When market incompleteness is preferable to market power. Insights from power markets. Online appendix. *

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A Appendix

A.1 Risk measures: additional details

Aversion to risk can be modeled in roughly two ways. The first way, which is popular among economists, uses expected concave utility functions to rank risk preferences. The present paper follows the second way, which is more popular in finance and Operations Research, by resorting to risk measures.[1] This choice is motivated by several factors. First, utility functions are often difficult to calibrate. Second, risk measures make it possible to account for hedging securities like contracts and options in a simple way. Third, risk measures can be formulated with optimization problems and might benefit from appreciable convexity properties. Fourth, some risk measures (those who are coherent as we will explain below) reformulate risk in terms of equivalent risk-adjusted expectations of loss, which eases their interpretation. Finally, risk measures can also price risk-free assets when they exist. Since being introduced in Artzner et al. [1999], risk measures have become standard in modeling risk aversion and the particular cases of convex or coherent risk measures are appealing because they are now well grounded in finance on the one hand and benefit from nice financial and mathematical properties on the other hand (examples include the above-mentioned literature on market incompleteness). We also refer the reader to Shapiro et al. [2021] for detailed treatment of the application of risk measures in stochastic programming.

Definition 1. *A convex risk measure (see Föllmer and Schied [2002]) ρ is a real valued function defined over $\mathcal{Z}$ that satisfies the following properties:*

- *Cash invariance: $\forall Z \in \mathcal{Z}$ and $\forall a \in \mathbb{R}$, $\rho(Z-a) = \rho(Z) + a$.*

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[1] A unified framework using Choquet integrals encompassing both concave utility functions and risk measures can be found in Bassett et al. [2004].

- *Convexity:* $\forall Z_1,\ Z_2 \in \mathcal{Z}$, *and* $\forall \lambda \in [0,1]$,

$$\rho(\lambda Z_1 + (1-\lambda)Z_2) \leq \lambda\rho(Z_1) + (1-\lambda)\rho(Z_2).$$

- *Monotonicity:* $\forall Z_1,\ Z_2 \in \mathcal{Z}, Z_1 \preceq Z_2 \Longrightarrow \rho(Z_1) \geq \rho(Z_2)$.

A coherent risk measure is convex and fulfills the positive homogeneity criterion:

Definition 2. *A coherent risk measure ρ is convex and satisfies:*

- *Positive hommogeneity:* $\forall Z \in \mathcal{Z}$ *and* $\forall a \in \mathbb{R}^+$, $\rho(aZ) = a\rho(Z)$.

A coherent risk measure expresses the risk of a stochastic payoff in the same monetary unit. Any coherent risk measure ρ can be expressed in an expectation-equivalent form via its dual representation (see the representation theorem in Föllmer and Schied [2002]),

$$\rho(Z) = \text{Max}_{\zeta \in \mathcal{M}} \left(-\mathbb{E}_{\zeta}[Z]\right), \tag{40}$$

where $\mathcal{M}$ is the so-called risk set: a compact and convex set of probability measures. The (coherent) risk of a random payoff Z is then expressed as the maximum expectation, over the set $\mathcal{M}$, of its associated loss. Because $\mathcal{M}$ is compact, Problem (2) always has a solution and when unique, the optimal probability measure, which we denote by $\zeta^* \in \mathcal{M}$, is called risk-adjusted probability because it expresses the risk as an equivalent expectation of the loss:

$$\rho(Z) = -\sum_{\omega \in \Omega} \zeta^*(\omega) Z(\omega). \tag{41}$$

The envelope theorem implies that the risk-adjusted probability ζ^* of a coherent risk measure equals the negative of the sub-gradient of the risk measure with respect to the payoff variable:

$$\partial \rho_Z(Z) = -\zeta^* = -\text{Argmax}_{\zeta \in \mathcal{M}} \left(-\mathbb{E}_{\zeta}[Z]\right). \tag{42}$$

When risk measure ρ is convex but not necessarily coherent, we still call risk-adjusted probability the negative of the sub-gradient of ρ with respect to the payoff variable Z. When the risk measure is coherent, we sometimes refer to the notion of risk-adjusted profit that we define as:

$$\sum_{\omega} \zeta^*(\omega) Z(\omega). \tag{43}$$

Therefore, in an optimization procedure, a risk-averse agent would strive to minimize its risk or, in an equivalent way, maximize its risk-adjusted profit. The degree of risk aversion, that is incorporated in the formulation of set $\mathcal{M}$, has a direct impact on the calculation of the risk and the formation of the risk-adjusted probabilities. As a consequence, unlike a risk-neutral agent who will value its payoff via the expectation under the natural distribution of scenarios θ, a risk-averse agent will distort θ in an endogenous way so that it will place more weight on scenarios with higher losses.

A.2 An equilibrium model for the fully incomplete market

In this section, we develop a model to represent competing risk-averse producers who invest in the first stage and operate their assets to trade in the market in the second stage when the market is fully incomplete. Agents compete on volume and we explicitly consider the possibility that some producers exercise market power. The market is fully incomplete as we assume here that there are no financial securities to enable market agents to share the risk: set $\mathcal{C}$ is empty.

A.2.1 Modeling consumers

Let us analyze the behavior of consumers. In each scenario $\omega \in \Omega$, consumers optimize their surplus by choosing their level of consumption $d^t(\omega)$:

$$\begin{aligned} CS(\omega) = \text{Max} \quad & \sum_{t=1}^{T} \left(a^t(\omega)d^t(\omega) - b^t(\omega)\frac{d^t(\omega)^2}{2} - p^t(\omega)d^t(\omega) \right) h^t \\ \text{s.t} \quad & d^t(\omega) \geq 0 \qquad \forall t = 1, ..., T. \end{aligned} \tag{44}$$

This optimization program is convex with linear constraints. It has the following first-order (or KKT) conditions (following the standard notation of complementarity problems, $a \perp b$ denotes $a.b = 0$),

$$0 \leq d^t(\omega) \perp \left(p^t(\omega) - a^t(\omega) + b^t(\omega)d^t(\omega)\right) \geq 0 \quad \forall t = 1, ..., T, \tag{45}$$

which is the physical price formation condition. In the fully incomplete market, there are no first-stage decisions for consumers to make as they do not have contracts to trade.

A.2.2 Modeling producers

Consider now a producer i who invests in capacity K_i in the first stage. In each scenario ω in the second stage, the optimal operations of its asset is tuned via the production variables $x_i^t(\omega), t = 1, ..., T$, for the purpose of maximizing profits (these are recourse variables).

When the producer is a price-taker, it does not know that it can influence the price by modifying its output: it considers the price as a given parameter with respect to which it strives to maximize profits. The producer's optimization problem is then written as follows (some relevant dual variables are written between brackets next to their constraints):

$$\begin{aligned}
\text{Max} \quad & \sum_{t=1}^{T} \left[\left(p^t(\omega) - CO_i^t(\omega) \right) x_i^t(\omega) \right] h^t \\
\text{s.t} \quad & x_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
& x_i^t(\omega) \leq K_i g_i^t(\omega) && (\mu_i^t(\omega) h^t), \quad \forall t = 1, ..., T.
\end{aligned} \tag{46}$$

Optimization program (46) is linear and has the following KKT conditions:

$$\begin{aligned}
0 \leq x_i^t(\omega) \perp & \quad CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) \geq 0 && \forall t = 1, ..., T \\
0 \leq \mu_i^t(\omega) \perp & \quad K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 && \forall t = 1, ..., T.
\end{aligned} \tag{47}$$

If the producer is a price-maker and exercises market power, we assume that it knows the demand curve and integrates it into its profit-maximization procedure. If we denote the total volume of electricity produced by other producers (i.e. not by i) by $x_{-i}^t(\omega)$, the profit-maximization problem transforms into the following:

$$\begin{aligned}
\text{Max} \quad & \sum_{t=1}^{T} \left[\left(a^t(\omega) - b^t(\omega) \left(x_i^t(\omega) + x_{-i}^t(\omega) \right) - CO_i^t(\omega) \right) x_i^t(\omega) \right] h^t \\
\text{s.t} \quad & x_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
& x_i^t(\omega) \leq K_i g_i^t(\omega) && (\mu_i^t(\omega) h^t), \quad \forall t = 1, ..., T.
\end{aligned} \tag{48}$$

Producer i has no control over $x_{-i}^t(\omega)$. Optimization program (48) is convex (quadratic) with linear constraints and has the following KKT conditions:

$$\begin{aligned}
0 \leq x_i^t(\omega) \perp & \quad CO_i^t(\omega) + \mu_i^t(\omega) - a^t(\omega) + b^t(\omega) \sum_j x_j^t(\omega) + b^t(\omega) x_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
0 \leq \mu_i^t(\omega) \perp & \quad K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 && \forall t = 1, ..., T.
\end{aligned} \tag{49}$$

