Electronic Companion to *Platform Disintermediation: Information Effects and Pricing Remedies*

The electronic companion is organized as follows: Section EC.1 presents preliminary results that are used throughout the main proofs. Sections EC.2, EC.3, EC.4, and EC.5 contain the proofs for Sections 3, 4, 5.1 and 5.2 respectively. In Section EC.6, we study an extension of our main model where buyers and sellers may engage in multiple transactions; the proofs from this section are present in Section EC.6.4.

EC.1. Preliminary Results

Section EC.1.1 characterizes relevant probabilities, offline prices, and possible cases for a seller's profit and optimal price; Section EC.1.2 characterizes the sellers' profit functions and optimal prices; and Section EC.1.3 provides conditions under which disintermediation occurs and defines the platform's revenue.

EC.1.1. Key Definitions

The main result in this section is Lemma EC.3, which describes possible cases for a seller's profit and optimal price, and is referred to extensively throughout the remainder of the electronic companion.

Lemma EC.1 **(Signal probabilities and sellers' beliefs)**. *The following statements hold for* $\sigma \in \{r, s\}$.

(i) The share of all buyers assigned the signal σ *is* η_σ, *where* $\eta_r := (1-\alpha)\lambda + \alpha(1-\lambda)$ *and* $\eta_s := \alpha\lambda + (1-\alpha)(1-\lambda)$. *Further,* η_r *and* η_s *strictly decrease and strictly increase in* α *on* $\alpha \in [\frac{1}{2}, 1]$, *respectively.*

(ii) A seller's posterior belief that a buyer with signal σ *is type-s is* ω_σ, *where* $\omega_r := \frac{(1-\alpha)\lambda}{\eta_r}$ *and* $\omega_s := \frac{\alpha\lambda}{\eta_s}$. *Further,* ω_r *and* ω_s *strictly decrease and increase in* α *on* $\alpha \in [\frac{1}{2}, 1]$, *respectively.*

Proof. For statement (i) , note $\lambda = \Pr(j = s)$ and $\alpha = \Pr(j = s|\sigma = s) = \Pr(j = r|\sigma = r)$ by definition. The expressions for η_r and η_s then follow by the total probability rule. Further, note $\frac{\partial}{\partial\alpha}\eta_r = 1-2\lambda < 0$ and $\frac{\partial}{\partial\alpha}\eta_s = 2\lambda - 1 > 0$. For statement (ii), by Bayes' rule we have

$$\omega_r = \Pr(j = s|\sigma = r) = \frac{(1-\alpha)\lambda}{(1-\alpha)\lambda + \alpha(1-\lambda)} = (1-\alpha)\lambda/\eta_r, \tag{EC.1}$$

$$\omega_s = \Pr(j = s|\sigma = s) = \frac{\alpha\lambda}{\alpha\lambda + (1-\alpha)(1-\lambda)} = \alpha\lambda/\eta_s. \tag{EC.2}$$

Further, note

$$\frac{\partial \omega_r}{\partial \alpha} = -\frac{(1-\lambda)\lambda}{(\alpha(1-\lambda)+(1-\alpha)\lambda)^2} < 0, \tag{EC.3}$$

$$\frac{\partial \omega_s}{\partial \alpha} = \frac{(1-\lambda)\lambda}{((1-\alpha)(1-\lambda)+\alpha\lambda)^2} > 0, \tag{EC.4}$$

where the strict inequalities follow because $\lambda \in [\frac{1}{2}, 1]$. □

LEMMA EC.2 **(Offline price for fixed commission rate γ and online price p)**. *Consider the transaction between a seller with online price p and a buyer with signal σ at a fixed commission rate $\gamma > 0$. The price in the offline channel is then given by*

$$b_\sigma(p) := \frac{1}{2}\left(\frac{p(1-\gamma+\omega_\sigma)}{\omega_\sigma} + \frac{\phi}{\omega_\sigma}\right). \tag{EC.5}$$

Proof. Under an offline price of b, the buyer and seller's expected surpluses from transacting offline are $p - b$ and $\omega_\sigma b - (1-\gamma)p - \phi$, respectively. The Nash bargaining function is thus $N(b) := (p-b)(\omega_\sigma b - (1-\gamma)p - \phi)$, which can be verified to be strictly concave in b. Solving $\frac{\partial}{\partial b}N(b) = 0$ yields the expression (EC.5). □

LEMMA EC.3 **(Sellers' profit and price cases)**. *Fix the commission rate γ and consider a unit mass of sellers with quality q and online price $p \le q$. Let $\Pi(p)$ be the sellers' expected profit (over buyer signals), let $\tilde{p}$ be the maximizer of $\Pi(p)$, and let $\tilde{\Pi}$ be the sellers' expected profit under price $\tilde{p}$.*

(a) If the sellers transact online with both $\sigma = r$ and $\sigma = s$ buyers,

$$\Pi(p) = \pi^a(p) := ((1-\gamma)p - (1-\lambda)c)\left(1 - \frac{p}{q}\right), \tag{EC.6}$$

$$\tilde{p} = p^a := \frac{1}{2}\left(q + \frac{(1-\lambda)c}{1-\gamma}\right), \tag{EC.7}$$

$$\tilde{\Pi} = \pi^a(p^a) = (1-\gamma)q\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q(1-\gamma)}\right)^2. \tag{EC.8}$$

(b) If the sellers reject $\sigma = r$ and transact online with $\sigma = s$,

$$\Pi(p) = \pi^b(p) := \eta_s((1-\gamma)p - (1-\omega_s)c)\left(1 - \frac{p}{q}\right), \tag{EC.9}$$

$$\tilde{p} = p^b := \frac{1}{2}\left(q + \frac{(1-\omega_s)c}{1-\gamma}\right), \tag{EC.10}$$

$$\tilde{\Pi} = \pi^b(p^b) = \eta_s(1-\gamma)q\left(\frac{1}{2} - \frac{(1-\omega_s)c}{2q(1-\gamma)}\right)^2. \tag{EC.11}$$

(c) If the sellers transact online with $\sigma = r$ and offline with $\sigma = s$,

$$\Pi(p) = \pi^c(p) := \left(\eta_r(1-\gamma)p + \eta_s(\omega_s b_s(p) - \phi) - (1-\lambda)c\right)\left(1 - \frac{p}{q}\right), \tag{EC.12}$$

$$\tilde{p} = p^c := \frac{1}{2}\left(q + \frac{2(1-\lambda)c + \eta_s\phi}{2\zeta}\right), \tag{EC.13}$$

$$\tilde{\Pi} = \pi^c(p^c) = q\zeta\left(\frac{1}{2} - \frac{2c(1-\lambda) + \eta_s\phi}{4q\zeta}\right)^2. \tag{EC.14}$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\omega_s)$.

(d) If the sellers reject $\sigma = r$ and transact offline with $\sigma = s$,

$$\Pi(p) = \pi^d(p) := \eta_s\left(1 - \frac{p}{q}\right)\left(\omega_s b_s(p) - \phi - (1-\omega_s)c\right), \tag{EC.15}$$

$$\tilde{p} = p^d := \frac{1}{2}\left(q + \frac{2c(1-\omega_s) + \phi}{1-\gamma+\omega_s}\right), \tag{EC.16}$$

$$\tilde{\Pi} = \pi^d(p^d) = \frac{\eta_s q}{2}(1-\gamma+\omega_s)\left(\frac{1}{2} - \frac{2c(1-\omega_s) + \phi}{2q(1-\gamma+\omega_s)}\right)^2. \tag{EC.17}$$

(e) If the sellers accept $\sigma = r$ and $\sigma = s$ offline,

$$\Pi(p) = \pi^e(p) := (\eta_s\omega_s b_s(p) + \eta_r\omega_r b_r(p) - \phi - (1-\lambda)c)\left(1 - \frac{p}{q}\right), \tag{EC.18}$$

$$\tilde{p} = p^e := \frac{1}{2}\left(q + \frac{2c(1-\lambda) + \phi}{1-\gamma+\eta_r\omega_r+\eta_s\omega_s}\right), \tag{EC.19}$$

$$\tilde{\Pi} = \pi^e(p^e) := \frac{q\zeta'}{2}\left(\frac{1}{2} - \frac{2c(1-\lambda) + \phi}{2q\zeta'}\right)^2, \tag{EC.20}$$

where $\zeta' = 1-\gamma+\omega_r\eta_r+\omega_s\eta_s$.

Proof. For each case $x \in \{a,b,c,d,e\}$, the profit expression $\pi^x(p)$ follows from the definitions of η_σ, ω_σ, and ω_σ given in Lemma EC.1. The profit maximizing price p^x follow by substituting the expression for the offline price $b_\sigma(p)$ (Lemma EC.2) into $\pi^x(p)$, noting that $\pi^x(p)$ is then strictly concave in p, and solving the first order condition $\frac{\partial}{\partial p}\pi^x(p) = 0$; the algebraic details are straightforward and omitted. □

EC.1.2. Sellers' Profit Functions and Optimal Prices

LEMMA EC.4 (**Disintermediation under fixed online price** p). *Consider a transaction between a seller with online price p and a buyer with signal σ. Both the buyer and seller prefer to transact offline if and only if $\gamma > \hat{\gamma}_\sigma(p)$, where*

$$\hat{\gamma}_\sigma(p) := 1 - \omega_\sigma + \frac{\phi}{p}.$$

Further, suppose $\gamma > 1-\omega_\sigma$. Then $\gamma > \hat{\gamma}_\sigma(p)$ holds if and only if $p > \hat{p}_\sigma$, where

$$\hat{p}_\sigma := \frac{\phi}{\gamma - (1-\omega_\sigma)}.$$

Proof. Under the offline price $b_\sigma(p)$ given in (EC.5), the seller has positive surplus from disintermediating if and only if $\omega_\sigma b_\sigma(p) - (1-\gamma)p - \phi > 0$, or equivalently, $\frac{1}{2}(-p(1-\gamma-\omega_\sigma)-\phi) > 0$. Re-arranging for γ, the seller's surplus is strictly positive at $b_\sigma(p)$ if and only if $\gamma > 1-\omega_\sigma + \frac{\phi}{p} = \hat{\gamma}_\sigma(p)$. Similarly, the buyer's surplus is strictly positive if and only if $p - b_\sigma(p) > 0$, or equivalently, $\frac{1}{2\omega_\sigma}(-p(1-\gamma-\omega_\sigma)-\phi) > 0$. Re-arranging for γ, the buyer's surplus is also positive if and only if $\gamma > \hat{\gamma}_\sigma(p)$. Lastly, in the case where $\gamma > 1-\omega_\sigma$, the definition of $\hat{p}_\sigma$ follows by re-arranging the inequality $\gamma > \bar{\gamma}_\sigma(p)$. □

LEMMA EC.5 **(Sellers' transaction decisions)**. *Let Assumption 1 hold. Then the following statements hold for all $\gamma \in [0,\gamma^m]$ and $\alpha \in [\frac{1}{2},1]$.*

(i) At their optimal price, type-L sellers reject $\sigma = r$ buyers.

(ii) At their optimal price, type-H sellers transact with both $\sigma = r$ and $\sigma = s$ buyers.

Proof. This proof uses the profit and price expressions from Lemma EC.3. ***(i)***. We show that the type-L seller's profit if they accept the $\sigma = r$ buyer is non-positive in both transaction channels for all $p \geq 0$ and $\gamma \geq 0$. Note the type-L seller's demand is $(1-\frac{p}{q_L})^+$, which implies their profit is zero for all $p \geq q_L$. For $p < q_L$, the seller's expected payment from a $\sigma = r$ buyer is at most p in either channel. It follows that the seller's profit from transacting with the $\sigma = r$ buyer is at most $p-(1-\omega_r)c$. Next, note $p-(1-\omega_r)c \leq q_L - (1-\omega_r)c \leq (1-\lambda)c - (1-\omega_r)c \leq 0$, where the third and fourth inequalities follow from Assumption 1 and because $\lambda \geq \omega_r$ for all $\alpha \in [\frac{1}{2},1]$, respectively.

(ii). It is straightforward to verify that $\omega_s \geq \omega_r$ (Lemma EC.1), which implies a seller accepts the $\sigma = r$ buyer only if they also accept the $\sigma = s$ buyer. Therefore, to show statement (ii), it suffices to show the type-H seller accepts the $\sigma = r$ buyer for all $\gamma \in [0,\gamma^m]$ and $\alpha \in [\frac{1}{2},1]$. Note there are two cases to consider depending on whether the $\sigma = s$ buyer transacts online or offline; thus, following Lemma EC.3, it suffices to show $\pi^a(p^a) \geq \pi^b(p^b) > 0$ and $\pi^c(p^c) \geq \pi^d(p^d) > 0$ both hold. We show $\pi^a(p^a) \geq \pi^b(p^b) > 0$ first. It is straightforward to verify that $\pi^b(p^b) > 0$ using the fact that $\pi^b(p^b)$ strictly decreases in q_H and $q_H \geq 4c$. Because p^a is the maximizer of $\pi^a(p)$, it suffices to show $\pi^a(p^b) - \pi^b(p^b) \geq 0$. Note

$$\pi^a(p^b) - \pi^b(p^b) = \left(1 - \frac{p^b}{q_H}\right)(\alpha c(1-\lambda) + (1-\gamma)p^b(\lambda - \alpha(2\lambda - 1)))$$

and that $\pi^b(p^b) > 0$ implies $p^b < q_H$. It remains to show $p^b \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}$, or equivalently,

$$\frac{1}{2}\left(q_H + \frac{c(1-\omega_s)}{1-\gamma}\right) \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}. \tag{EC.21}$$

Note the left hand side of (EC.21) decreases in α because ω_s and ω_s both increase in α (Lemma EC.1), and the right hand side of (EC.21) increases in α. Plugging in $\alpha = 1$, it follows that (EC.21) holds if $\frac{q_H}{2} \geq \frac{c}{1-\gamma}$ holds. The preceding inequality holds because $q_H \geq 4c$ by Assumption 1 and $\gamma \leq \frac{1}{2}$. We now show $\pi^c(p^c) \geq \pi^d(p^d) > 0$ using a similar argument. It is straightforward to verify that $\pi^d(p^d) > 0$ using the fact that $\pi^d(p^d)$ strictly decreases in q_H and $q_H \geq 4c$. Because p^c is the maximizer of $\pi^c(p)$, it suffices to show $\pi^c(p^d) - \pi^d(p^d) \geq 0$. Note

$$\pi^c(p^d) - \pi^d(p^d) = \left(1 - \frac{p^d}{q_H}\right)(\alpha c(1-\lambda) + (1-\gamma)p^d(\alpha(2\lambda-1)-\lambda)),$$

and that $\pi^d(p^d) > 0$ implies $p^d < q_H$. It remains to show $p^d \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}$. Because p^d increases in ϕ, letting $\phi = 0$ yields the lower bound $p^d \geq \frac{1}{2}\left(q_H + \frac{2c(1-\omega_s)}{1-\gamma+\omega_s}\right)$. Therefore, it remains to show

$$\frac{1}{2}\left(q_H + \frac{c(1-\omega_s)}{1-\gamma+\omega_s}\right) \geq \frac{c\alpha(1-\lambda)}{(1-\gamma)(\lambda-\alpha(2\lambda-1))}. \tag{EC.22}$$

Note the left hand side of (EC.22) decreases in α because ω_s and ω_s both increase in α (Lemma EC.1), and the right hand side of (EC.22) increases in α. The result follows by plugging in $\alpha = 1$ and noting $q_H \geq 4c$ and $\gamma \leq \frac{1}{2}$. □

LEMMA EC.6 **(Sellers' profit functions)**. *For $i \in \{L, H\}$, the profit function for the type-i seller is given by $\Pi^i(p)$, defined as follows.*

(i) If $\gamma \leq 1 - \omega_s$, then $\Pi^L(p) := \pi^b(p)$ and $\Pi^H(p) := \pi^a(p)$ for all $p \geq 0$.

(ii) If $\gamma > 1 - \omega_s$, then

$$\Pi^L(p) := \begin{cases} \pi^b(p), & \text{if } p \leq \hat{p}_s, \\ \pi^d(p), & \text{if } \hat{p}_s < p. \end{cases} \tag{EC.23}$$

and

$$\Pi^H(p) := \begin{cases} \pi^a(p), & \text{if } p \leq \hat{p}_s, \\ \pi^c(p), & \text{if } \hat{p}_s < p < \hat{p}_r, \\ \pi^e(p), & \text{if } \hat{p}_r < p. \end{cases} \tag{EC.24}$$

Proof. Note Lemma EC.3 defines a seller's profit based on the platform signal σ and the transaction channel, Lemma EC.4 provides conditions under which the transaction occurs offline, and Lemma EC.5 defines which signals $\sigma \in \{r, s\}$ are accepted by the type-L and type-H sellers. Combining Lemmas EC.3, EC.4, and EC.5 yields the profit expressions $\Pi^L(p)$ and $\Pi^H(p)$. □

LEMMA EC.7 **(No disintermediation with $\sigma = r$ buyers)**. *Let Assumption 2 hold. Then neither seller type transacts offline with the $\sigma = r$ buyer for all $\gamma \le \gamma^m$.*

Proof. Note by Lemma EC.5, the type-L seller never transacts with the $\sigma = r$ buyer, so it remains to prove the result for the type-H seller. First, if for some online price p the type-H seller transacts offline with the $\sigma = r$ buyer, then by Lemma EC.4 we must have $p > \hat{p}_r$, and by Lemma EC.6 the seller's profit is given by $\pi^e(p)$. Further, because $\pi^e(p)$ is maximized at p^e, it follows that $p^e \le \hat{p}_r$ is a sufficient condition for the transaction with the $\sigma = r$ buyer to be online. By definition of $\hat{p}_r$, the condition $p^e \le \hat{p}_r$ can be written equivalently as $\gamma \le \underline{\gamma}_r^H$, where $\underline{\gamma}_r^H$ is defined as the solution to

$$\gamma = 1 - \omega_r + \frac{\phi}{p^e(\gamma)}.$$

It remains to show $\underline{\gamma}_r^H > \gamma^m$ holds for all $\alpha \in [\frac{1}{2}, 1]$. Note that for any ϕ, we have the following lower bound on $\underline{\gamma}_r^H$:

$$\underline{\gamma}_r^H = 1 - \omega_r + \frac{\phi}{p^e(\underline{\gamma}_r^H)} \ge 1 - \omega_r. \tag{EC.25}$$

Next, note

$$\omega_r = \frac{(1-\alpha)\lambda}{\alpha(1-\lambda) + (1-\alpha)\lambda}. \tag{EC.26}$$

It is straightforward to show that the right hand side of (EC.26) strictly decreases in α. Plugging $\alpha = \frac{1}{2}$ into the right hand side of (EC.26) and combining with (EC.25) then yields $\underline{\gamma}_r^H \ge 1 - \lambda$ for all $\alpha \in [\frac{1}{2}, 1]$. It follows that $\lim_{\lambda \to \frac{1}{2}} \underline{\gamma}_r^H \ge \frac{1}{2} > \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$. □

LEMMA EC.8 **(Sellers' optimal prices)**. *Suppose for some fixed $\gamma \in (0, \gamma^m]$, neither seller type transacts offline with the $\sigma = r$ buyer at their optimal price. Then the following statements hold.*

(i) The type-H seller's optimal price satisfies $p^* \in \{p^a, p^c\}$, where $p^* = p^a$ if and only if the type-H seller transacts online with the $\sigma = s$ buyer.

(ii) The type-L seller's optimal price satisfies $p^* \in \{p^b, p^d\}$, where $p^* = p^b$ if and only if the type-L seller transacts online with the $\sigma = s$ buyer.

Proof. We focus on proving statement (i); the proof of (ii) follows by a similar argument and is briefly addressed afterward.

(i). We proceed in two steps. First, we show $p^* \in \{p^a, p^c\}$. Second, we show $p^* = p^a$ if and only if the type-H seller transacts online with the $\sigma = s$ buyer.

Step 1. Note the type-H seller's profit function $\Pi^H(p)$ is given in Lemma EC.6. It is straightforward to verify that $\pi^a(p)$, $\pi^c(p)$, and $\pi^e(p)$ are each strictly concave. Therefore, $\Pi^H(p)$ has five possible local maxima that are candidates for the optimal price p^*: p^a, p^c, and p^e, and the breakpoints $\hat{p}_s$ and $\hat{p}_r$. Note $p^* \neq p^e$ because no seller transacts offline with the $\sigma = r$ buyer by Lemma EC.7. Thus, we $p^* \in \{p^a, p^c\}$ by showing $p^* \notin \{\hat{p}_r, \hat{p}_s\}$. Consider $\hat{p}_s$ first. Note that if $\gamma < 1 - \omega_s$, then $\hat{p}_s < 0$, and the result holds trivially. Next, suppose $\gamma > 1 - \omega_s$ and assume by way of contradiction that $p^* = \hat{p}_s$. Then we must have

$$\lim_{p \to \hat{p}_s^-} \frac{\partial \Pi^H}{\partial p} \geq \lim_{p \to \hat{p}_s^+} \frac{\partial \Pi^H}{\partial p}, \tag{EC.27}$$

i.e., $\hat{p}_s$ must be a local maximum of $\Pi^H(p)$. Note that the piecewise profit function $\Pi^H(p)$ switches from $\pi^a(p)$ to $\pi^c(p)$ at $\hat{p}_s$. The inequality (EC.27) is therefore equivalent to

$$\left.\frac{\partial \pi^a}{\partial p}\right|_{p=\hat{p}_s} \geq \left.\frac{\partial \pi^c}{\partial p}\right|_{p=\hat{p}_s}. \tag{EC.28}$$

Using the profit expressions from Lemma EC.3, it is straightforward to show

$$\left.\left(\frac{\partial \pi^a}{\partial p} - \frac{\partial \pi^c}{\partial p}\right)\right|_{p=\hat{p}_s} = \frac{\eta_s(\phi - q_H(\gamma - (1-\omega_s)))}{2q_H}. \tag{EC.29}$$

Because $\gamma > 1 - \omega_s$, the expression in (EC.29) is non-negative if and only if

$$q_H \leq \frac{\phi}{\gamma - (1-\omega_s)}. \tag{EC.30}$$

Note that the right hand side (EC.30) is precisely $\hat{p}_s$ (Lemma EC.4). Therefore, $p^* = \hat{p}_s$ implies $\hat{p}_s \geq q_H$. However, the type-H seller generates zero demand for all prices $p \geq q_H$, which contradicts $p^* = \hat{p}_s$. We conclude $p^* \neq \hat{p}_s$. By a parallel argument, $p^* = \hat{p}_r$ implies $\gamma > 1 - \omega_r$ and

$$\left.\left(\frac{\partial \pi^c}{\partial p} - \frac{\partial \pi^e}{\partial p}\right)\right|_{p=\hat{p}_s} = \frac{(1-\eta_s)(\phi - q_H(\gamma - (1-\omega_r)))}{2q_H} \geq 0, \tag{EC.31}$$

which implies

$$q_H \leq \frac{\phi}{\gamma - (1-\omega_r)} = \hat{p}_r.$$

We again obtain a contradiction to $p^* = \hat{p}_r$ because the seller generates zero demand for all $p \geq q_H$. Therefore, $p^* \in \{p^a, p^c\}$.

Step 2. We now show $p^* = p^a$ if and only if the seller transacts online with the $\sigma = s$ buyer. Consider two cases: $\gamma \leq 1 - \omega_s$ and $\gamma > 1 - \omega_s$. Note if $\gamma \leq 1 - \omega_s$, by Lemma EC.4 the transaction is never offline because $\gamma \leq \hat{\gamma}_s(p)$ for all $p \geq 0$. In this case, the profit function is simply $\Pi^H(p) = \pi^a(p)$, and the result follows. Now suppose $\gamma > 1 - \omega_s$, which implies $\hat{p}_s > 0$. From Step 1, we know $\hat{p}_s$ cannot be a local maximum, and thus by concavity of $\pi^a(p)$ and $\pi^c(p)$, $p^c < \hat{p}_s < p^a$ cannot hold. It follows that if $\gamma > 1 - \omega_s$, one of three cases must hold: $\hat{p}_s \leq \min\{p^a, p^c\}$, $\hat{p}_s \geq \max\{p^a, p^c\}$, or $p^a < \hat{p}_s < p^c$. In the first case, $\hat{p}_s \leq p^a$ implies $\Pi^H(p)$ has a single local maximum at p^c, which implies $p^* = p^c > \hat{p}_s$. Because the transaction is online at p if $p \leq \hat{p}_s$ (Lemma EC.4), we conclude $\hat{p}_s \leq p^a$ implies both $p^* = p^c$ and that the transaction is offline. In the second case, $p^c \leq \hat{p}_s$ implies $\Pi^H(p)$ has a single local maximum at p^a, which implies $p^* = p^a < \hat{p}_s$, which implies the transaction is online. Lastly, if $p^a < \hat{p}_s < p^c$, $\Pi^H(p)$ has two local maxima at p^a and p^c. In this case, either $p^* = p^a < \hat{p}_s$ or $p^* = p^c > \hat{p}_s$ must hold. Thus, in all three cases, $p^* = p^a$ if and only if the transaction is online.

(ii). By Lemma EC.6, if $\gamma \leq 1 - \omega_s$ the type-L seller's profit function is simply $\Pi^L(p) = \pi^b(p)$ for all $p \geq 0$. If $\gamma > 1 - \omega_s$, the q_L seller's profit function is then

$$\Pi^L(p) = \begin{cases} \pi^b(p), & \text{if } p \leq \hat{p}_s, \\ \pi^d(p), & \text{if } \hat{p}_s < p. \end{cases} \tag{EC.32}$$

The result follows by parallel argument to the proof of statement (i), with $\Pi^L(p)$, p^b and p^d in place of $\Pi^H(p)$, p^a and p^c, respectively. □

EC.1.3. Commission Thresholds for Disintermediation and Platform Revenue

The main results in this section are Lemma EC.10, which defines thresholds on the commission rate γ that trigger disintermediation, and Lemma EC.11, which defines the platform's revenue function. Lemmas EC.12 and EC.13 provide useful properties of the platform revenue function that are used in later proofs.

LEMMA EC.9 **(Sufficient and necessary conditions for disintermediation)**. *Suppose a type-i seller transacts with a $\sigma = s$ buyer, where $i \in \{L, H\}$.*

(i) For the type-H seller, $\gamma \leq \hat{\gamma}_s(p^a)$ and $\gamma < \hat{\gamma}_s(p^c)$ are necessary and sufficient for the transaction to occur online, respectively.

(*ii*) *For the type-L seller,* $\gamma \le \hat{\gamma}_s(p^b)$ *and* $\gamma < \hat{\gamma}_s(p^d)$ *are necessary and sufficient for the transaction to occur online, respectively.*

Proof. The results largely follow from Lemma EC.4, which provides the definitions of $\hat{\gamma}_s(p)$ and $\hat{p}_s$. We briefly address statement (i); the proof of (ii) follows by parallel argument and is omitted. First, suppose the transaction is online, and consider two cases: $\gamma \le 1-\omega_s$ and $\gamma > 1-\omega_s$. If $\gamma \le 1-\omega_s$, then $\gamma \le \hat{\gamma}_s(p^a)$ must hold by definition of $\hat{\gamma}_s$. Now suppose $\gamma > 1-\omega_s$. Because the transaction is online, by Lemma EC.8 we have $p^* = p^a < \hat{p}_s$. By Lemma EC.4, $p^a < \hat{p}_s$ implies $\gamma < \hat{\gamma}_s(p^a)$, as desired. Now suppose $\gamma < \hat{\gamma}_s(p^c)$, and consider two cases: $\gamma \le 1-\omega_s$ and $\gamma > 1-\omega_s$. If $\gamma \le 1-\omega_s$, then $\gamma \le \hat{\gamma}(p)$ for all $p \ge 0$, which implies the transaction is online by Lemma EC.4. If $\gamma > 1-\omega_s$, then $\gamma < \hat{\gamma}_s(p^c)$ implies $p^c < \hat{p}_s$ by Lemma EC.4. Because p^c is the maximizer of $\pi^c(p)$ and $\pi^c(p)$ is strictly concave, $p^c < \hat{p}_s$ implies the seller's profit function $\Pi^H(p)$ strictly decreases in p on $[\hat{p}_s, \hat{p}_r]$. Further, $p^* \le p_r$ by Lemma EC.7, which implies $p^* \le \hat{p}_s$. It follows from Lemma EC.4 that the transaction occurs online. □

Lemma EC.10 **(Commission thresholds for disintermediation)**. *Let* $\underline{\gamma}_s^H$, $\bar{\gamma}_s^H$, $\underline{\gamma}_s^L$, *and* $\bar{\gamma}_s^L$ *be the solutions to* (EC.33a), (EC.33b), (EC.33c), *and* (EC.33d) *respectively:*

$$\gamma = 1-\omega_s + \frac{\phi}{p^c(\gamma)}, \tag{EC.33a}$$

$$\gamma = 1-\omega_s + \frac{\phi}{p^a(\gamma)}, \tag{EC.33b}$$

$$\gamma = 1-\omega_s + \frac{\phi}{p^d(\gamma)}, \tag{EC.33c}$$

$$\gamma = 1-\omega_s + \frac{\phi}{p^b(\gamma)}. \tag{EC.33d}$$

Suppose both type-L and type-H sellers accept the $\sigma = s$ *buyer for some* $\gamma \in (0, \gamma^m]$. *Then for each* $i \in \{L, H\}$, *there exists a unique threshold* $\gamma_s^i \in [\underline{\gamma}_s^i, \bar{\gamma}_s^i]$ *such that the type-i seller transacts offline with the* $\sigma = s$ *buyer under their optimal price if and only if* $\gamma > \gamma_s^i$. *Further,* $\gamma_s^H \le \gamma_s^L$ *holds for all* $\phi \ge 0$, *and* $\gamma_s^i = \underline{\gamma}_s^i = \bar{\gamma}_s^i = 1-\omega_s$ *for* $i \in \{L, H\}$ *if* $\phi = 0$.

