Online Appendix

Revisiting Stock Market Signals as a Lens for Patent Valuation

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This Online Appendix contains supplementary material for the main paper.

Abstract

Estimating the private value of patents is important, yet challenging. By developing a method based on stock market returns to produce estimates of individual patent values, Kogan, Papanikolaou, Seru, and Stoffman (2017) (KPSS) opened venues for new research. We characterize the measurement error in KPSS – the difference between the latent true patent value and the corresponding KPSS estimate – and show it is negatively correlated with the latent true patent value. We then investigate the use of KPSS estimates in two different applications. First, we show that using KPSS values to gauge differences in value between different patent groups is internally inconsistent and introduces attenuation bias. We offer two solutions: extending the original KPSS method to allow for patents to be drawn from two distinct value distributions, and using abnormal stock market returns. We compare both to the original KPSS estimates in several contexts relevant to the organizational scholars, such as patents by large and small teams, scientific and non-scientific patents, and offshored and domestically invented patents. Second, we show that KPSS yield unbiased estimates when used as explanatory variables. These analyses allow us to characterize the main trade-offs associated with each approach, and offer practical guidance to researchers.

Part 1: Appendix A
Estimating the value of patents with single patent type: the KPSS estimation

A1 KPSS patent value estimation methodology

KPSS make the following assumptions:

1. x_j is normally distributed truncated at zero $\mathcal{N}(0, \sigma^2_{xft})$
2. ϵ_j is normally distributed $\mathcal{N}(0, \sigma^2_{\varepsilon ft})$
3. σ^2_{xft} and $\sigma^2_{\varepsilon ft}$ vary across firms and across time but in constant proportions
4. x_j and ϵ_j are independent.

Under these assumptions, we can calculate $\mathbb{E}[x_j|r_j]$ using the distribution of x_j conditional on r_j. By independence, the joint density of $f(x_j, \epsilon_j)$ is product of their density:

$$f(x_j, \epsilon_j) = \frac{1}{(\pi\sigma_{xft}\sigma_{\varepsilon ft})} \exp\Big[\frac{-1}{2\sigma^2_{xft}} x_j^2 - \frac{1}{\sigma^2_{\varepsilon ft}} \epsilon_j^2\Big]; x_j > 0 \tag{13}$$

using the transformation $\epsilon_j = r_j - x_j$:

$$f(x_j, r_j) = \frac{1}{(\pi\sigma_{xft}\sigma_{\varepsilon ft})} \exp\Big[\frac{-1}{2\sigma^2_{vft}} x_j^2 - \frac{1}{2\sigma^2_{\varepsilon ft}} (r_j - x_j)^2\Big] \tag{14}$$

Using Aigner, Lovell, and Schmidt (1977), the density function of $f(r_j)$ is given by:

$$f(r_j) = \frac{2}{(\sqrt{2\pi}\sigma)} (\Phi(\frac{r_j\lambda}{\sigma})) \exp\Big[\frac{-1}{2\sigma^2} (r_j)^2\Big] \tag{15}$$

where $\sigma = \sigma^2_{xft} + \sigma^2_{\varepsilon ft}$, $\lambda = \frac{\sigma_{xft}}{\sigma_{\varepsilon ft}}$, and $\Phi(.)$ is CDF of standard normal distribution. As shown by Jondrow, Lovell, Materov, and Schmidt (1982), letting $\sigma^2_* = \frac{\sigma^2_{xft}\sigma^2_{\varepsilon ft}}{\sigma^2}$ we can calculate the conditional distribution and then expected values from:

$$f(x_j|r_j) = \frac{1}{(\Phi(\frac{r_j\lambda}{\sigma}))} \frac{1}{(\sqrt{2\pi}\sigma_*)} \exp\Big[\frac{-1}{2\sigma^2_*} (-x_j + \frac{\sigma^2_{xft} r_j}{\sigma^2})^2\Big] \tag{16}$$

The distribution of $f(x_j|r_j)$ is the same as a Normal distribution with $\mathcal{N}(\mu_*, \sigma^2_*)$ multiplied by $\frac{1}{(\Phi(\frac{r_j\lambda}{\sigma}))}$, where $\mu_* = \frac{\sigma^2_{xft} r_j}{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}$. The density is equivalent to the $\mathcal{N}(\mu_*, \sigma^2_*)$ distribution truncated at zero.

Using the expectation formula for truncated normal distribution:

$$\mathbb{E}[x_j|r_j] = \mu_* + \sigma_* \frac{\phi(R_j)}{1 - \Phi(R_j)} \tag{17}$$

where $R_j = \mu_*/\sigma^2_* = -\sqrt{\delta_j}\frac{r_j}{\sigma_{\varepsilon ft}}$ and $\delta_j = \frac{\sigma^2_{xft}}{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}$. Thus, μ_* can be written as $\delta_j r_j$ and σ^2_* becomes $\sqrt{\delta_j}\sigma_{\varepsilon ft}$, leading to the following formula:

$$\mathbb{E}[x_j|r_j] = \delta_j r_j + \sqrt{\delta_j}\sigma_{\varepsilon ft}\frac{\phi(R_j)}{1-\Phi(R_j)} \tag{18}$$

To estimate the value of patents from the formula, the parameters δ_j and $\sigma_{\varepsilon ft}$ need to be estimated. KPSS assume that δ_j, which is the ratio of the variance of x_j to the sum of the variance of x_j and ϵ_j, is constant across firms and over time. To compute δ, they estimate:

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma I_{fd} + \mu_{fd} \tag{19}$$

where r^l_{fd} is the idiosyncratic return of firm f centered on day d with a window of length l, a_{ft} is the firm-year fixed effect, and b_d is the day-of-week fixed effect. γ is equal to :

$$\mathbb{E}[ln(x_j + \epsilon_j)^2] - \mathbb{E}[ln(\epsilon_j^2)] = \gamma \tag{20}$$

Approximating the distribution of $x_j + \epsilon_j$ as a normal distribution, the square of a standard normal variable is distributed as $\chi^2(1)$.

$$\mathbb{E}\Big[\Big(ln(x_j + \epsilon_j)^2\Big)\Big(\frac{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}\Big)\Big] - \mathbb{E}\Big[ln\Big(\epsilon_j^2\Big)\Big(\frac{\sigma^2_{\varepsilon ft}}{\sigma^2_{\varepsilon ft}}\Big)\Big] = \gamma \tag{21}$$

Solving this and adjusting for the truncated variance of x_j leads to:[22]

$$\ln\Big[\frac{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}(1 - (\frac{\phi(0)}{1-\Phi(0)})^2)}{\sigma^2_{\varepsilon ft}}\Big] = \gamma \tag{22}$$

$$\Big[\frac{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}(1 - (\frac{\phi(0)}{1-\Phi(0)})^2)}{\sigma^2_{\varepsilon ft}}\Big] = e^\gamma \tag{23}$$

simplifying this using $\delta_j = \frac{\sigma^2_{xft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xft}}$ leads to:

$$\hat{\delta} = 1 - \Bigg(1 + \frac{1}{(1 - (\frac{\phi(0)}{1-\Phi(0)})^2)}(e^\gamma - 1)\Bigg)^{-1} = 1 - e^{-\gamma}$$

Next, KPSS need to recover the $\sigma^2_{\varepsilon ft}$. This is done non-parametrically using the sum of squared market-adjusted returns σ_{ft} if m_{ft} is the fraction of trading days with a patent grant in a firm-year.

$$\sigma^2_{\epsilon_{ft}} + m_{ft}\sigma^2_{xft}(1 + l) = \sigma^2_{ft}(1 + l)$$

[22] The variance of $\mathcal{N}(0, \sigma^2)$ truncated at d is $\sigma^2\lambda(d)(1 - \lambda(d))$ where λ is the inverse mill ratio $(\frac{\phi(d)}{(1-\Phi(d)})$. The square of standard Normal is distributed as $\chi^2(1)$.

using further simplification of equation 23 we get:

$$\sigma^2_{\epsilon_{ft}} = \frac{\sigma^2_{ft}(1+l)}{(1+m_{ft}(1+l)(e^{\hat{\gamma}}-1))}$$

A1.1 Distribution of KPSS values

Note that the KPSS values are distributed differently than the distribution of x itself. For a given firm-year, the covariance between $\mathbb{E}[x|r]$ & x is given by $\delta\sigma_x^2 + \sqrt{\delta}\sigma_\varepsilon Cov(\lambda(R),\ x)$. The term $\sqrt{\delta}\sigma_\varepsilon Cov(\lambda(R),\ x) \leq 0$ because $\lambda(R)$ is a decreasing function of r whereas x is an increasing function. Therefore, $Cov(\mathbb{E}[x|r], x) \leq \delta\sigma_x^2$.

