EC.1. Proof of Lemmas

EC.1.1. A Technical Lemma

We first state and prove a technical lemma that will be used repeatedly through the analysis, which essentially is a large deviation bound on the absolute magnitude of cumulative demand variability along arcs across consecutive time periods.

LEMMA EC.1. *For any $1 \le t_1 < t_2 \le T$, any arc $i \to j$, and any $y \ge 0$ and $r \in (0,1]$, it holds that*

$$\mathbf{P}\left(\max_{t_1 \le t \le t_2} \left|\sum_{s=t_1}^{t} \delta_{s,ij}\right| > y\right) \le 2\exp\left(\sum_{s=t_1}^{t_2} \min\{\lambda_{s,ij}, 1-\lambda_{s,ij}\} \cdot r^2 - ry\right) \tag{EC.1}$$

$$\le 2\exp((t_2 - t_1)r^2 - ry) \tag{EC.2}$$

Proof of Lemma EC.1. The inequality (EC.2) follows directly from inequality (EC.1) since $\lambda_{s,ij} \in [0,1]$. Next we prove inequality (EC.1). For any $r \in (0,1]$, the following holds

$$\begin{aligned}\mathbf{P}\left(\max_{t_1 \le t \le t_2} \left|\textstyle\sum_{s=t_1}^{t} \delta_{s,ij}\right| > y\right) &= \mathbf{P}\left(\max_{t_1 \le t \le t_2} \exp(r\left|\textstyle\sum_{s=t_1}^{t} \delta_{s,ij}\right|) > e^{ry}\right)\\ &\le \mathbf{P}\left(\max_{t_1 \le t \le t_2} \exp(r\textstyle\sum_{s=t_1}^{t} \delta_{s,ij}) > e^{ry}\right) + \mathbf{P}\left(\max_{t_1 \le t \le t_2} \exp(-r\textstyle\sum_{s=t_1}^{t} \delta_{s,ij}) > e^{ry}\right)\\ &\le \mathbf{E}\left[\exp(r\textstyle\sum_{s=t_1}^{t_2} \delta_{s,ij} - ry) + \exp(-r\textstyle\sum_{s=t_1}^{t_2} \delta_{s,ij} - ry)\right] \end{aligned}\tag{EC.3}$$

where the first inequality follows by the union bound and the second inequality follows by Doob's martingale inequality. Note that, by elementary algebra, we have

$$\mathbf{E}\exp(r\delta_{s,ij}) = (\lambda_{s,ij}e^r + 1 - \lambda_{s,ij})\exp(-r\lambda_{s,ij}) \le \exp((e^r - 1 - r)\lambda_{s,ij}) \le \exp(r^2\lambda_{s,ij}) \tag{EC.4}$$

where the equality follows since $\lambda_{s,ij} + \delta_{s,ij}$ is a binary random variable that equals 1 with probability $\lambda_{s,ij}$, the first inequality follows since $e^x \ge 1 + x$, the second inequality follows since $e^r - 1 - r \le r^2$ for all $r \in [-1,1]$. On the other hand, we obtain an alternative upper bound in the following way:

$$\begin{aligned}\mathbf{E}\exp(r\delta_{s,ij}) &= (\lambda_{s,ij}e^r + 1 - \lambda_{s,ij})\exp(-r\lambda_{s,ij}) = (1 - (1-\lambda_{s,ij})(1-e^{-r}))\exp(r(1-\lambda_{s,ij}))\\ &\le \exp((e^{-r} - 1 + r)(1-\lambda_{s,ij})) \le \exp(r^2(1-\lambda_{s,ij}))\end{aligned}\tag{EC.5}$$

where the second equality follows from elementary algebra, the first inequality holds since $1 - x \le e^{-x}$. We can apply similar techniques to obtain two more bounds as follows:

$$\mathbf{E}\exp(-r\delta_{s,ij}) = (\lambda_{s,ij}e^{-r} + 1 - \lambda_{s,ij})\exp(r\lambda_{s,ij}) \le \exp(e^{-r} - 1 + r)\lambda_{s,ij}) \le \exp(r^2\lambda_{s,ij}), \tag{EC.6}$$

$$\begin{aligned}\mathbf{E}\exp(-r\delta_{s,ij}) &= (\lambda_{s,ij}e^{-r} + 1 - \lambda_{s,ij})\exp(r\lambda_{s,ij}) = (1 + (1-\lambda_{s,ij})(e^r - 1))\exp(-r(1-\lambda_{s,ij}))\\ &\le \exp((e^{-r} - 1 - r)(1-\lambda_{s,ij})) \le \exp(r^2(1-\lambda_{s,ij}))\end{aligned}\tag{EC.7}$$

Inequality (EC.1) then follows by combining inequalities (EC.3) - (EC.7). □

EC.1.2. Proof of Lemma 1

Take any policy $\pi \in \Pi$ that is feasible to **SDP**. Define $\lambda^*_{t,ij} := \mathbf{E}^\pi[\lambda_{t,ij}(p^\pi_{t,ij})]$. Note that $\{\lambda^*_{t,ij}\}_{t,ij}$ is feasible to $\mathbf{DCP}(0)$. Indeed, (3) implies that under π, $\lambda_{t,ij}(p^\pi_{t,ij}) \in [0,1]$; thus, $\lambda^*_{t,ij} = \mathbf{E}^\pi[\lambda_{t,ij}(p^\pi_{t,ij})] \in [0,1]$. Moreover, by (2), the following holds for all i, t,

$$C_i - \sum_{j=1}^N \sum_{s=1}^t \lambda^*_{s,ij} + \sum_{j=1}^N \sum_{s=1}^{(t-\tau_{ji})^+} \lambda^*_{s,ji} = \mathbf{E}^\pi \left[C_i - \sum_{j=1}^N \sum_{s=1}^t D_{s,ij}(p^\pi_{s,ij}) + \sum_{j=1}^N \sum_{s=1}^{(t-\tau_{ji})^+} D_{s,ji}(p^\pi_{s,ji}) \right] \geq 0,$$

We have just shown that $\{\lambda^*_{t,ij}\}_{t,ij}$ is feasible to $\mathbf{DCP}(0)$. Finally, note that

$$\begin{aligned} \mathbf{E}^\pi\left[p^\pi_{t,ij} D_{t,ij}(p^\pi_{t,ij})\right] &= \mathbf{E}^\pi\left\{\mathbf{E}^\pi\left[p^\pi_{t,ij} D_{t,ij}(p^\pi_{t,ij}) \middle| \mathcal{F}_t\right]\right\} \\ &= \mathbf{E}^\pi\left[p^\pi_{t,ij}\lambda_{t,ij}(p^\pi_{t,ij})\right] = \mathbf{E}^\pi\left[r_{t,ij}(\lambda_{t,ij}(p^\pi_{t,ij}))\right] \\ &\leq r_{t,ij}\left(\mathbf{E}^\pi\left[\lambda_{t,ij}(p^\pi_{t,ij})\right]\right) = r_{t,ij}(\lambda^*_{t,ij}), \end{aligned}$$

where the inequality follows by the Jensen's inequality and A2. We then conclude that $\mathcal{J}(0) \geq \mathcal{J}^\star$ since $\mathcal{J}(0) \geq \sum_{t=1}^T \sum_{i=1}^N \sum_{j=1}^N r_{t,ij}(\lambda^*_{t,ij}) \geq \mathbf{E}^\pi[\sum_{t=1}^T \sum_{i=1}^N \sum_{j=1}^N p^\pi_{t,ij} D_{t,ij}(p^\pi_{t,ij})]$. $\square$

EC.1.3. Proof of Lemma 2

Note that $\mathbf{DCP(0)}$ is a feasible convex program and it satisfies the Slater's condition. Moreover, there exists $\bar{\zeta} > 0$ such that $\mathbf{DCP}(\boldsymbol{\zeta})$ is also feasible for all $\zeta \leq \bar{\zeta}$. Let Ψ'_0 denote the largest dual variable of $\mathbf{DCP(0)}$ for all the constraints in (6). Then, by strong duality, $\mathcal{J}^{Det} - \mathcal{J}(\zeta) = \mathcal{J}(0) - \mathcal{J}(\zeta) \leq 2\Psi'_0 N^2 T \zeta$ holds for all $\zeta \leq \bar{\zeta}$. Setting $\Psi_0 = 2\Psi'_0 N^2$ concludes the proof. $\square$.