Proof. The proof proceeds in four steps. First, we show the type-H seller's transaction is online if $\gamma \le \underline{\gamma}_s^H$ and offline if $\gamma > \bar{\gamma}_s^H$, which also establishes that $\underline{\gamma}_s^H < \bar{\gamma}_s^H$. Second, we show the existence of the threshold $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. Third, we show the analogous result for the type-L seller. Fourth, we show the final sentence of the lemma statement.

Step 1. By definition of $\underline{\gamma}_s^H$, we have

$$\underline{\gamma}_s^H = 1 - \omega_s + \frac{\phi}{p^c(\underline{\gamma}_s^H)}.$$

By inspecting the expression for $p^c(\gamma)$ (Lemma EC.3), it is straightforward to verify that $p^c(\gamma)$ strictly increases in γ. Therefore, $\gamma \le \underline{\gamma}_s^H$ implies

$$\gamma \le 1 - \omega_s + \frac{\phi}{p^c(\gamma)}.$$

By Lemma EC.9, the inequality above is a sufficient condition for the transaction to be online. Similarly, if $\gamma > \bar{\gamma}_s^H$, then by definition of $\bar{\gamma}_s^H$ and because $p^a(\gamma)$ increases in γ, we must have

$$\gamma > 1 - \omega_s + \frac{\phi}{p^a(\gamma)}.$$

By Lemma EC.9, the inequality above is a sufficient condition for the transaction to be offline. This completes the first step.

Step 2. We now show the existence of the threshold $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. To avoid the trivial case where transactions occur online for all $\gamma \in [0, \gamma^m]$, we assume $\gamma_s^H < \gamma^m$. Note that the first step of the proof and the definition of the seller's profit function $\Pi^H(p)$ (Lemma EC.6) implies $\pi^a(p^a) < \pi^c(p^c)$ for $\gamma > \bar{\gamma}_s^H$ and $\pi^a(p^a) > \pi^c(p^c)$ for $\gamma < \bar{\gamma}_s^H$. Therefore, because $\pi^a(p^a)$ and $\pi^c(p^c)$ are both continuous in γ, to prove statement (*i*) it suffices to show $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases in γ on $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. Differentiating the seller profit functions, for any γ on $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$ we have

$$\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) = \left(\frac{\partial \pi^a}{\partial p} \cdot \frac{dp^a}{d\gamma} + \frac{\partial \pi^a}{\partial \gamma}\right)\Bigg|_{p=p^a} - \left(\frac{\partial \pi^c}{\partial p} \cdot \frac{dp^c}{d\gamma} + \frac{\partial \pi^c}{\partial \gamma}\right)\Bigg|_{p=p^c} \tag{EC.34}$$

$$= \frac{\partial \pi^a}{\partial \gamma}\Bigg|_{p=p^a} - \frac{\partial \pi^c}{\partial \gamma}\Bigg|_{p=p^c} \tag{EC.35}$$

$$= -p^a\left(1 - \frac{p^a}{q_H}\right) + \left(1 - \frac{\eta_s}{2}\right) p^c \left(1 - \frac{p^c}{q_H}\right) \tag{EC.36}$$

$$\le \left(\left(1 - \frac{\eta_s}{2}\right) p^c - p^a\right)\left(1 - \frac{p^a}{q_H}\right). \tag{EC.37}$$

In the relations above, the second line follows from the envelope theorem, the third from evaluating the derivative algebraically, and the fourth because $p^c \ge p^a$ when $\gamma \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, which follows from the definitions of $\underline{\gamma}_s^H$ and $\bar{\gamma}_s^H$. Because $\left(1 - \frac{p^a}{q_H}\right) > 0$ and $\eta_s \in (0,1)$, to show $\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) < 0$ it suffices to show that $2p^a - p^c > 0$. Note we must have $p^c < q_H$ because the type-H seller earns zero profit for all prices above q_H. Therefore, it remains to show $2p^a > q_H$. This follows immediately from inspecting the expression for p^a (Lemma EC.3), which shows $p^a > q_H/2$. Therefore, $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases on $\gamma \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, as desired. Lastly, the result that $\gamma_s^H = \underline{\gamma}_s^H = \bar{\gamma}_s^H = 1 - \omega_s$ when $\phi = 0$ follows from the definition of $\underline{\gamma}_s^H$ and $\bar{\gamma}_s^H$.

Step 3. The proof follows similarly to the result for the type-H seller. Using the definitions of $\underline{\gamma}_s^L$ and $\bar{\gamma}_s^L$ and the fact that p^b and p^d are both strictly increasing in γ, it is straightforward to show the transaction is online if $\gamma < \underline{\gamma}_s^L$ and offline if $\gamma > \bar{\gamma}_s^L$. It remains to show $\pi^b(p^b) - \pi^d(p^d)$ strictly decreases in γ on $[\underline{\gamma}_s^L, \bar{\gamma}_s^L]$. Differentiating in γ, we have

$$\frac{d}{d\gamma}(\pi^b(p^b) - \pi^d(p^d)) = \left(\frac{\partial \pi^b}{\partial p} \cdot \frac{dp^b}{d\gamma} + \frac{\partial \pi^b}{\partial \gamma}\right)\bigg|_{p=p^b} - \left(\frac{\partial \pi^d}{\partial p} \cdot \frac{dp^d}{d\gamma} + \frac{\partial \pi^d}{\partial \gamma}\right)\bigg|_{p=p^d} \tag{EC.38}$$

$$= \frac{\partial \pi^b}{\partial \gamma}\bigg|_{p=p^b} - \frac{\partial \pi^d}{\partial \gamma}\bigg|_{p=p^d} \tag{EC.39}$$

$$= -\eta_s p^b \left(1 - \frac{p^b}{q_L}\right) + \frac{\eta_s}{2} p^d \left(1 - \frac{p^d}{q_L}\right) \tag{EC.40}$$

$$\le \left(-\eta_s p^b + \frac{\eta_s}{2} p^d\right)\left(1 - \frac{p^d}{q_L}\right), \tag{EC.41}$$

where the final inequality follows because $p^d \ge p^b$ when $\gamma \in [\underline{\gamma}_s^L, \bar{\gamma}_s^L]$, which follows from the definitions of $\underline{\gamma}_s^L$ and $\bar{\gamma}_s^L$. It remains to show $-\eta_s p^b + \frac{\eta_s}{2} p^d > 0$, or equivalently, $2p^b > p^d$. Note we must have $p^d < q_L$ because the type-L seller has zero demand for all prices above q_L, and further $2p^b > q_L$ by inspection of the expression for p^b. It follows that $2p^b > q_L > p^d$, as desired.

Step 4. We now show $\gamma_s^H \le \gamma_s^L$ for all $\phi \ge 0$. By Lemma EC.10, we have $\gamma_s^H \le \bar{\gamma}_s^H$ and $\underline{\gamma}_s^L \le \gamma_s^L$. Therefore, it suffices to show $\bar{\gamma}_s^H \le \underline{\gamma}_s^L$. Using the expressions in Lemma EC.10, $\bar{\gamma}_s^H \le \underline{\gamma}_s^L$ holds if and only if $p^a(\bar{\gamma}_s^H)\big|_{q=q_H} \ge p^d(\underline{\gamma}_s^L)\big|_{q=q_L}$. Next, note

$$p^a(\bar{\gamma}_s^H)\big|_{q=q_H} \ge \frac{q_H}{2} \ge 2c \ge 2(1-\lambda)c \ge 2q_L \ge p^d(\underline{\gamma}_s^L)\big|_{q=q_L}. \tag{EC.42}$$

The first inequality follows by inspecting the expression for p^a (Lemma EC.3), the second from Assumption 1, the third because $\lambda \ge 0$, and the fourth inequality follows from Assumption 1. To see that the final inequality holds, note the type-L seller has zero demand for all prices above q_L, which implies $p^d(\gamma) \le q_L$ for all $\gamma \ge 0$. It follows that $\gamma_s^H \le \gamma_s^L$. Lastly, the result that $\gamma_s^i = \underline{\gamma}_s^i = \bar{\gamma}_s^i = 1 - \omega_s$ for $i \in \{L, H\}$ when $\phi = 0$ follows by definition of $\underline{\gamma}_s^i$ and $\bar{\gamma}_s^i$. □

LEMMA EC.11 **(Platform's revenue function)**. *Let p^a, p^b, and p^c be as defined in Lemma EC.3, and define*

$$r^a(\gamma) := \gamma p^a \left(1 - \frac{p^a}{q_H}\right), \tag{EC.43}$$

$$r^b(\gamma) := \gamma\eta_s p^b\left(1 - \frac{p^b}{q_L}\right), \tag{EC.44}$$

$$r^c(\gamma) := \gamma\eta_r p^c\left(1 - \frac{p^c}{q_H}\right). \tag{EC.45}$$

Then the platform's commission revenue is given by $R(\gamma)$, where

$$R(\gamma) := \begin{cases} \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m], \end{cases} \tag{EC.46}$$

and $x^+ = \max\{0, x\}$.

Proof. To see that the piecewise function $R(\gamma)$ is the platform's commission revenue, note μ and $1-\mu$ are the shares of type-H and type-L sellers, respectively. By Lemma EC.5, the type-H seller accepts both $\sigma = s$ and $\sigma = r$ buyers and the type-L seller rejects the $\sigma = r$ buyer. Then combining Lemmas EC.3 and EC.10, the type-H sellers' contribution to platform revenue is $\mu r^a(\gamma)^+$ if $\gamma \le \gamma_s^H$ and $\mu r^c(\gamma)^+$ if $\gamma > \gamma_s^H$; similarly, the type-L sellers' contribution is $(1-\mu) r^b(\gamma)^+$ if $\gamma \le \gamma_s^L$ and 0 if $\gamma > \gamma_s^L$. Further, it is straightforward to verify algebraically that $r^a(\gamma) \ge 0$ and $r^c(\gamma) \ge 0$ for all $\alpha \in [\frac{1}{2}, 1]$ and $\gamma \ge 0$; the superscript $(\cdot)^+$ is suppressed accordingly. The function $R(\gamma)$ follows. □

Lemma EC.12 **(Revenue function properties)**. *Let $r^a(\gamma)$, $r^b(\gamma)$, and $r^c(\gamma)$ be as defined in Lemma EC.11. The following statements hold for all $\gamma \in [0, \frac{1}{2}]$ and $\alpha \in [\frac{1}{2}, 1]$.*

(i) $r^a(\gamma)$ is strictly concave and increasing in γ, and is independent of α and ϕ.

(ii) $r^b(\gamma)$ is strictly concave in γ, increases in α wherever $r^b(\gamma) > 0$, and is independent of ϕ.

(iii) $r^c(\gamma)$ is strictly concave in γ for all ϕ and is strictly increasing in γ for $\phi = 0$.

Proof. ***(i)***. Using the expressions for $r^a(\gamma)$ and p^a,

$$r^a(\gamma) = \gamma p^a\left(1 - \frac{p^a}{q_H}\right) = \frac{\gamma}{4}\left(q_H - \frac{c^2(1-\lambda)^2}{q_H(1-\gamma)^2}\right). \tag{EC.47}$$

Differentiating in γ, we have

$$\frac{\partial r^a}{\partial \gamma} = \frac{1}{4}\left(q_H - \frac{c^2(1-\lambda)^2}{q_H(1-\gamma)^2}\right) - \frac{\gamma}{4}\left(\frac{2c^2(1-\lambda)^2}{q_H(1-\gamma)^3}\right). \tag{EC.48}$$

By inspection, the first term on the right hand side of (EC.48) strictly decreases in γ and the second term strictly increases in γ. Therefore, $\frac{d}{d\gamma} r^a$ strictly decreases in γ, which implies $r^a(\gamma)$

is strictly concave in γ. Further, using the fact that $q_H \geq 4c$ (Assumption 1) it can be shown that $\lim_{\gamma\to\frac{1}{2}} \frac{d}{d\gamma} r^a > 0$. It follows that $r^a(\gamma)$ strictly increases in γ for $\gamma \in [0, \frac{1}{2}]$.

(ii). Using the expressions for p^b and $r^b(\gamma)$ (Lemmas EC.3 and EC.11), we have

$$r^b(\gamma) = \eta_s \gamma p^b \left(1 - \frac{p^b}{q_L}\right) = \frac{\eta_s \gamma}{4}\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right). \tag{EC.49}$$

Note for any $\gamma \in [0, \gamma^m]$, using the expressions for η_s and ω_s (Lemma EC.1) we have

$$\frac{\partial r^b}{\partial \alpha} = \frac{\partial r^b}{\partial \eta_s}\frac{\partial \eta_s}{\partial \alpha} + \frac{\partial r^b}{\partial \omega_s}\frac{\partial \omega_s}{\partial \alpha} \tag{EC.50}$$

$$= \frac{\gamma}{4}\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right)\frac{\partial \eta_s}{\partial \alpha} + \left(\frac{2c^2(1-\omega_s)}{q_L(1-\gamma)^2}\right)\frac{\partial \omega_s}{\partial \alpha} \tag{EC.51}$$

$$= \frac{\gamma}{4}\left(\frac{r^b}{\eta_s}\right)(2\lambda - 1) + \left(\frac{2c^2(1-\omega_s)}{q_L(1-\gamma)^2}\right)\frac{(1-\lambda)\lambda}{\eta_s^2} \tag{EC.52}$$

$$> 0, \tag{EC.53}$$

where the strict inequality follows from $\lambda \in [\frac{1}{2}, 1]$. Thus $r^b(\gamma)$ strictly increases in α if $r^b(\gamma) > 0$. Next, differentiating $r^b(\gamma)$ in γ yields

$$\frac{\partial r^b}{\partial \gamma} = \frac{\eta_s}{4}\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right) - \frac{\eta_s \gamma}{4}\left(\frac{2c^2(1-\omega_s)^2}{q_L(1-\gamma)^3}\right) = \frac{\eta_s}{4}\Bigg(q_L - \underbrace{\left(1 + \frac{2\gamma}{1-\gamma}\right)\frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}}_{g(\gamma)}\Bigg), \tag{EC.54}$$

where for convenience $g(\gamma)$ is defined as shown in (EC.54). Note by inspection that $g(\gamma)$ strictly increases in γ, which implies $\frac{d}{d\gamma} r^b$ strictly decreases in γ. Hence $r^b(\gamma)$ is strictly concave in γ.

(iii). Using the expressions for $r^c(\gamma)$ and p^c, for $q = q_H$ we have

$$r^c(\gamma) = \eta_r \gamma p^c \left(1 - \frac{p^c}{q_H}\right) = \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s \phi}{4 q_H \zeta}\right)^2\right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\omega_s)$. Differentiating in γ and using the fact that $\frac{d}{d\gamma}\zeta = -\eta_r - \frac{1}{2}\eta_s = \frac{1}{2}\eta_s - 1$, we have

$$\frac{\partial r^c}{\partial \gamma} = \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s\phi}{4q_H\zeta}\right)^2\right) - \frac{2\eta_r q_H}{\zeta^3}\gamma\left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s\phi}{4q_H}\right)^2\left(1 - \frac{\eta_s}{2}\right) \tag{EC.55}$$

$$= \eta_r q_H \left(\frac{1}{4} - \frac{h^2}{q_H^2}\left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3}\left(1 - \frac{\eta_s}{2}\right)\right)\right), \tag{EC.56}$$

where $h = \frac{1}{4}(2c(1-\lambda)+\eta_s\phi)$. Next, note (EC.56) strictly decreases in γ because $\frac{\partial}{\partial\gamma}\zeta = \frac{1}{2}\eta_s - 1 < 0$ and

$$\frac{\partial}{\partial\gamma}\left(\frac{\gamma}{\zeta^3}\right) = \frac{8(2+4\gamma-\eta_s(1+2\gamma-\omega_s))}{(2(1-\gamma)-\eta_s(1-\gamma-\omega_s))^4} > 0, \tag{EC.57}$$

where the strict inequality follows because $\gamma \in [0, \frac{1}{2}]$, $\eta_s \in [0,1]$ and $\omega_s \in [0,1]$. It follows that $r^c(\gamma)$ is strictly concave. Finally, consider the case when $\phi = 0$:

$$\frac{\partial r^c}{\partial\gamma} = \frac{\eta_r q_H}{4}\left(1 - \frac{c^2(1-\lambda)^2}{q_H^2}\left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3}\left(1-\frac{\eta_s}{2}\right)\right)\right). \tag{EC.58}$$

Since $\gamma \le \frac{1}{2}$, we have $\zeta \ge 1-\gamma \ge \frac{1}{2}$. Also note $\omega_s \ge \omega_s \ge \frac{1}{2}$. Utilizing this bound on ζ along with $q_H \ge 4c$, $\lambda \ge \frac{1}{2}$, and $\eta_s \ge \frac{1}{2}$, we can write

$$\frac{\partial r^c}{\partial\gamma} \ge \frac{\eta_r q_H}{4}\left(1 - \frac{c^2(1-\lambda)^2}{16c^2}\left(\frac{1}{(\frac{1}{2})^2} + \frac{2(\frac{1}{2})}{(\frac{1}{2})^3}\left(1-\frac{1}{4}\right)\right)\right) > 0. \tag{EC.59}$$

We conclude that $r^c(\gamma)$ strictly increases in $[0,\gamma^m]$ when $\phi = 0$. □

LEMMA EC.13. *The inequality* $\frac{8}{15}r^a(\gamma) > r^c(\gamma)$ *holds for all* $\gamma \in [0,\gamma^m]$ *and* $\phi \ge 0$.

Proof. We first show the following two inequalities hold:

$$\zeta \le 1-\gamma+\frac{\lambda\gamma}{2} \tag{EC.60a}$$

$$1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma+\frac{\lambda\gamma}{2})}\right)^2 \le \frac{16}{15}\left(1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2\right) \tag{EC.60b}$$

To see that (EC.60a) holds, note

$$\zeta = \eta_r(1-\gamma)+\frac{\eta_s}{2}(1-\gamma+\omega_s) \le \eta_r(1-\gamma)+\frac{\eta_s}{2}(1-\gamma+1) = 1-\gamma+\frac{\eta_s\gamma}{2} \le 1-\gamma+\frac{\lambda\gamma}{2}, \tag{EC.60c}$$

which follows because $\omega_s \le 1$, $\eta_r \le 1$, and $\eta_s \le \lambda$. Next, for (EC.60b) we have

$$\frac{1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma+\frac{\lambda\gamma}{2})}\right)^2}{1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2} \le \frac{1-\left(\frac{1-\lambda}{2+\lambda}\right)^2}{1-\left(\frac{1-\lambda}{2}\right)^2} \le \frac{16}{15}.$$

The first inequality above follows by noting the ratio in the left hand side is decreasing in q_H and increasing in γ, and because $q_H \ge 4c$ and $\gamma \le \frac{1}{2}$ by Assumptions 1 and 2. The second inequality follows because the intermediate expression decreases in λ and because $\lambda \geq \frac{1}{2}$. We can now prove the lemma statement. Note

$$\begin{aligned}
r^c(\gamma) &= \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s \phi}{4q_H\zeta} \right)^2 \right) \\
&\leq \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} \right)^2 \right) \\
&\leq \eta_r \gamma \frac{q_H}{4} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma+\frac{\lambda\gamma}{2})} \right)^2 \right) \\
&\leq \frac{16}{15} \eta_r \gamma \frac{q_H}{4} \left(1 - \left(\frac{(1-\lambda)c}{q_H(1-\gamma)} \right)^2 \right) \\
&= \frac{16}{15} \eta_r r^a(\gamma) \\
&< \frac{8}{15} r^a(\gamma).
\end{aligned}$$

The first line follows by definition of $r^c(\gamma)$, the second line follows because $r^c(\gamma)$ strictly decreases in ϕ, the third line follows using the upper bound on ζ from (EC.60a), the fourth line follows using the inequality (EC.60b), the fifth line follows from the definition of $r^a(\gamma)$, and the final line follows because $\eta_r \leq \frac{1}{2}$ since $\lambda \geq \frac{1}{2}$. □

EC.2. Proofs for Section 3: Optimal Commission and Platform Revenue

EC.2.1. Proof of Proposition 1

We first present two useful lemmas used in the proof of Proposition 1 and elsewhere: Lemma EC.14 presents comparative statics with respect to ϕ for different candidate solutions for the optimal commission rate γ^*, and Lemma EC.15 provides a characterization of the optimal commission rate γ^*. For use in the remainder of the electronic companion, define $\gamma^x := \text{argmax}_{\gamma \in [0,1]} r^x(\gamma)$ and $\gamma^{xy} := \text{argmax}_{\gamma \in [0,\gamma^m]} \{\mu r^x(\gamma) + (1-\mu) r^y(\gamma)\}$, where $x, y \in \{a, b, c\}$ and the $r^x(\gamma)$ functions are as defined in Lemma EC.11. For convenience, we describe these quantities informally below:

- $r^a(\gamma)$ is the platform's revenue from a type-H seller when the seller transacts only online, and γ^a is its unconstrained maximizer,
- $r^b(\gamma)$ is the platform's revenue from a type-L seller when they transact online with the $\sigma = s$ buyer (and reject $\sigma = r$ buyers), and γ^b is its unconstrained maximizer,
- $r^c(\gamma)$ is the platform's revenue from a type-H seller when the seller transacts online and offline with $\sigma = r$ and $\sigma = s$ buyers, respectively, and γ^c is its unconstrained maximizer,

- γ^{ab} – which is the only quantity of the form γ^{xy} used in the following proof – is the optimal commission rate when both type-H and type-L sellers transact online.

LEMMA EC.14. *The following statements hold.* (i) γ^a *and* γ^b *are both independent of* ϕ*,* (ii) γ^c *strictly decreases in* ϕ*, and* (iii) γ_s^H *and* γ_s^L *strictly increase in* ϕ*.*

Recall that γ_s^H and γ_s^L denote the smallest commission rates at which the type-H and type-L sellers transact offline with $\sigma = s$ buyers, respectively (Lemma EC.10). Intuitively, an increase in the switching cost leads to an increase in the commission rates at which sellers disintermediate, which is captured by statement (iii) in the lemma above.

Proof. $\boldsymbol{(i)}$. The result follows by noting that $r^a(\gamma)$ and $r^b(\gamma)$ are independent of ϕ (Lemma EC.12), and thus so are their maximizers γ^a and γ^b. This is because for fixed γ, the switching cost does not affect revenue when all transactions occur online.

$\boldsymbol{(ii)}$. Applying the implicit function theorem, we have

$$\frac{d\gamma^c}{d\phi} = -\left(\frac{\partial^2 r^c}{\partial\gamma\partial\phi}\right)\left(\frac{\partial^2 r^c}{\partial\gamma^2}\right)^{-1}\Bigg|_{\gamma=\gamma^c}. \tag{EC.61}$$

Note $\frac{\partial^2}{\partial\gamma^2}r^c < 0$ at $\gamma = \gamma^c$ because γ^c is the maximizer of r^c. Therefore, $\frac{d}{d\phi}\gamma^c$ has the same sign as $\frac{\partial^2}{\partial\gamma\partial\phi}r^c$. Using the expressions for p^c and $r^c(\gamma)$ from Lemmas EC.3 and EC.11, we have

$$r^c(\gamma) = \eta_r\gamma p^c\left(1 - \frac{p^c}{q_H}\right) = \eta_r\gamma q_H\left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s\phi}{4q_H\zeta}\right)^2\right),$$

where $\zeta = \eta_r(1-\gamma) + \frac{1}{2}\eta_s(1-\gamma+\omega_s)$. Differentiating in γ and using the fact that $\frac{d}{d\gamma}\zeta = \frac{1}{2}\eta_s - 1$, we have

$$\frac{\partial r^c}{\partial\gamma} = \eta_r q_H\left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s\phi}{4q_H\zeta}\right)^2\right) - \frac{2\eta_r q_H}{\zeta^3}\gamma\left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s\phi}{4q_H}\right)^2\left(1 - \frac{\eta_s}{2}\right) \tag{EC.62}$$

$$= \eta_r q_H\left(\frac{1}{4} - \frac{h^2}{q_H^2}\left(\frac{1}{\zeta^2} + \frac{2\gamma}{\zeta^3}\left(1 - \frac{\eta_s}{2}\right)\right)\right), \tag{EC.63}$$

where $h = \frac{1}{4}(2c(1-\lambda) + \eta_s\phi)$. Because h increases in ϕ, we conclude $\frac{\partial^2}{\partial\gamma\partial\phi}r^c < 0$ and thus $\frac{d}{d\phi}\gamma^c < 0$.

$\boldsymbol{(iii)}$. We first show $\frac{d}{d\phi}\gamma_s^H > 0$, followed by showing $\frac{d}{d\phi}\gamma_s^L > 0$. Note by the proof of Lemma EC.10, γ_s^H is the unique solution to $\pi^a(p^a) - \pi^c(p^c) = 0$. Note $\pi^a(p^a)$ depends on γ and $\pi^c(p^c)$ depends on both γ and ϕ. For convenience, we define the function $\pi^-(\gamma,\phi) := \pi^a(p^a) - \pi^c(p^c)$. It follows that $\pi^-(\gamma_s^H, \phi) = 0$. Taking the total derivative of this equation with respect to ϕ, we have

$$\frac{d\pi^-}{ds}\Bigg|_{\gamma=\gamma_s^H} = \left(\frac{\partial\pi^-}{\partial\gamma}\frac{d\gamma_s^H}{d\phi} + \frac{\partial\pi^-}{\partial\phi}\right)\Bigg|_{\gamma=\gamma_s^H} = 0.$$

Because γ_s^H is the unique solution to $\pi^-(\gamma,\phi)=0$ by Lemma EC.10, we must have $\frac{\partial}{\partial\gamma}\pi^- \neq 0$ at $\gamma=\gamma_s^H$. We can therefore re-arrange for $\frac{d}{d\phi}\gamma_s^H$ to obtain

$$\frac{d\gamma_s^H}{d\phi} = -\left(\frac{\partial \pi^-}{\partial \phi}\right)\left(\frac{\partial \pi^-}{\partial \gamma}\right)^{-1}\Bigg|_{\gamma=\gamma_s^H}. \tag{EC.64}$$

Next, we show $\frac{d}{d\phi}\gamma_s^H > 0$ by showing $\frac{\partial}{\partial\gamma}\pi^- < 0$ and $\frac{\partial}{\partial\phi}\pi^- > 0$ at $\gamma=\gamma_s^H$. First, the proof of Lemma EC.10 shows that $\pi^-(\gamma,\phi)$ strictly decreases in γ on the interval $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$, and that $\gamma_s^H \in [\underline{\gamma}_s^H, \bar{\gamma}_s^H]$. It follows that $\frac{\partial}{\partial\gamma}\pi^-(\gamma,\phi) < 0$ at $\gamma=\gamma_s^H$. Next, for all $\gamma\in[0,\gamma^m]$ we have

$$\begin{aligned}
\frac{\partial \pi^-}{\partial \phi} &= -\frac{\partial}{\partial \phi}\pi^c(p^c) && \text{(EC.65)}\\
&= -\frac{\partial}{\partial \phi}\left\{\frac{1}{2}\left(1-\frac{p^c}{q_H}\right)(p^c(2(1-\gamma)-\eta_s(1-\gamma-\omega_s))-2c(1-\lambda)-\eta_s\phi)\right\} && \text{(EC.66)}\\
&= \frac{\eta_s}{2}\left(1-\frac{p^c}{q_H}\right) && \text{(EC.67)}\\
&> 0. && \text{(EC.68)}
\end{aligned}$$

The first line follows by definition of $\pi^-(\gamma,\phi)$ and because $\frac{\partial}{\partial\phi}\pi^a(p^a)=0$, the second by plugging in the expression for $\pi^c(p^c)$, and the third from noting p^c is the maximizer of $\pi^c(p)$ and thus applying the envelope theorem to compute the partial derivative. The strict inequality follows because p^c maximizes $\pi^c(p)$ and the seller earns zero profit for all prices above q_H, which implies $p^c < q_H$. Because $\frac{\partial}{\partial\gamma}\pi^- < 0$ and $\frac{\partial}{\partial\phi}\pi^- > 0$ at $\gamma=\gamma_s^H$, we conclude $\frac{d}{d\phi}\gamma_s^H > 0$, as desired. Next, we show $\frac{d}{d\phi}\gamma_s^L > 0$ using a similar argument to that above for $\frac{d}{d\phi}\gamma_s^H > 0$. Note by the proof of Lemma EC.10, γ_s^L is the unique solution to $\pi^b(p^b)-\pi^d(p^d)$. Let $\bar{\pi}(\gamma,\phi)=\pi^b(p^b)-\pi^d(p^d)$, and note $\bar{\pi}(\gamma_s^L,\phi)=0$ by definition of γ_s^L. Taking the total derivative with respect to ϕ yields

$$\frac{d\bar{\pi}}{ds}\bigg|_{\gamma=\gamma_s^L} = \left(\frac{\partial\bar{\pi}}{\partial\gamma}\frac{d\gamma_s^L}{d\phi}+\frac{\partial\bar{\pi}}{\partial\phi}\right)\bigg|_{\gamma=\gamma_s^L} = 0.$$

Because γ_s^L is the unique solution to $\bar{\pi}(\gamma,\phi)=0$ by Lemma EC.10, we must have $\frac{\partial}{\partial\gamma}\bar{\pi}\neq 0$ at $\gamma=\gamma_s^L$. We can therefore re-arrange for $\frac{d}{d\phi}\gamma_s^L$ to obtain

$$\frac{d\gamma_s^L}{d\phi} = -\left(\frac{\partial \bar{\pi}}{\partial \phi}\right)\left(\frac{\partial \bar{\pi}}{\partial \gamma}\right)^{-1}\Bigg|_{\gamma=\gamma_s^L}. \tag{EC.69}$$

Next, we show $\frac{d}{d\phi}\gamma_s^L > 0$ by showing $\frac{\partial}{\partial\gamma}\bar{\pi} < 0$ and $\frac{\partial}{\partial\phi}\bar{\pi} > 0$ at $\gamma=\gamma_s^L$. First, the proof of Lemma EC.10 shows that $\bar{\pi}(\gamma,\phi)$ strictly decreases in γ on the interval $[\underline{\gamma}_s^L, \bar{\gamma}_s^L]$, and that $\gamma_s^L \in [\underline{\gamma}_s^L, \bar{\gamma}_s^L]$. It follows that $\frac{\partial}{\partial\gamma}\bar{\pi}(\gamma,\phi) < 0$ at $\gamma = \gamma_s^H$. Next, by parallel argument to the proof for $\frac{d}{d\phi}\gamma_s^H$ above, we have

$$\frac{\partial\bar{\pi}}{\partial\phi} = -\frac{d}{d\phi}\pi^d(p^d) \tag{EC.70}$$

$$= -\frac{d}{d\phi}\left\{\frac{1}{2}\eta_s\left(1-\frac{p^d}{q_L}\right)(p^d(1-\gamma+\omega_s)-2c(1-\omega_s)-\phi)\right\} \tag{EC.71}$$

$$= \frac{\eta_s}{2}\left(1-\frac{p^d}{q_L}\right) \tag{EC.72}$$

$$> 0. \tag{EC.73}$$

Because $\frac{\partial}{\partial\gamma}\bar{\pi} < 0$ and $\frac{\partial}{\partial\phi}\bar{\pi} > 0$ at $\gamma = \gamma_s^L$, we conclude $\frac{d}{d\phi}\gamma_s^L > 0$. □

LEMMA EC.15 **(Optimal commission rate)**. *The following statements hold.*

(i) There exists $\underline{\alpha} \in (\frac{1}{2}, 1]$ *such that* $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ *if* $\alpha \le \underline{\alpha}$ *and* $\phi \ge 0$.

