E-Companion to
"Optimal Design of Default Donations"

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EC.1. Proofs for Section 3

EC.1.1. Proof of Theorem 1.

For any fixed $n \geq 2$ and $\delta > 0$, consider an instance I with the utility function introduced in Example A, i.e., $v(\theta, t) = \theta \cdot t - t^2/2$. We remind that in this example, a donor with $\theta \geq 0$ will select the element of $\mathcal{M}$ that is closest to $t^\star(\theta) = \theta$ if and only if it is within $\sqrt{2\varepsilon}$ of θ. Otherwise, the donor will donate θ (see Lemma 1). If more than one donation amount maximizes utility, we remind that our convention is to break ties in favor of the largest such default.

Furthermore, suppose $\varepsilon \sim F$ is such that $\sqrt{2\varepsilon} \sim \text{Unif}[0, 2]$, and suppose independently $\theta \sim G$ where

$$G(\theta) = \begin{cases} 2i/M, & \theta \in [4(i-1), 4i), \quad \forall i \in \{1, \dots, n\} \\ 2n/M + \sum_{\ell=0}^{j} (2n - \ell\delta)/(M\delta), & \theta \in [4n + 2j, 4n + 2(j+1)), \quad \forall j \in \{0, \dots, \bar{j}\} \end{cases}, \tag{EC.1.1}$$

where $\bar{j} \triangleq \lfloor 2n/\delta \rfloor$ and $M = (2n + 2(\bar{j}+1)n/\delta - \bar{j}(\bar{j}+1)/2)^{-1}$. In other words, there are n point masses of size $2/M$ at $0, 4, \dots 4(n-1)$. At $4n$, there is a point mass of size $2n/(M\delta)$, and for each subsequent even integer, there is a point mass smaller than the previous one by $1/M$, until the next point mass would be negative. The constant M is appropriately defined so that the masses sum to 1. We illustrate the corresponding pdf g in Figure EC.1.1.

Claim EC.1 *In this instance, the greedily-selected menu is $\mathcal{M}^g = \{1, 5, \dots, 4(n-1)+1\}$, which obtains revenue gains of $E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}^g, \theta, \varepsilon)] = n/M$.*

Proof of Claim EC.1 We prove Claim EC.1 in two steps. First, we will show that (in the absence of nearby defaults) if we are putting a default m between any two consecutive generosity parameters θ_1, θ_2, revenue is maximized for $m = \theta_1 + 1$.[1] We then prove by induction that this leads to a greedily-selected menu of $\mathcal{M}^g = \{1, 5, \dots, 4(n-1)+1\}$ which obtains revenue gains of n/M.

[1] For completeness, we note that the optimal placement of a default m greater than $\theta^{\max} = 4n + 2\bar{j}$ is $m = \theta^{\max} + 1$, which leads to revenue gains of $\frac{1}{2M}$.

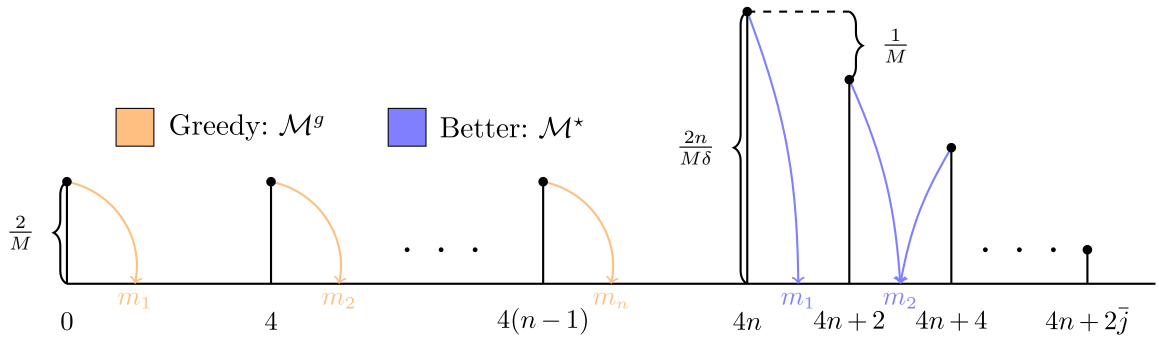


Figure EC.1.1 **Construction for the proof of Theorem 1.**

Step 1: Recall that (i) a utility-maximizing donor will select the element of $\mathcal{M}$ that is closest to $t^\star(\theta)=\theta$ if and only if it is within $\sqrt{2\varepsilon}$ of θ, (ii) with probability 1, $\sqrt{2\varepsilon}<2$, and (iii) the distance between any pair of consecutive generosity parameters θ_1 and θ_2 is at least 2. As a consequence, any default $m\in[\theta_1,\theta_2]$ will only impact donors with generosity parameter θ_1 or θ_2.

We now establish the base case of our inductive argument. Suppose there are no other defaults and consider placing a default at $m=\theta_1+x$, for $x\in[0,\theta_2-\theta_1]$. The revenue gains are given by the following:

$$\begin{aligned}
&g(\theta_1)\mathbf{P}(\sqrt{2\varepsilon}\geq x)\,x-g(\theta_2)\mathbf{P}(\sqrt{2\varepsilon}\geq \theta_2-\theta_1-x)(\theta_2-\theta_1-x)\\
&=g(\theta_1)\int_0^2\mathbf{1}\{u\geq x\}\,x\,\frac{1}{2}\,du\\
&\quad-g(\theta_2)\int_0^2\mathbf{1}\{u\geq \theta_2-\theta_1-x\}(\theta_2-\theta_1-x)\frac{1}{2}\,du\\
&=\frac{1}{2}\Big(g(\theta_1)x(2-x)^+-g(\theta_2)(\theta_2-\theta_1-x)(2-\theta_2+\theta_1+x)^+\Big).
\end{aligned}\tag{EC.1.2}$$

For $x\geq 2$, $(2-x)^+=0$, which implies that this expression is non-positive. Hence, we can focus our attention on $x\in[0,2]$. If $\theta_2-\theta_1=4$, the derivative of this expression with respect to x is $g(\theta_1)(1-x)$. Otherwise, if $\theta_2-\theta_1=2$, the derivative of this expression with respect to x is $g(\theta_1)(1-x)-g(\theta_2)(1-x)$, and we note that whenever $\theta_2-\theta_1=2$, we also have $g(\theta_1)>g(\theta_2)$. This implies that revenue gains are uniquely maximized at a choice of $x=1$, i.e., $m=\theta_1+1$.

Step 2: We can now assume without loss of generality that the first greedily-selected default is $m=\theta_1+1$ for some θ_1. If $\theta_1<4n$, then the revenue gains are $(1/2)\cdot(2/M)\cdot 1=1/M$ because the mass at θ_1 (which is $2/M$) will choose this default with probability 1/2, and the mass at θ_2 will never deviate. Alternatively, if $\theta_1\geq 4n$, the revenue gains are given by $(1/2)\cdot(g(\theta_1)-g(\theta_2))\cdot 1=\frac{1}{2M}$. Therefore, the first default selected must be in the set $\{1,5,\ldots,4(n-1)+1\}$.

We proceed via induction. Our inductive hypothesis holds that, for $m\leq n$, the first $m-1$ greedily-selected defaults are a subset of $\{1,5,\ldots,4(n-1)+1\}$ and they obtain revenue gains of

$(m-1)/M$. Consider the greedy placement of the m^{th} default. If we place the m^{th} default at an element of the set $\{1,5,\ldots,4(n-1)+1\}$ which is not currently in the menu, we will obtain revenue gains of $1/M$, by the same logic as before. Furthermore, no other default can obtain larger revenue gains: under the menu of our inductive hypothesis, the revenue gains given in Equation (EC.1.2) must be an upper bound on the revenue gains from adding a default to that interval. (To see why, note that the positive term cannot be larger because, for any θ_1, there is no default in $[\theta_1-2,\theta_1]$, while the negative term can be larger if there is a default at θ_2+1.) As a result, we have shown that the m^{th} greedily-selected default will be a new element of the set $\{1,5,\ldots,4(n-1)+1\}$ and will obtain revenue gains of $1/M$. As there are n defaults and n elements of the set, we have proved by induction that the greedily-selected menu is $\mathcal{M}^g=\{1,5,\ldots,4(n-1)+1\}$, which obtains revenue gains of n/M. □

Claim EC.2 *There is a feasible menu $\mathcal{M}^\star$ that obtains a value $(2n+\delta)/(2M\delta)$.*

Proof of Claim EC.2 Consider the menu $\mathcal{M}^\star=\{4n+1,4n+3\}$. Note that

$$\hat{t}(\mathcal{M}^\star,\theta,\varepsilon)=\begin{cases}\theta+1 \text{ if } \theta=4n, \sqrt{2\varepsilon}\geq 1,\\ \theta+1 \text{ if } \theta=4n+2, \sqrt{2\varepsilon}\geq 1,\\ \theta-1 \text{ if } \theta=4n+4, \sqrt{2\varepsilon}\geq 1,\\ \theta \qquad\qquad \text{o.w .}\end{cases}$$

In turn,

$$\begin{aligned}E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}^\star,\theta,\varepsilon)]&=E_{\theta,\varepsilon}[\hat{t}(\mathcal{M}^\star,\theta,\varepsilon)-t^\star(\theta)]\\&=\frac{2n}{M\delta}\cdot\frac{1}{2}+\frac{2n-\delta}{M\delta}\cdot\frac{1}{2}-\frac{2n-2\delta}{M\delta}\cdot\frac{1}{2}\\&=\frac{2n+\delta}{2M\delta}.\end{aligned}$$

This completes the proof of Claim EC.2. □

In order to conclude the proof of Theorem 1, note that

$$\delta E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}^\star,\theta,\varepsilon)]=\frac{2n+\delta}{2M}>\frac{n}{M}=E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}^g,\theta,\varepsilon)].$$

□

EC.1.2. Proof of Lemma 1.

Recall that in Section 3.1.1, the donor's utility function is given by $U(\mathcal{M},\theta,\varepsilon,t)=\theta\cdot t-t^2/2+\varepsilon\cdot\mathbf{1}\{t\in\mathcal{M}\}$. In the absence of any defaults, this reduces to $\theta\cdot t-t^2/2$, which is maximized at $t^\star(\theta)=\theta$. This obtains a utility of $\theta^2/2$.

We now consider the utility-maximizing donation from among the default options. Suppose the donor selects $m \in \mathcal{M}$. In that case, the donor's utility is given by

$$\begin{aligned} U(\mathcal{M},\theta,\varepsilon,m) &= \theta \cdot m - (m)^2/2 + \varepsilon \\ &= \theta^2/2 - \theta^2/2 + \theta \cdot m - (m)^2/2 + \varepsilon \\ &= \theta^2/2 - (\theta - m)^2/2 + \varepsilon \end{aligned}$$

We make the following observations: (i) this utility is decreasing in the distance $|\theta - m|$. Hence, the default closest to θ will be preferred. (ii) This utility exceeds $\theta^2/2$ (the utility from donating θ) if $\varepsilon > (\theta - m)^2/2$, or equivalently, if $\sqrt{2\varepsilon} > |\theta - m|$. Otherwise, if $\sqrt{2\varepsilon} < |\theta - m|$, donating θ obtains a higher utility. If $\sqrt{2\varepsilon} = |\theta - m|$, the donor is indifferent and we follow the convention of breaking ties in favor of the largest donation amount. $\square$

EC.1.3. Proof of Proposition 1.

Consider a donor with $t^\star(\theta) \in [m', m'']$, and consider two menus, $\mathcal{M}_1 = \{m', m''\}$ and $\mathcal{M}_2$ such that $\mathcal{M}_1 \subset \mathcal{M}_2$ and there are no other defaults in the interval $[m', m'']$. We proceed by contradiction: suppose $\hat{t}(\theta,\varepsilon,\mathcal{M}_1) \triangleq \hat{t}_1 \neq \hat{t}_2 \triangleq \hat{t}(\theta,\varepsilon,\mathcal{M}_2)$. There are two cases to consider, depending on whether or not $\hat{t}_2 \in \mathcal{M}_2 \setminus \mathcal{M}_1$.

Case (i): $\hat{t}_2 \in \mathcal{M}_2 \setminus \mathcal{M}_1$**.** We assume that v is strictly concave, which implies $U(\mathcal{M}_2,\theta,\varepsilon,\hat{t}_2) < v(\theta,m') + v_t(\theta,m')(\hat{t}_2 - m') + \varepsilon$ for any $\hat{t}_2 \neq m'$. Because $t^\star(\theta) \geq m'$, we must have $v_t(\theta,m') \geq 0$. This means that for any $\hat{t}_2 < m'$,

$$U(\mathcal{M}_2,\theta,\varepsilon,\hat{t}_2) \; < \; v(\theta,m') + \varepsilon \; = \; U(\mathcal{M}_2,\theta,\varepsilon,m').$$

Similarly, for any $\hat{t}_2 > m''$,

$$U(\mathcal{M}_2,\theta,\varepsilon,\hat{t}_2) \; < \; v(\theta,m'') + v_t(\theta,m'')(\hat{t}_2 - m'') + \varepsilon \; \leq \; v(\theta,m'') + \varepsilon \; = \; U(\mathcal{M}_2,\theta,\varepsilon,m'').$$

Thus, we have shown the suboptimality of any $\hat{t}_2 \notin [m', m'']$, which contradicts the assumption that there are no other defaults in the interval $[m', m'']$

Case (ii): $\hat{t}_2 \notin \mathcal{M}_2 \setminus \mathcal{M}_1$**.** Recall our assumption that $\hat{t}_1$ is optimal for $\mathcal{M}_1$. Then $U(\mathcal{M}_1,\theta,\varepsilon,\hat{t}_1) \geq U(\mathcal{M}_1,\theta,\varepsilon,\hat{t}_2)$. Because $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\hat{t}_2 \notin \mathcal{M}_2 \setminus \mathcal{M}_1$, $U(\mathcal{M}_2,\theta,\varepsilon,\hat{t}_1) \geq U(\mathcal{M}_1,\theta,\varepsilon,\hat{t}_1) \geq U(\mathcal{M}_1,\theta,\varepsilon,\hat{t}_2) = U(\mathcal{M}_2,\theta,\varepsilon,\hat{t}_2)$. If the inequality is strict, this contradicts the optimality of $\hat{t}_2$. Otherwise, we know that $\hat{t}_1$ and $\hat{t}_2$ obtain equal utility under both $\mathcal{M}_1$ and $\mathcal{M}_2$. Based on our convention that $\hat{t}$ breaks ties in favor of the largest donation amount, we must have $\hat{t}_1 = \hat{t}_2$ (because $\hat{t}_1$ is optimal for $\mathcal{M}_1$ while $\hat{t}_2$ is optimal for $\mathcal{M}_2$). This contradicts the assumption that the two amounts are different.

$\square$

EC.1.4. Proof of Theorem 2.

We first use induction to prove the formulas for the value-to-go functions, defined in Equations (7) and (8). Then, we prove that the nonprofit's optimal revenue gains are indeed given by the formula in Equation (9).

First, as a base case, note that when the menu $\mathcal{M}$ is restricted to at most 1 default with m as its minimum (i.e., only) value, we have the following series of equalities for the value-to-go function:

$$\begin{aligned}
\mathcal{V}_1(m) &= \max_{\mathcal{M}\in M^1}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m\}] : m=\min\mathcal{M}\right\}\\
&= E_{\theta,\varepsilon}[\mathcal{R}(\{m\},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m\}]\\
&= \int_{t^\star(\theta)\geq m}\int \mathcal{R}(\{m\},\theta,\varepsilon)\partial F(\varepsilon|\theta)\partial G(\theta)\\
&= \int_{t^\star(\theta)\geq m}\int \left(\hat{t}(\theta,\varepsilon,\{m\})-t^\star(\theta)\right)\partial F(\varepsilon|\theta)\partial G(\theta),
\end{aligned}$$

where the final equality comes from applying the definition of $\mathcal{R}$ from Equation (3). This proves the equality in Equation (8).

Now we proceed by induction and assume that $\mathcal{V}_{n-1}(m)$ is the value-to-go at m with $n-1$ defaults, i.e., the maximum revenue gains that can be obtained from donors whose ideal donation amount is above m when the menu has at most n options, the smallest of which is m. We will use this inductive hypothesis to prove Equation (7).

$$\begin{aligned}
\mathcal{V}_n(m) &= \max_{\mathcal{M}\in M^n}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m\}] : m=\min\mathcal{M}\right\}\\
&= \max_{m'\geq m, m'\in M}\ \max_{\mathcal{M}'\in M^{n-1}}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}'\cup\{m\},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m\}] : m'=\min\mathcal{M}'\right\}\\
&= \max_{m'\geq m, m'\in M}\ \max_{\mathcal{M}'\in M^{n-1}}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\{m,m'\},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\in[m,m']\}]\right.\\
&\qquad\left. + E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}',\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m'\}]\right\} : m'=\min\mathcal{M}'\Big\} \qquad \text{(EC.1.3)}\\
&= \max_{m'\geq m, m'\in M}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\{m,m'\},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\in[m,m']\}]\right.\\
&\qquad\left. + \max_{\mathcal{M}'\in M^{n-1}}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M}',\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\geq m'\}] : m'=\min\mathcal{M}'\right\}\right\}\\
&= \max_{m'\geq m, m'\in M}\left\{\int_{t^\star(\theta)\in[m,m']}\int \left(\hat{t}(\theta,\varepsilon,\{m,m'\})-t^\star(\theta)\right)\partial F(\varepsilon|\theta)\partial G(\theta)+\mathcal{V}_{n-1}(m')\right\}
\end{aligned}$$

The key step in Line (EC.1.3) involves separating the expectation into two groups, depending on whether $t^\star(\theta)\in[m,m']$ or whether $t^\star(\theta)\geq m'$. By Proposition 1, the first group will only be impacted by the defaults at m and m'. Similarly, the second group will not be impacted by the default at m. The final step comes from applying the definition of $\mathcal{R}$ as well as our inductive hypothesis. This proves the equality in Equation (7).

To conclude the proof of Theorem 2, we must show that the nonprofit's objective is equivalent to the expression in Equation (9). We start with the definition of the nonprofit's objective in (OMD).

$$\begin{aligned}
\mathcal{R}_I^\star &= \max_{\mathcal{M}\in M^n} E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M},\theta,\varepsilon)] \\
&= \max_{m\in M}\max_{\mathcal{M}\in M^n}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M},\theta,\varepsilon)] : m=\min\mathcal{M}\right\} \\
&= \max_{m\in M}\max_{\mathcal{M}\in M^n}\left\{E_{\theta,\varepsilon}[\mathcal{R}(\{m\},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\le m\}]+E_{\theta,\varepsilon}[\mathcal{R}(\mathcal{M},\theta,\varepsilon)\mathbf{1}\{t^\star(\theta)\ge m\}] : m=\min\mathcal{M}\right\} \quad \text{(EC.1.4)} \\
&= \max_{m\in M}\left\{\int_{t^\star(\theta)\le m}\int\left(\hat{t}(\theta,\varepsilon,\{m\})-t^\star(\theta)\right)\partial F(\varepsilon|\theta)\partial G(\theta)+\mathcal{V}_n(m)\right\}
\end{aligned}$$

The key step in Line (EC.1.4) separates the expectation into two groups and again relies on Proposition 1 to remove the dependence on the set $\mathcal{M}$ for the first group. The final step comes from applying the definition of $\mathcal{R}$ as well as the definition of our value-to-go function $\mathcal{V}_n(m)$. This completes the proof of Theorem 2. □

EC.2. Omitted Details from Section 4

EC.2.1. Omitted Details from Section 4.1

EC.2.1.1. Proof of Proposition 2. Consider a donor of type (θ,ε). By Equation (11), $\bar{t}(\theta,\varepsilon)$ is the largest t satisfying

$$v(\theta,t)+\varepsilon-v(\theta,t^\star(\theta))\ge 0. \quad \text{(EC.2.1)}$$

To show that a unique $\bar{t}(\theta,\varepsilon)$ must exist, recall our assumptions that v is strictly concave and that the marginal donated dollar eventually generates a net loss in utility (i.e., there exists a t' such that $v_t(\theta,t')<0$). For $t=t^\star(\theta)$, the left hand side of (EC.2.1) is positive. Furthermore, for $t\ge t'$, the left hand side is strictly decreasing, which means that there exists a large enough t such that the left hand side is negative. Because it is a continuous and strictly concave function, there must be a unique zero greater than $t^\star(\theta)$. This is $\bar{t}(\theta,\varepsilon)$.

By construction, the donor is indifferent between selecting $\bar{t}(\theta,\varepsilon)$ from the menu or donating $t^\star(\theta)$. Based on our tie-breaking convention, they select the larger amount, leading to revenue gains of $\bar{t}(\theta,\varepsilon)-t^\star(\theta)$. To see that this menu is optimal, note that including any lower amount in the menu can only reduce revenue, while any higher default would simply be ignored due to the limits of the donor's suggestibility parameter. In other words, for any $t'>\bar{t}(\theta,\varepsilon)$ and any $\mathcal{M}$, we must have $U(\theta,\varepsilon,\mathcal{M},t')<U(\theta,\varepsilon,\mathcal{M},t^\star(\theta))$. □

EC.2.1.2. Characterizing the Full-Information Benchmark. In the following proposition, we provide a partial characterization of the full-information revenue gains from a donor.

