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Optimal trade execution under endogenous order flow (e-Appendix)

This e-Appendix provides supplementary materials, including extended technical details and additional results, to support the findings presented in the main paper. It is intended to enhance transparency, reproducibility, and provide deeper insights for interested readers.

Appendix A: Optimal execution with generalized Hawkes kernels: The sum of exponential case

In this section, we broaden the scope of our preceding analyses by considering a more general class of Hawkes processes, focusing particularly on the sum of exponential kernels. This approach offers several key advantages in financial modeling: superior fit to market data, efficient estimation, and versatility in approximation.

A.1. Properties and approximation of sum of exponential kernels

The sum of exponential kernels is given by a function of the form $h_n(s) = \sum_{i=1}^{n} \alpha_i e^{-\beta_i s}$ that satisfies the integrability condition $\int_0^\infty h_n(s)ds < 1$. We assume that market buy/sell order arrivals follow independent Hawkes processes $N^{b/s}$ with respective intensities

$$I_t^{b/s} = \mu + \int_0^t h_n(t-u)\, dN_u^{b/s} \tag{A.1}$$

For simplification and without compromising generality, we focus our analysis on a non-side specific Hawkes process N, governed by the intensity function:

$$I_t = \mu + \int_0^t h_n(t-u)\, dN_u. \tag{A.2}$$

The sum of exponential kernels offers three primary advantages for financial modeling:

1. **Superior Fit**: According to Lallouache and Challet (2016), this kernel demonstrates excellent goodness-of-fit, surpassing other tested kernels like the power law with an exponential cut-off and modified power law. Notably, the sum of exponential kernel, even with just two terms, provides better fit than mono-exponential kernels and rivals more complex alternatives for fitting high frequency data on an hourly basis.

2. **Efficient Estimation**: Its Maximum Likelihood Estimation (MLE) can be efficiently approximated in O(N) time, offering a significant improvement over the O(N^2) complexity of other methods. This approach has been adopted in literature, for instance Hardiman et al. (2013), to estimate Hawkes processes with a power law kernel.

3. **Versatility in Approximation**: The sum of exponentials kernel can approximate any continuous Hawkes kernel with an increased number of exponential terms, as we demonstrate below.

The following lemma formalizes the approximation capability of sums of exponential functions:

LEMMA 1 **(Zhu (2015), Lemma 11).** *If $h(t) > 0$, $\int_0^\infty h(t)\, dt < \infty$, $\lim_{t\to\infty} h(t) = 0$, and h is continuous, then h can be approximated by a sum of exponential functions, both in the L^1 and the L^∞ norm.*

Let h_n be an excitation function of the form

$$h_n(s) = \sum_{i=1}^{n} \alpha_i e^{-\beta_i s} \tag{A.3}$$

and let us denote by N^n the Hawkes process with intensity

$$I_t^n = \mu + \int_0^t h_n(t-u)\, dN_u^n. \tag{A.4}$$

It follows from (Zhu 2015, Theorem 12) (and its proof) that the log moment generating function of N_n asymptotically approximates that of N_g as $n \to \infty$. More precisely, the following holds:

THEOREM 1. *Let P, respectively P_n, denote the probability measures under which N, respectively N^n, follows the Hawkes process with exciting function h, respectively h_n, such that $h_n \to h$ as $n \to \infty$ in both L^1 and L^∞ norms. For any $\theta \in \mathbb{R}$ let*

$$\Gamma(\theta) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}^P[e^{\theta N_t}] \quad \textit{and} \quad \Gamma_n(\theta) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}^{P_n}[e^{\theta N_t^n}]. \tag{A.5}$$

Then

$$\lim_{n\to\infty} \Gamma_n(\theta) = \Gamma(\theta). \tag{A.6}$$

The previous theorem establishes that a Hawkes process with a kernel comprising a sum of exponential functions serves as a practical approximation for empirically relevant Hawkes processes, including those with power-law kernels that are frequently used in empirical finance literature.

A.2. Risk-neutral liquidation with sum of exponential kernels

We now present the key results for the optimal liquidation problem when market order arrivals follow a Hawkes process with a sum of K exponential kernels. The objective function to minimize is:

$$L(\xi) := \eta \int_0^T \xi_t^2 dt - \sum_{i=1}^K \gamma_i \int_0^T \xi_t \int_0^t \xi_s e^{-\omega_i(t-s)} ds dt. \tag{A.7}$$

Let $A_j = -\frac{\gamma_j}{2\eta}$ and $B_j = -\omega_j$. The necessary condition for optimality is:

$$\xi(t) + \sum_{j=1}^K A_j I_j(t) = C \tag{A.8}$$

where C is a positive constant and:

$$I_j(t) = \int_0^T \xi(s) e^{B_j|t-s|} ds = \int_0^t e^{B_j(t-s)} \xi(s) ds + \int_t^T e^{B_j(s-t)} \xi(s) ds \tag{A.9}$$

Through differentiation of equation (A.9), we can establish:

$$I_j^{(2)}(t) = 2B_j \xi(t) + B_j^2 I_j(t) \tag{A.10}$$

This allows us to transform the integral equation into an ODE. By further differentiation and substitution, we obtain a linear ODE of order $2K$:

$$\xi^{2K}(t) + \kappa_{(K,1)}\xi^{(2(K-1))}(t) + \cdots + \kappa_{(K,K)}\xi(t) = (-1)^K \sum_{j=1}^{K} B_{K-j}^2 C \tag{A.11}$$

where the coefficients κ can be computed recursively.

The full verification that our candidate strategy is indeed optimal follows the same approach as in the classical case. We can prove that for sufficiently large instantaneous impact parameter η, specifically when $\eta > \sum_{i=1}^{n} \frac{\gamma_i}{\omega_i}$, the cost function $L(\xi)$ is strictly positive for any admissible strategy. This verification extends our previous results while maintaining the essential linear-quadratic structure of the problem.