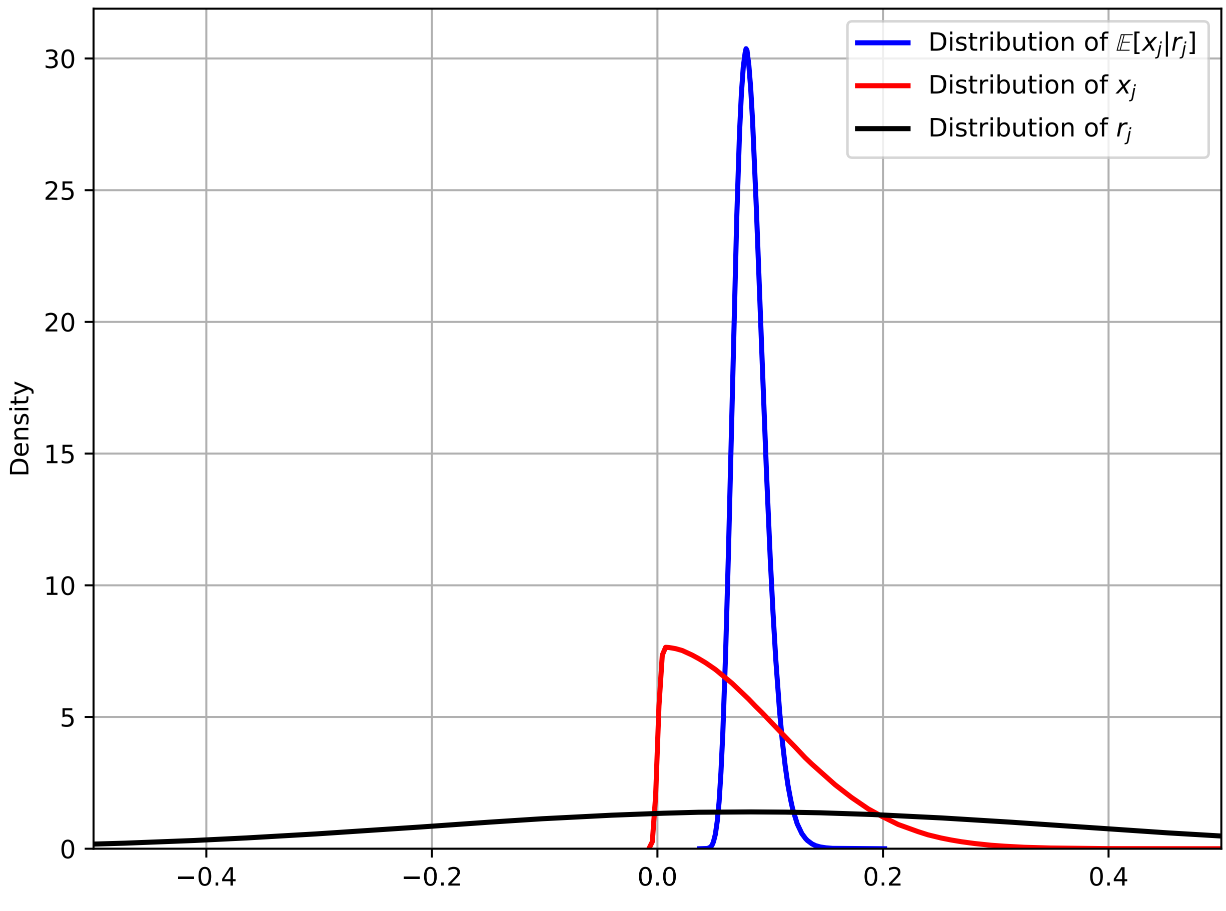

Figure 1: Simulation distribution of x, r, and $\mathbb{E}(x|r)$

Numerical simulations indicate that $\mathbb{E}[x|r]$ has a has a less skewed distribution and smaller variance than x.[23] The median of the distribution of KPSS values is greater than the median of x but also has less probability mass in the tails.

[23] Where $x \sim \mathcal{N}^+(0, \sigma_x^2)$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$, with $\sigma_x^2 = 0.011$ and $\sigma_\epsilon^2 = 0.077$. More details on the simulation are provided in Appendix D, where we extend to simulation to two types of patents.

A1.2 Variance of $\mathbb{E}[x \mid r]$

By the law of total variance,

$$\text{Var}(x) = \mathbb{E}[\text{Var}(x \mid r)] + \text{Var}(\mathbb{E}[x \mid r])\,.$$

Since the first term is strictly positive whenever patent values are not perfectly revealed by r, it follows immediately that

$$\text{Var}(\mathbb{E}[x \mid r]) < (x).$$

Thus, KPSS values, defined as conditional expectations, necessarily exhibit lower dispersion than the underlying distribution of true patent values.

Numerical simulations confirm the mechanical compression implied by conditional expectation. In our baseline calibration, the variance of KPSS values is an order of magnitude smaller than the variance of true patent values. In addition, while the underlying patent value distribution is strongly right-skewed, mapping values into conditional expectations substantially attenuates skewness. The right tail is compressed as extreme realizations of patent value are shrunk toward the mean, although positive skewness remains. These patterns illustrate how KPSS values smooth both dispersion and tail asymmetry relative to the underlying distribution.

A2 Revisiting the KPSS method to handle multiple patents granted in a single day

$$r = x_1 + x_2 + \varepsilon$$

Where x_1 and x_2 are two patents drawn from the same distribution (e.g., both are science-based or non-science-based).[24]

First, assuming only two patents are granted on a given day. Following the KPSS method as described in Appendix A section A1.

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1 I_{1fd} + \gamma_2 I_{2fd} + \mu_{fd} \tag{24}$$

I_{1fd} indicates that there is only 1 patent on grant day; similarly, I_{2fd} indicates two patents on the grant day.

$$\ln\Big[\frac{\sigma^2_{\varepsilon ft} + \sigma^2_{xft}}{\sigma^2_{\varepsilon ft}}\Big] = \gamma_1, \ln\Big[\frac{\sigma^2_{\varepsilon ft} + 2 * \sigma^2_{xft}}{\sigma^2_{\varepsilon ft}}\Big] = \gamma_2 \tag{25}$$

[24] We dropped the index for patent day etc. to keep the notation simple.

This implies

$$1 + \theta^2 = e^{\gamma_1}$$
$$1 + 2\theta^2 = e^{\gamma_2}$$

where $\theta = \frac{\sigma_{xft}^2}{\sigma_{\varepsilon ft}^2}$

We solve for θ^2 from the first equation:

$$\theta^2 = e^{\gamma_1} - 1$$

Substitute θ^2 into the second equation:

$$1 + 2(e^{\gamma_1} - 1) = e^{\gamma_2}$$

Simplify the left-hand side:

$$1 + 2e^{\gamma_1} - 2 = e^{\gamma_2} 2e^{\gamma_1} - 1 = e^{\gamma_2}$$

Thus, we have the relationship between γ_1 and γ_2:

$$e^{\gamma_2} = 2e^{\gamma_1} - 1$$

Alternatively, we can express γ_2 in terms of γ_1:

$$\gamma_2 = \ln(2e^{\gamma_1} - 1)$$

Approximation for Small γ

Consider the relationship:

$$e^{\gamma_2} = 2e^{\gamma_1} - 1$$

Using the Taylor expansion for e

$$1 + \gamma_2 = 2(1 + \gamma_1) - 1$$

So, for small γ_1, we can approximate:

For a more exact representation, the original expression $\gamma_2 = \ln(2e^{\gamma_1} - 1)$ is the most accurate, but the approximation $\gamma_2 \approx 2\gamma_1$ provides a useful simplification under certain conditions.