Proposition EC.1 (Value of Full-Information Gains from a Donor) *Suppose $v(\theta,\cdot)$ is $\mathbf{C}^3$ and suppose there exists some $\alpha>0$ such that $|v_{tt}(\theta,t)|>\alpha$ for all θ,t. Given a donor with generosity parameter θ where $t^\star(\theta)>0$, there exists a positive constant c_1 such that for any suggestibility parameter ε,*

$$\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \quad \leq \quad c_1\cdot\sqrt{\varepsilon}.$$

Moreover,

$$\lim_{\varepsilon\downarrow 0}\frac{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)}{\sqrt{\varepsilon}} \quad = \quad \sqrt{\frac{2}{-v_{tt}(\theta,t^\star(\theta))}}.$$

When the utility from selecting a default is small relative to the overall utility from donating (i.e., fixing the donor's utility function and taking the limit as ε approaches 0), then the optimal revenue gains approximately equal $\sqrt{(2\varepsilon)/(-v_{tt}(\theta,t^\star(\theta)))}$. Comparative statics of this expression reveal insights about the two key drivers of the value of defaults: (i) If the donor's utility from selecting a default (ε) is larger, then the nonprofit can extract additional revenue by increasing the default. (ii) If the donor's change in marginal utility at their ideal donation ($v_{tt}(\theta,t^\star(\theta))$) is smaller in magnitude, then the nonprofit can persuade them to donate more by increasing the default, thereby increasing revenue.

To illustrate the accuracy of this characterization, let us revisit Example A, where $v_{tt}(\theta,t^\star(\theta))=-1$ for all $\theta>0$. According to Proposition EC.1, the full-information gains from this donor should be approximately $\sqrt{2\varepsilon}$ (when ε is sufficiently small). In fact, by Lemma 1 the full-information gains are *exactly* equal to $\sqrt{2\varepsilon}$, regardless of the value of ε. For intuition as to why equality holds in this example even for large ε, we note that the proof of Proposition EC.1 relies on a quadratic approximation of v using a Taylor expansion at $t^\star(\theta)$. Hence, for Example A — where v is quadratic — our approximation is exact.

Proof of Proposition EC.1 We first prove the upper bound on $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$ which holds for any ε. Then, we show that the value $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$ converges to a simple expression in the limit as $\varepsilon\to 0$.

Upper Bound on $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$: Recall the assumption that $-v_{tt}(\theta,t)>\alpha>0$. By bounding the curvature of the donor's concave utility function, we can bound the utility $v(\theta,t^\star(\theta)+z)$:

$$v(\theta,t^\star(\theta)+z) \quad\leq\quad v(\theta,t^\star(\theta))+v_t(\theta,t^\star(\theta))\cdot z-\alpha\cdot\frac{z^2}{2}. \tag{EC.2.2}$$

Because we assume that this donor's ideal donation is strictly positive ($t^\star(\theta)>0$), the first-order condition $v_t(\theta,t^\star(\theta))=0$ must hold which implies

$$v(\theta,t^\star(\theta)+z) \quad\leq\quad v(\theta,t^\star(\theta))-\alpha\cdot\frac{z^2}{2}. \tag{EC.2.3}$$

Suppose $z > \sqrt{2\varepsilon/\alpha}$. In that case,

$$v(\theta, t^\star(\theta) + z) \quad \leq \quad v(\theta, t^\star(\theta)) - \alpha \cdot \frac{z^2}{2} \quad < \quad v(\theta, t^\star(\theta)) - \varepsilon.$$

This implies that $t^\star(\theta) + z$ is not feasible in Equation (11) for any $z > \sqrt{2\varepsilon/\alpha}$. Therefore, $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \leq \sqrt{2\varepsilon/\alpha}$. Defining $c_1 \triangleq \sqrt{2/\alpha}$ proves the upper bound in Proposition EC.1.

Limiting value of $\mathcal{R}^{\mathbf{FI}}(\theta,\varepsilon)$: To solve $\bar{t}(\theta,\varepsilon)$ in Equation (11), i.e., the maximum amount a donor would be willing to select from a menu, it is sufficient to find the unique value of $z \geq 0$ satisfying

$$v(\theta, t^\star(\theta)) - v(\theta, t^\star(\theta) + z) = \varepsilon,$$

where uniqueness follows Proposition 2. This value of z represents the maximum amount the donor is willing to deviate from their ideal donation; hence, $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) = z$.

Taking a Taylor expansion of $v(\theta,t)$ at $t^\star(\theta)$ and using the mean value theorem, we have that for some $\hat{t} \in [t^\star(\theta), t^\star(\theta) + z]$:

$$v(\theta, t^\star(\theta) + z) \quad = \quad v(\theta, t^\star(\theta)) + v_t(\theta, t^\star(\theta)) \cdot z + v_{tt}(\theta, t^\star(\theta)) \cdot \frac{z^2}{2} + v_{ttt}(\theta, \hat{t}) \cdot \frac{z^3}{6}. \qquad \text{(EC.2.4)}$$

Plugging in the Taylor expansion and recalling that the first-order condition $v_t(\theta, t^\star(\theta)) = 0$ must hold, we have

$$\begin{aligned} v(\theta, t^\star(\theta)) - v(\theta, t^\star(\theta) + z) &= v(\theta, t^\star(\theta)) - v(\theta, t^\star(\theta)) - v_t(\theta, t^\star(\theta)) \cdot z \\ &\quad - v_{tt}(\theta, t^\star(\theta)) \cdot \frac{z^2}{2} - v_{ttt}(\theta, \hat{t}) \cdot \frac{z^3}{6} \\ &= -v_{tt}(\theta, t^\star(\theta)) \cdot \frac{z^2}{2} - v_{ttt}(\theta, \hat{t}) \cdot \frac{z^3}{6}. \end{aligned}$$

Based on the first part of the proof, we know $z \leq \sqrt{2\varepsilon/\alpha}$. Furthermore, since $v(\theta, \cdot)$ is $\mathbf{C}^3$ we can always find a positive constant $\hat{M}$ such that $|v_{ttt}(\theta,t)| \leq \hat{M}$ for all t in a bounded interval $[t^\star(\theta), t^\star(\theta) + c_2]$, where c_2 is an arbitrary constant. Therefore, when ε is sufficiently small (i.e., $\varepsilon \leq c_2^2 \cdot \alpha/2$), $|v_{ttt}(\theta,t)| \leq \hat{M}$ for any $t \in [t^\star(\theta), t^\star(\theta) + z]$. In other words, for any ε less than a fixed constant, the value of z satisfying Equation (EC.2.4) must satisfy:

$$\varepsilon - \frac{\hat{M}(2\varepsilon)^{3/2}}{6\alpha^{3/2}} \quad \leq \quad -v_{tt}(\theta, t^\star(\theta)) \cdot \frac{z^2}{2} \quad \leq \quad \varepsilon + \frac{\hat{M}(2\varepsilon)^{3/2}}{6\alpha^{3/2}}. \qquad \text{(EC.2.5)}$$

Solving for $z/\sqrt{\varepsilon}$, we must have

$$\sqrt{\frac{2}{-v_{tt}(\theta, t^\star(\theta))}\left(1 - \frac{\hat{M}(2\varepsilon)^{1/2}}{6\alpha^{3/2}}\right)} \quad \leq \quad \frac{z}{\sqrt{\varepsilon}} \quad \leq \quad \sqrt{\frac{2}{-v_{tt}(\theta, t^\star(\theta))}\left(1 + \frac{\hat{M}(2\varepsilon)^{1/2}}{6\alpha^{3/2}}\right)} \qquad \text{(EC.2.6)}$$

In the limit as $\varepsilon \to 0$, the upper and lower bounds on $z/\sqrt{\varepsilon}$ both converge to $\sqrt{2/(-v_{tt}(\theta, t^\star(\theta)))}$, which completes the proof of Proposition EC.1. $\square$

EC.2.2. Upper Bounds on Attainable Performance in General Instances

Here we provide two instances which highlight the need for additional structural assumptions in order to provide meaningful guarantees. In particular, these examples motivate the introduction of Assumption 1 in our partial information setting (Section 4.2) and the necessity of Assumption 2 in our general setting with only distributional information (Section 4.3).

Upper Bound on Performance in Section 4.2 without Assumption 1. Consider an instance $I \in \hat{\mathcal{I}}$ where the donor's utility function is given as in Example A, i.e., $v(\theta,t) = \theta \cdot t - t^2/2$. In this instance, let $\theta = t^\star(\theta) = 0$ deterministically, and suppose $\varepsilon \sim F$ where $F(x) = 0$ for $x < 1$ and $F(x) = 1 - 1/\sqrt{x} + 1/\sqrt{L}$ for $x \in [1, L]$ for some large constant L (i.e., there is a point mass of size $1/\sqrt{L}$ at $\varepsilon = 1$). Furthermore, since $I \in \hat{\mathcal{I}}$, the set of allowable defaults is given by $M = \mathbb{R}_+$.

Based on Lemma 1, in this instance a donor will select the default closest to $t^\star(\theta) = 0$ if it is at most $\sqrt{2\varepsilon}$; otherwise, they will donate 0. As such, we have $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) = \sqrt{2\varepsilon}$ for all donors. This yields

$$\mathcal{R}_I^{\text{FI}} \;=\; \mathbf{E}_{\theta,\varepsilon}[\mathcal{R}^{\text{FI}}(\theta,\varepsilon)] \;\geq\; \int_1^L \sqrt{2x} f(x)\; dx \;=\; \log(L)/\sqrt{2} \tag{EC.2.7}$$

In contrast, to characterize the optimal revenue gains under partial information, it is sufficient to only consider singleton menus (because the donor would only select the default closest to $t^\star(\theta) = 0$). Suppose the default closest to $t^\star(\theta) = 0$ is denoted by m. The donor will select m if and only if $m \leq \sqrt{2\varepsilon}$, or equivalently, if $\varepsilon \geq m^2/2$. If m is selected, the nonprofit receives revenue gains of m; otherwise, revenue gains are 0. Therefore, the optimal revenue gains are given by

$$\mathcal{R}_I^{\text{PI}} \;=\; \max_m m \cdot (1 - F(m^2/2)) \;<\; \sqrt{2} \tag{EC.2.8}$$

Therefore, $\mathcal{R}_I^{\text{PI}} / \mathcal{R}_I^{\text{FI}} < 2/\log(L)$, which can be arbitrarily small for an appropriate choice of M.

Upper Bound on Performance in Section 4.3 without Assumption 2. Here we present a similar example, but using a different donor utility function where ε is deterministic and the variability in $\mathcal{R}^{\text{FI}}(\theta,\varepsilon)$ is instead driven by θ. Specifically, consider an instance I where $v(\theta,t) = -t^2/\theta$, which meets our minimal assumptions on v as it is strictly concave in t (for all $\theta > 0$). Suppose ε is deterministically 1, and suppose suppose $\theta \sim G$ where $G(x) = 0$ for $x < 1$ and $G(x) = 1 - 1/\sqrt{x} + 1/\sqrt{L}$ for $x \in [1, L]$ for some large constant L (i.e., there is a point mass of size $1/\sqrt{L}$ at $\theta = 1$).

For any θ, we have $t^\star(\theta) = 0$, and for all donors, we have $\varepsilon = 1$. A donor will select the default closest to $t^\star(\theta) = 0$, which we denote with m, if and only if $-m^2/\theta + 1 \geq 0$, or equivalently, if $m \leq \sqrt{\theta}$. As such, we have $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) = \sqrt{\theta}$. In such an instance,

$$\mathcal{R}_I^{\text{FI}} \;=\; \mathbf{E}_{\theta,\varepsilon}[\mathcal{R}^{\text{FI}}(\theta,\varepsilon)] \;\geq\; \int_1^L \sqrt{x} g(x)\; dx \;=\; \log(L)/2 \tag{EC.2.9}$$

In contrast, to characterize the optimal revenue gains with access only to distributional information, it is sufficient to only consider singleton menus (because the donor would only select the default closest to $t^\star(\theta)=0$). Suppose the default closest to $t^\star(\theta)=0$ is denoted by m. The donor will select m if and only if $m \le \sqrt{\theta}$, or equivalently, if $\theta \ge m^2$. If m is selected, the nonprofit receives revenue gains of m; otherwise, revenue gains are 0. Therefore, the optimal revenue gains are given by

$$\mathcal{R}_I^\star \;=\; \max_m m\cdot(1-G(m^2)) \;<\; 1 \tag{EC.2.10}$$

Therefore, $\mathcal{R}_I^\star/\mathcal{R}_I^{\mathrm{FI}} < 2/\log(M)$, which can be arbitrarily small for an appropriate choice of M.

EC.2.3. Proof of Proposition 3

We prove this result in two steps. In Step 1, we explicitly construct an instance of a posted pricing problem whose optimal value is equivalent to the nonprofit's optimal revenue gains when the generosity parameter is known. Then, in Step 2, we show that the MHR assumption on the buyer's value distribution is satisfied in the instance of posted pricing that we construct.

Step 1: Fixing any θ, we now show how the nonprofit's problem under partial information (ODD-PI) maps into an instance of the posted pricing problem. This requires us to identify the buyer's value as well as the seller's pricing problem, and establish an equivalence between the two revenue functions.

Given a fixed θ, the nonprofit can elect to offer an arbitrary menu, but it is without loss of optimality to assume the nonprofit offers a single default $\bar{k}(\theta) \ge t^\star(\theta)$.[2] The nonprofit's revenue is equal to $\bar{k}(\theta) - t^\star(\theta)$ if $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \ge \bar{k}(\theta) - t^\star(\theta)$, and equal to 0 otherwise. Letting H denote the CDF of $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$ given the fixed θ, the nonprofit's revenue is $(\bar{k}(\theta)-t^\star(\theta))\cdot\big(1-H(\bar{k}(\theta)-t^\star(\theta))\big)$.[3]

To see the connection to posted pricing, let the buyer's value u be given by $\bar{t}(\theta,\varepsilon) - t^\star(\theta)$, or equivalently, $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$. In that case, u also follows a distribution with CDF H. For any posted price p, the revenue is given by $p\cdot(1-H(p))$. Setting $p = \bar{k}(\theta) - t^\star(\theta)$ obtains the same revenue gains as in the nonprofit's problem. In other words, if the donor's value is less than the difference between the default and their ideal donation, the donor enters their ideal donation and the nonprofit obtains no revenue gain. Similarly, this would imply that the buyer's value u is less than the seller's price p, and no sale occurs. The range of allowable prices ($p \ge 0$) corresponds to the range of reasonable defaults ($\bar{k}(\theta) \ge t^\star(\theta)$). Thus, we have shown that the optimal expected revenue in this constructed

[2] To see that this is without loss of optimality, suppose the nonprofit offers multiple defaults. If any defaults are below the donor's ideal donation, it is weakly optimal to remove those defaults. Among the remaining defaults, the donor will ignore all but the smallest (i.e., the closest to their ideal donation), which implies that all other defaults can be removed without impacting revenue.

[3] For a fixed θ, the distribution H is uniquely determined by the conditional distribution of ε, i.e., $F(\varepsilon|\theta)$.

posted pricing problem is equivalent to the nonprofit's optimal revenue gains when the generosity parameter θ is known.

Step 2: For the instance of posted pricing that we constructed in the previous step, we now establish a structural result about the buyer's value distribution. This will allow us to apply results from Hartline et al. (2008) to our constructed problem.

Claim EC.3 *For a fixed θ, if ε is drawn from a known monotone hazard rate (MHR) distribution $F(\varepsilon|\theta)$, then the distribution of $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$ also satisfies MHR.*

Proof of Claim EC.3. The donor's type uniquely determines the revenue gains under full information $\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon)$. Therefore, for a fixed θ, we can think of $\mathcal{R}^{\mathrm{FI}}$ as being drawn from a distribution with cumulative distribution function (CDF) H. We aim to show that H is MHR by leveraging its relationship with ε. For ease of notation, we fix an arbitrary generosity parameter θ and omit the dependence on that parameter in the remainder of this step.

We begin by characterizing the CDF $H(x)=\mathbf{P}(\mathcal{R}^{\mathrm{FI}}(\varepsilon)\le x)$. We first note that this is simply a shifted version of the CDF of $\bar{t}(\varepsilon)$, i.e., $H(x)=\mathbf{P}(\bar{t}(\varepsilon)\le x+t^\star)$.

Furthermore, based on Equation (11), we must have that $\bar{t}(\varepsilon)$ is the unique solution to

$$\varepsilon \quad = \quad v(t^\star)-v(\bar{t}(\varepsilon)). \tag{EC.2.11}$$

We note that for $\bar{t}(\varepsilon)\ge t^\star$, the right hand side is a strictly increasing function of $\bar{t}$ due to the strict concavity of v and the optimality of $t^\star$ (which ensures that $v_t(\bar{t}(\varepsilon))\le 0$). Hence, $\bar{t}(\varepsilon)$ has a well-defined inverse which we denote with $\psi(\bar{t})$:

$$\psi(\bar{t}) \quad \triangleq \quad v(t^\star)-v(\bar{t}). \tag{EC.2.12}$$

Since ψ is a strictly increasing function for $\bar{t}\ge t^\star$, we have that for $x\ge 0$,

$$H(x) \;=\; \mathbf{P}(\bar{t}(\varepsilon)\le x+t^\star) \;=\; \mathbf{P}(\psi(\bar{t}(\varepsilon))\le \psi(x+t^\star)) \;=\; \mathbf{P}(\varepsilon\le\psi(x+t^\star)) \;=\; F(\psi(x+t^\star)) \tag{EC.2.13}$$

Having established the relationship between the distribution of $\mathcal{R}^{\mathrm{FI}}$ and the distribution of ε, we are now ready to compute the derivative of the hazard rate of H:

$$\frac{\partial}{\partial x}\frac{h(x)}{1-H(x)} \quad = \quad \frac{\partial}{\partial x}\frac{\psi'(x+t^\star)f(\psi(x+t^\star))}{1-F(\psi(x+t^\star))} \tag{EC.2.14}$$

$$=\psi'(x+t^\star)\frac{\partial}{\partial x}\frac{f(\psi(x+t^\star))}{1-F(\psi(x+t^\star))}+\psi''(x+t^\star)\frac{f(\psi(x+t^\star))}{1-F(\psi(x+t^\star))} \tag{EC.2.15}$$

The first equality comes from the relationship between the CDFs H and F which holds for all $x\ge 0$, while the second equality comes from applying the product rule.

We now argue that all terms on the right hand side of Equation (EC.2.15) must be non-negative. Recall the definition of the function ψ in Equation (EC.2.12). Based on the concavity of the function v and the optimality of $t^\star$, we must have $\psi'(x+t^\star)\geq 0$ and $\psi''(x+t^\star)\geq 0$ for all $x\geq 0$. Furthermore, because we assume ε is drawn from an MHR distribution F, its hazard rate must be non-decreasing. Mathematically, $\frac{\partial}{\partial x}\frac{f(\psi(x+t^\star))}{1-F(\psi(x+t^\star))}\geq 0$. Finally, because F is a well-defined distribution, we must have $\frac{f(\psi(x+t^\star))}{1-F(\psi(x+t^\star))}\geq 0$. We have shown that the CDF of $\mathcal{R}^{\text{FI}}(\theta,\varepsilon)$ is MHR, which completes the proof of the claim. $\square$

In light of Claim EC.3 and the mapping to the posted pricing setting established in Step 1, we can apply the following result from Hartline et al. (2008):

Lemma EC.1 (Hartline et al. 2008 Lemma A.1) *Suppose that X is a non-negative random variable with CDF F, where F has a monotone hazard rate. Then $1-F(E[X])\geq 1/e$.*

This result follows from properties of MHR distributions first presented in Barlow et al. (1963), and it establishes that if the buyer's value v is drawn from an MHR distribution, the expected revenue obtained by posting a price of $E[v]$ is within a $1/e$ fraction of $E[v]$. Therefore, we have shown that under Assumption 1, the nonprofit can offer a personalized default based only on the donor's generosity parameter θ that in expectation obtains a $1/e$ fraction of the maximum possible revenue gains under full information for a donor with that generosity parameter. Taking the expectation over all generosity parameters completes the proof of Proposition 3. $\square$

EC.2.4. Proof of Theorem 3

We prove Theorem 3 in four parts. First, in Part (i) we place a lower bound on $\inf_{I\in\hat{\mathcal{I}}_\rho}\mathcal{R}^\star_I/\mathcal{R}^{\text{FI}}_I$, which corresponds to the first part of Theorem 3. Then, in Parts (ii) - (iv), we construct three different families of instances which provide upper bounds on $\inf_{I\in\hat{\mathcal{I}}_\rho}\mathcal{R}^\star_I/\mathcal{R}^{\text{FI}}_I$.