(ii) There exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ *such that* $\gamma^* = \min\{\gamma^c, \gamma^m\}$ *if* $\alpha \in [\bar{\alpha}, 1]$ *and* $\phi = 0$.

Unpacking the above lemma, part (i) states that when information quality α is low, the platform chooses a commission rate no greater than γ_s^H, and so all transactions occur on-platform. At high values of α under no switching cost (part (ii)) , the only transactions that occur on-platform are between the type-H seller and $\sigma = r$ buyer.

Proof. $\boldsymbol{(i)}$. Note for any $\gamma > 0$ and $\alpha = \frac{1}{2}$, the type-L seller's profit is

$$\pi^b(p^b) = (1-\gamma)q_L\left(\frac{1}{2}-\frac{(1-\omega_s)c}{2q_L(1-\gamma)}\right)^2 \le (1-\gamma)q_L\left(\frac{1}{2}-\frac{(1-\omega_s)c}{2q_L}\right)^2 = (1-\gamma)q_L\left(\frac{1}{2}-\frac{(1-\lambda)c}{2q_L}\right)^2 \le 0,$$

which follows because at $\gamma > 0$, $\omega_s = \lambda$ at $\alpha = \frac{1}{2}$, and $q_L \le (1-\lambda)c$ by Assumption 1. Since $\pi^b(p^b) \le 0$ implies that the type-L seller rejects all buyers, we conclude the type-L seller's contribution to platform revenue at $\alpha = \frac{1}{2}$ is $r^b(\gamma)^+ = 0$. It follows by continuity of $\pi^b(p^b)$ in α that there exists $\tilde{\alpha}$ such that $r^b(\gamma)^+ = 0$ for all $\alpha \le \tilde{\alpha}$. Thus, for $\alpha \le \tilde{\alpha}$, the platform's revenue is given by (Lemma EC.11)

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^H, \gamma^m]. \end{cases} \tag{EC.74}$$

Next, using the expressions for ω_s and ω_r (Lemma EC.1), we have $\omega_s = \omega_r = \lambda$ for $\alpha = \frac{1}{2}$, which implies $\omega_s = \omega_r$, and thus $\gamma_s^H = \gamma_r^H$. It follows from Lemma EC.7, that $\gamma_s^H = \gamma_r^H > \gamma^m$ at $\alpha = \frac{1}{2}$. Further, because $r^a(\gamma)$ strictly increases in γ (Lemma EC.12), we conclude $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ at

$\alpha = \frac{1}{2}$. Finally, the existence of the threshold $\underline{\alpha} \le \tilde{\alpha}$ follows because γ_s^H is continuous in α and $r^a(\gamma) > r^c(\gamma)$ for $\gamma > 0$ by Lemma EC.13.

$(\boldsymbol{ii})$. By Lemma EC.11, the platform's revenue function is

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m]. \end{cases} \tag{EC.75}$$

Note $r^c(\gamma)$ is strictly concave in γ by Lemma EC.12. Therefore, to show $\gamma^* = \min\{\gamma^c, \gamma^m\}$ it suffices to show that the following three inequalities hold:

$$\min\{\gamma^c, \gamma^m\} > \gamma_s^L, \tag{EC.76a}$$

$$\mu r^c(\gamma^m) > \max_{\gamma \in [0, \gamma_s^H]} \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+, \tag{EC.76b}$$

$$\mu r^c(\gamma^m) > \max_{\gamma \in (\gamma_s^H, \gamma_s^L]} \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+. \tag{EC.76c}$$

We show $\gamma^* = \min\{\gamma^c, \gamma^m\}$ holds at $\alpha = 1$. Note $\phi = 0$ and $\alpha = 1$ implies $\omega_s = 1$ and thus $\gamma_s^H = \gamma_s^L = 0$ by Lemma EC.10. Therefore, in this setting the right hand sides of (EC.76b) and (EC.76c) are both zero. It remains to show $r^c(\gamma^m) > 0$. Note

$$\zeta = \eta_r(1-\gamma^m) + \frac{\eta_s}{2}(1-\gamma^m+\omega_s) \ge \frac{1}{2}(\eta_r+\eta_s)(1-\gamma^m) = \frac{1-\gamma^m}{2} \ge \frac{1}{4}.$$

Because $\zeta \ge \frac{1}{4}$ and $q_H \ge 4c$ by Assumption 1, we have $q_H \zeta \ge c$. Using this inequality, we can write

$$r^c(\gamma^m) = \eta_r \gamma^m q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2 q_H \zeta} \right)^2 \right) \ge \eta_r \gamma^m q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)}{2} \right)^2 \right) > 0,$$

where the final inequality follows because $\lambda < 1$. Therefore, $\gamma^* = \min\{\gamma^c, \gamma^m\}$ if $\phi = 0$ and $\alpha = 1$. Finally, the existence of the threshold $\bar{\alpha} < 1$ follows because γ_s^H and γ_s^L and the functions $r^a(\gamma)$, $r^b(\gamma)$ and $r^c(\gamma)$ are all continuous in α. □

PROPOSITION 1. *Let $\gamma^*(\phi)$ be the platform's optimal commission rate under switching cost ϕ. There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

(i) Suppose information quality is low, i.e., $\alpha \le \underline{\alpha}$. Then the optimal commission rate $\gamma^*(\phi)$ weakly increases in the switching cost ϕ for all $\phi \ge 0$.

(ii) Suppose information quality is high, i.e., $\alpha > \bar{\alpha}$*. Then there exists* $\bar{\phi} > 0$ *such that for each* $\phi \geq \bar{\phi}$*, the optimal commission rate is higher in the absence of switching costs,* $\gamma^*(0) \geq \gamma^*(\phi)$*, where the inequality is strict if* $\gamma^*(\phi) < \gamma^m$*. Further, there exists* $\underline{\phi} \in (0, \bar{\phi}]$ *such that* $\gamma^*(\phi)$ *strictly decreases in* ϕ *on* $\phi \in [0, \underline{\phi}]$ *wherever* $\gamma^*(\phi) < \gamma^m$*.*

Proof. This proof makes use of Lemmas EC.14 and EC.15. $\boldsymbol{(i)}$. By Lemma EC.15 there exists $\underline{\alpha} \in [\frac{1}{2}, 1]$ such that if $\alpha \leq \underline{\alpha}$, then $\gamma^* = \min\{\gamma^a, \gamma_s^H\}$ for all $\phi \geq 0$. That is, the optimal commission rate γ^* ensures that all transactions occur on-platform, e.g., see role of γ_s^H in Figure 1. By Lemma EC.14, γ^a is independent of ϕ and γ_s^H strictly increases in ϕ. It follows that $\gamma^* = \min\{\gamma^a, \gamma_s^H\}$ weakly increases in ϕ for all $\phi \geq 0$. In summary, when the information quality is low, all transactions occur on-platform. Further, any increase in the switching cost only strengthens the platform's pricing power, as sellers are less inclined to go off-platform, even at higher commission rates.

$\boldsymbol{(ii)}$. The proof proceeds in two steps. First we show $\gamma^*(0) \geq \gamma^*(\phi)$ holds for sufficiently large ϕ and α. Second, we address the comparative statics result.

Step 1. Note by Lemma EC.11, the platform's revenue function is

$$R(\gamma) = \begin{cases} \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) & \text{if } \gamma \in (\gamma_s^L, \gamma^m]. \end{cases} \tag{EC.77}$$

Because $\omega_s = 1$ at $\alpha = 1$ (Lemma EC.1) and $\gamma_s^H = \gamma_s^L = 1 - \omega_s$ at $\phi = 0$ (Lemma EC.10), we have $\gamma_s^H = \gamma_s^L = 0$ at $\phi = 0$ and $\alpha = 1$. When $\alpha = 1$, transactions with the $\sigma = s$ buyer will always occur offline, as the commission thresholds for disintermediation are zero. By continuity of γ_s^H and γ_s^L in α, it follows that there exists $\tilde{\alpha} \in [\frac{1}{2}, 1)$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ if $\phi = 0$ and $\alpha \geq \tilde{\alpha}$. Note that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ implies that the only transactions that occur online are between the type-H seller and $\sigma = r$ buyer. Similarly, because γ_s^H strictly increases in ϕ (Lemma EC.14), when the switching cost is sufficiently high, the platform can set its commission rate to guarantee that all transactions occur online. Formally, there exists $\bar{\phi} > 0$ such that $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ if $\phi \geq \bar{\phi}$. Because γ^{ab} does not depend on ϕ, it remains to show there exists $\bar{\alpha} \in [\tilde{\alpha}, 1)$ such that $\gamma^{ab} < \gamma^c$ if $\alpha \geq \bar{\alpha}$ and $\phi = 0$. In other words, the platform's commission rate at $\phi = 0$ (γ^c) is larger than its commission rate when switching costs are large (γ^{ab}).

To show this, we first define an auxiliary function $\ell^x(\gamma)$ for each $x \in \{a, b, c\}$. To define $\ell^a(\gamma)$, we differentiate $r^a(\gamma)$ in γ to obtain

$$\frac{\partial r^a}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\gamma p^a\left(1 - \frac{p^a}{q_H}\right)\right\} = \frac{\partial}{\partial \gamma}\left\{\frac{\gamma}{4}\left(q_H - \frac{(\rho c(1-\lambda))^2}{q_H(1-\gamma)^2}\right)\right\} = \frac{1}{4q_H}\underbrace{\left(q_H^2 - \frac{c^2(1+\gamma)(1-\lambda)^2}{(1-\gamma)^3}\right)}_{\ell^a(\gamma)}. \tag{EC.78}$$

Similarly, for $\ell^b(\gamma)$ we have

$$\frac{\partial r^b}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\gamma \eta_s p^b\left(1 - \frac{p^b}{q_H}\right)\right\} = \frac{\partial}{\partial \gamma}\left\{\frac{\gamma\eta_s}{4}\left(q_L - \frac{(\rho c(1-\omega_s))^2}{q_L(1-\gamma)^2}\right)\right\} = \frac{\eta_s}{4}\underbrace{\left(q_L - \left(1 + \frac{2\gamma}{1-\gamma}\right)\frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right)}_{\ell^b(\gamma)}. \tag{EC.79}$$

For $\ell^c(\gamma)$,

$$\frac{\partial r^c}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\gamma\eta_r p^c\left(1 - \frac{p^c}{q_H}\right)\right\} \tag{EC.80}$$

$$= \frac{\partial}{\partial \gamma}\left\{\gamma\eta_r q_H\left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta}\right)^2\right)\right\} \tag{EC.81}$$

$$= \gamma q_H \underbrace{\left(\frac{\partial \eta_r}{\partial \alpha}\gamma q_H\left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta}\right)^2\right) + \frac{2\eta_r}{\zeta}\left(\frac{(1-\lambda)c}{2q_H\zeta}\right)^2\frac{\partial \zeta}{\partial \alpha}\right)}_{\ell^c(\gamma)}, \tag{EC.82}$$

Note because $r^x(\gamma)$ is strictly concave in γ for each $x \in \{a, b, c\}$ (Lemma EC.12), $\ell^x(\gamma)$ strictly decreases in γ and $\ell^x(\gamma^x) = 0$ for each $x \in \{a, b, c\}$ by definition. Using these properties, it is straightforward to verify that $\gamma^b \le \gamma^a$ for all $\alpha \in [\frac{1}{2}, 1]$. Further, because $r^a(\gamma)$ and $r^b(\gamma)$ are both strictly concave in γ, we must have $\gamma^b \le \gamma^{ab} \le \gamma^a$ for all $\alpha \in [\frac{1}{2}, 1]$. It is then sufficient to show there exists $\bar{\alpha} \in [\tilde{\alpha}, 1)$ such that $\gamma^a < \gamma^c$ for $\alpha \ge \bar{\alpha}$. Using $\frac{\partial}{\partial \alpha}\eta_r = 1 - 2\lambda$ and $\frac{\partial}{\partial \alpha}\zeta = \frac{1}{2}((1-\lambda) - \gamma(2\lambda - 1))$, it can be shown algebraically that

$$\lim_{\alpha \to 1}\{\ell^c(\gamma) - \ell^a(\gamma)\} = 4c^2(1-\lambda)^2(2 - \gamma(2-\lambda)) + q_H^2((2 - \gamma(2-\lambda))^3 - 1) + \frac{c^2(1+\gamma)(1-\lambda)^2}{(1-\gamma)^3} > 0, \tag{EC.83}$$

where the strict inequality follows because $\gamma \in (0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. By continuity of $\ell^c(\gamma)$ and $\ell^a(\gamma)$ in α, it follows that there exists $\bar{\alpha} \in [\tilde{\alpha}, 1)$ such that $\ell^a(\gamma) < \ell^c(\gamma)$ for all $\gamma \in [0, \gamma^m]$ if $\alpha \ge \bar{\alpha}$. Therefore, for all $\alpha \ge \bar{\alpha}$ we have $\ell^a(\gamma^a) = 0 < \ell^c(\gamma^a)$, which implies $\gamma^a < \gamma^c$. Because

$\gamma^{ab} \leq \gamma^a$, we conclude $\gamma^{ab} < \gamma^c$ if $\alpha \geq \bar{\alpha}$ and $\phi = 0$. Because $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ for all $\phi \geq \bar{\phi}$ and $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\phi = 0$ as established at the beginning of the proof, we conclude $\gamma^*(0) \geq \gamma^*(\phi)$ for all $\phi \geq \bar{\phi}$ and $\alpha \geq \bar{\alpha}$. Finally, to see that the inequality is strict wherever $\gamma^*(\phi) < \gamma^m$, note $\gamma^*(\phi) < \gamma^m$ implies $\gamma^*(\phi) = \gamma^{ab} < \min\{\gamma^c, \gamma^m\} = \gamma^*(0)$.

Step 2. From part (ii), we have $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\phi = 0$ and $\alpha \geq \bar{\alpha}$. Further, since the thresholds γ_s^H and γ_s^L and the revenue functions $r^a(\gamma)$, $r^b(\gamma)$, and $r^c(\gamma)$ are each continuous in ϕ, it follows that there exists $\underline{\phi} > 0$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for all $\phi \leq \underline{\phi}$ and $\alpha \geq \bar{\alpha}$. The result follows because γ^c strictly decreases in ϕ (Lemma EC.14). □

EC.2.2. Proof of Proposition 2

PROPOSITION 2. *Let $R(\gamma^*)$ be the platform's revenue under the optimal commission rate γ^*. There exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

(i) Suppose information quality is low, $\alpha \leq \underline{\alpha}$. Then the platform's optimal revenue $R(\gamma^*)$ weakly increases in the switching cost ϕ on $\phi \in [0, \infty)$.

(ii) Suppose information quality is high, $\alpha \geq \bar{\alpha}$. Then there exists $\underline{\phi}$ such that the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the switching cost ϕ on $\phi \in [0, \underline{\phi}]$.

Proof. **(i)**. By Lemma EC.15, there exists $\underline{\alpha}$ such that if $\alpha \leq \underline{\alpha}$, then $\gamma^* = \min\{\gamma^a, \gamma_s^H, \gamma^m\}$ for all $\phi \geq 0$ and $r^b(\gamma^*)^+ = 0$. That is, the low information quality ensures that type-L sellers do not transact at all and it is optimal for the platform to choose a commission rate where type-H sellers fully transact online as the disintermediation threshold γ_s^H is sufficiently large (e.g., see Figure 1). Pick $\alpha \leq \underline{\alpha}$. It is straightforward to show that because γ^a, γ_s^H, $r^a(\gamma)$ and $r^c(\gamma)$ are all continuous in ϕ, so is $R(\gamma^*)$. It remains to show $R(\gamma^*)$ weakly increases in ϕ in three separate cases: $\gamma^* = \gamma^a$, $\gamma^* = \gamma_s^H$ and $\gamma^* = \gamma^m$.

<u>Case I:</u> $\gamma^* = \gamma^a$. Following the proof of Lemma EC.15, we have $r^b(\gamma)^+ = 0$ for all $\gamma \geq 0$ if $\alpha \leq \underline{\alpha}$, which implies $R(\gamma) = \mu r^a(\gamma)$. Then

$$\left.\frac{dR}{d\phi}\right|_{\gamma=\gamma^a} = \mu\left(\frac{\partial r^a}{\partial \gamma}\frac{d\gamma^*}{d\phi} + \frac{\partial r^a}{\partial \phi}\right)\bigg|_{\gamma=\gamma^a} = \mu \left.\frac{\partial r^a}{\partial \phi}\right|_{\gamma=\gamma^a} = 0. \tag{EC.84}$$

The first equality follows from taking the total derivative with respect to ϕ, the second equality follows from the envelope theorem because γ^a is the unconstrained maximizer of r^a, and the third equality follows because r^a is independent of ϕ (Lemma EC.12). Hence $R(\gamma^*)$ is independent of ϕ when $\gamma^* = \gamma^a$.

Case II: $\gamma^* = \gamma^m$. In this case, we again obtain (EC.84), except the second equality holds because $\frac{d}{d\phi}\gamma^* = 0$ for $\gamma^* = \gamma^m$ instead of by the envelope theorem. Hence $R(\gamma^*)$ is independent of ϕ when $\gamma^* = \gamma^m$.

Case III: $\gamma^* = \gamma_s^H$. Because $\frac{\partial}{\partial\phi} r^a = 0$ (Lemma EC.12), we have

$$\left.\frac{dR}{d\phi}\right|_{\gamma=\gamma_s^H} = \mu\left.\left(\frac{\partial r^a}{\partial\gamma}\frac{d\gamma^*}{d\phi} + \frac{\partial r^a}{\partial\phi}\right)\right|_{\gamma=\gamma_s^H} = \mu\left.\frac{\partial r^a}{\partial\gamma}\frac{d\gamma^*}{d\phi}\right|_{\gamma=\gamma_s^H} > 0. \tag{EC.85}$$

To see why the strict inequality holds, note γ_s^H strictly increases in ϕ (Lemma EC.14) and $\gamma^* = \gamma_s^H$ implies we must have $\frac{\partial}{\partial\gamma} r^a > 0$ at $\gamma = \gamma_s^H$. We conclude that $R(\gamma^*)$ strictly increases in ϕ if $\gamma^* = \gamma_s^H$. Statement (i) thus follows.

$\boldsymbol{(ii)}$. By Lemma EC.15 and the continuity of the thresholds γ_s^H and γ_s^L in ϕ, there exists $\underline{\phi} > 0$ such that $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for $\alpha \ge \bar{\alpha}$ and $\phi \le \underline{\phi}$. Pick $\alpha \ge \bar{\alpha}$. We show $R(\gamma^*)$ strictly decreases in ϕ at each $\phi \in [0, \underline{\phi}]$ by considering $\gamma^* = \gamma^c$ and $\gamma = \gamma^m$ as separate cases.

Case I: $\gamma^* = \gamma^c$. In this case, we have

$$\left.\frac{dR}{d\phi}\right|_{\gamma=\gamma^c} = \mu\left.\left(\frac{\partial r^c}{\partial\gamma}\frac{d\gamma^*}{d\phi} + \frac{\partial r^c}{\partial\phi}\right)\right|_{\gamma=\gamma^c} = \mu\left.\frac{\partial r^c}{\partial\phi}\right|_{\gamma=\gamma^c}, \tag{EC.86}$$

where the first equality follows because $R(\gamma^*) = \mu r^c(\gamma^*)$ for $\gamma^* = \gamma^c$, and the second equality follows by the envelope theorem because γ^c is the maximizer of $r^c(\gamma)$. Next, using the expressions for $r^c(\gamma)$ and p^c from Lemma EC.3, we have

$$\frac{\partial r^c}{\partial\phi} = \frac{\partial}{\partial\phi}\left\{\eta_r\gamma p^c\left(1 - \frac{p^c}{q_H}\right)\right\} \tag{EC.87}$$

$$= \frac{\partial}{\partial\phi}\left\{\eta_r\gamma q_H\left(\frac{1}{4} - \left(\frac{2(1-\lambda)c + \eta_s\phi}{4q_H\zeta}\right)^2\right)\right\} \tag{EC.88}$$

$$= -\frac{\eta_s\eta_r\gamma(2(1-\lambda)c + \eta_s\phi)}{8q_H\zeta^2} \tag{EC.89}$$

$$< 0. \tag{EC.90}$$

It follows that $R(\gamma^*)$ strictly decreases in ϕ if $\gamma^* = \gamma^c$.

Case II: $\gamma^* = \gamma^m$. Similar to Case I, we have

$$\left.\frac{dR}{d\phi}\right|_{\gamma=\gamma^*} = \mu\left.\left(\frac{\partial r^c}{\partial\gamma}\frac{d\gamma^m}{d\phi} + \frac{\partial r^c}{\partial\phi}\right)\right|_{\gamma=\gamma^m} = \mu\left.\frac{\partial r^c}{\partial\phi}\right|_{\gamma=\gamma^m} < 0, \tag{EC.91}$$

where the second equality follows because $\frac{d}{d\phi}\gamma^m = 0$ and the strict inequality follows from (EC.90). Therefore, $R(\gamma^*)$ strictly decreases in ϕ if $\gamma^* = \gamma^m$. □

EC.3. Proofs for Section 4: Optimal Information Quality

EC.3.1. Proof of Lemma 4

There exist thresholds $\underline{\phi} > 0$ and $\bar{\phi} \geq \underline{\phi}$ such that the following statements hold.

(i) Suppose the switching cost is high, $\phi > \bar{\phi}$. Then the platform's optimal revenue $R(\gamma^*)$ weakly increases in information quality α on $\alpha \in [\frac{1}{2}, 1]$.

(ii) Suppose the switching cost is low, $\phi \leq \underline{\phi}$. Then there exists $\underline{\alpha} \in (\frac{1}{2}, 1]$, $\bar{\alpha} \in [\underline{\alpha}, 1)$ and $\bar{\lambda} \in [\frac{1}{2}, 1)$ such that the platform's optimal revenue $R(\gamma^*)$ weakly increases in α on $\alpha \in [\frac{1}{2}, \underline{\alpha}]$ for all $\lambda \in [\frac{1}{2}, 1]$ and strictly decreases in α on $\alpha \in [\bar{\alpha}, 1]$ if $\lambda \geq \bar{\lambda}$.

Proof. $\boldsymbol{(i)}$. By Lemma EC.14, γ_s^H and γ_s^L are both strictly increasing in ϕ for each $\alpha \in [\frac{1}{2}, 1]$. It follows that there exists $\bar{\phi}$ such that $\gamma_s^H \geq \gamma^m$ and $\gamma_s^L \geq \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, by Lemma EC.11, for each $\phi \geq \bar{\phi}$ the platform's revenue is given by $R(\gamma) = \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+$ for all $\alpha \in [\frac{1}{2}, 1]$ and $\gamma \in [0, \gamma^m]$. Let γ^* be the maximizer of $R(\gamma)$ on $\gamma \in [0, \gamma^m]$. Because $r^b(\gamma)$ increases in α for each $\gamma \in [0, \gamma^m]$ such that $r^b(\gamma) > 0$ (Lemma EC.12), there exists $\tilde{\alpha} \in [\frac{1}{2}, 1]$ such that $r^b(\gamma^*) > 0$ if and only if $\alpha > \tilde{\alpha}$. Note $r^a(\gamma)$ is independent of α. Therefore, for each $\alpha \leq \tilde{\alpha}$, $R(\gamma^*)$ is also independent of α. Next, for each $\alpha > \tilde{\alpha}$, we consider two further cases: $\gamma^* < \gamma^m$ and $\gamma^* = \gamma^m$. If $\gamma^* < \gamma^m$, then

$$\left.\frac{dR}{d\alpha}\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial R}{\partial \gamma}\frac{d\gamma}{d\alpha} + \frac{\partial R}{\partial \alpha}\right)\right|_{\gamma=\gamma^*} = \left.\frac{\partial R}{\partial \alpha}\right|_{\gamma=\gamma^*} = (1-\mu)\left.\frac{\partial r^b}{\partial \alpha}\right|_{\gamma=\gamma^*} > 0$$

where the second equality follows from the envelope theorem, the third equality follows because $\frac{\partial}{\partial \alpha} r^a = 0$, and the strictly inequality follows because $\frac{\partial}{\partial \alpha} r^b > 0$, as established above. If $\gamma^* = \gamma^m$, then $\frac{\partial}{\partial \alpha}\gamma = 0$ at $\gamma = \gamma^m$, and we again obtain $\frac{d}{d\alpha} R > 0$ at $\gamma = \gamma^*$.

$\boldsymbol{(ii)}$. The proof proceeds in three steps. First, we show that for $\gamma \in [0, \gamma^m]$ and $\phi = 0$, r^c strictly decreases in α for all $\lambda \geq \bar{\lambda} = 0.52$. Second, we address the upper threshold $\bar{\alpha}$. Third, we address the lower threshold $\underline{\alpha}$.

Step 1. Using $\gamma \leq \frac{1}{2}$, we have the following lower bound on ζ:

$$\zeta = (1-\eta_s)(1-\gamma) + \frac{\eta_s}{2}(1-\gamma+\omega_s) \geq \frac{1-\eta_s}{2} + \frac{\eta_s}{2}\left(\frac{1}{2} + \omega_s\right) = \frac{1}{4}(1+\alpha+\lambda) \geq \frac{3+2\lambda}{8}. \quad \text{(EC.92)}$$

Suppose $\phi = 0$. Then

$$\frac{\partial r^c}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left\{\eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2 q_H \zeta}\right)^2\right)\right\} \quad \text{(EC.93)}$$

$$= \frac{\partial \eta_r}{\partial \alpha} \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H \zeta} \right)^2 \right) + \frac{2\eta_r \gamma q_H}{\zeta} \left(\frac{(1-\lambda)c}{2q_H \zeta} \right)^2 \frac{\partial \zeta}{\partial \alpha} \tag{EC.94}$$

$$\leq 4c\gamma \left((1-2\lambda) \left(\frac{1}{4} - \left(\frac{1-\lambda}{8\zeta} \right)^2 \right) + \frac{2\eta_r}{\zeta} \left(\frac{1-\lambda}{8\zeta} \right)^2 \frac{1-\lambda}{2} \right) \tag{EC.95}$$

$$\leq 4c\gamma \left((1-2\lambda) \left(\frac{1}{4} - \left(\frac{1-\lambda}{3+2\lambda} \right)^2 \right) + \frac{4}{3+2\lambda} \left(\frac{1-\lambda}{3+2\lambda} \right)^2 (1-\lambda) \right). \tag{EC.96}$$

The third line follows because $\frac{\partial}{\partial \alpha}\eta_r = 1-2\lambda$, $\frac{\partial}{\partial \alpha}\zeta = \frac{1}{2}((1-\lambda)-\gamma(2\lambda-1)) \leq \frac{1}{2}(1-\lambda)$ and $q_H \geq 4c$, and the fourth line follows from the lower bound on ζ from (EC.92). Next, let $\bar{\lambda} = 0.52$. Plugging in $\lambda = 0.52$, we obtain the bound

$$4c\gamma \left((1-2\lambda) \left(\frac{1}{4} - \left(\frac{1-\lambda}{3+2\lambda} \right)^2 \right) + \frac{4}{3+2\lambda} \left(\frac{1-\lambda}{3+2\lambda} \right)^2 (1-\lambda) \right) \leq -\frac{c\gamma}{100} < 0. \tag{EC.97}$$

Next, note that the upper bound on $\frac{\partial}{\partial \alpha} r^c$ in (EC.96) strictly decreases in λ on $\lambda \in [\frac{1}{2}, 1]$, and is strictly negative for $\lambda = 0.52$. It follows that for any $\gamma \in [0, \gamma^m]$ and $\phi = 0$, $r^c(\gamma)$ strictly decreases in α for all $\lambda \geq \bar{\lambda} = 0.52$.