Using parameter $\delta_i \in \{0, 1\}$, which specifies when agent i exercises market power, we can unify conditions (47) and (49) into the following,

$$\begin{aligned}
0 \leq x_i^t(\omega) \perp & \quad CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
0 \leq \mu_i^t(\omega) \perp & \quad K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 && \forall t = 1, ..., T,
\end{aligned} \tag{50}$$

which can in turn be reformulated as the following utility optimization problem,

$$
\begin{aligned}
Z_i(\omega, K_i) = \text{Max} \quad & \sum_{t=1}^{T} \left[\left(p^t(\omega) - CO_i^t(\omega)\right) x_i^t(\omega) - \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right] h^t \\
\text{s.t} \quad & x_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
& x_i^t(\omega) \leq K_i g_i^t(\omega) && (\mu_i^t(\omega) h^t), \quad \forall t = 1, ..., T,
\end{aligned}
\tag{51}
$$

where $Z_i(\omega, K_i)$ denotes the general optimal utility function of a potentially profit-maximizing Cournot producer. We make the utility function depend explicitly on the first-stage investment decision K_i. Optimization problem (51), which is convex with linear constraints, has the following dual formulation:

$$
\begin{aligned}
Z_i(\omega, K_i) = \text{Min} \quad & \sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t K_i + \sum_{t=1}^{T} \delta_i \frac{\left(p^t(\omega) - CO_i^t(\omega) - \mu_i^t(\omega) + s_i^t(\omega)\right)^2}{2b^t(\omega)} h^t \\
\text{s.t} \quad & \mu_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
& s_i^t(\omega) \geq 0 && \forall t = 1, ..., T \\
& CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) - s_i^t(\omega) = 0 && \forall t = 1, ..., T.
\end{aligned}
\tag{52}
$$

Inasmuch as producer i is risk-averse, it will value its second-stage stochastic utility $Z_i()$ via a coherent measure of risk ρ_i associated with a risk set that we denote by $\mathcal{M}_i$. Using the representation theorem (see relationship (2)), the producer will measure its risk as follows:

$$
\rho_i(K_i) = \text{Max}_{\zeta_i \in \mathcal{M}_i} \left(- \sum_{\omega \in \Omega} \zeta_i(\omega) Z_i(\omega, K_i) \right). \tag{53}
$$

When deciding on its investment K_i in the first stage, the producer will strive to maximize its risk-adjusted utility net of the investment cost (or, in an equivalent way, minimize the investment cost plus the risk of its second-stage stochastic utility):

$$
\text{Min}_{K_i \geq 0} \ \left(\rho_i(K_i) + CI_i K_i\right). \tag{54}
$$

It can be shown that because we resort to convex risk measures, $\rho_i(.)$ is a convex function of variable K_i. Therefore, problem (54) is convex and has the following KKT condition:

$$
0 \leq K_i \perp \qquad CI_i + \partial \rho_i(K_i) \geq 0. \tag{55}
$$

Using the envelope theorem applied to problem (53) that gives the expression of the risk ρ_i and the dual formulation of the utility Z_i in (52), we can rewrite (55) as the following,

$$0 \leq K_i \perp \qquad CI_i - \sum_{\omega \in \Omega} \zeta_i^*(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \geq 0, \tag{56}$$

where ζ_i^* is the risk-adjusted probability for producer i that solves optimization problem (53). This condition has a straightforward economic interpretation: the investment K_i is undertaken only when the investment cost can be recovered, in a risk-adjusted expectation, from the spot market through scarcity rents $\mu_i^t(\omega)$. In line with economic theory, this implies that, if agent i is competitive (or a price-taker), its total risk-adjusted profit (comprising first- and second-stage revenues and costs) is always null. On the other hand, when agent i is a price-maker, the inclusion of the Cournot premium $b^t(\omega)x_i^t(\omega)$ in the formation of the scarcity margin makes the total risk-adjusted profit positive.

A.2.3 Market-clearing and the Nash equilibrium

We write the clearing condition of the physical spot market (equating demand with supply at each time step and in each scenario), whose dual variable represents the market price as follows:

$$d^t(\omega) - \sum_{i \in I} x_i^t(\omega) = 0 \quad \forall (\omega, t) \in \Omega \times \{1, ..., T\}.$$

We consider the market to be at equilibrium when all agents simultaneously optimize their (risk-adjusted) utility functions and the spot market clears. This property frames the Nash equilibrium of the game defined by the set of market players (comprising producers and consumers), each striving to optimize their utility in a fully incomplete market, as described in sections A.2.1 and A.2.2. The equilibrium can be found by simultaneously solving the KKT conditions for all agents along with the market clearing equation, which leads to the following complementarity problem,

$$\begin{aligned}
0 \leq x_i^t(\omega) \perp &\quad CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \geq 0 &&\forall (i, \omega, t) \in I \times \Omega \times \{1, ..., T\} \\
0 \leq \mu_i^t(\omega) \perp &\quad K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 &&\forall (i, \omega, t) \in I \times \Omega \times \{1, ..., T\} \\
0 \leq K_i \perp &\quad CI_i - \sum_{\omega \in \Omega} \zeta_i^*(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \geq 0 &&\forall i \in I \\
\zeta_i^* = &\quad \text{Argmax}_{\zeta_i \in \mathcal{M}_i} \left(- \sum_{\omega \in \Omega} \zeta_i(\omega) Z_i(\omega) \right) &&\forall i \in I \\
0 \leq d^t(\omega) \perp &\quad p^t(\omega) - a^t(\omega) + b^t(\omega) d^t(\omega) \geq 0 &&\forall (\omega, t) \in \Omega \times \{1, ..., T\} \\
p^t(\omega) \perp &\quad d^t(\omega) - \sum_{i \in I} x_i^t(\omega) = 0 &&\forall (\omega, t) \in \Omega \times \{1, ..., T\}.
\end{aligned} \tag{57}$$

Utility $Z_i(\omega)$ is derived from (51):

$$Z_i(\omega) = \sum_{t=1}^{T} \left[\left(p^t(\omega) - CO_i^t(\omega) \right) x_i^t(\omega) - \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right] h^t. \tag{58}$$

The formulation of the fully incomplete benchmark in the particular case where risk is valued by the CVaR is provided in Section A.3.1, which is what we use in our numerical application.

A.3 The complementarity formulation when risk aversion is modeled by the CVaR

A.3.1 The fully incomplete market

In this section, we rewrite the equilibrium problem of the fully incomplete market (57) when risk is measured with the CVaR (see relationship (7) of Example 2). We denote by $\alpha_i \in [0,1)$ the degree of risk aversion of producer i, $i \in I$ and by $\alpha_d \in [0,1)$ that of consumers. The problem can then be rewritten as follows:

$$
\begin{aligned}
0 \le x_i^t(\omega) \perp \quad & CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \ge 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \le \mu_i^t(\omega) \perp \quad & K_i g_i^t(\omega) - x_i^t(\omega) \ge 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \le K_i \perp \quad & CI_i - \sum_{\omega\in\Omega} \zeta_i^*(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \ge 0 && \forall i \in I \\
0 \le \zeta_i^*(\omega) \perp \quad & Z_i(\omega) - \lambda_i + \gamma_i(\omega) \ge 0 && \forall (i,\omega) \in I \times \Omega \\
\lambda_i \perp \quad & \sum_{\omega\in\Omega} \zeta_i^*(\omega) - 1 = 0 && \forall i \in I \\
0 \le \gamma_i(\omega) \perp \quad & \frac{\theta(\omega)}{1-\alpha_i} - \zeta_i^*(\omega) \ge 0 && \forall (i,\omega) \in I \times \Omega \\
0 \le d^t(\omega) \perp \quad & p^t(\omega) - a^t(\omega) + b^t(\omega) d^t(\omega) \ge 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\} \\
p^t(\omega) \perp \quad & d^t(\omega) - \sum_{i\in I} x_i^t(\omega) = 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\}.
\end{aligned} \tag{59}
$$

with

$$
Z_i(\omega) = \sum_{t=1}^{T} \left[\left(p^t(\omega) - CO_i^t(\omega) \right) x_i^t(\omega) - \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right] h^t. \tag{60}
$$

