Online Companion for

"Marketable Pollution Permits in Oligopolistic Markets with Transaction Costs"

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Table 1: **Example 1 - Input Data**

Production cost parameters

Firm i	c_{i1}	c_{i2}	K_{i1}	K_{i2}	β_{i1}	β_{i2}
1	2.	5.	5.	4.	1.2	1.9
2	6.	7.	3.	6.	1.9	1.8

Emission cost parameters

Firm i	$g1_{i1}$	$g1_{i2}$	$g2_{i1}$	$g2_{i2}$	$g3_{i1}$	$g3_{i2}$	$g4_{i1}$	$g4_{i2}$
1	1.4	1.8	-10.	-20.	1.5	1.5	10.	14.
2	1.4	2.7	-15.	-5.	3.5	2.5	5.	15.

Transaction cost parameters

Firm i	$\phi1_{i11}$	$\phi1_{i12}$	$\phi1_{i21}$	$\phi1_{i22}$	$\phi2_{i11}$	$\phi2_{i12}$
1	0.09	0.05	0.05	0.01	-7.	-8.
2	0.03	0.04	0.09	0.05	-8.	-7.

Firm i	$\phi2_{i21}$	$\phi2_{i22}$	α_{i11}	α_{i12}	α_{i21}	α_{i22}
1	-1.	-5.	0.004	0.009	0.005	0.005
2	-5.	-8.	0.002	0.006	0.003	0.006

Appendices for Nagurney and Dhanda

Marketable Pollution Permits in Oligopolistic Markets with Transaction Costs

Table 2: **Example 1 - Output Data**

Equilibrium production and emission vectors

Firm i	${q_{i1}}^*$	${q_{i2}}^*$	e_i^{1*}	e_i^{2*}
1	75.710	86.970	0.516	1.165
2	66.547	72.257	5.371	0.017

Equilibrium license and marginal abatement cost vectors

Firm i	l_{i1}^{1*}	l_{i1}^{2*}	l_{i2}^{1*}	l_{i2}^{2*}
1	0.257	5.831	1.291	5.831
2	5.734	0.169	4.709	0.169

Firm i	λ_{i1}^{1*}	λ_{i1}^{2*}	λ_{i2}^{1*}	λ_{i2}^{2*}
1	0.700	0.061	3.282	3.100
2	0.000	0.491	0.000	0.000

Equilibrium price vector

Receptor Point j	p_j^{1*}	p_j^{2*}
1	7.654	7.477
2	4.153	7.983

New equilibrium price vector

Receptor Point j	p_j^{1*}	p_j^{2*}
1	6.415	6.708
2	2.230	7.127

Table 3: **Example 2 - Input Data**

Production cost parameters

Firm i	c_{i1}	c_{i2}	K_{i1}	K_{i2}	β_{i1}	β_{i2}
1	2.	5.	5.	4.	1.2	1.9
2	6.	7.	3.	6.	1.9	1.8
3	4.9	6.4	2	4	2.5	2.1

Emission cost parameters

Firm i	$g1_{i1}$	$g1_{i2}$	$g2_{i1}$	$g2_{i2}$	$g3_{i1}$	$g3_{i2}$	$g4_{i1}$	$g4_{i2}$
1	1.4	1.8	-10.	-20.	1.5	1.5	10.	14.
2	1.4	2.7	-15.	-5.	3.5	2.5	5.	15.
3	1.7	2.3	-5.	-10.	4.1	3.1	6.	2.

Transaction cost parameters

Firm i	$\phi 1_{i11}$	$\phi 1_{i12}$	$\phi 1_{i21}$	$\phi 1_{i22}$	$\phi 2_{i11}$	$\phi 2_{i12}$
1	0.09	0.05	0.05	0.01	-7.	-8.
2	0.03	0.04	0.09	0.05	-8.	-7.
3	0.07	0.04	0.03	0.06	-5.	-1.