We can impose the restriction $\gamma_2 = \ln(2e^{\gamma_1} - 1)$ in equation 24 which becomes

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1 I_{1fd} + (\ln(2e^{\gamma_1} - 1))I_{2fd} + \mu_{fd} \tag{26}$$

This can be estimated by GMM or non-linear least squares. If we impose the approximation $\gamma_2 = 2\gamma_1$, equation 24 simplifies to

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_1(I_{1fd} + \frac{2}{1}I_{2fd}) + \mu_{fd} \tag{27}$$

This can also extend to n-patent granted on a given day.

$$ln(r^l_{fd})^2 = a_0 + a_{ft} + b_d + \gamma n + \mu_{fd} \tag{28}$$

A2.1 Correcting the bias in KPSS estimates if more than one patent is granted on a single day

Suppose n patents are granted on a particular day, where n is a random variable. As before, we assume that the probability that any patent is granted in a given day is $p > 0$. We show that the KPSS version of equation 27 yields an upward biased estimate, where the bias is given by $\gamma_1(1 - N)$, where $N = E(n|I_{1fd} = 1) = np$. Put differently, one needs to divide the KPSS estimate of γ_1 by the expected number of patents conditional on a patent being granted.

To see this, consider a simplified version of of equation 27, where we dispense with the subscripts to reduce notational clutter.

$$Y = a_0 + \gamma n + \zeta \tag{29}$$

where Y is the square of the log returns, and n is the number patents granted, $n = 0, 1, 2, ...$ Consider the analog of the KPSS estimating equation

$$Y = a_0 + \beta I + \zeta \tag{30}$$

where I is a dummy variable that takes value one if there is a patent granted and zero otherwise.

Then the expected value of b, the OLS estimate of β is given by

$$
\begin{aligned}
\mathbb{E}(b) &= \frac{Cov(Y,I)}{Var(I)} \\
Cov(Y,I) &= E(YI) - E(Y)E(I) = \gamma np - (a_0 + \gamma np)p \\
\implies \mathbb{E}(b) &= \gamma N
\end{aligned}
\tag{31}
$$

This implies that KPSS estimates of γ should be divided by the mean number of patents per day, conditional on patents being granted.

Figure 2: Distribution of average patent value by number of patents on the grant day

Notes: The following plot shows the average patent value calculated using the KPSS assumption (where the total invention value of all patents granted on a given day follows a half-normal distribution) compared to a scenario where we assume a patent value distribution and adjust for the number of patents when calculating the signal-to-noise ratio. The x-axis represents the number of patents granted on a given day. The KPSS method tends to overestimate the patent value compared to the average when the number of patents granted is low, while the modified approximation, which adjusts for the number of patents, underestimates the patent value compared to the average in the same scenario.

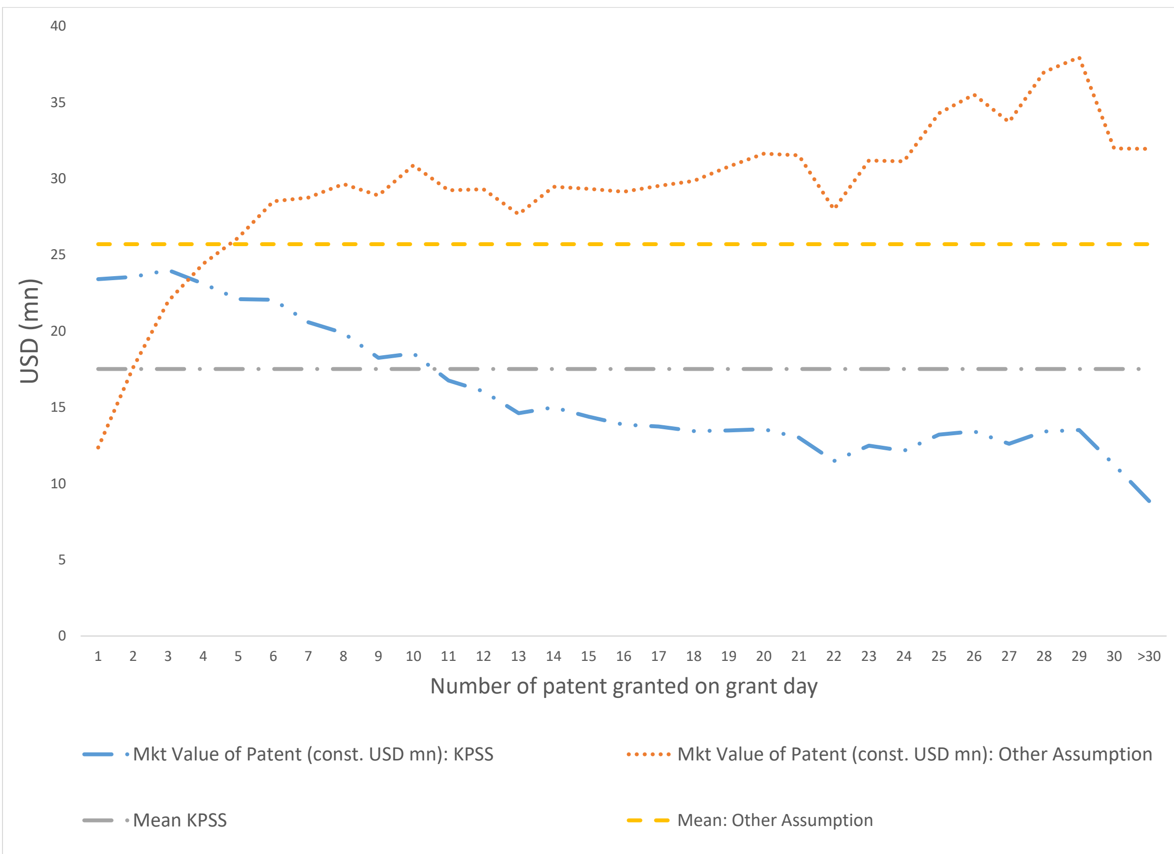

Table A1: Patent private value estimation and KPSS assumptions

	Log(1+Fwd citations)							Breakthrough				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log (Pat val KPSS (Baseline))		0.053***			0.083***			0.059***			0.149***	
		(0.001)			(0.001)			(0.008)			(0.013)	
Log (Pat val KPSS (New Assump.))			0.011***			0.103***			-0.079***			0.166***
			(0.001)			(0.001)			(0.008)			(0.016)
Constant	1.826***	0.981***	1.648***	1.826***	0.512***	0.154***	1.283***	0.357***	2.574***	1.283***	-1.073***	-1.413***
	(0.001)	(0.013)	(0.012)	(0.001)	(0.019)	(0.024)	(0.009)	(0.133)	(0.133)	(0.009)	(0.207)	(0.262)
Avg of DV	1.826	1.826	1.826	1.826	1.826	1.826	1.283	1.283	1.283	1.283	1.283	1.283
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
R^2	0.1317	0.1345	0.1318	0.2080	0.2105	0.2105	0.0052	0.0052	0.0053	0.0418	0.0419	0.0419
N	1,443,877	1,443,866	1,443,866	1,443,262	1,443,251	1,443,251	1,443,877	1,443,866	1,443,866	1,443,262	1,443,251	1,443,251

Note: The dependent variable in odd columns (1-6) is the *Log (1 + patent Forward citation)*, which is the natural log of the patent's forward citations garnered over a five-year period. The dependent variable in even columns (7-12) is a binary variable equal to one if the focal patent received a number of forward citations in the top 99th percentile among the patents granted in the same year and within the same patent class, as sourced from Squicciarini et al. (2013). Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Appendix B
Estimating the value of patents with multiple patent types

B1 Estimating the Value of Patents with Multiple Patent Types

We make a similar assumption to KPSS, with a slight variation:

