Multiple Organization Goals with Feedback from Shared Technological Task Environments

Appendix A: VAR with panel data[1]

A VAR is a multivariate simultaneous equation system, in which each variable under study is regressed on the current values of all other variables and a finite number of lags of all variables jointly considered. The VAR approach is useful when the intention is to analyze a phenomenon without having any strong priors about competing explanations of it. The method focuses on deriving a good statistical representation of the interactions between variables, letting the data determine the model. In a simple bivariate system, a first-order vector autoregression model can be written as follows:

$$y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \qquad (2)$$

$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \qquad (3)$$

The time path of $\{y_t\}$ is affected by current and past realizations of the $\{z_t\}$ sequence, and the time path of $\{z_t\}$ is affected by current and past realizations of the $\{y_t\}$ sequence. Both y_t and z_t are stationary. The errors $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ are uncorrelated white-noise disturbances with standard variances. The model presented by Equations (2) and (3) is called "*structural*" or "*primitive*" VAR since y_t has a contemporaneous effect ($-b_{21}$) on z_t and z_t has a contemporaneous effect ($-b_{12}$) on y_t. If b_{21} is not equal to zero, ε_{yt} has an indirect contemporaneous effect on z_t, and, if b_{12} is not equal to zero, ε_{zt} has an indirect contemporaneous effect on y_t. In this case, these equations *cannot* be estimated directly due to the correlation of y_t with ε_{yt} and of z_t with ε_{zt}. In short, the structural form describes the theoretical economic relationship between variables. However, it is not designed to get the estimates of model coefficients, due to endogeneity problems when one endogenous variable is regressed on another.

Fortunately, Equations (2) and (3) can be rewritten in the compact form using matrix algebra:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

or

[1] The materials on VAR with panel data are largely adapted from Chapter 5 of Enders' (2010) book.

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t \qquad (4)$$

where $B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$, $x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$, $\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$, $\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$

Premultiplication by B^{-1} allows us to obtain the VAR model in *standard* or *reduced* form:

$$x_t = A_0 + A_1 x_{t-1} + e_t \qquad (5)$$

where $A_0 = B^{-1}\Gamma_0$, $A_1 = B^{-1}\Gamma_1$, and $e_t = B^{-1}\varepsilon_t$.

For notational purpose, we can define a_{i0} as element i of the vector A_0, a_{ij} as the element in row i and column j of the matrix A_1, and e_{it} as element i of the vector e_t. Thus, we can rewrite Equation (5) in the equivalent form:

$$y_t = a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t} \qquad (6)$$

$$z_t = a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t} \qquad (7)$$

In the *standard* or *reduced* form of the model, the errors e_t are composites of the white-noise processes ε_t and therefore have zero means and constant variances and are individually serially uncorrelated. However, the covariance of the e_{1t} and e_{2t} shocks is not in general equal to zero. The VAR model in reduced form does not present the estimation problems of structural form. The OLS method gives unbiased estimates of the elements of the matrices A_0 and A_1 and of the variance-covariance matrix of the errors $\{e_t\}$. The reduced form is designed to get the unbiased estimates of the model. Often, we need to recover the information regarding estimates of the structural model from the estimates of the reduced model. However, the estimation of the *reduced* model yields fewer estimates than the number of parameters of the *structural* model. For instance, estimating (6) and (7) yields six coefficient estimates (a_{10}, a_{11}, a_{12}, a_{20}, a_{21}, and a_{22}) and the calculated values of var(e_{1t}), var(e_{2t}), and cov(e_{1t},e_{2t}). By contrast, the primitive system (2) and (3) contains ten parameters: the two intercept coefficients b_{10} and b_{20}, the four autoregressive coefficients γ_{11}, γ_{12}, γ_{21}, and γ_{22}, the two feedback coefficients b_{12} and b_{21}, and the two standard deviations σ_y and σ_z. Overall, the structural system contains 10 parameters, whereas the VAR estimation yields only nine parameters. Unless one of the parameters is restricted, it is not possible to identify the primitive system; that is, equations (2) and (3) are under-identified. Sim (1980) proposes to use a recursive system by imposing some restrictions on the primitive system such that the coefficient b_{21} is assumed to be equal to zero in equation 3, to recover the structural VAR (equations 2 and 3) from the estimate of the model in standard form (equations 6 and 7).