A.3.2 The partially incomplete market

In this section, we rewrite the equilibrium problem of the partially incomplete market (23) when risk is measured with the CVaR,

$$
\begin{aligned}
0 \le x_i^t(\omega) \perp \quad & CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \ge 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \le \mu_i^t(\omega) \perp \quad & K_i g_i^t(\omega) - x_i^t(\omega) \ge 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \le K_i \perp \quad & CI_i - \sum_{\omega\in\Omega} \zeta_i^*(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \ge 0 && \forall i \in I \\
w_c^i \perp \quad & p_c^1 - \sum_{\omega\in\Omega} \zeta_i^*(\omega) p_c^2(\omega) = 0 && \forall (i,c) \in I \times \mathcal{C} \\
0 \le \zeta_i^*(\omega) \perp \quad & Z_i(\omega) - \lambda_i + \gamma_i(\omega) \ge 0 && \forall (i,\omega) \in I \times \Omega \\
\lambda_i \perp \quad & \sum_{\omega\in\Omega} \zeta_i^*(\omega) - 1 = 0 && \forall i \in I \\
0 \le \gamma_i(\omega) \perp \quad & \frac{\theta(\omega)}{1-\alpha_i} - \zeta_i^*(\omega) \ge 0 && \forall (i,\omega) \in I \times \Omega \\
0 \le d^t(\omega) \perp \quad & p^t(\omega) - a^t(\omega) + b^t(\omega) d^t(\omega) \ge 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\} \\
w_c^d \perp \quad & p_c^1 - \sum_{\omega\in\Omega} \zeta_d^*(\omega) p_c^2(\omega) = 0 && \forall c \in \mathcal{C} \\
0 \le \zeta_d^*(\omega) \perp \quad & CS(\omega) - \lambda_d + \gamma_d(\omega) \ge 0 && \forall \omega \in \Omega \\
\lambda_d \perp \quad & \sum_{\omega\in\Omega} \zeta_d^*(\omega) - 1 = 0 && \\
0 \le \gamma_d(\omega) \perp \quad & \frac{\theta(\omega)}{1-\alpha_d} - \zeta_d^*(\omega) \ge 0 && \forall \omega \in \Omega \\
p^t(\omega) \perp \quad & d^t(\omega) - \sum_{i\in I} x_i^t(\omega) = 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\} \\
p_c^1 \perp \quad & \sum_{i\in I} w_c^i + w_c^d = 0 && \forall c \in \mathcal{C},
\end{aligned}
\tag{61}
$$

with

$$
Z_i(\omega) = \sum_{t=1}^{T} \left[\left(p^t(\omega) - CO_i^t(\omega) \right) x_i^t(\omega) - \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right] h^t + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^i \tag{62}
$$

and

$$
CS(\omega) = \sum_{t=1}^{T} \left(a^t(\omega) d^t(\omega) - b^t(\omega) \frac{d^t(\omega)^2}{2} - p^t(\omega) d^t(\omega) \right) h^t + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^d. \tag{63}
$$

A.3.3 The fully complete market

In this section, we rewrite the problem of the fully complete market (31) when risk is measured with the CVaR as an equilibrium problem. In that case, the overall level of risk aversion derived from the system's risk set $\mathcal{M}$ is denoted by $\alpha \in [0,1)$,[2]

$$
\begin{aligned}
0 \leq x_i^t(\omega) \perp \quad & CO_i^t(\omega) + \mu_i^t(\omega) - a^t(\omega) + b^t(\omega)\sum_{i\in I} x_i^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \geq 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \leq \mu_i^t(\omega) \perp \quad & K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \leq K_i \perp \quad & CI_i - \sum_{\omega\in\Omega} \zeta^*(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \geq 0 && \forall i \in I \\
0 \leq \zeta^*(\omega) \perp \quad & SU(\omega) - \lambda + \gamma(\omega) \geq 0 && \forall \omega \in \Omega \\
\lambda \perp \quad & \sum_{\omega\in\Omega} \zeta^*(\omega) - 1 = 0 && \\
0 \leq \gamma(\omega) \perp \quad & \frac{\theta(\omega)}{1-\alpha} - \zeta^*(\omega) \geq 0 && \forall \omega \in \Omega,
\end{aligned}
$$

with

$$
SU(\omega) = \sum_{t=1}^{T} \left(\left(a^t(\omega) - CO_i^t(\omega)\right) \sum_{i\in I} x_i^t(\omega) - b^t(\omega) \frac{\left(\sum_{i\in I} x_i^t(\omega)\right)^2}{2} - \sum_{i\in I} \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right) h^t. \tag{64}
$$

A.4 Sketch of the proof of existence of equilibria

To construct the general proof, one would begin by proving that the feasibility sets of all our problems' variables belong to a compact set (after checking that they are not empty). One first proves this proposition for the physical transactions, prices, and investments by showing that second-stage sales are all bounded by a limit set by the linear demand curves (a limit above which prices become negative and KKT conditions can no longer hold, provided that total OPEX + CAPEX costs are positive, which we naturally assume). This in turn will demonstrate that spot prices and investment decisions have to be bounded as well. Risk-adjusted probabilities also belong to a compact set by definition. Second, one has to prove that, when they exist, financial transactions are also bounded, which is the most difficult step in the proof: contract prices are always bounded because of the absence of arbitrage conditions and financial transactions are also bounded because of the non-emptiness of the interior of the system's risk set $\mathcal{M}$. Finally, once one has proven that all our problems' variables can be cast into a compact set, one can resort to standard fixed-points arguments (Brouwer's for instance) to demonstrate the existence of a Nash equilibrium for all our problems. Uniqueness is usually not guaranteed but some results might be drawn in the particular case of polyhedral risk measures (like the CVaR) inasmuch as the problems can all be reformulated as Linear Complementarity Problems (LCPs).

A.5 An efficient algorithm to solve the equilibrium problems

For ease of exposition, we present the algorithm in detail for the case of the partially incomplete market presented in Section 2.3, as its extension to the other cases is quite natural. We start with the KKT conditions of the partially incomplete market reported in (23). We split these conditions into two sets of equations. The

[2] It can be shown that $\alpha = \text{Min}(\text{Min}_{i\in I}\alpha_i, \alpha_d)$.

first corresponds to the physical spot market equilibrium with fixed risk-adjusted probabilities $\zeta_i^q(\omega)$, $i \in I$ (for producers) and $\zeta_d^q(\omega)$ (for consumers), where $q \in \mathbb{N}$ denotes the iteration of the algorithm (by abuse of notation, we might omit writing the dependence of these probabilities on ω when there is no risk of confusion). These equations are the following:

$$
\begin{aligned}
0 \leq x_i^t(\omega) \perp \quad & CO_i^t(\omega) + \mu_i^t(\omega) - p^t(\omega) + \delta_i b^t(\omega) x_i^t(\omega) \geq 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \leq \mu_i^t(\omega) \perp \quad & K_i g_i^t(\omega) - x_i^t(\omega) \geq 0 && \forall (i,\omega,t) \in I \times \Omega \times \{1,...,T\} \\
0 \leq K_i \perp \quad & CI_i - \sum_{\omega\in\Omega} \zeta_i^q(\omega) \left(\sum_{t=1}^{T} \mu_i^t(\omega) g_i^t(\omega) h^t \right) \geq 0 && \forall i \in I \\
0 \leq d^t(\omega) \perp \quad & p^t(\omega) - a^t(\omega) + b^t(\omega) d^t(\omega) \geq 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\} \\
p^t(\omega) \perp \quad & d^t(\omega) - \sum_{i\in I} x_i^t(\omega) = 0 && \forall (\omega,t) \in \Omega \times \{1,...,T\},
\end{aligned} \tag{65}
$$

which defines a complementarity problem that we denote by P-Physical(ζ_i^q, ζ_d^q). This is our first subproblem. Problem P-Physical(ζ_i^q, ζ_d^q) does not explicitly depend on ζ_d^q because, in the Cournot model of physical trade, consumers are passive agents. To make the notation consistent, however, we keep ζ_d^q in the notation of the problem. The obtained problem is in general easy to tackle by standard commercial solvers like PATH (the problem has the same complexity as a standard stochastic Cournot model with investments). From problem P-Physical(ζ_i^q, ζ_d^q), we derive *at the equilibrium* the two following parameters: the physical consumer surplus,

$$PCS^q(\omega) = \sum_{t=1}^{T} \left(a^t(\omega) d^t(\omega) - b^t(\omega) \frac{d^t(\omega)^2}{2} - p^t(\omega) d^t(\omega) \right) h^t, \tag{66}$$

and the physical utility of producers:

$$PZ_i^q(\omega) = \sum_{t=1}^{T} \left(\left(p^t(\omega) - CO_i^t(\omega) \right) x_i^t(\omega) - \delta_i \frac{b^t(\omega)}{2} x_i^t(\omega)^2 \right) h^t, \tag{67}$$

and we keep track of the invested capacities at iteration q, which we denoted by K_i^q, with $i \in I$, and which we concatenate into a vector $\mathbf{K}^q \in \mathbb{R}^n$.