Step 2. It follows from Lemma EC.15 and the continuity of γ_s^H and γ_s^L that there exists $\underline{\phi} > 0$ and $\bar{\alpha} < 1$ such that for each $\phi \leq \underline{\phi}$, $\gamma^* = \min\{\gamma^c, \gamma^m\}$ for all $\alpha \in [\bar{\alpha}, 1]$. Further, following the proof of Lemma EC.15, we also have $\gamma^c > \gamma_s^L$ for $\phi \leq \underline{\phi}$ and $\alpha \in [\bar{\alpha}, 1]$. Now let $\lambda \geq \bar{\lambda}$ and consider two cases: $\gamma^* < \gamma^m$ and $\gamma^* = \gamma^m$. If $\gamma^* < \gamma^m$, then $\gamma^* = \gamma^c$, and we have

$$\left.\frac{dR}{d\alpha}\right|_{\gamma=\gamma^c} = \mu \left.\frac{dr^c}{d\alpha}\right|_{\gamma=\gamma^c} = \mu \left.\frac{\partial r^c}{\partial \alpha}\right|_{\gamma=\gamma^c} < 0. \tag{EC.98}$$

The first equality follows because $\gamma^c > \gamma_s^L$ implies $R(\gamma^c) = \mu r^c(\gamma^c)$ (Lemma EC.11), the second equality follows from the envelope theorem because γ^c is a maximizer of $r^c(\gamma)$, and the strict inequality follows from Step 1. It follows that if $\phi \leq \underline{\phi}$ and $\lambda \geq \bar{\lambda}$, then $R(\gamma^*)$ strictly decreases in α on $\alpha \in [\bar{\alpha}, 1]$. In the case where $\gamma^* = \gamma^m$, we have $\frac{d}{d\alpha}\gamma^* = 0$, from which (EC.98) again follows.

Step 3. For the lower threshold $\underline{\alpha}$, first suppose $\phi = 0$ and $\alpha = \frac{1}{2}$. Note that $\alpha = \frac{1}{2}$ implies $\omega_s = \omega_r$, and thus $\gamma_s^H = \gamma_r^H$. By Lemma EC.10 we have $\gamma_r^H = 1 - \omega_r > \gamma^m$, which implies $\gamma_s^H > \gamma^m$. Further, because γ_s^H strictly increases in ϕ by Lemma EC.14 and is continuous in α, there exists $\underline{\alpha} \in [\frac{1}{2}, \bar{\alpha}]$ such that $\gamma_s^H > \gamma^m$ holds for all $\phi \leq \underline{\phi}$ and $\alpha \leq \bar{\alpha}$. It follows from Lemma EC.11 that the platform's revenue is given by $R(\gamma) = \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+$. Let γ^* be the maximizer of $R(\gamma)$. Note $r^a(\gamma)$ is independent of α and $r^b(\gamma)$ strictly increases in α (Lemma EC.12). The result that $R(\gamma^*)$ weakly increases in α then follows by an identical argument to the proof of statement (i). □

EC.3.2. Proof of Proposition 3

We first present two supporting results that are used to prove Proposition 3: Lemma EC.16 presents comparative statics with respect to α for different candidate solutions for the optimal commission rate γ^*, and Lemma EC.17 proves a useful property that holds at the optimal information quality and commission rate (α^*, γ^*).

LEMMA EC.16. *The following statements hold.* (i) γ^a *is independent of* α *for all* ϕ, (ii) γ^b *strictly increases in* α *for all* ϕ, (iii) γ^c *strictly increases in* α *if* $\phi < \bar{\phi}$ *for some* $\bar{\phi} > 0$, (iv) γ_s^H *strictly decreases in* α, *and* (v) γ_s^L *strictly decreases in* α *if* $\phi < \bar{\phi}$ *for some* $\bar{\phi} > 0$.

Proof. $\boldsymbol{(i)}$. Note $r^a(\gamma)$ is independent of α (Lemma EC.12), which implies the maximizer γ^a is also independent of α.

$\boldsymbol{(ii)}$. Note $r^b(\gamma)$ does not depend on ϕ, which implies $\frac{d}{d\alpha}\gamma^b$ has the same sign for all ϕ. By the implicit function theorem, we have

$$\frac{d\gamma^b}{d\alpha} = -\left(\frac{\partial^2 r^b}{\partial\gamma\partial\alpha}\right)\left(\frac{\partial^2 r^b}{\partial\gamma^2}\right)^{-1}\Bigg|_{\gamma=\gamma^b}. \tag{EC.99}$$

By Lemma EC.12, $r^b(\gamma)$ is strictly concave in γ, which implies $\frac{\partial^2}{\partial\gamma^2}r^b < 0$ at $\gamma = \gamma^b$. It follows that $\frac{d}{d\alpha}\gamma^b$ has the same sign as $\frac{\partial^2}{\partial\gamma\partial\alpha}r^b$ at $\gamma = \gamma^b$. Next, using the expressions in Lemmas EC.3 and EC.11 and plugging p^b into r^b, we have

$$\frac{\partial r^b}{\partial\gamma} = \frac{\partial}{\partial\gamma}\left\{\frac{\eta_s\gamma}{4}\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right)\right\} \tag{EC.100}$$

$$= \frac{\eta_s}{4}\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right) - \frac{\eta_s\gamma}{4}\left(\frac{2c^2(1-\omega_s)^2}{q_L(1-\gamma)^3}\right) \tag{EC.101}$$

$$= \frac{\eta_s}{4}\left(q_L - \left(1 + \frac{2\gamma}{1-\gamma}\right)\frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2}\right). \tag{EC.102}$$

Note η_s and ω_s both strictly increase in α by Lemma EC.1. Thus, by inspecting (EC.102) it can be verified that $\frac{\partial}{\partial\gamma}r^b$ also strictly increases in α. It follows that $\frac{\partial^2}{\partial\gamma\partial\alpha}r^b > 0$, and thus γ^b strictly increases in α.

$\boldsymbol{(iii)}$. By the implicit function theorem, we have

$$\frac{d\gamma^c}{d\alpha} = -\left(\frac{\partial^2 r^c}{\partial\gamma\partial\alpha}\right)\left(\frac{\partial^2 r^c}{\partial\gamma^2}\right)^{-1}\Bigg|_{\gamma=\gamma^c}. \tag{EC.103}$$

By Lemma EC.12, r^c is strictly concave in γ, which implies $\frac{\partial^2}{\partial\gamma^2}r^c < 0$ at $\gamma = \gamma^c$. It follows that $\frac{d}{d\alpha}\gamma^c$ has the same sign as $\frac{\partial^2}{\partial\gamma\partial\alpha}r^c$ at $\gamma = \gamma^c$. Next, note

$$r^c = \eta_r \gamma q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s\phi}{4q_H\zeta}\right)^2\right), \tag{EC.104}$$

where $\zeta = \eta_r(1-\gamma) + \eta_s\frac{1-\gamma+\omega_s}{2}$. Differentiating in γ, we have

$$\begin{aligned}\frac{\partial r^c}{\partial\gamma} &= \eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s\phi}{4q_H\zeta}\right)^2\right) - \frac{2\eta_r q_H\gamma}{\zeta^3}\left(\frac{(1-\lambda)c}{2q_H} + \frac{\eta_s\phi}{4q_H}\right)^2\left(1 - \frac{\eta_s}{2}\right)\\ &= \eta_r \underbrace{q_H\left(\frac{1}{4} - \frac{h^2}{q_H^2}\left(\frac{1}{\zeta^2} + 2\frac{\gamma}{\zeta^3}\left(1-\frac{\eta_s}{2}\right)\right)\right)}_{g(\alpha,\gamma)}\end{aligned}$$

where $h = \frac{1}{4}(2c(1-\lambda)+\eta_s\phi)$ and $g(\alpha,\gamma)$ is defined as above for convenience. Differentiating again in α and evaluating at $\gamma = \gamma^c$ yields

$$\left.\frac{\partial^2 r^c}{\partial\gamma\partial\alpha}\right|_{\gamma=\gamma^c} = \left.\left(\eta_r\frac{\partial g}{\partial\alpha} + \frac{\partial\eta_r}{\partial\alpha}g\right)\right|_{\gamma=\gamma^c} = \left.\left(\eta_r\frac{\partial g}{\partial\alpha}\right)\right|_{\gamma=\gamma^c}. \tag{EC.105}$$

The second equality above follows because γ^c is the maximizer of $r^c(\gamma)$, which implies $\frac{\partial}{\partial\gamma}r^c = 0$ at $\gamma = \gamma^c$. By (EC.104), this implies $g(\alpha,\gamma^c) = 0$. It remains to show $\frac{\partial}{\partial\alpha}g(\alpha,\gamma^c) > 0$. Note by inspection that $g(\alpha,\gamma)$ is increasing in ζ and η_s for each γ; therefore, it suffices to show $\frac{\partial}{\partial\alpha}\zeta > 0$ and $\frac{\partial}{\partial\alpha}\eta_s > 0$. Using the expressions for η_s and ω_s (Lemma EC.1), it can be shown algebraically that

$$\frac{\partial\zeta}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left\{(1-\gamma)(1-\eta_s) + \frac{\eta_s(1-\gamma+\omega_s)}{2}\right\} = \frac{1}{2}(\gamma(2\lambda-1)+(1-\lambda)) \geq 0, \tag{EC.106}$$

where the final inequality follows because $\lambda \geq \frac{1}{2}$. By Lemma EC.1, we also have $\frac{\partial}{\partial\alpha}\eta_s = 2\lambda - 1 > 0$ when $\lambda \geq \frac{1}{2}$. Because $\frac{\partial}{\partial\alpha}\zeta > 0$ and $\frac{\partial}{\partial\alpha}\eta_s > 0$ for $\phi = 0$ and $\lambda \geq \frac{1}{2}$, we conclude $g(\alpha,\gamma^c)$ strictly increases in α. This establishes that $\frac{d}{d\alpha}\gamma^c < 0$ for $\phi = 0$. The existence of the threshold $\bar{\phi} > 0$ follows because $r^c(\gamma)$ is continuous in ϕ, which by (EC.103) implies $\frac{d}{d\alpha}\gamma^c$ is continuous in ϕ.

(iv). By Lemma EC.10, γ_s^H is the unique solution to $\pi^a(p^a) - \pi^c(p^c) = 0$. For convenience, define the function $\pi^-(\alpha,\gamma) := \pi^a(p^a) - \pi^c(p^c)$. By definition, for each $\alpha \in [\frac{1}{2}, 1]$ we have $\pi^-(\alpha,\gamma_s^H) = 0$. It is straightforward to verify that $\pi^a(p^a)$ and $\pi^c(p^c)$ are both differentiable in α and γ. Therefore, we can differentiate $\pi^-(\alpha,\gamma)$ with respect to α to obtain

$$\left.\frac{d\pi^-}{d\alpha}\right|_{\gamma=\gamma_s^H} = \left.\left(\frac{\partial\pi^-}{\partial\gamma}\frac{d\gamma_s^H}{d\alpha} + \frac{\partial\pi^-}{\partial\alpha}\right)\right|_{\gamma=\gamma_s^H} = 0. \tag{EC.107}$$

Because $\pi^-(\alpha,\gamma)=0$ strictly decreases in γ on the interval $[\underline{\gamma}_s^H, \bar{\gamma}_s^H]$ by the proof of Lemma EC.10, we have $\frac{\partial}{\partial\gamma}\pi^- < 0$ at $\gamma=\gamma_s^H$. We can therefore re-arrange for $\frac{d}{d\alpha}\gamma_s^H$ to obtain

$$\frac{d}{d\alpha}\gamma_s^H = -\left(\frac{\partial\pi^-}{\partial\alpha}\right)\left(\frac{\partial\pi^-}{\partial\gamma}\right)^{-1}\Bigg|_{\gamma=\gamma_s^H}. \tag{EC.108}$$

Because $\frac{\partial}{\partial\gamma}\pi^- < 0$, $\frac{d}{d\alpha}\gamma_s^H < 0$ holds if $\frac{\partial}{\partial\alpha}\pi^- < 0$; we show the latter inequality holds. Note

$$\frac{\partial\pi^-}{\partial\alpha} = \frac{d\pi^a(p^a)}{d\alpha} - \frac{d\pi^c(p^c)}{d\alpha} = -\frac{d\pi^c(p^c)}{d\alpha}, \tag{EC.109}$$

where the second equality follows because π^a and p^a are both independent of α, and the right hand side is the total derivative of $\pi^c(p^c)$ with respect to α. Further, we have

$$\frac{d\pi^c}{d\alpha}\bigg|_{(\gamma,p)=(\gamma_s^H,p^c)} = \left(\frac{\partial\pi^c}{\partial p}\frac{\partial p^c}{\partial\alpha} + \frac{\partial\pi^c}{\partial\alpha}\right)\bigg|_{(\gamma,p)=(\gamma_s^H,p^c)} = \frac{\partial\pi^c}{\partial\alpha}\bigg|_{(\gamma,p)=(\gamma_s^H,p^c)},$$

because $\frac{\partial}{\partial p}\pi^c = 0$ at $p=p^c$ by the envelope theorem. Therefore, it remains to show $\frac{\partial}{\partial\alpha}\pi^c > 0$ at $(\gamma,p)=(\gamma_s^H,p^c)$. Writing out this partial derivative using the expression for $\pi^c(p)$, we have

$$\frac{\partial\pi^c}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left\{\eta_s\left(\omega_s b_s(p) - \phi - (1-\gamma)p\right)\left(1-\frac{p}{q_H}\right)\right\} \tag{EC.110}$$

$$= \frac{\partial\eta_s}{\partial\alpha}\left(\omega_s b_s(p) - \phi - (1-\gamma)p\right)\left(1-\frac{p}{q_H}\right) + \eta_s\frac{\partial\omega_s b_s(p)}{\partial\alpha}\left(1-\frac{p}{q_H}\right). \tag{EC.111}$$

Next, we show the expression in (EC.111) is strictly positive at $(\gamma,p)=(\gamma_s^H,p^c)$. Note we must have $\left(1-\frac{p^c}{q_H}\right) > 0$ because the type-H seller has positive demand at $p=p^c$. Note also that γ_s^H is the commission rate at which the seller is indifferent between transacting online and offline; using this fact it is straightforward to show that $(\omega_s b_s(p) - \phi - (1-\gamma)p) \geq 0$ and $b_s(p) \geq 0$ at $(\gamma,p)=(\gamma_s^H,p^c)$. Because $\frac{\partial}{\partial\alpha}\eta_s = 2\lambda - 1 > 0$, we conclude the first term in (EC.111) is positive. For the second term, note

$$\frac{\partial\omega_s b_s(p)}{\partial\alpha} = \frac{1}{2}\frac{\partial\omega_s}{\partial\alpha} = \frac{(1-\lambda)\lambda}{2(1-\lambda+(\alpha(2\lambda-1))^2)} > 0,$$

where the first equality follows using the expression for $b_s(p)$ (Lemma EC.2) and the strict inequality follows because $\lambda \in [\frac{1}{2}, 1]$. Therefore, $\frac{\partial}{\partial\alpha}\pi^c(p) > 0$ at $(\gamma,p)=(\gamma_s^H,p^c)$, as desired. We conclude γ_s^H strictly decreases in α.

(*v*). Let $\phi=0$. It follows from Lemma EC.10 that

$$\gamma_s^L = 1-\omega_s = \frac{(1-\alpha)(1-\lambda)}{\alpha(1-\lambda)+(1-\alpha)\lambda}. \tag{EC.112}$$

Differentiating in α,

$$\frac{d\gamma_s^L}{d\alpha} = -\frac{(1-\lambda)\lambda}{(\alpha(1-\lambda)+(1-\alpha)\lambda)^2} < 0. \tag{EC.113}$$

Therefore, γ_s^L strictly decreases in α on $\alpha \in [\frac{1}{2}, 1]$ when $\phi = 0$. Finally, to see that there exists a threshold $\bar{\phi} > 0$ such that $\frac{d}{d\alpha}\gamma_s^L < 0$ for $\phi \le \bar{\phi}$, note that $\frac{d}{d\alpha}\gamma_s^L$ can be shown to be continuous in ϕ using the fact that γ_s^L is the solution to $\pi^b(p^b) - \pi^d(p^d) = 0$, where $\pi^b(p^b)$ does not depend on ϕ and $\pi^d(p^d)$ is continuous in ϕ. □

Lemma EC.17 **(Optimal information quality and commission rate)**. *Suppose $\gamma^* < \gamma^m$ holds at the optimal information quality and commission rate (α^*, γ^*). Then $\gamma^* \in [\gamma_s^H, \gamma_s^L]$.*

Proof. We show neither $\gamma^* < \gamma_s^H$ nor $\gamma^* > \gamma_s^L$ can hold if $\gamma^* < \gamma^m$. By way of contradiction, suppose $\gamma^* < \gamma_s^H$ holds at (α^*, γ^*). Because $\gamma_s^H = 0$ at $\alpha = 1$, $\gamma^* < \gamma_s^H$ implies $\alpha^* < 1$. Further, because $r^a(\gamma)$ strictly increases in γ on $\gamma \in [0, \gamma^m]$ (Lemma EC.12), we must have $r^b(\gamma^*) > 0$; otherwise, we obtain a contradiction to $\gamma^* < \gamma_s^H$. Because γ_s^H is continuous and strictly decreasing in α (Lemma EC.16), there exists $\tilde{\alpha} > \alpha^*$ such that $\gamma_s^H = \gamma^*$ at $\alpha = \tilde{\alpha}$. To make dependence on α explicit, we slightly abuse notation and write $R(\alpha, \gamma)$ to denote the platform's revenue. Then we can write

$$R(\alpha^*, \gamma^*) = \mu r^a(\alpha^*, \gamma^*) + (1-\mu) r^b(\alpha^*, \gamma^*)^+ < \mu r^a(\tilde{\alpha}, \gamma^*) + (1-\mu) r^b(\tilde{\alpha}, \gamma^*)^+ = R(\tilde{\alpha}, \gamma^*), \tag{EC.114}$$

where the strict inequality follows because $r^a(\gamma)$ and $r^b(\gamma)$ are independent of and increasing in α, respectively (Lemma EC.12), and the two equalities follow by noting $\gamma^* \le \gamma_s^H$ at both α^* and $\tilde{\alpha}$, and applying the definition of $R(\gamma)$ from Lemma EC.11. Note (EC.114) contradicts the optimality of α^*, which implies $\gamma^* < \gamma_s^H$ cannot hold. Next, suppose $\gamma^* > \gamma_s^L$. Then

$$R(\alpha^*, \gamma^*) = \mu r^c(\alpha^*, \gamma^*) < \mu r^a(\alpha^*, \gamma^*) = \mu r^a\left(\tfrac{1}{2}, \gamma^*\right) = R\left(\tfrac{1}{2}, \gamma^*\right). \tag{EC.115}$$

The first equality follows because $\gamma^* > \gamma_s^L$, the strictly inequality follows by Lemma EC.13, the second equality follows because $r^a(\gamma)$ is independent of α, and the final equality follows because $\gamma_s^H \ge \gamma^m$ for $\alpha = \frac{1}{2}$. Note (EC.115) contradicts the optimality of α^*, and so $\gamma^* > \gamma_s^L$ cannot hold. The result follows. □

Proposition 3. *Let α^* be the platform's revenue-maximizing information quality when jointly optimized with the commission rate. There exist thresholds $\bar{\mu} \in [0, 1]$, $\bar{\phi} > 0$, $\underline{\alpha} \in (\frac{1}{2}, 1)$ and $\bar{\alpha} \in (\underline{\alpha}, 1)$ such that the following statements hold.*

(i) A no-information policy is optimal $\alpha^* = \frac{1}{2}$ *if the share of type-H sellers is large* $\mu > \bar{\mu}$ *and there is no switching cost* $\phi = 0$*.*

(ii) A partial-information policy is optimal $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ *if the share of type-H sellers is small* $\mu \leq \bar{\mu}$ *and there is no switching cost* $\phi = 0$*. Further,* α^* *strictly decreases in* μ *for all* $\mu \in [0, \bar{\mu}]$*.*

(iii) A full-information policy is optimal $\alpha^* = 1$ *for all* $\mu \in [0,1]$ *if the switching cost is high* $\phi \geq \bar{\phi}$*.*

Proof. The proof proceeds in five steps. First, we define two thresholds, α^b and $\tilde{\alpha}$, which are used in the remainder of the proof. Second, we show that if $\phi = 0$, then there exists $\bar{\mu} \in [0,1]$ such that the optimal information quality satisfies $\alpha^* \leq \min\{\alpha^b, \tilde{\alpha}\}$ if $\mu \geq \bar{\mu}$ and $\alpha^* \geq \max\{\alpha^b, \tilde{\alpha}\}$ if $\mu \leq \bar{\mu}$, which is used to prove statement (i). Third, we define the thresholds $\underline{\alpha}$ and $\bar{\alpha}$ and show $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if $\mu \leq \bar{\mu}$ and $\phi = 0$. Fourth, we show α^* is decreasing in μ when $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$, which combined with the third step proves statement (ii). Fifth, we prove statement (iii). Note that by Lemma EC.17, the platform's optimal policy (α^*, γ^*) satisfies $\alpha^* \in [\frac{1}{2}, 1]$ and $\gamma^* \in [\gamma_s^H, \gamma_s^L] \cup \gamma^m$; therefore, we restrict attention to those sets throughout the proof.

Step 1. Note $\alpha = \frac{1}{2}$ implies $\omega_s = \omega_r$ and thus $\gamma_r^H = \gamma_s^H$. Because $\gamma_r^H > \gamma^m$ by Lemma EC.7, and γ_s^H strictly decreases in α (Lemma EC.16), it follows there exists a unique threshold $\tilde{\alpha} \in (\frac{1}{2}, 1)$ such that $\gamma_s^H \geq \gamma^m$ if and only if $\alpha \leq \tilde{\alpha}$. Next, let $\gamma^*(\alpha)$ be the optimal commission rate for fixed α. We show there exists a unique threshold $\alpha^b \in (\frac{1}{2}, 1)$ such that $r^b(\gamma^*) \geq 0$ if and only if $\alpha \geq \alpha^b$. To see this, note

$$r^b(\gamma) = \frac{\eta_s \gamma}{4} \underbrace{\left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right)}_{h(\gamma)},$$

where for convenience we define $h(\gamma)$ to be the expression inside the parentheses. Next, differentiating h in α yields

$$\left.\frac{dh}{d\alpha}\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial h}{\partial \gamma}\frac{d\gamma^*}{d\alpha} + \frac{\partial h}{\partial \alpha} \right)\right|_{\gamma=\gamma^*}.$$

By inspection of h, we have $\frac{\partial}{\partial \gamma} h < 0$ and $\frac{\partial}{\partial \alpha} h > 0$ for any $\gamma \in [0, \gamma^m]$ and $\alpha \in [\frac{1}{2}, 1]$. Further, because $\phi = 0$, we have $\gamma_s^H = \gamma_s^L$, which combined with Lemma EC.17 implies $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$. Because γ_s^H decreases in α, we must have $\frac{d}{d\alpha}\gamma^* \leq 0$. It follows that $\frac{d}{d\alpha} h > 0$ at $\gamma = \gamma^*(\alpha)$ for all $\alpha \in [\frac{1}{2}, 1]$. Because $r^b(\gamma) \geq 0$ if and only if $h(\gamma) \geq 0$, we conclude there exists a unique threshold $\alpha^b \in [\frac{1}{2}, 1]$ such that $r^b(\gamma^*(\alpha)) \geq 0$ if and only if $\alpha \geq \alpha^b$. Lastly, it can be verified algebraically that $r^b(\gamma) < 0$ at $\alpha = \frac{1}{2}$ and $r^b(\gamma) > 0$ at $\alpha = 1$ for any $\gamma \in [0, \gamma^m]$, which implies $\alpha^b \in (\frac{1}{2}, 1)$.

Step 2. We now show there exists $\bar{\mu} \in [0,1]$ such that $\alpha^* \le \min\{\alpha^b, \tilde{\alpha}\}$ if $\mu > \bar{\mu}$ and $\alpha^* \ge \max\{\alpha^b, \tilde{\alpha}\}$ if $\mu \le \bar{\mu}$. There are two cases to consider: $\alpha^b \ge \tilde{\alpha}$ and $\alpha^b < \tilde{\alpha}$.

Case I: $\alpha^b \ge \tilde{\alpha}$. In this case, the platform's revenue as a function of α can be written as

$$R(\gamma^*) = \begin{cases} \mu r^a(\gamma^m), & \text{for } \alpha \in [\frac{1}{2}, \tilde{\alpha}), \\ \mu r^a(\gamma_s^H), & \text{for } \alpha \in [\tilde{\alpha}, \alpha^b), \\ \mu r^a(\gamma_s^H) + (1-\mu) r^b(\gamma_s^H)^+, & \text{for } \alpha \in [\alpha^b, 1]. \end{cases} \tag{EC.116}$$

Note $r^a(\gamma)$ is independent of α and strictly increasing in γ on $\gamma \in [0, \gamma^m]$ (Lemma EC.12) and γ_s^H is strictly decreasing in α (Lemma EC.16). Combining these with the revenue expression above imply either $\alpha^* \le \tilde{\alpha}$ or $\alpha^* \ge \alpha^b$ must hold for all $\mu \in [0,1]$. Because $r^a(\gamma^m)$ does not depend on α, it follows that $\alpha^* \le \tilde{\alpha}$ if and only if the following inequality holds

$$\max_{\alpha \ge \alpha^b} \left\{ \mu(r^a(\gamma_s^H) - r^a(\gamma^m)) + (1-\mu) r^b(\gamma_s^H)^+ \right\} \le 0. \tag{EC.117}$$

It remains to show (EC.117) holds if and only if $\mu > \bar{\mu}$ for some $\mu \in (0,1)$. Note if $\mu = 0$, then (EC.117) cannot hold because $\max_{\alpha \ge \alpha^b} r^b(\gamma_s^H) > 0$. If $\mu = 1$, then (EC.117) holds strictly because $\gamma_s^H < \gamma^m$ on $\alpha \ge \alpha^b$ and r^a is strictly increasing in γ. Lastly, note the argument in (EC.117) is strictly decreasing in μ for every value of α because $r^a(\gamma_s^H) \le r^a(\gamma^m)$ by Lemma EC.12, which implies the left hand side of (EC.117) is also strictly decreasing in μ. Because (EC.117) does not hold at $\mu = 0$, holds strictly at $\mu = 1$, and the left hand side is strictly decreasing in μ, we conclude there exists a unique $\bar{\mu} \in (0,1)$ such that (EC.117) holds if and only if $\mu > \bar{\mu}$. The result follows because $\tilde{\alpha} = \min\{\alpha^b, \tilde{\alpha}\}$ and $\alpha^b = \max\{\alpha^b, \tilde{\alpha}\}$ in this case.

Case II: $\alpha^b < \tilde{\alpha}$. In this case, the platform's revenue is

$$R(\gamma^*) = \begin{cases} \mu r^a(\gamma^m), & \text{for } \alpha \in [\frac{1}{2}, \alpha^b), \\ \mu r^a(\gamma^m) + (1-\mu) r^b(\gamma^m)^+, & \text{for } \alpha \in [\alpha^b, \tilde{\alpha}), \\ \mu r^a(\gamma_s^H) + (1-\mu) r^b(\gamma_s^H)^+, & \text{for } \alpha \in [\tilde{\alpha}, 1]. \end{cases} \tag{EC.118}$$

Note that because r^b strictly increases in α (Lemma EC.12) , either $\alpha^* \le \alpha^b$ or $\alpha^* \ge \tilde{\alpha}$ must hold for all $\mu \in [0,1]$. Similar to Case I, because $r^a(\gamma^m)$ does not depend on α, it follows that $\alpha^* \le \alpha^b$ holds if and only if

$$\max_{\alpha \ge \alpha'} \left\{ \mu(r^a(\gamma_s^H) - r^a(\gamma^m)) + (1-\mu) r^b(\gamma_s^H)^+ \right\} \le 0. \tag{EC.119}$$

Because $r^a(\gamma)$ strictly increases in γ on $\gamma \in [0, \gamma^m]$ and does not depend on α (Lemma EC.12), and because $\gamma_s^H \leq \gamma^m$ if $\alpha \geq \tilde{\alpha}$, it is straightforward to verify that (EC.119) holds only if $\mu = 1$. The result follows by setting $\bar{\mu} = 1$ and noting $\alpha^b = \min\{\alpha^b, \tilde{\alpha}\}$ and $\tilde{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$.