EC.1.4. Proof of Lemma 3

We prove by induction. For the base case, $\mathbb{H}_1$ holds since, by definition, $C^n_{1,i} = nC_{1,i} \geq nN \geq N$; so $p^\pi_{1,ij} = p^n_{1,ij}(\lambda^{n,\zeta,D}_{1,ij} - \epsilon)$ by the definition of SPC.

For the inductive step, suppose that $\mathbb{H}_s$ holds for all $s \leq t$. We now show that $\mathbb{H}_{t+1}$ holds as well. Note that the number of supply in node i cannot increase before period $\min_j \tau^n_{ji}$, since no supply units will flow in to node i before that period. Therefore, we need to argue for the case when $t < \min_j \tau^n_{ji}$ and when $t \geq \min_j \tau^n_{ji}$ separately. If $t < \min_j \tau^n_{ji}$, then we have

$$\begin{aligned} C^n_{t+1,i} &= C^n_{1,i} - \sum_{j=1}^N \sum_{s=1}^t D^n_{s,ij} = C^n_{1,i} - \sum_{j=1}^N \sum_{s=1}^t (\lambda^{n,\zeta,D}_{s,ij} + D^n_{s,ij} - \lambda^{n,\zeta,D}_{s,ij}) \\ &\geq \sum_{j=1}^N \sum_{s=1}^{\min_j \tau^n_{ji}} \lambda^{n,\zeta,D}_{s,ij} - \sum_{j=1}^N \sum_{s=1}^t \lambda^{n,\zeta,D}_{s,ij} - \sum_{j=1}^N \sum_{s=1}^t (D^n_{s,ij} - \lambda^{n,\zeta,D}_{s,ij}) \\ &= \sum_{j=1}^N \left(\sum_{s=t+1}^{\min_j \tau^n_{ji}} \lambda^{n,\zeta,D}_{s,ij} + \sum_{s=1}^t (\lambda^{n,\zeta,D}_{s,ij} - \lambda^n_{s,ij}(p^\pi_{s,ij})) - \sum_{s=1}^t \delta^n_{s,ij} \right) \end{aligned}$$

$$
\begin{aligned}
&\geq N\left(\left(\min_j \tau^n_{ji} - t\right)\cdot\zeta + t\cdot\epsilon\right) - \sum_{j=1}^N \left|\sum_{s=1}^t \delta^n_{s,ij}\right| \\
&\geq N\epsilon\underline{\tau}^n - N(\epsilon\underline{\tau}^n - 1) = N
\end{aligned}
\tag{EC.8}
$$

where the first inequality holds by inequality (5) at period $\min_j \tau^n_{ji}$; the second inequality holds by the inductive assumption $\mathbb{H}_s$ and the fact that $\lambda^{n,\zeta,D}_{s,ij} \geq \zeta$ (inequality (6)), the third equality holds by the definition of $\mathcal{S}_{ij}$ and $\zeta = \epsilon$. On the other hand, if $t \geq \min_j \tau^n_{ji}$, we know that

$$
\begin{aligned}
C^n_{t+1,i} &= C^n_{1,i} - \sum_{j=1}^N \sum_{s=1}^t D^n_{s,ij} + \sum_{j=1}^N \sum_{s=1}^{(t-\tau^n_{ji})^+} D^n_{s,ji} \\
&= C^n_{1,i} - \sum_{j=1}^N \sum_{s=1}^t (\lambda^{n,\zeta,D}_{s,ij} + D^n_{s,ij} - \lambda^{n,\zeta,D}_{s,ij}) + \sum_{j=1}^N \sum_{s=1}^{(t-\tau^n_{ji})^+} (\lambda^{n,\zeta,D}_{s,ji} + D^n_{s,ji} - \lambda^{n,\zeta,D}_{s,ji}) \\
&\geq -\sum_{j=1}^N \sum_{s=1}^t (D^n_{s,ij} - \lambda^{n,\zeta,D}_{s,ij}) + \sum_{j=1}^N \sum_{s=1}^{(t-\tau^n_{ji})^+} (D^n_{s,ji} - \lambda^{n,\zeta,D}_{s,ji}) \\
&\geq \sum_{j=1}^N \left[\sum_{s=1}^t (\lambda^{n,\zeta,D}_{s,ij} - \lambda^n_{s,ij}(p^\pi_{s,ij})) - \sum_{s=1}^{(t-\tau^n_{ji})^+} (\lambda^{n,\zeta,D}_{s,ji} - \lambda^n_{s,ji}(p^\pi_{s,ji}))\right] - \sum_{j=1}^N \left(\left|\sum_{s=1}^t \delta^n_{s,ij}\right| + \left|\sum_{s=1}^{(t-\tau^n_{ji})^+} \delta^n_{s,ji}\right|\right) \\
&\geq \epsilon\left(Nt - \sum_{j=1}^N (t-\tau^n_{ji})^+\right) - N(\epsilon\underline{\tau}^n - 1) \\
&\geq N\epsilon\underline{\tau}^n - N(\epsilon\underline{\tau}^n - 1) = N
\end{aligned}
\tag{EC.9}
$$

where the first inequality follows since $\{\lambda^{n,\zeta,D}_{t,ij}\}_{t,ij}$ is feasible to $\mathbf{DCP}(n,\zeta)$, the second inequality follows since $D^n_{s,ij} = \lambda^n_{s,ij}(p^\pi_{s,ij}) + \delta^n_{s,ij}$, the third inequality follows by the inductive assumption and the definition of $\mathcal{S}_{ij}$, and the fourth inequality holds since $t \geq \min_j \tau^n_{ji} \geq \underline{\tau}^n$. Moreover, inequalities (EC.8) and (EC.9) implies that $p^\pi_{t+1,ij} = p^n_{t+1,ij}(\lambda^{n,\zeta,D}_{t+1,ij} - \epsilon)$ by the definition of SPC. Hence, $\mathbb{H}_{t+1}$ holds which completes the inductive step.

