Appendix

Let $a_t^{i,j}$ be the attraction of the j^{th} wrong choice chosen by agent i in trial t. j varies from 1 to $(m-1)$. Let A_k^i be the attraction of the right choice chosen by agent i in trial t.

Let PFN_t be the probability of a "false negative", PTN_t be the probability of a "true negative" and PTP_t be the probability of a "true positive" in trial t respectively.

Proof of Proposition 1:

We need to show

$$PFN_1^{NN} > PFN_1^{BB},\ PFN_1^{GB} > PFN_1^{BB} \text{ and}$$

$$PTN_1^{BB} > PTN_1^{NN},\ PTN_1^{BB} > PTN_1^{GB}.$$

For the No-No representations (Case 0), the initial attractions of the agents wrong and right choices using Equation 1 are:

$$a_1^{1,j} = a_1^{2,j} = a_1 = A_1 = p\Pi_{max} + (1-p)\Pi_{min}.$$

Substituting $\Pi_{max} = \Pi_{min} + \Delta$ and using Equation 2, we calculate

$$PFN_1^{NN} = \frac{2(m-1)}{m^2}, \tag{4}$$

$$PTN_1^{NN} = \left[\frac{m-1}{m}\right]^2. \tag{5}$$

For the Bad-Bad representations (Case 2), the initial attractions of the agents $(m-2)$ wrong choices are

$$a_1^{1,j} = a_1^{2,j} = p\Pi_{min} + (1-p)\Pi_{max},$$

and for the remaining wrong choice is

$$a_1^{1,j'} = a_1^{2,j'} = p\Pi_{max} + (1-p)\Pi_{min}.$$

The initial attractions of the agents right choices are

$$A_1^1 = A_1^2 = p\Pi_{min} + (1-p)\Pi_{max}.$$

Substituting $\Pi_{max} = \Pi_{min} + \Delta$ and using Equation 2, we calculate

$$PFN_1^{BB} = \frac{2e^{\frac{\Delta}{\tau}}\left(e^{\frac{2p_0\Delta}{\tau}} + (m-2)e^{\frac{\Delta}{\tau}}\right)}{\left(e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta}{\tau}}\right)^2},$$

$$PTN_1^{BB} = \frac{\left(e^{\frac{2p_0\Delta}{\tau}} + (m-2)e^{\frac{\Delta}{\tau}}\right)^2}{\left(e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta}{\tau}}\right)^2}.$$

For the Good-Bad representations (Case 4), the first agent has a good representation and the second agent has a bad representation. The initial attractions of the first agent's wrong choices are

$$a_1^{1,j} = p\Pi_{min} + (1-p)\Pi_{max},$$

and for the right choice is

$$A_1^1 = p\Pi_{max} + (1-p)\Pi_{min}.$$

The initial attractions of the second agent's $(m-2)$ wrong choices are

$$a_1^{2,j} = p\Pi_{min} + (1-p)\Pi_{max},$$

for the remaining wrong choice is

$$a_1^{2,j'} = p\Pi_{max} + (1-p)\Pi_{min},$$

and for the right choice is

$$A_1^2 = p\Pi_{min} + (1-p)\Pi_{max}.$$

Substituting $\Pi_{max} = \Pi_{min} + \Delta$ and using Equation 2, we calculate

$$PFN_1^{GB} = \frac{e^{\frac{4p_0\Delta}{\tau}} + (m-2)e^{\frac{\Delta+2p_0\Delta}{\tau}} + (m-1)e^{\frac{2\Delta}{\tau}}}{\left(e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta}{\tau}}\right)^2},$$

$$PTN_1^{GB} = \frac{(m-1)e^{\frac{\Delta}{\tau}}\left(e^{\frac{2p_0\Delta}{\tau}} + (m-2)e^{\frac{\Delta}{\tau}}\right)}{\left(e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta}{\tau}}\right)^2}.$$

Now,

$$PFN_1^{NN} - PFN_1^{BB} = N_1/D_1$$

$$PFN_1^{GB} - PFN_1^{BB} = N_2/D_1$$

where

$$\begin{aligned} N_1 &= -2\left(e^{\frac{\Delta}{\tau}} - e^{\frac{2p_0\Delta}{\tau}}\right)\left((m-1)e^{\frac{2p_0\Delta}{\tau}} + (m^2-3m+1)e^{\frac{\Delta}{\tau}}\right), \\ N_2 &= m^2\left(e^{\frac{4p_0\Delta}{\tau}} - (m-3)e^{\frac{2\Delta}{\tau}} + (m-4)e^{\frac{\Delta+2p_0\Delta}{\tau}}\right), \end{aligned}$$

and

$$D_1 = m^2\left(e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta}{\tau}}\right)^2.$$