Step 3. We now show there exists $\underline{\alpha} \in [\frac{1}{2}, 1)$ and $\bar{\alpha} \in (\underline{\alpha}, 1)$ such that $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ if $\mu \leq \bar{\mu}$. Define $\underline{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$. Note the result that $\alpha^* \geq \underline{\alpha}$ if $\mu \leq \bar{\mu}$ follows immediately from Step 2. It remains to show the existence of $\bar{\alpha} < 1$ such that $\alpha^* \leq \bar{\alpha}$ for $\mu \leq \bar{\mu}$. We do so by showing $\alpha^* = 1$ cannot hold for any $\mu \in [0, 1]$. Note that if $\mu = 0$ and $\alpha = 1$, then trivially we have $\gamma_s^L = 0$ and thus $R(\alpha, \gamma) = 0$ for all $\gamma \in [0, \gamma^m]$, which implies $\alpha^* < 1$. Now let $\mu > 0$. Suppose by way of contradiction that $\alpha^* = 1$ and let γ^* be the corresponding optimal commission rate. Further, define $\alpha' = \frac{1}{2}$ and let γ' be the optimal commission rate under α'. Then we have

$$R(\alpha', \gamma') \geq \mu r^a(\alpha', \gamma') = \max_{\gamma \leq \gamma^m} \mu r^a(\alpha', \gamma) = \max_{\gamma \leq \gamma^m} \mu r^a(\alpha^*, \gamma) > \max_{\gamma \leq \gamma^m} \mu r^c(\alpha^*, \gamma) = R(\alpha^*, \gamma^*).$$

The relations above follow because $\gamma_s^H \geq \gamma^m$ at $\alpha = \frac{1}{2}$, which implies $R(\gamma) \geq \mu r^a(\gamma)$ for all $\gamma \in [0, \gamma^m]$, by definition of γ', because $r^a(\gamma)$ is independent of α, because $r^a(\gamma) > r^c(\gamma)$ for all $\gamma \leq \gamma^m$ (Lemma EC.13), and because $\alpha = 1$ and $\phi = 0$ imply $\gamma_s^H = \gamma_s^L = 0$. Note $R(\alpha', \gamma') > R(\alpha^*, \gamma^*)$ contradicts the optimality of (α^*, γ^*). Therefore, $\alpha^* = 1$ cannot hold at $\phi = 0$ for any $\mu > 0$. It follows that there exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that $\alpha^* \leq \bar{\alpha}$ for all $\mu \leq \bar{\mu}$.

Step 4. We now show that α^* strictly decreases in μ if $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$. From the revenue expressions in Step 2 and the definition of $\underline{\alpha} = \max\{\alpha^b, \tilde{\alpha}\}$, $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ implies the platform's optimal revenue is

$$R(\gamma_s^H) = \mu r^a(\gamma_s^H) + (1 - \mu) r^b(\gamma_s^H)^+. \tag{EC.120}$$

It can be shown that $R(\gamma_s^H)$ is continuous and differentiable in α using the definitions of $r^a(\gamma)$ and $r^b(\gamma)$ (Lemma EC.11) and γ_s^H (see proof of Lemma EC.16(iv)). Because α^* is a local maximizer of $R(\gamma_s^H)$, we have

$$\left.\frac{dR}{d\alpha}\right|_{(\alpha,\gamma)=(\alpha^*,\gamma_s^H)} = \left.\left(\mu \frac{dr^a}{d\alpha} + (1 - \mu) \frac{dr^b}{d\alpha}\right)\right|_{(\alpha,\gamma)=(\alpha^*,\gamma_s^H)} = 0. \tag{EC.121}$$

Then by the implicit function theorem,

$$\frac{d\alpha^*}{d\mu} = -\left.\left(\frac{d^2 R}{d\alpha d\mu}\right)\left(\frac{d^2 R}{d\alpha^2}\right)^{-1}\right|_{(\alpha,\gamma)=(\alpha^*,\gamma_s^H)}. \tag{EC.122}$$

Because α^* maximizes $R(\gamma_s^H)$ and $\alpha^* \in (\frac{1}{2}, 1)$, we have $\frac{d^2}{d\alpha^2}R < 0$ at $(\alpha, \gamma) = (\alpha^*, \gamma_s^H)$. Therefore, $\frac{d}{d\mu}\alpha^*$ has the same sign as $\frac{d^2}{d\alpha d\mu}R$. Further, because $r^a(\gamma)$, $r^b(\gamma)$, and γ_s^H do not depend on μ, we have

$$\frac{d^2 R}{d\alpha d\mu} = \frac{dr^a}{d\alpha} - \frac{dr^b}{d\alpha}. \tag{EC.123}$$

It remains to show $\frac{d}{d\alpha}r^a - \frac{d}{d\alpha}r^b < 0$ at (α^*, γ_s^H). First, because $\frac{\partial}{\partial\alpha}r^a = 0$ (Lemma EC.12), we have

$$\left.\frac{dr^a}{d\alpha}\right|_{\gamma=\gamma_s^H} = \left.\left(\frac{\partial r^a}{\partial \gamma}\frac{d\gamma_s^H}{d\alpha}\right)\right|_{\gamma=\gamma_s^H} < 0,$$

where the strict inequality follows because $\frac{d}{d\alpha}\gamma_s^H < 0$ (Lemma EC.16) and $\frac{\partial}{\partial\gamma}r^a > 0$ for all $\gamma \in [0, \gamma^m]$ (Lemma EC.12). Because $\frac{d}{d\alpha}r^a < 0$, it follows from (EC.121) that $\frac{d}{d\alpha}r^b > 0$ at (α^*, γ_s^H). Finally, re-arranging (EC.121) yields

$$\left.\left(\mu\left(\frac{dr^a}{d\alpha} - \frac{dr^b}{d\alpha}\right) + \frac{dr^b}{d\alpha}\right)\right|_{(\alpha,\gamma)=(\alpha^*,\gamma_s^H)} = 0. \tag{EC.124}$$

Because $\frac{d}{d\alpha}r^b > 0$, it follows that $\mu\left(\frac{d}{d\alpha}r^a - \frac{d}{d\alpha}r^b\right) < 0$, and thus $\frac{d}{d\alpha}r^a - \frac{d}{d\alpha}r^b < 0$ at (α^*, γ_s^H), as desired. We conclude α^* strictly decreases in μ if $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$.

Step 5. We now show statement (*iii*). Note γ_s^H strictly increases in ϕ for all $\alpha \in [\frac{1}{2}, 1]$ (Lemma EC.14), strictly decreases in α (Lemma EC.16), and $\gamma_s^L \geq \gamma_s^H$ (Lemma EC.11). It follows that there exists $\bar{\phi} > 0$ such that $\gamma_s^H \geq \gamma^m$ for all $\alpha \in [\frac{1}{2}, 1]$ if $\phi \geq \bar{\phi}$. Thus, for any $\phi \geq \bar{\phi}$, the platform's optimal revenue for fixed $\alpha \in [\frac{1}{2}, 1]$ is given by

$$R(\gamma^*) = \max_{\gamma \leq \gamma^m} \left\{\mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+\right\}.$$

Because $r^b(\gamma)$ strictly increases in α and $r^a(\gamma)$ is independent of α (Lemma EC.12), it follows that $R(\gamma^*)$ is strictly increasing in α for $\phi \geq \bar{\phi}$. The result follows. □

EC.3.3. Proof of Corollary 1

LEMMA EC.18. *If $\mu > \frac{2}{9}$, then the inequality $\mu r^a(\gamma) > \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+$ holds for all $\alpha \in [\frac{1}{2}, 1]$, $\gamma \in (0, \gamma^m]$ and $\phi \geq 0$.*

Proof. Before proving the lemma, we show the following bound holds for any $\gamma \in (0, \gamma^m]$:

$$\frac{r^b(\gamma)}{r^a(\gamma)} \leq \frac{2}{15}. \tag{EC.125}$$

To see that (EC.125) holds, note

$$r^b(\gamma) = \frac{q_L \eta_s \gamma}{4}\left(1 - \frac{c^2(1-\omega_s)^2}{q_L^2(1-\gamma)^2}\right) \tag{EC.126}$$

$$\leq \frac{q_L \eta_s \gamma}{4} \tag{EC.127}$$

$$\leq \frac{16}{15}\frac{q_L \eta_s \gamma}{4}\left(1 - \frac{c^2(1-\lambda)^2}{q_H^2(1-\gamma)^2}\right) \tag{EC.128}$$

$$\leq \frac{16}{15}\cdot\frac{1}{8}\frac{q_H \eta_s \gamma}{4}\left(1 - \frac{c^2(1-\lambda)^2}{q_H^2(1-\gamma)^2}\right) \tag{EC.129}$$

$$\leq \frac{2}{15} r^a(\gamma). \tag{EC.130}$$

The third line above follows because $\left(1 - \frac{c^2(1-\lambda)^2}{q_H^2(1-\gamma)^2}\right)$ is minimized at $\lambda = \frac{1}{2}$, $q_H = 4c$, and $\gamma = \frac{1}{2}$, which corresponds to a minimal value of $\frac{15}{16}$. The fourth line follows because $8q_L \leq q_H$ by Assumption 1. We can now write

$$\mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+ \leq \frac{8}{15}\mu r^a(\gamma) + (1-\mu)\frac{2}{15} r^a(\gamma) < \frac{8}{15}\mu r^a(\gamma) + \frac{7}{2}\mu \cdot \frac{2}{15} r^a(\gamma) = \mu r^a(\gamma),$$

where the first inequality follows by combining Lemma EC.13 and (EC.125), and the second inequality follows because $\mu > \frac{2}{9}$ implies $(1-\mu) \leq \frac{7\mu}{2}$. □

Corollary 1. *There exist thresholds $\bar{\mu} \in [0,1)$, $\underline{\phi} \geq 0$, and $\bar{\phi} > \underline{\phi}$ such that partial-information is optimal $\alpha^* \in (\frac{1}{2}, 1)$ if the switching cost is moderate $\phi \in [\underline{\phi}, \bar{\phi}]$ and the share of type-H sellers is large $\mu > \bar{\mu}$.*

Proof. The proof proceeds in two steps. First, we show that under the optimal policy (α^*, γ^*), $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ holds for all $\phi \geq 0$ if $\mu > \frac{2}{9}$. Second, we prove the main result. With a slight abuse of notation, we write the platform's revenue as $R(\alpha, \gamma)$ to make dependence on α explicit.

Step 1. We focus on the most general case where $\gamma^m > \gamma_s^L$ holds at $\alpha = \alpha^*$; the cases where $\gamma^m \leq \gamma_s^H$ and $\gamma^m \in (\gamma_s^H, \gamma_s^L)$ follow by parallel argument and are omitted. First, note $\gamma^* \notin (0, \gamma_s^H)$ follows directly from Lemma EC.17. It remains to show $\gamma^* \notin (\gamma_s^H, \gamma_s^L]$ and $\gamma^* \notin (\gamma_s^L, \gamma^m]$. Note any $\gamma \in (\gamma_s^H, \gamma_s^L]$ and $\mu > \frac{2}{9}$,

$$R(\alpha^*, \gamma) = \mu r^c(\alpha^*, \gamma) + (1-\mu)\mu r^b(\alpha^*, \gamma) < \mu r^a(\alpha^*, \gamma) \leq \mu r^a\left(\tfrac{1}{2}, \gamma^m\right) = R\left(\tfrac{1}{2}, \gamma^m\right),$$

where the first inequality follows from Lemma EC.18 and the second follows because $r^a(\gamma)$ is independent of α and increases in γ (Lemma EC.12). Hence, $\gamma^* \notin (\gamma_s^H, \gamma_s^L]$ for $\mu > \frac{2}{9}$. Similarly, by again using Lemma EC.18, for any $\gamma > \gamma_s^L$ and $\mu > \frac{2}{9}$ we have

$$R(\alpha^*, \gamma) = \mu r^c(\alpha^*, \gamma) < \mu r^a(\alpha^*, \gamma) \leq R(\tfrac{1}{2}, \gamma^m), \tag{EC.131}$$

which implies $\gamma^* \leq \gamma_s^L$. We have thus shown $\gamma^* = \gamma_s^H$ if $\mu > \frac{2}{9}$ and $\gamma_s^L < \gamma^m$. Because $\gamma_s^H \leq \gamma_s^L$ (Lemma EC.11), we conclude $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ if $\mu > \frac{2}{9}$.

Step 2. To begin, define α_0 to be the solution to $1 - \omega_s = \frac{q_L \lambda}{c}$, and note $\alpha_0 < 1$ since ω_s strictly increases in α and $\omega_s = 1$ at $\alpha = 1$ (Lemma EC.1). Further, define $\hat{\alpha} = \alpha_0 + \epsilon < 1$ for any $\epsilon \in (0, 1 - \alpha_0)$ and define $\hat{\phi} \geq 0$ to be the smallest switching cost such that $\gamma_s^H \geq \gamma^m$ for all $\phi \geq \hat{\phi}$ at $\alpha = \hat{\alpha}$. Note $\hat{\phi}$ exists because γ_s^H strictly increases in ϕ for all $\alpha \in [\frac{1}{2}, 1]$ (Lemma EC.14); let $\phi = \hat{\phi}$ be fixed in the remainder. To prove the corollary statement, it suffices to show there exists $\bar{\mu} < 1$ such that $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(\frac{1}{2}, \gamma^*(\frac{1}{2}))$ and $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(1, \gamma^*(1))$ both hold for all $\mu \geq \bar{\mu}$ at $\phi = \hat{\phi}$, i.e., the platform's revenue at $\alpha = \hat{\alpha}$ is strictly higher than the revenue at both $\alpha = \frac{1}{2}$ and $\alpha = 1$, under the corresponding optimal commission rate γ^* in each instance.

<u>Case I:</u> Comparison with $\alpha = \frac{1}{2}$. First, at $\alpha = \alpha_0$ we have

$$r^b(\alpha_0, \gamma^m) = \frac{q_L \eta_s \gamma}{4}\left(1 - \left(\frac{(1-\omega_s)c}{q_L(1-\gamma^m)}\right)^2\right) = \frac{q_L \eta_s \gamma}{4}\left(1 - \left(\frac{\lambda}{1-\gamma^m}\right)^2\right) \geq 0,$$

where the second equality follows by construction of α^0 and the inequality follows because $\gamma^m \leq 1 - \lambda$ by Assumption 2. For all $\mu > \frac{2}{9}$ we can now write

$$R(\hat{\alpha}, \gamma^*(\hat{\alpha})) \geq R(\hat{\alpha}, \gamma^m) = \mu r^a(\hat{\alpha}, \gamma^m) + (1-\mu) r^b(\hat{\alpha}, \gamma^m)^+ > \mu r^a(\hat{\alpha}, \gamma^m) = R\left(\tfrac{1}{2}, \gamma^m\right) = R\left(\tfrac{1}{2}, \gamma^*(\tfrac{1}{2})\right). \tag{EC.132}$$

To see that the strict inequality in (EC.132) holds, note that $r^b(\alpha^0, \gamma^m) \geq 0$ as established above, $\hat{\alpha} > \alpha_0$, and $r^b(\alpha, \gamma^m)$ increases in α because ω_s and ω_s both increase in α (Lemma EC.1). The second equality in (EC.132) holds because $r^a(\gamma)$ is independent of α, and the final equality follows because $\gamma_s^H \geq \gamma^m$ at $\alpha = \frac{1}{2}$ and $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ by Step 1. We conclude $R(\hat{\alpha}, \gamma^*(\hat{\alpha})) > R(\frac{1}{2}, \gamma^*(\frac{1}{2}))$ for $\mu > \frac{2}{9}$.

<u>Case II:</u> Comparison with $\alpha = 1$. First, note $\gamma_s^H < \gamma^m$ at $(\alpha, \phi) = (1, \hat{\phi})$. To see this, note $\gamma_s^H(\hat{\alpha}) = \gamma^m$ at $(\alpha, \phi) = (\hat{\alpha}, \hat{\phi})$ by definition of $\hat{\phi}$, γ_s^H is strictly decreasing in α (Lemma EC.16), and $\hat{\alpha} < 1$. Next, let $\mu = 1$. We can now write

$$R(1, \gamma^*(1)) = \mu r^a(1, \gamma_s^H) + (1-\mu) r^b(1, \gamma_s^H)^+ < \mu r^a(\hat{\alpha}, \gamma^m) \leq R(\hat{\alpha}, \gamma^m) \leq R(\hat{\alpha}, \gamma^*(\hat{\alpha})). \tag{EC.133}$$

The first equality in (EC.133) follows because $\gamma_s^H < \gamma^m$ implies $\gamma^* = \gamma_s^H$ by Step 1. To see that the strict inequality holds at $\mu = 1$, note $r^a(\gamma)$ is strictly increasing in γ and independent of α

(Lemmas EC.12 and EC.16), and $\gamma_s^H < \gamma^m$ as established above. We have thus shown $R(1,\gamma^*(1)) < R(\hat{\alpha},\gamma^*(\hat{\alpha}))$ holds for $\mu = 1$. It follows by continuity of $R(\alpha,\gamma)$ in μ that there exists $\bar{\mu} \in [\frac{2}{9}, 1)$ such that $R(1,\gamma^*(1)) < R(\hat{\alpha},\gamma^*(\hat{\alpha}))$ for all $\mu \geq \bar{\mu}$. Because $\mu \geq \frac{2}{9}$, it follows that $R(\hat{\alpha},\gamma^*(\hat{\alpha})) > R(\frac{1}{2},\gamma^*(\frac{1}{2}))$ and $R(\hat{\alpha},\gamma^*(\hat{\alpha})) > R(1,\gamma^*(1))$ for all $\mu \geq \bar{\mu}$ for $\phi = \hat{\phi}$, as desired. Finally, the thresholds $\underline{\phi}$ and $\bar{\phi}$ can be shown to exist using the continuity of $R(\alpha,\gamma)$ in ϕ. □

EC.4. Proofs for Section 5.1: Platform-Access Fees

In section EC.4.1, we first present several supporting results (Lemmas EC.19–EC.23) that are needed for the proof of Proposition 4. Throughout this section, we use $R_C(\alpha,\gamma)$ and $R_A(\alpha,\psi)$ to denote the platform's revenue under the commission and access fee mechanisms, respectively. For conciseness, define $R_C^* := R_C(\alpha^*,\gamma^*)$ and $R_A^* := R_A(\alpha^*,\psi^*)$, where (α^*,γ^*) and (α^*,ψ^*) are the optimal policies under commission and access fees, respectively.

EC.4.1. Preliminary Results for Proposition 4

Lemma EC.19 **(Characterization of access fees)**. *Let Π_0^i be a type-i seller's on-platform profit under a commission rate of $\gamma = 0$.*

(i) For each $\alpha \in [\frac{1}{2}, 1]$,

$$\Pi_0^H := \frac{q_H}{4}\left(1 - \frac{(1-\lambda)c}{q_H}\right)^2, \tag{EC.134a}$$

$$\Pi_0^L := \frac{\eta_s q_L}{4}\left(1 - \frac{(1-\omega_s)c}{q_L}\right)^2. \tag{EC.134b}$$

(ii) Π_0^H is independent of α, Π_0^L strictly increases in α, and $0 < \Pi_0^L < \Pi_0^H$ for all $\alpha \in [\frac{1}{2}, 1]$.

(iii) For each $\psi > 0$, the platform's revenue under access fees $R_A(\psi)$ is weakly increasing in α on $\alpha \in [\frac{1}{2}, 1]$.

(iv) The platform's optimal access fee satisfies $\psi^* \in \{\Pi_0^H, \Pi_0^L\}$, with corresponding optimal revenue $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$.

Proof. ***(i)***. By Lemma EC.3, the profit for the type-H and type-L sellers are given by $\pi^a(p^a)$ and $\pi^b(p^b)$, respectively. Thus, the on-platform earnings under access fees are given by setting $\gamma = 0$ in $\pi^a(p^a)$ and $\pi^b(p^b)$, which yields the expressions in statement (i).

(ii). By inspection, Π_0^H is independent of α. Because $\frac{\partial}{\partial\alpha}\eta_s > 0$ and $\frac{\partial}{\partial\alpha}\omega_s > 0$ (Lemma EC.1), we have $\frac{d}{d\alpha}\Pi_0^L > 0$. Finally, $0 < \Pi_0^L < \Pi_0^H$ follows from the expressions in part (i) and because $q_H \geq 8q_L$ by Assumption 1.

(iii). Define $\bar{\Pi}_0^L := \lim_{\alpha\to 1}\Pi_0^L$, and note $\bar{\Pi}_0^L < \Pi_0^H$ by part (ii). We consider three cases: $\psi \le \bar{\Pi}_0^L$, $\psi \in (\bar{\Pi}_0^L, \Pi_0^H]$, and $\psi > \Pi_0^H$. First, if $\psi \le \bar{\Pi}_0^L$, because Π_0^L strictly increases in α, for each $\psi \le \bar{\Pi}_0^L$ there exists $\alpha_L \in [\frac{1}{2}, 1]$ such the type-L seller joins the platform if and only if $\alpha \ge \alpha_L$. Further, the type-H seller joins for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, if $\psi \le \bar{\Pi}_0^L$, the platform's revenue under access fees is

$$R_A(\psi) = \begin{cases} \mu\psi, & \text{if } \alpha < \alpha_L, \\ \psi, & \text{if } \alpha \ge \alpha_L. \end{cases} \tag{EC.135}$$

Because $\mu \le 1$, $R_A(\psi)$ weakly increases in α. Next, if $\psi \in (\Pi_0^L, \Pi_0^H]$, then the type-$L$ seller does not join for any $\alpha \in [\frac{1}{2}, 1]$, which implies $R_A(\psi) = \mu\psi$ for all $\alpha \in [\frac{1}{2}, 1]$, and thus $R_A(\psi)$ is independent of α. Finally, if $\psi > \Pi_0^H$, then neither seller type joins, which implies $R_A(\psi) = 0$ for all $\alpha \in [\frac{1}{2}, 1]$. Therefore, in all three cases the platform's revenue is weakly increasing in α.

(iv). By part (i), if $\psi \le \Pi_0^L$ then both seller types join, which generates a revenue of ψ. If $\psi \in [\Pi_0^L, \Pi_0^H]$, then only the type-$H$ seller joins the platform, which generates revenue $\mu\psi$. It follows that $\psi^* \in \{\Pi_0^H, \Pi_0^L\}$ and thus $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$. □

LEMMA EC.20. *If $\phi \ge 0$, then the following inequalities hold for all $\alpha \in [\frac{1}{2}, 1]$:*

$$\frac{r^a(\gamma)}{\Pi_0^H} \le \frac{9\gamma}{7} \quad \gamma \in [0, \gamma^m], \tag{EC.136a}$$

$$\frac{r^b(\gamma)}{\Pi_0^L} \le \frac{1}{2-\gamma} \quad \gamma \in [0, \gamma^m]. \tag{EC.136b}$$

Further, if $\phi = 0$ and $\gamma \le 1 - \omega_s$, then the following inequality holds for all $\alpha \in [\frac{1}{2}, 1]$:

$$\frac{r^b(\gamma)}{\Pi_0^L} \le \begin{cases} \frac{1}{2-\gamma} & \gamma \in [0, 2-\sqrt{3}], \\ \gamma\frac{(1+\gamma)(1-3\gamma)}{(1-\gamma)^2(1-2\gamma)^2} & \gamma \in [2-\sqrt{3}, \gamma^m]. \end{cases} \tag{EC.137}$$

Proof. We first show that (EC.136a) holds. Note

$$\frac{1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2}{\left(1-\frac{(1-\lambda)c}{q_H}\right)^2} \le \frac{1-\left(\frac{(1-\lambda)c}{q_H}\right)^2}{\left(1-\frac{(1-\lambda)c}{q_H}\right)^2} \le \frac{9}{7}, \tag{EC.138}$$

where the second inequality above follows because $\lambda \ge \frac{1}{2}$ and $q_H \ge 4c$. Then we can write

$$r^a(\gamma) = \frac{q_H\gamma}{4}\left(1-\left(\frac{(1-\lambda)c}{q_H(1-\gamma)}\right)^2\right) \le \frac{q_H\gamma}{4}\cdot\frac{9}{7}\left(1-\frac{(1-\lambda)c}{q_H}\right)^2 = \frac{9\gamma}{7}\Pi_0^H, \tag{EC.139}$$

where the first and second equalities follow by definition of $r^a(\gamma)$ and Π_0^H, and the inequality follows from (EC.138). Next, we show (EC.136b) and (EC.137). In the case where $r^b(\gamma) \le 0$, (EC.136b) and (EC.137) follow trivially; we assume $r^b(\gamma) > 0$ for the remainder of the proof. Let $z = \frac{(1-\omega_s)c}{q_L}$. We first show $z \le 1-\gamma$ for all $\phi \ge 0$ and $z \ge 2\gamma$ if $\phi = 0$. To see $z \le 1-\gamma$ for $\phi \ge 0$, note $r^b(\gamma) > 0$ implies $q_L(1-\gamma) > (1-\omega_s)c$ (Lemma EC.3), which by definition of z implies $z \le 1-\gamma$. To see that $z \ge 2\gamma$ when $\phi = 0$, note

$$z = \frac{(1-\omega_s)c}{q_L} \ge \frac{(1-\omega_s)c}{(1-\lambda)c} \ge 2(1-\omega_s) \ge 2\gamma,$$

which follows because $q_L \le (1-\lambda)c$, $\lambda \ge \frac{1}{2}$, and $\gamma \le \gamma_s^L = 1-\omega_s \le 1-\omega_s$ at $\phi = 0$. Thus, $z \le 1-\gamma$ for $\phi \ge 0$ and $z \ge 2\gamma$ for $\phi = 0$. We now show (EC.137) assuming $\phi = 0$; (EC.136b) follows by a similar argument. First, using the defintion of z, we have

$$\frac{r^b(\gamma)}{\Pi_0^L} = \frac{\gamma q_L^2}{(q_L - (1-\omega_s)c)^2}\left(1 - \left(\frac{(1-\omega_s)c}{q_L(1-\gamma)}\right)^2\right) = \frac{\gamma}{(1-z)^2}\left(1 - \frac{z^2}{(1-\gamma)^2}\right). \tag{EC.140}$$

Differentiating in z yields

$$\frac{\partial}{\partial z}\left(\frac{r^b(\gamma)}{\Pi_0^L}\right) = \frac{\gamma}{(1-z)^4}\left((1-z)^2\left(-2\frac{z}{(1-\gamma)^2}\right) + 2\left(1 - \frac{z^2}{(1-\gamma)^2}\right)(1-z)\right) = \frac{2\gamma}{(1-z)^3}\left(1 - \frac{z}{(1-\gamma)^2}\right),$$

which implies the ratio (EC.140) strictly increases in z on $z \in [0, (1-\gamma)^2]$ and has a single maximizer at $z = (1-\gamma)^2 < 1-\gamma$. Because $z \in [2\gamma, 1-\gamma]$ when $\phi = 0$, we have two cases to consider: If $2\gamma < (1-\gamma)^2$, the maximizer of the ratio (EC.140) on the interval $z \in [2\gamma, 1-\gamma]$ is $z = (1-\gamma)^2$; if $2\gamma \ge (1-\gamma)^2$, the maximizer is $z = 2\gamma$. Note $2\gamma \ge (1-\gamma)^2$ if and only if $\gamma \ge 2-\sqrt{3}$. Plugging $z = 2\gamma$ and $z = (1-\gamma)^2$ into (EC.140) yields (EC.137), as desired. Lastly, in the case where $\phi \ge 0$, (EC.136b) follows by a similar argument where only $z \le 1-\gamma$ is assumed to hold. □

LEMMA EC.21. *The platform's optimal commission revenue is strictly smaller than the optimal access fee revenue, $R_C^* < R_A^*$, if at least one the following conditions holds at the optimal commission rate γ^*:*

(i) The type-H seller transacts offline with the $\sigma = s$ buyer, $\gamma^* > \gamma_s^H$.

(ii) The type-L seller does not transact with any buyer, $r^b(\gamma^*) \le 0$.

Proof. **(i)**. Note $\gamma_s^L \ge \gamma_s^H$ by Lemma EC.10. We therefore consider two cases: $\gamma^* > \gamma_s^L$ and $\gamma^* \le \gamma_s^L$. If $\gamma^* > \gamma_s^L$, then the platform's revenue under commission fees is given by $R_C(\gamma^*) = \mu r^c(\gamma^*)$ (Lemma EC.11). Then

$$R_C^* = \mu r^c(\gamma^*) < \mu r^a(\gamma^*) < \mu \Pi_0^H \le R_A^*, \tag{EC.141}$$

where the equality follows by Lemma EC.11 because $\gamma > \gamma_s^L$, and the next three inequalities follow from Lemmas EC.13, EC.20, and EC.19, in order. Now suppose $\gamma^* \in (\gamma_s^H, \gamma_s^L]$. Then there are two further cases to consider: $\gamma^* \leq \gamma_r^H$ and $\gamma^* > \gamma_r^H$.