We continue to prove the probability bound. By Lemma EC.1 and letting $r = (\epsilon\underline{\tau}^n - 1)/(4T^n) = \sqrt{\log(n\underline{\tau})/(nT)} \in (0,1]$, we have:

$$
\begin{aligned}
\mathbf{P}(\mathcal{S}^c) &\leq \sum_{i=1}^N \sum_{j=1}^N \mathbf{P}(\mathcal{S}^c_{ij}) \leq 2N^2 \exp\left(T^n r^2 - r\frac{\epsilon\underline{\tau}^n - 1}{2}\right) \\
&= 2N^2 \exp\left(-\frac{(\epsilon\underline{\tau}^n - 1)^2}{16T^n}\right) = 2N^2\exp(-\log(n\underline{\tau})) = \frac{2N^2}{n\underline{\tau}}. \quad \square
\end{aligned}
$$

EC.1.5. Proof of Lemma 4

We first show that, for all sample paths on $\mathcal{A}$, it holds that $\hat{\lambda}^\pi_{t,ij} = \lambda^{n,\zeta,D}_{t,ij} - \epsilon - u_{t,ij}\cdot\bar{\delta}_{ij}^{\kappa_{ij}(t)-1} \in (0,1)$. In other words, the targeted demand rate after adjustment is still in the interior of feasible region for demand rate. Fix i, j and k. Consider two cases. For any $t \in \mathcal{T}^k_{ij}$, if $\bar{\delta}^{k-1}_{ij} \geq 0$, then we have

$$
1 > \lambda^{n,\zeta,D}_{t,ij} - \epsilon \geq \lambda^{n,\zeta,D}_{t,ij} - \epsilon - u_{t,ij}\cdot\bar{\delta}^{k-1}_{ij}
$$

$$
\begin{aligned}
&= \frac{\lambda_{t,ij}^{n,\zeta,D}}{2} - \epsilon + \frac{\lambda_{t,ij}^{n,\zeta,D}}{2} - \frac{\lambda_{t,ij}^{n,\zeta,D}(1-\lambda_{t,ij}^{n,\zeta,D})}{\sum_{v\in\mathcal{T}_{ij}^{k}} \lambda_{v,ij}^{n,\zeta,D}(1-\lambda_{v,ij}^{n,\zeta,D})} \cdot \bar{\delta}_{ij}^{k-1} \\
&\geq \frac{\zeta}{2} - \epsilon + \lambda_{t,ij}^{n,\zeta,D} \cdot \left(\frac{1}{2} - \frac{\eta}{b}\right) > 0,
\end{aligned}
$$

where the second inequality holds since $\bar{\delta}_{ij}^{k-1} \geq 0$, the third inequality holds by the definition of batch $\mathcal{T}_{ij}^{k}$ and event $\mathcal{A}$, and the last inequality holds by condition **C2**. On the other hand, if $\bar{\delta}_{ij}^{k-1} < 0$, then we have

$$
\begin{aligned}
0 &\leq \lambda_{t,ij}^{n,\zeta,D} - \epsilon \leq \lambda_{t,ij}^{n,\zeta,D} - \epsilon - u_{t,ij} \cdot \bar{\delta}_{ij}^{k-1} = \lambda_{t,ij}^{n,\zeta,D} - \epsilon - \frac{\lambda_{t,ij}^{n,\zeta,D}(1-\lambda_{t,ij}^{n,\zeta,D})}{\sum_{v\in\mathcal{T}_{ij}^{k}} \lambda_{v,ij}^{n,\zeta,D}(1-\lambda_{v,ij}^{n,\zeta,D})} \cdot \bar{\delta}_{ij}^{k-1} \\
&\leq \lambda_{t,ij}^{n,\zeta,D} - \epsilon + \left(1-\lambda_{t,ij}^{n,\zeta,D}\right) \cdot \frac{\eta}{b} < \lambda_{t,ij}^{n,\zeta,D} - \epsilon + 1 - \lambda_{t,ij}^{n,\zeta,D} < 1
\end{aligned}
$$

where the second inequality holds since $\bar{\delta}_{ij}^{k-1} < 0$, the third inequality holds by the definition of $\mathcal{A}$, definition of batch $\mathcal{T}_{ij}^{k}$, and condition **C2**, and the fourth inequality holds since $\eta/b < 1/2 < 1$. Therefore, we can conclude that $\lambda_{t,ij}^{\zeta,D} - \epsilon - u_{s,ij} \cdot \bar{\delta}_{ij}^{\kappa_{ij}(t)} \in (0,1)$ holds on event $\mathcal{A}$.

We now proceed to prove condition $\mathbb{H}_t$ by induction. When $t=1$, $C_{1,i}^{n} = nC_{1,i} \geq N$ by definition, which also implies that $p_{1,ij}^{\pi} = p_{1,ij}^{n}(\hat{\lambda}_{t,ij}) = p_{1,ij}^{n}(\lambda_{t,ij}^{n,\zeta,D} - \epsilon)$ by the definition of ABC. This completes the inductional basis. Suppose that $\mathbb{H}_s$ holds for all $s \leq t$, we now show that $\mathbb{H}_{t+1}$ holds as well. Note that the following holds for all sample paths on $\mathcal{A}$:

$$
\begin{aligned}
C_{t+1,i}^{n} &\geq N\epsilon\underline{\tau}^{n} + \sum_{j=1}^{N}\left(\sum_{s=1}^{(t-\tau_{ji}^{n})^{+}} \delta_{s,ji}^{n} + \sum_{s=1}^{(t-\tau_{ji}^{n})^{+}} u_{s,ji} \cdot \bar{\delta}_{ji}^{\kappa_{ji}(s)-1}\right) - \sum_{j=1}^{N}\left(\sum_{s=1}^{t} \delta_{s,ij}^{n} + \sum_{s=1}^{t} u_{s,ij} \cdot \bar{\delta}_{ij}^{\kappa_{ij}(s)-1}\right) \\
&\geq N\epsilon\underline{\tau}^{n} + \sum_{j=1}^{N} \sum_{s\in\mathcal{T}_{ji}^{\kappa_{ji}((t-\tau_{ji}^{n})^{+})}} \delta_{s,ji}^{n} + \sum_{j=1}^{N}\left(1 - \sum_{s=\min\left\{v\in\mathcal{T}_{ji}^{\kappa_{ji}((t-\tau_{ji}^{n})^{+})}\right\}}^{(t-\tau_{ji}^{n})^{+}} u_{s,ji}\right) \cdot \bar{\delta}_{ji}^{\kappa_{ji}((t-\tau_{ji}^{n})^{+})-1} \\
&\quad - \sum_{j=1}^{N} \sum_{s\in\mathcal{T}_{ij}^{\kappa_{ij}(t)}} \delta_{s,ij}^{n} + \sum_{j=1}^{N}\left(1 - \sum_{s=\min\left\{v\in\mathcal{T}_{ij}^{\kappa_{ij}(t)}\right\}}^{t} u_{s,ij}\right) \cdot \bar{\delta}_{ij}^{\kappa_{ij}(t)-1} \qquad \text{(EC.10)}
\end{aligned}
$$

where the first inequality follows by the same argument as in the proof of (EC.8) and (EC.9) and the inductional hypothesis, and the last inequality follows by the definition of $\bar{\delta}_{ij}^{k}$ and the fact that $\sum_{t\in\mathcal{T}_{ij}^{k}} u_{t,ij} = 1$ for all i, j and k. Note that the absolute value of the second until the fifth terms after the last equality in (EC.10) are all bounded from above by $\sum_{j=1}^{N} \eta = N\eta$ on event $\mathcal{A}$. Therefore, we can bound

$$
C_{t+1,i}^{n} \geq N\epsilon\underline{\tau}^{n} - 4N\eta = N \qquad \text{(EC.11)}
$$

where the equality follows by the definition of η. Since the supply level is positive at node i, by Step 2.b of ABC, we know that $p_{t+1,ij}^{\pi} = p_{t+1,ij}^{n}(\hat{\lambda}_{t+1,ij})$. This completes the inductional step.

