## EC.1. Proof of Theorem 3.

*Proof of Theorem 3* *Finite Termination.* We show that CnP terminates in a finite number of steps. The calls to solve the NCP in Step 4, and to the ESO in Step 10 terminate in a finite number of steps because we assume the players' feasible sets are computationally convexifiable (in particular, condition (i) in Assumption 1). The only loop that could potentially not terminate is the `repeat` starting in Step 2.

First, we restrict to the case where the set $\mathcal{X}^i$ is bounded or finite for any player $i$. Any PAG $\widetilde{G}$ necessarily admits a PNE if the approximations $\widetilde{\mathcal{X}}^i_t$ are compact. Therefore, the algorithm never triggers Step 5. Thus, Step 11 is the only step refining the sets $\widetilde{\mathcal{X}}^i_t$ for some player $i$. As $\mathcal{X}^i$ is computationally convexifiable for every player $i$, the ESO can, in a finite number of steps, refine the approximations $\widetilde{\mathcal{X}}^i_t$ to $\text{conv}(\mathcal{X}^i)$. As a consequence, the algorithm converges (in the worst case) to $\overline{G}$ (*i.e.*, the exact convex approximation), and the correctness of the resulting MNE follows from Theorem 1.

Second, if $\mathcal{X}^i$ is unbounded for some player $i$, then a $PNE$ for a given PAG $\widetilde{G}$ may not exist, and the algorithm may enter Step 5. However, because $\mathcal{X}^i$ is computationally convexifiable for every player $i$, the branching step and the ESO can refine, in a finite number of steps, the approximations $\widetilde{\mathcal{X}}^i_t$ to $\text{conv}(\mathcal{X}^i)$. Therefore, even in the unbounded case, the algorithm correctly returns an MNE, similarly to the bounded case.

*Proof of statements (i) and (ii).* We show that $\widehat{\sigma}$ is an MNE for $G$. If the algorithm returns $\widehat{\sigma}$, then there exists an approximate game $\widetilde{G}$ in Step 3 with a PNE $\widetilde{x}$. We associate the PNE $\widetilde{x}$ with the mixed strategy profile $\widehat{\sigma}$. Let the iteration associated with $\widetilde{G}$ having the PNE $\widetilde{x}$ be denoted with $t=\theta$, and, for each player $i$, let $\widetilde{\mathcal{X}}^i_\theta$ be the associated feasible region $i$. Then, for any player $i$ and $\overline{x}^i \in \widetilde{\mathcal{X}}^i_\theta$, it follows that $f^i(\widehat{\sigma}^i;\widehat{\sigma}^{-i}) = f^i(\widetilde{x}^i;\widetilde{x}^{-i}) \leq f^i(\overline{x}^i;\widehat{\sigma}^{-i})$, *i.e.*, no player $i$ has the incentive to deviate from $\widehat{\sigma}^i$ to any other strategy $\overline{x}^i \in \widetilde{\mathcal{X}}^i_\theta$ in $\widetilde{G}$. Because $\text{conv}(\mathcal{X}^i) \subseteq \widetilde{\mathcal{X}}^i_\theta$ for any $i$, then the previous inequality holds for any $\overline{x}^i \in \text{conv}(\mathcal{X}^i)$. Moreover, by construction, $\widetilde{x}^i \in \text{conv}(\mathcal{X}^i)$; otherwise, the ESO would have returned `no` for player $i$.