For any fixed admissible strategy ξ we set $\Xi_t := \mathbf{1}_{[0,T]}(t)\xi_t$ for $t \in \mathbb{R}$ and get that

$$\begin{aligned} L(\xi) &= \eta \int_0^T \xi_t^2 - \sum_{i=1}^{n} \left[\gamma_i \int_0^T \xi_t \int_{t>s} \xi_s e^{-\omega_i(t-s)} ds dt \right] \\ &= \eta \int_{\mathbb{R}} {\Xi_t}^2 - \sum_{i=1}^{n} \left[\gamma_i \int_{\mathbb{R}} \Xi_t \int_{t>s} \Xi_s e^{-\omega_i(t-s)} ds dt \right] \end{aligned}$$

Using Plancherel's theorem and the symmetry of the exponential kernel, we have:

$$L(\xi) = \eta \int_{\mathbb{R}} |\hat{\Xi}_z|^2 dz - \sum_{i=1}^{n} \left[\gamma_i \frac{1}{2} \int_{\mathbb{R}} \Xi_t \int_{\mathbb{R}} \Xi_s e^{-\omega_i |t-s|} ds dt \right]$$

Since the exponential decay function is the Fourier transform of a Lorentzian function, this yields:

$$\begin{aligned} L(\xi) &= \eta \int_{\mathbb{R}} |\hat{\Xi}_z|^2 dz - \sum_{i=1}^{n} \left[\gamma_i \frac{1}{2} \int_{\mathbb{R}} \Xi_t \int_{\mathbb{R}} \Xi_s \int_{\mathbb{R}} \frac{2\omega_i}{\omega_i^2 + 4\pi^2 z^2} e^{-2\pi i(t-s)z} dz ds dt \right] \\ &= \eta \int_{\mathbb{R}} |\hat{\Xi}_z|^2 dz - \sum_{i=1}^{n} \left[\gamma_i \int_{\mathbb{R}} \Xi_t e^{-2\pi i z t} dt \int_{\mathbb{R}} \Xi_s e^{2\pi i z s} ds \int_{\mathbb{R}} \frac{\omega_i}{\omega_i^2 + 4\pi^2 z^2} dz \right] \\ &= \eta \int_{\mathbb{R}} |\hat{\Xi}_z|^2 dz - \sum_{i=1}^{n} \left[\gamma_i \left(\int_{\mathbb{R}} \Xi_t e^{-2\pi i z t} dt \right) \left(\int_{\mathbb{R}} \overline{\Xi_s e^{-2\pi i z s}} ds \right) \int_{\mathbb{R}} \frac{\omega_i}{\omega_i^2 + 4\pi^2 z^2} dz \right] \\ &= \eta \int_{\mathbb{R}} |\hat{\Xi}_z|^2 dz - \sum_{i=1}^{n} \left[\gamma_i \int_{\mathbb{R}} |\hat{\Xi}_z|^2 \frac{\omega_i}{\omega_i^2 + 4\pi^2 z^2} dz \right] \\ &= \int_{\mathbb{R}} |\hat{\Xi}_z|^2 \left(\eta - \sum_{i=1}^{n} \left[\gamma_i \frac{\omega_i}{\omega_i^2 + 4\pi^2 z^2} \right] \right) dz. \end{aligned}$$

Thus, if $\eta > \sum_{i=1}^{n} \frac{\gamma_i}{\omega_i}$, then $L(\xi) > 0$

Appendix B: Instantaneous impact factor estimation from stochastic orderbook

The instantaneous impact parameter has been estimated by several works before, where the executed price is assumed to be linearly proportional to market order size and the instantaneous factor can be estimated by feeding a hypothetical market order of large size by walking the LOB; see Cartea and Jaimungal (2015) and Chen et al. (2018). While this offers a tractable representation of the orderbook for mathematical model, it is only an approximation of the visible (observed) orderbook for two reasons:

- **Price discretization** Stocks are often transacted in tick size, which determines the minimum price amount a security can change. Consequently, prices in the orderbook can only be multiples of tick size. Thus, the executed price is a step function of the total executed shares.
- **Liquidity fluctuation** As mentioned in Tóth et al. (2011), the liquidity posted in the visible orderbook may or may not reflect the true supply and demand of the market. Rather, it is often a consequence of high frequency market makers adjusting their quotes to avoid adverse selection cost, and therefore only a small amount of liquidity posted near the best bid-offer is capturable at any one time.

Given the stochastic nature of liquidity, one cannot directly use the orderbook data to estimate the instantaneous cost factor. This advocates to measure the average instantaneous market impact based on a steady-state profile of the orderbook. We follow the previous works to model orderbook, but account for price discretization and random fluctuation in standing volume.

We consider a Limit Order Book (LOB) model with total depth of S price levels. The LOB is described by the bid/ask side liquidities $L_i^{b/a}$ standing at the i-th price level away from the best bid/offer price. Instead of assuming the price level grid is fixed with equidistance (e.g. 1 tick) which may introduce "holes" in the book with 0 standing liquidity ($L_{i_0}^{b/a} = 0$ for some i_0) for certain stocks and bring variations in book reconstruction, we build (bid side) LOB samples with positive standing liquidities at any price level as follows:

Let $M_0 := L_0^b$. By definition $M_0 > 0$. Let $\Delta P_0 := 0$ and $S_0 := \#\{i \in \{0, ..., S\} : L_i^b > 0\}$. We now define a sequence of price offsets ΔP_i and and standing liquidities M_i recursively by

$$\Delta P_i := \inf\{j > \Delta P_{i-1} : L_j^b > 0\} \quad \text{and} \quad M_i := L_{\Delta P_i}^b, \qquad i = 1, ..., S_0.$$

In particular, the liquidity standing ΔP_i ticks into the book is given by M_i. The aggregate liquidity up to price level ΔP_i is denoted

$$m_i = \sum_{j=0}^{i} M_j.$$

The state of the LOB at any given time can be fully described by a random vector $\{(M_i, \Delta P_i)\}_{i=0,\dots S_0}$.

Compared to simply taking averages of, say 10-second LOB snapshots, a nonparametric density estimation provides comprehensive stochastic behaviours of the random variables, and more importantly, a better estimate of transaction cost, especially when the distribution deviates form Gaussian with asymmetry and/or heavy tails. Estimating joint density of multi vector is challenging. Given that the price grid is determined by the availability of standing liquidity, we replace the random offsets by their empirical averages, still denoted ΔP_i. In this case, the offsets are assumed to be deterministic, the distribution of the vector $\{M_i\}_{i=0,\dots,S_0}$ allows to compute the cost function

$$P(v) := \frac{1}{v}\mathbb{E}\left[\sum_{j=1}^{S} \mathbf{1}_{\{m_{j-1}<v\le m_j\}} \left\{\sum_{i=1}^{j} M_{i-1}\Delta P_{i-1} + (v-m_j)\Delta P_j\right\}\right].$$

This is still too complicated; estimating the high dimensional joint distribution of the standing liquidity is prone to the curse of dimensionality; see Gramacki (2017). To balance bias and variance tradeoff, we considered the modified function

$$\hat{P}(v) = \frac{1}{v}\sum_{i=0}^{S} \mathbb{E}\left[m_{i-1}\Delta P_{i-1} + (v-m_i)\Delta P_i \middle| m_i < v \le m_{i+1}\right] \mathbb{P}(m_i < v \le m_{i+1}).$$

The modified version is clearly biased. It assumes that our order, when clearing up to level i, will be priced at the most aggressive price ΔP_i instead of the average price $\frac{\sum_{j=0}^{i} M_j \Delta P_j}{\sum_{j=0}^{i} M_j}$. This inflates the price impact in the estimation, leading to larger value of η. Conversely, it provides a very conservative estimator of the cost improvement of our trading strategy over TWAP. The numerical benefit of using the modified function is that it only needs to estimate separately the joint distributions at two levels (m_i, m_{i+1}) each time, which lowers estimation variance and speeds up computation.