As a high-level overview, our lower bound in Part (i) leverages the fact that the nonprofit can present a menu with many evenly-spaced defaults in an interval $[m,t_H]$ in order to obtain some revenue gains (from donors increasing their donation from $t^\star(\theta)$ to m) while mostly avoiding revenue losses (because there will be a default very close to $t^\star(\theta)$ if $t^\star(\theta)>m$). The first instance for our upper bound, presented in Part (ii), builds on the intuition developed in the partial information setting, where a lack of information about the suggestibility of donors limits the attainable revenue gains. In the second instance for our upper bound, presented in Part (iii), we use a uniform distribution of ideal donations to ensure that revenue gains from lower defaults are offset by corresponding revenue losses. This part of the proof introduces Claim EC.4, which establishes sufficient conditions under which the optimal menu only includes defaults at or above the mode. (We defer the formal proof of Claim EC.4 to Appendix EC.2.5.) The third instance, presented in Part (iv), involves constructing a distribution of ideal donations such that each additional default leads to a bounded net increase in revenue.

Part (i): We now provide a lower bound on $\mathcal{R}_I^\star/\mathcal{R}_I^{\mathrm{FI}}$ for any instance in the set $\hat{\mathcal{I}}_\rho$, i.e., the set of instances with range-to-gains ratio ρ (see Definition 1), with no restrictions on the allowable defaults (i.e., $M=\mathbb{R}_+$), and which satisfy Assumption 2.

Consider any instance $I\in\hat{\mathcal{I}}_\rho$. For this instance, suppose that the smallest possible ideal donation is t_L while the largest is t_H. We will denote the range of possible ideal donations with $t_R \triangleq t_H - t_L$. (Note that because $I\in\hat{\mathcal{I}}_\rho$, we must have $\rho = t_R/\mathcal{R}_I^{\mathrm{FI}}$.) Characterizing the value of the *optimal* menu for this instance is challenging; instead, we lower-bound the value by focusing on a particular family of menus, which allows us to exploit a connection to the partial information setting studied in Section 4.2 (where the nonprofit knows the donor's ideal donation).

Specifically, consider a family of menus parameterized by $m\ge t_L$, where m is the lowest default, and then the remaining defaults (if any) are evenly spaced between m and t_H. In that case, the nonprofit obtains revenue gains from any donor who would increase their donation to m. Furthermore, for any donor with $t^\star(\theta)\ge m$, the closest default below $t^\star(\theta)$ for such donors is within $(t_H-m)/n$. As a consequence, the total expected revenue losses cannot exceed $(t_H-t_L)/n$. Roughly speaking, this family of menus can be thought of as a feasible (but not necessarily optimal) solution to Equation (9). The nonprofit selects m for the first default, and then — rather than optimally placing the remaining defaults and obtaining a value of $\mathcal{V}_n(m)$ — bounds the potential revenue losses above m with $(t_H-t_L)/n$.

The value of a menu in this family heavily depends on the choice of m. In order to establish guarantees for a broad class of instances, rather than try to characterize the revenue gains from the optimal choice of m, we instead characterize the *average* gains over a range of choices for m, i.e., for $m\in[m_L,m_H]$, where $m_L\triangleq t_L+\mathcal{R}_I^{\mathrm{FI}}(1-\sqrt{\rho})^+$ and $m_H\triangleq t_H+\mathcal{R}_I^{\mathrm{FI}}$.

The gains from the optimal choice of m must exceed the average gains over this interval. In other words, we place the following lower bound on the value of the optimal menu for any instance I in $\hat{\mathcal{I}}_\rho$, where we use $m_R\triangleq t_R+\mathcal{R}_I^{\mathrm{FI}}\big(1-(1-\sqrt{\rho})^+\big)$ to denote the range of m.

$$
\begin{aligned}
\mathcal{R}_I^\star &= \max_{m\ge t_L}\left\{\int_{t^\star(\theta)\in[t_L,m]}\int\left(\hat{t}(\theta,\varepsilon,\{m\})-t^\star(\theta)\right)dF(\varepsilon|\theta)dG(\theta)+\mathcal{V}_n(m)\right\}\\
&\ge \max_{m\ge t_L}\left\{\int_{t^\star(\theta)\in[t_L,m]}\int\left(\hat{t}(\theta,\varepsilon,\{m\})-t^\star(\theta)\right)dF(\varepsilon|\theta)dG(\theta)-t_R/n\right\}\\
&\ge \frac{1}{m_R}\left(\int_{m_L}^{m_H}\int_{t^\star(\theta)\in[t_L,m]}\int\left(\hat{t}(\theta,\varepsilon,\{m\})-t^\star(\theta)\right)dF(\varepsilon|\theta)dG(\theta)dm\right)-t_R/n \qquad \text{(EC.2.16)}\\
&\triangleq \hat{\mathcal{R}}_I
\end{aligned}
$$

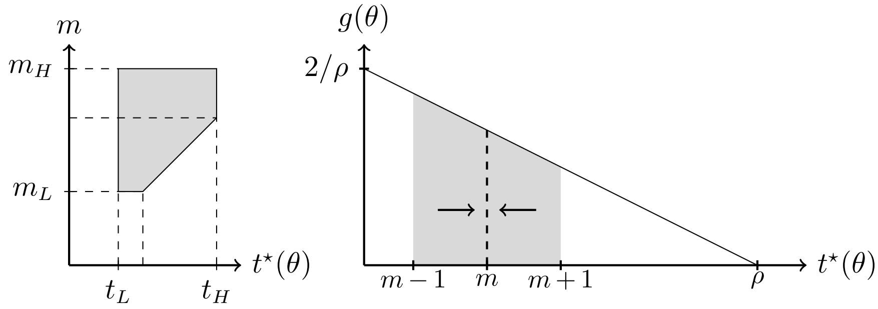


Figure EC.2.1 **(Left) Region of integration in Equations (EC.2.17) and (EC.2.18). (Right) Illustration of instance I_4.**

The first line is the characterization of the value of the optimal menu given in Theorem 2. We then replace $\mathcal{V}_n(m)$ with our lower bound $(-t_R/n)$, and in the final inequality, we bound the value of the integral for the optimal choice of m with its average value for $m \in [m_L, m_H]$. The remainder of the proof involves placing a lower bound on the value of $\hat{\mathcal{R}}_I$.

To that end, we first replace the integral in Equation (EC.2.16) with an equivalent expression. Consider any donor with type (θ, ε) where $m \geq t^\star(\theta)$. For such donors, $\hat{t}(\theta, \varepsilon, \{m\}) - t^\star(\theta) = m - t^\star(\theta)$ if and only if $\mathcal{R}^{\mathrm{FI}}(\theta, \varepsilon) \geq m - t^\star(\theta)$, and it is equal to 0 otherwise. Consequently,

$$\hat{\mathcal{R}}_I = \frac{1}{m_R}\left(\int_{m_L}^{m_H}\int_{t^\star(\theta)\in[t_L,m]}\int \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq m - t^\star(\theta)\}\cdot(m - t^\star(\theta))\ dF(\varepsilon|\theta)dG(\theta)dm\right) - t_R/n \tag{EC.2.17}$$

We illustrate the region of integration in Figure EC.2.1. Changing the order of integration yields:

$$\hat{\mathcal{R}}_I = \frac{1}{m_R}\left(\int\int\int_{\max\{t^\star(\theta),m_L\}}^{m_H} \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq m - t^\star(\theta)\}\cdot(m - t^\star(\theta))\ dmdF(\varepsilon|\theta)dG(\theta)\right) - t_R/n \tag{EC.2.18}$$

$$\geq \frac{1}{m_R}\left(\int\int\int_{\max\{t^\star(\theta),m_L\}}^{\mathcal{R}_I^{\mathrm{FI}}+t^\star(\theta)} \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq m - t^\star(\theta)\}\cdot(m - t^\star(\theta))\ dmdF(\varepsilon|\theta)dG(\theta)\right) - t_R/n \tag{EC.2.19}$$

$$\geq \frac{1}{m_R}\left(\int\int\int_{\max\{t^\star(\theta),m_L\}}^{\mathcal{R}_I^{\mathrm{FI}}+t^\star(\theta)} \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\mathrm{FI}}\}\cdot(m - t^\star(\theta))\ dmdF(\varepsilon|\theta)dG(\theta)\right) - t_R/n \tag{EC.2.20}$$

$$= \frac{1}{m_R}\left(\int\int \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\mathrm{FI}}\}\left(\frac{(\mathcal{R}_I^{\mathrm{FI}})^2 - (\max\{0, m_L - t^\star(\theta)\})^2}{2}\right)dF(\varepsilon|\theta)dG(\theta)\right) - t_R/n \tag{EC.2.21}$$

Inequality (EC.2.19) comes from further restricting the limits of integration, using the observation that the integrand is weakly positive for any m in the region of integration. Inequality (EC.2.20) comes from strengthening the condition within the indicator. Note that because $m - t^\star(\theta) \leq \mathcal{R}_I^{\mathrm{FI}}$ for any m within the new limits of integration, $\mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq m - t^\star(\theta)\} \geq \mathbf{1}\{\mathcal{R}^{\mathrm{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\mathrm{FI}}\}$. To explain these restrictions in words, we are only counting the revenue gains from donors who are willing to increase their donation by more than the average amount (i.e., donors with $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\text{FI}}$), and even then, we are only counting the revenue gains when the default is within this average amount from the donor's ideal donation (i.e., when $m \leq t^\star(\theta)+\mathcal{R}_I^{\text{FI}}$). We then evaluate the interior integral in Line (EC.2.21). Focusing on the remaining integrals, we have

$$\begin{aligned}
&\int\int \mathbf{1}\{\mathcal{R}^{\text{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\text{FI}}\} \left(\frac{(\mathcal{R}_I^{\text{FI}})^2 - (\max\{0, m_L - t^\star(\theta)\})^2}{2}\right) dF(\varepsilon|\theta)dG(\theta) \\
&\quad \geq \int\int \mathbf{1}\{\mathcal{R}^{\text{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\text{FI}}\} \left(\frac{(\mathcal{R}_I^{\text{FI}})^2 - (\mathcal{R}_I^{\text{FI}}(1-\sqrt{\rho})^+)^2}{2}\right) dF(\varepsilon|\theta)dG(\theta) \qquad \text{(EC.2.22)} \\
&= \left(\frac{(\mathcal{R}_I^{\text{FI}})^2(1-((1-\sqrt{\rho})^+)^2)}{2}\right) \int\int \mathbf{1}\{\mathcal{R}^{\text{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\text{FI}}\}\ dF(\varepsilon|\theta)dG(\theta) \\
&\geq \left(\frac{(\mathcal{R}_I^{\text{FI}})^2(1-((1-\sqrt{\rho})^+)^2)}{2}\right)(1/e) \qquad \text{(EC.2.23)}
\end{aligned}$$

In Line (EC.2.22), we plug in the definition of $m_L = t_L + \mathcal{R}_I^{\text{FI}}(1-\sqrt{\rho})^+$, and bound the expression by noting that $t_L \leq t^\star(\theta)$ for any θ. We then move any terms that no longer depend on θ or ε outside the remaining integrals. The key step in Line (EC.2.23) relies on the fact that I satisfies Assumption 2, which allows us to apply Lemma EC.1. Specifically, note that $\mathcal{R}^{\text{FI}}(\theta,\varepsilon)$ is a random variable whose value is determined by the joint distribution of θ and ε. Under Assumption 2, this random value follows an MHR distribution. Furthermore, its expected value is given by $\mathcal{R}_I^{\text{FI}}$, and according to Lemma EC.1, the probability that an MHR random variable exceeds its expected value is at least $1/e$. Therefore, the expected value of the indicator (taken over the joint distribution of θ and ε) must be at least $1/e$.

Based on Equations (EC.2.21) and (EC.2.23), we have

$$\begin{aligned}
\hat{\mathcal{R}}_I &\geq \left(\frac{1}{m_R}\right)\left(\frac{(\mathcal{R}_I^{\text{FI}})^2(1-((1-\sqrt{\rho})^+)^2)}{2}\right)(1/e) - t_R/n \\
&= \left(\frac{1}{\mathcal{R}_I^{\text{FI}} + t_R - \mathcal{R}_I^{\text{FI}}(1-\sqrt{\rho})^+}\right)\left(\frac{(\mathcal{R}_I^{\text{FI}})^2(1-((1-\sqrt{\rho})^+)^2)}{2e}\right) - t_R/n \\
&= \mathcal{R}_I^{\text{FI}}\left(\frac{1-((1-\sqrt{\rho})^+)^2}{(2e)(1+\rho-(1-\sqrt{\rho})^+)} - \frac{\rho}{n}\right) \qquad \text{(EC.2.24)}
\end{aligned}$$

The final step comes from applying the definition of the range-to-gains ratio ρ, i.e., $\rho = t_R/\mathcal{R}_I^{\text{FI}}$ (see Definition 1). Therefore,

$$\mathcal{R}_I^\star/\mathcal{R}_I^{\text{FI}} \quad \geq \quad \hat{\mathcal{R}}_I/\mathcal{R}_I^{\text{FI}} \quad \geq \quad \frac{1-((1-\sqrt{\rho})^+)^2}{(2e)(1+\rho-(1-\sqrt{\rho})^+)} - \frac{\rho}{n}$$

Coupled with the observation that the optimal revenue gains must be non-negative (because displaying *no* menu is always an option), this corresponds to the piecewise lower bound in Part (i) of Theorem 3, thereby completing the proof of this step. □

Part (ii): To establish the first piece of the upper bound in Theorem 3, we construct a family of instances parameterized by the range-to-gains ratio ρ. For any $\rho \in \mathbb{R}_+$, consider an instance $I_2(\rho)$ where the set of allowable menus is given by $\mathbb{R}_+^n$ for an arbitrary n. (For ease of notation, we henceforth suppress the dependence of I_2 on ρ.) The donor's utility function is given as in Example A, i.e., $v(\theta,t) = \theta \cdot t - t^2/2$. As established in Lemma 1, for this utility function, a donor with $\theta \geq 0$ will select the default closest to $t^\star(\theta)$ if it is within $\sqrt{2\varepsilon}$; otherwise, they will donate $t^\star(\theta)$. As such, we have $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) = \sqrt{2\varepsilon}$.

In instance I_2, the distribution of generosity parameters $G(\theta)$ is uniform in the range $[0,\rho]$. The distribution of suggestibility parameters is given by $F(\varepsilon|\theta) = 1 - \exp(-\sqrt{2\varepsilon})$ for $\varepsilon \geq 0$. Based on this distribution of donor types, for any $y \geq 0$ we have

$$\begin{aligned}
\mathbf{P}(\mathcal{R}^{\text{FI}}_{I_2}(\theta,\varepsilon) \leq y) &= \mathbf{P}(\sqrt{2\varepsilon} \leq y) \\
&= \mathbf{P}(\varepsilon \leq y^2/2) \\
&= 1 - \exp(-\sqrt{2 \cdot y^2/2}) \\
&= 1 - \exp(-y)
\end{aligned}$$

In other words, the distribution of $\mathcal{R}^{\text{FI}}_{I_2}(\theta,\varepsilon)$ is exponential with rate 1.

We begin by establishing that instance I_2 satisfies the three criteria for inclusion into $\hat{\mathcal{I}}_\rho$, as specified in Definition 1. First, the range of ideal donations is given by ρ while the expected value of $\mathcal{R}^{\text{FI}}_{I_2}(\theta,\varepsilon)$ (i.e., our benchmark $\mathcal{R}^{\text{FI}}_{I_2}$) is given by 1. Hence, the range-to-gains ratio is equal to ρ. Second, by construction the set of allowable defaults is $M = \mathbb{R}_+$. Third, $\mathcal{R}^{\text{FI}}_{I_2}(\theta,\varepsilon)$ follows an exponential distribution, which has a monotone hazard rate; hence, this instance satisfies Assumption 2. We now upper-bound the attainable revenue gains in this instance.

Lemma EC.2 (Upper Bound on Revenue Gains in I_2) *In instance I_2, $\mathcal{R}^\star_{I_2} \leq e^{-1}$.*

Given that our full-information benchmark $\mathcal{R}^{\text{FI}}_{I_2}$ is equal to 1, Lemma EC.2 (which we prove below) establishes that the relative performance of the optimal menu is bounded by e^{-1}. This is sufficient to prove the first piece of the upper bound of Theorem 3.

Proof of Lemma EC.2. Suppose that the nonprofit presents a menu of $\mathcal{M}$. Focusing on a donor with a fixed generosity parameter θ, let m represent the closest default to $t^\star(\theta)$ that exceeds $t^\star(\theta)$. The donor would never prefer a larger default (by Proposition 1), but could prefer a default below $t^\star(\theta)$. Hence, the expected revenue gain from this donor (where the expectation is taken over ε) is bounded by the expected gain if m was the only default. This is given by

$$(m - t^\star(\theta)) \cdot \mathbf{P}(\mathcal{R}^{\text{FI}}_{I_2}(\theta,\varepsilon) \leq m - t^\star(\theta)) \quad = \quad (m - t^\star(\theta)) \cdot \exp\big(-(m - t^\star(\theta))\big),$$

where the right hand side uses the fact that the distribution of $\mathcal{R}^{\mathrm{FI}}_{I_2}(\theta,\varepsilon)$ is exponential (and independent of θ). This expression is maximized at $m = t^\star(\theta)+1$, which obtains a value of e^{-1}. In other words, we have shown that for any donor with a fixed generosity parameter θ, the revenue gains are bounded by e^{-1}. Taking expectation over θ completes the proof of Lemma EC.2.

Part (iii): To establish the second piece of the upper bound in Theorem 3, we again construct a family of instances parameterized by the range-to-gains ratio ρ. For any $\rho \in \mathbb{R}_+$, consider an instance $I_3(\rho)$ where the set of allowable menus is given by $\mathbb{R}^n_+$ for an arbitrary n. (As in Part (ii), we henceforth suppress the dependence of I_3 on ρ.) The donor's utility function is given as in Example A, i.e., $v(\theta,t) = \theta \cdot t - t^2/2$. As established in Lemma 1, for this utility function, a donor with $\theta \geq 0$ will select the default closest to $t^\star(\theta)$ if it is within $\sqrt{2\varepsilon}$; otherwise, they will donate $t^\star(\theta)$. As such, we have $\mathcal{R}^{\mathrm{FI}}_{I_3}(\theta,\varepsilon) = \sqrt{2\varepsilon}$.

In instance I_3, the distribution of generosity parameters $G(\theta)$ is uniform in the range $[0,\rho]$. The suggestibility parameter ε is deterministically equal to $1/2$, which implies that $\mathcal{R}^{\mathrm{FI}}_{I_3}(\theta,\varepsilon) = 1$.

We begin by establishing that instance I_3 satisfies the three criteria for inclusion into $\hat{\mathcal{I}}_\rho$, as specified in Definition 1. First, the range of ideal donations is given by ρ while the expected value of $\mathcal{R}^{\mathrm{FI}}_{I_3}(\theta,\varepsilon)$ (i.e., our benchmark $\mathcal{R}^{\mathrm{FI}}_{I_3}$) is equal to 1. Hence, the range-to-gains ratio is equal to ρ. Second, by construction the set of allowable defaults is $M = \mathbb{R}_+$. Third, $\mathcal{R}^{\mathrm{FI}}_{I_3}(\theta,\varepsilon)$ is deterministic; hence, this instance satisfies Assumption 2.

We now characterize the optimal revenue gains in such an instance, i.e., $\mathcal{R}^\star_{I_3}$.

Lemma EC.3 (Upper Bound on Revenue Gains in I_3) *In instance I_3 with $\rho \geq 1$, we have $\mathcal{R}^\star_{I_3} \leq 1/2\rho$.*

Given that our full-information benchmark is equal to 1 in instance I_3, Lemma EC.3 (which we prove below) establishes that the relative performance of the optimal menu is given by $1/2\rho$. This is sufficient to prove the second piece of the upper bound in Theorem 3.

Proof of Lemma EC.3. The proof of Lemma EC.3 relies primarily on Claim EC.4, which establishes a general property of the optimal menu for the class of quasi-symmetric instances (which includes instance I_3). To define this class, we make use of the following three definitions, which together will allow us to express the distribution of donor types more directly in the context of donation amounts:

$$\tilde{G}(t) = \int \mathbf{1}\{t^\star(\theta) \leq t\}\ g(\theta)d\theta \tag{EC.2.25}$$

$$\tilde{g}(t) = \frac{\partial \tilde{G}(t)}{\partial t} \tag{EC.2.26}$$

$$\tilde{F}(y|t) = \frac{\int F(v(\theta,t^\star(\theta)) - v(\theta,t^\star(\theta)+y)|\theta)\delta(t-t^\star(\theta))\ g(\theta)d\theta^{4}}{\tilde{g}(t)} \tag{EC.2.27}$$

In words, $\tilde{G}(t)$ represents the CDF of ideal donations, $\tilde{g}(t)$ represents the corresponding PDF, and $\tilde{F}(y|t)$ represents the marginal probability that – conditional on having an ideal donation at t – a donor is *unwilling* to deviate from their ideal donation at t in favor of a default at $t+y$ (i.e., they prefer their ideal donation). The functions $\tilde{G}(t)$ and $\tilde{g}(t)$ can be thought of as the analogs to $G(\theta)$ and $g(\theta)$ in the domain of donation amounts, as opposed to donor types. The relationship between $\tilde{F}(y|t)$ and $F(\varepsilon|\theta)$ is similar: the former can be thought of as the CDF for donors' unwillingness to positively deviate from their ideal donation by an amount $y \geq 0$, though we note that it is also defined for $y \leq 0$. (Donors' deviations from their ideal donations can be either positive or negative.)