- x_{sj} & x_{nj} denote science-based and non science patent values respectively for firm f and time t.
- x_{sfti} & x_{nftk} are normally distributed truncated at zero but have different variance ($\mathcal{N}^+(0, \sigma^2_{xsft})$ and $\mathcal{N}^+(0, \sigma^2_{xnft})$).
- ε_j is normally distributed $\mathcal{N}(0, \sigma^2_{\varepsilon ft})$.
- σ^2_{xft}, σ^2_{xnft} and $\sigma^2_{\varepsilon ft}$ vary across firms and across time but in constant proportions.
- $\delta_{sft} = \frac{\sigma^2_{xsft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xsft}} = \delta_s$ and $\delta_{nft} = \frac{\sigma^2_{xnft}}{\sigma^2_{\varepsilon ft}+\sigma^2_x nft} = \delta_n$
- x_{sift}, x_{njft} and ϵ_{ft} are independent for a given firm and time.

we can derive the conditional expectation of the scientific contribution to a patent's value.

$$E(x_{sj}|r_{sj}) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)} r_j + \frac{\left\{2\frac{(1+\theta_2^2)\sigma^2_{\varepsilon ft}}{\omega_1}\phi(r_j/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r_j\right) - \frac{2\theta_1^2\sigma^2_{\varepsilon ft}}{\omega_2}\phi\left(\frac{r_j}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r_j\right)\right\}}{\left[\Phi\left(\frac{r_j}{\omega_1}\right) - 2T\left(\frac{r_j}{\omega_1}, \lambda_1\right) + \Phi\left(\frac{r_j}{\omega_2}\right) - 2T\left(\frac{r_j}{\omega_2}, \lambda_2\right)\right]} \quad (32)$$

Where:

- $\theta_1 = \frac{\sigma_{xsft}}{\sigma_{\varepsilon ft}}$ and $\theta_2 = \frac{\sigma_{xnft}}{\sigma_{\varepsilon ft}}$
- $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$ and $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
- $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$ and $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$
- $s = \sqrt{\sigma^2_{\varepsilon ft} + (\sigma_{xsft})^2 + (\sigma_{xnft})^2} = \sigma_{\varepsilon j}\sqrt{1+\theta_1^2+\theta_2^2}$
- T = Owen's T function (Owen, 1980)

To derive the $\mathbb{E}[x_j|r_j]$ it is useful to analyze the sum of random variables. Let $r = u+v+w$ where v and w are normally distributed, truncated at zero, but have different variances: $v \sim \mathcal{N}^+(0, \sigma_1^2)$ and $w \sim \mathcal{N}^+(0, \sigma_2^2)$ and u is normally distributed $u \sim \mathcal{N}(0, \sigma_u^2))$.[25] We follow Papadopoulos (2015), which entails these steps:

[25]The relationship to patent values is readily apparent when r represents market returns, u represents the noise term, u represents value of science-based patents, and v represents the value of non science-based patents. This substitution simplifies notation, particularly subscripts.

- Derive the distribution of $z = v + w$

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) + \Phi\left(\frac{\sigma_2}{\sigma_1}(z/s_h)\right) - 1\right] \tag{33}$$

- Derive the distribution of r

$$f_r(r) = \frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right] \tag{34}$$

Where:

- $s^2 = \sigma_u^2 + \sigma_1^2 + \sigma_2^2$
- $s_h^2 = \sigma_2^2 + \sigma_1^2$
- $\lambda_1 = \frac{s\lambda}{\sigma_u}$ & $\lambda_2 = \frac{s}{\lambda\sigma_u}$ where $\lambda = \frac{\sigma_1}{\sigma_2}$
- $a = \frac{s_h}{s\sigma_u}$

- Derive the conditional density $f(v|r)$

$$f_{v|r}(v|r) = A^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right) \tag{35}$$

Where:

- $\frac{\sigma_1}{ss_2} = \frac{\theta_1}{s\sqrt{1+\theta_2^2}} = \frac{1}{\omega_1}$
- $\frac{\theta_2}{s_2} = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}\frac{1}{\omega_1} = \frac{\lambda_1}{\omega_1}$
- $\omega_v = \frac{\sigma_1 s_2}{s}$

- Derive the condition expectation $\mathbb{E}(v|r)$

$$E(v|r) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)}r + \frac{\left\{2\frac{(1+\theta_2^2)\sigma_u^2}{\omega_1}\phi(r/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2\theta_1^2\sigma_u^2}{\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}}{\left[\Phi\left(\frac{r}{\omega_1}\right) - 2T\left(\frac{r}{\omega_1}, \lambda_1\right) + \Phi\left(\frac{r}{\omega_2}\right) - 2T\left(\frac{r}{\omega_2}, \lambda_2\right)\right]} \tag{36}$$

We will use the following parameterization, where:

- $\theta_1 = \frac{\sigma_1}{\sigma_u}$
- $\theta_2 = \frac{\sigma_2}{\sigma_u}$
- $s = \sqrt{\sigma_u^2 + \sigma_1^2 + \sigma_2^2} = \sigma_u\sqrt{1+\theta_1^2+\theta_2^2}$, with
- $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$

- $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
- $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$
- $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$

- Derive the two signal-to-noise ratios. We can estimate the γ_s and γ_n and recover δ_s as $1-e^{-\gamma_s}$ and δ_n as $1-e^{-\gamma_n}$ from the following regression:

$$ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_s I_{sfd} + \gamma_n I_{nfd} + \nu_{fd} \quad (37)$$

- Derive the variance of ϵ_j. We can estimate it as follows:

$$\sigma^2_{\varepsilon ft} + \mu_s \sigma^2_{sft}(1+l) + \mu^N \sigma^2_{nft}(1+l) = \sigma^2_{ft}(1+l) \quad (38)$$

Derive the distribution of $v + w$

$z = v + w$. Both v and w are truncated normal at zero with variance σ_1 & σ_2, so z is the sum of two truncated normal distributions. Further, both v and w are iid so the distribution of z is:

$$F_z(z) = \mathbb{P}(v + w \leq z)$$
$$f_Z(z) = \int_0^z f(z-w)f(w)\,dw$$
$$f_Z(z) = \frac{2}{\pi\sigma_1\sigma_2}\int_0^z \left(\exp\left(-\frac{(z-w)^2}{2\sigma_1^2}\right)\right)\left(\exp\left(-\frac{w^2}{2\sigma_2^2}\right)\right)dw$$

The inside term can written as:

$$\frac{2}{\pi\sigma_1\sigma_2}\exp\left[-\frac{1}{2}\left(\frac{z}{\sigma_1}\right)^2\right]\exp\left[-\frac{1}{2}\frac{s_h^2}{\sigma_1^2\sigma_2^2}w^2 + \frac{z}{\sigma_1^2}w\right]$$

where $s_h^2 = \sigma_1^2 + \sigma_2^2$

Imposing restriction on the domain of w, with the integral between 0 and z, we have to calculate the integral from

$$\int_0^z \left(\exp\left(-\frac{1}{4\delta}\omega^2 + \gamma\omega\right)\right)d\omega$$

Where $\delta = \frac{\sigma_1^2\sigma_2^2}{2s_h^2}$ and $\gamma = \frac{z}{\sigma_1}$

We can write the above expression in the following form:

$$f_Z(z) = \frac{2}{\pi\sigma_1\sigma_2} \exp\left[\frac{1}{2}\left(\frac{z}{s_h^2}\right)^2\right] \sqrt{2\pi\left(\frac{\sigma_1\sigma_2}{s_h}\right)^2} \int_0^z \frac{1}{\sqrt{2\pi\left(\frac{\sigma_1\sigma_2}{s_h}\right)^2}} \exp\left[-\frac{1}{2}\frac{(-(w-\frac{z\sigma_1^2}{s_h^2})^2}{(\frac{\sigma_1\sigma_2}{s_h})^2}\right] dw$$

The density function can be simplified to

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) + \Phi\left(\frac{\sigma_2}{\sigma_1}(z/s_h)\right) - 1\right]$$

$$f_Z(z) = \frac{4}{s_h}\phi(z/s_h)\left[\Phi\left(\frac{\sigma_1}{\sigma_2}(z/s_h)\right) - \Phi\left(-\frac{\sigma_2}{\sigma_1}(z/s_h)\right)\right]$$