We use kernel density to estimate the joint probability density function, denoted by $f_{x,y}$, of two random variable $x = m_i$ and $y = m_{i+1}$. Letting $\mathbf{X_t} = (x_1, x_2, \cdots, x_n)$ and $\mathbf{Y_t} = (y_1, y_2, \cdots, y_n)$ be samples drawn from the distribution of m_i and m_{i+1} we have:

$$\hat{f}_{x,y} = n^{-1} \sum_{t=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{X_t})(\mathbf{y} - \mathbf{Y_t})$$

where $K_{\mathbf{H}}(\mathbf{x}, \mathbf{y}) = |\mathbf{H}|^{-1/2} K(\mathbf{x}, \mathbf{y}) |\mathbf{H}|^{-1/2}$ is the kernel and $\mathbf{H}$ is the bandwidth matrix. We consider the Gaussian kernel[1] in our estimation.

To measure our instantaneous market impact, we first simulate the orderbook in its stationary state. We then submit orders with different size v (multiples of 1, .., 20 average market order sizes[2]) and compute the market impact $\hat{P}(v)$ of each order on the simulated book. Finally, we estimate the instantaneous market impact factor η in the linear relationship

$$\hat{P}(v) = \eta v + \epsilon$$

where $\epsilon \sim N(0, \sigma_\epsilon)$ and $\hat{P}(v)$ is the change in price per share after we submit our market orders of size v.

In the estimation, the density of liquidity is calibrated using kernel density estimation based on the quote data snapshots at $\{t_i\}_{i=1}^{n}$ sampled within each bin of $T = 1$. We consider up to 20 times the size of AMO or the linearity is not violated[3], which in all cases lead up to first $S = 5$ levels of orderbook data.

Appendix C: Permanent impact factor estimation using order flow imbalance

In the theoretical framework, orders are supposed to have the same permanent impact. In other words, the value of permanent impact will not be influenced by individual agent's action and market timing. While one can measure the change in fundamental price at each trade directly, it often leads to large variance due to the presence of microstructure noise. To obtain a robust estimation, we measure the average change in fundamental price after a certain amount of liquidity has been removed from the market. Specifically, we adopt the idea of Cont et al. (2013), which estimates

[1] Note that the kernel only serves as a smoothing function for data within the bandwidth, and it does not mean we assume normal distribution, or any distribution in particular.

[2] To ensure the assumption of orderbook recovered within Δt reasonable, we only submit orders of the average market order's size, with a trading rate at a fraction of the average market order's arrival rate.

[3] We check for linearity in the regression and only use up to X number of AMOs until the linearity is violated.

permanent impact based on the linear relation between price change and *order flow imbalance* (OFI), defined as the imbalance between supply and demand at the best bid and ask prices.

Over a time interval $[t_{i-1}, t_i]$, the order flow imbalance is a sum of changes for every liquidity event:

$$OFI_i = \sum_{s=C(t_{i-1})+1}^{C(t_i)} e_s$$

where $C(\cdot)$ denotes the count of all the best-bid offer (BBO) change events (add/delete/trade) and e_s denotes the changes in liquidity at BBO. For a detailed guide how e_s is measured, we refer to the original paper Cont et al. (2013). The permanent market impact factor is estimated with a linear regression equation

$$\Delta P_i = \lambda OFI_i + \epsilon_i$$

where λ is the price impact coefficient applied to each share, ΔP_i refers to the price change over the time interval, and ϵ_i is a noise term. In other words, we estimate the permanent impact at frequency Δt, that is, the same duration for every interval $[t_{i-1}, t_i]$, and balance the bias and variance tradeoff in the market microstructure. We report the average R-squared of the linear regression for each stock in Appendix. Similar to the results reported in Cont et al. (2013), the regression fits data reasonably well, with R-squared around 41.01% – 71.10% .

Appendix D: Hawkes process estimation using maximum likelihood estimation

We use Hawkes process to model the dynamics of order arrivals. There are several issues to be addressed in the estimation:

- **Artificial trade clustering**. While the self-exciting effects of Hawkes process are related to the degree of endogeneity of how much parent orders trigger child orders, there is no detailed information in public data. In other words, we cannot distinguish the parent-child relationship among the orders. This is further complicated by the nature of high frequency market data, in which events are clustering together not due to their relationship, but possibly due to the distribution mechanism of market data (Filimonov and Sornette (2015)). For example, during a general market event such as index movement, it is reasonable to expect a flurry of limit orders submitted at almost the same time as traders are to hedge their position to reduce adverse selection risk. The exchange will first match each order in FIFO order, then redistribute the trade to every market participant. This creates an artificial stream of trades that resembles herding effects. We aggregate multiple trades over a super-short interval of 100 milliseconds to avoid capturing such artificial herding effects in estimation induced by the microstructure effect.

- **Mixture of herding and splitting effect**. In addition to endogeneity, the self-exciting effects of Hawkes process is attributed to the splitting of a big meta-order into multiple child orders (Tóth et al. (2014)). Without proprietary data, it is hard to obtain the detailed information. Filimonov and Sornette (2015) shows that the splitting effect results in an overestimation for the Hawkes process, with the estimated branching ratio close to the critical value of 1. The overestimation will become more serious for a long sample period. We assume that the Hawkes process is stationary[4]. In our estimation, we slice data into the hourly bins and estimate the parameters over each bin. This alleviates the bias in estimation, but potentially at the cost of larger variance. We take the median of all the estimates as a robust estimate to balance the bias and variance tradeoff.
- **Misspecification of Hawkes's kernel** We chose an exponential kernel in the estimation, in accordance with the model set up. [5] To verify the accuracy, we perform goodness-of-fit testing of the residuals of the fitted Hawkes process with the exponential kernel. In general, the exponential Hawkes process fits the data reasonably well, as the test in general cannot reject the null hypothesis at 10% significance level.
- **Seasonality effect**: Diurnal pattern is known in trading, and can be vital in estimating Hawkes process's branching ratio using high frequency data. The average number of trades is not constant over the day, but often follows a U-shape with peaks of trading volumes near the opening and close auctions. As shown in Wehrli et al. (2021), not controlling the seasonality effect can lead to critical Hawkes's branching ratio. Consequently, by removing the open and close and using data from 10 AM to 3 PM only, we lower the disturbance of seasonal effect to stationarity.