We still need to prove the probability bound. Note that, for each arc $i \to j$, the maximum number of batches K_{ij} is at most T^n/b since expected demand rate in one period is at most one. For each $i \to j$ and any $K_{ij}+1 \le k \le T^n/b$, we further define $\mathcal{T}_{ij}^k = \emptyset$. Then, applying union bound we have

$$
\begin{aligned}
\mathbf{P}(\mathcal{A}^c) &= \mathbf{P}\left(\cup_{i=1}^N \cup_{j=1}^N \cup_{k=1}^{K_{ij}} \mathcal{A}_{ijk}^c\right) \le \mathbf{P}\left(\cup_{i=1}^N \cup_{j=1}^N \cup_{k=1}^{T^n/b} \mathcal{A}_{ijk}^c\right) \le \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^{T^n/b} \mathbf{P}(\mathcal{A}_{ijk}^c) \\
&\le \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^{T^n/b} \mathbf{P}\left\{ \max_{t\in\mathcal{T}_{ij}^k} \left| \sum_{s=\min\{v\in\mathcal{T}_{ij}^k\}}^{t} \delta_{s,ij}^n \right| \ge \eta \right\} \\
&\le \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^{T^n/b} 2\exp\left(\sum_{s\in\mathcal{T}_{ij}^k} \min\{\lambda_{s,ij}, 1-\lambda_{s,ij}\} \cdot r^2 - \eta \cdot r \right) \\
&\le \frac{2N^2T^n}{b} \exp\left(4(b+1)r^2 - \eta\cdot r\right) \le \frac{2N^2T^n}{b} \exp\left(-\frac{(\epsilon \underline{T}^n - 1)^2}{256(b+1)}\right) \\
&\le \frac{2N^2}{bT^n}
\end{aligned}
$$

where the fourth inequality holds by (EC.1) in Lemma EC.1, the third to the last inequality holds by the the fact that $\min\{x, 1-x\} \le 2x(1-x)$ holds for any $x \in (0,1)$ and the definition of $\mathcal{T}_{ij}^k$, the second to the last inequality holds by setting $r = \eta/(8(b+1)) \in (0,1)$ (the inclusion follows by **C2**), the last inequality follows by the definition of ϵ. $\square$

EC.2. Proof of Theorem 1: Lower Bound on Static Policy

Fix n. Note that the objective function of the deterministic problem $\mathbf{DCP}(n,0)$ equals $\sum_{t=1}^{nT}[p_{t,12}(1-p_{t,12}) + p_{t,21}(1-p_{t,21})]$, and it achieves the unconstrained optimum at $p_{t,12} = p_{t,21} = 0.5$ for all t with an optimal revenue of $nT/2$. Moreover, it can be easily verified that this set of prices satisfies the constraints of $\mathbf{DCP}(n,0)$: Specifically, the capacity at node i decreases at a rate of 0.5 units per time unit for a total of $n\tau_{ij} = 2n$ time units, reaching a capacity level of 0 at the end of period $n\tau_{ij}$; afterwards, the incoming flow and outgoing flow at node i balances out so the capacity is kept at 0 at each node. Take any $\pi \in \Pi$, then,

$$
\begin{aligned}
nT\mathcal{L}^\pi &= \mathcal{J}(n,0) - \mathbf{E}^\pi[R^\pi(n)] = \frac{nT}{2} - \mathbf{E}^\pi[R^\pi(n)] \\
&= \frac{n\tau}{2} - \mathbf{E}^\pi\left[\sum_{t=1}^{n\tau}\sum_{i,j} D_{t,ij}(p_{t,ij}^\pi)p_{t,ij}^\pi\right] + \frac{n(T-\tau)}{2} - \mathbf{E}^\pi\left[\sum_{t=n\tau+1}^{nT}\sum_{i,j} D_{t,ij}(p_{t,ij}^\pi)p_{t,ij}^\pi\right] \\
&\ge \frac{n\tau}{2} - \mathbf{E}^\pi\left[\sum_{t=1}^{n\tau}\sum_{i,j} D_{t,ij}(p_{t,ij}^\pi)p_{t,ij}^\pi\right],
\end{aligned}
\tag{EC.12}
$$

where the inequality follows since

$$
\mathbf{E}^\pi\left[\sum_{t=n\tau+1}^{nT}\sum_{i,j} D_{t,ij}(p_{t,ij}^\pi)p_{t,ij}^\pi\right] = \sum_{t=n\tau+1}^{nT} \mathbf{E}^\pi\left[\mathbf{E}^\pi\left[\sum_{i,j} D_{t,ij}(p_{t,ij}^\pi)p_{t,ij}^\pi \,\middle|\, \mathcal{F}_t\right]\right]
$$

$$= \sum_{t=n\tau+1}^{nT} \mathbf{E}^{\pi}\left[\sum_{i,j} \lambda_{t,ij}(p^{\pi}_{t,ij}) p^{\pi}_{t,ij}\right] \leq \sum_{t=n\tau+1}^{nT} \sum_{i,j} \lambda_{t,ij}(\mathbf{E}^{\pi}[p^{\pi}_{t,ij}])\mathbf{E}^{\pi}[p^{\pi}_{t,ij}]$$

$$\leq \sum_{t=n\tau+1}^{nT} \max_{x_{t,12}, x_{t,21}} \sum_{i,j} \lambda_{t,ij}(x_{t,ij}) x_{t,ij} = \frac{n(T-\tau)}{2},$$

where the first inequality follows by Jensen's inequality and the second inequality follows by principle of optimization. Finally, note that since the resource cannot reach the destination within $n\tau$ days, this means that (EC.12) equals the revenue gap between the deterministic upper bound and the revenue of a static pricing policy for a classic network revenue management problem with two resources each with capacity $n\tau/2 = n$, two products each with an identical and independent demand function $\lambda(p) = 1 - p$, a capacity consumption matrix $[1\ 0; 0\ 1]$, and a planning horizon of $n\tau = 2n$ periods, which is known in the literature to have a revenue loss in the order of $\sqrt{n}$ (see Remark 2 in Jasin (2014)), i.e., there exists some constant $\check{\psi}$ independent of n such that

$$\frac{n\tau}{2} - \mathbf{E}^{\pi}\left[\sum_{t=1}^{n\tau} \sum_{i,j} D_{t,ij}(p^{\pi}_{t,ij}) p^{\pi}_{t,ij}\right] \leq \check{\psi}\sqrt{n\tau}.$$

The stated result follows by letting $\tilde{\Psi}_0 = \check{\psi}\sqrt{\tau}/T$. $\square$

EC.3. Additional Details to the Numerical Studies

In this section, we report additional details of the numerical studies. All the experiments are implemented on a Windows desktop with Intel(R) Xeon(R) W-2145 CPU and 64 GB RAM.