Fixing an instance, these three functions are uniquely defined. We are now ready to define the class of quasi-symmetric instances and state the claim.

[Quasi-Symmetric Instances] An instance is weakly (strictly) *quasi-symmetric* if $M = \mathbb{R}_+$ and if it satisfies the following three properties:

1. The distribution of ideal donations $\tilde{g}(t)$ is weakly (strictly) unimodal over its support.
2. Donors' (un)willingness to deviate is independent of their ideal donation, i.e., $\tilde{F}(x|t) = \tilde{F}(x|t')$ for all $t, t', x \in \mathbb{R}$.
3. Donors' utility is symmetric around their ideal donation $t^\star(\theta)$, i.e., $v(\theta, t^\star(\theta) + x) = v(\theta, t^\star(\theta) - x)$ for all θ and all $x \leq t^\star(\theta)$.

Claim EC.4 (Optimality of Only Having Defaults Above the Mode) *For any weakly quasi-symmetric instance I with a mode at t_{mode}, there exists an optimal menu $\mathcal{M}_I^*$ such that* $\arg\min_{m \in \mathcal{M}_I^*} m \geq t_{mode}$.

Claim EC.4 identifies a class of instances where it is without loss of optimality to only include default options that are at least as big as the most common ideal donation amount. We note that this result holds for any cardinality constraint, and it can be applied to any mode in cases where a weakly quasi-symmetric instance has multiple modes. Though immediately helpful in the proof of Lemma EC.3, this claim may be of broader interest as it can be thought of as a rule of thumb for the placement of defaults. The properties in Claim EC.4 represent *sufficient* conditions under which it is optimal to only include "large" defaults. Instances that do not satisfy these conditions may nevertheless have optimal menus with exclusively "large" defaults.[5] We defer the proof of Claim EC.4 to Appendix EC.2.5.

To leverage Claim EC.4 in the proof of Lemma EC.3, we first must establish that instance I_3 is weakly quasi-symmetric. For Property (1), each donor's ideal donation $t^\star(\theta)$ is equal to their

[4] Here, $\delta(x)$ is the Dirac delta. If $\tilde{g}(t) = 0$, this expression is undefined, and we will follow the convention that $\tilde{F}(y|t) = \tilde{F}(y|t_{\text{mode}})$ where t_{mode} is the mode of $\tilde{g}$.

[5] Regarding the necessity of these conditions, we note that for any one of the three conditions, we can construct a counterexample where only that condition is violated and where the optimal menu includes a default below t_{mode}.

generosity parameter θ, which is drawn from a uniform (i.e., weakly unimodal) distribution. Importantly, any value between 0 and ρ is a mode of the distribution. For Property (2), $F(\cdot|\theta)$ is a step function that does not depend on θ, and $v(\theta, t^\star(\theta)) - v(\theta, t^\star(\theta) + y) = y^2/2$ regardless of the value of θ. Property (3) holds based on similar algebra: $v(\theta, t^\star(\theta) + y) = v(\theta, t^\star(\theta) - y) = \theta^2/2 - y^2/2$.

Given that instance I_3 satisfies these properties and has a mode at ρ, Claim EC.4 establishes that it is without loss of optimality to only consider menus with defaults at least equal to ρ. For such menus, any donor — who in instance I_3 must satisfy $t^\star(\theta) \in [0, \rho]$ — will only consider the smallest default, as any other default is further from $t^\star(\theta)$ (see Proposition 1). Thus, we can restrict our attention to singleton menus with $m \geq \rho$. This allows us to bound the revenue gains in instance I_3 with the solution to a single-variable optimization problem:

$$
\begin{aligned}
\mathcal{R}^\star_{I_3} &\leq \max_{m \geq \rho} \int_0^\infty (m-\theta)\mathbf{1}\{m-\theta \leq 1\} \cdot (1/\rho)\mathbf{1}\{\theta \leq \rho\}\ d\theta \\
&= \max_{m \in [\rho, \rho+1]} \int_{m-1}^{\rho} (1/\rho)(m-\theta)\ d\theta && \text{(EC.2.28)} \\
&= \max_{m \in [\rho, \rho+1]} (1/\rho)\big(m \cdot \theta - \theta^2/2\big)\big|_{\theta=m-1}^{\rho} \\
&= \max_{m \in [\rho, \rho+1]} (1/\rho)\big(m \cdot \rho - \rho^2/2 - m \cdot (m-1) + (m-1)^2/2\big) \\
&= \max_{m \in [\rho, \rho+1]} (1/\rho)\big(m \cdot \rho - \rho^2/2 - m^2/2 + 1/2\big) \\
&= \max_{m \in [\rho, \rho+1]} (1/2\rho)\big(1 - (m-\rho)^2\big) && \text{(EC.2.29)} \\
&= 1/2\rho && \text{(EC.2.30)}
\end{aligned}
$$

In Line (EC.2.28), we restrict the limits of integration to match the region where both indicators are equal to 1, and we correspondingly restrict the range of the maximization to ensure that the region of integration is non-negative (i.e., to ensure that the lower limit of integration does not exceed the upper limit, in which case the expression would simply be equal to 0). The expression in Line (EC.2.29) is maximized at $m = \rho$, thereby obtaining a value of $1/2\rho$ and completing the proof of Lemma EC.3.

Part (iv): To establish the third piece of the upper bound in Theorem 3, we construct a family of instances which is nearly identical to family of instances used in Part (iii); the only difference comes from the distribution of generosity parameters G. Instead of a uniform probability density function (pdf), we construct a linearly decreasing pdf as shown in Figure EC.2.1. Intuitively, under such a distribution, the revenue gains from any single default will be largely offset by corresponding revenue losses, especially when the range-to-gains ratio ρ is large. This allows us to bound the revenue gains from each additional default.

To be specific, for any $\rho \geq 0$, consider an instance $I_4(\rho)$ where the set of allowable menus is given by $\mathbb{R}^n_+$ for an arbitrary n. (We henceforth suppress the dependence of I_4 on ρ.) The donor's utility function is given as in Example A, i.e., $v(\theta,t)=\theta\cdot t-t^2/2$. As established in Lemma 1, for this utility function, a donor with $\theta\geq 0$ will select the default closest to $t^\star(\theta)$ if it is within $\sqrt{2\varepsilon}$; otherwise, they will donate $t^\star(\theta)$. As such, we have $\mathcal{R}^{\mathrm{FI}}_{I_4}(\theta,\varepsilon)=\sqrt{2\varepsilon}$.

In instance I_4, the distribution of generosity parameters $G(\theta)$ has a pdf $g(\theta)=(2/\rho^2)(\rho-\theta)$ for all $\theta\in[0,\rho]$, and $g(\theta)=0$ otherwise. One can verify that this is a well-defined density function, with $\int_0^\rho g(\theta)=1$. The suggestibility parameter ε is deterministically equal to $1/2$, which implies that $\mathcal{R}^{\mathrm{FI}}_{I_4}(\theta,\varepsilon)=1$.

We begin by establishing that instance I_4 satisfies the three criteria for inclusion into $\hat{\mathcal{I}}_\rho$, as specified in Definition 1. First, the range of ideal donations is given by ρ while the expected value of $\mathcal{R}^{\mathrm{FI}}_{I_4}(\theta,\varepsilon)$ (i.e., our benchmark $\mathcal{R}^{\mathrm{FI}}_{I_4}$) is given by 1. Hence, the range-to-gains ratio is equal to ρ. Second, the set of allowable defaults is $M=\mathbb{R}_+$. Third, $\mathcal{R}^{\mathrm{FI}}_{I_4}(\theta,\varepsilon)$ is deterministic; hence, this instance satisfies Assumption 2.

We now characterize the optimal revenue gains in such an instance, i.e., $\mathcal{R}^\star_{I_4}$.

Lemma EC.4 (Upper Bound on Revenue Gains in I_4) *In instance I_4, $\mathcal{R}^\star_{I_4}\leq 4n/3\rho^2$.*

Given that our full-information benchmark is equal to 1 in instance I_4, Lemma EC.4 (which we prove below) establishes that the relative performance of the optimal menu is given by $4n/3\rho^2$. This is sufficient to prove the third piece of the upper bound in Theorem 3. By combining the three upper bounds in Parts (ii) - (iv) and identifying the domain where each upper bound is the minimum, we can confirm the piecewise upper bound of Theorem 3, thereby completing the proof.

□

Proof of Lemma EC.4 To aid in this proof, we make the following three observations, which correspond to our insight that any revenue gains from a default m are largely offset by corresponding revenue losses.

First, we have that for any $m\in[0,\rho]$ and any $x\in[0,1]$,

$$\begin{aligned}\int_m^{m+x} g(t)\cdot(m-t)\ dt \quad &\leq \quad \int_m^{m+x}(2/\rho^2)(\rho-t)(m-t)\ dt\\ &= (x/\rho)^2(m-\rho+2x/3) \end{aligned}\qquad \text{(EC.2.31)}$$

The first inequality follows from noting that $g(t)\geq(2/\rho^2)(\rho-t)$ while $m-t\leq 0$ for all t within the limits of integration, i.e., for all $t\in[m,m+x]$ where $m\in[0,\rho]$. This integral corresponds to a bound on the revenue losses from the shaded area to the right of the default in Figure EC.2.1. Evaluating the integral yields the upper bound in Line (EC.2.31).

Second, we have that for any $m\in[0,\rho]$ and any $x\in[0,1]$,

$$\begin{aligned}\int_{m-x}^{m} g(t)\cdot(m-t)\ dt \quad &\leq \quad \int_{m-x}^{m}(2/\rho^2)(\rho-t)(m-t)\ dt\\ &= (x/\rho)^2(-m+\rho+2x/3) \end{aligned}\qquad \text{(EC.2.32)}$$

The first inequality follows from noting that $g(t) \leq (2/\rho^2)(\rho - t)$ and $m - t \geq 0$ for all t within the limits of integration, i.e., for all $t \in [m-x, m]$ where $m \in [0, \rho]$. This integral corresponds to a bound on the revenue gains from the shaded area to the left of the default m in Figure EC.2.1. Note that when the range-to-gains ratio ρ is large, the slope of this distribution will be relatively flat and the revenue gains will be mostly offset by revenue losses. Evaluating the integral yields the upper bound in Line (EC.2.32).

Similarly, for any $m > \rho$ and any $x \in [0, 1]$,

$$\int_{m-x}^{m} g(t) \cdot (m-t)\ dt \quad \leq \quad \int_{\min\{m-x,\rho\}}^{\rho} (2/\rho^2)(\rho - t)(m-t)\ dt$$

The right hand side is equal to 0 for $m \geq \rho + x$, and it is decreasing in m for $m \in (\rho, \rho + x)$.[6] Therefore, the expression is bounded by its value when $m = \rho$. This implies that for for any $m > \rho$ and any $x \in [0, 1]$,

$$\int_{m-x}^{m} g(t) \cdot (m-t)\ dt \quad \leq \quad (x/\rho)^2 (2x/3) \tag{EC.2.33}$$

We now use these observations to bound the revenue gains from the optimal menu. Similar to the proof of Lemma EC.3, we leverage properties of the dynamic programming solution established in Theorem 2. Specifically, we first bound the value of $\mathcal{V}_n(m)$ as a function of n and m: we claim that for all $n \in \mathbb{N}$ and all $m \in [0, \rho]$, $\mathcal{V}_n(m) \leq (1/\rho)^2 (m - \rho + (4n-2)/3)$. Furthermore, for all $m > \rho$, $\mathcal{V}_n(m) = 0$. We then use these bounds and Equation (9) to place an upper bound on $\mathcal{R}^\star_{I_4}$.

We proceed via induction on n. We begin with the base case of $n = 1$. If $m > \rho$, then there are no donors with $t^\star(\theta) \geq m$, in which case $\mathcal{V}_1(m) = 0$. Otherwise, we have

$$\mathcal{V}_1(m) \quad = \quad \int_{m}^{m+1} g(t) \cdot (m-t) dt \quad \leq \quad (1/\rho)^2 (m - \rho + 2/3), \tag{EC.2.34}$$

where the inequality follows from setting $x = 1$ in Equation (EC.2.31).

We now assume that the inductive hypothesis is true for i, namely that $\mathcal{V}_i(m) \leq (1/\rho)^2 (m - \rho + (4i-2)/3)$. To complete the proof by induction, we must show a similar result for $\mathcal{V}_{i+1}(m)$. Once again, if $m > \rho$, then we have $\mathcal{V}_{i+1}(m) = 0$ by definition. Otherwise,

$$\mathcal{V}_{i+1}(m) \ = \ \max_{m' \geq m} \int_{m}^{m + \min\{1, \frac{m'-m}{2}\}} g(t) \cdot (m-t)\ dt + \int_{m' - \min\{1, \frac{m'-m}{2}\}}^{m'} g(t) \cdot (m'-t)\ dt + \mathcal{V}_i(m') \tag{EC.2.35}$$

This equality follows from properties of instance I_4, i.e., a donor prefers the closest default within 1 of $t^\star(\theta)$. There are now two cases to consider.

[6] One can verify that the derivative with respect to m is given by $(2/\rho^2)[(m-\rho)^2 - x^2]$, which is weakly negative for $m \in [\rho, \rho + x]$.

Case (i): $m' \leq \rho$. In this case,

$$\mathcal{V}_{i+1}(m) \leq \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(m - \rho + \frac{2\min\{1, \frac{m'-m}{2}\}}{3}\right) + \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(-m' + \rho + \frac{2\min\{1, \frac{m'-m}{2}\}}{3}\right) + \left(\frac{1}{\rho}\right)^2 \left(m' - \rho + \frac{4i-2}{3}\right) \tag{EC.2.36}$$

$$= \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(m - m' + \frac{4\min\{1, \frac{m'-m}{2}\}}{3}\right) + \left(\frac{1}{\rho}\right)^2 \left(m' - \rho + \frac{4i-2}{3}\right) \tag{EC.2.37}$$

The three parts of the first inequality come from the upper bounds given by Equation (EC.2.31), Equation (EC.2.32), and the inductive hypothesis, respectively. We now further divide this case into two sub-cases.

Case (ia): $\min\{1, (m'-m)/2\} = 1$. In this case, the expression in (EC.2.37) is equal to

$$\left(\frac{1}{\rho}\right)^2 \left(m - m' + \frac{4}{3}\right) + \left(\frac{1}{\rho}\right)^2 \left(m' - \rho + \frac{4i-2}{3}\right) = \left(\frac{1}{\rho}\right)^2 \left(m - \rho + \frac{4(i+1)-2}{3}\right), \tag{EC.2.38}$$

as desired. Therefore, the inductive hypothesis holds in Case (ia).

Case (ib): $\min\{1, (m'-m)/2\} = (m'-m)/2$. In this case, the expression in (EC.2.37) is equal to

$$\left(\frac{m'-m}{2\rho}\right)^2 \left(m - m' + \frac{2(m'-m)}{3}\right) + \left(\frac{1}{\rho}\right)^2 \left(m' - \rho + \frac{4i-2}{3}\right)$$
$$= \left(\frac{-(m'-m)^3}{12\rho^2}\right) + \left(\frac{1}{\rho}\right)^2 (m'-m) + \left(\frac{1}{\rho}\right)^2 \left(m - \rho + \frac{4i-2}{3}\right) \tag{EC.2.39}$$

Taking the derivative of Line (EC.2.39) with respect to m', we get $-(m'-m)^2/(2\rho)^2 + (1/\rho)^2$. This is non-negative whenever $(m'-m)/2 \leq 1$ (which must be true in this case). Hence, given that we are in Case (ib), the expression in Line (EC.2.39) is upper-bounded by its value when m' is at its largest, i.e., $(m'-m)/2 = 1$. This is equivalent to the expression in Line (EC.2.38), which implies that the inductive hypothesis also holds in Case (ib).

Case (ii): $m' > \rho$. In this case, starting from Line (EC.2.35), we have

$$\mathcal{V}_{i+1}(m) \leq \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(m - \rho + \frac{2\min\{1, \frac{m'-m}{2}\}}{3}\right) + \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(\frac{2\min\{1, \frac{m'-m}{2}\}}{3}\right) + 0$$

$$= \left(\frac{\min\{1, \frac{m'-m}{2}\}}{\rho}\right)^2 \left(m - \rho + \frac{4\min\{1, \frac{m'-m}{2}\}}{3}\right) \tag{EC.2.40}$$

The three parts of the first inequality come from the upper bounds given by Equation (EC.2.31), Equation (EC.2.33), and the inductive hypothesis for $m' > \rho$, respectively. We again consider two sub-cases.

Case (ia): $\min\{1,(m'-m)/2\}=1$. In this case, the expression in (EC.2.40) is equal to $(1/\rho)^2(m-\rho+4/3)$, which is less than $(1/\rho)^2(m-\rho+(4(i+1)-2)/3)$ for all $i\geq 1$. Therefore, our inductive hypothesis holds in this case.

Case (iib): $\min\{1,(m'-m)/2\}=(m'-m)/2$. In this case, the expression in (EC.2.40) is quasiconvex in $m'-m$.[7] This implies that the expression is maximized at a boundary point for m', either its minimum in this case ($m'=\rho$) or its maximum in this case ($m'=2+m$). The former returns us to Case (i) because $m'\in[0,\rho]$, while the latter returns us to Case (iia) because $m'=2+m$ implies $\min\{1,(m'-m)/2\}=1$. Therefore, the inductive hypothesis must hold in this case as well.

This completes the proof by induction: we have shown that for all $n\in\mathbb{N}$ and all $m\in[0,\rho]$, $\mathcal{V}_n(m)\leq(1/\rho)^2(m-\rho+(4n-2)/3)$. Furthermore, for all $m>\rho$, $\mathcal{V}_n(m)=0$. We now use this characterization of $\mathcal{V}_n(m)$ in instance I_4 to establish an upper bound on the attainable revenue gains.

By Theorem 2, the nonprofit's optimal gains in instance I_4 are given by

$$\mathcal{R}^{\star}_{I_4}=\max_{m\geq 0}\left\{\int_{m-1}^{m} g(t)\,(m-t)\,dt+\mathcal{V}_n(m)\right\} \tag{EC.2.41}$$

For any $m\leq\rho$, we have the following upper bound on the expression in Line (EC.2.41):

$$\begin{aligned}
&\max_{m\in[0,\rho]}\left\{\int_{m-1}^{m} g(t)\,(m-t)\,dt+\mathcal{V}_n(m)\right\}\\
&\quad\leq \max_{m\in[0,\rho]}\left(\frac{1}{\rho}\right)^2\left(-m+\rho+\frac{2}{3}\right)+\left(\frac{1}{\rho}\right)^2\left(m-\rho+\frac{4n-2}{3}\right) &\text{(EC.2.42)}\\
&\quad=\left(\frac{1}{\rho}\right)^2\left(\frac{4n}{3}\right) &\text{(EC.2.43)}
\end{aligned}$$

The inequality in Line (EC.2.42) comes from applying the bound in Line (EC.2.32) as well as our inductive bound on $\mathcal{V}_n(m)$. The bound in Line (EC.2.43) is equivalent to the upper bound on $\mathcal{R}^{\star}_{I_4}$ in Lemma EC.4.

To complete the proof, we note that for any $m>\rho$, we have a tighter upper bound of

$$\int_{m-1}^{m} g(t)\,(m-t)\,dt+\mathcal{V}_n(m) \quad\leq\quad \left(\frac{1}{\rho}\right)^2\left(\frac{2}{3}\right), \tag{EC.2.44}$$

where the bound on the integral comes from Inequality (EC.2.33) and the bound of $\mathcal{V}_n(m)\leq 0$ comes from our inductive hypothesis. This completes the proof of Lemma EC.4.

[7] To see this, note that $f(y)=b\cdot y^2(y-a)$ is quasiconvex in y for $y\geq 0$ and constants $a,b\geq 0$.

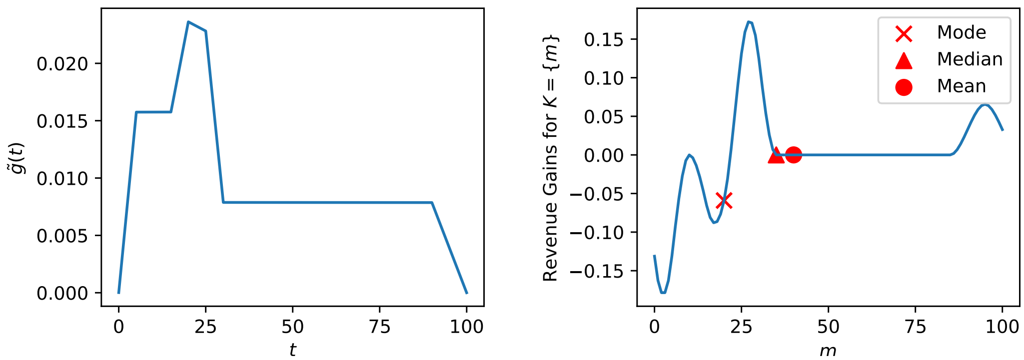


Figure EC.2.2 Example of an instance satisfying the conditions of Claim EC.4 with a mode at 20, along with its corresponding revenue gains for singleton menus.