For each sample over $T = 1$ hour, we conduct the maximum likelihood estimation for the Hawkes process. The log-likelihood function is as follows:

$$\log L(t_1, t_2, \cdots, t_n) = -\mu t_n + \sum_{i=1}^{n} \left[\frac{\alpha}{\beta} (e^{-\beta(t_n - t_i)} - 1) \right] + \sum_{i=1}^{n} \log(\mu + \alpha A(i))$$

where $A(i) = \sum_{t_j < t_i} e^{-\beta(t_i - t_j)}$, and μ is the average number of market orders over the sample period. The parameter α is the strength of the self-exciting effect. For $\alpha = 1.0$, it means a 100% self-exciting effect, in the sense that each immigrant order will be replicated fully over the sample period if there is no decay. The parameter β is the strength of the decaying effect. For $\beta = 1.0$, it expects a

[4] This is a necessary condition to obtain closed-form solutions.

[5] We are aware that many other kernels including power law kernels have been suggested in the literature; see, e.g. Gatheral (2011) and Hawkes (2018) among many others. The choice of exponential kernels allows us to obtain closed-form solutions.

100% decaying effect in the exponential term. We report the mean and statistical significance of the estimates using Anderson Darling test across subsamples of each stock in the e-Appendix. The statistics show that the fits of Hawkes process are reasonable.

Appendix E: Square-root law market impact and benchmark

A standard method to measure market impact is the square-root formula, which is:

$$\Delta P = c\sigma\sqrt{\frac{x_0}{V}}$$

where ΔP is the price change from executing x_0 shares, with σ is the volatility of the fundamental price and V is average daily market volume. c is a constant that depends on market, but often of order unity; see, for example, Tóth et al. (2011) for more details. The corresponding square root cost formula is then

$$c\sigma\sqrt{\frac{x_0}{V}}x_0.$$

While the square root does not factor directly in our estimation, it serves as a good benchmark for reasonable estimation of market impact. We would expect the market impact generated by square-root law to be in the same magnitude as our fixed cost component in the total cost equation. The fixed cost can be decomposed into a market linear impact component and the number of shares executed as

$$(\gamma+\lambda)\frac{x_0}{2}x_0$$

The results (assuming $c = 1$) are presented in Table 10 and Table 11. In general, the fixed cost and square-root cost are in the same magnitude, implying our cost calculation is reasonable.

Appendix F: Goodness-of-fit testing for univariate Hawkes process using Anderson-Darling test

The goodness-of-fit for the estimation of Hawkes processes can formally be tested using residual analysis. Specifically, consider a univariate Hawkes process N with conditional intensity $i(t)$, and the corresponding compensator function $I(t)$

$$I(t) = \int_0^t i(s)ds$$

Define a scaled point process S for all $\{t_i : i \in N_t\}$ where

$$S(t_i) = I(t_i)$$

Table 1 Anderson-Darling critical value table for standard exponential distribution for 200 observations.

Significance level	10%	5%	2.5%	1%
Critical value	1.075	1.337	1.601	1.951

then S is a standard Poisson process (Daley and Vere-Jones (2003)). The goodness-of-fit test can then be performed on the residual process R where

$$R(t_i) = S_i - S_{i-1}$$

under the null hypothesis that R is exponentially distributed with rate 1. We use the Anderson-Darling test (Anderson and Darling (1952)) to test if sample residuals came from a population with a standard exponential distribution. While the exact critical values depend on the number of observations in each stock, a rough estimation of critical values against significance levels is provided in Table (1).

Appendix G: Empirical results

Table 2 The average of goodness of fit measures and the general statistics per symbol (from index 1 to 27). Residual statistic refers to the Anderson-Darling test statistic for the Hawkes process, while OFI R2 refers to the R-squared measure of the linear regression for the permanent impact factors. In general, the goodness of fit measures are in the expected range.

symbol	Tick	Volume	Volatility	Residual statistic	OFI R2
ANSS	0.470	93.920	1.695	1.069	36.563
JKHY	0.700	55.491	1.181	1.040	29.121
FFIV	0.730	90.035	1.797	0.972	39.152
AZPN	0.780	51.697	2.252	1.108	26.838
PFPT	0.810	84.907	2.254	1.419	30.714
BLUE	0.910	68.981	2.648	1.294	42.912
NDAQ	1.010	70.441	1.431	0.990	59.320
MRTX	1.110	53.630	3.688	0.990	25.594
GRMN	1.210	85.427	1.585	1.362	40.873
BPMC	1.210	35.373	2.770	0.958	38.654
UTHR	1.230	40.201	1.741	1.094	35.246
INCY	1.250	86.779	1.942	1.064	54.772
VSAT	1.290	25.463	1.354	0.932	25.436
QRVO	1.350	85.128	1.738	1.180	38.454
EXPD	1.370	66.622	1.503	1.199	53.799
OLLI	1.410	91.053	3.901	1.041	47.076
AAXJ	1.490	85.619	0.942	1.139	65.364
YY	1.670	56.451	2.201	1.144	31.539
FIVN	1.790	41.790	3.102	1.014	28.398
PFG	1.790	60.807	1.660	1.235	60.055
BPOP	1.840	27.665	1.457	1.052	29.692
XRAY	1.860	84.242	1.249	0.751	80.312
AMBA	1.900	35.495	2.948	1.254	36.716
FLIR	1.940	39.007	1.636	1.038	63.165
GBT	1.960	37.025	2.534	0.925	25.530
APPN	2.140	32.233	3.514	1.061	33.078
HUBG	2.280	11.090	2.058	0.641	25.799

Table 3 The average of goodness of fit measures and general statistics per symbol (from index 28 to 54). Residual statistic refers to the Anderson-Darling test statistic for the Hawkes process, while OFI R2 refers to the R-squared measure of the linear regression for the permanent impact factors. In general, the goodness of fit measures are in the expected range.