Case I: $\gamma^* \leq \gamma_r^H$. Note by Lemma EC.11 the platform's optimal commission revenue in this case is given by $R_C^* = \mu r^c(\gamma^*) + (1-\mu) r^b(\gamma^*)^+$. We show $R_C^* \leq R_A^*$ by first establishing upper bounds on $r^b(\gamma^*)$ and $r^c(\gamma^*)$. First, for $r^b(\gamma^*)$ we have

$$r^b(\gamma^*) = \frac{q_L \eta_s \gamma^*}{4}\left(1 - \frac{c^2(1-\omega_s)^2}{q_L^2(1-\gamma)^2}\right) \leq \frac{q_L \eta_s \gamma^*}{4} \leq \frac{q_L \eta_s \gamma^m}{4} = \frac{q_L \eta_s}{8}.$$

For $r^c(\gamma^*)$,

$$\begin{aligned}
r^c(\gamma^*) &= \eta_r \gamma^* q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s \phi}{4 q_H \zeta}\right)^2\right) \\
&\leq \frac{\eta_r \gamma^* q_H}{4}\left(1 - \frac{c^2(1-\lambda)^2}{q_H^2 \zeta^2}\right) \\
&\leq \frac{\eta_r \gamma^* q_H}{4}\left(1 - \frac{(1-\lambda)c}{q_H \zeta}\right)^2 \cdot \frac{9}{7} \\
&\leq \frac{9}{7} \cdot \frac{q_H}{16}\left(1 - \frac{(1-\lambda)c}{q_H\zeta}\right)^2,
\end{aligned}$$

where the second line follows by setting $\phi = 0$, the third line from (EC.138), and the final line follows because $\gamma^* \leq \gamma^m = \frac{1}{2}$ and $\eta_r \leq \frac{1}{2}$. Combining the bounds above, we can then write

$$\begin{aligned}
\mu r^c(\gamma^*) + (1-\mu) r^b(\gamma^*)^+ &\leq \mu \cdot \frac{9}{7} \cdot \frac{q_H}{16}\left(1 - \frac{(1-\lambda)c}{q_H\zeta}\right)^2 + (1-\mu)\frac{q_L \eta_s}{8} \\
&\leq 2\max\left\{\mu \cdot \frac{9}{7} \cdot \frac{q_H}{16}\left(1 - \frac{(1-\lambda)c}{q_H\zeta}\right)^2, (1-\mu)\frac{q_L\eta_s}{8}\right\} \\
&\leq \max\left\{\mu \cdot \frac{9}{7} \cdot \frac{q_H}{8}\left(1 - \frac{(1-\lambda)c}{q_H\zeta}\right)^2, (1-\mu)\frac{q_L\eta_s}{4}\right\} \\
&\leq \max\left\{\mu \cdot \frac{q_H}{5}\left(1 - \frac{(1-\lambda)c}{q_H\zeta}\right)^2, (1-\mu)\frac{q_L\eta_s}{4}\right\} \\
&< \max\left\{\mu \Pi_0^H, \Pi_0^L\right\} \\
&= R_A^*.
\end{aligned}$$

In the first inequality, we used the fact that $x + y \leq 2\max\{x, y\}$ for any non-negative x and y. This completes the proof for the case $\gamma^* \leq \gamma_r^H$.

<u>Case II:</u> $\gamma^* > \gamma_r^H$. In this case, the platform's revenue is given by $R_C^* = (1-\mu)r^b(\gamma^*)^+$. Then we have

$$R_C^* = (1-\mu)r^b(\gamma^*) < (1-\mu)\Pi_0^L \leq \Pi_0^L \leq R_A^*,$$

where the first and third inequalities follow from Lemmas EC.20 and EC.19, respectively.

(ii). Because $r^b(\gamma^*) \leq 0$, only the type-H seller transacts online. Note that if $r^b(\gamma^*) \leq 0$ implies $\gamma^* \leq \gamma_r^H$, otherwise the platform's commission revenue is zero. Then there are two cases to consider: $\gamma^* \leq \gamma_s^H$ and $\gamma^* > \gamma_s^H$. If $\gamma^* \leq \gamma_s^H$, then by Lemma EC.11 the platform's commission revenue is $R_c(\gamma^*) = \mu r^a(\gamma^*)$. Then we have

$$R_C^* = \mu r^a(\gamma^*) \leq \mu \Pi_0^H \leq R_A^*, \tag{EC.142}$$

where the first and second inequalities follow from Lemma EC.20 and EC.19, respectively. If $\gamma^* > \gamma_s^H$, then $R_C(\gamma^*) = \mu r^c(\gamma^*)$, and

$$R_C^* = \mu r^c(\gamma^*) \leq \mu r^a(\gamma^*) \leq \mu \Pi_0^H \leq R_A^*,$$

where the first, second, and third inequalities follow from Lemmas EC.13, EC.20, and EC.19, respectively. □

LEMMA EC.22. *Define $\hat{\mu} = \Pi_0^L/\Pi_0^H$. If the inequality $R_A^* < R_C^*$ holds for some $\mu \neq \hat{\mu}$, then it also holds for $\mu = \hat{\mu}$.*

Proof. To make dependence on μ explicit, we write $R_C^*(\mu)$, $R_A^*(\mu)$ and $\gamma^*(\mu)$ to denote the platform's optimal revenue under commission fees, optimal revenue under access fees, and optimal commission rate, respectively. We also let $R_C(\gamma, \mu)$ be the platform's commission revenue for fixed γ and optimal α, where $R_C^*(\mu) = R_C(\gamma^*(\mu), \mu)$.First, suppose $R_A^*(\mu) < R_C^*(\mu)$ for some $\mu \neq \hat{\mu}$. We show $R_A^*(\hat{\mu}) < R_C^*(\hat{\mu})$ must also hold. We consider two cases: $\mu > \hat{\mu}$ and $\mu < \hat{\mu}$.

<u>Case I:</u> $\mu > \hat{\mu}$. Note

$$\begin{aligned}
R_C^*(\hat{\mu}) &= R_C^*(\mu) + R_C^*(\hat{\mu}) - R_C^*(\mu) \\
&\geq R_C^*(\mu) + R_C(\gamma^*(\mu), \hat{\mu}) - R_C^*(\mu) \\
&= R_C^*(\mu) + \hat{\mu} r^a(\gamma^*(\mu)) + (1-\mu) r^b(\gamma^*(\mu))^+ - R_C^*(\mu) \\
&= R_C^*(\mu) + (\hat{\mu} - \mu) r^a(\gamma^*(\mu)) + (\mu - \hat{\mu}) r^b(\gamma^*(\mu))^+ \\
&> R_C(\mu) + (\hat{\mu} - \mu)\Pi_0^H \\
&= R_C(\mu) + R_A^*(\hat{\mu}) - R_A^*(\mu) \\
&> R_A^*(\hat{\mu}).
\end{aligned}$$

The second line follows by optimality of $\gamma^*(\hat{\mu})$ under $\mu = \hat{\mu}$, the third because $R_C^*(\mu) > R_A^*(\mu)$ implies $\gamma^*(\mu) \leq \gamma_s^H$ by Lemma EC.21 and thus $R_C(\gamma^*(\mu), \mu') = \mu' r^a(\gamma^*(\mu)) + (1-\mu')r^b(\gamma^*(\mu))^+$ for any $\mu' \in [0,1]$, the fourth by expanding $R_C^*(\mu)$, the fifth because $r^a(\gamma) < \Pi_0^H$ for any $\gamma \leq \gamma^m$ by Lemma EC.20, and because $(\mu - \hat{\mu})r^b(\gamma^*(\mu))^+ \geq 0$, the sixth because $R_A^*(\mu) = \Pi_0^H$ for all $\mu \geq \hat{\mu}$ by Lemma EC.19, and the seventh because $R_A^*(\mu) < R_C^*(\mu)$ by assumption. Therefore, $R_A^*(\mu) < R_C^*(\mu)$ for $\mu > \hat{\mu}$ implies $R_A^*(\hat{\mu}) < R_C^*(\hat{\mu})$.

<u>Case II:</u> $\mu < \hat{\mu}$. Following a similar argument to Case I, we have

$$\begin{aligned}
R_C^*(\hat{\mu}) &= R_C^*(\mu) + R_C^*(\hat{\mu}) - R_C^*(\mu) \\
&\geq R_C^*(\mu) + R_C(\gamma^*(\mu), \hat{\mu}) - R_C^*(\mu) \\
&= R_C^*(\mu) + (\hat{\mu} - \mu) r^a(\gamma^*(\mu)) + (\mu - \hat{\mu}) r^b(\gamma^*(\mu))^+ \\
&> R_C^*(\mu) \\
&\geq R_A^*(\mu) \\
&= R_A^*(\hat{\mu}),
\end{aligned}$$

where the fourth line follows because $r^a(\gamma) > r^b(\gamma)$ for all $\gamma \in [0, \gamma^m]$. □

LEMMA EC.23. *Let $\hat{\mu} = \Pi_0^L / \Pi_0^H$. There exists $\bar{\phi} > 0$ such that for each $\phi \geq \bar{\phi}$, $R_C^* > R_A^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$, where $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$.*

Proof. The proof proceeds in three steps. First, we show that the following two inequalities hold at $\alpha = 1$:

$$2r^b(\gamma^m) \geq \Pi_0^L, \tag{EC.143a}$$

$$2r^a(\gamma^m) - \Pi_0^H > \Pi_0^L. \tag{EC.143b}$$

Second, we show that there exists $\bar{\phi} > 0$ such that $R_C^* > R_A^*$ for all $\phi \geq \bar{\phi}$ at $\mu = \hat{\mu}$. Third, we show the existence of the thresholds $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$ for each $\phi \geq \bar{\phi}$.

Step 1. First, consider the expressions for Π_0^L and $r^b(\gamma)$:

$$\Pi_0^L = \eta_s q_L \left(\frac{1}{2} - \frac{(1-\omega_s)c}{2q_L} \right)^2, \tag{EC.144}$$

$$r^b(\gamma) = \frac{\eta_s \gamma}{4} \left(q_L - \frac{c^2(1-\omega_s)^2}{q_L(1-\gamma)^2} \right). \tag{EC.145}$$

Using $\lim_{\alpha\to 1}\eta_s=\lambda$ and $\lim_{\alpha\to 1}\omega_s=1$, we have:

$$\lim_{\alpha\to 1}\left\{2r^b(\gamma^m)-\Pi_0^L\right\}=\frac{\lambda\gamma^m q_L}{2}-\frac{\lambda q_L}{4}=0. \tag{EC.146}$$

It follows that (EC.143a) holds if $\alpha=1$. Next, to see that (EC.143b) holds, note

$$\begin{aligned}
2r^a(\gamma^m)-\Pi_0^H &= \left(q_H-\frac{4c^2(1-\lambda)^2}{q_H}\right)-\left(\frac{q_H}{4}\left(1-\frac{(1-\lambda)c}{q_H}\right)^2\right) && \text{(EC.147a)}\\
&= q_H\left(1-\frac{4c^2(1-\lambda)^2}{q_H^2}\right)-\left(\frac{q_H}{4}\left(1-\frac{(1-\lambda)c}{q_H}\right)^2\right) && \text{(EC.147b)}\\
&= \frac{q_H}{4}\left(\left(1-\frac{4(1-\lambda)^2c^2}{q_H^2}\right)-\left(1-\frac{(1-\lambda)c}{q_H}\right)^2\right)\\
&= \frac{q_H}{4}\left(\frac{2(1-\lambda)c}{q_H}-\frac{5(1-\lambda)^2c^2}{q_H^2}\right)\\
&= \frac{(1-\lambda)c}{2}\left(1-\frac{5(1-\lambda)c}{2q_H}\right)\\
&\geq \frac{q_L}{2}\left(1-\frac{5(1-\lambda)c}{2q_H}\right) && \text{(EC.147c)}\\
&> \frac{q_L\lambda}{4} && \text{(EC.147d)}\\
&\geq \frac{q_L\eta_s}{4}\left(1-\frac{(1-\omega_s)c}{q_L}\right)^2\\
&= \Pi_0^L.
\end{aligned}$$

The first five lines follow by algebra and using $\gamma^m=\frac{1}{2}$ and $q_H\geq 4c$. The inequality (EC.147c) follows because $q_L\leq(1-\lambda)c$ by Assumption 1 and (EC.147d) follows algebraically using $q_H\geq 4c$. The final inequality follows because $\eta_s\leq\lambda$ and because $r^b(\gamma^m)>0$ by (EC.143a) for $\alpha=1$, which implies $q_L\geq(1-\omega_s)c$ and thus $\left(1-\frac{(1-\omega_s)c}{q_L}\right)^2\leq 1$.

Step 2. We now show that (EC.143a) and (EC.143b) imply there exists $\bar{\phi}>0$ and $\hat{\mu}$ such that $R_C^*>R_A^*$ at $\hat{\mu}$ for all $\phi\geq\bar{\phi}$. By Lemmas EC.10 and EC.11, there exists $\bar{\phi}>0$ such that for all $\phi\geq\bar{\phi}$ the platform's commission revenue is given by $R_C(\gamma)=\mu r^a(\gamma)+(1-\mu)r^b(\gamma)^+$. Further, by Lemma EC.19, under the access fee mechanism it is optimal to set $\alpha=1$, with corresponding optimal revenue given by $R_A^*=\max\left\{\mu\Pi_0^H,\Pi_0^L\right\}$. Therefore, to show $R_C^*>R_A^*$, it suffices to find $\gamma\in[0,\gamma^m]$ such that at $\alpha=1$,

$$\hat{\mu}r^a(\gamma)+(1-\hat{\mu})r^b(\gamma)^+>\max\left\{\hat{\mu}\Pi_0^H,\Pi_0^L\right\}. \tag{EC.148}$$

We show the inequality above holds for $\gamma = \gamma^m$. Note

$$\begin{aligned}\hat{\mu} r^a(\gamma^m) + (1-\hat{\mu}) r^b(\gamma^m)^+ &> \frac{\hat{\mu}}{2}\Pi_0^H (1+\hat{\mu}) + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\ &= \frac{\hat{\mu}}{2}\Pi_0^H + \frac{\hat{\mu}}{2}\Pi_0^L + \frac{(1-\hat{\mu})}{2}\Pi_0^L \\ &= \frac{\Pi_0^L}{2} + \frac{\Pi_0^L}{2} \\ &\geq R_A^*. \end{aligned} \tag{EC.149}$$

The first line follows from (EC.143a) and (EC.143b), and the second and third lines follow by definition of $\hat{\mu}$. Therefore, $R_C^* > R_A^*$ at $\hat{\mu}$.

Step 3. We now show the existence of the thresholds $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$. Fix $\phi \geq \bar{\phi}$ and $\alpha = 1$. Because $R_C^* > R_A^*$ at $\hat{\mu}$ and $R_C^* - R_A^*$ is continuous in μ, it suffices to show $R_C^* - R_A^* = 0$ has exactly one solution in μ on each of the intervals $[0, \hat{\mu})$ and $(\hat{\mu}, 1]$. First consider the interval $[0, \hat{\mu})$. Note $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$ by Lemma EC.19 and $\hat{\mu} = \Pi_0^L/\Pi_0^H$ by definition. It follows that $R_A^* = \Pi_0^L$ for all $\mu \in [0, \hat{\mu})$. Next, because $R_C^* = \mu r^a(\gamma^*) + (1-\mu) r^b(\gamma^*)^+$, we must also have $\gamma^* \leq \gamma_s^H$ by Lemma EC.11. We can then write

$$\frac{d}{d\mu}\left\{R_C^* - R_A^*\right\} = \frac{\partial}{\partial\gamma}\left\{R_C^* - R_A^*\right\}\frac{d\gamma^*}{d\mu} + \frac{\partial}{\partial\mu}\left\{R_C^* - R_A^*\right\} \tag{EC.150}$$

$$= \frac{\partial}{\partial\gamma}\left\{\mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+\right\}\frac{d\gamma^*}{d\mu} + r^a(\gamma^*) - r^b(\gamma^*) \tag{EC.151}$$

$$= r^a(\gamma^*) - r^b(\gamma^*) \tag{EC.152}$$

$$> 0, \tag{EC.153}$$

where the first line follows from taking the total derivative and the second line follows because $\frac{d}{d\mu}\Pi_0^L = 0$. To see why the third line holds, consider the two cases $\gamma^* < \gamma_s^H$ and $\gamma^* = \gamma_s^H$. If $\gamma^* < \gamma_s^H$, then $\frac{\partial}{\partial\gamma}\left\{\mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+\right\} = 0$ at γ^* by the envelope theorem; if $\gamma^* = \gamma_s^H$, then $\frac{d}{d\mu}\gamma^* = \frac{d}{d\mu}\gamma_s^H = 0$ because γ_s^H does not depend on μ (Lemma EC.10). The final line follows because $r^a(\gamma) > r^b(\gamma)$ for all $\gamma \in [0, \gamma^m]$. It follows that $R_C^* - R_A^*$ strictly increases in μ on $[0, \hat{\mu})$ wherever $R_C^* - R_A^* > 0$. Hence $R_C^* - R_A^* = 0$ has at most one solution on the interval $[0, \hat{\mu})$. Further, note $R_C^* < R_A^*$ for $\mu = 0$ as a consequence of Lemma EC.21 and $R_C^* > R_A^*$ at $\hat{\mu}$ by Step 2. It follows that $R_C^* - R_A^* = 0$ has exactly one solution $\underline{\mu} \in (0, \hat{\mu})$ on the interval $[0, \hat{\mu})$. Next, we address the interval $(\hat{\mu}, 1]$. Because $R_A^* = \max\{\mu\Pi_0^H, \Pi_0^L\}$ by Lemma EC.19 and $\hat{\mu} = \Pi_0^L/\Pi_0^H$ by definition, we have $R_A^* = \mu\Pi_0^H$ for all $\mu \in (\hat{\mu}, 1]$. Similar to the $\mu \in [0, \hat{\mu})$ case above, it follows that $R_C^* - R_A^* = \mu(r^a(\gamma^*) - \Pi_0^H) + (1-\mu) r^b(\gamma^*)^+$ and $\gamma^* \le \gamma_s^H$. Differentiating in μ yields

$$\frac{d}{d\mu}\left\{R_C^* - R_A^*\right\} = \frac{\partial}{\partial\gamma}\left\{R_C^* - R_A^*\right\}\frac{d\gamma^*}{d\mu} + \frac{\partial}{\partial\mu}\left\{R_C^* - R_A^*\right\} \tag{EC.154}$$

$$= \frac{\partial}{\partial\gamma}\left\{\mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+\right\}\frac{d\gamma^*}{d\mu} + r^a(\gamma^*) - r^b(\gamma^*)^+ \tag{EC.155}$$

$$= r^a(\gamma^*) - \Pi_0^H - r^b(\gamma^*)^+ \tag{EC.156}$$

$$< 0, \tag{EC.157}$$

where the second line follows because $\frac{d}{d\mu}\Pi_0^H = 0$, the third line follows by a parallel argument to the $\mu \in [0, \hat{\mu})$ case above, and the final line follows because $r^a(\gamma) < \Pi_0^H$ for all $\gamma \in [0, \gamma^m]$ by Lemma EC.20. It follows that $R_C^* - R_A^*$ strictly decreases in μ on $(\hat{\mu}, 1]$. Hence $R_C^* - R_A^* = 0$ has at most one solution on the interval $(\hat{\mu}, 1]$. Further, note $R_C^* < R_A^*$ at $\mu = 1$ by Lemma EC.21 and $R_C^* > R_A^*$ at $\hat{\mu}$ by Step 2. It follows that $R_C^* - R_A^* = 0$ has exactly one solution $\bar{\mu} \in (\hat{\mu}, 1]$ on the interval $(\hat{\mu}, 1]$. Because $R_C^* - R_A^* = 0$ has one solution $\underline{\mu}$ on the interval $[0, \hat{\mu})$, one solution $\bar{\mu}$ on the interval $(\hat{\mu}, 1]$, and $R_C^* > R_A^*$ at $\hat{\mu}$, we conclude $R_C^* > R_A^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$. □

EC.4.2. Proof of Proposition 4

PROPOSITION 4. *Let R_A^* and R_C^* be the platform's revenue under the optimal pricing and information policies for access and commission fees, respectively. Let Π_0^i be the on-platform profit of a type-i seller under a commission rate of $\gamma = 0$. There exists $\bar{\phi} > 0$ such that the following statements hold.*

(i) Suppose the switching cost is low, $\phi \le \bar{\phi}$. Then access fees generate higher revenue than commission fees $R_A^* \ge R_C^*$ for all $\mu \in [0, 1]$.

(ii) Suppose the switching cost is high, $\phi > \bar{\phi}$. Then there exists $\underline{\mu} \in \left(0, \frac{\Pi_0^L}{\Pi_0^H}\right)$ and $\bar{\mu} \in \left(\frac{\Pi_0^L}{\Pi_0^H}, 1\right)$ such that access fees generate lower revenue than commission fees $R_A^* < R_C^*$ if and only if the share of type-H sellers is moderate $\mu \in [\underline{\mu}, \bar{\mu}]$.

Proof. The proof of part (i) uses Lemmas EC.20–EC.22. The proof of part (ii) follows almost immediately from Lemma EC.23; our focus here is to show the threshold $\bar{\phi}$ from Lemma EC.23 and part (i) of the proposition statement are the same.

(i). Fix $\phi = 0$. By Lemma EC.21, $R_A^* \ge R_C^*$ if either $r^b(\gamma^*) \le 0$ or $\gamma^* > \gamma_s^H$, from which the result immediately follows. It remains to address the case where $r^b(\gamma^*) > 0$ and $\gamma^* \le \gamma_s^H$. By Lemma EC.22, if $R_A^* > R_C^*$ holds under $\mu = \hat{\mu}$, then $R_A^* > R_C^*$ also holds for any $\mu \in [0, 1]$, where

$\hat{\mu} = \Pi_0^L/\Pi_0^H$. Therefore, it suffices to show $R_A^* > R_C^*$ at $\mu = \hat{\mu}$ and $\phi = 0$. We consider two cases defined by the piecewise upper bound on $\frac{r^b(\gamma)}{\Pi_0^L}$ given in Lemma EC.20. First, suppose $\gamma^* \le 2 - \sqrt{3}$. Because $r^b(\gamma^*) > 0$ and $\gamma^* \le \gamma_s^H$, by Lemma EC.11 the platform's optimal commission revenue is given by $R_C^* = \mu r^a(\gamma^*) + (1-\mu) r^b(\gamma^*)^+$. We can then write

$$\begin{aligned}
R_C^* &= \hat{\mu} r^a(\gamma^*) + (1-\hat{\mu}) r^b(\gamma^*)^+ \\
&\le \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \frac{1}{2-\gamma^*}(1-\hat{\mu})\Pi_0^L \\
&< \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \frac{1}{2-\gamma^*}\Pi_0^L \\
&= \hat{\mu}\Pi_0^H \left(\gamma^* \frac{9}{7} + \frac{1}{2-\gamma^*}\right) \\
&\le \hat{\mu}\Pi_0^H \left((2-\sqrt{3})\frac{9}{7} + \frac{1}{\sqrt{3}}\right) \\
&< \hat{\mu}\Pi_0^H \\
&= R_A^*.
\end{aligned}$$

The relations above uses the bounds Lemma EC.20, the definition of $\hat{\mu}$, and the observation that $\left(\gamma \frac{9}{7} + \frac{1}{2-\gamma}\right)$ strictly increases in γ on $\gamma \in [0, 2-\sqrt{3}]$. Next, suppose $\gamma^* \in [2-\sqrt{3}, \gamma^m]$. Again applying the bounds from Lemma EC.20, we have

$$\begin{aligned}
R_C^* &= \hat{\mu} r^a(\gamma^*) + (1-\hat{\mu}) r^b(\gamma^*)^+ \\
&\le \gamma^* \frac{9}{7} \hat{\mu} \Pi_0^H + \gamma \frac{(1+\gamma^*)(1-3\gamma^*)}{(1-\gamma^*)^2(1-2\gamma^*)^2}\Pi_0^L \\
&= \hat{\mu}\Pi_0^H \underbrace{\left(\gamma^* \frac{9}{7} + \gamma^* \frac{(1+\gamma^*)(1-3\gamma^*)}{(1-\gamma^*)^2(1-2\gamma^*)^2}\right)}_{g(\gamma^*)} \\
&< \hat{\mu}\Pi_0^H \\
&= R_A^*.
\end{aligned}$$

The fourth line follows because $g(\gamma^*) \le 1$ in the interval $\gamma \in [2-\sqrt{3}, \gamma^m]$, and the final line follows from Lemma EC.19. It follows that $R_A^* > R_C^*$ for all $\mu \in [0,1]$ at $\phi = 0$. Next, it is straightforward to verify that R_A^* does not depend on ϕ and R_C^* is continuous in ϕ. It follows that there exists $\bar{\phi} > 0$ such that for $\phi \le \bar{\phi}$, $R_A^* \ge R_C^*$ for all $\mu \in [0,1]$. Statement (i) follows. In the remainder of the proof, let $\bar{\phi}$ be the largest threshold such that $R_A^* \ge R_C^*$ holds for all $\mu \in [0,1]$ and $\phi \le \bar{\phi}$.

(ii). Let $\hat{\mu} = \frac{\Pi_0^L}{\Pi_0^H}$. By Lemma EC.23, there exists $\underline{\phi} > 0$ and $\underline{\mu} \in (0, \hat{\mu})$ and $\bar{\mu} \in (\hat{\mu}, 1)$ such that if $\phi > \underline{\phi}$ then $R_A^* < R_C^*$ if and only if $\mu \in [\underline{\mu}, \bar{\mu}]$. Pick $\underline{\phi}$ to be the smallest such threshold. It remains to show $\underline{\phi} = \bar{\phi}$. Suppose by way of contradiction that $\underline{\phi} > \bar{\phi}$. Then there exists $\hat{\phi} \in (\bar{\phi}, \underline{\phi})$ such that $R_A^* < R_C^*$ holds at $\phi = \hat{\phi}$ for $\mu \in [\underline{\mu}, \bar{\mu}]$. Further, note that $R_A^* < R_C^*$ implies $\gamma^* \leq \gamma_s^H$ by Lemma EC.21. Thus, using the expression for platform commission revenue (Lemma EC.11), $R_A^* < R_C^*$ holds if

$$R_A^* < \max_{\alpha \in [\frac{1}{2}, 1],\, \gamma \leq \gamma_s^H} \mu r^a(\gamma) + (1 - \mu) r^b(\gamma)^+. \tag{EC.158}$$

Next, note that because γ_s^H increases in ϕ (Lemma EC.14), so does the right hand side of (EC.158). It follows that (EC.158) and thus $R_A^* < R_C^*$ hold for all $\phi \geq \hat{\phi}$. However, this yields a contradiction to the selection of $\underline{\phi}$ as the smallest such threshold. We conclude $\underline{\phi} = \bar{\phi}$. □

EC.5. Proofs for Section 5.2: Should Platforms Ban Sellers?

EC.5.1. Proof of Proposition 5

Before proving the main result, we first characterize the commission thresholds for disintermediation in each period, analogous to Lemma EC.10.

LEMMA EC.24 **(Commission thresholds for disintermediation)**. *For each $i \in \{L, H\}$, $\sigma \in \{r, s\}$, and $t \in \{1, 2\}$, there exists a unique threshold γ_σ^{it} such that a type-i seller disintermediates with a signal-σ buyer in period t if and only if $\gamma > \gamma_\sigma^{it}$. Further,*

(i) $\gamma_\sigma^{i2} := 1 - \omega_\sigma$ for $\sigma \in \{r, s\}$, and

(ii) γ_s^{i1} weakly increases in d on $d \in [0, 1]$, where $\gamma_s^{i1} \geq \gamma_s^{i2}$ for all $d \in [0, 1]$ and $\gamma_s^{i1} = \gamma_s^{i2}$ for $d = 0$.

Proof. We focus on proving the main lemma statement for the type-H seller, and note the results for the type-L seller follow by similar arguments. For (i), we assume without loss of generality that the seller is undetected in period 1; otherwise, no transaction occurs in period 2.

(i). The result follows closely from Lemmas EC.2 and EC.4. Let p and b be an arbitrary online and offline price in period 2, respectively. Because the game ends after period 2, the buyer's and seller's surplus from transacting offline are simply $p - b$ and $\omega_\sigma b - (1 - \gamma)p$, respectively, which yields a Nash bargaining solution of $b_\sigma(p) = \frac{p(1-\gamma+\omega_\sigma)}{2\omega_\sigma}$. Because $\phi = 0$, it follows by parallel argument to Lemma EC.4 that both $p - b_\sigma(p) > 0$ and $\omega_\sigma b_\sigma(p) - (1 - \gamma)p > 0$ hold if and only if $\gamma > 1 - \omega_s$.

$\boldsymbol{(ii)}$. The result follows by parallel argument to the proof of Lemma EC.10; we verify the main steps here. Analogous to Lemma EC.7, it can be shown that Assumption 2 implies $\gamma_r^{i2} > \gamma^m$, meaning the type-i seller never transacts offline with the $\sigma = r$ buyer. Therefore, we focus on the threshold γ_s^{i2}, beginning with $i = H$. In particular, we proceed in two steps: First, we show there exists $\underline{\gamma}$ and $\bar{\gamma} > \underline{\gamma}$ such that the type-H seller transacts online with the $\sigma = s$ buyer if $\gamma < \underline{\gamma}$ and offline if $\gamma > \bar{\gamma}$. Second, we show there exists a unique $\gamma_s^H \in [\underline{\gamma}, \bar{\gamma}]$ such that the transaction is offline if and only if $\gamma > \gamma_s^H$.