The second set of equations corresponds to the financial and risk valuation model. With this model, we compute the risk-adjusted probabilities and the hedging volumes, given the second-stage stochastic utilities of the agents obtained from problem P-Physical(ζ_i^q, ζ_d^q): $PCS^q(\omega)$ and $PZ_i^q(\omega)$. These equations that define problem P-Risk(PCS^q, PZ^q) are as follows:

$$
\begin{aligned}
&w_c^i && \perp p_c^1 - \sum_{\omega\in\Omega} \zeta_i^*(\omega) p_c^2(\omega) && = 0 \quad \forall (i,c) \in I \times \mathcal{C} \\
&\zeta_i^* && = \text{Argmax}_{\zeta_i \in \mathcal{M}_i} \left(- \sum_{\omega\in\Omega} \zeta_i(\omega) \left(PZ_i^q(\omega) + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^i \right) \right) && \forall i \in I \\
&w_c^d && \perp p_c^1 - \sum_{\omega\in\Omega} \zeta_d^*(\omega) p_c^2(\omega) && = 0 \quad \forall c \in \mathcal{C} \\
&\zeta_d^* && = \text{Argmax}_{\zeta_d \in \mathcal{M}_d} \left(- \sum_{\omega\in\Omega} \zeta_d(\omega) \left(PCS^q(\omega) + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^d \right) \right) && \\
&p_c^1 && \perp \sum_{i\in I} w_c^i + w_c^d && = 0 \quad \forall c \in \mathcal{C}.
\end{aligned} \tag{68}
$$

The risk model P-Risk(PCS^q, PZ^q) cannot in general be reformulated as a single convex optimization problem. We found, here again, that the problem is always easy to solve numerically in its equilibrium form by standard commercial solvers like PATH. A particular instance of P-Risk(PCS^q, PZ^q) when risk is valued via the CVaR is interesting inasmuch as the sub-problem can then be reformulated as a single linear optimization problem as we demonstrate in Appendix A.6. From problem P-Risk(PCS^q, PZ^q), we derive the risk-adjusted probabilities of the next iteration, ζ_i^{q+1} and ζ_d^{q+1}, which leads to the following iterative algorithm:

- **Initialization.** Initialize the risk-adjusted probabilities by setting them equal to the natural probabilities θ:

$$
\begin{aligned}
\zeta_i^0(\omega) &= \theta(\omega) \quad \forall (i,\omega) \in I \times \Omega \\
\zeta_d^0(\omega) &= \theta(\omega) \quad \forall \omega \in \Omega.
\end{aligned} \tag{69}
$$

- **Iteration.** At iteration q, solve the equilibrium problem P-Physical($\zeta_i^{q-1}, \zeta_d^{q-1}$), then compute $PSC^{q-1}(\omega)$ (for all scenarios) and $PZ_i^{q-1}(\omega)$ (for all producers and all scenarios) as in relationships (66) and (67). Then solve problem P-Risk(PCS^{q-1}, PZ^{q-1}) to obtain the risk-adjusted probabilities $\zeta_d^*(\omega)$ and $\zeta_i^*(\omega)$ at optimality. Then compute ζ_i^q and ζ_d^q as follows,

$$
\begin{aligned}
\zeta_i^q(\omega) &= (1-\gamma)\zeta_i^{q-1}(\omega) + \gamma \zeta_i^*(\omega) \quad \forall (i,\omega) \in I \times \Omega \\
\zeta_d^q(\omega) &= (1-\gamma)\zeta_d^{q-1}(\omega) + \gamma \zeta_d^*(\omega) \quad \forall \omega \in \Omega,
\end{aligned} \tag{70}
$$

  with $\gamma \in (0,1]$.

- **Stopping criterion.** Carry on the iteration phase until the invested capacities converge, as in

$$
\|\mathbf{K}^q - \mathbf{K}^{q-1}\| \leq \epsilon, \tag{71}
$$

  where $\epsilon > 0$ is the convergence tolerance level and $\|.\|$ is a norm defined over the space $\mathbb{R}^n$.

By construction, it can be shown that, when convergence is reached, the algorithm finds an equilibrium solution. We were not, however, able to prove that convergence is always reached because of a lack of contracting properties. Numerically, convergence can be facilitated by the introduction of the learning rate γ in the updating of the risk-adjusted probabilities (with $\gamma \in (0,1]$, see equations (70)) which retains some memory in the algorithm's updating procedure. In that case, the convergence point, however, becomes sensitive to the initialization values. Therefore, we amend the initialization step of the algorithm to test various initial probability distributions to verify that the convergence point remains the same.

A.6 The integrability of sub-problem P-Risk(PCS^q, PZ^q) in the case of the CVaR

In this section, we prove that problem P-Risk(PCS^q, PZ^q) (see expression (68)) can be reformulated as a single linear optimization problem when risk is valued via the CVaR risk measure presented in relationship (7) of Example 2. In that case, we will exploit the reformulation of the problem provided in Section A.3.2. We remind readers that we denote by α_i the degree of risk aversion of producer i and by α_d that of consumers. Problem P-Risk(PCS^q, PZ^q) can hence be rewritten as follows:

$$
\begin{aligned}
& w_c^i && \perp p_c^1 - \sum_{\omega\in\Omega} \zeta_i^*(\omega) p_c^2(\omega) && = 0 \quad \forall (i,c) \in I \times \mathcal{C} \\
0 \leq\ & \zeta_i^*(\omega) && \perp \left(PZ_i^q(\omega) + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^i \right) - \lambda_i + \gamma_i(\omega) && \geq 0 \quad \forall (i,\omega) \in I \times \Omega \\
& \lambda_i && \perp \sum_{\omega\in\Omega} \zeta_i^*(\omega) - 1 && = 0 \quad \forall i \in I \\
0 \leq\ & \gamma_i(\omega) && \perp \frac{\theta(\omega)}{1-\alpha_i} - \zeta_i^*(\omega) && \geq 0 \quad \forall (i,\omega) \in I \times \Omega \\
& w_c^d && \perp p_c^1 - \sum_{\omega\in\Omega} \zeta_d^*(\omega) p_c^2(\omega) && = 0 \quad \forall c \in \mathcal{C} \\
0 \leq\ & \zeta_d^*(\omega) && \perp \left(PCS^q(\omega) + \sum_{c\in\mathcal{C}} \left(p_c^2(\omega) - p_c^1 \right) w_c^d \right) - \lambda_d + \gamma_d(\omega) && \geq 0 \quad \forall \omega \in \Omega \\
& \lambda_d && \perp \sum_{\omega\in\Omega} \zeta_d^*(\omega) - 1 && = 0 \\
0 \leq\ & \gamma_d(\omega) && \perp \frac{\theta(\omega)}{1-\alpha_d} - \zeta_d^*(\omega) && \geq 0 \quad \forall \omega \in \Omega \\
& p_c^1 && \perp \sum_{i\in I} w_c^i + w_c^d && = 0 \quad \forall c \in \mathcal{C}.
\end{aligned} \tag{72}
$$

It can be verified that it is equivalent to the following linear problem (dual variables are written next to their constraints):

$$
\begin{aligned}
\text{Min} \quad & \sum_{i\in I}\left(-\lambda_i + \sum_{\omega\in\Omega}\gamma_i(\omega)\frac{\theta(\omega)}{1-\alpha_i}\right) + \left(-\lambda_d + \sum_{\omega\in\Omega}\gamma_d(\omega)\frac{\theta(\omega)}{1-\alpha_d}\right) \\
\text{s.t} \quad & \gamma_i(\omega) \geq 0 \qquad \forall (i,\omega) \in I\times\Omega \\
& \gamma_d(\omega) \geq 0 \qquad \forall \omega \in \Omega \\
& w_c^i \in \mathbb{R} \qquad \forall (i,c) \in I\times\mathcal{C} \\
& w_c^d \in \mathbb{R} \qquad \forall c \in \mathcal{C} \\
& \lambda_i \in \mathbb{R} \qquad \forall i \in I \\
& \lambda_d \in \mathbb{R} \\
& \left(PZ_i^q(\omega) + \sum_{c\in\mathcal{C}} p_c^2(\omega) w_c^i\right) - \lambda_i + \gamma_i(\omega) \geq 0 \qquad (\zeta_i^*(\omega)), \qquad \forall (i,\omega) \in I\times\Omega \\
& \left(PCS^q(\omega) + \sum_{c\in\mathcal{C}} p_c^2(\omega) w_c^d\right) - \lambda_d + \gamma_d(\omega) \geq 0 \qquad (\zeta_d^*(\omega)), \qquad \forall \omega \in \Omega \\
& \sum_{i\in I} w_c^i + w_c^d = 0 \qquad (p_c^1), \qquad \forall c \in \mathcal{C}.
\end{aligned} \tag{73}
$$

This reformulation can be leveraged to accelerate the convergence of our algorithm presented in Section A.5. Indeed, to mitigate large fluctuations in contracted volumes during the algorithm's iterations, we can add the classical regularization term to the objective function of optimization problem (73), of the form $reg \sum_{c\in\mathcal{C}} \left((w_c^d)^2 + \sum_{i\in I}(w_c^i)^2\right)$, with $reg \geq 0$. In our numerical applications, we set parameter reg to the very small value of 10^{-4} to accelerate numerical convergence without altering the problem.