symbol	Tick	Volume	Volatility	Residual statistic	OFI R2
FGEN	2.360	22.600	2.532	1.063	30.846
PLAY	2.480	38.940	2.038	1.167	46.816
ACGL	2.500	50.322	1.155	0.836	83.466
RGNX	2.520	22.328	3.918	0.894	42.310
PCRX	2.530	27.176	2.207	1.229	35.101
ADPT	2.670	18.939	4.951	1.169	27.943
MMSI	2.690	32.375	5.064	1.124	31.065
SHOO	2.950	22.166	2.649	1.239	31.250
MOMO	2.960	77.758	2.851	0.919	74.486
CORE	3.010	8.411	2.075	0.761	27.877
BECN	3.010	20.283	3.074	1.122	31.468
TLRY	3.060	51.187	4.724	1.100	26.106
ACIW	3.180	21.586	1.839	0.754	61.509
ARWR	3.220	40.316	3.298	1.080	52.136
CATM	3.300	14.756	2.682	0.791	26.466
SLGN	3.320	12.144	1.170	0.939	37.447
PPC	3.420	31.036	1.986	1.084	68.245
PDCE	3.550	42.968	3.670	1.224	51.914
APLS	3.670	14.353	3.517	0.844	28.977
FHB	3.810	18.201	1.469	1.073	65.118
VCYT	3.880	16.668	3.658	1.126	26.943
NSTG	3.970	12.034	3.546	1.054	27.579
RDUS	4.000	13.151	2.773	0.760	31.261
HCSG	4.010	21.587	3.593	0.866	41.445
NTNX	4.110	89.122	3.396	1.085	76.948
BCOR	4.160	9.861	3.002	0.954	36.839
IRDM	4.230	18.724	2.902	0.987	31.307
SABR	4.340	37.143	1.384	0.873	76.742

Table 4 The average of goodness of fit measures and general statistics per symbol (from index 55 to 81). Residual statistic refers to the Anderson-Darling test statistic for the Hawkes process, while OFI R2 refers to the R-squared measure of the linear regression for the permanent impact factors. In general, the goodness of fit measures are in the expected range.

symbol	Tick	Volume	Volatility	Residual statistic	OFI R2
SFIX	4.400	65.906	3.297	0.790	58.067
DENN	4.490	10.146	1.269	1.181	38.205
AERI	4.510	25.962	3.934	1.286	36.199
CARA	4.530	15.945	3.230	1.125	35.094
AIMT	4.610	20.501	3.713	1.018	33.422
IOVA	4.680	26.899	2.828	1.316	46.464
HAIN	4.730	24.093	2.570	0.947	80.205
MTSI	4.980	11.976	2.728	1.185	42.182
BLDR	5.130	21.257	1.955	1.010	82.385
PENN	5.210	31.640	2.149	0.730	78.538
GOSS	5.240	8.684	4.808	0.921	31.964
INSM	5.330	22.642	3.376	1.303	36.882
HOMB	5.360	10.983	1.771	1.000	65.677
HRTX	5.400	17.204	3.162	1.160	34.804
LSCC	5.410	34.718	3.318	1.084	67.626
FATE	5.580	13.129	3.669	1.158	31.986
PETS	5.610	16.283	4.946	0.774	39.979
SVMK	5.690	21.277	2.900	1.410	60.608
ONB	5.830	15.219	1.349	0.744	84.091
TVTY	5.910	11.451	2.986	1.043	62.479
LAUR	6.050	20.994	2.021	1.174	80.312
UMPQ	6.140	20.685	1.597	0.748	80.308
CVET	6.150	31.268	7.033	1.378	79.299
HALO	6.170	11.764	1.647	1.062	42.199
RTRX	6.620	9.330	4.129	0.686	43.557
UPWK	6.670	13.584	2.429	1.060	59.427
MRNA	6.690	31.120	3.235	1.128	62.285
IMMU	6.760	30.598	3.724	1.010	79.406

Table 5 The average of goodness of fit measures and general statistics per symbol (from index 82 to 110). Residual statistic refers to the Anderson-Darling test statistic for the Hawkes process, while OFI R2 refers to the R-squared measure of the linear regression for the permanent impact factors. In general, the goodness of fit measures are in the expected range.

symbol	Tick	Volume	Volatility	Residual statistic	OFI R2
GLNG	6.900	18.958	3.405	1.161	68.000
FEYE	6.940	50.983	2.055	1.050	81.132
REGI	7.040	10.635	2.856	0.705	46.641
GT	7.290	47.864	2.767	0.847	82.756
NWSA	7.320	33.603	1.575	1.143	77.699
CHNG	7.450	17.828	3.017	0.911	43.489
QNST	7.500	8.155	4.457	0.846	72.596
NAVI	7.660	24.800	2.417	0.962	79.896
WIFI	7.760	9.220	3.411	1.293	47.949
SONO	7.870	15.684	2.778	0.766	74.574
PAYS	7.930	19.838	6.256	0.889	45.796
ISBC	8.810	20.013	1.309	1.139	79.714
TTMI	8.980	11.311	3.035	0.717	74.316
VLY	9.220	17.048	1.531	0.880	80.777
AMAG	9.280	11.548	5.279	0.784	50.451
SGMO	9.500	15.949	2.624	0.890	74.397
MDRX	9.640	16.961	2.095	0.908	77.541
FLEX	9.750	50.250	2.835	0.921	76.403
PBYI	10.040	14.636	5.170	0.958	55.007
IRWD	10.540	16.607	2.541	1.074	61.217
TELL	13.470	13.972	5.112	1.209	56.174
APPS	15.760	12.742	3.546	1.061	66.321
CPRX	18.930	10.320	4.923	0.634	76.050
VRAY	19.990	10.496	9.004	0.977	67.951
OAS	27.260	43.796	6.663	0.823	79.271
ENDP	28.970	28.630	7.775	0.917	82.565
GPOR	31.790	16.424	5.200	0.954	79.379

Table 6 The estimated paramaters λ, η and α per symbol (from index 1 to 27). The significance level of the estimated parameters is denoted in number of stars next to the number. Three, two, and one star(s) are correspoding to the 1%, 5% and 10% significance levels. Note that ζ, ω and θ are computed from the estimated paramaters and thus no significance is reported. Similarly, β has the same significance level as α given the residual test in Subsection (F), and therefore is omitted here.