Firm i	$\phi 2_{i21}$	$\phi 2_{i22}$	α_{i11}	α_{i12}	α_{i21}	α_{i22}
1	-1.	-5.	0.004	0.009	0.005	0.005
2	-5.	-8.	0.002	0.006	0.003	0.006
3	-8.	-3.	0.004	0.007	0.006	0.009

Table 4: **Example 2 - Output Data**

Equilibrium production and emission vectors

Firm i	${q_{i1}}^*$	${q_{i2}}^*$	e_i^{1*}	e_i^{2*}
1	71.768	83.491	0.000	1.440
2	55.597	61.956	4.815	0.171
3	67.693	61.693	0.340	0.000

Equilibrium license and marginal abatement cost vectors

Firm i	l_{i1}^{1*}	l_{i1}^{2*}	l_{i2}^{1*}	l_{i2}^{2*}
1	0.000	7.290	0.000	7.196
2	8.490	1.170	2.408	1.804
3	0.510	0.000	6.592	0.000

Firm i	λ_{i1}^{1*}	λ_{i1}^{2*}	λ_{i2}^{1*}	λ_{i2}^{2*}
1	0.000	0.000	4.151	2.964
2	0.000	0.408	3.037	0.000
3	2.562	1.852	0.000	0.000

Equilibrium price vector

Receptor Point j	p_j^{1*}	p_j^{2*}
1	7.491	7.271
2	7.604	7.820

Table 5: **Example 3 - Input Data**

Production cost parameters

Firm i	c_{i1}	c_{i2}	K_{i1}	K_{i2}	β_{i1}	β_{i2}
1	2.	5.	5.	4.	1.2	1.9
2	6.	7.	3.	6.	1.9	1.8
3	4.9	6.4	2	4	2.5	2.1

Emission cost parameters

Firm i	$g1_{i1}$	$g1_{i2}$	$g2_{i1}$	$g2_{i2}$	$g3_{i1}$	$g3_{i2}$	$g4_{i1}$	$g4_{i2}$
1	1.4	1.8	-10.	-20.	1.5	1.5	10.	14.
2	1.4	2.7	-15.	-5.	3.5	2.5	5.	15.
3	1.7	2.3	-5.	-10.	4.1	3.1	6.	2.

Transaction cost parameters

Firm i	$\phi 1_{i11}$	$\phi 1_{i12}$	$\phi 1_{i21}$	$\phi 1_{i22}$	$\phi 1_{i31}$	$\phi 1_{i32}$
1	0.09	0.05	0.05	0.01	0.08	0.02
2	0.03	0.04	0.09	0.05	0.07	0.09
3	0.07	0.04	0.03	0.06	0.03	0.08

Firm i	$\phi 2_{i11}$	$\phi 2_{i12}$	$\phi 2_{i21}$	$\phi 2_{i22}$	$\phi 2_{i31}$	$\phi 2_{i32}$
1	-7.	-8.	-1.	-5.	-2.	-5.
2	-8.	-7.	-5.	-8.	-6.	-5.
3	-5.	-1.	-8.	-3.	-4.	-5.

Table 6: **Example 3 - Output Data**

Equilibrium production and emission vectors

Firm i	${q_{i1}}^*$	${q_{i2}}^*$	e_i^{1*}	e_i^{2*}
1	71.768	83.498	0.000	1.440
2	55.595	61.953	2.699	0.171
3	67.514	61.688	0.008	0.000

Equilibrium license and marginal abatement cost vectors

Firm i	l_{i1}^{1*}	l_{i1}^{2*}	l_{i2}^{1*}	l_{i2}^{2*}	l_{i3}^{1*}	l_{i3}^{2*}
1	0.000	7.291	0.000	7.197	0.000	6.111
2	8.987	1.709	1.349	1.803	8.997	1.360
3	0.013	0.000	7.652	0.000	0.003	1.530

Firm i	λ_{i1}^{1*}	λ_{i1}^{2*}	λ_{i2}^{1*}	λ_{i2}^{2*}	λ_{i3}^{1*}	λ_{i3}^{2*}
1	0.000	0.000	3.198	2.964	1.659	0.000
2	0.000	0.408	2.782	0.000	1.815	0.000
3	2.462	1.499	0.000	0.000	2.556	2.434

Equilibrium price vector

Receptor Point j	p_j^{1*}	p_j^{2*}
1	7.461	7.271
2	7.540	7.820
3	6.556	4.755