EC.3.1. Synthetic Data

The travel times among different nodes and the initial server distribution are given as follows.

$$[\tau_{ij}] = \begin{bmatrix} 0 & 7 & 10 & 8 & 2 \\ 8 & 0 & 9 & 4 & 10 \\ 9 & 11 & 0 & 6 & 9 \\ 7 & 5 & 6 & 0 & 9 \\ 2 & 9 & 10 & 8 & 0 \end{bmatrix}, \qquad C = [5, 3, 4, 6, 5]^{\top}.$$

Table EC.1 reports the performance of different pricing policies (as compared with $\mathcal{J}(\zeta)$) when stationary optimal deterministic solution is used as the baseline control. In particular, the only difference between SPC (resp. ABC) and S-SPC (resp. S-ABC) is that the second policy uses $\{\lambda^{\zeta,SD}_{t,ij}\}_{t,ij}$ in Step 1 as the baseline control, where $\lambda^{\zeta,SD}_{t,ij} \equiv \lambda^{\zeta,SD,*}_{ij}$ for all t and $\lambda^{\zeta,SD,*}_{ij}$ is the solution to the following optimization problem:

$$\textbf{(S-DCP}(\zeta)\textbf{)} \quad \mathcal{J}^{\text{STAT}}(\zeta) := \max_{(\lambda_{ij})_{(i,j)}} \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} r_{t,ij}(\lambda_{ij})$$

$$s.t. \quad C_i - \sum_{j=1}^{N} t \cdot \lambda_{ij} + \sum_{j=1}^{N} (t - \tau_{ji})^{+} \lambda_{ji} \geq 0, \forall i, t$$

$$\zeta \leq \lambda_{ij} \leq 1 - \zeta, \forall i, j$$

As a reference, $(\mathcal{J}(\zeta)-\mathcal{J}^{\mathrm{STAT}}(\zeta))/\mathcal{J}(\zeta)=6.93\%$. As we can observe, the performance of S-ABC is worse than that of SPC, which suggests the necessity of using non-stationary baseline control even if the prices are adjusted dynamically.

	S-SPC			S-ABC			
n	Loss $	Loss %	Rev. $	Loss $	Loss %	Rev. $	Rev. Incr. %
40	14.30	12.08	124879	9.41	7.95	130748	4.70
80	12.58	10.63	253890	8.87	7.50	262777	3.50
120	11.82	9.98	383568	8.76	7.40	394564	2.87
160	11.40	9.63	513411	8.68	7.33	526483	2.55
200	11.16	9.43	643207	8.65	7.31	658286	2.34
240	10.95	9.25	773407	8.55	7.23	790648	2.23
280	10.83	9.15	903266	8.54	7.21	922565	2.14
320	10.66	9.01	1033951	8.48	7.17	1054859	2.02
360	10.54	8.91	1164494	8.45	7.13	1187129	1.94
400	10.49	8.86	1294496	8.40	7.10	1319549	1.94

Table EC.1 Performance of proposed pricing policies under varying n using stationary optimal deterministic solution as baseline

We further report the experiment results under different demand models: Table EC.2 contains the results under exponential demand models (where $\lambda_{t,ij}=\mu_{ij}\cdot\exp(-\beta_{ij}\cdot p_{t,ij})$); Table EC.3 contains the results under logit demand models (where $\lambda_{t,ij}(p_{t,ij})=\mu_{ij}\cdot\frac{\exp(\alpha-\beta_{ij}\cdot p_{t,ij})}{1+\exp(\alpha-\beta_{ij}\cdot p_{t,ij})}$). The parameters are set in the same way as in the case with linear demand models. The observations from these two tables are qualitatively similar to those drawn under the linear demand models.

	SPC				ABC					
n	Loss $	Loss %	Rev. $	Admit	Loss $	Loss %	Rev. $	Rev. Incr. %	Admit Incr. %	Avg Price Incr. %
40	4.86	5.41	101974	1950	2.58	2.88	104706	2.68	1.61	1.05
80	3.66	4.08	206821	3948	1.72	1.91	211493	2.26	1.51	0.74
120	3.11	3.46	312239	5957	1.20	1.34	319095	2.20	1.54	0.65
160	2.78	3.09	417889	7972	1.02	1.14	426312	2.02	1.44	0.56
200	2.53	2.82	523853	9981	0.88	0.98	533775	1.89	1.23	0.66
240	2.31	2.57	630184	11996	0.74	0.83	641484	1.79	1.15	0.63
280	2.29	2.55	735412	13996	0.73	0.81	748529	1.78	0.94	0.84
320	2.04	2.28	842825	16023	0.71	0.79	855643	1.52	0.82	0.69
360	1.78	1.98	951051	18046	0.63	0.70	963443	1.30	0.62	0.68
400	1.65	1.83	1058311	20070	0.53	0.59	1071652	1.26	0.65	0.60

Table EC.2 Performance of proposed pricing policies under varying n with exponential demand function

	SPC				ABC					
n	Loss $	Loss %	Rev. $	Admit	Loss $	Loss %	Rev. $	Rev. Incr. %	Admit Incr. %	Avg Price Incr. %
40	12.84	3.70	400475	3293	6.74	1.95	407786	1.83	1.25	0.57
80	10.25	2.96	807153	6631	4.53	1.31	820887	1.70	1.21	0.49
120	8.95	2.58	1215424	9980	3.72	1.07	1234247	1.55	1.11	0.43
160	8.32	2.40	1623596	13331	3.02	0.87	1648998	1.56	1.10	0.46
200	7.82	2.26	2032477	16686	2.80	0.81	2062588	1.48	1.09	0.39
240	6.62	1.91	2447594	20036	2.31	0.67	2478602	1.27	0.84	0.42
280	6.15	1.77	2859492	23404	2.07	0.60	2893758	1.20	0.78	0.42
320	6.10	1.76	3268465	26751	2.00	0.58	3307832	1.20	0.66	0.54
360	5.83	1.68	3679970	30117	1.96	0.56	3721759	1.14	0.64	0.49
400	5.57	1.61	4091908	33487	1.62	0.47	4139397	1.16	0.61	0.55

Table EC.3 Performance of proposed pricing policies under varying n with logit demand function

At last, we test the performance of our proposed policies by varying the length of the planning horizon. Specifically, we fix the scaling parameter $n = 100$ and set $T = \{30, 40, 50, 60, 70, 80\}$. Table EC.4 reports the numerical results. We find that, as T increases, both SPC and ABC incur higher revenue loss, but the loss of ABC grows slower than that of SPC. In other words, the performance of ABC is more robust under different lengths of planning horizon than that of SPC.

		SPC			ABC			
T	$\mathcal{J}(1,0)$	Loss $	Loss %	Rev. $	Loss $	Loss %	Rev. $	Rev. Incr. %
30	3551	5.81	4.91	3377	3.33	2.82	3451	2.20
40	4639	6.28	5.41	4388	3.35	2.89	4505	2.67
50	5720	6.70	5.85	5385	3.47	3.03	5547	3.00
60	6807	6.82	6.01	6398	3.52	3.10	6595	3.09
70	7891	7.08	6.28	7396	3.56	3.16	7642	3.33
80	8976	7.33	6.53	8389	3.55	3.16	8692	3.61

Table EC.4 Performance of pricing policies under varying T

EC.3.2. New York Taxi Dataset

Table EC.5 summarize how we construct regions by grouping adjacent regions defined in New York City (2020). Roughly speaking, the bigger the difference of the region indexes is, the farther two regions are apart. Table EC.6 provides summary statistics for the travel time from region 1 to all the other regions. In general, the travel time is proportional to the distance.