We need to show that if the algorithm returns $\emptyset$, then the original game $G$ has no MNE. We prove this via its contrapositive, *i.e.*, we show that if $G$ has an MNE, then there exists an iteration $t$ where $\widetilde{G}$ has a PNE and Algorithm 1 returns the associated MNE in Step 12. Since the feasible set $\mathcal{X}^i$ of each player $i$ is computationally convexifiable, we can take the last step $t_{\text{last}}$ of the finitely many steps taken to convexify all players' feasible sets. From the definition of computational convexiability, PAG $\widetilde{G}$ at step $t_{\text{last}}$ corresponds to the game where each player's feasible set is replaced by its convex hull, *i.e.*, $\overline{G} = \widetilde{G}$. From Theorem 1, we know that the convexified version of the game has a PNE if and only if the original game has an MNE. Thus, Algorithm 1 has to find this PNE in its last iteration. $\square$

## EC.2. Proof of Theorem 4.

*Proof of Theorem 4* The ESO inner approximates the polyhedron $\text{conv}(\mathcal{X})$ with its $\mathcal{V}$-representation, which is made of finitely many extreme rays and vertices. Hence, we have to prove that the ESO never finds, at any step, a vertex $\nu$ (ray $r$) in Step 13 (Step 10) so that $\nu$ is already in $V$ ($r$ is already in $R$). This implies that the repeat loop in Algorithm 2 terminates.

The inequality after the else statement in Step 8 is valid for $\mathcal{W}$ if and only if $v^\top \overline{\pi} \leq \overline{\pi}_0$ for any $v \in V$, and $r^\top \overline{\pi} \leq 0$ for any $r \in R$ as of (6b) and (6c). Also, because the latter inequality is a separating hyperplane between $\mathcal{W}$ and $\overline{x}$, then $\overline{\pi}^\top \overline{x} > \overline{\pi}_0$. However, it may not necessarily be a valid inequality for any element in $\text{ext}(\text{conv}(\mathcal{X}))$ and $\text{rec}(\text{conv}(\mathcal{X}))$. Therefore, we must consider the optimization problem $\mathcal{G}$ in Step 9. On the one hand, if $\mathcal{G}$ is bounded, let $\nu$ be its optimal solution. Then, either (i.) $\overline{\pi}^\top \nu < \overline{\pi}^\top \overline{x}$, with $\overline{\pi}^\top x \leq \overline{\pi}^\top \nu$ being a separating hyperplane between $\text{conv}(\mathcal{X})$ and $\overline{x}$, and the algorithm terminates and returns `no`, or (ii.) $\overline{\pi}^\top \nu \geq \overline{\pi}^\top \overline{x}$, $\nu$ is necessarily a vertex of $\text{ext}(\text{conv}(\mathcal{X})) \setminus V$ violating $\overline{\pi}^\top x \leq \overline{\pi}_0$, and the algorithm updates $V \leftarrow V \cup \{\nu\}$. On the other hand, if $\mathcal{G}$ is unbounded, then there exists an extreme ray $r$ so that $r^\top \overline{\pi} > 0$. Then, $r$ is necessarily in $\text{rec}(\text{conv}(\mathcal{X})) \setminus R$, $\overline{\pi}^\top r > \overline{\pi}_0$, and the algorithm updates $R \leftarrow R \cup \{r\}$ and returns to Step 6. As there are finitely many extreme rays and vertices, the algorithm terminates. $\square$

## EC.3. IPGs Results

Tables EC.1 and EC.2 presents the full computational results for our experiments. The column names are analogous to those of Table 1, with the addition of a few columns. Specifically, we report the value of the social welfare in $SW$ and the average numbers of: (i.) cuts from the ESO excluding value cuts ($ESOCuts$), (ii.) value cuts from the ESO ($VCuts$). Finally, in the time column, we report in parenthesis the time the algorithm spent to compute the first MNE; this is relevant when CnP optimizes the social welfare function via a MIP solver.