With the restriction imposed on the primitive systems, the estimates of the $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ sequences in the primitive VAR can also be recovered from the estimates of the standard VAR. The assumption $b_{21} = 0$ means that y_t does not have a contemporaneous effect on z_t. The restriction manifests that both ε_{yt} and ε_{zt} shocks affect the contemporaneous values of y_t, but only ε_{zt} shocks affect the contemporaneous value of z_t. The observed values of e_{2t} are completely attributed to pure shocks to the $\{z_t\}$ sequence. Such decomposition of the variance/covariance matrix of residuals procedure in the triangular fashion is called Choleski decomposition. In an *n*-variable VAR, exact identification requires $\frac{n(n-1)}{2}$ restrictions to be placed on the relationship between the regression residuals and the structural innovation. The Choleski decomposition forces exactly $\frac{n(n-1)}{2}$ values of the *B* matrix to equal to zero, since it is triangular. For instance, in our three-goal variable system, exact identification requires three restrictions to be placed on the six coefficients of the contemporaneous effects. That means, we can assume any three out of the six coefficients of the contemporaneous effects to be zero for the estimation purpose. This results in six different sets of coefficient restrictions and requires robustness test of all these possibilities. For more details, please refer to Enders' (2010) book *Applied Econometric Time Series*, Chapter 5.

A vector autoregression can be written as a vector moving average (VMA) presentation as follows:

$$x_t = \mu + \sum_{i=0}^{\infty} A_1^i e_{t-i} \qquad (8)$$

where *u* is a function of the parameters of the model and A_1^i is the i^{th} power of the matrix A_1 from equation (5). However, equation (8), expressing y_t and z_t in terms of the $\{e_{1t}\}$ and $\{e_{2t}\}$ sequences, is not very useful to study the effect of changes in, say, e_t on either $\{y_t\}$ or $\{z_t\}$, because the errors are correlated and therefore tend to move together. It is thus insightful to rewrite (8) in terms of $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ sequences. We can rewrite x_t as:

$$x_t = \mu + \sum_{i=0}^{\infty} \emptyset_i \varepsilon_{t-i} \qquad (9)$$

The coefficients of $Ø_i$ are the impulse-response functions. The coefficients of $Ø_i$ can be used to generate the effects of ε_{yt} and ε_{zt} shocks on the entire paths of the $\{y_t\}$ and $\{z_t\}$ sequences. For example, the coefficient $Ø_{12}$ (0) is the instantaneous impact of a one-unit change in ε_{zt} on y_t. By the same token, the elements $Ø_{11}$ (1) and $Ø_{12}$ (1) are the one-period responses of unit changes in ε_{yt-1} and ε_{zt-1} on y_t, respectively. To qualify the cumulative response of an element of y_t to an unpredicted innovation in some component of ε_t, the components of ε_t must be orthogonal. If we assume that the $\Omega = E(\varepsilon_t \varepsilon_t')$ is positive definite, then there exists a unique lower triangular matrix K with ones along the principal diagonal and a unique diagonal matrix D with positive entries along the principal diagonal, such that:

$$\Omega = KDK' \qquad (10)$$

Let

$$\omega_t = K^{-1}\varepsilon_t \qquad (11)$$

Then $E(\omega_t \omega_t') = K^{-1}\Omega\ (K^{-1})'$ = D. Since $\varepsilon_t = K\omega_t$, the vector $\{x_t\}$ has a moving average representation in terms of $\{\omega_t\}$ sequences:

$$x_t = \mu + \sum_{i=0}^{\infty} K\emptyset_i \omega_{t-i} \qquad (12)$$

For example, in the two-variable case, we will have

$$\frac{\partial z_t}{\partial \omega_{y,t-s}} = \emptyset_s K_y \qquad (13)$$

where K_y is the first column of the matrix K. The plot of (13) as a function of $s > 0$ is an orthogonalized impulse response function.