symbol	Tick	α	ω	λ	η	ζ	θ
ANSS	0.470	3.84***	5.137	0.00300***	0.00048***	4.725	-2.116
JKHY	0.700	5.23***	6.228	0.00312***	0.00047***	5.643	-3.642
FFIV	0.730	3.88**	5.370	0.00451***	0.00064***	5.069	-1.620
AZPN	0.780	2.81***	4.258	0.00375**	0.00059***	4.160	-0.418
PFPT	0.810	4.93***	5.504	0.00384**	0.00063***	5.493	-0.058
BLUE	0.910	7.12***	7.862	0.00793***	0.00102***	7.023	-6.597
NDAQ	1.010	5.73**	6.900	0.00475***	0.00058***	6.765	-0.929
MRTX	1.110	5.82**	6.870	0.01155*	0.00154***	6.370	-3.433
BPMC	1.210	2.74**	4.905	0.02186***	0.00278***	4.402	-2.465
GRMN	1.210	3.23***	4.467	0.00266***	0.00043***	4.442	-0.112
UTHR	1.230	4.33***	5.987	0.00673**	0.00091***	5.366	-3.716
INCY	1.250	3.17***	5.189	0.00531***	0.00064***	5.062	-0.662
VSAT	1.290	4.75**	5.666	0.00429**	0.00072***	5.029	-3.609
QRVO	1.350	3.75***	5.063	0.00300**	0.00047***	4.699	-1.846
EXPD	1.370	4.71***	5.806	0.00423***	0.00059***	5.800	-0.030
OLLI	1.410	5.22***	6.539	0.00464**	0.00060***	6.135	-2.638
AAXJ	1.490	5.51***	6.776	0.00060***	0.00008***	6.025	-5.087
YY	1.670	5.29***	6.335	0.00560*	0.00074***	6.294	-0.263
PFG	1.790	6.17***	7.040	0.00452***	0.00060***	6.629	-2.894
FIVN	1.790	4.17**	5.219	0.00626***	0.00097***	5.156	-0.332
BPOP	1.840	3.37***	4.682	0.00409*	0.00065***	4.532	-0.703
XRAY	1.860	4.22**	5.590	0.00289***	0.00043***	5.051	-3.015
AMBA	1.900	4.61***	6.085	0.00461***	0.00067***	5.245	-5.115
FLIR	1.940	4.65***	6.066	0.00486***	0.00064***	5.822	-1.479
GBT	1.960	5.84**	6.452	0.00719*	0.00112***	5.803	-4.185
APPN	2.140	4.46***	5.869	0.00766**	0.00108***	5.395	-2.780
HUBG	2.280	5.30*	6.180	0.00630*	0.00092***	5.871	-1.909

Table 7 The estimated paramaters λ, η and α per symbol (from index 28 to 54). The significance level of the estimated parameters is denoted in number of stars next to the number. Three, two, and one star(s) are correspoding to the 1%, 5% and 10% significance levels. Note that ζ, ω and θ are computed from the estimated paramaters and thus no significance is reported. Similarly, β has the same significance level as α given the residual test in Subsection (F), and therefore is omitted here.

symbol	Tick	α	ω	λ	η	ζ	θ
FGEN	2.360	3.72***	4.938	0.00951**	0.00151***	4.763	-0.865
PLAY	2.480	3.82***	5.149	0.00629***	0.00097***	4.816	-1.713
ACGL	2.500	5.07**	5.930	0.00237***	0.00036***	5.567	-2.152
RGNX	2.520	5.67**	6.462	0.01711***	0.00250***	5.999	-2.992
PCRX	2.530	4.12***	5.260	0.00572**	0.00091***	4.927	-1.749
ADPT	2.670	3.67***	5.251	0.01618*	0.00236***	4.793	-2.404
MMSI	2.690	6.50***	6.587	0.00567**	0.00091***	6.133	-2.993
SHOO	2.950	7.06***	7.160	0.00425*	0.00061***	6.829	-2.369
MOMO	2.960	4.33**	6.338	0.00387***	0.00044***	6.081	-1.630
CORE	3.010	5.10**	5.986	0.00881**	0.00136***	5.522	-2.782
BECN	3.010	3.80***	5.061	0.00523*	0.00083***	4.725	-1.697
TLRY	3.060	7.45***	8.457	0.01177**	0.00126***	8.236	-1.869
ACIW	3.180	4.81**	5.945	0.00484**	0.00070***	5.618	-1.946
ARWR	3.220	5.28***	6.404	0.01086***	0.00142***	6.314	-0.577
CATM	3.300	3.04**	4.489	0.00603*	0.00096***	4.246	-1.089
SLGN	3.320	5.26**	5.826	0.00545**	0.00087***	5.630	-1.140
PPC	3.420	4.39***	5.450	0.00505***	0.00076***	5.355	-0.514
PDCE	3.550	5.65***	6.518	0.00692***	0.00099***	6.038	-3.123
APLS	3.670	4.11**	5.422	0.01067**	0.00157***	5.151	-1.470
FHB	3.810	5.11***	5.881	0.00531***	0.00079***	5.835	-0.271
VCYT	3.880	5.63***	6.046	0.01208**	0.00191***	5.874	-1.040
NSTG	3.970	4.88***	5.941	0.01186**	0.00171***	5.706	-1.396
RDUS	4.000	4.10**	5.912	0.01138**	0.00146***	5.408	-2.980
HCSG	4.010	6.29**	6.826	0.00686**	0.00102***	6.196	-4.296
NTNX	4.110	5.79***	7.481	0.00474***	0.00049***	7.454	-0.205
BCOR	4.160	3.95**	5.435	0.00974**	0.00146***	4.860	-3.123
IRDM	4.230	5.52**	6.595	0.00635**	0.00088***	6.062	-3.516
SABR	4.340	4.95**	6.003	0.00269***	0.00038***	5.807	-1.177

Table 8 The estimated paramaters λ, η and α per symbol (from index 55 to 81). The significance level of the estimated parameters is denoted in number of stars next to the number. Three, two, and one star(s) are correspoding to the 1%, 5% and 10% significance levels. Note that ζ, ω and θ are computed from the estimated paramaters and thus no significance is reported. Similarly, β has the same significance level as α given the residual test in Subsection (F), and therefore is omitted here.

symbol	Tick	α	ω	λ	η	ζ	θ
SFIX	4.400	10.74**	12.832	0.00724***	0.00048***	12.722	-1.401
DENN	4.490	4.45***	5.748	0.00558**	0.00083***	5.205	-3.121
AERI	4.510	6.51***	7.294	0.00969***	0.00119***	7.259	-0.256
CARA	4.530	3.10***	4.918	0.01091***	0.00142***	4.822	-0.474
AIMT	4.610	7.87**	7.914	0.00999**	0.00129***	7.699	-1.704
IOVA	4.680	5.75***	7.160	0.01087***	0.00129***	6.798	-2.590
HAIN	4.730	4.56**	5.572	0.00410***	0.00064***	5.226	-1.931
MTSI	4.980	5.14***	6.343	0.00758**	0.00105***	5.862	-3.050
BLDR	5.130	5.41**	6.413	0.00393***	0.00055***	6.063	-2.240
PENN	5.210	4.11*	5.514	0.00502***	0.00071***	5.292	-1.225
GOSS	5.240	4.27**	6.026	0.01731**	0.00220***	5.572	-2.737
INSM	5.330	4.38***	5.595	0.00976**	0.00142***	5.382	-1.194
HOMB	5.360	3.37**	4.668	0.00648***	0.00105***	4.474	-0.905
HRTX	5.400	9.91***	9.513	0.00616**	0.00073***	8.809	-6.698
LSCC	5.410	6.28***	7.018	0.00628***	0.00085***	6.633	-2.705
FATE	5.580	3.61***	5.377	0.01092**	0.00149***	4.918	-2.466
PETS	5.610	2.34**	3.967	0.00952**	0.00145***	3.875	-0.366
SVMK	5.690	6.95***	7.904	0.00614***	0.00072***	7.481	-3.345
ONB	5.830	4.81*	6.263	0.00272***	0.00034***	6.068	-1.221
TVTY	5.910	5.57***	7.792	0.00883***	0.00085***	7.404	-3.020
LAUR	6.050	2.96***	4.775	0.00244***	0.00038***	3.948	-3.947
UMPQ	6.140	3.26*	4.837	0.00242***	0.00035***	4.678	-0.771
CVET	6.150	2.71***	4.967	0.00681***	0.00094***	3.930	-5.154
HALO	6.170	3.60***	5.092	0.00901***	0.00131***	4.864	-1.158
RTRX	6.620	4.99*	6.186	0.01371***	0.00197***	5.611	-3.559
UPWK	6.670	3.13***	5.142	0.00749***	0.00108***	4.239	-4.646
MRNA	6.690	9.04***	9.712	0.01044***	0.00108***	8.991	-7.004
IMMU	6.760	7.01**	8.540	0.00705***	0.00071***	8.106	-3.698