EC.2.5. Proof of Claim EC.4

We will prove this claim by contradiction. Consider any quasi-symmetric instance (i.e., satisfying weak unimodality of ideal donations, independence of donors' willingness to deviate from their ideal donation, and symmetric donor utility), and consider any menu $\mathcal{M}$ where the smallest default m_1 is strictly less than a mode t_{mode}. We will show the existence of a menu that obtains weakly larger revenue gains, where all defaults in this alternative menu are weakly greater than t_{mode}.

Though the result may seem fairly intuitive for quasi-symmetric instances, the proof is non-trivial. To illustrate the challenge, consider the example shown in Figure EC.2.2. The left panel displays the distribution of $\tilde{g}$, which is continuous and weakly unimodal with support on $[0,100]$ and a mode at 20. The utility function is given by Example A, and ε is deterministically 50. The right panel displays the expected revenue gains for a single default m, as a function of m. We make the following observations: (i) the expected revenue gains are neither concave nor convex as a function of m, (ii) there is a local maximum at a default below the mode, and (iii) moving the default from that local maximum to the mode would reduce the expected revenue gains. Each of these features pose challenges when proving that it is without loss of optimality to only consider defaults weakly greater than the mode. Furthermore, we observe that both the median and the mean of $\tilde{g}$ exceed the optimal single default, which implies that a similar result does not hold for either of those two statistics of the distribution.

To aid in this proof, we will make use of the following implication of Properties (2) and (3) of Definition EC.2.4: for any $t, t', y \in \mathbb{R}$, $\tilde{F}(y|t) = \tilde{F}(-y|t')$, i.e., every donor is equally likely to be (un)willing to select a default y units away from their ideal donation, for any y and any ideal donation. To see this, we can apply Property (3) to establish that $\tilde{F}(y|t) = \tilde{F}(-y|t)$, and then we can apply Property (2) to establish that $\tilde{F}(y|t) = \tilde{F}(y|t')$. Because $\tilde{F}(y|t)$ does not depend on t

when these properties are satisfied, we will omit this dependence in the remainder of the proof. We will also make frequent use of the symmetry around 0, i.e., $\tilde{F}(y) = \tilde{F}(-y)$.

There are two cases to consider, depending on the existence of the second-largest default in the menu. The first case is more straightforward and covers menus where $|\mathcal{M}| = 1$. In this case, we show that the revenue can be weakly improved by replacing the default less than t_{mode} with a default (weakly) greater than t_{mode}. The second case, where $|\mathcal{M}| > 1$, is more complex and either (a) removes the smallest default, or (b) moves the smallest default above t_{mode}. This process can be applied iteratively until all remaining defaults are at least as large as t_{mode}.

Case (1): $|\mathcal{M}| = 1$. In this case, let us define $\bar{m}_1$ such that $\bar{m}_1 = \sup\{t : \tilde{g}(t) \geq \tilde{g}(m_1)\}$. Due to weak unimodality, we must have $\bar{m}_1 \geq t_{\text{mode}}$. If $\tilde{g}$ is continuous, then $\bar{m}_1$ is a value on the other side of t_{mode} from m_1 with the same likelihood of being the donor's ideal donation.

We claim that a singleton menu consisting of $\bar{m}_1$ is weakly better than a default at m_1. To see this, note that the revenue gains under $\mathcal{M} = \{m_1\}$ are given by

$$
\begin{aligned}
\mathcal{R}(\{m_1\}) &= \int \Big(m_1 - t^\star(\theta)\Big)\Big(1 - F\big(v(\theta, t^\star(\theta)) - v(\theta, m_1)\big)\Big)\ g(\theta) d\theta \\
&= \int_0^\infty \Big(m_1 - t\Big)\Big(1 - \tilde{F}(m_1 - t)\Big)\tilde{g}(t)\ dt && \text{(EC.2.45)} \\
&\leq \int_0^\infty \Big(m_1 - t\Big)\Big(1 - \tilde{F}(m_1 - t)\Big)\tilde{g}(t + \bar{m}_1 - m_1)\ dt && \text{(EC.2.46)} \\
&= \int_{\bar{m}_1 - m_1}^\infty \Big(\bar{m}_1 - t'\Big)\Big(1 - \tilde{F}(\bar{m}_1 - t')\Big)\tilde{g}(t')\ dt' && \text{(EC.2.47)} \\
&\leq \int_0^\infty \Big(\bar{m}_1 - t'\Big)\Big(1 - \tilde{F}(\bar{m}_1 - t')\Big)\tilde{g}(t')\ dt' && \text{(EC.2.48)} \\
&= \mathcal{R}(\{\bar{m}_1\})
\end{aligned}
$$

Line (EC.2.45) applies the definitions of the functions $\tilde{g}$ and $\tilde{F}$, as given in Equations (EC.2.26) and (EC.2.27). The key step of the proof is the inequality Line (EC.2.46). To prove that this inequality holds, it is sufficient to establish that $\tilde{g}(t) \geq$ (resp. $\leq$)$\tilde{g}(\bar{m}_1 - m_1 + t)$ if $t >$ (resp. $<$)m_1. We partition the possible values of t into four groups:

- For $t \in (-\infty, 2m_1 - \bar{m}_1]$, we have $t < \bar{m}_1 - m_1 + t \leq m_1 < t_{\text{mode}}$. Because $\tilde{g}$ is weakly increasing until t_{mode}, we must have $\tilde{g}(t) \leq \tilde{g}(\bar{m}_1 - m_1 + t)$.
- For $t \in (2m_1 - \bar{m}_1, m_1)$, we have $t < m_1 < \bar{m}_1 - m_1 + t < \bar{m}_1$. By the unimodality of $\tilde{g}$ and the definition of $\bar{m}$, we must have $\tilde{g}(t) \leq \tilde{g}(m_1) \leq \tilde{g}(\bar{m}_1) \leq \tilde{g}(\bar{m}_1 - m_1 + t)$.
- For $t \in (m_1, \bar{m}_1]$, we have $m_1 < t \leq \bar{m}_1 < \bar{m}_1 - m_1 + t$. By the same logic as the prior case, we must have $\tilde{g}(t) \geq \tilde{g}(\bar{m}_1) \geq \tilde{g}(\bar{m}_1 - m_1 + t)$.
- For $t \in (\bar{m}_1, \infty)$, we have $t_{\text{mode}} \leq \bar{m}_1 < t < \bar{m}_1 - m_1 + t$. Because $\tilde{g}$ is weakly decreasing after t_{mode}, we must have $\tilde{g}(t) \geq \tilde{g}(\bar{m}_1 - m_1 + t)$.

In each of these four intervals, we have $(m_1 - t) \cdot \tilde{g}(t) \leq (m_1 - t) \cdot \tilde{g}(\bar{m}_1 - m_1 + t)$. This proves the upper bound in Line (EC.2.46).

Line (EC.2.47) applies the change of variables $t = t' + m_1 - \bar{m}_1$. Then in Line (EC.2.48), we expand the region of integration (expanding it to include only positive terms), which leads to a further upper bound. This expression is exactly equal to the expected revenue gains given a menu of $\mathcal{M} = \{\bar{m}_1\}$, which completes the proof for Case (1).

Case (2): $|\mathcal{M}| > 1$. We will partition menus with $|\mathcal{M}| > 1$ into two different sub-cases, based on the density between the smallest and second-smallest default. Let us denote the smallest (resp. second-smallest) default as m_1 (resp. m_2) and define $\Delta_m := m_2 - m_1$. If the density at m_1 does not exceed the density at $m_1 + \Delta_m/2$, then we are in Case (2a), and we will see how we can safely remove m_1 without harming revenue. Otherwise, we are in Case (2b), and we will apply a similar procedure as in Case (1), where we replace the default at m_1 with a default between t_{mode} and $m_1 + \Delta_m$. By construction, this replacement default would be above t_{mode} yet would still be the smallest default.

If we iteratively apply this procedure, we will either stay in Case (2a) and eventually remove enough defaults and get to Case (1), or after one application of Case (2b), we will have created a menu exclusively containing defaults weakly greater than t_{mode}, all while weakly improving revenue gains. This is sufficient to prove Claim EC.4.

Case (2a): $\tilde{g}(m_1) \leq \tilde{g}(m_1 + \Delta_m/2)$. In this case, we will show that it is weakly better to remove the default at m_1. We remind that $\Delta_m = m_2 - m_1$, where m_2 is the second-smallest default. Based on the criteria for this case and the unimodality of $\tilde{g}(\cdot)$, the density $\tilde{g}(x)$ for any x in the interval $[m_1, m_1 + \Delta_m/2]$ must be weakly greater than the density for any x in the interval $[0, m_1]$. Intuitively, this will allow us to show that the *additional upward movement* due to m_1 is more than offset by the mass moving *down* to m_1. (There may be more upward movement than downward movement to m_1, but by removing m_1 we would then get additional upward movement to m_2.) Formally,

$$\mathcal{R}(\mathcal{M}) - \mathcal{R}(\mathcal{M} \setminus \{m_1\}) = \int\int \Big(\hat{t}(\mathcal{M}, \theta, \varepsilon) - \hat{t}(\mathcal{M} \setminus \{m_1\}, \theta, \varepsilon)\Big) f(\varepsilon)\; g(\theta) d\varepsilon d\theta$$

$$= \int \bigg(\Big(m_1 - t^\star(\theta)\Big)\Big(1 - F(v(\theta, t^\star(\theta)) - v(\theta, m_1))\Big) - \Big(m_2 - t^\star(\theta)\Big)\Big(1 - F(v(\theta, t^\star(\theta)) - v(\theta, m_2))\Big)\bigg)\; \mathbf{1}\{t^\star(\theta) \leq m_1 + \Delta_m/2\} g(\theta) d\theta \tag{EC.2.49}$$

$$= \int_0^{m_1 + \Delta_m/2} \bigg(\Big(m_1 - t\Big)\Big(1 - \tilde{F}(m_1 - t)\Big) - \Big(m_2 - t\Big)\Big(1 - \tilde{F}(m_2 - t)\Big)\bigg)\, \tilde{g}(t)\; dt \tag{EC.2.50}$$

$$\leq \int_0^{m_1 - \Delta_m/2} \Big(m_1 - t\Big)\Big(1 - \tilde{F}(m_1 - t)\Big)\tilde{g}(t)\; dt - \int_{\Delta_m}^{m_1 + \Delta_m/2} \Big(m_2 - t\Big)\Big(1 - \tilde{F}(m_2 - t)\Big)\tilde{g}(t)\; dt + \int_{m_1 - \Delta_m/2}^{m_1 + \Delta_m/2} \Big(m_1 - t\Big)\Big(1 - \tilde{F}(m_1 - t)\Big)\tilde{g}(t)\; dt \tag{EC.2.51}$$

$$
\begin{aligned}
&= \int_0^{m_1-\Delta_m/2} \Big(m_1 - t\Big)\Big(1-\tilde{F}(m_1-t)\Big)\tilde{g}(t)\ dt - \int_0^{m_1-\Delta_m/2} \Big(m_1 - t'\Big)\Big(1-\tilde{F}(m_1-t')\Big)\tilde{g}(t'+\Delta_m)\ dt' \\
&\quad + \int_{-\Delta_m/2}^{\Delta_m/2} (-x)\Big(1-\tilde{F}(-x)\Big)\tilde{g}(m_1+x)\ dx \qquad \text{(EC.2.52)} \\
&= \int_0^{m_1-\Delta_m/2} \Big(m_1 - t\Big)\Big(1-\tilde{F}(m_1-t)\Big)\Big(\tilde{g}(t)-\tilde{g}(t+\Delta_m)\Big)\ dt \\
&\quad + \int_0^{\Delta_m/2} (-x)\Big(1-\tilde{F}(x)\Big)\Big(\tilde{g}(m_1+x)-\tilde{g}(m_1-x)\Big)\ dx \qquad \text{(EC.2.53)} \\
&\le 0
\end{aligned}
$$

Line (EC.2.49) uses Property (3) of Claim EC.4, which states that donors' utility function is symmetric. In that case, any donor with $t^\star(\theta) \ge m_1 + \Delta_m/2$ will prefer a default greater than m_1. Hence, the only differences between the two menus occur for donors with $t^\star(\theta) \le m_1 + \Delta_m/2$. Line (EC.2.50) applies the definitions of the functions $\tilde{g}$ and $\tilde{F}$, as given in Equations (EC.2.26) and (EC.2.27). Line (EC.2.51) separates the integral into three pieces and restricts the limits of integration for the second piece (which increases the value of the entire expression). We note that if $m_1 < \Delta_m/2$, the upper limit of integration for the first integral may be negative; in that case, the entire integral will evaluate to 0, as $\tilde{g}(t) = 0$ for all t within the limits of integration.

The key step in Line (EC.2.52) involves two changes of variables. In the second integral, we substitute $t' = t - \Delta_m$. In the third integral, we substitute $x = t - m_1$. In Line (EC.2.53) we combine the first two integrals, and then we exploit the symmetry of the third integral, using the fact that $\tilde{F}(-x) = \tilde{F}(x)$.

We obtain that both terms of the expression are non-positive by leveraging the criteria for Case (2a). Specifically, the assumption of weak unimodality and the conditions for this case imply that in the first term of Line EC.2.53, we must have $\tilde{g}(t+\Delta_m) \ge \tilde{g}(t)$ for any $t \le m_1 - \Delta_m/2$. By similar logic, in the second term we must have $\tilde{g}(m_1+x) \ge \tilde{g}(m_1-x)$ for $x \in [0, \Delta_m/2]$. This proves that removing the smallest default weakly improves revenue in Case (2a).

Case (2b): $\tilde{g}(m_1) > \tilde{g}(m_1 + \Delta_m/2)$**.** In this case, we will show that it is weakly better to add a default at $\bar{m}_1$, defined the same way as in Case (1), i.e., $\bar{m}_1 = \sup\{t : \tilde{g}(t) \ge \tilde{g}(m_1)\}$. Based on the conditions of Case (2b) and the definition of this new default, we must have $\bar{m}_1 \in [t_{\text{mode}}, m_1 + \Delta_m/2]$. Intuitively, the upward movement to $\bar{m}_1$ will weakly exceed the upward movement to m_1, and we will also be able to show that the revenue losses between m_1 and m_2 weakly exceed the revenue losses between $\bar{m}_1$ and m_2. Formally, recall that $\Delta_m = m_2 - m_1$, and let $\bar{\Delta}_m = m_2 - \bar{m}_1$ and $\bar{\mathcal{M}} = \mathcal{M} \cup \{\bar{m}_1\} \setminus \{m_1\}$. Then,

$$
\mathcal{R}(\mathcal{M}) - \mathcal{R}(\bar{\mathcal{M}}) \quad = \quad \int\int \big(\hat{t}(\mathcal{M},\theta,\varepsilon) - \hat{t}(\bar{\mathcal{M}},\theta,\varepsilon)\big) f(\varepsilon)\ g(\theta) d\varepsilon d\theta
$$

$$
\begin{aligned}
&= \int_0^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt \\
&\quad + \int_{m_1+\Delta_m/2}^{\bar{m}_1+\bar{\Delta}_m/2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt - \int_0^{\bar{m}_1+\bar{\Delta}_m/2} \big(\bar{m}_1 - t\big)\big(1 - \tilde{F}(\bar{m}_1 - t)\big)\tilde{g}(t)\ dt && \text{(EC.2.54)} \\
&= \int_{\bar{m}_1-m_1}^{\bar{m}_1+\bar{\Delta}_m/2} \big(\bar{m}_1 - t'\big)\big(1 - \tilde{F}(\bar{m}_1 - t')\big)\tilde{g}(t' + m_1 - \bar{m}_1)\ dt' \\
&\quad + \int_{m_1+\bar{\Delta}_m/2}^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt \\
&\quad + \int_{m_2-\Delta_m/2}^{m_2-\bar{\Delta}_m/2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt - \int_0^{\bar{m}_1+\bar{\Delta}_m/2} \big(\bar{m}_1 - t\big)\big(1 - \tilde{F}(\bar{m}_1 - t)\big)\tilde{g}(t)\ dt && \text{(EC.2.55)} \\
&\le \int_{\bar{m}_1-m_1}^{\bar{m}_1+\bar{\Delta}_m/2} \big(\bar{m}_1 - t'\big)\big(1 - \tilde{F}(\bar{m}_1 - t')\big)\tilde{g}(t')\ dt' + \int_{m_1+\bar{\Delta}_m/2}^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt \\
&\quad - \int_0^{\bar{m}_1+\bar{\Delta}_m/2} \big(\bar{m}_1 - t\big)\big(1 - \tilde{F}(\bar{m}_1 - t)\big)\tilde{g}(t)\ dt + \int_{m_2-\Delta_m/2}^{m_2-\bar{\Delta}_m/2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt && \text{(EC.2.56)} \\
&\le \int_{m_1+\bar{\Delta}_m/2}^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt + \int_{m_2-\Delta_m/2}^{m_2-\bar{\Delta}_m/2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt && \text{(EC.2.57)} \\
&\le \int_{m_1+\bar{\Delta}_m/2}^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt \\
&\quad + \int_{m_2-\Delta_m/2}^{m_2-\bar{\Delta}_m/2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(m_1 + m_2 - t)\ dt && \text{(EC.2.58)} \\
&= \int_{m_1+\bar{\Delta}_m/2}^{m_1+\Delta_m/2} \big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big)\tilde{g}(t)\ dt - \int_{m_1+\Delta_m/2}^{m_1+\bar{\Delta}_m/2} \big(t' - m_1\big)\big(1 - \tilde{F}(t' - m_1)\big)\tilde{g}(t')\ dt' && \text{(EC.2.59)} \\
&= 0
\end{aligned}
$$

Line (EC.2.54) is similar to Lines (EC.2.49) and (EC.2.50), in that it narrows the focus onto the donors who are impacted by the change in defaults. Specifically, by Property (3) of Claim EC.4, donors are symmetric, which means that before the change, donors with $t^\star(\theta) \in [0, m_1 + \Delta_m]$ prefer m_1 to m_2. After the change, donors with $t^\star(\theta) \in [0, \bar{m}_1 + \bar{\Delta}_m]$ prefer $\bar{m}_1$ to m_2. The behavior of all donors with $t^\star(\theta) \ge \bar{m}_1 + \bar{\Delta}_m$ will not change. In addition to this observation, we also apply the definitions of the functions $\tilde{g}$ and $\tilde{F}$, as given in Equations (EC.2.26) and (EC.2.27). In Line (EC.2.55), we split the first term into two parts, and for the first part, we make the substitution $t' = t - m_1 + \bar{m}_1$. We also equivalently restate the limits of integration for the third term (corresponding to donors who select m_2).

The key inequality in Line (EC.2.56) follows from the observation that $\tilde{g}(x) - \tilde{g}\big(x - (\bar{m}_1 - m_1)\big)$ can cross 0 at most once due to unimodality, and by construction it is non-negative for $x < \bar{m}_1$ and negative for $x > \bar{m}_1$. Hence, we have $(\bar{m}_1 - x)\Big(\tilde{g}(x) - \tilde{g}\big(x - (\bar{m}_1 - m_1)\big)\Big) \geq 0$.

In Line (EC.2.57), we cancel out the first and second term, which yields an upper bound as the second term covers a slightly larger range where the integrand is non-negative. In Line (EC.2.58), we upper bound the second term by upper-bounding the density. To see that this is an upper bound, note that in Case (2b) the density function $\tilde{g}(\cdot)$ must satisfy $\tilde{g}(x) \geq \tilde{g}(y)$ for any $x \in [m_1 + \bar{\Delta}_m/2, m_1 + \Delta_m/2]$ and any $y \in [m_2 - \Delta_m/2, m_2 - \bar{\Delta}_m/2]$, due to unimodality and the fact that $\tilde{g}(m_1) > \tilde{g}(m_1 + \Delta_m)$. We then make the substitution $t' = m_1 + m_2 - t$ in Line (EC.2.59) to complete the proof that replacing m_1 with $\bar{m}_1$ weakly improves revenue gains.

This establish Case (2b). Together, these cases establish that for quasi-symmetric instances, for any menu that includes a default below the mode, one can always find a weakly better (and weakly smaller) menu without any defaults below the mode.

EC.2.6. Additional Properties of Quasi-Symmetric Instances

We now introduce two additional properties that hold for quasi-symmetric instances: first, we show that for quasi-symmetric instances in $\hat{\mathcal{I}}_\rho$ with $\rho < 1/(e+1)$, there exists an optimal menu with a single default (Claim EC.5). We then show that for strictly quasi-symmetric instances, either there exists an optimal menu with one default or the optimal menu is cardinality-constrained (Claim EC.6).