Appendix B: Simultaneous-Equations Model (SEM) and Vector Autoregression (VAR)

In this appendix, we first introduce SEM with three examples. We then compare SEM with VAR by discussing their similarities and differences.

SEM

SEM is a type of statistical model in the form of a set of linear simultaneous equations to capture *sets/systems* of relationships. The classic example is market equilibrium (supply and demand) in economics. Below we replicate this example and two other examples illustrated by Greene in his Econometric Analysis book (2003, Chapter 15, p. 378-381, 5th edition).

The first example is a familiar example of a system of simultaneous equations – a model of market equilibrium, consisting of the following:

demand equation: $q_{d,t} = \alpha_1 p_t + \alpha_2 x_t + \varepsilon_{d,t}$,
supply equation: $q_{s,t} = \beta_1 p_t + \varepsilon_{s,t}$,
equilibrium condition: $q_{d,t} = q_{s,t} = q_t$.

These equations are structural equations in that they are *derived from theory* and each purports to describe a particular aspect of the economy. Since the model is one of joint determination of price and quantity, they are labeled jointly endogenous variables. Income *x* is assumed to be determined outside of the model, which makes it exogenous. The disturbances are added to the usual textbook description to obtain an econometric model. All three equations are needed to determine the equilibrium price and quantity, so the system is interdependent. Because least squares regression generates inconsistent estimates (the simultaneous-equation bias, for more details please refer to Greene' book), we might instead use an instrumental variable estimator.

The second example is a small macroeconomic model. It is illustrated as follows:

consumption: $c_t = \alpha_0 + \alpha_1 y_t + \alpha_2 c_{t-1} + \varepsilon_{t1}$,
investment: $i_t = \beta_0 + \beta_1 r_t + \beta_2 (y_t - y_{t-1}) + \varepsilon_{t2}$,
demand: $y_t = c_t + g_t + i_t$.

The model contains an autoregressive consumption function, an investment equation based on interest and the growth in output, and an equilibrium condition. The model determines the values of the three endogenous variables c_t, i_t, and y_t. This model is a dynamic mode. In addition to the exogenous variables, r_t, and g_t, it contains two predetermined variables, c_{t-1} and y_{t-1}. These are obviously not exogenous, but with regard to the current values of the endogenous variables, they may be regarded as having already been determined.

The preceding two examples illustrate systems in which there are *behavioral equations* and *equilibrium conditions*. The latter are distinct in that even in an econometric model, they have no disturbances.

The third model is Klein's Model I. It may be written

$$
\begin{aligned}
C_t &= \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_3 \left(W_t^p + W_t^g\right) + \varepsilon_{1t} && \text{(consumption)},\\
I_t &= \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 K_{t-1} + \varepsilon_{2t} && \text{(investment)},\\
W_t^p &= \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 A_t + \varepsilon_{3t} && \text{(private wages)},\\
X_t &= C_t + I_t + G_t && \text{(equilibrium demand)},\\
P_t &= X_t - T_t + W_t^p && \text{(private profits)},\\
K_t &= K_{t-1} + I_t && \text{(capital stock)}.
\end{aligned}
$$

The endogenous variables are each on the left-hand side of equation and are labeled on the right. The exogenous variables are G_t= government nonwage spending, T_t= indirect business taxes plus net exports, W_t^g= government wage bill, A_t= time trend measured as years, and the constant term. There are also three predetermined variables: the lagged values of the capital stock, private profits, and total demand. The model contains three behavioral equations, an equilibrium condition, and two accounting identities. This model provides an excellent example of a small, dynamic model of the economy.

SEM needs specific theory guiding the equation structures. The three examples above not only include an *equilibrium condition* (the last equation(s) exclude the error term in each system), but more importantly, theory is needed to guide SEM to specify the exogenous variables, endogenous variables, and equilibrium conditions. This imposes two main limitations on SEM (Enders, 2010: 294). "The first concerns the goal of fitting a parsimonious model. Obviously, a parsimonious model is preferable to an overparameterized model. In the relatively small samples usually encountered in economic data, estimating an unrestricted model may so severely limit degrees of freedom as to render forecasts useless.