A.7 Additional numerical results for Section 4

Figure 9 shows the evolution of total consumer surplus with the level of risk aversion in the cases of a fully incomplete market and a partially incomplete market for risk, under market power, with the objective of better visualizing the drop in consumer surplus after partial completion.

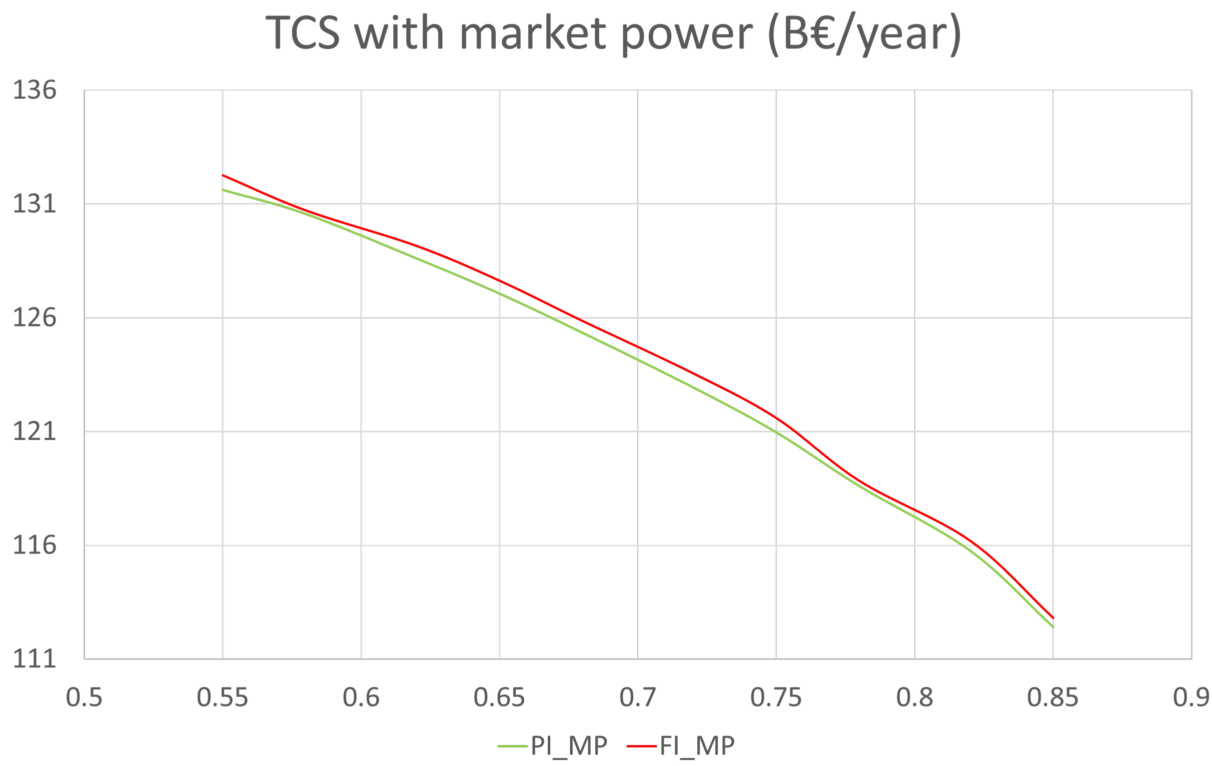

Figure 9: The risk-adjusted consumer surplus as a function of the degree of risk aversion for an elasticity of -0.15 when there is market power.

A.8 Mathematical developments of the theoretical illustration of Section 5

In this section, we develop in detail the mathematical illustration of Section 5. We simplify the representation of the market and the agents' risk aversion to obtain closed-form results. More precisely, we depart from the general models of Section 2 as follows: i) we assume that only the incumbent operates in the market. We hence eliminate index i from the notation. We also consider only one time-block in the second stage, which allows us to eliminate index t from the notation. ii) We include investment costs (CAPEX) directly in total production costs (as in an LCOE approach). We hence overlook the capacity constraint in the second stage. (This situation is relevant in the case of oversupply.) Basic Cournot withholding strategy can still occur in the spot market. iii) We model risk aversion using the convex entropic risk measure as defined in Example 1. We denote the degree of risk aversion by ν_p for the producer and ν_d for the consumer. iv) We consider the partially incomplete market (because we are interested in the contracting strategy) and the existence of one hedging contract. We hence eliminate the contract index c from the notation. More precisely, we use $w_p = w$ to denote the contract volume purchased by the producer from consumers (the corresponding variable for consumers is hence $w_d = -w$ because of the financial clearing). The rest of the notation remains unchanged. Thanks to these simplifications, we could derive a number of theoretical results pertaining to the level of contract exchanged in the industry which already provides a better understanding of how a market player leverages its contracting decision when it exercises market power. We could not, unfortunately, derive clean theoretical results on social welfare. Therefore, only a number of numerical simulations are offered to highlight conditions under which social welfare might decrease when one partially completes the market.

All intermediary calculations are presented in Appendix A.9.1. The resulting equilibrium problem, which we denote by $P(\delta)$ is: find the *contract volume* $w \in \mathbb{R}$ and the *contract price* $p^1 \in \mathbb{R}$ such that:

$$\begin{aligned} F_1(w, p^1, \delta) &:= \sum_{\omega \in \Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega) + \left(p^2(\omega) - p^1\right) w\right)\right) \left(p^1 - p^2(\omega)\right) &= \quad 0 \\ F_2(w, p^1, \delta) &:= \sum_{\omega \in \Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega) - \left(p^2(\omega) - p^1\right) w\right)\right) \left(p^1 - p^2(\omega)\right) &= \quad 0. \end{aligned} \tag{74}$$

with $\Pi(\omega)$ and $\tau(\omega)$ being expressed in relationships (80) and (87). To assess the impact of market power on the contract volume, we explicitly write the dependence of problem $P(\delta)$ with respect to parameter $\delta \in \{0, 1\}$.[3] We concatenate functions F_1 and F_2 into a single function $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ with

$$F(w, p^1, \delta) := \begin{pmatrix} F_1(w, p^1, \delta) \\ F_2(w, p^1, \delta) \end{pmatrix}, \quad \forall (w, p^1, \delta) \in \mathbb{R}^3, \tag{75}$$

to rewrite problem $P(\delta)$ as:

$$\text{Find } w \in \mathbb{R} \text{ and } p^1 \in \mathbb{R} \text{ such that } F(w, p^1, \delta) = 0. \tag{76}$$

In the following, we treat parameter δ as a continuous number belonging to $[0, 1]$. We subsequently apply and interpret our results by considering only the realistic cases where δ equals either 0 or 1. All proofs can be found in Section A.9.2 of the appendix.

Theorem 1. *For any $\delta \in [0, 1]$, problem $P(\delta)$ has a unique solution that we denote by $(w(\delta), p^1(\delta))$.*

The next theorem provides a result pertaining to the variations in the contract volume $w(.)$ with respect to the market power variable δ.

Theorem 2. *Functions $p^1(.)$ and $w(.)$ are continuous and differentiable with respect to δ. Moreover, for sufficiently high levels of risk aversion ν_p and ν_d, the exchanged contract volume $|w(.)|$ is a decreasing function of δ:*

$$\exists A \geq 0 \text{ such that } \nu_p \geq A \text{ and } \nu_d \geq A \Longrightarrow |w(.)| \text{ decreases with } \delta, \tag{77}$$

which implies in particular that $|w(\delta = 0)| \geq |w(\delta = 1)|$.

Economically, Theorem 2 states that under a high degree of risk aversion, the hedging contract is less used when the producer exercises market power. This is in line with our numerical results obtained with the French case in Section 4. Of course, this threshold depends on the level of elasticity of consumption with respect to the price.

[3] Recall that this parameter determines whether the producer exercises market power or not.

	CO	p_{ref}	q_{ref}	p_2	θ
ω_1	0	0.1	1	0.5	0.33
ω_2	0	0.2	1.1	1.5	0.33
ω_3	0	0.6	2	2	0.33

Table 1: The dataset considered in the numerical application.