Derive the distribution of $r = u + v + w$

$r = u + z$ (where u is normal distribution with variance σ_u). The domain of u is $(-\infty, \infty)$, while the domain of z is $(0, \infty)$:

$$F_r(r) = \int_0^\infty \int_{-\infty}^{r-z} f_{u,z}(u, z) du dz$$

$$f_r(r) = \frac{d}{dr}F_r(r) = \int_0^\infty f_{u,z}(r - z, z)dz$$

Since the variables are independent, joint density is the product of the two marginal densities (one normal and one derived in the previous section). We use $1 - \Phi(x) = \Phi(-x)$

$$f_r(r) = \int_0^\infty \frac{1}{\sigma_u}\phi\left(\frac{r-z}{\sigma_u}\right)\frac{4}{s_h}\phi(z/s_h)\left[1 - \Phi\left(-\frac{\sigma_1}{\sigma_2}(z/s_h)\right) - \Phi\left(-\frac{\sigma_2}{\sigma_1}(z/s_h)\right)\right] dz$$

Following Papadopoulos (2015) we get

$$f_r(r) = \frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]$$

Where:

- $s^2 = \sigma_u^2 + \sigma_1^2 + \sigma_2^2$
- $s_h^2 = \sigma_2^2 + \sigma_1^2$

- $\lambda_1 = \frac{s\lambda}{\sigma_u}$ & $\lambda_2 = \frac{s}{\lambda\sigma_u}$
- $\lambda = \frac{\sigma_1}{\sigma_2}$
- $a = \frac{s_h}{s\sigma_u}$
- T = Owen's T function (Owen, 1980)

Derive the conditional density $f(v|r)$

$$f_{v|r}(v|r) = \frac{f_{v,u+w}(v, u+w)}{F_r(r)}$$

Note: v and w are independent of each other.
$R = \zeta + v$, where $\zeta = u + w$

$$f_{v|r}(v|r) = \frac{f_v(v) f_\zeta(r-v)}{F_r(r)}$$

$$f_{v|r}(v|r) = \frac{\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_1}\exp\left(\frac{-1}{2}(\frac{v}{\sigma_1})^2\right)\frac{2}{s_2}\phi((r-v)/s_2)\Phi\left(\theta_2\frac{r-v}{s_2}\right)}{\frac{2}{s}\phi(r/s)\left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]}$$

Where $s_2^2 = \sigma_2^2 + \sigma_u^2$ and $\theta_2 = \frac{\sigma_2}{\sigma_u}$ & $\theta_1 = \frac{\sigma_1}{\sigma_u}$

$$A = \left[\Phi\left(\frac{ar}{\sqrt{1+\lambda_1^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_1^2}}, \lambda_1\right) + \Phi\left(\frac{ar}{\sqrt{1+\lambda_2^2}}\right) - 2T\left(\frac{ar}{\sqrt{1+\lambda_2^2}}, \lambda_2\right)\right]$$

We can write this in the following form:

$$f_{v|r}(v|r) = A^{-1}\sqrt{\frac{2}{\pi}}\frac{s}{\sigma_1 s_2}\exp\left\{-\frac{1}{2}\left(\frac{v}{\sigma_1}\right)^2 - \frac{1}{2}\left(\frac{r-v}{s_2}\right)^2 + \frac{1}{2}\left(\frac{r}{s}\right)^2\right\}\Phi\left(\theta_2\frac{r-v}{s_2}\right)$$

Following Papadopoulos (2015), we can write
$\frac{\sigma_1}{ss_2} = \frac{\theta_1}{s\sqrt{1+\theta_2^2}} = \frac{1}{\omega_1}$

$\frac{\theta_2}{s_2} = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}\frac{1}{\omega_1} = \frac{\lambda_1}{\omega_1}$

$\omega_v = \frac{\sigma_1 s_2}{s}$

$$f_{v|r}(v|r) = A^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right)$$

The conditional expected value $\mathbb{E}(v|r)$

$$\mathbb{E}(v|r) = \int_0^\infty v f_{v|r}(v|r)dv = \int_0^\infty vA^{-1}\frac{2}{\omega_v}\phi\left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)\Phi\left(\lambda_1\frac{r-v}{\omega_1}\right)dv$$

Using the substitution:$v^* = \left(\frac{v}{\omega_v} - \frac{r}{\omega_1}\right)$

$v = \omega_v V^* + (\omega_v/\omega_1)r$

$dv = \omega_v dv^*$

$v = 0 \rightarrow -\frac{r}{\omega_1}$

Substituting:

$$\mathbb{E}(v|r) = (\omega_v/\omega_1)r + (2\omega_v A^{-1})\int_{-\frac{r}{\omega_1}}^\infty (v^*\phi(v^*)\Phi\left\{-\left(\frac{\lambda_1 r(\omega_v - \omega_1)}{\omega_1^2} + \frac{\lambda_1\omega_v}{\omega_1}v^*\right)\right\}dv^*$$

Similar to Papadopoulos (2015), this integral can be written as

$$\int_{-\frac{r}{\omega_1}}^\infty x\phi(x)\Phi(a+bx)dx$$

From Owen (1980):

$$\int_c^\infty x\phi(x)\Phi(a+bx) = \frac{b}{\sqrt{1+b^2}}\phi\left(\frac{a}{\sqrt{1+b^2)}}\right)\Phi\left(\frac{c+b(a+bc)}{-\sqrt{1+b^2}}\right) + \phi(c)\Phi(a+bc)$$

Where:

- $a = \frac{-\lambda_1(\omega_v - \omega_1)}{\omega_1^2}$
  $b = \frac{-\lambda_1\omega_v}{\omega_1}$
  $c = -r/\omega_1$
- $a + bc = \frac{\lambda_1}{\omega_1}r$
- $c + b(a+bc) = -\frac{1}{\omega_1}(1+\omega_2^2)r = \frac{-\theta_1\sqrt{(1+\theta_2^2)}}{s}r$
- $\sqrt{1+b^2} = \frac{s_2}{s}\sqrt{1+\theta_1^2}$

Solving for $\frac{c+b(a+bc)}{\sqrt{1+b^2}} = \frac{\lambda_2}{\omega_2} r$:

- $\frac{a}{\sqrt{1+b^2}} = -\frac{r}{\omega_2}$
- $\frac{b}{\sqrt{1+b^2}} = -\frac{s^2\omega_v}{s_2^2\omega_2}$

Finally, we obtain.

$$E(v|r) = (\omega_v/\omega_1)r + A^{-1}\left\{2\omega_v \frac{\omega_1}{\omega_1}\phi(r/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2s^2\omega_v^2}{s_2^2\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}$$

Note, this can be further simplified to
$\omega_v/\omega_1 = \frac{\sigma_1^2}{s^2}$
$\omega_v * \omega_1 = \sigma_u^2 + \sigma_2^2$
$\frac{s^2\omega_v^2}{s_2^2} = \sigma_1^2$

$$E(v|r) = \frac{\sigma_1^2}{s^2} r + A^{-1}\left\{(s^2 - \sigma_1^2)g_1 - \sigma_1^2 g_2\right\}$$

using $\frac{a}{\sqrt{1+\lambda_1^2}} = \frac{1}{\omega_1}$ & $\frac{a}{\sqrt{1+\lambda_2^2}} = \frac{1}{\omega_2}$

$$E(v|r) = \frac{\theta_1^2}{(1+\theta_1^2+\theta_2^2)} r + \frac{\left\{2\frac{(1+\theta_2^2)\sigma_u^2}{\omega_1}\phi(R/\omega_1)\Phi\left(\frac{\lambda_1}{\omega_1}r\right) - \frac{2\theta_1^2\sigma_u^2}{\omega_2}\phi\left(\frac{r}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r\right)\right\}}{\left[\Phi\left(\frac{r}{\omega_1}\right) - 2T\left(\frac{r}{\omega_1},\lambda_1\right) + \Phi\left(\frac{r}{\omega_2}\right) - 2T\left(\frac{r}{\omega_2},\lambda_2\right)\right]} \tag{39}$$