We now give a formal definition for the deterministic relaxation problem used in Section 6.2 where the optimal static solution is piece-wise stationary. Define U to be a positive integer and assume without loss of generality that T is an integral multiply of U. In particular, U should be interpreted

Region ID	**Original Zone Number and Names**
1	12-Battery Park, 13-Battery Park City , 261-World Trade Center
2	87-Financial District North, 88-Financial District South , 209-Seaport
3	125-Hudson Sq, 211-SoHo , 231-TriBeCa/Civic Center
4	45-Chinatown, 144-Little Italy/NoLiTa , 148-Lower East Side
5	158-Meatpacking/West Village West, 249-West Village
6	79-East Village, 113-Greenwich Village North , 114-Greenwich Village South
7	4-Alphabet City, 232-Two Bridges/Seward Park
8	68-East Chelsea, 246-West Chelsea/Hudson Yards
9	90-Flatiron, 100-Garment District , 186-Penn Station/Madison Sq West
10	107-Gramercy, 224-Stuy Town/Peter Cooper Village , 234-Union Sq
11	137-Kips Bay, 164-Midtown South , 170-Murray Hill
12	48-Clinton East, 50-Clinton West
13	161-Midtown Center, 163-Midtown North , 230-Times Sq/Theatre District
14	162-Midtown East, 229-Sutton Place/Turtle Bay North , 233-UN/Turtle Bay South
15	142-Lincoln Square East, 143-Lincoln Square West , 140-Lenox Hill East
16	140-Lenox Hill East, 141-Lenox Hill West , 237-Upper East Side South
17	238-Upper West Side North, 239-Upper West Side South
18	236-Upper East Side North, 262-Yorkville East , 263-Yorkville West
19	24-Bloomingdale, 151-Manhattan Valley
20	43-Central Park

Table EC.5 Construction of Region IDs used in the Numerical Experiments

Destination	Median	25%-quantile	75%-quantile	10%-quantile	90%-quantile
2	529.75	508.45	561.43	478.73	587.21
3	502.61	474.71	530.46	451.09	557.15
4	915.61	864.21	966.23	809.54	1022.73
5	638.67	609.86	679.01	585.03	708.49
6	951.55	906.91	1009.68	862.89	1046.54
7	899.40	835.18	964.57	781.50	1027.38
8	929.53	872.28	998.34	819.12	1081.04
9	1300.08	1199.28	1397.63	1112.33	1503.27
10	1213.90	1145.57	1268.94	1075.84	1321.19
11	1452.32	1351.50	1580.42	1260.00	1676.11
12	1506.24	1359.06	1674.97	1259.11	1842.51
13	1778.76	1608.01	1937.79	1491.31	2081.46
14	1412.46	1297.01	1535.53	1168.00	1666.09
15	1692.00	1552.61	1877.09	1402.41	2037.61
16	1747.87	1570.98	1911.89	1364.52	2060.91
17	1853.75	1682.08	2061.70	1493.60	2275.25
18	1992.82	1782.82	2196.30	1610.23	2367.80
19	1971.50	1748.31	2258.25	1502.13	2621.10
20	2055.18	1870.63	2239.57	1681.13	2411.45

Table EC.6 Average Travel Time From Region 1 in Seconds

as the duration during which the demand rate is piece-wise stationary. In our experiment, since one period equals 5 seconds and the optimal demand rate is required to be stationary during each 5-minute time window, $U = 5 \times 60/5 = 60$. Let $\mathcal{U} = \{\mathcal{U}_k\}$ to be a partition of $[T]$, where $\mathcal{U}_k = \{(k-1)t+1,\dots,kt\}$. Denote by Π^U the class of policies where the prices for all the periods in $\mathcal{U}_k$ are the same, i.e. $\Pi(U) = \{\pi : p^{\pi}_{t,ij} = p_{s,ij}, \forall s,t \in \mathcal{U}_k, \forall k\}$. In other words, $\Pi(U) \subset \Pi$ is the class of admissible policies where the price trajectories are piece-wise stationary (in other words, we restrict our attention to the class of pricing policies under which the prices cannot be adjusted more frequently than every 5 minutes). Denote by $\mathcal{J}^*(U)$ the optimal expected profit given that the policy is chosen from $\Pi(U)$. It is straightforward to verify that the following deterministic relaxation is an upper bound of $\mathcal{J}^*(U)$:

$$
\begin{aligned}
(\mathbf{DCP(}\zeta\mathbf{)})(U) \quad \mathcal{J}(\zeta,U) := \max_{\boldsymbol{\lambda}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} r_{t,ij}(\lambda_{t,ij}) \\
s.t. \quad & \sum_{j=1}^{N}\sum_{s=1}^{t}\lambda_{s,ij} - \sum_{j=1}^{N}\sum_{s=1}^{(t-\tau_{ji})^+}\lambda_{s,ji} \le C_i, \forall i,t && \text{(EC.13a)} \\
& \zeta \le \lambda_{t,ij} \le 1-\zeta, \forall t,i,j && \text{(EC.13b)} \\
& \lambda_{s,ij} = \lambda_{t,ij}, \forall s,t \in \mathcal{U}_k, \forall k && \text{(EC.13c)}
\end{aligned}
$$

The dimension of the problem above can be reduced by eliminating the last constraint (i.e. the piecewise stationarity constraint). Define $\mathcal{U}^{\dagger} := \{1 \le t \le T : t = k \cdot U \text{ for some } k \in \mathbf{N}_+\}$ and $\mathcal{U}_{ij} := \{1 \le t \le T : t - \tau_{ij} = k \cdot U \text{ for some } k \in \{0\} \cup \mathbf{N}_+\}$ for each $i,j \in [N]$. For a fixed i, constraint (EC.13a) only needs to be considered on the set $\mathcal{U}_i = \mathcal{U}^{\dagger} \cup (\cup_{j=1}^{N} \mathcal{U}_{ji})$. To see why this is the case, consider a period $t \in [T] \backslash \mathcal{U}_i$. Let $\{u_k\}_{k=1}^{K}$ be the elements in $\mathcal{U}_i$ indexed from smallest to the largest. If $t < u_1$, it is not difficult to check that constraint (EC.13a) at t is implied by constraint (EC.13a) at u_1, since $u_1 = \min\{U, \min_{j=1}^{N} \tau_{ji}\}$. Suppose that $u_k < t < u_{k+1}$. Note that, the difference between the LHS of constraint (EC.13a) at t and constraint (EC.13a) at $t+1$ is simply the net outflow, given by

$$
\Delta_{it} := \sum_{j=1}^{N} \lambda_{t+1,ij} - \sum_{j:t \ge \tau_{ji}} \lambda_{t+1-\tau_{ji},ji}
$$