In Figure 10, we show via numerical simulations how this result translates in terms of welfare. The equilibrium model (74) is solved with the Newton algorithm. We consider three scenarios with the generic data presented in Table 1, where $p_{ref}(\omega)$ is the reference price and $q_{ref}(\omega)$ is the reference consumption which we use to build the inverse demand functions. We examine the evolution of total market surplus as a function of ν_p and ν_d (levels of risk aversion) when the market is fully incomplete and when the contract is added to partially complete the market. The left-hand side of Figure 10 shows in black the region where market surplus decreases after the introduction of the contract for an elasticity of -0.1 while the right-hand side shows this region for an elasticity of -0.2. As a sanity check, we verify beforehand that when the incumbent does not exercise market power, this region is always empty confirming, at least for the small dataset we consider in this section, the result that proper risk-sharing contracts always increase welfare in competitive markets. These results confirm our findings of Section 4.2: when there exists market power, partially completing the market can decrease welfare and increasing the level of elasticity of consumption with respect to the price reduces the likelihood of this issue. Consistent with Theorem 2, we also observe that the problem arises when market agents become too risk-averse.

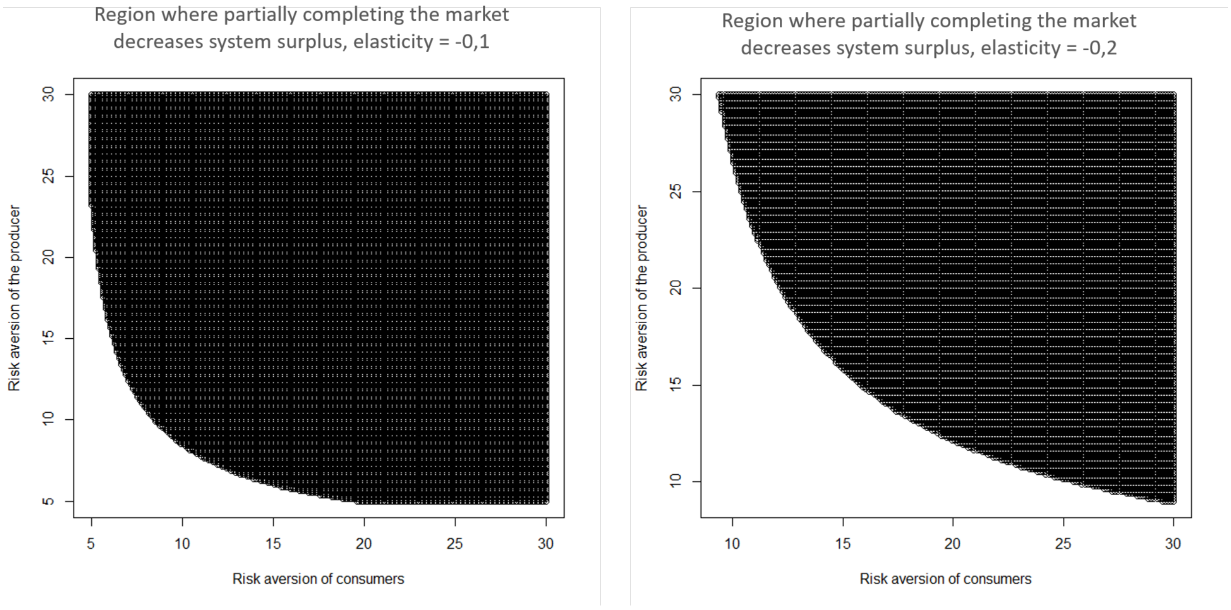

Figure 10: Region where market surplus decreases after adding the contract. Left for an elasticity of -0.1 and right for an elasticity of -0.2.

A.9 Intermediary calculations and proofs of Section 5

A.9.1 The elaboration of the equilibrium problem in the case of a monopoly and the entropic risk measure

The second-stage objective of the producer is to maximize its utility $Z(\omega, w_p)$, which we derive from expression (18),

$$
\begin{aligned}
Z(\omega, w_p) = & \\
\text{Max} \quad & (p(\omega) - CO(\omega))\, x(\omega) - \delta \frac{b(\omega)}{2} x(\omega)^2 + \left(p^2(\omega) - p^1\right) w_p \\
\text{s.t} \quad & x(\omega) \geq 0, \qquad (78)
\end{aligned}
$$

which leads to the expression of optimal production $x(\omega)$ denoted below. (We assume that the intercept of the inverse demand function is always superior to the total production cost of electricity: $a(\omega) \geq CO(\omega)$ for all scenarios.) Optimal production is

$$
x(\omega) = \frac{a(\omega) - CO(\omega)}{b(\omega)(1+\delta)}, \qquad (79)
$$

and, if we use $\Pi(\omega)$ to denote the revenue accrued from the spot market at the optimum,

$$
\Pi(\omega) := \frac{\delta}{(1+\delta)^2} \frac{\left(a(\omega) - CO(\omega)\right)^2}{b(\omega)}, \qquad (80)
$$

the utility will equal:

$$
Z(\omega, w_p) = \Pi(\omega) + \left(p^2(\omega) - p^1\right) w_p. \qquad (81)
$$

The producer values its second-stage stochastic utility $Z(\omega, w_p)$ via the convex entropic risk measure $\rho^{entr}_{\nu_p}$ with parameter ν_p to select its risk-minimizing contracting decision w_p,

$$
\begin{aligned}
\text{Min} \quad & \rho^{entr}_{\nu_p}\left(\Pi(\omega) + \left(p^2(\omega) - p^1\right) w_p\right) \\
\text{s.t} \quad & w_p \in \mathbb{R}, \qquad (82)
\end{aligned}
$$

and the equivalent KKT conditions are,

$$
w_p \perp \qquad p^1 - \sum_{\omega \in \Omega} \zeta^{entr}_{\nu_p}(\omega) p^2(\omega) = 0, \qquad (83)
$$

where $\zeta^{entr}_{\nu_p}$ is the entropic risk-adjusted probability for the producer that we derive from expression (6):

$$\zeta^{entr}_{\nu_p}(\omega) = \frac{\theta(\omega)\exp\left(-\nu_p\Big(\Pi(\omega) + (p^2(\omega) - p^1)\, w_p\Big)\right)}{\sum_{\omega'\in\Omega}\theta(\omega')\exp\left(-\nu_p\Big(\Pi(\omega') + (p^2(\omega') - p^1)\, w_p\Big)\right)}, \quad \forall\omega\in\Omega. \tag{84}$$

Relationship (83) can then be reformulated as:

$$\sum_{\omega\in\Omega}\theta(\omega)\exp\left(-\nu_p\Big(\Pi(\omega) + \Big(p^2(\omega) - p^1\Big)\, w_p\Big)\right)\Big(p^1 - p^2(\omega)\Big) = 0. \tag{85}$$

The second-stage consumer surplus can be derived from relationship (8) by replacing the consumption variable with the expression of the supply obtained in relationship (79), as a consequence of the physical clearing of the spot market:

$$CS(\omega, w_d) = \frac{1}{2(1+\delta)^2}\frac{\Big(a(\omega) - CO(\omega)\Big)^2}{b(\omega)} + \Big(p^2(\omega) - p^1\Big)\, w_d. \tag{86}$$

If we denote the physical surplus by $\tau(\omega)$ as in

$$\tau(\omega) := \frac{1}{2(1+\delta)^2}\frac{\Big(a(\omega) - CO(\omega)\Big)^2}{b(\omega)}, \tag{87}$$

we can rewrite the consumer surplus as follows:

$$CS(\omega, w_d) = \tau(\omega) + \Big(p^2(\omega) - p^1\Big)\, w_d. \tag{88}$$

In the first stage, consumers optimize their hedging contract volume w_d by maximizing their risk-adjusted surplus estimated via the convex entropic risk measure $\rho^{entr}_{\nu_d}$ with parameter ν_d,

$$\begin{aligned} \text{Min} \quad & \rho^{entr}_{\nu_d}\Big(\tau(\omega) + \Big(p^2(\omega) - p^1\Big)\, w_d\Big) \\ \text{s.t} \quad & w_d \in \mathbb{R}, \end{aligned} \tag{89}$$

and the equivalent KKT conditions are,

$$w_d \perp \qquad p^1 - \sum_{\omega\in\Omega}\zeta^{entr}_{\nu_d}(\omega)p^2(\omega) = 0, \tag{90}$$

with

$$\zeta^{entr}_{\nu_d}(\omega) = \frac{\theta(\omega)\exp\left(-\nu_d\Big(\tau(\omega) + (p^2(\omega) - p^1)\, w_d\Big)\right)}{\sum_{\omega'\in\Omega}\theta(\omega')\exp\left(-\nu_d\Big(\tau(\omega') + (p^2(\omega') - p^1)\, w_d\Big)\right)}, \quad \forall\omega\in\Omega. \tag{91}$$

Relationship (90) can then be reformulated as:

$$\sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\Big(\tau(\omega) + \Big(p^2(\omega) - p^1\Big)\, w_d\Big)\right) \Big(p^1 - p^2(\omega)\Big) = 0. \tag{92}$$

If we use w to denote the contract volume purchased by the producer w_p and if we replace the volume w_d by $-w$, our equilibrium problem, which we denote by $P(\delta)$, can be simplified into the following: find $w \in \mathbb{R}$ and $p^1 \in \mathbb{R}$ such that:

$$\begin{aligned} F_1(w, p^1, \delta) &:= \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\Big(\Pi(\omega) + \Big(p^2(\omega) - p^1\Big)\, w\Big)\right) \Big(p^1 - p^2(\omega)\Big) = 0 \\ F_2(w, p^1, \delta) &:= \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\Big(\tau(\omega) - \Big(p^2(\omega) - p^1\Big)\, w\Big)\right) \Big(p^1 - p^2(\omega)\Big) = 0. \end{aligned} \tag{93}$$

A.9.2 Proof of theorem 1

Existence can be obtained by standard fixed-point theorems. We elaborate on an alternative analytical proof, however, to introduce some notation and results that will help us present the remaining developments.