Let
$\theta_1 = \frac{\sigma_1}{\sigma_u}$
$\theta_2 = \frac{\sigma_2}{\sigma_u}$
$s = \sqrt{\sigma_u^2 + \sigma_1^2 + \sigma_2^2} = \sigma_u\sqrt{1+\theta_1^2+\theta_2^2}$

Following Papadopoulos (2015) we get:

- $\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}$
- $\omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}$
- $\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1+\theta_1^2+\theta_2^2}$
- $\lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1+\theta_1^2+\theta_2^2}$

If $\sigma_2^2 \to 0$, we obtain $\theta_1 = \frac{\sigma_1}{\sigma_u}$; $\theta_1 = 0$; $s = \sqrt{\sigma_u^2 + \sigma_1^2}$; $\frac{1}{\omega_1} = \frac{\sigma_1}{\sigma_u\sqrt{\sigma_u^2+\sigma_1^2}}$. It follows that:

$$E(v|r) = \frac{\sigma_1^2}{\sigma_u^2 + \sigma_1^2} r + \frac{\sigma_1}{\sqrt{\sigma_u^2 + \sigma_1^2}} \sigma_u \frac{\phi\left(\frac{\sigma_1}{\sigma_u\sqrt{\sigma_u^2+\sigma_1^2}} r\right)}{\Phi\left(\frac{\sigma_1}{\sigma_u\sqrt{\sigma_u^2+\sigma_1^2}} r\right)}$$

This is same as Kogan et al. (2017) upon noting the $1 - \Phi(-x) = \Phi(x)$ & $\phi(x) = \phi(-x)$ and $\delta = \frac{\sigma_1^2}{\sigma_u^2+\sigma_1^2}$:

$$E(v|r) = \delta r + \sqrt{\delta}\sigma_u \frac{\phi\left(\frac{\sqrt{\delta}}{\sigma_u} r\right)}{\Phi\left(\frac{\sqrt{\delta}}{\sigma_u} r\right)}$$

B2 Step-by-step implementation guide for Generalised KPSS

B2.1 Baseline KPSS: Estimating Private Patent Value

In KPSS's approach, which we fully characterize in Appendix A1, the private value of patent j granted to firm f on day t is given by:

$$\hat{A}_{jft} = \frac{1}{K_{ft}}(1-\pi)^{-1} \cdot S \cdot \mathbb{E}[x_j \mid r_j], \tag{40}$$

where K_{ft} represents the number of patents granted on the day, π is the ex-ante probability that the patent application is successful, and S denotes the market capitalization of the firm the day prior to the patent grants.[26]

The key estimand is $\mathbb{E}[x_j \mid r_j]$, the expected value of the patent (x_j) conditional on the observed stock market reaction around the patent grant announcement date (r_j). KPSS decompose the observed return into two unobserved components:

$$r_j = x_j + \varepsilon_j, \tag{41}$$

where x_j is the patent-induced change in firm value and ε_j captures contemporaneous, non-patent value-relevant events. KPSS impose the following distributional assumptions:

$$x_j \sim \mathcal{N}^+(0, \sigma^2_{xft}), \tag{42}$$

$$\varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon ft}). \tag{43}$$

Under these assumptions, the conditional expectation takes the form:[27]

$$\mathbb{E}[x \mid r] = \delta\, r + \sqrt{\delta}\,\sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)} = \delta\, r + \lambda(R, \delta, \sigma_\varepsilon), \tag{44}$$

where

$$R = -\sqrt{\delta}\,\frac{r}{\sigma_\varepsilon}, \qquad \delta = \frac{\sigma_x^2}{\sigma_\varepsilon^2 + \sigma_x^2}, \qquad \lambda(R, \delta, \sigma_\varepsilon) = \sqrt{\delta}\,\sigma_\varepsilon \frac{\phi(R)}{1-\Phi(R)}. \tag{45}$$

Equation 44 has two attractive features: (i) the estimated patent value is higher for larger stock-market reactions at the grant date (r_j); and (ii) this relation is stronger when markets are more informative about patent value (i.e., when δ is larger).

[26] We use a 3-day time window over which the stock market response to the patent grant is measured.
[27] To simplify notation, subscripts denoting patent, year, and firm are omitted.

Step 1: Estimate the signal-to-noise ratio. Implementing (44) requires estimating (i) δ_{ft} (signal-to-noise) and (ii) $\sigma^2_{\varepsilon ft}$ (noise variance). KPSS assume δ_{ft} is constant across firms and time, $\delta_{ft} = \delta$, and estimate it using:

$$\ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma I_{fd} + \mu_{fd}, \tag{46}$$

where r_{fd} denotes the three-day idiosyncratic return for firm f starting on day d, I_{fd} is an indicator for patent-grant days, and a_{ft} and b_d are firm-year and day-of-week fixed effects. The coefficient γ captures the patent announcement effect on idiosyncratic volatility and can be used to recover

$$\hat{\delta} = 1 - e^{-\hat{\gamma}}, \tag{47}$$

using the approximation described in KPSS.

Step 2: Estimate the noise variance. KPSS estimate $\sigma^2_{\varepsilon ft}$ non-parametrically using realized idiosyncratic squared returns. Denote the resulting estimate by $\widehat{\sigma^2_{\varepsilon ft}}$. Specifically, this is done using the sum of squared market-adjusted returns σ^2_{ft}, where m_{ft} is the fraction of trading days with a patent grant in a firm-year:

$$\sigma^2_{\varepsilon ft} + m_{ft}\sigma^2_{xft}(1 + l) = \sigma^2_{ft}(1 + l).$$

Using further simplification of the volatility decomposition, we obtain:

$$\sigma^2_{\varepsilon ft} = \frac{\sigma^2_{ft}(1 + l)}{1 + m_{ft}(1 + l)\left(e^{\hat{\gamma}} - 1\right)}.$$

Step 3: Compute baseline KPSS values. Given $\hat{\delta}$ and $\widehat{\sigma^2_{\varepsilon ft}}$, compute $\mathbb{E}[x_j \mid r_j]$ from (44), and then compute $\hat{A}_{jft}$ from (40) [28].

B2.2 Generalised KPSS: Multiple Patent Types (Science vs. Non-science)

We extend KPSS to settings where a firm may receive multiple patent types on a given grant day. Let x_{sj} and x_{nj} denote the latent values of science-based and non-science patents, respectively, granted to firm f on day t. We maintain KPSS's distributional structure with a slight variation:

- x_{sj} and x_{nj} denote science-based and non-science patent values for firm f at time t.

[28] Similar to KPSS, we set the ex-ante probability of patent grant to $\pi = 0.55$.

- $x_{sj} \sim \mathcal{N}^{+}(0, \sigma^2_{xsft})$ and $x_{nj} \sim \mathcal{N}^{+}(0, \sigma^2_{xnft})$.
- $\varepsilon_j \sim \mathcal{N}(0, \sigma^2_{\varepsilon ft})$.
- σ^2_{xsft}, σ^2_{xnft}, and $\sigma^2_{\varepsilon ft}$ vary across firms and time but in constant proportions.
- $\delta_{sft} = \frac{\sigma^2_{xsft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xsft}} = \delta_s$ and $\delta_{nft} = \frac{\sigma^2_{xnft}}{\sigma^2_{\varepsilon ft}+\sigma^2_{xnft}} = \delta_n$.
- x_{sj}, x_{nj}, and ε_j are independent for a given firm and time.

B2.2.1 Different-day case: only one type is granted on a day

When patent grant days are "pure" (only S-type patents or only N-type patents are granted on a given day), the problem reduces to baseline KPSS applied separately by type.