Table 9 The estimated paramaters λ, η and α per symbol (from index 82 to 110). The significance level of the estimated parameters is denoted in number of stars next to the number. Three, two, and one star(s) are correspoding to the 1%, 5% and 10% significance levels. Note that ζ, ω and θ are computed from the estimated paramaters and thus no significance is reported. Similarly, β has the same significance level as α given the residual test in Subsection (F), and therefore is omitted here.

symbol	Tick	α	ω	λ	η	ζ	θ
GLNG	6.900	3.64***	5.368	0.00601***	0.00081***	5.017	-1.889
FEYE	6.940	4.92***	5.842	0.00137***	0.00022***	5.316	-3.072
REGI	7.040	8.47*	8.100	0.01099***	0.00164***	6.984	-9.037
GT	7.290	5.38**	6.061	0.00199***	0.00029***	6.022	-0.236
NWSA	7.320	3.98***	5.670	0.00094***	0.00014***	4.775	-5.077
CHNG	7.450	5.63**	6.525	0.00698***	0.00106***	5.693	-5.427
QNST	7.500	5.29**	6.698	0.00898***	0.00125***	5.654	-6.996
NAVI	7.660	4.22**	6.228	0.00129***	0.00017***	5.104	-7.000
WIFI	7.760	5.88***	7.277	0.01230***	0.00167***	5.942	-9.714
SONO	7.870	8.47**	9.878	0.00457***	0.00044***	8.894	-9.722
PAYS	7.930	3.69**	5.592	0.01323***	0.00179***	4.858	-4.106
ISBC	8.810	5.83***	7.009	0.00114***	0.00014***	6.729	-1.963
TTMI	8.980	5.34*	6.111	0.00575***	0.00092***	5.454	-4.011
VLY	9.220	5.28**	6.311	0.00121***	0.00017***	6.145	-1.050
AMAG	9.280	5.66**	6.712	0.01436***	0.00188***	6.442	-1.813
SGMO	9.500	4.33**	5.845	0.01025***	0.00142***	5.363	-2.817
MDRX	9.640	5.30**	6.134	0.00237***	0.00037***	5.537	-3.666
FLEX	9.750	4.37**	5.754	0.00077***	0.00012***	4.983	-4.435
PBYI	10.040	5.31**	7.146	0.01393***	0.00157***	6.604	-3.875
IRWD	10.540	6.86***	7.059	0.00823***	0.00124***	6.444	-4.336
TELL	13.470	8.43***	9.317	0.01094***	0.00117***	8.436	-8.214
APPS	15.760	4.88***	6.745	0.00760***	0.00092***	5.981	-5.153
CPRX	18.930	5.45*	6.993	0.00612***	0.00075***	6.352	-4.488
VRAY	19.990	6.50**	6.749	0.00577**	0.00089***	6.252	-3.351
OAS	27.260	2.17**	3.823	0.00104***	0.00016***	3.702	-0.459
ENDP	28.970	4.59**	6.136	0.00189***	0.00025***	5.732	-2.480
GPOR	31.790	4.34**	6.150	0.00202***	0.00026***	5.388	-4.682

Table 10 The cost measures per symbol (from index 1 to 27). C_R is the square-root cost per share, while C_S is the fixed cost per share, as explained in Subsection (E)

symbol	Tick	C_R (bps/share)	C_S (bps/share)	TWAP	Opt	r_{TWAP}(%)
ANSS	0.470	68.748	37.985	0.141	0.122	13.361
JKHY	0.700	45.798	33.821	0.120	0.108	10.099
FFIV	0.730	83.012	108.220	0.287	0.234	18.521
AZPN	0.780	98.851	48.599	0.109	0.071	34.843
PFPT	0.810	87.327	75.574	0.082	0.031	62.834
BLUE	0.910	113.948	176.070	0.552	0.511	7.394
NDAQ	1.010	55.335	91.990	0.102	0.065	36.548
MRTX	1.110	135.234	171.453	0.437	0.381	12.716
BPMC	1.210	77.781	114.860	0.462	0.414	10.407
GRMN	1.210	59.715	67.420	0.107	0.051	52.416
UTHR	1.230	71.092	95.864	0.357	0.324	9.181
INCY	1.250	79.090	154.203	0.247	0.164	33.657
VSAT	1.290	53.640	40.780	0.190	0.173	8.583
QRVO	1.350	66.244	87.375	0.303	0.257	15.008
EXPD	1.370	55.822	96.190	0.087	0.029	66.382
OLLI	1.410	159.046	177.617	0.410	0.346	15.506
AAXJ	1.490	35.559	19.649	0.072	0.066	7.803
YY	1.670	93.979	177.333	0.164	0.073	55.151
PFG	1.790	69.884	163.553	0.346	0.292	15.746
FIVN	1.790	104.424	95.314	0.139	0.076	45.489
BPOP	1.840	64.878	71.080	0.166	0.118	29.154
XRAY	1.860	49.156	123.477	0.482	0.431	10.416
AMBA	1.900	154.226	149.746	0.734	0.688	6.241
FLIR	1.940	72.717	128.638	0.240	0.184	23.456
GBT	1.960	92.819	133.398	0.500	0.455	9.101
APPN	2.140	119.067	106.777	0.339	0.297	12.408
HUBG	2.280	72.651	36.870	0.082	0.066	19.405