Therefore, for $m \geq 2, p_0 > 0.5, \Delta > 0$,

$$PFN_1^{NN} > PFN_1^{BB} \text{ and } PFN_1^{GB} > PFN_1^{BB}.$$

Now,

$$PTN_1^{NN} - PTN_1^{BB} = N_3/D_1$$

$$PTN_1^{GB} - PTN_1^{BB} = N_4/D_1$$

where

$$\begin{aligned} N_3 &= \left(e^{\frac{\Delta}{\tau}} - e^{\frac{2p_0\Delta}{\tau}}\right)\left((2m-1)e^{\frac{2p_0\Delta}{\tau}} + (2m^2-4m+1)e^{\frac{\Delta}{\tau}}\right), \\ N_4 &= m^2\left(e^{\frac{\Delta}{\tau}} - e^{\frac{2p_0\Delta}{\tau}}\right)\left(e^{\frac{2p_0\Delta}{\tau}} + (m-2)e^{\frac{\Delta}{\tau}}\right), \end{aligned}$$

and D_1 is defined before.

Therefore, for $m \geq 2, p_0 > 0.5, \Delta > 0$,

$$PTN_1^{BB} > PTN_1^{NN} \text{ and } PTN_1^{BB} > PTN_1^{GB}.$$

Remark 1

$$\lim_{p_0 -> \frac{1}{2}} \left[PFN_1^{NN} - PFN_1^{BB}\right] = \lim_{p_0 -> \frac{1}{2}} \left[PFN_1^{GB} - PFN_1^{BB}\right]$$
$$= \lim_{p_0 -> \frac{1}{2}} \left[PTN_1^{BB} - PTN_1^{NN}\right] = \lim_{p_0 -> \frac{1}{2}} \left[PTN_1^{BB} - PTN_1^{GB}\right] \to 0.$$

Proof of Proposition 2:

We need to show

$$\frac{\partial}{\partial \phi}\left[\frac{PTP_2^{BB}}{PTP_2^{NN}}\right] > 0 \quad \text{and} \quad \frac{\partial}{\partial \phi}\left[\frac{PTP_2^{BB}}{PTP_2^{GB}}\right] > 0.$$

Let the agents pick the most likely choices in the first trial. Then the most likely outcome for the Good-Bad case is false negative, and for the Bad-Bad and No-No cases it is true negative.

Conditional on the most likely outcome in the first trial, the probability of true positive in the second trial for different cases are as follows ($\Pi_{min} + \Delta$ is substituted for Π_{max}):

$$\begin{aligned} PTP_2^{NN} &= \frac{1}{\left(m - 1 + e^{-\frac{p_0 \Delta \phi}{\tau}}\right)^2}, \\ PTP_2^{GB} &= \frac{e^{\frac{\Delta(1+p_0(2+\phi))}{\tau}}}{\left((m-1)e^{\frac{\Delta(1+p_0\phi)}{\tau}} + e^{\frac{2p_0\Delta}{\tau}}\right)^2}, \\ PTP_2^{BB} &= \frac{e^{\frac{2\Delta(1+p_0\phi)}{\tau}}}{\left((m-1)e^{\frac{\Delta(1+p_0\phi)}{\tau}} + e^{\frac{2p_0\Delta}{\tau}}\right)^2}. \end{aligned}$$

Now,

$$\frac{\partial}{\partial\phi}\left[\frac{PTP_2^{BB}}{PTP_2^{NN}}\right] = \frac{2p_0\Delta(m-1)e^{\frac{\Delta(2+p_0\phi)}{\tau}}\left[e^{\frac{2p_0\Delta}{\tau}} - e^{\frac{\Delta}{\tau}}\right]\left[1+(m-1)e^{\frac{p_0\Delta\phi}{\tau}}\right]}{\tau\left[e^{\frac{2p_0\Delta}{\tau}} + (m-1)e^{\frac{\Delta(1+p_0\phi)}{\tau}}\right]^3},$$

and

$$\frac{\partial}{\partial\phi}\left[\frac{PTP_2^{BB}}{PTP_2^{GB}}\right] = \frac{p_0\Delta}{\tau}e^{\frac{\Delta(1+p_0(\phi-2))}{\tau}}.$$

Therefore, for $m \geq 2, p_0 > 0.5, \Delta > 0$ and $\tau > 0$,

$$\frac{\partial}{\partial\phi}\left[\frac{PTP_2^{BB}}{PTP_2^{NN}}\right] > 0 \ \text{ and } \ \frac{\partial}{\partial\phi}\left[\frac{PTP_2^{BB}}{PTP_2^{GB}}\right] > 0.$$

Remark 2 *We condition on the most likely outcome in the first trial because we assume that the probability of occurrences of other outcomes is negligible. It is easy to show that Proposition 2 holds even if we do not assume that the probability of false negative in the first trial for the No-No case is negligible.*

Remark 3 $PTP_2^{BB}|_{\phi'} = PTP_2^{GB}|_{\phi'}$ *for* $\phi' = \frac{2p_0-1}{p_0}$. *For all* $\phi > \phi', PTP_2^{BB} > PTP_2^{GB}$.