Step 1: Estimate separate signal-to-noise ratios. We recover δ_s and δ_n using:

$$\ln(r_{fd})^2 = a_0 + a_{ft} + b_d + \gamma_s I_{sfd} + \gamma_n I_{nfd} + \nu_{fd}, \tag{48}$$

where $I_{sfd} = 1$ if at least one S-type patent is granted on day d and 0 otherwise, while $I_{nfd} = 1$ if at least one N-type patent and no S-type patent is granted on day d and 0 otherwise. We then recover:

$$\hat{\delta}_s = 1 - e^{-\hat{\gamma}_s}, \qquad \hat{\delta}_n = 1 - e^{-\hat{\gamma}_n}. \tag{49}$$

Step 2: Estimate the noise variance. We compute $\sigma^2_{\varepsilon ft}$ using the accounting identity:

$$\sigma^2_{\varepsilon ft} + \mu_{sft}\sigma^2_{xsft}(1+l) + \mu_{nft}\sigma^2_{xnft}(1+l) = \sigma^2_{ft}(1+l), \tag{50}$$

where μ_{sft} is the share of firm-year days with S-type patent grants, μ_{nft} is the share of firm-year days with only N-type patent grants, σ^2_{ft} is the realized mean idiosyncratic squared returns of firm f in year t, and l is the event window length in days.[29]

Step 3: KPSS on S-only days (or N-only days). On S-only days, compute the conditional expectation using the baseline KPSS formula with $(\delta, \sigma_\varepsilon) = (\delta_s, \sigma_{\varepsilon ft})$:

$$\mathbb{E}[x_s \mid r] = \delta_s r + \sqrt{\delta_s}\,\sigma_{\varepsilon ft}\frac{\phi(R_s)}{1-\Phi(R_s)}, \qquad R_s = -\sqrt{\delta_s}\frac{r}{\sigma_{\varepsilon ft}}. \tag{51}$$

[29] The term $(1+l)$ (3-day window in our case) reflects the event window over which the market incorporates patent information, whereas noise is integrated continuously throughout the year.

Analogously, on N-only days, replace δ_s with δ_n.

B2.2.2 Same-day case: both S and N patents are granted on the same day

When both S- and N-type patents are granted on the same day, the observed stock reaction loads on both latent components:

$$r_j = x_{sj} + x_{nj} + \varepsilon_j. \tag{52}$$

The object of interest becomes the conditional expectation of the *scientific* contribution:

$$\mathbb{E}[x_{sj} \mid r_j]. \tag{53}$$

Step 1: Separate signal-to-noise ratios. Estimate $\hat{\delta}_s$ and $\hat{\delta}_n$ as above using (48).

Step 2: Noise variance. Recover $\widehat{\sigma^2_{\varepsilon ft}}$ using the firm-year identity above.

Step 3: Generalised KPSS formula for $\mathbb{E}[x_{sj} \mid r_j]$. Define:

$$\theta_1 = \frac{\sigma_{xsft}}{\sigma_{\varepsilon ft}}, \qquad \theta_2 = \frac{\sigma_{xnft}}{\sigma_{\varepsilon ft}}, \tag{54}$$

$$s = \sqrt{\sigma^2_{\varepsilon ft} + \sigma^2_{xsft} + \sigma^2_{xnft}} = \sigma_{\varepsilon ft}\sqrt{1 + \theta_1^2 + \theta_2^2}, \tag{55}$$

$$\omega_1 = \frac{s\sqrt{1+\theta_2^2}}{\theta_1}, \qquad \omega_2 = \frac{s\sqrt{1+\theta_1^2}}{\theta_2}, \tag{56}$$

$$\lambda_1 = \frac{\theta_2}{\theta_1}\sqrt{1 + \theta_1^2 + \theta_2^2}, \qquad \lambda_2 = \frac{\theta_1}{\theta_2}\sqrt{1 + \theta_1^2 + \theta_2^2}. \tag{57}$$

Then the conditional expectation of the scientific contribution is:

$$\mathbb{E}(x_{sj} \mid r_j) = \frac{\theta_1^2}{1+\theta_1^2+\theta_2^2} r_j + \frac{2\left\{\frac{(1+\theta_2^2)\sigma^2_{\varepsilon ft}}{\omega_1}\phi\left(\frac{r_j}{\omega_1}\right)\Phi\left(\frac{\lambda_1}{\omega_1}r_j\right) - \frac{\theta_1^2\sigma^2_{\varepsilon ft}}{\omega_2}\phi\left(\frac{r_j}{\omega_2}\right)\Phi\left(\frac{\lambda_2}{\omega_2}r_j\right)\right\}}{\left[\Phi\left(\frac{r_j}{\omega_1}\right) - 2T\left(\frac{r_j}{\omega_1}, \lambda_1\right) + \Phi\left(\frac{r_j}{\omega_2}\right) - 2T\left(\frac{r_j}{\omega_2}, \lambda_2\right)\right]}, \tag{58}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal pdf and cdf, respectively, and $T(\cdot,\cdot)$ is Owen's T function.

Step 4: Private value scaling (generalised case). Given $\mathbb{E}[x_{sj} \mid r_j]$ from (58), compute the private value contribution using the same scaling as KPSS:

$$\hat{A}^{(s)}_{jft} = \frac{1}{K_{ft}}(1-\pi)^{-1} \cdot S \cdot \mathbb{E}[x_{sj} \mid r_j], \tag{59}$$

and analogously for the non-scientific component $\hat{A}^{(n)}_{jft}$.

Replication code. Detailed step-by-step code for implementing the baseline and generalized KPSS estimators—covering applications to team size, R&D offshoring, reliance on science, and vertical integration—is available at:
https://github.com/jaypnagar/ABFN_Relication.

Appendix C
Measurement error in KPSS

C1 Comparing differences in value of groups of patents using KPSS values

Consider two types of patents, N and S. Letting r_s represent the market returns for S type patents and r_n for N patents, so that $r_{is} = x_{is} + \epsilon_i$, and $r_{jn} = x_{jn} + \epsilon_j$. If ε_i and ε_j have a normal distribution with mean zero and standard deviation σ_ε, we have

$$\mathbb{E}[r_s] = \mathbb{E}[x_s] = \sigma_s\sqrt{\frac{2}{\pi}} > \mathbb{E}[r_n] = \mathbb{E}[x_n] = \sigma_n\sqrt{\frac{2}{\pi}} \tag{60}$$

The sample mean of S type patents for that firm and year is:

$$\bar{y}_s = \delta\frac{1}{S}\sum_{i=1}^{i=S} r_{is} + \sqrt{\delta}\sigma_\varepsilon\frac{1}{S}\sum_{i=1}^{i=S}\lambda(R_{is}) \tag{61}$$

The other term in equation 61 involves $\lambda(R)$. Taking expectations, equation 2 implies that

$$\mathbb{E}[\lambda(R_s)] = \mathbb{E}[r_s]\frac{(1-\delta_s)}{\sqrt{\delta_s}\sigma_\varepsilon}; \quad \mathbb{E}[\lambda(R_n)] = \mathbb{E}[r_n]\frac{(1-\delta_n)}{\sqrt{\delta_n}\sigma_\varepsilon} \tag{62}$$

Using equation 5 one gets

$$\begin{aligned}
\bar{y}_s &= \delta\frac{1}{S}\sum_{i-1}^{i=S} r_{is} + \sqrt{\delta}\sigma_\varepsilon\frac{1}{S}\sum_{i-1}^{i=S}\lambda\left(R_s\sqrt{\frac{\delta}{\delta_s}}\right)\\
\mathbb{E}[\bar{y}_s] &= \delta\,\mathbb{E}[r_s] + \sqrt{\delta}\sigma_\varepsilon\,\mathbb{E}\left[\lambda\left(R_s\sqrt{\frac{\delta}{\delta_s}}\right)\right]\\
&\leq \delta\,\mathbb{E}[r_s] + \sqrt{\delta}\sigma_\varepsilon\,\mathbb{E}[\lambda\left(R_s\right)] \text{ (because } \lambda \text{ increases with } R \text{ and } \delta_s > \delta)\\
&= \delta\,\mathbb{E}[r_s] + (1-\delta_s)\,\mathbb{E}[r_s]\frac{\sqrt{\delta}}{\sqrt{\delta_s}}(\text{ by using equation 62})\\
&\leq \mathbb{E}[r_s] \text{ because } \delta_s \geq \delta\\
\implies \mathbb{E}[\bar{y}_s] &\leq \mathbb{E}[x_s]
\end{aligned} \tag{63}$$

That is, the sample mean for S type patents using KPSS values will underestimate their true mean. A similar logic implies that the sample mean for N type patents using KPSS values with over-estimate their true value, $\mathbb{E}[\bar{y}_n] > \mathbb{E}[x_n]$. It follows that

$$\mathbb{E}[x_s] - \mathbb{E}[x_n] \geq \mathbb{E}[\bar{y}_s] - \mathbb{E}[\bar{y}_n] \tag{64}$$

Appendix D
Simulated data where signal values are drawn from two different distributions

D1 Simulated Data with Heterogeneous Signal Distributions

We conduct a simulation to compare the performance of the KPSS method with our proposed generalization when patent values are drawn from different signal distributions. The goal is to assess how imposing a common signal-to-noise ratio affects the estimated difference in average patent values across patent types.