Claim EC.5 (Optimality of a Single Default with Sufficient Segmentation) *Suppose instance I is quasi-symmetric and $I \in \hat{\mathcal{I}}_\rho$ with $\rho < 1/(e+1)$. Then there exists an optimal menu with a single default.*

Proof of Claim EC.5. We first provide an upper bound on the value of any menu that contains a default within the support of donors' ideal donations. We then provide a (weakly larger) lower bound on the value of a menu with a single default beyond the support of donors' ideal donations. This implies that it is without loss of optimality to only consider menus with defaults beyond the support of donors' ideal donations, and for such menus only the lowest default matters. Hence, there is an optimal menu with a single default.

Step 1: Upper bound on menus overlapping the support. Suppose the support of donors' ideal donations is $[t_L, t_H]$ and suppose menu $\mathcal{M}$ contains a default $m \in [t_L, t_H]$. Because this instance is quasi-symmetric, donors will prefer the closest default to their ideal donation. Hence, given the presence of m, no donor will increase their donation by more than $t_H - t_L$. This represents an upper bound on the overall revenue gains from a menu with a default overlapping the support of ideal donations.

Step 2: Lower bound. To establish this lower bound (which we note does not rely on the properties of quasi-symmetric instances), recall that $\mathcal{R}^{\text{FI}}(\theta,\varepsilon)$ is a random variable which is assumed to follow an MHR distribution (as $I \in \hat{\mathcal{I}}_\rho$). Its expected value is given by $\mathcal{R}_I^{\text{FI}}$, and by Lemma EC.1, the probability that it exceeds its expected value is at least $1/e$.

Consider a menu with a single default at $m' = t_L + \mathcal{R}_I^{\text{FI}}$. This default will be chosen with probability at least $1/e$, as any donor with $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) \geq \mathcal{R}_I^{\text{FI}}$ will select the default. In the worst case where the only donors who select the default had an ideal donation of t_H, this corresponds to revenue gains of

$$\begin{aligned}
\mathcal{R}_I(\{m'\}) &\geq \Big(\mathcal{R}_I^{\text{FI}} + t_L - t_H\Big)/e \\
&\geq \Big((e+1)(t_H - t_L) - (t_H - t_L)\Big)/e \\
&= t_H - t_L
\end{aligned}$$

The second inequality comes from the claim's assumption that $\rho \leq 1/(e+1)$, where $\rho = (t_H - t_L)/\mathcal{R}_I^{\text{FI}}$.

This establishes that a single default at $t_L + \mathcal{R}_I^{\text{FI}}$ performs at least as well as any menu with a default overlapping the support of ideal donations. Hence, there exists an optimal menu with a single default.

Claim EC.6 (Optimality of Small or Large Menus) *For any* strictly *quasi-symmetric instance I, the minimum-cardinality optimal menu is either of size* 1 *or of size n.*

Proof of Claim EC.6. Let $\mathcal{M}$ be a minimum-cardinality optimal menu where $\mathcal{M}$ has k elements denoted $\{m_1,\ldots,m_k\}$ with $m_1 < \cdots < m_k$. Suppose that $1 < k < n$. We will show that we can either find a menu of size 1 with at least the same revenue as $\mathcal{M}$, or we can find a menu of size $k+1$ that strictly improves revenue. This would contradict the fact that $\mathcal{M}$ is a minimum-cardinality optimal menu, thereby completing the proof.

By Claim EC.4, we can assume that $m_1 \geq t_{\text{mode}}$, otherwise we can find a weakly better menu with the same cardinality (or with a smaller cardinality, which would already be a contradiction). To aid in this proof, we will again use $\Delta_m = m_2 - m_1$.

Case 1: $\int_{m_2-\Delta_m/2}^{m_2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt = 0$. In this case, m_2 can be removed, as it has no associated revenue gains. Specifically, positive revenue gains could only come from donors where $t \in [(m_1+m_2)/2, m_2)$, as donors with lower ideal donations will prefer m_1. Given the criteria from Case 1, for each such t we must have either $\tilde{g}(t) = 0$ (i.e., there is no mass) or $\tilde{F}(m_2 - t) = 1$ (i.e., donors are not sufficiently suggestible) almost everywhere. This means there are no positive revenue gains associated with m_2, and its removal will not reduce revenue.

Case 2: $\int_{(m_1+m_2)/2}^{m_2} \big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big)\tilde{g}(t)\ dt > 0$**.** In this case, we can add a new default at $m' := (m_1 + m_2)/2$ that strictly improves revenue. Intuitively, we are placing a default in a region where we know *some* donors will be influenced (as guaranteed by Case 2). Furthermore, the mass of donors influenced *positively* will exceed the mass of donors influenced *negatively* (as guaranteed by strict unimodality and the location of all defaults relative to the mode). Formally,

$$
\begin{aligned}
\mathcal{R}(\mathcal{M}) - \mathcal{R}(\mathcal{M} \cup \{m'\}) \quad &= \quad \int\!\!\int \big(\hat{t}(\mathcal{M}, \theta, \varepsilon) - \hat{t}(\mathcal{M} \cup \{m'\}, \theta, \varepsilon)\big) f(\varepsilon)\ g(\theta) d\varepsilon d\theta \\
&= \int_{m' - \Delta_m/4}^{m'} \Big(\big(m_1 - t\big)\big(1 - \tilde{F}(m_1 - t)\big) - \big(m' - t\big)\big(1 - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \\
&\quad + \int_{m'}^{m' + \Delta_m/4} \Big(\big(m_2 - t\big)\big(1 - \tilde{F}(m_2 - t)\big) - \big(m' - t\big)\big(1 - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \qquad \text{(EC.2.60)} \\
&= \int_{m' - \Delta_m/4}^{m'} \Big(\big(m_1 - m'\big)\big(1 - \tilde{F}(m_1 - t)\big) - \big(m' - t\big)\big(\tilde{F}(m_1 - t) - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \\
&\quad + \int_{m'}^{m' + \Delta_m/4} \Big(\big(m_2 - m'\big)\big(1 - \tilde{F}(m_2 - t)\big) - \big(m' - t\big)\big(\tilde{F}(m_2 - t) - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \qquad \text{(EC.2.61)} \\
&= \int_{m' - \Delta_m/4}^{m'} \Big(\big(-\Delta_m/2\big)\big(1 - \tilde{F}(m_1 - t)\big) - \big(m' - t\big)\big(\tilde{F}(m_1 - t) - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \\
&\quad + \int_{m' - \Delta_m/4}^{m'} \Big(\big(\Delta_m/2\big)\big(1 - \tilde{F}(m_1 - t')\big) - \big(t' - m'\big)\big(\tilde{F}(m_1 - t') - \tilde{F}(m' - t')\big)\Big)\, \tilde{g}(2m' - t')\ dt' \qquad \text{(EC.2.62)} \\
&< \int_{m' - \Delta_m/4}^{m'} \Big(\big(-\Delta_m/2\big)\big(1 - \tilde{F}(m_1 - t)\big) - \big(m' - t\big)\big(\tilde{F}(m_1 - t) - \tilde{F}(m' - t)\big)\Big)\, \tilde{g}(t)\ dt \\
&\quad + \int_{m' - \Delta_m/4}^{m'} \Big(\big(\Delta_m/2\big)\big(1 - \tilde{F}(m_1 - t')\big) - \big(t' - m'\big)\big(\tilde{F}(m_1 - t') - \tilde{F}(m' - t')\big)\Big)\, \tilde{g}(t')\ dt' \qquad \text{(EC.2.63)} \\
&= 0
\end{aligned}
$$

Line (EC.2.60) is similar to Lines (EC.2.49) and (EC.2.50), in that it narrows the focus onto the donors who are impacted by the new default. Specifically, by Property (3) of Definition EC.2.4, donors are symmetric, which means any donor with $t^\star(\theta) \in [m' - \Delta_m/4, m' + \Delta_m/4]$ will (weakly) prefer m' to any other default. The behavior of all other donors will not change. In addition to this observation, we also apply the definitions of the functions $\tilde{g}$ and $\tilde{F}$, as given in Equations (EC.2.26) and (EC.2.27). We rearrange the expressions algebraically in Line (EC.2.61). In Line (EC.2.62), we first use the fact that $m' - m_1 = m_2 - m' = \Delta_m/2$. We also adjust the second integral using the change of variable $t' = 2m' - t$ (which implies $m_2 - t = t' - m_1$) and the symmetry of $\tilde{F}$, i.e., $\tilde{F}(x) = \tilde{F}(-x)$.

The key step in Line (EC.2.63) relies on the criteria for Case (2), which ensures that the second integrand is strictly positive.[8] Thus, to prove the strict inequality, it is sufficient to show that $\tilde{g}(2m'-t') < \tilde{g}(t')$, as this is the only change between Lines (EC.2.62) and (EC.2.63). By strict unimodality and the fact that m_1 cannot be less than the mode, for any $t_1 \in [m_1 + \Delta_m/4, m_1 + \Delta_m/2)$ and any $t_2 \in (m_1 + \Delta_m/2, m_1 + 3\Delta_m/4]$, we must have $\tilde{g}(t_1) > \tilde{g}(t_2)$. Equivalently, for any $t' \in [m' - \Delta_m/4, m')$, $\tilde{g}(t') \geq \tilde{g}(2m'-t')$. We apply this upper bound to the second integral. We conclude by observing that the two integrals are additive complements. This proves that adding a default at m' strictly improves revenue.

We have thus shown that if there does not exist an optimal single-default menu, then revenue can be increased by adding a default between two existing defaults. This completes the proof of Claim EC.6.

EC.3. Example of Extended Model (Section 5.1)

In this section, we introduce a simple game-theoretic example that is a special case of Edwards List (2014) and describe how it can also be viewed as a special case of our extended model in Section 5. This equivalence provides insight into the feasibility of using our framework to efficiently optimize the menu when donors strategically adjust their donation based on others' behavior.

Specifically, if donors have full information and form a Nash equilibrium based on the menu, then our dynamic program will no longer be efficient (i.e., the state space would need to include the full menu). However, if donors use simpler heuristics (e.g., if their utility depends only on the Nash equilibrium in the absence of menu), then our optimization approach may remain efficient.

Instance of Edwards List (2014). In their model, donor's utility (U) is broken into three components: an altruistic component ($\tilde{U}$), a warm-glow component (f^{WG}), and a social norms component (f^{NORM}). We now introduce a specific, simple version of their model, which allows us to illustrate the extent to which our model can capture similar effects.

Suppose there are n participants. Each player j has an endowment I_j and must decide on a donation amount t_j, given their suggestibility ε_j, the menu $\mathcal{M}$, and the total donations of others $T_{-j}(\mathcal{M})$. We assume that each player maximizes their utility, given by the following:

$$U_j\big(t_j, I_j, \varepsilon_j, \mathcal{M}, T_{-j}(\mathcal{M})\big) = \alpha \log(I_j - t_j) + (1-\alpha)\log\big(\gamma \cdot T_{-j}(\mathcal{M}) + t_j\big) + \varepsilon_j \cdot \mathbf{1}\{t \in \mathcal{M}\}, \quad \text{(EC.3.1)}$$

[8] To prove this formally, note that the second integrand in Line (EC.2.63) is equivalent to the second integrand in Line (EC.2.61). By Case (2), either the first part of that integrand must be strictly positive, or $\int_{m'+\Delta_m/4}^{m'+\Delta_m/2}(m_2 - t)(1-\tilde{F}(m_2-t))\tilde{g}(t)dt = \int_{m'}^{m'+\Delta_m/4}(t'-m')(1-\tilde{F}(t'-m'))\tilde{g}(m'+m_2-t')dt' > 0$, where the equality comes from the substitution $t' = m' + m_2 - t$. By strict unimodality and the symmetry of $\tilde{F}(\cdot)$, this would imply that the second part of that integrand is strictly positive. Thus, at least one component must be strictly positive, and the other must be non-negative.

where $\alpha, \gamma \in (0,1)$. The first two terms collectively account for the altruistic and warm-glow components. The first term is the donor's utility from consumption, while the second is their utility from the nonprofit's revenue. Because $\gamma < 1$, the donor derives more utility from their donations than from the donations of others, which is our way of capturing the warm glow effect. The third term represents the social norms component of utility.

Equivalence to instance of our extended model. To establish an equivalence, we will create one donor type for each of the n players. Assuming each endowment is unique, I_j can simply correspond to the donor's generosity parameter, and ε_j corresponds to their suggestibility parameter. Framed in terms of the distributions over donor types, $g(\theta)$ has a point mass of 1 on each I_j and $f(\varepsilon|I_j)$ has a point mass of 1 on ε_j.

To account for game-theoretic considerations, for $\theta = I_j$, let us define $T^*_{-\theta} = T_{-j}(\emptyset)$. This represents the total contributions of all other players *in the absence of a menu.* Furthermore, let us define $\bar{T}_{-\theta}(\mathcal{M}) = T_{-j}(\mathcal{M})$. This represents the total contributions of all other players *given that player j sees the menu $\mathcal{M}$.*

We now introduce the following specifications for the functions $v(\cdot)$ and $\tilde{v}(\cdot)$:

$$\begin{aligned} v(\theta, t) &= \alpha \log(\theta - t) + (1-\alpha)\log(\gamma \cdot T^*_{-\theta} + t) \\ \tilde{v}(\mathcal{M}, \theta, t) &= (1-\alpha)\log\left(\gamma \cdot \bar{T}_{-\theta}(\mathcal{M}) + t\right) - (1-\alpha)\log(\gamma \cdot T^*_{-\theta} + t) \end{aligned}$$

We emphasize that this construction satisfies all of the assumptions of our extended model: $v(\cdot)$ is strictly concave in t for $\alpha \in (0,1)$, and $\tilde{v}(\emptyset, \theta, t) = 0$.[9] Given this constructed instance of our extended model, the donor utility is given by

$$\tilde{U}(\mathcal{M}, \theta, \varepsilon, z, t) = \alpha \log(\theta - t) + (1-\alpha)\log\left(\gamma \cdot \bar{T}_{-\theta}(\mathcal{M}) + t\right) + \varepsilon \cdot \mathbf{1}\{t \in \mathcal{M}\} \tag{EC.3.2}$$

This is equivalent to the special case of Edwards List (2014) introduced in Equation (EC.3.1). Given this framing, we remind that the nonprofit's optimization problem, ODD-EXT, involves optimizing revenue subject to the constraint that donors choose their utility-maximizing donation, i.e.,

$$\begin{aligned} \max_{\mathcal{M} \in M^n} \quad & E_{\theta, \varepsilon, z}[\tilde{t}(\mathcal{M}, \theta, \varepsilon, z) - t^\star(\theta)] \\ \text{subject to} \quad & \tilde{t}(\mathcal{M}, \theta, \varepsilon, z) \in \arg\max_{t \geq 0} \tilde{U}(\mathcal{M}, \theta, \varepsilon, z, t) \end{aligned}$$

[9] To exactly match our extended model — where $\tilde{v}$ depends on z instead of θ — we can simply construct the marginal distribution of z such that it deterministically is equal to θ.

Equilibrium without a menu. To solve for the Nash equilibrium donations in the absence of a menu, we will first identify each donor's best response, fixing the donations of others $\bar{T}^*_{-\theta}$. In that case, the first-order condition for a donor with generosity θ is given by

$$\alpha\big(\gamma\cdot\bar{T}^*_{-\theta}+t^\star(\theta)\big) \;=\; (1-\alpha)(\theta-t^\star(\theta)) \tag{EC.3.3}$$

$$\iff \qquad t^\star(\theta) \;=\; (1-\alpha)\cdot\theta-\alpha\cdot\gamma\cdot\bar{T}^*_{-\theta} \tag{EC.3.4}$$

$$\iff \qquad (1-\alpha\cdot\gamma)\cdot t^\star(\theta) \;=\; (1-\alpha)\cdot\theta-\alpha\cdot\gamma\cdot\sum_{\theta}t^\star(\theta) \tag{EC.3.5}$$

If γ is sufficiently small and each θ is sufficiently large, this constraint will bind for all donor types. Summing over all θ, we can solve for the total donation amount:

$$(1-\alpha\cdot\gamma)\cdot\sum_{\theta}t^\star(\theta) \;=\; (1-\alpha)\cdot\sum_{\theta}\theta-n\cdot\alpha\cdot\gamma\cdot\sum_{\theta}t^\star(\theta) \tag{EC.3.6}$$

$$\iff \qquad \sum_{\theta}t^\star(\theta) \;=\; \frac{(1-\alpha)\cdot\sum_{\theta}\theta}{1-(n+1)\cdot\alpha\cdot\gamma} \tag{EC.3.7}$$

This in turn allows us to solve for $t^\star(\theta)$.

Optimizing the menu. Under this model, the donations of other donors — i.e., $\bar{T}_{-\theta}(\mathcal{M})$ — can depend on the entire set of defaults in the menu. Intuitively, each item in the menu may influence the donation of other donors, which in turn can impact the entire equilibrium (and may even impact the existence of an equilibrium). Thus, using the criteria in Section 5.2, we would place this example in Category (iii). In other words, optimizing the menu may require enumerating over all possible menus and selecting the one with the best equilibrium outcome.

As an alternative that is both more tractable and potentially well-motivated, consider a model where donors' utility does not depend on the actual donations of others but instead depends on the Nash equilibrium in the absence of a menu. Specifically, suppose we were to replace $\bar{T}_{-\theta}(\mathcal{M})$ in Equation (EC.3.2) with $\bar{T}^*_{-\theta}$, which does not depend on the menu. (Roughly speaking, this could be viewed as a setting where donors do not account for the possibility that the menu will induce changes in *other* donors' decisions.) This alternative model belongs to Category (i); assuming the nonprofit can solve for the Nash equilibrium in the absence of a menu, $\bar{T}^*_{-\theta}$ could be treated as a constant and the nonprofit can efficiently optimize the menu using our optimization framework. As another example, if we were to replace $\bar{T}_{-\theta}(\mathcal{M})$ with a combination of $\bar{T}^*_{-\theta}$ and a menu-specific heuristic (e.g., a function of the minimum default), then the model would potentially belong to Category (ii), which would likewise lend itself to efficient optimization under our framework.

EC.4. Technical Details for Section 6

EC.4.1. Estimation of Instance Parameters

For our numerical calibration, we follow a similar approach to the one employed in Altmann et al. (2019). In particular, we consider a given functional form for the donor's utility and we estimate the distributions G and F using the experimental data provided in Altmann et al. (2019). In what follows, we provide the details of this approach.

Utility model. As stated in the body of the paper, we use the utility function given in Example A. For ease of exposition, we restate $v(\theta,t)$ and the corresponding utility function here:

$$v(\theta,t)=\theta\cdot t-t^2/2.$$

The corresponding utility function is given by

$$U(\mathcal{M},\theta,\varepsilon,t)=\theta\cdot t-t^2/2+\varepsilon\cdot\mathbf{1}\{t\in\mathcal{M}\}.$$

Note that as characterized in Lemma 1, the utility maximizing donation for a donor of type θ when $\mathcal{M}$ is the empty set is $t^\star(\theta)=\theta$. When there is a single default option $\mathcal{M}=\{m\}$, the optimal donation $\hat{t}$ when the type is $\theta>0$ is:

$$\hat{t}=\begin{cases} m & \text{if } (\theta-m)^2<2\varepsilon; \\ \theta & \text{if } (\theta-m)^2\geq 2\varepsilon. \end{cases}$$

Donors of type $\theta=0$ are assumed to be ungenerous donors who will not donate regardless of the value of the default, i.e., for these donors we have $\varepsilon=0$.

Estimation. To estimate the instance parameters, we need to be able to estimate two objects: $G(\theta)$ and $F(\varepsilon|\theta)$. The first of them, $G(\theta)$, can be observed from the data. In fact, we can identify this distribution non-parametrically from the observed distribution in the control group. For $F(\varepsilon|\theta)$, we follow the approach in Altmann et al. (2019). First, we separate between donors who are suggestible and those who are not. By assumption, ungenerous donors are never suggestible, and generous donors may not be suggestible either. Let $z\in\{0,1\}$ denote a variable that takes the value 0 with probability λ_1 (to be estimated) when a generous donor is not suggestible, and that takes the value 1 with probability $1-\lambda_1$ otherwise. In the former case, $\varepsilon=0$, which implies that those donors will choose to donate their preferred donation amount θ. Generous donors that are suggestible have a suggestibility ε that is exponentially distributed with parameter ν_2. Note that $\lambda_2=\mathbf{E}_{\varepsilon\sim F}[\sqrt{2\varepsilon}|\varepsilon>0]=\sqrt{\pi/(2\nu_2)}$ (c.f. Section EC.4.3), so that there is a one-to-one mapping between λ_2 (to be estimated) and ν_2. The density function of $F(\varepsilon|\theta)$ is given by

$$f(\varepsilon|\theta)=\mathbf{1}\{\theta=0\}\delta_0(\varepsilon)+\mathbf{1}\{\theta>0\}\left(\lambda_1\delta_0(\varepsilon)+(1-\lambda_1)\nu_2e^{-\nu_2\varepsilon}\right),$$

where $\delta_0(\varepsilon)$ is the Dirac delta function.