**Table EC.1** IPGs complete results, first set.

| Algorithm | O | C | Time (s) | #TL | SW | #It | Cuts | ESOCuts | VCuts | MIPCuts |
|---|---|---|---|---|---|---|---|---|---|---|
| `n=3 m=10` | | | | | | | | | | |
| **SGM** | - | - | 2.11 | 0 | 632.99 | 10.00 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 0.47 (0.23) | 0 | 812.48 | 4.50 | 5.0 | 2.0 | 3.0 | 0.0 |
| **CnP-MIP** | Q | 0 | 0.31 (0.14) | 0 | 812.98 | 4.60 | 4.8 | 2.0 | 1.1 | 1.7 |
| **CnP-MIP** | Q | 1 | 0.20 (0.08) | 0 | 820.71 | 2.60 | 7.2 | 0.5 | 1.1 | 5.6 |
| **CnP-PATH** | F | -1 | 0.02 | 0 | 706.66 | 5.00 | 5.9 | 2.0 | 3.9 | 0.0 |
| **CnP-PATH** | F | 0 | 0.02 | 0 | 718.13 | 4.50 | 4.9 | 2.0 | 1.5 | 1.4 |
| **CnP-PATH** | F | 1 | 0.03 | 0 | 742.87 | 2.00 | 5.4 | 0.3 | 0.7 | 4.4 |
| `n=2 m=20` | | | | | | | | | | |
| **SGM** | - | - | 0.01 | 0 | 658.31 | 5.40 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 0.96 (0.25) | 0 | 684.19 | 6.40 | 6.3 | 4.4 | 1.9 | 0.0 |
| **CnP-MIP** | Q | 0 | 0.93 (0.29) | 0 | 683.91 | 6.10 | 5.9 | 3.0 | 1.2 | 1.7 |
| **CnP-MIP** | Q | 1 | 0.75 (0.18) | 0 | 682.69 | 3.70 | 7.6 | 1.4 | 0.9 | 5.3 |
| **CnP-PATH** | F | -1 | 0.05 | 0 | 645.44 | 5.30 | 5.5 | 3.1 | 2.4 | 0.0 |
| **CnP-PATH** | F | 0 | 0.04 | 0 | 664.44 | 4.90 | 4.7 | 1.8 | 1.2 | 1.7 |
| **CnP-PATH** | F | 1 | 0.03 | 0 | 656.44 | 3.10 | 6.2 | 1.2 | 0.4 | 4.6 |
| `n=3 m=20` | | | | | | | | | | |
| **SGM** | - | - | 0.20 | 0 | 1339.98 | 9.90 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 29.74 (1.49) | 0 | 1488.96 | 12.50 | 17.4 | 7.0 | 10.4 | 0.0 |
| **CnP-MIP** | Q | 0 | 27.22 (0.66) | 0 | 1473.46 | 6.50 | 8.7 | 4.0 | 1.2 | 3.5 |
| **CnP-MIP** | Q | 1 | 29.61 (0.61) | 0 | 1476.85 | 4.20 | 14.0 | 2.0 | 0.5 | 11.5 |
| **CnP-PATH** | F | -1 | 1.04 | 0 | 1327.47 | 12.50 | 19.2 | 6.3 | 12.9 | 0.0 |
| **CnP-PATH** | F | 0 | 0.08 | 0 | 1325.23 | 6.40 | 8.1 | 3.4 | 1.6 | 3.1 |
| **CnP-PATH** | F | 1 | 0.07 | 0 | 1361.74 | 4.60 | 15.0 | 2.2 | 0.5 | 12.3 |
| `n=2 m=40` | | | | | | | | | | |
| **SGM** | - | - | 1.26 | 0 | 1348.56 | 13.70 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 27.87 (5.11) | 0 | 1433.13 | 16.70 | 21.9 | 11.1 | 10.8 | 0.0 |
| **CnP-MIP** | Q | 0 | 25.58 (3.53) | 0 | 1434.09 | 12.80 | 13.4 | 8.2 | 1.1 | 4.1 |
| **CnP-MIP** | Q | 1 | 29.72 (2.16) | 0 | 1405.30 | 10.50 | 18.7 | 6.4 | 0.7 | 11.6 |
| **CnP-PATH** | F | -1 | 0.89 | 0 | 1355.26 | 16.80 | 20.7 | 9.5 | 11.2 | 0.0 |
| **CnP-PATH** | F | 0 | 0.70 | 0 | 1355.01 | 10.00 | 9.9 | 7.1 | 0.8 | 2.0 |
| **CnP-PATH** | F | 1 | 0.62 | 0 | 1355.21 | 7.80 | 14.1 | 5.1 | 0.3 | 8.7 |