By definition of $\mathcal{U}_i$, it is not difficult to see that, for any $u_k < s < v \le u_{k+1}$, $\lambda_{s,ij} = \lambda_{v,ij}$ for any i and j. Therefore, we know that $\Delta_{i,u_k} = \Delta_{i,u_k+1} = \cdots = \Delta_{i,u_{k+1}-1}$. As a result, the LHS of constraint (EC.13a) at t is smaller than the LHS of constraint (EC.13a) at u_k if $\Delta_{it} < 0$, and is smaller than the LHS of constraint (EC.13a) at u_{k+1} if $\Delta_{it} > 0$. The argument above suggests that, after variable reduction, (EC.13a) can be equivalently reformulated as follows:

$$
\begin{aligned}
\max_{\boldsymbol{\lambda}} \quad & \sum_{k=1}^{T/K}\sum_{t\in\mathcal{U}_k}\sum_{i=1}^{N}\sum_{j=1}^{N} r_{t,ij}(\lambda_{k,ij}) \\
\text{s.t.} \quad & C_i \ge \sum_{j=1}^{N}\left(\sum_{k=1}^{\lfloor t/U \rfloor} U \cdot \lambda_{k,ij} + (t - U \cdot \lfloor t/U \rfloor)\,\lambda_{\lceil t/U \rceil,ij}\right) && \text{(EC.14a)}
\end{aligned}
$$

$$-\sum_{j:\tau_{ji}<t}\left(\sum_{k=1}^{\lfloor (t-\tau_{ji})^+/U\rfloor} U\cdot\lambda_{k,ji}+\left((t-\tau_{ji})^+-U\cdot\lfloor (t-\tau_{ji})^+/U\rfloor\right)\lambda_{\lceil (t-\tau_{ji})^+/U\rceil,ji}\right),\forall i,t\in\mathcal{U}_i$$

$$\zeta\le\lambda_{k,ij}\le 1-\zeta,\qquad \forall i,j,k \tag{EC.14b}$$

Note that, while the original optimization problem DCP has about NT constraints and N^2T decision variables, (EC.14) has at most $N^2(T/U)$ constraints and $N^2(T/U)$ decision variables. For the model calibrated using the Manhattan yellow taxi dataset, $T=7,200$ and $N=20$, which means the there are about $144,000$ constraints and $2,880,000$ decision variables and the original DCP is intractable using standard desktops. If we restrict the baseline optimal demand rate to be piecewise stationary for each 5-minute interval, (EC.14) has about $48,000$ constraints and $48,000$ decision variables. Using `fmincon` function in MATLAB, problem of such scale can be solved in hours.

EC.4. Performance Analysis of R-ABC: Proof of Theorem 4

Let $\pi=$R-ABC. Since the proof is similar to the proof of Theorem 3, for brevity, we will primarily focus on the differences. By an argument that is similar with the Proof of Lemma 2, we know that there exist some positive constants $\bar{\zeta}$ and Ψ_0 such that, for any $\zeta,\zeta^q<\bar{\zeta}$, we have $\mathcal{J}^R(0,0)-\mathcal{J}^R(\zeta,\zeta^q)\le\Psi_0N^2T(\zeta+\zeta^q)$. We further define $\eta=(\epsilon\underline{\tau}^n-1)/8$ and $\eta^q=(\epsilon^q\underline{\tau}^n-1)/8$. We prove the results by considering two cases.

Analysis for small n

By definition, we know that both ζ and η/b converges to zero as n goes to infinity. Therefore, define $\Omega:=\max\{n\in\mathbb{Z}_{++}:(\underline{\tau}^n)^{2/3}\le\max\{8N^2/(T^n)^2,512\log T^n,256\sqrt{\log T^n}/\bar{\zeta}\}\}$ (if the right hand side is an empty set, let $\Omega:=0$). Similar to the proof of Theorem 2, when $n\le\Omega$, $\mathcal{L}^\pi(n)\le M_1(\log T^n)^{2/3}/(\underline{\tau}^n)^{2/3}$ where M_1 is independent of n, T, and τ_{ij}.

Analysis for large n

When $n>\Omega$, by the definition of Ω, (17) and (18), the following holds:

$$\mathbf{C3}:\quad \bar{\zeta}>\max\{\zeta,\zeta^q\},\zeta=2\epsilon,\zeta^q=2\epsilon^q,\text{ and }\frac{2\eta}{b}<1. \tag{EC.15}$$

Define the following sequence of events for all arc $i\to j$ and batch k

$$\mathcal{R}_{ijk}:=\left\{\max_{t\in\mathcal{T}_{ij}^k}\left|\sum_{s=\min\{v\in\mathcal{T}_{ij}^k\}}^{t}\delta_{s,ij}^n\right|<\eta\right\}\cap\left\{\max_{t\in\mathcal{T}_{ij}^{q,k}}\left|\sum_{s=\min\{v\in\mathcal{T}_{ij}^{q,k}\}}^{t}\delta_{s,ij}^{q,n}\right|<\eta^q\right\}$$

Further define $\mathcal{R}:=\cap_{i,j,k}\mathcal{R}_{ijk}$. We stat a lemma that says that condition **C3** implies that supply at all the nodes will not run out throughout the planning horizon with high probability. In fact, this lemma should be considered as the analogue of Lemma 4 in the analysis of ABC.

LEMMA EC.2. *If **C3** holds, then for all sample paths on $\mathcal{R}$, the following condition holds for all t,*

$$\mathbb{H}_t: \quad C^n_{t,i} \geq N \text{ for all } i, \text{ and } p^\pi_{t,ij} = p^n_{t,ij}(\hat{\lambda}_{t,ij}) \text{ and } \mathbf{E}^\pi[Q^\pi_{t,ij} | \mathcal{F}_t] = \hat{q}_{t,ij} \text{ for all } i \to j.$$

where $\hat{\lambda}_{t,ij} = \lambda^{n,\zeta,\zeta^q,D}_{t,ij} - \epsilon - u_{t,ij} \cdot \bar{\delta}^{\kappa_{ij}(t)-1}_{ij} \in (0,1)$ *and* $\hat{q}_{t,ij} = q^{n,\zeta,\zeta^q,D}_{t,ij} - \epsilon^q - u^q_{t,ij} \cdot \bar{\delta}^{q,\kappa^q_{ij}(t)-1}_{ij} > 0$. *Moreover,* $\mathbf{P}(\mathcal{R}) \geq 1 - 4N^2 \cdot b^{-1}(T^n)^{-3}$.

Proof. First, it is straightforward to verify that, on all sample paths on $\mathcal{R}$, it holds that $\hat{\lambda}_{t,ij} \in (0,1)$ and $\hat{q}_{t,ij} > 0$. In particular, when $\bar{\delta}^{q,\kappa^q_{ij}(t)-1}_{ij} > 0$, $\hat{q}_{t,ij} > 0$ holds trivially; if $\bar{\delta}^{q,\kappa^q_{ij}(t)-1}_{ij} < 0$, we further have

$$\begin{aligned} \hat{q}_{t,ij} &= \frac{q^{n,\zeta,\zeta^q,D}_{t,ij}}{2} - \epsilon^q + \frac{q^{n,\zeta,\zeta^q,D}_{t,ij}}{2} - \frac{q^{n,\zeta,\zeta^q,D}_{t,ij}}{\sum_{v \in \mathcal{T}^{q,k}_{ij}} q^{n,\zeta,\zeta^q,D}_{v,ij}} \cdot \bar{\delta}^{q,k-1}_{ij} \\ &\geq \frac{\zeta^q}{2} - \epsilon^q + q^{n,\zeta,\zeta^q,D}_{t,ij} \cdot \left(\frac{1}{2} - \frac{\eta^q}{b} \right) > 0, \end{aligned}$$

where the last inequality holds by condition **C3**. We now proceed to prove condition $\mathbb{H}_t$ by induction. When $t = 1$, $C^n_{1,i} = nC_{1,i} \geq N$ by definition, which also implies the rest two identities by the definition of R-ABC. This completes the inductional basis. Suppose that $\mathbb{H}_s$ holds for all $s \leq t$, we now show that $\mathbb{H}_{t+1}$ holds as well. At the beginning of period $t+1$, we know that

$$C^n_{t+1,i} = C^n_{1,i} - \sum_{j=1}^{N} \sum_{s=1}^{t} (D^n_{s,ij} + Q^n_{s,ij}) + \sum_{j=1}^{N} \sum_{s=1}^{(t-\tau^n_{ji})^+} (D^n_{s,ji} + Q^n_{s,ji})$$