We estimate the parameters λ_1 and λ_2 via maximum likelihood estimation. For ease of exposition, in what follows, we write the maximum likelihood function as a function of λ_1 and ν_2 (but we perform the estimation over λ_1 and λ_2). Let $\mathbf{P}(X=t)$ be the probability of donating t, we have

$$\begin{aligned}
\mathbf{P}(X=t) &= \mathbf{P}(X=t \mid \theta=0)\mathbf{P}(\theta=0) + \mathbf{P}(X=t \mid \theta\neq 0)\mathbf{P}(\theta\neq 0) \\
&= \mathbf{1}\{t=0\}\mathbf{P}(\theta=0) + \mathbf{P}(X=t \mid z=0,\theta\neq 0)\mathbf{P}(z=0 \mid \theta\neq 0)\mathbf{P}(\theta\neq 0) \\
&\quad + \mathbf{P}(X=t \mid z=1,\theta\neq 0)\mathbf{P}(z=1 \mid \theta\neq 0)\mathbf{P}(\theta\neq 0) \\
&= \mathbf{1}\{t=0\}\mathbf{P}(\theta=0) + \mathbf{P}(\theta=t \mid \theta\neq 0)\lambda_1\mathbf{P}(\theta\neq 0) \\
&\quad + \mathbf{P}(X=t \mid z=1,\theta\neq 0)(1-\lambda_1)\mathbf{P}(\theta\neq 0) \\
&= \mathbf{1}\{t=0\}\mathbf{P}(\theta=0) + \lambda_1\mathbf{1}\{t\neq 0\}\mathbf{P}(\theta=t) \\
&\quad + (1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=t \mid z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta).
\end{aligned}$$

We compute the expression above for different values of t. We make the following observations which we will later use when explicitly computing the expression above:

- $\mathbf{P}(\theta=t)$: we observe this in the data from the control group.
- $\mathbf{P}(X=t|z=1,\theta=m)\mathbf{P}(\theta=m)=\mathbf{1}\{t=m\}\mathbf{P}(\theta=m)$: when the default is m, a type m agent will always donate the default.

Let A be the event where the utility of the agent from donating $t=\theta$ is greater than the utility of donating the default m:

$$A=\{(\varepsilon,\theta): v(\theta,m)+\varepsilon< v(\theta,\theta)\}=\{(\varepsilon,\theta):\varepsilon<\frac{\theta^2+m^2}{2}-m\theta\}$$

Let us also define:

$$\Delta(\theta,m)=\frac{\theta^2+m^2}{2}-m\theta.$$

There are now three cases to consider:

Case 1: For $t=0$, we have

$$\begin{aligned}
\mathbf{P}(X=0) &= \mathbf{1}\{0=0\}\mathbf{P}(\theta=0)+\lambda_1\mathbf{1}\{0\neq 0\}\mathbf{P}(\theta=0) \\
&\quad + (1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=0 \mid z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta) \\
&= \mathbf{P}(\theta=0)+(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=0 \mid z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta) \\
&= \mathbf{P}(\theta=0).
\end{aligned}$$

where the final equality comes from observing that any donor with $\theta>0$ will prefer donating θ to donating 0. Hence, for any $\hat\theta>0$, $\mathbf{P}(X=0|\theta=\hat\theta)=0$.

Case 2: For $t=m>0$, we have

$$\begin{aligned}\mathbf{P}(X=m) &= \mathbf{1}\{m=0\}\mathbf{P}(\theta=0)+\lambda_1\mathbf{1}\{m\neq 0\}\mathbf{P}(\theta=m)\\ &\quad +(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=m\mid z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta)\\ &= \lambda_1\mathbf{P}(\theta=m)\\ &\quad +(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(A^c\mid z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta)\\ &= \lambda_1\mathbf{P}(\theta=m)\\ &\quad +(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}\big(\varepsilon\geq\Delta(\hat\theta,m)\mid\theta=\hat\theta\big)\mathbf{P}(\theta=\hat\theta)\\ &= \lambda_1\mathbf{P}(\theta=m)\\ &\quad +(1-\lambda_1)\sum_{\hat\theta>0}e^{-\nu_2\Delta(\hat\theta,m)}\mathbf{P}(\theta=\hat\theta).\end{aligned}$$

Case 3: For $t\neq m, t>0$, we have

$$\begin{aligned}\mathbf{P}(X=t) &= \mathbf{1}\{t=0\}\mathbf{P}(\theta=0)+\lambda_1\mathbf{1}\{t\neq 0\}\mathbf{P}(\theta=t)+(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=t|z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta)\\ &= \lambda_1\mathbf{P}(\theta=t)+(1-\lambda_1)\sum_{\hat\theta>0}\mathbf{P}(X=t|z=1,\theta=\hat\theta)\mathbf{P}(\theta=\hat\theta)\\ &= \lambda_1\mathbf{P}(\theta=t)+(1-\lambda_1)\mathbf{P}(X=t|z=1,\theta=t)\mathbf{P}(\theta=t)\\ &= \lambda_1\mathbf{P}(\theta=t)+(1-\lambda_1)\mathbf{P}(A|z=1,\theta=t)\mathbf{P}(\theta=t)\\ &= \lambda_1\mathbf{P}(\theta=t)+(1-\lambda_1)\mathbf{P}(\varepsilon<\Delta(t,m))\mathbf{P}(\theta=t)\\ &= \lambda_1\mathbf{P}(\theta=t)+(1-\lambda_1)(1-e^{-\nu_2\Delta(t,m)})\mathbf{P}(\theta=t)\end{aligned}$$

Let t_i^m be an observed donation under menu $\mathcal{M}=\{m\}$ then if we let $g(\cdot)$ be the probability density function of agent types, we have

$$\begin{aligned}\mathbf{P}(t_i^m|m,\lambda_1,\nu_2,g(\cdot)) &= \mathbf{1}\{t_i^m=0\}g(t_i^m)\\ &\quad +\mathbf{1}\{t_i^m=m\}\left\{\lambda_1 g(m)+(1-\lambda_1)\sum_{\hat\theta>0}g(\hat\theta)e^{-\nu_2\Delta(\hat\theta,m)}\right\}\\ &\quad +\mathbf{1}\{t_i^m>0,t_i^m\neq m\}g(t_i^m)\left\{\lambda_1+(1-\lambda_1)(1-e^{-\nu_2\Delta(t_i^m,m)})\right\}.\end{aligned}$$

Hence, to obtain the maximum likelihood estimator, we optimize the following log-likelihood function

$$\mathcal{L}(\lambda_1,\nu_2|g(\cdot))=\sum_{m\in\{10,20,50\}}\sum_i \log\left(\mathbf{P}(t_i^m|m,\lambda_1,\nu_2,g(\cdot))\right).$$

We present the estimated parameters in Table EC.4.1 together with their corresponding error. The standard errors are computed using the BHHH algorithm to approximate the Fisher Information matrix (Berndt et al., 1974).

Table EC.4.1 Parameter Estimates with Standard Errors: Estimates computed from maximum likelihood estimation using observation $\theta \in [0,300]$.

Parameter	Estimate	Standard Error
λ_1	89.6×10^{-2}	73.1×10^{-4}
λ_2	109.07	38.29

EC.4.2. Equivalence of Extended Instance and Altmann et al. (2019)

In this section, we formally establish the equivalence between our extended instance I_E in Section 6.3 and the model estimated in Altmann et al. (2019).

In the model estimated in Altmann et al. (2019), donors are characterized by a type (ρ, δ, α) where ρ represents a donor's ideal donation, δ is a deviation cost and α is an opt-out cost. More precisely, a donor who chooses to donate their ideal donation ρ can experience a deviation cost δ and a donor who chooses to opt out (i.e., donate zero) can experience an opt-out cost α. Overall, donor utility from a donation t under this model is given by

$$U_A(\mathcal{M}, \rho, \delta, \alpha, t) \;=\; \rho \cdot t - t^2 - \delta \cdot \mathbf{1}\{t \notin \mathcal{M} \cup \{0\}\} - \alpha \cdot \mathbf{1}\{t = 0\}$$

In comparison, in instance I_E in Section 6.3, donor utility is given by

$$\begin{aligned}\tilde{U}(\mathcal{M}, \theta, \varepsilon, z, t) \;&=\; \theta \cdot t - t^2/2 + \varepsilon \cdot \mathbf{1}\{t \in \mathcal{M}\} + z \cdot \mathbf{1}\{t = 0\} \\ &=\; \theta \cdot t - t^2/2 + \varepsilon - \varepsilon \cdot \mathbf{1}\{t \notin \mathcal{M} \cup \{0\}\} + (z - \varepsilon) \cdot \mathbf{1}\{t = 0\}\end{aligned}$$

We establish a direct relationship between the two utility models by setting $\theta = \rho$, $\varepsilon = \delta$, $z = \delta - \alpha$, and shifting utility in our model downward by δ for all t (which does not affect a donor's relative preference of donation amounts). This establishes that for a fixed donor type, the utility function will be the same up to an additive constant under both models; hence, the utility-maximizing decision will be identical. We now establish the equivalence of the estimated distributions over donor types.

In Altmann et al. (2019), the distribution ρ is estimated non-parametrically using data on donations between 0 and 300. We follow the same approach to estimate the distribution of θ. For "ungenerous" donors (with $\rho = 0$) ε and α are assumed to be 0. Otherwise, for generous donors, the distribution of δ has a point mass at 0 of size λ_1 and then follows an exponential distribution with rate λ_2. Similarly, conditional on the donor being generous, α follows an exponential distribution with rate ν_3. We make identical modeling assumptions, and using maximum likelihood estimation, we arrive at the same parameter estimates, see Table EC.4.2 (a). (For a full description of their estimation approach, see Section D.1 in the online appendix of Altmann et al. (2019).) This establishes the equivalence between the two settings.

Table EC.4.2 presents the estimate parameters with corresponding standard errors. The standard errors are computed using the BHHH algorithm to approximate the Fisher Information matrix (Berndt et al., 1974). Note that in the estimation of λ_3 we make use of the following relation

$$\begin{aligned}
\lambda_3 &= \mathbf{E}[\sqrt{2(\varepsilon-\alpha)}\mathbf{1}\{\varepsilon>\alpha\}] \\
&= \int_0^\infty \int_\alpha^\infty \sqrt{2(\varepsilon-\alpha)}\nu_2\nu_3 e^{-\nu_2\varepsilon}e^{-\nu_3\alpha}d\varepsilon d\alpha \\
&= \int_0^\infty \int_0^\infty \sqrt{2u}\nu_2\nu_3 e^{-\nu_2(u+\alpha)}e^{-\nu_3\alpha}dud\alpha \\
&= \left(\nu_3\int_0^\infty e^{-(\nu_2+\nu_3)\alpha}d\alpha\right)\left(\nu_2\int_0^\infty \sqrt{2u}e^{-\nu_2 u}du\right) \\
&= \frac{\nu_3}{\nu_2+\nu_3}\sqrt{\frac{\pi}{2\nu_2}}.
\end{aligned}$$

Table EC.4.2 Extended Instance Parameter Estimates with Standard Errors: Estimates computed from maximum likelihood estimation using observation $\theta \in [0,300]$.

Parameter	Estimate	Standard Error
λ_1	88.9×10^{-2}	65.3×10^{-4}
ν_2	63.05×10^{-6}	50.5×10^{-6}
ν_3	98.8×10^{-5}	34.6×10^{-5}

(a) Altmann et al. (2019) estimated parameters

Parameter	Estimate	Standard Error
λ_1	88.9×10^{-2}	65.3×10^{-4}
λ_2	157.82	63.30
λ_3	148.35	67.15

(b) Our estimated parameters

EC.4.3. Deriving the Optimal Revenue Gains with Personalization

Here we formally describe the computation of $\mathcal{R}^{\text{FI}}$ and $\mathcal{R}^{\text{PI}}$ in our case study, both for our base model in Section 6.2 as well as our extended model in Section 6.3. The values are presented in Table 1.

Base Model. Our model in Section 6.2 uses the utility function from Example A, which implies that each donor would be willing to exceed their ideal donation by $\sqrt{2\varepsilon}$. In other words, $\mathcal{R}^{\text{FI}}(\theta,\varepsilon) = \sqrt{2\varepsilon}$ (see Lemma 1 as well as the discussion after Proposition EC.1).

To compute our benchmark $\mathcal{R}^{\text{FI}}$, we take the expected value of $\sqrt{2\varepsilon}$ over the distribution of ε. In the instance constructed, $\mathbf{P}(\varepsilon=0) = \mathbf{P}(\theta=0) + \lambda_1\Big(1-\mathbf{P}(\theta=0)\Big) \triangleq c$. In other words, $\varepsilon=0$ for all "ungenerous" donors as well as a λ_1 fraction of "generous" donors. The probability density function for $\varepsilon>0$ is given by $f(\varepsilon) = (1-c)\big(\nu_2\cdot e^{-\nu_2\cdot\varepsilon}\big)$. Consequently,

$$\mathcal{R}^{\text{FI}} \;=\; \mathbf{E}_{\varepsilon\sim F}[\sqrt{2\varepsilon}] \;=\; \int_0^\infty \sqrt{2\varepsilon}(1-c)\big(\nu_2\cdot e^{-\nu_2\cdot\varepsilon}\big)d\varepsilon \;=\; (1-c)\sqrt{\frac{\pi}{2\nu_2}}$$

To compute the optimal revenue gains under partial information, $\mathcal{R}^{\mathrm{PI}}$, we first must solve for the optimal single default to present as a function of θ. For a given θ, the revenue gains from presenting a default $\theta + p$ are given by

$$p \cdot \mathbf{P}(\sqrt{2\varepsilon} \geq p) \;=\; p \cdot \mathbf{P}(\varepsilon \geq p^2/2) \;=\; p \cdot (1-c)\big(e^{-\nu_2 \cdot p^2/2}\big)$$

This is maximized when $p = \nu_2^{-1/2}$ (regardless of the value of θ) which obtains revenue gains of $\mathcal{R}^{\mathrm{PI}} = (1-c)(\nu_2 \cdot e)^{-1/2}$.

Extended Model. Our model in Section 6.3 uses an extended version of the utility function from Example A. In this extended model, the donor can now choose to opt out (i.e., donate 0) and obtain utility of $z = \varepsilon - \alpha$. This is preferable if it exceeds the utility of giving their ideal donation (given by $\theta^2/2$) as well as the utility of selecting the default (given by $\theta^2/2 - (m-\theta)^2/2 + \varepsilon$).

In an extended full information setting where the nonprofit knows the type $(\theta, \varepsilon, \alpha)$, the nonprofit will offer the largest default that will be accepted. As in our base setting, the donor still prefers a default $m \geq t^\star(\theta)$ to their ideal donation $t^\star(\theta)$ if and only if $m \leq \theta + \sqrt{2\varepsilon}$. However, the donor also must prefer the default to 0, or equivalently, $m \leq \theta + \sqrt{\theta^2 + 2\varepsilon - 2(\varepsilon - \alpha)}$. Consequently, the nonprofit will offer the menu

$$\mathcal{M}^{\mathrm{FI}}(\theta, \varepsilon, \alpha) \;=\; \theta + \min\{\sqrt{2\varepsilon}, \sqrt{\theta^2 + 2\varepsilon - 2(\varepsilon - \alpha)}\} \;=\; \theta + \min\{\sqrt{2\varepsilon}, \sqrt{\theta^2 + 2\alpha}\}.$$

This menu obtains revenue gains of $\mathbf{E}[\min\{\sqrt{2\varepsilon}, \sqrt{\theta^2 + 2\alpha}\}]$. We compute this expectation numerically to an accuracy of three digits by repeatedly sampling from the distributions of θ, ε, and α. The estimate is reported in Table 1.

In an extended partial information setting where the nonprofit only knows the generosity parameter θ, we must solve for the optimal single default to present as a function of θ. For donors who are not suggestible (i.e., donors with $\theta = 0$ and a λ_1 fraction of donors with $\theta > 0$), the menu does not impact revenue because we assume $\varepsilon = \alpha = 0$. For suggestible donors, we can directly express the revenue gains from presenting a default $\theta + p$ as a function of the distributions of ε and α ($F(\cdot)$ and $H(\cdot)$, respectively). We note that conditional on a donor being suggestible, these distributions are independent of each other and of θ; thus, in the following, we suppress the conditioning for ease of notation.

$$\begin{aligned} &p \cdot \mathbf{P}(\sqrt{2\varepsilon} \geq p \text{ and } \sqrt{\theta^2 + 2\alpha} \geq p \mid \theta) - \theta \cdot \mathbf{P}(\varepsilon - \alpha \geq \theta^2/2 \text{ and } \sqrt{\theta^2 + 2\alpha} \leq p \mid \theta) \\ &\quad = \; p \cdot \mathbf{P}(\varepsilon \geq p^2/2 \text{ and } \alpha \geq (p^2 - \theta^2)/2 \mid \theta) - \theta \cdot \mathbf{P}(\alpha \leq \min\{\varepsilon - \theta^2/2, p^2/2 - \theta^2/2\} \mid \theta) \end{aligned} \tag{EC.4.1}$$

The second term comes from the two conditions that need to be satisfied for the donor to opt out (i.e., select 0). We now consider the two terms in Line (EC.4.1) separately. First, note that

$$p \cdot \mathbf{P}(\varepsilon \geq p^2/2 \text{ and } \alpha \geq (p^2-\theta^2)/2 \mid \theta) \;=\; p\Big(\bar{G}(p^2/2 \mid \theta)\Big)\Big(\bar{H}((p^2-\theta^2)/2)\Big), \tag{EC.4.2}$$

where $\bar{G}(\varepsilon|\theta)$ denotes the complement of the CDF of ε, while $\bar{H}$ denotes the complement of the distribution of α. This follows from the fact that ε and α are independent random variables, conditional on the donor being suggestible.

Next we consider the second term in Line (EC.4.1), which accounts for the revenue loss from donors opting out of donating altogether.

$$\begin{aligned}
&\theta \cdot \mathbf{P}(\alpha \leq \min\{\varepsilon - \theta^2/2, p^2/2 - \theta^2/2\} \mid \theta) \\
&\qquad = \theta\Big(\mathbf{P}(\alpha \leq \varepsilon - \theta^2/2 \mid \theta) - \mathbf{P}(\alpha \leq \varepsilon - \theta^2/2 \text{ and } \alpha \geq p^2/2 - \theta^2/2 \mid \theta)\Big) \\
&\qquad = \theta\left((\bar{G}(\theta^2/2 \mid \theta)\left(\frac{\nu_3}{\nu_2+\nu_3}\right) - \bar{G}(\max\{\theta^2/2, p^2/2\} \mid \theta)\bar{H}((p^2-\theta^2)/2)\left(\frac{\nu_3}{\nu_2+\nu_3}\right)\right) && \text{(EC.4.3)} \\
&\qquad = \theta \cdot \mathbf{1}\{p > \theta\} \cdot \Big(\bar{G}(\theta^2/2 \mid \theta) - \bar{G}(p^2/2 \mid \theta)\bar{H}((p^2-\theta^2)/2)\Big)\left(\frac{\nu_3}{\nu_2+\nu_3}\right) && \text{(EC.4.4)}
\end{aligned}$$

The first equality comes from a different way of accounting for the probability that the event occurs. To unpack Line (EC.4.3), note that the first event can only hold if $\varepsilon \geq \theta^2$. In addition, conditional on $\varepsilon \geq \theta^2/2$, the distribution of $\varepsilon - \theta^2/2$ is exponential with rate ν_2. Based on properties of exponential random variables, $\mathbf{P}(\alpha \leq \varepsilon - \theta^2/2 \mid \varepsilon > \theta^2/2) = \nu_3/(\nu_2+\nu_3)$. For the second event to hold, we also need $\varepsilon \geq p^2/2 - \theta^2/2$ and $\alpha \geq p^2 - \theta^2$, in which case we can apply similar logic. We then simplify to yield the expression in Line (EC.4.4).

Combining Line (EC.4.2) and Line (EC.4.4) gives us

$$\begin{aligned}
\mathcal{R}(\{\theta + p\}) \quad = \quad & p\Big(\bar{G}(p^2/2 \mid \theta)\Big)\Big(\bar{H}((p^2-\theta^2)/2)\Big) \\
& - \theta \cdot \mathbf{1}\{p > \theta\} \cdot \Big(\bar{G}(\theta^2/2 \mid \theta) - \bar{G}(p^2/2 \mid \theta)\bar{H}((p^2-\theta^2)/2)\Big)\left(\frac{\nu_3}{\nu_2+\nu_3}\right)
\end{aligned}$$

Choosing p to maximize this expression is a single-dimensional optimization problem that we can solve numerically for each θ. This gives the optimal revenue gains under partial information for suggestible donors with that θ, i.e., $\mathcal{R}^{\mathrm{PI}}(\theta)$. We then take the expectation of this value over the non-parametric distribution of θ and scale by the fraction of donors that are suggestible to arrive at an estimate for $\mathcal{R}^{\mathrm{PI}}$. We report our estimate in Table 1.