**Table EC.2** IPGs complete results, second set.

| Algorithm | O | C | Time (s) | #TL | SW | #It | Cuts | ESOCuts | VCuts | MIPCuts |
|---|---|---|---|---|---|---|---|---|---|---|
| n=3 m=40 | | | | | | | | | | |
| **SGM** | - | - | 27.04 | 2 | 2339.79 | 20.10 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 140.33 (5.49) | 0 | 2991.76 | 20.20 | 28.5 | 13.2 | 15.3 | 0.0 |
| **CnP-MIP** | Q | 0 | 128.74 (3.06) | 0 | 3016.22 | 11.60 | 15.6 | 8.9 | 1.9 | 4.8 |
| **CnP-MIP** | Q | 1 | 162.20 (2.58) | 0 | 2980.69 | 9.30 | 21.9 | 6.7 | 0.9 | 14.3 |
| **CnP-PATH** | F | -1 | 2.35 | 0 | 2882.45 | 17.60 | 24.9 | 12.6 | 12.3 | 0.0 |
| **CnP-PATH** | F | 0 | 0.87 | 0 | 2906.33 | 10.80 | 14.0 | 8.8 | 1.4 | 3.8 |
| **CnP-PATH** | F | 1 | 0.79 | 0 | 2898.04 | 9.00 | 21.1 | 6.6 | 0.8 | 13.7 |
| n=2 m=80 | | | | | | | | | | |
| **SGM** | - | - | 14.97 | 1 | 2676.52 | 19.40 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 29.83 (11.47) | 0 | 3127.96 | 7.60 | 6.7 | 5.4 | 1.3 | 0.0 |
| **CnP-MIP** | Q | 0 | 27.02 (7.27) | 0 | 3127.97 | 7.80 | 7.0 | 5.3 | 0.7 | 1.0 |
| **CnP-MIP** | Q | 1 | 36.71 (10.06) | 0 | 3124.63 | 6.10 | 8.6 | 3.6 | 0.5 | 4.5 |
| **CnP-PATH** | F | -1 | 7.71 | 0 | 2914.36 | 8.80 | 8.1 | 6.7 | 1.4 | 0.0 |
| **CnP-PATH** | F | 0 | 5.45 | 0 | 2926.82 | 7.00 | 6.1 | 4.5 | 0.4 | 1.2 |
| **CnP-PATH** | F | 1 | 4.93 | 0 | 2936.52 | 5.80 | 7.4 | 3.4 | 0.4 | 3.6 |
| n=2 m=100 | | | | | | | | | | |
| **SGM** | - | - | 77.13 | 3 | 2861.20 | 21.10 | - | - | - | - |
| **CnP-MIP** | Q | -1 | 102.57 (36.29) | 0 | 3750.38 | 10.30 | 10.9 | 7.4 | 3.5 | 0.0 |
| **CnP-MIP** | Q | 0 | 105.97 (33.07) | 1 | 3454.41 | 14.30 | 14.5 | 9.4 | 1.2 | 3.9 |
| **CnP-MIP** | Q | 1 | 107.04 (30.86) | 0 | 3771.62 | 12.00 | 18.0 | 6.3 | 0.8 | 10.9 |
| **CnP-PATH** | F | -1 | 23.02 | 0 | 3496.86 | 11.22 | 11.67 | 8.33 | 3.33 | 0.0 |
| **CnP-PATH** | F | 0 | 14.46 | 0 | 3488.44 | 10.70 | 11.0 | 7.1 | 1.2 | 2.7 |
| **CnP-PATH** | F | 1 | 14.56 | 0 | 3507.71 | 10.30 | 14.8 | 6.4 | 0.7 | 7.7 |