Let us fix a $\delta \in [0, 1]$ and study the system of equations $F = 0$ as functions of variables (w, p^1):

$$\begin{aligned} F_1(w, p^1) &= \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\Big(\Pi(\omega) + \Big(p^2(\omega) - p^1\Big)\, w\Big)\right) \Big(p^1 - p^2(\omega)\Big) = 0 \\ F_2(w, p^1) &= \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\Big(\tau(\omega) - \Big(p^2(\omega) - p^1\Big)\, w\Big)\right) \Big(p^1 - p^2(\omega)\Big) = 0. \end{aligned} \tag{94}$$

These functions are indefinitely differentiable, with:[4]

$$\frac{\partial F_1}{\partial w}(w, p^1) = \nu_p \textstyle\sum_{\omega\in\Omega} \theta(\omega) \exp\Big(-\nu_p\Big(\Pi(\omega) + \big(p^2(\omega) - p^1\big) w\Big)\Big) \Big(p^1 - p^2(\omega)\Big)^2 > 0 \tag{95}$$

$$\frac{\partial F_2}{\partial w}(w, p^1) = -\nu_d \textstyle\sum_{\omega\in\Omega} \theta(\omega) \exp\Big(-\nu_d\Big(\tau(\omega) - \big(p^2(\omega) - p^1\big) w\Big)\Big) \Big(p^1 - p^2(\omega)\Big)^2 < 0 \tag{96}$$

$$\frac{\partial F_1}{\partial p^1}(w, p^1) = \textstyle\sum_{\omega\in\Omega} \theta(\omega) \exp\Big(-\nu_p\Big(\Pi(\omega) + \big(p^2(\omega) - p^1\big) w\Big)\Big) + \nu_p w F_1(w, p^1) \tag{97}$$

$$\frac{\partial F_2}{\partial p^1}(w, p^1) = \textstyle\sum_{\omega\in\Omega} \theta(\omega) \exp\Big(-\nu_d\Big(\tau(\omega) - \big(p^2(\omega) - p^1\big) w\Big)\Big) - \nu_d w F_2(w, p^1). \tag{98}$$

Consider then p_1 fixed in $\mathbb{R}$. We know that the limit of $F_1(., p^1)$ is $-\infty$ when $w \longrightarrow -\infty$ and that $F_1(., p^1) \longrightarrow +\infty$ when $w \longrightarrow +\infty$. Because $\frac{\partial F_1}{\partial w}(w, p^1) > 0$ for all $w \in \mathbb{R}$, we derive that there exists a unique $w_1(p^1) \in \mathbb{R}$ such that $F_1(w_1(p^1), p^1) = 0$. Because $\frac{\partial F_1}{\partial w}(w(p^1), p^1) > 0$, we can resort to the implicit functions theorem to

[4] We assume that the contract price $p^2(\omega)$ is not constant across scenarios. Otherwise, the contract would not generate any risk-mitigation interest.

state that $w_1(.)$ is a continuous function of p^1 whose derivative can be calculated as:

$$\frac{\partial w_1}{\partial p^1}(p^1) = -\frac{\frac{\partial F_1}{\partial p^1}(w_1(p^1), p^1)}{\frac{\partial F_1}{\partial w}(w_1(p^1), p^1)} = -\frac{\sum_{\omega\in\Omega}\theta(\omega)\exp\Big(-\nu_p\Big(\Pi(\omega)+\big(p^2(\omega)-p^1\big)w\Big)\Big)}{\nu_p\sum_{\omega\in\Omega}\theta(\omega)\exp\Big(-\nu_p\Big(\Pi(\omega)+\big(p^2(\omega)-p^1\big)w\Big)\Big)\Big(p^1-p^2(\omega)\Big)^2} < 0, \quad (99)$$

which shows that $w_1(.)$ is a strictly decreasing function of p^1. A similar line of reasoning will show that there exists a unique $w_2(p^1) \in \mathbb{R}$ such that $F_2(w_2(p^1), p^1) = 0$ and that $w_2(.)$ is a continuous, differentiable, and strictly increasing function of p^1.

In particular, $w_1(.)$ has a limit when $p_1 \longrightarrow -\infty$. We know that $F_1(w_1(p^1), p^1) = 0$ and, given the expression of F_1 in relationship (94), we can show that the limit of $w_1(.)$ cannot be finite when $p^1 \longrightarrow -\infty$ and, because $w_1(.)$ is a decreasing function, this limit must then equate $+\infty$. In a similar manner, we prove that $\lim_{p^1\longrightarrow+\infty} w_1(p^1) = -\infty$, $\lim_{p^1\longrightarrow-\infty} w_2(p^1) = -\infty$, and $\lim_{p^1\longrightarrow+\infty} w_2(p^1) = +\infty$. Therefore, if we consider function $\Big(w_1 - w_2\Big)(.)$, we can state that it is continuously differentiable with respect to p^1, that it is strictly decreasing, and that it has limit $+\infty$ when $p^1 \longrightarrow -\infty$ and $-\infty$ when $p^1 \longrightarrow +\infty$. This implies the existence of a unique $p^1 \in \mathbb{R}$ such that $w_1(p^1) - w_2(p^1) = 0$, which in turn induces the existence of a unique couple (w, p^1) (with $w := w_1(p^1) = w_2(p^1)$) verifying $F(w, p^1) = 0$. □

A.9.3 Proof of theorem 2

For a given $\delta \in \mathbb{R}$, variables w and p^1 are implicitly given as follows,

$$F(w, p^1, \delta) = \begin{pmatrix} F_1(w, p^1, \delta) \\ F_2(w, p^1, \delta) \end{pmatrix} = 0, \quad (100)$$

with

$$\begin{aligned} F_1(w, p^1, \delta) &= \sum_{\omega\in\Omega}\theta(\omega)\exp\Big(-\nu_p\Big(\Pi(\omega,\delta) + \Big(p^2(\omega) - p^1\Big)w\Big)\Big)\Big(p^1 - p^2(\omega)\Big) = 0 \\ F_2(w, p^1, \delta) &= \sum_{\omega\in\Omega}\theta(\omega)\exp\Big(-\nu_d\Big(\tau(\omega,\delta) - \Big(p^2(\omega) - p^1\Big)w\Big)\Big)\Big(p^1 - p^2(\omega)\Big) = 0, \end{aligned} \quad (101)$$

where we have made explicit the dependence of the producer's profit $\Pi(\omega, \delta)$ and the consumer's spot market surplus $\tau(\omega, \delta)$ with respect to δ, as shown in relationships (80) and (87). Function F is continuously differentiable and its Jacobian matrix with respect to variables (w, p^1) expressed at a point (w, p^1, δ) satisfying the implicit relationship (100) can be obtained from relationships (95)-(98),

$$\partial_{(w,p^1)}F(w, p^1, \delta) = \begin{pmatrix} \frac{\partial F_1}{\partial w}(w, p^1, \delta) & \frac{\partial F_1}{\partial p^1}(w, p^1, \delta) \\ \frac{\partial F_2}{\partial w}(w, p^1, \delta) & \frac{\partial F_2}{\partial p^1}(w, p^1, \delta) \end{pmatrix}, \quad (102)$$

with

$$\frac{\partial F_1}{\partial w}(w,p^1,\delta) = \nu_p \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega,\delta)+\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)^2 > 0 \quad (103)$$

$$\frac{\partial F_2}{\partial w}(w,p^1,\delta) = -\nu_d \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega,\delta)-\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)^2 < 0 \quad (104)$$

$$\frac{\partial F_1}{\partial p^1}(w,p^1,\delta) = \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega,\delta)+\left(p^2(\omega)-p^1\right)w\right)\right) > 0 \quad (105)$$

$$\frac{\partial F_2}{\partial p^1}(w,p^1,\delta) = \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega,\delta)-\left(p^2(\omega)-p^1\right)w\right)\right) > 0, \quad (106)$$

and its determinant $|\partial_{(w,p^1)}F(w,p^1,\delta)|$ is therefore always positive. One can then state that the Jacobian matrix $\partial_{(w,p^1)}F(w,p^1,\delta)$ is always invertible and invoke the implicit functions theorem to obtain:

- Variables (w,p^1) can be expressed locally as continuously differentiable functions of δ. This result has already been proved *globally* in Theorem 1, where we have also proved the unicity of those functions.
- The gradient of (w,p^1) with respect to δ satisfies

$$\begin{pmatrix} \frac{\partial w}{\partial \delta}(\delta) \\ \frac{\partial p^1}{\partial \delta}(\delta) \end{pmatrix} = -\left(\partial_{(w,p^1)}F(w,p^1,\delta)\right)^{-1} \begin{pmatrix} \frac{\partial F_1}{\partial \delta}(w,p^1,\delta) \\ \frac{\partial F_2}{\partial \delta}(w,p^1,\delta) \end{pmatrix}, \quad (107)$$

  where $\left(\partial_{(w,p^1)}F(w,p^1,\delta)\right)^{-1}$ is the inverse matrix of $\partial_{(w,p^1)}F(w,p^1,\delta)$.