EC.4.4. Additional Model Predictions for the Distribution of Donations

In this section, we present predictions for the menus $\mathcal{M} = \{10\}$ and $\mathcal{M} = \{20\}$ (cf., Figure 4). We also extend the discussion about the goodness of fit of the extended instance.

Figures EC.4.1 and EC.4.2 show the model predictions. Our estimated parameters in Table EC.4.1 and Table EC.4.2 indicate a larger willingness to select a default in the extended instance (i.e., a larger ε in expectation), explaining its higher frequency of donations at the default values in Figures EC.4.1 and EC.4.2 (10 and 20, respectively). Overall, in all predictions, we observe that both instances are able to capture the movement of donors towards defaults. However, as discussed in the main text, the case of $\mathcal{M} = \{50\}$ in Figure 4 also shows that the base instance fails to capture the option to opt out for less generous donors.

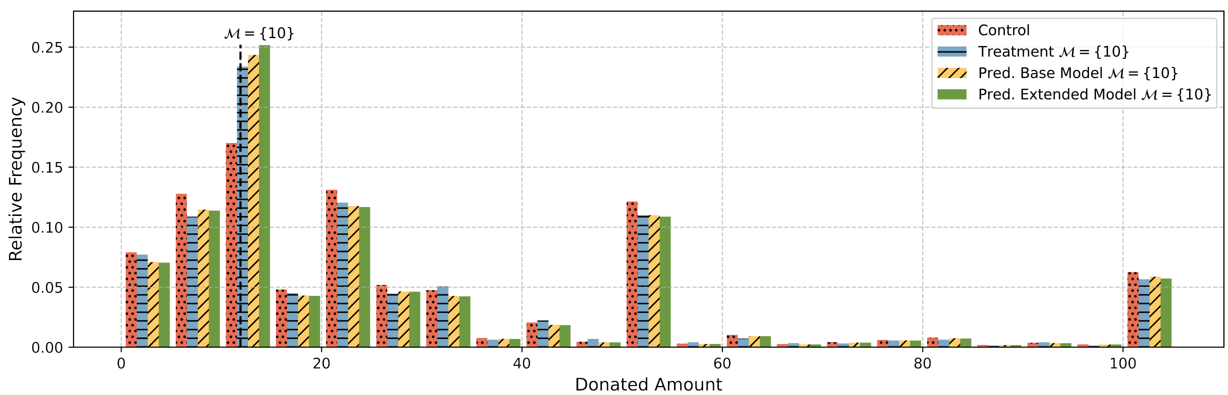


Figure EC.4.1 Relative frequency of donations in the range $(0, 100]$, grouped in increments of €5.

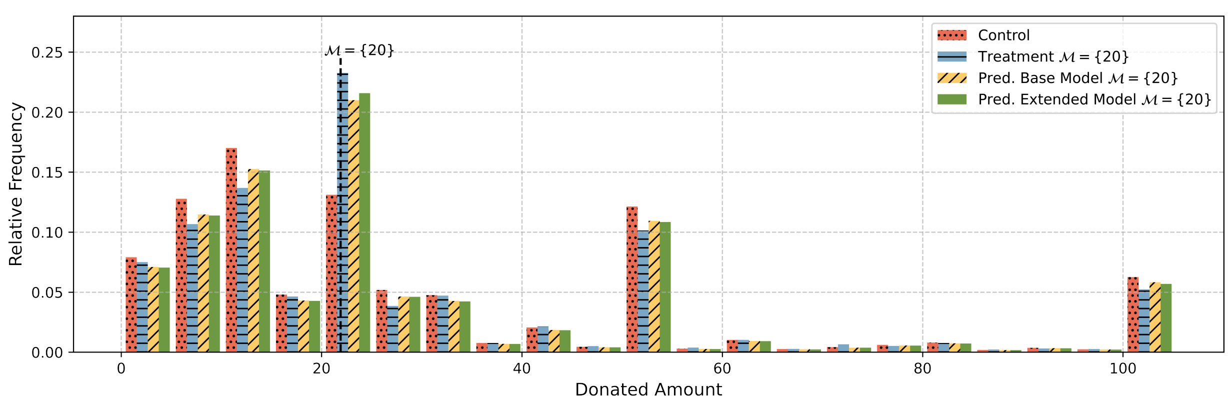
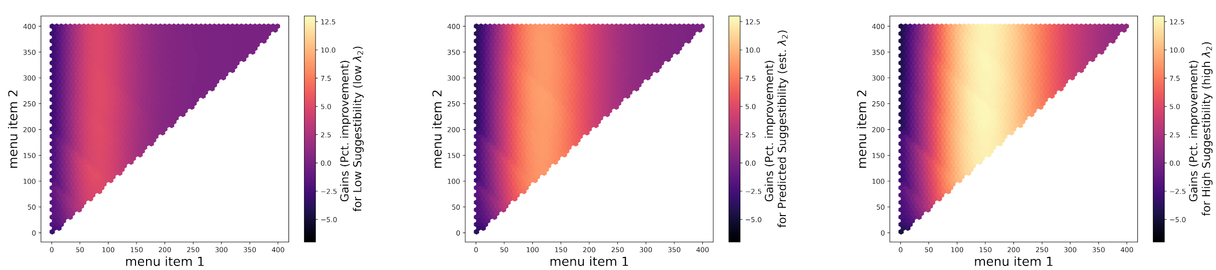


Figure EC.4.2 Relative frequency of donations in the range $(0, 100]$, grouped in increments of €5.

EC.4.5. Robustness of Model Predictions in Case Study (Section 6)

In this section, we numerically assess the robustness of the results in our case study from three different perspectives. First, we explore the extent to which parameter misestimation impacts our results. Then, we evaluate the performance of the hypothesized optimal menu under various types of structural misspecification. Finally, using additional data to partition donors into two groups, we analyze the impact of another type of misspecification, where the donor population is comprised of two distinct distributions over types. This latter analysis also highlights the value of market segmentation.

EC.4.5.1. Robustness to Parameter Misestimation In Figure 5 of Section 6, we depict the revenue gains for a menu of size $n = 1$ for three different values of λ_2, which is a parameter representing the expected willingness to deviate from the ideal donation (WDID, for short). Specifically, we present results using the estimated value of the WDID, as well as plus/minus one standard deviation, for both our base instance (Instance I_B in Section 6.2) and our extended instance (Instance in I_E Section 6.3). We now extend this analysis to menus of size $n = 2$. Figure EC.4.3 depicts the revenue gains in our base instance under the same three scenarios (i.e., when the WDID is low, when it is exactly as estimated, and when it is high). We note that positive revenue gains across all three parameter regimes are achievable for a broad set of possible menus. We conduct a similar analysis for our extended instance in Figure EC.4.4. In comparison to our base instance (see Figure EC.4.3), we note that the set of menus that obtain positive revenue gains is smaller; nevertheless, there are still several menus of size $n = 2$ that yield positive revenue gains in all three regimes, including our predicted optimal menu of size 2, i.e., $\mathcal{M} = \{47, 152\}$.



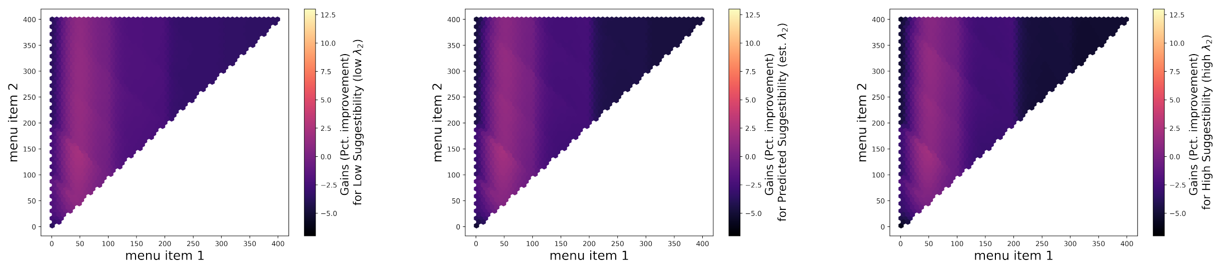
(a) Low Expected Willingness to Deviate

(b) Predicted Expected Willingness to Deviate

(c) High Expected Willingness to Deviate

Figure EC.4.3 **Pct. improvement of the revenue gains for the estimated λ_2 (plus and minus one standard deviation) in Instance I_B (base instance) for a menu of size $n = 2$**

To get a more complete sense for robustness to misestimation, we now conduct the following thought experiment, starting with our base instance. Suppose the parameters we estimate in



(a) Low Expected Willingness to Deviate

(b) Predicted Expected Willingness to Deviate

(c) High Expected Willingness to Deviate

Figure EC.4.4 **Pct. improvement of the revenue gains for the estimated λ_2 (plus and minus one standard deviation) in Instance I_E (extended instance) for a menu of size $n = 2$**

instance I_B are the ground truth, but the nonprofit presents the optimal menu under (i) the low estimate of the WDID (estimated value minus one standard deviation) or (ii) the high estimate of the WDID (estimated value plus one standard deviation). When $n = 1$, as mentioned in Section 6.2, the optimal menu is $\{119\}$ (assuming our estimated parameters are the ground truth). However, under the low and high WDID parameters, the optimal menus are predicted to be $\{84\}$ and $\{152\}$, respectively. When we evaluate these menus in the ground truth instance, we obtain expected gains of 3.652 and 3.863, representing approximately 84% and 89% of the revenue gains from the optimal menu, respectively. For the case of $n = 2$, the optimal menu is $\{118, 281\}$ (again, assuming our estimated parameters are the ground truth). However, under the low and high WDID parameters, the menus we obtain are $\{76, 123\}$ and $\{148, 251\}$, respectively. When we evaluate these menus in the ground truth instance, we obtain expected gains of 3.648 and 4.131, representing approximately 80% and 90% of the revenue gains from the optimal menu, respectively. As expected, misestimating the instance parameters hurts performance. However, in all the cases tested above, we still obtain significant revenue gains, which exceed the gains from the best treatment group, i.e., $\mathcal{M} = \{50\}$ (cf. Table 1). Table EC.4.3 presents the results.

We can conduct the same thought experiment for our extended instance, assuming that the parameters we estimate in instance I_E are the ground truth. We find that for both $n = 1$ and $n = 2$, the optimal menus are essentially the same even when the expected willingness to deviate parameter is misestimated. As a result, the expected gains remain largely the same (fixing a menu size), with the worst relative performance in this thought experiment (the case of $n = 1$ and a low estimate of the WDID) still obtaining 99.2% of the corresponding optimal revenue gains. Table EC.4.4 presents the results. Interestingly, compared to the base instance, the extended instance is more robust to misestimates of the WDID, at least in the context of this case study.

Table EC.4.3 Robustness of optimal menus under different assumptions for the expected willingness to deviate from the ideal donation (WDID) parameter in the base instance.

n = 1			
WDID	**Optimal Menu**	**Revenue Gains**	**% of** $\mathcal{R}^\star_{I_B,n=1}$
True	{119}	4.325	100%
Low	{84}	3.652	84%
High	{152}	3.863	89%
n = 2			
WDID	**Optimal Menu**	**Revenue Gains**	**% of** $\mathcal{R}^\star_{I_B,n=2}$
True	{118, 281}	4.543	100%
Low	{76, 123}	3.648	80%
High	{148, 251}	4.131	90%

Table EC.4.4 Robustness of optimal menus under different assumptions for the expected willingness to deviate from the ideal donation (WDID) parameter in the extended instance.

n = 1			
WDID	**Optimal Menu**	**Revenue Gains**	**% of** $\mathcal{R}^\star_{I_E,n=1}$
True	{52}	0.274	100%
Low	{50}	0.272	99.2%
High	{53}	0.273	99.6%
n = 2			
WDID	**Optimal Menu**	**Revenue Gains**	**% of** $\mathcal{R}^\star_{I_E,n=2}$
True	{47, 152}	1.156	100%
Low	{47, 152}	1.156	100%
High	{47, 152}	1.156	100%

EC.4.5.2. Robustness to Structural Misspecification To evaluate the robustness of our approach to our three main structural assumptions (an exponential distribution for suggestibility, independence of generosity and suggestibility, and a quadratic utility function), we consider various alternative structural models, described below.

- **(Unif)** This model assumes a uniform distribution for ε (and, in our extended instance, also assumes a uniform distribution for α).
- **(Pareto)** This model allows ε to be drawn from a Pareto distribution, which is a generalization of the exponential distribution with an extra parameter ξ that indicates whether we have a bounded, a light, or a heavy tail. The cumulative distribution function is given by

$$\Phi(\varepsilon|\xi,\nu_2) = \begin{cases} 1-(1+\nu_2\xi\varepsilon)^{-1/\xi} & \text{if } \xi \neq 0; \\ 1-\exp(-\nu_2\varepsilon) & \text{if } \xi = 0. \end{cases}$$

When $\xi = 0$, we recover an exponential distribution.

- **(Corr-1 and Corr-2)** These models allow for correlation between generosity and suggestibility. Specifically, in **Corr-1** we allow the proportion of suggestible donors to be a linear function of θ, i.e., $\lambda_1(\theta) = \hat{\lambda}_{1,0} + \hat{\lambda}_{1,1} \cdot \theta$. In **Corr-2** we allow the rate of the exponential distribution of suggestibility to be a linear function of θ, i.e., $\nu_2(\theta) = \hat{\nu}_{2,0} + \hat{\nu}_{2,1} \cdot \theta$.
- **(Asym)** This model directly allows for asymmetry in the utility function. Specifically, we set

$$v(\theta, t) = \begin{cases} \theta^2/2 - (\theta - t)^2/2 & \text{if } t \geq \theta; \\ \theta^2/2 - \lambda_4 \cdot (\theta - t)^2/2 & \text{if } t < \theta. \end{cases}$$

  This utility function is smooth and continuous, and it allows for donors to prefer, e.g., the closest default below $t^\star(\theta)$ even if there is a default above $t^\star(\theta)$ which is closer.

Using each of these structural models, we construct instances of our base model (with no donor opt-out) and our extended model (with donor opt-out) by deriving maximum likelihood estimates of model parameters (identical to the procedure described in Section 6). Then, to evaluate the robustness of our model predictions, we take the optimal menu of size 1 and the optimal menu of size 2 from our original instances and test those menus in these newly-constructed instances. We present the full results in Table EC.4.5, alongside the additive change in log likelihood under each of the alternative instances, using the log likelihood in Instance I_B as a baseline (similar results hold for larger menus). As a reference, the first column of Table EC.4.5 reports the predicted revenue gains of each menu assuming our originally-constructed instances (see Sections 6.2 and 6.3) are correctly specified.

Table EC.4.5 Robustness of optimal menus to various forms of structural misspecification, as well as the log likelihood in these alternative instances.

Base Instance (without donor opt-out)						
	Instance I_B	**Unif**	**Pareto**	**Corr-1**	**Corr-2**	**Asym**
Revenue Gains of $\mathcal{M} = \{119\}$	4.325	6.613	4.374	3.951	0.963	0.890
Revenue Gains of $\mathcal{M} = \{118, 281\}$	4.543	6.881	4.731	4.201	1.696	1.115
Change in Log Likelihood from I_B	0	-0.070	+2.979	+0.076	+0.800	+2.831
Extended Instance (with donor opt-out)						
	Instance I_E	**Unif**	**Pareto**	**Corr-1**	**Corr-2**	**Asym**
Revenue Gains of $\mathcal{M} = \{52\}$	0.274	0.952	0.334	0.326	0.297	0.294
Revenue Gains of $\mathcal{M} = \{47, 152\}$	1.156	1.862	1.192	1.195	1.181	1.171
Change in Log Likelihood from I_B	+7.890	+8.973	+8.744	+7.917	+7.892	+7.899

We make the following observations about the results in Table EC.4.5:

- **Comparison to Unif.** In all cases, our proposed menus obtain larger revenue gains under **Unif**. The difference stems from the relatively larger weight above the mean of a uniform distribution, which makes the donor more likely to select a default (and less likely to opt out). We note that in our extended model, this alternative instance obtains a *larger* log likelihood, indicating a better fit. This suggests that our reported results could be somewhat pessimistic. (We note that, despite meaningfully different predictions for revenue gains, our proposed menus remain near-optimal under **Unif**.)

- **Other comparisons under our base model.** For many of the other instances — which all add one additional parameter — we observe substantially lower revenue gains (e.g., revenue gains decrease by more than 75% in **Asym**). These alternative instances also generally provide a better fit to the data, as evidenced by statistically significant results in some log likelihood tests (e.g., a p-value of 0.017 when comparing against **Asym**). In general, we believe this is due to our base model's inability to capture donor opt-out. The extra parameter in these alternative instances can capture this behavior to some extent (e.g., by allowing the donor to asymmetrically prefer downward movement). Our extended instance — which likewise adds a single parameter — obtains a much sharper increase in log likelihood (an increase of 7.890).
- **Other comparisons under our extended model.** Once we incorporate donor opt-out, our model predictions become quite robust. Indeed, our predicted revenue gains increase in each of these alternative instances (by at most 21.9% for $n = 1$ and at most 3.4% for $n = 2$). Furthermore, for each of these alternative instances, a likelihood ratio test does not find statistically significant evidence that the extra parameter improves fit (p-value ¿ 0.1 in all cases).

In sum, this exercise reinforces the potential lack of robustness of model predictions *under our base instance*. However, the optimal menus *under our extended instance* appear to be quite robust to structural misspecification, and our predicted revenue gains are likely pessimistic if anything.

EC.4.5.3. Robustness of Results to Heterogeneity in Donors' Type Distributions

In this section, we use additional data from the context of our case study to explore how possible sources of heterogeneity in donors' preferences can impact the nonprofit's revenue gains. Specifically, we control for the recipient type for each donation (below, we describe the different types) and allow donors to differ based on the recipient type. From a mathematical perspective, we partition the data into two subsets, and for each subset we estimate model parameters and compute the optimal menu. We then aggregate the results and discuss how they compare to the results obtained with the pooled dataset in the main text.

Our dataset includes the type of recipient for a given donation. There are three possible types: Project, Project Element, and Fundraising Event. A Project is characterized by a well-defined goal and a prespecified amount that an organization or project director would like to reach. Project

Elements are subcategories in a project that define what is needed in the project and allow donors to select specific parts of it to which they would like to contribute. A Fundraising Event is typically designed to support a particular event or action. Due to the relatively small size of the Fundraising Event category, we partition the data into two subsets. The first includes all donations to a Project, with an average donation per generous donor of 60.70 and a standard deviation of 122.66 (this subset consists of about one third of all donations). The second includes donations directed toward more specific goals — Fundraising Event or a Project Element — with an average donation per generous donor of 43.17 and a standard deviation of 95.49 (this subset consists of the remaining two-third of donations). Table EC.4.6 shows the estimated parameters for each of the two subsets. The table also reports the estimated parameters for a dataset with both types.

Table EC.4.6 Parameter estimates with standard errors. Estimates computed from maximum likelihood estimation using observations with $\theta \in [0,300]$.

Parameter	Project		Element + Event		Pooled data	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
λ_1	89.8×10^{-2}	92.7×10^{-4}	89.3×10^{-2}	98.2×10^{-4}	89.6×10^{-2}	73.1×10^{-4}
λ_2	195.27	132.98	79.27	26.33	109.07	38.29
λ_1	88.9×10^{-2}	99.4×10^{-4}	89.0×10^{-2}	88.5×10^{-4}	88.9×10^{-2}	65.3×10^{-4}
λ_2	202.18	128.43	124.89	64.39	157.82	63.30
λ_3	196.20	132.30	111.47	71.32	148.35	67.15

Table EC.4.7 shows the revenue gains for the different receiver types under the base and extended models. For each subset, the results are consistent with those presented in the main text in Table 1: in the base model, a single default provides close-to-optimal gains, while in the extended model, a larger optimal menu leads to a substantial improvement over using a single default. For both models and both menu sizes, we note that the average revenue gains across the two groups exceed the revenue gains attainable using pooled data. In the former case, we construct an instance for each group and then optimize the menu for that group, which explains the improved performance and highlights the potential value of personalization.

Table EC.4.7 Revenue gains per visitor (generous donors only) under the parameter estimates in Table EC.4.6. Avg Type indicates the weighted average gains between Project and Element+Event.

	Project		Element + Event		Avg Type		Pooled data	
	$\mathcal{R}^\star_{n=1}$	$\mathcal{R}^\star$	$\mathcal{R}^\star_{n=1}$	$\mathcal{R}^\star$	$\mathcal{R}^\star_{n=1}$	$\mathcal{R}^\star$	$\mathcal{R}^\star_{n=1}$	$\mathcal{R}^\star$
Base Model	8.221	8.609	3.035	3.410	4.789	5.168	4.325	4.650
Extended Model	0	1.617	0.701	1.734	0.464	1.695	0.274	1.669

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