Table 11 The cost measures per symbol (from index 28 to 54). C_R is the square-root cost per share, while C_S is the fixed cost per share, as explained in Subsection (E)

symbol	Tick	C_R (bps/share)	C_S (bps/share)	TWAP	Opt	r_{TWAP}(%)
FGEN	2.360	102.550	145.804	0.340	0.248	27.136
PLAY	2.480	69.588	123.334	0.387	0.323	16.558
ACGL	2.500	53.453	118.593	0.321	0.269	16.398
RGNX	2.520	140.136	231.109	0.640	0.555	13.379
PCRX	2.530	78.489	88.737	0.275	0.229	16.757
ADPT	2.670	150.501	128.465	0.476	0.419	12.078
MMSI	2.690	169.474	109.786	0.306	0.264	13.783
SHOO	2.950	119.596	112.714	0.210	0.169	19.458
MOMO	2.960	130.585	314.685	0.520	0.401	22.901
CORE	3.010	121.410	141.637	0.453	0.395	12.806
BECN	3.010	118.883	83.621	0.275	0.231	16.275
TLRY	3.060	137.281	293.005	0.285	0.204	28.571
ACIW	3.180	79.399	112.002	0.272	0.223	18.090
ARWR	3.220	118.246	330.965	0.362	0.206	43.249
CATM	3.300	102.179	71.481	0.228	0.182	20.266
SLGN	3.320	44.417	60.186	0.118	0.086	27.140
PPC	3.420	80.417	158.833	0.241	0.145	39.838
PDCE	3.550	156.319	357.596	0.993	0.865	12.958
APLS	3.670	157.965	199.408	0.497	0.396	20.421
FHB	3.810	60.171	115.412	0.129	0.061	52.359
VCYT	3.880	147.612	245.324	0.422	0.295	30.059
NSTG	3.970	167.450	230.058	0.456	0.347	23.924
RDUS	4.000	107.158	151.290	0.472	0.417	11.629
HCSG	4.010	100.177	88.628	0.290	0.262	9.752
NTNX	4.110	105.975	299.902	0.178	0.066	63.005
BCOR	4.160	145.431	161.679	0.690	0.625	9.487
IRDM	4.230	121.029	160.520	0.466	0.412	11.597
SABR	4.340	45.760	86.474	0.152	0.110	27.456

Table 12 The cost measures per symbol (from index 55 to 81). C_R is the square-root cost per share, while C_S is the fixed cost per share, as explained in Subsection (E)

symbol	Tick	C_R (bps/share)	C_S (bps/share)	TWAP	Opt	r_{TWAP}(%)
SFIX	4.400	114.110	461.443	0.136	0.071	47.740
DENN	4.490	52.704	77.782	0.292	0.261	10.508
AERI	4.510	139.967	272.170	0.190	0.076	59.864
CARA	4.530	128.853	204.231	0.332	0.207	37.727
AIMT	4.610	205.796	578.042	0.674	0.482	28.483
IOVA	4.680	112.071	387.380	0.689	0.565	17.936
HAIN	4.730	90.818	105.984	0.308	0.257	16.613
MTSI	4.980	156.338	268.951	0.765	0.667	12.728
BLDR	5.130	83.686	144.894	0.323	0.266	17.654
PENN	5.210	67.629	142.904	0.302	0.229	24.155
GOSS	5.240	193.095	216.793	0.606	0.526	13.173
INSM	5.330	126.153	293.323	0.604	0.453	25.056
HOMB	5.360	81.269	138.495	0.372	0.281	24.588
HRTX	5.400	237.154	656.579	1.268	1.140	10.113
LSCC	5.410	129.356	339.207	0.697	0.581	16.745
FATE	5.580	160.337	255.260	0.882	0.774	12.228
PETS	5.610	134.171	101.666	0.241	0.158	34.440
SVMK	5.690	95.742	152.292	0.261	0.219	16.086
ONB	5.830	58.194	79.312	0.122	0.088	28.042
TVTY	5.910	144.991	241.729	0.356	0.294	17.408
LAUR	6.050	88.242	95.596	0.690	0.654	5.263
CVET	6.150	279.666	319.957	2.484	2.390	3.810
HALO	6.170	65.524	176.743	0.431	0.332	22.830
RTRX	6.620	170.683	261.373	0.905	0.812	10.257
UPWK	6.670	82.056	124.614	0.829	0.788	4.911
MRNA	6.690	102.223	419.080	0.748	0.674	9.950
IMMU	6.760	148.241	421.324	0.604	0.506	16.224

Table 13 The cost measures per symbol (from index 82 to 110). C_R is the square-root cost per share, while C_S is the fixed cost per share, as explained in Subsection (E)

symbol	Tick	C_R (bps/share)	C_S (bps/share)	TWAP	Opt	r_{TWAP}(%)
GLNG	6.900	134.107	204.667	0.583	0.489	16.051
FEYE	6.940	76.399	123.090	0.455	0.405	11.013
REGI	7.040	99.314	203.394	0.850	0.803	5.603
GT	7.290	71.808	88.165	0.091	0.041	55.051
NWSA	7.320	37.404	22.306	0.129	0.122	5.403
CHNG	7.450	87.843	146.305	0.657	0.613	6.649
QNST	7.500	170.589	144.054	0.731	0.695	4.899
NAVI	7.660	71.902	36.506	0.215	0.206	4.129
WIFI	7.760	181.352	450.175	2.511	2.421	3.570
SONO	7.870	122.412	203.433	0.446	0.404	9.526
PAYS	7.930	131.132	151.647	0.720	0.669	7.069
ISBC	8.810	36.387	28.502	0.045	0.035	22.266
TTMI	8.980	88.596	93.264	0.389	0.355	8.677
VLY	9.220	40.365	24.387	0.036	0.025	31.227
AMAG	9.280	133.055	180.220	0.308	0.239	22.543
SGMO	9.500	80.458	254.180	0.814	0.715	12.150
MDRX	9.640	66.643	72.959	0.276	0.249	9.756
FLEX	9.750	69.527	39.776	0.202	0.189	6.755
PBYI	10.040	147.196	289.481	0.675	0.595	11.861
IRWD	10.540	83.468	306.640	0.943	0.846	10.220
TELL	13.470	154.455	357.936	0.823	0.759	7.745
APPS	15.760	106.590	237.706	0.832	0.769	7.605
CPRX	18.930	192.866	326.459	0.944	0.853	9.657
VRAY	19.990	241.404	170.761	0.480	0.419	12.692
OAS	27.260	125.666	68.967	0.190	0.134	29.635
ENDP	28.970	195.489	172.958	0.439	0.373	15.002
GPOR	31.790	144.696	139.360	0.583	0.541	7.194

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