Matrix $\left(\partial_{(w,p^1)}F(w,p^1,\delta)\right)^{-1}$ can be calculated from (102) as follows

$$\left(\partial_{(w,p^1)}F(w,p^1,\delta)\right)^{-1} = \frac{1}{|\partial_{(w,p^1)}F(w,p^1,\delta)|}\begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (108)$$

with

$$A = \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega,\delta)-\left(p^2(\omega)-p^1\right)w\right)\right) \quad (109)$$

$$B = -\sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega,\delta)+\left(p^2(\omega)-p^1\right)w\right)\right) \quad (110)$$

$$C = \nu_d \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega,\delta)-\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)^2 \quad (111)$$

$$D = \nu_p \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega,\delta)+\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)^2, \quad (112)$$

and the derivatives of F_1 and F_2 with respect to δ can be expressed using the explicit dependence of Π and τ with respect to δ as reported in equations (80) and (87),

$$\begin{pmatrix} \frac{\partial F_1}{\partial \delta}(w,p^1,\delta) \\ \frac{\partial F_2}{\partial \delta}(w,p^1,\delta) \end{pmatrix} = \begin{pmatrix} -\nu_p \frac{1-\delta}{(1+\delta)^3} \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_p\left(\Pi(\omega,\delta)+\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)\frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \\ \nu_d \frac{1}{(1+\delta)^3} \sum_{\omega\in\Omega} \theta(\omega) \exp\left(-\nu_d\left(\tau(\omega,\delta)-\left(p^2(\omega)-p^1\right)w\right)\right)\left(p^1-p^2(\omega)\right)\frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \end{pmatrix}, \quad (113)$$

which, given relationships (107) and (108), implies that:

$$\frac{\partial w}{\partial \delta}(\delta) = -\frac{1}{|\partial_{(w,p^1)} F(w,p^1,\delta)|}\Big(A\frac{\partial F_1}{\partial \delta}(w,p^1,\delta) + B\frac{\partial F_2}{\partial \delta}(w,p^1,\delta)\Big). \tag{114}$$

After simplifications, one obtains (for ease of notation, we denote the term $|\partial_{(w,p^1)} F(w,p^1,\delta)|$ by $|\partial_{(w,p^1)} F|$ in the remaining developments)

$$\frac{\partial w}{\partial \delta}(\delta) = \frac{1}{|\partial_{(w,p^1)} F|}\frac{1}{(1+\delta)^3}\Bigg(\sum_{\omega\in\Omega}\theta(\omega)\exp(-\nu_p\phi(\omega))\Bigg)\Bigg(\sum_{\omega\in\Omega}\theta(\omega)\exp(-\nu_d\psi(\omega))\Bigg)\Big(E+F\Big), \tag{115}$$

with variables $\phi(\omega)$ and $\psi(\omega)$ defined as

$$\phi(\omega) := \Pi(\omega) + \Big(p^2(\omega) - p^1\Big)w \tag{116}$$

$$\psi(\omega) := \tau(\omega) - \Big(p^2(\omega) - p^1\Big)w, \tag{117}$$

and terms E and F defined as

$$E := \nu_p(1-\delta)\mathbb{E}_{\zeta^{entr}_{\nu_p}}\Big[\Big(p^1 - p^2(\omega)\Big)\frac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big] \tag{118}$$

$$F := \nu_d\mathbb{E}_{\zeta^{entr}_{\nu_d}}\Big[\Big(p^1 - p^2(\omega)\Big)\frac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big], \tag{119}$$

where $\mathbb{E}(.)$ denotes the expectation operator and $\zeta^{entr}_{\nu_p}$ and $\zeta^{entr}_{\nu_d}$ denote the entropic risk-adjusted probabilities of the producer and consumers, respectively:

$$\zeta^{entr}_{\nu_p}(\omega) = \frac{\theta(\omega)\exp\Big(-\nu_p\Big(\Pi(\omega,\delta) + (p^2(\omega)-p^1)\,w\Big)\Big)}{\sum_{\omega'\in\Omega}\theta(\omega')\exp\Big(-\nu_p\Big(\Pi(\omega',\delta) + (p^2(\omega')-p^1)\,w\Big)\Big)} \tag{120}$$

$$\zeta^{entr}_{\nu_d}(\omega) = \frac{\theta(\omega)\exp\Big(-\nu_d\Big(\tau(\omega,\delta) - (p^2(\omega)-p^1)\,w\Big)\Big)}{\sum_{\omega'\in\Omega}\theta(\omega')\exp\Big(-\nu_d\Big(\tau(\omega',\delta) - (p^2(\omega')-p^1)\,w\Big)\Big)}. \tag{121}$$

As a consequence, when one multiplies both terms of relationship (115) by w, one can state that $w\frac{\partial w}{\partial \delta}$ has the sign of $(E+F)w$, a term that we denote by G and which is equal to:

$$G = \nu_p(1-\delta)\mathbb{E}_{\zeta^{entr}_{\nu_p}}\Big[\Big(p^1 - p^2(\omega)\Big)\,w\tfrac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big] + \nu_d\mathbb{E}_{\zeta^{entr}_{\nu_d}}\Big[\big(p^1 - p^2(\omega)\big)\,w\tfrac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big]. \tag{122}$$

From the expression of $\zeta^{entr}_{\nu_p}$ (see equation (120)), one can deduce that $\nu_p(1-\delta)\mathbb{E}_{\zeta^{entr}_{\nu_p}}\Big[\big(p^1 - p^2(\omega)\big)\,w\tfrac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big]$ converges to zero when $\nu_p \longrightarrow +\infty$. In that case, G becomes equivalent to the second term:

$$G \sim \nu_d\mathbb{E}_{\zeta^{entr}_{\nu_d}}\Big[\Big(p^1 - p^2(\omega)\Big)\,w\tfrac{(a(\omega)-CO(\omega))^2}{b(\omega)}\Big]. \tag{123}$$

The right-hand side of relationship (123) can be split in two terms:

$$\nu_d \left(\sum_{\omega/\psi(\omega)>0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right] + \sum_{\omega/\psi(\omega)\leq 0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right] \right)$$

Given the expression of $\zeta_{\nu_d}^{entr}$ (see equation (121)), one can state that the first term

$$\sum_{\omega/\psi(\omega)>0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right]$$

becomes negligible with respect to the second term

$$\sum_{\omega/\psi(\omega)\leq 0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right]$$

when $\nu_d \longrightarrow +\infty$. This implies that $w\frac{\partial w}{\partial \delta}$ has the sign of

$$\sum_{\omega/\psi(\omega)\leq 0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right]$$

for sufficiently high ν_p and ν_d.

If $\psi(\omega) \leq 0$, then by relationship (117), one can write that:

$$\begin{aligned}
& \tau(\omega) - \left(p^2(\omega) - p^1\right) w && \leq 0 && (124)\\
\Longrightarrow\quad & \left(p^1 - p^2(\omega)\right) w && \leq -\tau(\omega) && (125)\\
\Longrightarrow\quad & \left(p^1 - p^2(\omega)\right) w && \leq 0 && (126)\\
\Longrightarrow\quad & \textstyle\sum_{\omega/\psi(\omega)\leq 0} \zeta_{\nu_d}^{entr}(\omega) \left[\left(p^1 - p^2(\omega)\right) w \frac{(a(\omega)-CO(\omega))^2}{b(\omega)} \right] && \leq 0. && (127)
\end{aligned}$$

To summarize, we have demonstrated that, for sufficiently high values of ν_p and ν_d, $w\frac{\partial w}{\partial \delta}$ is non-positive. Therefore, w^2 is a decreasing function of δ, and so is $|w|$. □

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