Moreover, the possible inclusion of large but insignificant coefficients will add variability to the model's forecasts." However, in paring down the form of the model, two equally skilled researchers will likely arrive at two different structural models. "Although one model may have a better 'fit' (in terms of the AIC or SBC), the residuals of the other may have better diagnostic properties." There is substantial truth to the consensus opinion that fitting a SEM has many characteristics of an 'art form'. There is a potential cost to using a parsimonious model. This is related to the second problem of SEM which concerns the assumption of no feedback of one or certain variables to another (or other) variables. Since SEM requires the specification of exogenous variables ({X}) and endogenous variables ({Y}), it assumes that there is no feedback from {Y} to {X}. "Although certain economic models may assert that policy variables (such as money supply or government spending) are exogenous, there may be feedback such that the policy variables are set with specific reference to the state of other variables in the system." In other words, {Y} may have some feedback to {X} and this will bias the estimation of the system. (Enders, 2010: 294)

The needs to restrict the form of SEM and the problem of feedback or "reverse causality" lead Sims (1980) to propose VAR.

VAR

A VAR is a multivariate simultaneous equation system, in which each variable under study is regressed on current values of all other variables and a finite number of lags of all variables jointly considered. The VAR approach is useful when the intention is to analyze a phenomenon without having any strong priors about competing explanations of it. The method focuses on deriving a good statistical representation of the interactions between variables, letting the data determine the model. The PVAR model combines the traditional VAR approach which captures co-evolution and *interdependencies among multiple time series* and treats all the variables in the system as endogenous, with the panel data approach which corrects for unobserved individual heterogeneity.

Canova & Ciccarelli (2013) have recently conducted a survey of papers using PVAR methods and provided a nice summary of PVAR. "Panel VARs are particularly suited to analyze the transmission of idiosyncratic shocks across units and time." (Canova & Ciccarelli, 2013: 211-212) This is exactly what we do in our paper: examine the feedback interdependency of multiple operational goals across time.

Similarities, difference, and technical limitation

If we consider SEM as a general form of a set of linear simultaneous equations to capture *sets/systems* of relationships, we can consider VAR as a special type of SEM form. The above three examples of SEM can be considered as structural multivariate models (referred to as 'transfer function' by Enders, 2010). Common issues arise in interpreting and estimating models such as the problem of identification – the fundamental question of whether the parameters of interest in the model are even estimable must be resolved before estimation can even been considered. For example, both SEM and VAR may rely on instrumental variables to estimate the equations. As a result, both may encounter the difficulty in recovering the structural form from the reduced form due to the identification restrictions.

SEM and VAR have different theory grounds. SEM as currently applied in most studies focuses more on the simultaneity across equations. VAR focuses more on the dynamic interdependency; it is a *time series model* in nature. It is more interested in the interval feedback among different endogenous variables. It considers all variables in the systems as endogenous and let data determine the system and the relationship across equations or endogenous variables. As mentioned above, "VAR approach has the desirable property that all variables are treated systematically so that the econometrician does not rely on any incredible identifications restrictions" (Enders, 2010: 325). PVAR is to apply VAR model to panel data.

Both methods have limited available software packages. We use the PVAR code in Stata written by Love and Zicchino (2006). The paper has been cited by 445 research papers (Abrigo & Love, 2015). "For example, these programs have been used in studies recently published in The American Economic Review (Head, Lloyd-Ellis and Sun, 2014), Applied Economics (Mora and Logan, 2012), Journal of Macroeconomics (Carpenter and Demiralp, 2012) and The Journal of Economic History (Neumann, Fishback and Kantor, 2010), among others." The online access link of the program is as follows https://sites.google.com/a/hawaii.edu/inessalove/home/pvar

In conclusion, PVAR is the most appropriate method for the research question we study. Indeed, we are interested in the inter-temporal feedback process of one endogenous variable on another, that is, how the performance feedback against one goal variable (any endogenous variable in our system) generates distorting (unintentional) feedback to other two goal variables.