Step 1. Let π^{x2} denote the type-H seller's expected optimal profit in period 2 conditional on non-detection in period 1, where $x \in \{a, c\}$ as per Lemma EC.3. For online price p, offline price b, and detection probability d, the seller's surplus from transacting offline in period 1 is $\omega_\sigma b - d\pi^{x2} - (1-\gamma)p$, and the buyer's surplus is again $p - b$. Solving for the Nash bargaining price yields $b_\sigma^x(p) = \frac{p(1-\gamma+\omega_\sigma)+d\pi^{x2}}{2\omega_\sigma}$. Analogous to Lemma EC.4, it follows that the buyer and seller both have strictly positive surplus from transacting offline if and only if

$$\gamma > 1 - \omega_\sigma + \frac{d\pi^{x2}}{p}. \tag{EC.159}$$

Note EC.159 implies $\gamma_\sigma^{i1} = \gamma_\sigma^{i2}$ for $i \in \{L, H\}$ and $\sigma = \{r, s\}$ follows. Further, analogous to Lemma $EC.9$, note the inequalities $\gamma \le 1 - \omega_\sigma + \frac{d\pi^{x2}}{p^a}$ and $\gamma > 1 - \omega_\sigma + \frac{d\pi^{x2}}{p^c}$ are a necessary and sufficient condition for the transaction to be online, respectively. Next, if the type-H seller transacts online with a $\sigma = s$ buyer in period 1, their expected profit over both periods is

$$\pi^a(p) := ((1-\gamma)p - (1-\lambda)c)\left(1 - \frac{p}{q_H}\right) + \pi^{x2}. \tag{EC.160}$$

If the seller transacts offline with the $\sigma = s$ buyer in period 1, their expected profit over both periods is

$$\pi^c(p) := \left(1 - \frac{p}{q_H}\right)\Big(((1-\gamma)p - (1-\lambda)c) + \eta_s(\omega_s b_s(p) - d\pi^{x2} - (1-\gamma)p)\Big) + \pi^{x2}. \tag{EC.161}$$

The unconstrained maximizers of $\pi^a(p)$ and $\pi^c(p)$ are then given by p^a and p^c, where

$$p^a := \frac{1}{2}\left(q_H + \frac{(1-\lambda)c}{1-\gamma}\right),$$

$$p^c := \frac{1}{2}\left(q_H + \frac{(1-\lambda)c}{\zeta} + \frac{\eta_s d\pi^{x2}}{2\zeta}\right),$$

and where $\zeta = \eta_r(1-\gamma) + \eta_s\frac{1-\gamma+\omega_s}{2}$. It remains to show there exists a unique threshold γ_s^{H1} such that $\pi^a(p^a) < \pi^c(p^c)$ if and only if $\gamma > \gamma_s^{H1}$. First, define $\bar{\gamma}$ to be the largest solution to $\gamma = 1 - \omega_s + \frac{d\pi^{x2}}{p^a}$.

Because p^a increases in γ and π^{x2} decreases in γ, it follows that $\gamma > \bar{\gamma}$ implies (EC.159) holds at $p = p^a$, and thus the type-H seller transacts offline with the $\sigma = s$ buyer. Similarly, define $\underline{\gamma}$ to be the smallest solution to

$$\gamma = 1 - \omega_s + \frac{d\pi^{x2}(\gamma)}{p^c} = 1 - \omega_s + \frac{d}{\frac{1}{2}\left(\frac{q_H}{\pi^{x2}} + \frac{(1-\lambda)c}{\pi^{x2}\zeta} + \frac{\eta_s d}{2\zeta}\right)}. \tag{EC.162}$$

Note ζ and π^{x2} both decrease in γ, which implies the right hand side of (EC.162) decreases in γ. As a result, for any $\gamma < \underline{\gamma}$, (EC.159) cannot hold at $p = p^c$, which implies the transaction with the $\sigma = s$ buyer occurs online. We have thus shown the transaction between the type-H seller and $\sigma = s$ buyer occurs offline if $\gamma > \bar{\gamma}$ and online if $\gamma < \underline{\gamma}$. This completes Step 1.

Step 2. First, note $p^c \geq p^a$ for all $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. To see why, suppose by contradiction that $p^c < p^a$ for some $\gamma' \in [\underline{\gamma}, \bar{\gamma}]$, and define

$$f(p, \gamma) := \gamma - \left(1 - \omega_s + \frac{d\pi^{x2}}{p}\right).$$

By definition of $\underline{\gamma}$ and $\bar{\gamma}$, we have $f(\gamma', p^a) < 0$ and $f(\gamma', p^c) \geq 0$. However, $f(p, \gamma)$ increases in p for each $\gamma \in (0, \gamma^m]$, which yields a contradiction. We conclude $p^c \geq p^a$ for all $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. Analogous to the proof of Lemma EC.10, we show that $\pi^a(p^a) - \pi^c(p^c)$ strictly decreases in γ on $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, from which the existence of the unique threshold γ_s^{H1} follows. Differentiating $\pi^a(p^a) - \pi^c(p^c)$ in γ, we have

$$\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) = \left(\frac{\partial \pi^a}{\partial p} \cdot \frac{dp^a}{d\gamma} + \frac{\partial \pi^a}{\partial \gamma}\right)\Bigg|_{p=p^a} - \left(\frac{\partial \pi^c}{\partial p} \cdot \frac{dp^c}{d\gamma} + \frac{\partial \pi^c}{\partial \gamma}\right)\Bigg|_{p=p^c} \tag{EC.163}$$

$$= \frac{\partial \pi^a}{\partial \gamma}\Bigg|_{p=p^a} - \frac{\partial \pi^c}{\partial \gamma}\Bigg|_{p=p^c} \tag{EC.164}$$

$$= -p^a\left(1 - \frac{p^a}{q_H}\right) + \left(1 - \frac{\eta_s}{2}\right)p^c\left(1 - \frac{p^c}{q_H}\right) + d\left(1 - \frac{p^c}{q_H}\right)\eta_s \frac{\partial \pi^{x2}}{\partial \gamma} \tag{EC.165}$$

$$\leq \left(\left(1 - \frac{\eta_s}{2}\right)p^c - p^a\right)\left(1 - \frac{p^a}{q_H}\right). \tag{EC.166}$$

The second line above follows from the envelope theorem and the third line follows from evaluating the derivative algebraically. To see that the inequality in the fourth line holds, note π^{x2} decreases in γ and $p^c \geq p^a$ for $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, as established above. Since $\left(1 - \frac{p^a}{q}\right) > 0$ and $\eta_s \in (0, 1)$, to show $\frac{d}{d\gamma}(\pi^a(p^a) - \pi^c(p^c)) < 0$ it suffices to show that $2p^a - p^c > 0$. The preceding inequality follows immediately from the observations that $p^c \leq q$ and $2p^a > q$. This establishes the existence of the unique threshold γ_s^{H1}. Finally, to see that γ_s^{H1} increases in d, note that $\pi^a(p^a)$ is independent of d, and $\pi^c(p^c)$ can be written as

$$\pi^c(p^c) = q_H\zeta\left(\frac{1}{2} - \frac{(1-\lambda)c}{2q_H\zeta} - \frac{\eta_s d\pi^{x2}}{4q_H\zeta}\right)^2.$$

Clearly, $\pi^c(p^c)$ decreases in d wherever $\pi^c(p^c) > 0$. Because γ_s^{H1} is the solution to $\pi^a(p^a) = \pi^c(p^c)$ and $\pi^a(p^a) - \pi^c(p^c)$ decreases in γ on $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, γ_s^{H1} also decreases in d. The proof for the threshold γ_s^{L1} follows by parallel argument and is omitted. □

Proposition 5. *Let $R^0(\gamma^*)$ and $R^d(\gamma^*)$ be the platform's optimal revenue under the blind eye and banning policies, respectively. Then, there exist thresholds $\underline{\alpha} \in (\frac{1}{2}, 1]$ and $\bar{\alpha} \in [\underline{\alpha}, 1)$ such that the following statements hold.*

(i) Suppose information quality is low $\alpha \le \underline{\alpha}$. Then for all detection probabilities $d \in [0,1]$, the banning policy generates weakly higher revenue than the blind eye policy, $R^0(\gamma^*) \le R^d(\gamma^*)$.

(ii) Suppose information quality is high $\alpha \ge \bar{\alpha}$. Then there exists $\bar{d} \in (0,1)$ such that if the detection probability is low $d \in (0, \bar{d}]$, the banning policy generates strictly lower revenue than the blind eye policy, i.e., $R^0(\gamma^*) > R^d(\gamma^*)$.

Proof. **(*i*)**. First suppose $\alpha = \frac{1}{2}$ and $d = 0$. Then by Lemma EC.24 and Assumption 2, we have $\gamma^m \le \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$. Further, because γ_s^{i1} and γ_s^{i2} are weakly increasing in and independent of d, respectively, it follows by continuity of γ_s^{i1} and γ_s^{i2} in α that there exists $\underline{\alpha} \in (0,1]$ such that if $\alpha \le \underline{\alpha}$, then $\gamma^* \le \gamma_s^{it}$ for $i \in \{L, H\}$, $t \in \{1, 2\}$ and all $d \in [0,1]$, i.e., all transactions occur online. Then by Lemma EC.24, the platform's revenue is given by

$$R(\gamma^*) = 2(\mu r^a(\gamma^*) + (1-\mu) r^b(\gamma^*))^+, \tag{EC.167}$$

where $r^a(\gamma)$ and $r^b(\gamma)$ are as defined in Lemma EC.11. It follows that for $\alpha \le \underline{\alpha}$ and $d \in [0,1]$, we have

$$\left.\frac{dR}{dd}\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial R}{\partial \gamma}\frac{d\gamma^*}{dd} + \frac{\partial R}{\partial d}\right)\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial R}{\partial \gamma}\frac{d\gamma^*}{dd}\right)\right|_{\gamma=\gamma^*} \ge 0. \tag{EC.168}$$

The second equality above follows because $r^a(\gamma)$ and $r^b(\gamma)$ are both independent of d, which implies $\frac{\partial}{\partial d}R = 0$. To see that the inequality holds, note $\gamma_s^{i1} \ge \gamma_s^{i2}$ holds for all $d \in [0,1]$ by Lemma EC.24. Thus, there are two cases to consider: $\gamma^* < \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$ and $\gamma^* = \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$. If $\gamma^* < \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$, then $\frac{\partial}{\partial \gamma}R = 0$ holds at $\gamma = \gamma^*$ by the envelope theorem, and the weak inequality in (EC.168) holds as an equality. If $\gamma^* = \min\{\gamma_s^{L2}, \gamma_s^{H2}\}$, then $\frac{\partial}{\partial\gamma} R \geq 0$ must hold at $\gamma = \gamma^*$ because γ^* is the optimal commission rate. Further, by Lemma EC.24 both γ_s^{L2} and γ_s^{H2} weakly increase in d, which implies $\frac{d}{dd}\gamma^* \geq 0$. The inequality in (EC.168) again follows.

(ii). First suppose $\alpha = 1$ and $d = 0$. Then by Lemma EC.24 we have $\gamma^* > \gamma_s^{it} = 0$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$, i.e., all transactions with $\sigma = s$ buyers occur offline. It follows by continuity of γ_s^{it} in α and d that there exists $\bar{\alpha} \in [\underline{\alpha}, 1)$ and $\bar{d} > 0$ such that $\gamma^* > \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$ if $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$. Note that $\gamma^* > \gamma_s^{it}$ for $i \in \{L, H\}$ and $t \in \{1, 2\}$ implies the platform only extracts commissions from type-H sellers. Further, type-H sellers are banned after period 1 with probability d only if they transact with a $\sigma = s$ buyer in period 1, and thus the probability a type-H seller is undetected following period 1 is $\eta_r + \eta_s(1-d)$. It follows from Lemma EC.24 that the platform's revenue for $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$ is

$$R(\gamma^*) = \mu r^c(\gamma^*) + \mu(\eta_r + \eta_s(1-d)) r^c(\gamma^*), \tag{EC.169}$$

where

$$r^c(\gamma) = \gamma\eta_r p^c \left(1 - \frac{p^c}{q_H}\right) = \gamma\eta_r q_H \left(\frac{1}{4} - \left(\frac{(1-\lambda)c}{2q_H\zeta} + \frac{\eta_s d\pi^{x2}}{4q_H\zeta}\right)^2\right). \tag{EC.170}$$

It follows that for $\alpha \geq \bar{\alpha}$ and $d \leq \bar{d}$,

$$\left.\frac{dR}{dd}\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial R}{\partial\gamma}\frac{d\gamma^*}{dd} + \frac{\partial R}{\partial d}\right)\right|_{\gamma=\gamma^*} = \left.\frac{\partial R}{\partial d}\right|_{\gamma=\gamma^*} = \left.\left(-\mu\eta_s r^c(\gamma) + \mu(1 + \eta_r + \eta_s(1-d))\frac{\partial r^c}{\partial\gamma}\right)\right|_{\gamma=\gamma^*} < 0. \tag{EC.171}$$

To see the second equality above, note that if $\gamma^* = \gamma^m$, then $\frac{d}{dd}\gamma^* = 0$, and if $\gamma^* < \gamma^m$ then $\frac{\partial}{\partial\gamma} R = 0$ holds at $\gamma = \gamma^*$ by the envelope theorem. The strict inequality follows because $\frac{\partial}{\partial d} r^c < 0$ by inspection of the expression for $r^c(\gamma)$ in (EC.170). Because $R(\gamma^*)$ strictly decreases in d on $d \in [0, \bar{d}]$, statement (ii) follows. □

EC.6. Repeat Interactions with Returning Buyers

Our main model examines disintermediation in a single-shot setting where sellers transact with a given buyer at most once. In practice, sellers may interact with the same buyer repeatedly, allowing them to learn the buyer's type from earlier transactions. In this section, we consider a dynamic variant of our main model, in which sellers form beliefs about buyers' types through an initial transaction, instead of through the platform signal. The purpose of this section is to establish that our main insights hold when information about buyers is transmitted in this alternative manner.

EC.6.1. Model Setup

Formally, we consider a two-period model where each seller is matched to the same buyer in both periods, and a share $1-\lambda$ of buyers are type-r (i.e., risky). As before, type-s buyers impose the transaction cost $c_s = 0$ on sellers. However, in contrast to our main model, the transaction cost imposed by type-r buyers is stochastic in each period, where $c_r = c > 0$ with probability $\rho \in [0,1]$ and $c_r = 0$ with probability $1-\rho$. As a consequence of this cost structure, when the realized transaction cost is c, the seller immediately learns the buyer is type-r; when the realized transaction cost is 0, the seller's posterior belief that the buyer is type-r is $\frac{(1-\lambda)(1-\rho)}{(1-\lambda)(1-\rho)+\lambda}$ (see Lemma EC.25). The parameter ρ thus captures sellers' *ability to learn* - as ρ increases, sellers form stronger posterior beliefs and can distinguish buyer types more accurately. To isolate the effect of the learning parameter ρ, we assume the platform's signal is uninformative ($\alpha = \frac{1}{2}$).

At the beginning of the horizon, the platform sets the commission rate γ. Each seller then commits to an online price p for both periods, and is randomly matched to one buyer. In each period, a seller's choices are to transact online, offline, or not at all. If the seller rejects the buyer in period 1, they earn zero profit and are not matched to a new buyer. This setup assumes that a seller's decision to complete a follow-up transaction with a buyer is independent of potential matches with new buyers (for example, in freelance marketplaces, sellers typically juggle multiple projects simultaneously, so the possibility of future contracts does not impact transactions with current buyers). We adopt notation from the main model and re-define σ to denote the signal generated by the period 1 transaction, where the seller observes $\sigma = r$ if the realized transaction cost from period 1 is $c > 0$ and observes $\sigma = s$ if the realized cost is 0.

To facilitate our analysis, we make two assumptions that are analogous to Assumptions 1 and 2 from the main model. First, we again assume that the two seller types are well-separated with respect to their quality level:

Assumption EC.1. *The seller qualities satisfy* $q_H \geq 4c$ *and* $q_L \in \left[\frac{2(1-\lambda)(2-\rho)\rho}{2-(1-\lambda)\rho}c, \rho c\right]$.

The above assumption implies that the type-H seller transacts with all buyers, whereas the type-L seller always rejects the $\sigma = r$ buyer. In addition, we impose the requirement the quality level of the type-L seller is high enough to guarantee that this seller transacts on the platform in period 1 for every value of $\gamma \in [0, \gamma^m]$; this assumption precludes the less interesting case where only the type-H seller participates. We also impose an assumption that allows us to focus on the case where sellers do not disintermediate prior to forming a belief about the buyer's type (i.e., in period 1).

Assumption EC.2. *The maximum commission rate γ^m, share of type-H sellers λ, switching cost ϕ, and type-H seller quality q_H satisfy the inequality $\gamma^m \leq 1 - \lambda + \frac{\phi}{q_H}$.*

Assumption EC.2 is similar to Assumption 2 from the main model, and similarly implies that sellers do not disintermediate with $\sigma = r$ buyers.

Although we assume the learning parameter ρ to be exogenous, in practice it may be impacted by platform design decisions. For example, in the context of freelance marketplaces like Upwork, a low value of ρ may be the result of features that smooth out differences in how buyers interact with sellers, including customer support, a common structure on job postings, or AI-assisted communication (Upwork 2024a). Similarly, a high value of ρ may correspond to a less regulated environment in which sellers can more accurately screen buyers, as a consequence of risky buyers being more likely to “reveal" themselves. Note that ρ plays a similar role to α from our main model, since it captures the accuracy with which sellers can learn buyers' types.

EC.6.2. Seller Learning and Platform Revenue

With repeated interactions, it is natural to expect the threat of disintermediation to depend on the accuracy with which sellers can infer buyers' types from an initial transaction. To the extent that platforms can influence seller learning (e.g., through features or policies that change how buyers and sellers interact), it is valuable to understand the impact of the sellers' learning parameter ρ on platform revenue.

Proposition EC.1. *Suppose the switching cost is $\phi = 0$. Then there exists $\bar{\mu} \in [0, 1)$ and $\bar{\rho} \in [0, 1)$ such that if the share of type-H sellers is large $\mu \geq \bar{\mu}$, the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the learning parameter ρ on $\rho \in [0, \bar{\rho})$ and strictly increases in ρ on $\rho \in (\bar{\rho}, 1]$.*

All proofs for this section are in Section EC.6.4. Similar to our prior results, the non-monotonic behavior in Proposition EC.1 can be understood by considering the effects induced by a change in the learning parameter ρ, which we briefly outline. When ρ is small ($\rho < \bar{\rho}$), sellers cannot easily identify safe (type-s) buyers, leading all transactions to occur online in equilibrium in both periods. In this case, an increase in ρ increases the threat of disintermediation, which degrades the platform's pricing power and lowers revenue. However, when ρ is large ($\rho > \bar{\rho}$), sellers disintermediate in period 2 with the zero-cost buyers (which consists of both type-r and type-s buyers), but transact on-platform with the costly buyers (consisting only of type-r buyers). In this setting, further increases in ρ allow sellers to identify type-r buyers with increased accuracy and avoid transacting off-platform with them, which increases on-platform transaction volume and boosts revenue. In summary, although the ability to screen buyers is necessary for disintermediation, Proposition EC.1 suggests that in settings where sellers already transact off-platform (large ρ), further improvements to seller learning can help revenue by stemming the flow of any additional disintermediation.

EC.6.3. Optimal Commission Rate

We conclude this section by showing that a variation of our main result from Proposition 1 holds when sellers learn through an initial transaction instead of via the platform signal.

Proposition EC.2. *Let $\gamma^*(\phi)$ be the optimal commission rate under switching cost ϕ. There exists $\bar{\phi} > 0$ and $\bar{\rho} \in [0, 1)$ such that for any $\rho \geq \bar{\rho}$ and $\phi \geq \bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0) \geq \gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi) < \gamma^m$.*

Proposition EC.2 mirrors Proposition 1(ii) by showing that in a disintermediation-prone environment (i.e., no switching cost and high ρ), it is optimal for the platform to "double down" on the on-platform transactions - that is, choose a commission rate higher than the corresponding rate when there is no disintermediation. The intuition follows similarly to our discussion in Section 3.1.

EC.6.4. Proofs

This section contains the proofs for Propositions EC.1 and EC.2. We first provide some supporting lemmas that are analogous to those presented in Section EC.1 for the main model.

Preliminary Results First, Lemmas EC.25 – EC.29 below characterize the relevant probabilities, commission thresholds for disintermediation, the sellers' profit functions, and the platform's revenue function. These results are analogous to those presented in Section EC.1 for the main model; the proofs follow similarly and are omitted to avoid repetition.

Lemma EC.25 **(Signal probabilities and sellers' beliefs)**. *The following statements hold for $\sigma \in \{r, s\}$.*

(i) The probability a seller receives the signal σ following the period 1 transaction is η_σ, where $\eta_s := \lambda + (1-\lambda)(1-\rho)$ and $\eta_r := (1-\lambda)\rho$.

(ii) The seller's posterior belief that a buyer with signal σ has true type $j = s$ is ω_σ, where $\omega_s := \lambda/\eta_s$ and $\omega_r := 0$.

(iii) The probability a buyer with signal σ pays the seller if transacting offline is ω_σ.

LEMMA EC.26 **(Disintermediation thresholds fixed p)**. *Let Assumption EC.2 hold. In period 1, all transactions occur online for all $\gamma \in [0, \gamma^m]$. In period 2, given an online price $p > 0$, both seller types transact offline with the $\sigma = s$ buyer if and only if $\gamma \geq \hat{\gamma}_s(p)$, where*

$$\hat{\gamma}_s(p) := 1 - \omega_s + \frac{\phi}{p}$$

and the offline price is given by

$$b_s(p) := \frac{p(1-\gamma+\omega_s)+\phi}{2\omega_s}.$$

Further, neither seller transacts offline with the $\sigma = r$ buyer in period 2 for all $\gamma \in [0, \gamma^m]$ and $p > 0$.

LEMMA EC.27 **(Sellers' profit and price cases)**. *Fix the commission rate γ and consider a unit mass of sellers with quality q and online price $p \leq q$. Let $\Pi(p)$ be the sellers' profit and let $\tilde{p}$ be the maximizer of $\Pi(p)$.*

(i) If the sellers transacts online in period 2 with both $\sigma = r$ and $\sigma = s$ buyers,

$$\Pi(p) = \pi^a(p) := 2((1-\gamma)p - (1-\lambda)\rho c)\left(1 - \frac{p}{q}\right) \tag{EC.172}$$

$$\tilde{p} = p^a := \frac{1}{2}\left(q + \frac{\rho c(1-\lambda)}{1-\gamma}\right). \tag{EC.173}$$

(ii) If the sellers reject $\sigma = r$ and transact online with $\sigma = s$,

$$\Pi(p) = \pi^b(p) := ((1-\gamma)p - (1-\lambda)\rho c + \eta_s((1-\gamma)p - (1-\omega_s)\rho c))\left(1 - \frac{p}{q}\right) \tag{EC.174}$$

$$\tilde{p} = p^b := \frac{1}{2}\left(q + \frac{\rho c(1-\lambda)(2-\rho)}{(1-\gamma)(2-(1-\lambda)\rho)}\right). \tag{EC.175}$$

(iii) If the seller transact online with $\sigma = r$ and offline with $\sigma = s$,

$$\Pi(p) = \pi^c(p) := ((1-\gamma)p - (1-\lambda)\rho c + \eta_s(\omega_s b - (1-\omega_s)\rho c - \phi) + \eta_r((1-\gamma)p - \rho c))\left(1 - \frac{p}{q}\right) \tag{EC.176}$$

$$\tilde{p} = p^c := \frac{1}{2}\left(q + \frac{4\rho c(1-\lambda) + \phi(1-(1-\lambda)\rho)}{(1-\gamma)(3+(1-\lambda)\rho)+\lambda}\right). \tag{EC.177}$$

(iv) If the sellers reject $\sigma = r$ and transact offline with $\sigma = s$,

$$\Pi(p) = \pi^d(p) := ((1-\gamma)p - (1-\lambda)\rho c + \eta_s(\omega_s b - (1-\omega_s)\rho c - \phi))\left(1 - \frac{p}{q}\right) \tag{EC.178}$$

$$\tilde{p} = p^d := \frac{1}{2}\left(q + \frac{2\rho c(1-\lambda)(2-\rho) + \phi(1-(1-\lambda)\rho)}{(1-\gamma)(3-(1-\lambda)\rho)+\lambda}\right). \tag{EC.179}$$

Lemma EC.28 **(Disintermediation threshold and platform revenue)**. *For each seller type $i \in \{L, H\}$, there exists a unique threshold γ_s^i such that the type-i seller transacts offline with the $\sigma = s$ buyer if and only if $\gamma > \gamma_s^i$. Further, $\gamma_s^H \le \gamma_s^L$ for all $\phi \ge 0$, and $\gamma_s^H = \gamma_s^L = 1 - \omega_s$ if $\phi = 0$.*

Lemma EC.29 **(Platform's revenue function)**. *Let p^x for $x \in \{a, b, c, d\}$ be as defined in Lemma EC.27, and define*

$$r^a(\gamma) := 2\gamma p^a \left(1 - \frac{p^a}{q_H}\right), \tag{EC.180}$$

$$r^b(\gamma) := \gamma(1 + \lambda + (1-\lambda)(1-\rho))p^b \left(1 - \frac{p^b}{q_L}\right), \tag{EC.181}$$

$$r^c(\gamma) := \gamma(1 + (1-\lambda)\rho)p^c \left(1 - \frac{p^c}{q_H}\right), \tag{EC.182}$$

$$r^d(\gamma) := \gamma p^d \left(1 - \frac{p^d}{q_L}\right). \tag{EC.183}$$

Then the platform's commission revenue is given by $R(\gamma)$, where

$$R(\gamma) := \begin{cases} \mu r^a(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in [0, \gamma_s^H], \\ \mu r^c(\gamma) + (1-\mu) r^b(\gamma)^+ & \text{if } \gamma \in (\gamma_s^H, \gamma_s^L], \\ \mu r^c(\gamma) + (1-\mu) r^d(\gamma)^+ & \text{if } \gamma \in (\gamma_s^L, \gamma^m], \end{cases} \tag{EC.184}$$

and $x^+ = \max\{0, x\}$.

Proof of Proposition EC.1 Before presenting the proof of Proposition EC.1, we first present Lemmas EC.30 and EC.31 which describe useful properties of the platform's revenue function and the optimal commission rate, respectively. For the remainder of Section EC.6.4, define $\gamma^x := \operatorname{argmax}_{\gamma \in [0,1]} r^x(\gamma)$ and $\gamma^{xy} := \operatorname{argmax}_{\gamma \in [0, \gamma^m]} \{\mu r^x(\gamma) + (1-\mu) r^y(\gamma)\}$, where $x, y \in \{a, b, c\}$ and the $r^x(\gamma)$ functions are defined in Lemma EC.29.

Lemma EC.30 **(Revenue function properties)**. *For any $\gamma \in (0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1)$, (i) $r^a(\gamma)$ and $r^b(\gamma)$ both strictly decrease in ρ on $\rho \in [0,1]$, and (ii) $r^c(\gamma)$ strictly increases in ρ on $\rho \in [0,1]$.*

Proof. $\boldsymbol{(i)}$. First note for any $\gamma \in (0, \gamma^m]$,

$$\frac{\partial r^a}{\partial \rho} = \frac{\partial}{\partial \rho} \left\{ \frac{\gamma}{2} \left(q_H - \frac{(c(1-\lambda)\rho)^2}{q_H(1-\gamma)^2} \right) \right\} = -\frac{c^2 \gamma (1-\lambda)^2 \rho}{(1-\gamma)^2 q_H} < 0. \tag{EC.185}$$

For $r^b(\gamma)$, we have

$$\frac{\partial r^b}{\partial \rho} = \frac{\partial}{\partial \rho} \left\{ \frac{\gamma}{4} \left(q_L(2 - (1-\lambda)\rho) - \frac{(c(1-\lambda)(2-\rho)\rho)^2}{q_L(1-\gamma)^2(2-(1-\lambda)\rho)} \right) \right\} \tag{EC.186}$$

$$= \frac{\gamma}{4}\left(-q_L(1-\lambda) - \frac{c^2(1-\lambda)^2(2-\rho)\rho\left(8+3(1-\lambda)\rho^2-2(5-\lambda)\rho\right)}{q_L(1-\gamma)^2(2-(1-\lambda)\rho)^2}\right) \tag{EC.187}$$

$$< 0, \tag{EC.188}$$

where the strict inequality follows because $\rho \in [0,1]$ and $\lambda \in [\frac{1}{2}, 1]$ imply $8+3(1-\lambda)\rho^2 > 2(5-\lambda)\rho$. Thus, $r^a(\gamma)$ and $r^b(\gamma)$ both strictly decrease in ρ.