By a very similar derivation as in the proof of (EC.10) and (EC.11), it is straightforward to verify that $C^n_{t+1,i} \geq N\underline{T}^n(\epsilon + \epsilon^q) - 4N(\eta + \eta^q) = N$, which implies that $p^\pi_{t,ij} = p^n_{t,ij}(\hat{\lambda}_{t,ij})$ and $\mathbf{E}^\pi[Q^\pi_{t,ij} | \mathcal{F}_t] = \tilde{q}_{t,ij} = \hat{q}_{t,ij}$ by the definition of R-ABC. At last, we prove the probability bound. Similar to the proof of Lemma 4, we will apply the union bound to obtain an upper bound on the probability of the complement of $\mathcal{R}$. Since the upper bounds on the demand variability has been derived already, we only need to focus on the variability due to sampling the relocation quantity. For any batch $\mathcal{T}^{q,k}_{ij}$, we have

$$\begin{aligned} &\mathbf{P}\left\{ \max_{t \in \mathcal{T}^{q,k}_{ij}} \left| \sum_{s=\min\{v \in \mathcal{T}^{q,k}_{ij}\}}^{t} \delta^{n,q}_{s,ij} \right| \geq \eta^q \right\} \\ &\leq 2\exp\left(\sum_{s \in \mathcal{T}^{q,k}_{ij}} \min\left\{ q^{n,\zeta,\zeta^q,D}_{t,ij} - \lfloor q^{n,\zeta,\zeta^q,D}_{t,ij} \rfloor, \lceil q^{n,\zeta,\zeta^q,D}_{t,ij} \rceil - q^{n,\zeta,\zeta^q,D}_{t,ij} \right\} \cdot r^2 - \eta^q \cdot r \right) \\ &\leq 2\exp\left(\left(\sum_{s \in \mathcal{T}^{q,k}_{ij}} q^{n,\zeta,\zeta^q,D}_{t,ij} \right) \cdot r^2 - \eta^q \cdot r \right) \\ &\leq 2\exp\left((b+1) \cdot r^2 - \eta^q \cdot r \right) = 2\exp\left(-\frac{(\epsilon^q \underline{T}^n - 1)^2}{256(b+1)} \right) = 2(T^n)^{-4} \end{aligned}$$

where the first inequality follows from (EC.1), the third inequality holds by the definition of $\mathcal{T}_{ij}^{q,k}$, the first equality holds by setting $r = \eta^q/(2(b+1)) \in (0,1)$ (the inclusion follows by **C3**), the last equality follows by the definition of ϵ^q. Combining the bound above with the upper bounds on the demand variability gives us the probability upper bound. $\square$

We now move on to bound the profit loss. Define two positive constants $Q_{\max} := \max_i C_i/T$ and $c_{\max} := \max_{t,i,j} c_{t,ij}$. It is straightforward to check that the feasible relocation quantity in each period is upper bounded by $NQ_{\max}T^n$. Similar to (12), we have

$$\begin{aligned}
&\mathcal{J}^R(n,\zeta,\zeta^q) - \mathbf{E}^\pi\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} p_{t,ij}^\pi D_{t,ij}(p_{t,ij}^\pi) - c_{t,ij}Q_{t,ij}^\pi\right] \\
&\le \sum_{t=1}^{T^n}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{E}^\pi\left[r_{t,ij}^n(\lambda_{t,ij}^{n,\zeta,D}) - r_{t,ij}^n(\hat{\lambda}_{t,ij})|\mathcal{R}\right] + r_{\max}\cdot N^2 T^n \mathbf{P}(\mathcal{R}^c) \\
&\qquad \sum_{t=1}^{T^n}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{E}^\pi\left[c_{t,ij}(\hat{q}_{t,ij} - q_{t,ij}^{n,\zeta,\zeta^q,D})|\mathcal{R}\right] + c_{\max}Q_{\max}N^3(T^n)^2\mathbf{P}(\mathcal{R}^c)
\end{aligned} \tag{EC.16}$$

The first and the second term can be bounded using the same argument as in the proof of Theorem 3. As for the second term, since the sampling procedure across periods are independent, we have $\mathbf{E}^\pi[\bar{\delta}_{ij}^{q,\kappa_{ij}^q(t)-1}] = 0$. Therefore, we further know that

$$\begin{aligned}
\sum_{t=1}^{T^n}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{E}^\pi\left[c_{t,ij}(\hat{q}_{t,ij} - q_{t,ij}^{n,\zeta,\zeta^q,D})|\mathcal{R}\right] &\le 0 - \sum_{t=1}^{T^n}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{E}^\pi\left[c_{t,ij}u_{t,ij}^q \cdot \bar{\delta}_{ij}^{q,\kappa_{ij}^q(t)-1}\middle|\,\mathcal{R}\right] \\
&= \sum_{t=1}^{T^n}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{P}(\mathcal{R})^{-1}\cdot \mathbf{E}^\pi\left[c_{t,ij}u_{t,ij}^q \cdot \bar{\delta}_{ij}^{q,\kappa_{ij}^q(t)-1}\middle|\,\mathcal{R}^c\right]\mathbf{P}(\mathcal{R}^c) \le \frac{4c_{\max}N^2}{b\zeta^q T^n(1-2N^2b^{-1}(T^n)^{-2})}
\end{aligned}$$

where the second inequality follows by Lemma EC.2 and the fact that $u_{t,ij}^q \le 1$, $|\delta_{t,ij}^q| < 1$ almost surely and $q_{t,ij}^{n,\zeta,\zeta^q,D} \ge \zeta^q$. Combining the bounds above with the upper bound on the revenue loss (see the proof of Theorem 3), we have

$$\begin{aligned}
\mathcal{L}^\pi(n) &= (\mathcal{J}^R(n,0,0) - \mathcal{J}^R(n,\zeta,\zeta^q) + \mathcal{J}^R(n,\zeta,\zeta^q) - \mathbf{E}^\pi[R^\pi(n)])/T^n \\
&\le \Psi_0(\zeta+\zeta^q) + N^2\left[\Psi_3\left(\epsilon + 2\epsilon^2 + \frac{64\log T^n}{b}\right) + \frac{4(\Psi_3+c_{\max})N^2}{b\zeta T^n(1-2N^2b^{-1}(T^n)^{-2})} + \frac{2N^2(r_{\max}+c_{\max}Q_{\max}N)}{b}\right] \\
&\le M_2\cdot\frac{1+(\log T^n)^{\frac{3}{2}}}{(\underline{T}^n)^{2/3}}
\end{aligned}$$

where $M_2 = 64\Psi_0 + 160N^2\Psi_3 + 4N^4(\Psi_3+c_{\max}) + 2N^4(r_{\max}+c_{\max}Q_{\max}N)$ is independent of n. Setting $\tilde{\Psi}_3 = \max\{M_1, M_2\}[(\log(2))^{-3/2} + (1+\log(T)/\log(2))^{3/2}]\underline{T}^{-2/3}$ completes the proof. $\square$