), \alpha)$ whenever $\left(\mu, \hat{V}(\mu, N(\mu), \alpha)\right) \in B$, by definition of extreme point.[15] Take now any other function l satisfying this requirement and such that $l(\mu) < \widetilde{V}(\mu)$ for some μ. Clearly $\mu \notin \{0, N_{min}+1, \ldots, N_{max}, 1\}$, but since $\widetilde{V}$ is affine, l must be non-concave in the cell of the partition containing μ. Thus, we cannot find another concave function that is smaller than $\widetilde{V}$ and that passes through all the points in B, which implies that $\widetilde{V} = V^*$. At this point, following arguments identical to those employed in Theorem 1 it follows that the given policy delivers a value equal to V^*.

[15] Here $N(\mu)$ denotes the number of entrants when the belief is μ, determined according to our equilibrium rule.

References

Anunrojwong, Jerry, Krishnamurthy Iyer, David Lingenbrink. 2020. Persuading risk-conscious agents: A geometric approach. *Available at SSRN 3386273* .

Aumann, Robert J, Michael Maschler. 1995. *Repeated games with incomplete information*. MIT press.

Bimpikis, Kostas, Davide Crapis, Alireza Tahbaz-Salehi. 2019. Information sale and competition. *Management Science* **65**(6) 2646–2664.

Hedlund, Jonas. 2017. Bayesian persuasion by a privately informed sender. *Journal of Economic Theory* **167** 229–268.

Kamenica, Emir, Matthew Gentzkow. 2011. Bayesian persuasion. *American Economic Review* **101**(6) 2590–2615. doi:10.1257/aer.101.6.2590.

Lariviere, Martin A. 2006. A note on probability distributions with increasing generalized failure rates. *Operations Research* **54**(3) 602–604. doi:10.1287/opre.1060.0282.

Martínez-De-Albéniz, Victor, Kalyan Talluri. 2011. Dynamic Price Competition with Fixed Capacities. *Management Science* **57**(6) 1078–1093. doi:10.1287/mnsc.lll0.1337.

Myatt, David P, Chris Wallace. 2015. Cournot competition and the social value of information. *Journal of Economic Theory* **158** 466–506.

Salop, Steven C. 1979. Monopolistic Competition with Outside Goods. *The Bell Journal of Economics* **10**(1) 141. doi:10.2307/3003323.