(ii). To prove the result, we first show that $\frac{\partial}{\partial\rho} r^c > 0$ at $\rho = 1$. We then show $\frac{\partial^2}{\partial\rho^2} r^c \leq 0$ for all $\rho \in [0,1]$, which implies $\frac{\partial}{\partial\rho} r^c > 0$ for all $\rho \in [0,1]$. To begin, note

$$\frac{\partial r^c}{\partial\rho} = \frac{\gamma}{4}\cdot\frac{\partial}{\partial\rho}\left\{(1+(1-\lambda)\rho)\left(q_H - \frac{(4c(1-\lambda)\rho)^2}{q_H((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2}\right)\right\} \tag{EC.189}$$

$$= \frac{\gamma}{4}\left((1-\lambda)\left(q_H - \frac{(4c(1-\lambda)\rho)^2}{q_H((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2}\right) - (1+(1-\lambda)\rho)\left(\frac{32c^2(1-\lambda)^2(3(1-\gamma)+\lambda)\rho}{q_H((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}\right)\right) \tag{EC.190}$$

$$\geq \gamma c\left((1-\lambda)\left(1 - \frac{((1-\lambda)\rho)^2}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2}\right) - (1+(1-\lambda)\rho)\left(\frac{2(1-\lambda)^2(3(1-\gamma)+\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}\right)\right), \tag{EC.191}$$

where the third line above follows because $\frac{d}{d\rho} r^c$ increases in q_H and $q_H \geq 4c$ by Assumption EC.1. Then substituting $\rho = 1$,

$$\left(\frac{\partial r^c}{\partial\rho}\right)\bigg|_{\rho=1} \geq \gamma c\left((1-\lambda)\left(1 - \frac{(1-\lambda)^2}{((1-\gamma)(4-\lambda)+\lambda)^2}\right) - (2-\lambda)\left(\frac{2(1-\lambda)^2(3(1-\gamma)+\lambda)}{((1-\gamma)(4-\lambda)+\lambda)^3}\right)\right). \tag{EC.192}$$

Next, it can be shown algebraically that (EC.192) holds if $g(\gamma) > 0$, where

$$g(\gamma) := (1-\lambda)\left(((1-\gamma)(4-\lambda)+\lambda)^2 - (1-\lambda)^2\right)((1-\gamma)(4-\lambda)+\lambda) - (2-\lambda)\left(2(1-\lambda)^2(3(1-\gamma)+\lambda)\right).$$

With some effort, it can be verified that $\frac{\partial^2}{\partial\gamma^2} g > 0$ and $\lim_{\gamma\to\frac{1}{2}} \frac{\partial}{\partial\gamma} g \leq 0$, which together imply $g(\gamma)$ strictly decreases in γ on $\gamma \in [0, \gamma^m]$. It is straightforward to verify that $g(\gamma^m) > 0$ because $\lambda \in [\frac{1}{2}, 1)$ and $\gamma^m \leq \frac{1}{2}$. It follows that $g(\gamma) > 0$ for all $\gamma \in [0, \gamma^m]$. Therefore, $\frac{\partial}{\partial\rho} r^c > 0$ at $\rho = 1$. It remains to show $\frac{\partial^2}{\partial\rho^2} r^c \leq 0$. Following (EC.191), we have

$$\frac{\partial r^c}{\partial\rho} \geq \gamma c\left((1-\lambda)\left(1 - \frac{((1-\lambda)\rho)^2}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2}\right) - (1+(1-\lambda)\rho)\left(\frac{2(1-\lambda)^2(3(1-\gamma)+\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}\right)\right) \tag{EC.193}$$

$$= \gamma c(1-\lambda)\left(1 - \frac{((1-\lambda)\rho)^2}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2} - (1+(1-\lambda)\rho)\left(\frac{2(1-\lambda)(3(1-\gamma)+\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}\right)\right) \tag{EC.194}$$

$$= \gamma c(1-\lambda)\left(1 - \frac{(1-\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3} \cdot k(\rho)\right) \tag{EC.195}$$

$$= \gamma c(1-\lambda)\left(1 - h(\rho)\cdot k(\rho)\right), \tag{EC.196}$$

where

$$k(\rho) := (1-\lambda)\rho((1-\gamma)(3+(1-\lambda)\rho)+\lambda) + 2(1+(1-\lambda)\rho)(3(1-\gamma)+\lambda), \tag{EC.197}$$

$$h(\rho) := \frac{(1-\lambda)\rho}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}. \tag{EC.198}$$

Observe that $k(\rho)$ weakly increases in ρ for all $\gamma \in [0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. It remains to show $h(\rho)$ also increases in ρ. Note

$$\frac{\partial h}{\partial \rho} = \frac{(1-\lambda)(((1-\gamma)(3+(1-\lambda)\rho)+\lambda) - 3\rho(1-\gamma)(1-\lambda))}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^4} = \frac{(1-\lambda)(3(1-\gamma)+\lambda-2\rho(1-\gamma)(1-\lambda))}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^4} \geq 0, \tag{EC.199}$$

where the inequality follows because $\gamma \in [0, \frac{1}{2}]$ and $\lambda \in [\frac{1}{2}, 1]$. The result follows. □

Lemma EC.31 **(Optimal commission rate)**. *Suppose that $\mu = 1$ and $\phi = 0$. Then there exists $\underline{\rho} \in [0,1)$ and $\bar{\rho} \in (\underline{\rho}, 1)$ such that $\gamma^* > \gamma_s^H$ if $\rho > \bar{\rho}$, $\gamma^* = \gamma_s^H$ if $\rho \in [\underline{\rho}, \bar{\rho}]$, and $\gamma^* = \gamma^m$ if $\rho < \underline{\rho}$.*

Proof. Note γ_s^H is continuous and decreasing in ρ and $\lim_{\rho\to 0}\gamma_s^H = 1-\lambda \geq \gamma^m$. It follows that there exists $\underline{\rho} \in (0,1]$ such that $\gamma_s^H < \gamma^m$ if and only if $\rho > \underline{\rho}$. Therefore, if $\rho \leq \underline{\rho}$, then platform revenue is given by $R(\gamma) = r^a(\gamma)$ for $\gamma \leq \gamma^m$. Because $r^a(\gamma)$ strictly increases in γ (Lemma EC.12), we have $\gamma^* = \gamma^m$ for $\rho \leq \underline{\rho}$, as desired. The remainder of the proof addresses the interval $(\underline{\rho}, 1]$. We focus on showing there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $\gamma^* > \gamma_s^H$ if and only if $\rho > \bar{\rho}$; we later strengthen this result to show $\gamma^* = \gamma_s^H$ if $\rho \in (\underline{\rho}, \bar{\rho}]$. For $\mu = 1$, the platform's revenue is

$$R(\gamma) = \begin{cases} r^a(\gamma) & \text{if } \gamma \leq \gamma_s^H, \\ r^c(\gamma) & \text{if } \gamma > \gamma_s^H. \end{cases}$$

Next, for convenience define the function

$$h(\rho) := \max_{\gamma \geq \gamma_s^H} r^c(\gamma) - \max_{\gamma \leq \gamma_s^H} r^a(\gamma), \tag{EC.200}$$

Note $\gamma^* > \gamma_s^H$ if and only if $h(\rho) > 0$. To see why, note $\gamma^* > \gamma_s^H$ immediately implies $h(\rho) > 0$ by definition of $R(\gamma)$. For the reverse direction, it is straightforward to show $r^a(\gamma) > r^c(\gamma)$ for all $\gamma \in (0, \gamma^m]$, meaning $r^c(\gamma)$ cannot attain its maximum over $\gamma \geq \gamma_s^H$ at $\gamma = \gamma_s^H$. Thus, $h(\rho) > 0$ implies $\gamma^* > \gamma_s^H$. It remains to show there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $h(\rho) > 0$ if and only if $\rho \geq \bar{\rho}$. Note for fixed γ, $r^a(\gamma)$ strictly decreases in ρ and $r^c(\gamma)$ strictly increases in ρ (Lemma EC.30). Further, γ_s^H strictly decreases in ρ. It follows that $\max_{\gamma \leq \gamma_s^H} r^a(\gamma)$ strictly decreases in ρ and $\max_{\gamma \leq \gamma_s^H} r^c(\gamma)$ strictly increases in ρ. Hence, $h(\rho)$ strictly increases in ρ. Further, because $\lim_{\rho \to \underline{\rho}} \gamma_s^H = \gamma^m$ and $\lim_{\rho \to 1} \gamma_s^H = 0$, we have $\lim_{\rho \to \underline{\rho}} h(\rho) = -\max_{\gamma \leq \gamma^m} r^a(\gamma) < 0$ and $\lim_{\rho \to 1} h(\rho) = \max_{\gamma \geq 0} r^c(\gamma) > 0$. It follows that there exists $\bar{\rho} \in (\underline{\rho}, 1)$ such that $h(\rho) > 0$ if and only if $\rho > \bar{\rho}$. We conclude $\gamma^* > \gamma_s^H$ if and only if $\rho > \bar{\rho}$. Lastly, note that $\rho \in (\underline{\rho}, \bar{\rho}]$ implies $\gamma^* = \operatorname{argmax}_{\gamma \leq \gamma_s^H} r^a(\gamma)$. Using the expressions in Lemma EC.27, it is straightforward to show $r^a(\gamma)$ strictly increases in γ. It follows that $\gamma^* = \gamma_s^H$ for $\rho \in (\underline{\rho}, \bar{\rho}]$. □

Proposition EC.1. *Suppose the switching cost is $\phi = 0$. Then there exists $\bar{\mu} \in [0, 1)$ and $\bar{\rho} \in [0, 1)$ such that if the share of type-H sellers is large $\mu \geq \bar{\mu}$, the platform's optimal revenue $R(\gamma^*)$ strictly decreases in the learning parameter ρ on $\rho \in [0, \bar{\rho})$ and strictly increases in ρ on $\rho \in (\bar{\rho}, 1]$.*

Proof. The proof largely follows from Lemmas EC.30 and EC.31. Let $\mu = 1$, and let $\bar{\rho}$ be as defined in Lemma EC.31. First suppose $\rho \leq \bar{\rho}$, which implies $\gamma^* = \min\{\gamma_s^H, \gamma^m\}$ by the statement and proof of Lemma EC.31; we consider $\gamma^* = \gamma_s^H$ and $\gamma^* = \gamma^m$ separately. By Lemma EC.28, $\gamma^* = \gamma_s^H$ implies the platform's optimal revenue is $R(\gamma^*) = \mu r^a(\gamma^*) + (1-\mu) r^b(\gamma^*)^+$. Then, for $\rho \in [0, \bar{\rho}]$ we have

$$\left.\frac{dR}{d\rho}\right|_{\gamma=\gamma^*} = \left.\left(\frac{\partial R}{\partial \gamma}\frac{d\gamma_s^H}{d\rho} + \frac{\partial R}{\partial \rho}\right)\right|_{\gamma=\gamma_s^H} \tag{EC.201}$$

$$= \left.\left(\left(\mu\frac{\partial r^a}{\partial \gamma} + (1-\mu)\frac{\partial r^b}{\partial \gamma}\right)\frac{d\gamma_s^H}{d\rho} + \left(\mu\frac{\partial r^a}{\partial \rho} + (1-\mu)\frac{\partial r^b}{\partial \rho}\right)\right)\right|_{\gamma=\gamma_s^H} \tag{EC.202}$$

$$< \left.\left(\mu\frac{\partial r^a}{\partial \gamma} + (1-\mu)\frac{\partial r^b}{\partial \gamma}\right)\frac{d\gamma_s^H}{d\rho}\right|_{\gamma=\gamma_s^H}, \tag{EC.203}$$

where the strict inequality follows because $\frac{\partial}{\partial\rho} r^a < 0$ and $\frac{\partial}{\partial\rho} r^b < 0$ by Lemma EC.30. It remains to show

$$\left.\left(\mu\frac{\partial r^a}{\partial \gamma} + (1-\mu)\frac{\partial r^b}{\partial \gamma}\right)\frac{d\gamma_s^H}{d\rho}\right|_{\gamma=\gamma_s^H} \leq 0. \tag{EC.204}$$

Because $\phi = 0$, we have $\gamma_s^H = 1 - \omega_s$ by Lemma EC.10, which implies

$$\frac{d}{d\rho}\gamma_s^H = -\frac{(1-\lambda)\lambda}{(1-(1-\lambda)\rho)^2} < 0.$$

Further, because $\left(\mu\frac{\partial r^a}{\partial\gamma}+(1-\mu)\frac{\partial r^b}{\partial\gamma}\right)\geq 0$ must hold at $\gamma=\gamma^*$, we conclude that (EC.204) holds. Therefore, $R(\gamma^*)$ strictly decreases in ρ for $\rho\in[0,\bar{\rho})$ if $\gamma^*=\gamma_s^H$. The case where $\gamma^*=\gamma^m$ follows by a similar argument, where the condition (EC.204) (with γ^m in place of γ_s^H) holds trivially because $\frac{d}{d\rho}\gamma^m=0$. We conclude $R(\gamma^*)$ strictly decreases in ρ for $\rho\in[0,\bar{\rho})$. It remains to show that $R(\gamma^*)$ increases in ρ on $(\bar{\rho},1]$. By Lemma EC.31, $\rho>\bar{\rho}$ implies $\gamma_s^H<\gamma^*$, and thus $R(\gamma^*)=\mu r^c(\gamma^*)$. Therefore, we have

$$\left.\frac{dR}{d\rho}\right|_{\gamma=\gamma^*}=\mu\left.\left(\frac{\partial r^c}{\partial\gamma}\frac{d\gamma^*}{d\rho}+\frac{\partial r^c}{\partial\rho}\right)\right|_{\gamma=\gamma^*}=\mu\left.\frac{\partial r^c}{\partial\rho}\right|_{\gamma=\gamma^*}>0. \tag{EC.205}$$

To see why the second equality holds, consider two cases: $\gamma^*=\gamma^m$ and $\gamma^*<\gamma^m$. If $\gamma^*=\gamma^m$, then $\frac{d}{d\rho}\gamma^*=0$; if $\gamma^*<\gamma^m$, then $\frac{\partial}{\partial\rho}r^c=0$ at $\gamma=\gamma^*$ by the envelope theorem. Finally, the strict inequality follows because $\frac{\partial}{\partial\rho}r^c>0$ by Lemma EC.30. We have thus shown $R(\gamma^*)$ strictly increases in ρ on $\rho\in(\bar{\rho},1]$ for $\mu=1$. The existence of the threshold $\bar{\mu}<1$ in the proposition statement then follows by continuity of $\frac{d}{d\rho}R(\gamma^*)$ in μ. □

Proof of Proposition EC.2

Proposition EC.2. *Let $\gamma^*(\phi)$ be the optimal commission rate under switching cost ϕ. There exists $\bar{\phi}>0$ and $\bar{\rho}\in[0,1)$ such that for any $\rho\geq\bar{\rho}$ and $\phi\geq\bar{\phi}$, the optimal commission rate is higher in the absence of switching costs, $\gamma^*(0)\geq\gamma^*(\phi)$, where the inequality is strict if $\gamma^*(\phi)<\gamma^m$.*

Proof. Note by Lemma EC.29, the platform's revenue is

$$R(\gamma)=\begin{cases}\mu r^a(\gamma)+(1-\mu)r^b(\gamma)^+ & \text{if } \gamma\in[0,\gamma_s^H],\\ \mu r^c(\gamma)+(1-\mu)r^b(\gamma)^+ & \text{if } \gamma\in(\gamma_s^H,\gamma_s^L],\\ \mu r^c(\gamma)+(1-\mu)r^d(\gamma)^+ & \text{if } \gamma\in(\gamma_s^L,\gamma^m].\end{cases} \tag{EC.206}$$

Further, by Lemma EC.28, we have $\gamma_s^H=\gamma_s^L=0$ at $\phi=0$ and $\rho=1$. By continuity of γ_s^H and γ_s^L in ρ, it follows that there exists $\hat{\rho}\in[0,1]$ such that $\gamma^*=\min\{\gamma^{cd},\gamma^m\}$ if $\phi=0$ and $\rho\geq\hat{\rho}$. Similarly, because γ_s^H strictly increases in ϕ (Lemma EC.28), there exists $\bar{\phi}>0$ such that $\gamma^*=\min\{\gamma^{ab},\gamma^m\}$ if $\phi\geq\bar{\phi}$. Because γ^{ab} does not depend on ϕ, it remains to show there exists $\bar{\rho}\geq\hat{\rho}$ such that $\gamma^{ab}<\gamma^{cd}$ if $\rho\geq\bar{\rho}$ and $\phi=0$, which we do in four steps. First, we define four auxilliary functions, $\ell^x(\gamma)$ for $x\in\{a,b,c,d\}$, which have the useful property that $\ell^x(\gamma^x)=0$. Second, we show there exists $\bar{\rho}\in[\hat{\rho},1)$ such that $\ell^a(\gamma)<\ell^c(\gamma)$ for all $\gamma\in[0,\gamma^m]$ if $\rho\geq\bar{\rho}$. Third, we show $\ell^b(\gamma)<\ell^d(\gamma)$ for all $\gamma\in[0,\gamma^m]$ and $\rho\in[0,1]$. Fourth, we combine the first three steps to prove the proposition statement.

Step 1. Fix $\phi = 0$. We begin by defining four auxiliary functions $\ell^a(\gamma)$, $\ell^b(\gamma)$, $\ell^c(\gamma)$, and $\ell^d(\gamma)$. Note differentiating $r^a(\gamma)$ in γ yields

$$\frac{\partial r^a}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\gamma\left(\frac{q_H}{2} - \frac{(\rho c(1-\lambda))^2}{2q_H(1-\gamma)^2}\right)\right\} = \frac{q_H}{2}\underbrace{\left(1 - \frac{(\rho c(1-\lambda))^2}{q_H^2}\cdot\frac{(1+\gamma)}{(1-\gamma)^3}\right)}_{\ell^a(\gamma)}, \tag{EC.207}$$

where $\ell^a(\gamma)$ is defined as shown in (EC.207). Similarly, for $r^b(\gamma)$,

$$\frac{\partial r^b}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\frac{q_L(2-(1-\lambda)\rho)}{4}\left(\gamma - \frac{(c(1-\lambda)(2-\rho)\rho)^2}{q_L^2(2-(1-\lambda)\rho)^2}\cdot\frac{\gamma}{(1-\gamma)^2}\right)\right\} \tag{EC.208}$$

$$= \frac{q_L(2-(1-\lambda)\rho)}{4}\underbrace{\left(1 - \frac{(c(1-\lambda)(2-\rho)\rho)^2}{q_L^2(2-(1-\lambda)\rho)^2}\cdot\frac{(1+\gamma)}{(1-\gamma)^3}\right)}_{\ell^b(\gamma)}. \tag{EC.209}$$

For $r^c(\gamma)$,

$$\frac{\partial r^c}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\frac{q_H}{4}(1+(1-\lambda)\rho)\left(\gamma - \frac{(4c(1-\lambda)\rho)^2}{q_H^2}\cdot\frac{\gamma}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^2}\right)\right\} \tag{EC.210}$$

$$= \frac{q_H}{4}(1+(1-\lambda)\rho)\underbrace{\left(1 - \frac{(4c(1-\lambda)\rho)^2}{q_H^2}\cdot\frac{(1+\gamma)(3+(1-\lambda)\rho)+\lambda}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}\right)}_{\ell^c(\gamma)}. \tag{EC.211}$$

Lastly, for $r^d(\gamma)$,

$$\frac{\partial r^d}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\frac{q_L}{4}\left(\gamma - \frac{(2c(1-\lambda)(2-\rho)\rho)^2}{q_L^2}\cdot\frac{\gamma}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2}\right)\right\} \tag{EC.212}$$

$$= \frac{q_L}{4}\underbrace{\left(1 - \frac{(2c(1-\lambda)(2-\rho)\rho)^2}{q_L^2}\cdot\frac{(1+\gamma)(3-(1-\lambda)\rho)+\lambda}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^3}\right)}_{\ell^d(\gamma)}. \tag{EC.213}$$

Further, note for $x \in \{a,b,c,d\}$ we have $\ell^x(\gamma^x) = 0$.

Step 2. We now show there exists $\bar{\rho} \in [\hat{\rho}, 1)$ such that $\ell^a(\gamma) < \ell^c(\gamma)$ for all $\gamma \in [0, \gamma^m]$ if $\rho \geq \bar{\rho}$. Using the expressions given in (EC.207) and (EC.211), note $\ell^a(\gamma) < \ell^c(\gamma)$ holds if and only if

$$\frac{(\rho c(1-\lambda))^2}{q_H^2}\cdot\frac{1+\gamma}{(1-\gamma)^3} > \frac{(4c(1-\lambda)\rho)^2}{q_H^2}\cdot\frac{(1+\gamma)(3+(1-\lambda)\rho)+\lambda}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3}, \tag{EC.214}$$

or equivalently, $h(\gamma) < 0$, where

$$h(\gamma) = \frac{16((1+\gamma)(3+(1-\lambda)\rho)+\lambda)}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^3} - \frac{1+\gamma}{(1-\gamma)^3}.$$

It remains to show that for any $\gamma \in (0, \gamma^m]$, $h(\gamma) < 0$ if $\rho > \bar{\rho}$ for some $\bar{\rho} \in [\hat{\rho}, 1)$. First, observe that

$$\begin{aligned}
\lim_{\rho \to 1} h(\gamma) &= 16 \frac{(1+\gamma)(4-\lambda)+\lambda}{((1-\gamma)(4-\lambda)+\lambda)^3} - \frac{1+\gamma}{(1-\gamma)^3} \\
&= 16 \frac{1+\gamma}{(1-\gamma)^3} \frac{(4-\lambda)+\frac{\lambda}{1+\gamma}}{((4-\lambda)+\frac{\lambda}{1-\gamma})^3} - \frac{1+\gamma}{(1-\gamma)^3} \\
&< 16 \frac{1+\gamma}{(1-\gamma)^3} \frac{(4-\lambda)+\lambda}{((4-\lambda)+\lambda)^3} - \frac{1+\gamma}{(1-\gamma)^3} \\
&= 16 \frac{1+\gamma}{(1-\gamma)^3} \frac{1}{4^2} - \frac{1+\gamma}{(1-\gamma)^3} \\
&= 0,
\end{aligned}$$

where the strict inequality follows because $\gamma \in (0, \gamma^m]$. We also have

$$\frac{\partial h}{\partial \rho} = -\frac{32(1-\lambda)\left((1-\gamma^2)(3+(1-\lambda)\rho)+\lambda(1-2\gamma)\right)}{((1-\gamma)(3+(1-\lambda)\rho)+\lambda)^4} \leq 0,$$

where the inequality follows because $\gamma \in [0, \frac{1}{2}]$. We have thus shown that for any $\gamma \in [0, \gamma^m]$, $h(\gamma)$ strictly decreases in ρ and $\lim_{\rho \to 1} h(\gamma) < 0$. It follows that for each $\gamma \in [0, \gamma^m]$, there exists $\bar{\rho}(\gamma) \in [0, 1)$ such that $h(\gamma) < 0$ and thus $\ell^a(\gamma) < \ell^c(\gamma)$ if $\rho > \bar{\rho}(\gamma)$. The result follows by choosing $\bar{\rho}$ to be the larger of $\max_{\gamma \geq 0} \bar{\rho}(\gamma)$ and the threshold $\hat{\rho}$ defined at the beginning of the proof.

Step 3. The proof is similar to Step 2. Using the expressions from (EC.208) and (EC.213), it can be shown that $\ell^b(\gamma) < \ell^d(\gamma)$ holds if and only if $g(\gamma) < 0$, where

$$g(\gamma) = \frac{4((1+\gamma)(3-(1-\lambda)\rho)+\lambda)}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^3} - \frac{1+\gamma}{(1-\gamma)^3}.$$

We show that for any $\gamma \in (0, \gamma^m]$, $g(\gamma) < 0$ for all $\rho \in [0, 1]$ and $\lambda \in [\frac{1}{2}, 1]$. We first show $g(\gamma)$ decreases in λ. To see this, note

$$\begin{aligned}
\frac{\partial g}{\partial \lambda} &= \frac{\partial}{\partial \lambda} \left\{ \frac{4}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2} \cdot \frac{(1+\gamma)(3-(1-\lambda)\rho)+\lambda}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)} - \frac{1+\gamma}{(1-\gamma)^3} \right\} && \text{(EC.215)} \\
&= \frac{\partial}{\partial \lambda} \left\{ \frac{4}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2} \right\} \cdot \frac{(1+\gamma)(3-(1-\lambda)\rho)+\lambda}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)} && \text{(EC.216)} \\
&\quad + \frac{4}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2} \cdot \frac{\partial}{\partial \lambda} \left\{ \frac{(1+\gamma)(3-(1-\lambda)\rho)+\lambda}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)} \right\} && \text{(EC.217)} \\
&< -\frac{4}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2} \cdot \frac{2\gamma(3-\rho)}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2} && \text{(EC.218)} \\
&< 0. && \text{(EC.219)}
\end{aligned}$$

The first strict inequality above follows because $(1-\gamma)(3-(1-\lambda)\rho)+\lambda$ strictly increases in λ, which implies $\frac{\partial}{\partial\lambda}\left\{\frac{4}{((1-\gamma)(3-(1-\lambda)\rho)+\lambda)^2}\right\}<0$. Because $g(\gamma)$ strictly decreases in λ, plugging in $\lambda=0$ yields the upper bound

$$g(\gamma)<\frac{4(1+\gamma)(3-\rho)}{((1-\gamma)(3-\rho))^3}-\frac{1+\gamma}{(1-\gamma)^3}=\frac{1+\gamma}{(1-\gamma)^3}\left(\frac{4}{(3-\rho)^2}-1\right)\leq 0$$

for $\rho\in[0,1]$ and $\lambda\in[\frac{1}{2},1]$. We have thus shown $g(\gamma)<0$, which implies $\ell^b(\gamma)<\ell^d(\gamma)$.

Step 4. We now complete the proof by showing $\gamma^{cd}>\gamma^{ab}$ holds for $\phi=0$ and $\rho\geq\bar{\rho}$. Analogous to Lemma EC.12, it can be shown that $r^x(\gamma)$ is strictly concave in γ for $x\in\{a,b,c,d\}$. It follows that $\mu r^a(\gamma)+(1-\mu)r^b(\gamma)$ and $\mu r^c(\gamma)+(1-\mu)r^d(\gamma)$ are both strictly concave in γ. Therefore, to show $\gamma^{cd}>\gamma^{ab}$, it suffices to show

$$\left(\mu\frac{\partial r^c}{\partial\gamma}+(1-\mu)\frac{\partial r^d}{\partial\gamma}\right)\bigg|_{\gamma=\gamma^{ab}}>0. \tag{EC.220}$$

Using the expressions for $\ell^a(\gamma)$ and $\ell^b(\gamma)$, it is straightforward to verify that $\gamma^b\leq\gamma^a$. Further, because $r^a(\gamma)$ and $r^b(\gamma)$ are both strictly concave, we must have $\gamma^b\leq\gamma^{ab}\leq\gamma^a$. By Step 2, we have $\gamma^a<\gamma^c$ for $\rho\geq\bar{\rho}$, which implies $\gamma^{ab}<\gamma^c$ and thus $\frac{\partial}{\partial\gamma}r^c>0$ at $\gamma=\gamma^{ab}$ for $\rho\geq\bar{\rho}$. Therefore, (EC.220) follows immediately if $\frac{\partial}{\partial\gamma}r^d\geq 0$ at $\gamma=\gamma^{ab}$. It remains to show (EC.220) also holds when $\frac{\partial}{\partial\gamma}r^d<0$ at $\gamma=\gamma^{ab}$. By Step 2, for each $\gamma\in(0,\gamma^m]$ we have $\ell^a(\gamma)<\ell^c(\gamma)$ for $\rho\geq\bar{\rho}$. It follows that

$$\frac{\partial r^c/\partial\gamma}{\partial r^a/\partial\gamma}=\frac{1+(1-\lambda)\rho}{2}\cdot\frac{\ell^c(\gamma)}{\ell^a(\gamma)}>\frac{1+(1-\lambda)\rho}{2}. \tag{EC.221}$$

Similarly, by Step 3 we have $\ell^b(\gamma)<\ell^d(\gamma)$ for any $\gamma\in(0,\gamma^m]$, which implies

$$\frac{\partial r^d/\partial\gamma}{\partial r^b/\partial\gamma}=\frac{1}{2-(1-\lambda)\rho}\cdot\frac{\ell^d(\gamma)}{\ell^b(\gamma)}>\frac{1}{2-(1-\lambda)\rho}. \tag{EC.222}$$

We can now write for $\rho\geq\bar{\rho}$,

$$\left(\mu\frac{\partial r^c}{\partial\gamma}+(1-\mu)\frac{\partial r^d}{\partial\gamma}\right)\bigg|_{\gamma=\gamma^{ab}}>\left(\mu\frac{\partial r^a}{\partial\gamma}\cdot\frac{1+(1-\lambda)\rho}{2}+(1-\mu)\frac{\partial r^b}{\partial\gamma}\cdot\frac{1}{2-(1-\lambda)\rho}\right)\bigg|_{\gamma=\gamma^{ab}} \tag{EC.223}$$

$$\geq\frac{1+(1-\lambda)\rho}{2}\left(\mu\frac{\partial r^a}{\partial\gamma}+(1-\mu)\frac{\partial r^b}{\partial\gamma}\right)\bigg|_{\gamma=\gamma^{ab}} \tag{EC.224}$$

$$\geq 0, \tag{EC.225}$$

where the strict inequality follows from applying (EC.221) and (EC.222) and noting that $\gamma^{ab}\leq\gamma^a\leq\gamma^c$ implies $\frac{\partial}{\partial\gamma}r^a\geq 0$ and $\frac{\partial}{\partial\gamma}r^c\geq 0$ at $\gamma=\gamma^{ab}$; the second inequality holds because $\frac{1+(1-\lambda)\rho}{2}\geq$

$\frac{1}{2-(1-\lambda)\rho}$ for all $\lambda \in [\frac{1}{2}, 1]$ and $\rho \in [0,1]$, and because $\gamma^b \le \gamma^{ab}$ implies $\frac{\partial}{\partial \gamma} r^b \le 0$ at $\gamma = \gamma^{ab}$; and the final inequality follows by definition of γ^{ab}. Because $\gamma^* = \min\{\gamma^{ab}, \gamma^m\}$ for all $\phi \ge \bar{\phi}$ and $\gamma^* = \min\{\gamma^{cd}, \gamma^m\}$ for $\phi = 0$ as established at the beginning of the proof, we conclude $\gamma^*(0) \ge \gamma^*(\phi)$ for all $\phi \ge \bar{\phi}$ and $\rho \ge \bar{\rho}$. Finally, to see that the inequality is strict wherever $\gamma^*(\phi) < \gamma^m$, note $\gamma^*(\phi) < \gamma^m$ implies $\gamma^*(\phi) = \gamma^{ab} < \min\{\gamma^{cd}, \gamma^m\} = \gamma^*(0)$. □