D1.1 Data Generating Process

We simulate data for 5,000 firms observed daily over 40 years, yielding 365×40 observations per firm. For each firm i, we draw a firm-specific scale parameter σ_i uniformly from the interval $(0, 1)$. Patent-specific signals and noise are then generated as follows.

For N-type (non-science) patents, the latent signal is

$$x_n \sim \mathcal{N}^+(0, \sigma_i),$$

while for S-type (science-based) patents, the signal is

$$x_s \sim \mathcal{N}^+(0, 1.1\,\sigma_i),$$

implying $\sigma_{si} = 1.1\,\sigma_i$. The noise term follows

$$\epsilon \sim \mathcal{N}(0, 5\,\sigma_i).$$

On each day, a firm generates an N-type patent with probability 15% and an S-type patent with probability 10%. The realized return is given by

$$r_j = I(n)\,x_n + I(s)\,x_s + \epsilon,$$

where $I(n)$ and $I(s)$ are indicators for N-type and S-type patents, respectively.

D1.2 Estimation Procedure

Using the simulated returns, we first compute $\mathbb{E}[x_j \mid r_j]$ following the KPSS methodology, imposing a common signal-to-noise ratio estimated via equation 3. We then estimate separate signal-to-noise ratios for N-type and S-type patents using equation 7, and compute $\mathbb{E}[x_j \mid r_j]$ separately by patent type.

D1.3 True and Estimated Signal-to-Noise Ratios

In the simulated data, the true coefficients implied by the data-generating process are

$$\gamma_s = \ln\left(\frac{(1.1)^2 + 5^2}{5^2}\right) = 0.047 \quad \text{and} \quad \gamma_n = \ln\left(\frac{1^2 + 5^2}{5^2}\right) = 0.039.$$

When imposing a common signal-to-noise ratio (KPSS), the estimated coefficient is $\hat{\gamma} = 0.042$. In contrast, estimating separate ratios yields $\hat{\gamma}_s = 0.047$ and $\hat{\gamma}_n = 0.038$, closely matching the true values.

D1.4 Results

The simulation illustrates that imposing a common signal-to-noise ratio substantially attenuates differences in estimated patent values across types. In the data-generating process, the true difference in mean latent values satisfies

$$\mathbb{E}[x_s] - \mathbb{E}[x_n] = 0.04$$

Under the KPSS restriction of a common signal-to-noise ratio, the estimated difference is close to zero. Allowing signal-to-noise ratios to differ across patent types recovers a difference of 0.0447 , closely aligned with the true value.

Appendix Tables D1–D5 report the estimated signal-to-noise ratios and the correlations between true and estimated latent values. Overall, the simulation confirms the analytical prediction that measurement error induced by pooling heterogeneous signal distributions can materially bias inference when using KPSS values.

We note that allowing for heterogeneous signal-to-noise ratios produces estimates that are slightly larger than the true effect in the simulation. This difference does not reflect systematic overestimation, but instead arises from estimation error in recovering the signal-to-noise ratio and noise variance from realized returns. As shown in Appendix A, both KPSS and our extension rely on simplifying approximations when mapping return moments to these parameters. When the true signal-to-noise ratios and noise variances used in the data-generating process are supplied directly, as in Column (4) of Appendix Table D3, the estimated science patent premium is statistically indistinguishable from the true value. This confirms that the small upward deviation observed when parameters are estimated reflects finite-sample parameter estimation noise rather than bias introduced by our approach.

Table D1: Estimated coefficients of signal-to-noise ratio: One distribution (KPSS) vs. two distributions

	KPSS: One Distribution	Modified: Two distribution
	(1)	(2)
Science Patent Dummy		0.0475***
		(0.0010)
Patent Dummy	0.0420***	
	(0.0006)	
No Science Patent Dummy		0.0384***
		(0.0006)
Constant	-0.0319***	-0.0319***
	(0.0001)	(0.0001)
Firm*Year Fixed Effects	Yes	Yes
Return Day	Yes	Yes
R^2	0.443	0.443
N	72,995,000	72,995,000

Note: In column 1, we estimate the singal-to-noise ratio using equation 3. In column 2 we have two different signal dummies based on the distribution type and used the equation similar to equation 7 To simulate data where signal values are drawn from two different distributions, we proceed as follows. We create 5,000 firms and, for each firm, generate daily data for 365×40 days. For each firm, the ε values are drawn from a normal distribution with a mean of 0 and a standard deviation defined as a random number between 0 and 1 for each firm, multiplied by five, denoted as $\mathcal{N}(0, \sigma_i \times 5)$. The patent value x values are then generated as follows: on most days, the signal value $r = 0$. For 15% of the days, r is drawn from a half-normal distribution $\mathcal{N}^+(0, \sigma_i)$, representing nonscience patents. For 10% of the days, r is drawn from a half-normal distribution $\mathcal{N}^+(0, \sigma_i \times 1.1)$, representing science patents. The actual Science Patent dummy coefficient in our simulated data $ln(\frac{(1.1)^2+5^2}{5^2}) = 0.047$ and the No Science Patent dummy coefficient is $ln(\frac{(1)^2+5^2}{5^2}) = 0.039$ Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table D2: Difference in True vs. Estimated patent values: One distribution vs. two distributions

	True value (x_i)		Estimated value of $\mathbb{E}(x_i \vert r_i)$			
			One distribution		Two distribution	
	(1)	(2)	(3)	(4)	(5)	(6)
Science Patent Dummy	0.0401***	0.0399***	0.0008***	0.0006***	0.0449***	0.0447***
	(0.0002)	(0.0002)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
Constant	0.3992***	0.3992***	0.4131***	0.4132***	0.3950***	0.3950***
	(0.0001)	(0.0001)	(0.0001)	(0.0000)	(0.0001)	(0.0000)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	Yes	No	Yes	No	Yes
R^2	0.002	0.303	0.000	0.960	0.008	0.957
N	18,247,117	18,247,117	18,247,117	18,247,117	18,247,117	18,247,117
Mean DV	0.415	0.415	0.413	0.413	0.413	0.413

Note: In columns 1 and 2, the dependent variable is the simulated values of x. In columns 3 and 4, we estimate $\mathbb{E}(x|r)$ using the KPSS methodology, while in columns 5 and 6, the dependent variable is $\mathbb{E}(x|r)$ estimated using two different signal-to-noise ratios for x_s and x_n. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table D3: Difference Between True and Estimated Patent Values: One vs. Two Distributions and Signal-to-Noise Assumptions

	Simulated Data	One distribution	Two distribution	Using Actual SNR and Variance
	(1)	(2)	(3)	(4)
Science Patent Dummy	0.040***	0.001***	0.045***	0.041***
	(0.000)	(0.000)	(0.000)	(0.000)
Constant	0.399***	0.413***	0.395***	0.403***
	(0.000)	(0.000)	(0.000)	(0.000)
Year Fixed Effects	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes
R^2	0.303	0.960	0.957	0.962
N	18,247,117	18,247,117	18,247,117	18,247,117
Mean DV	0.415	0.413	0.413	0.420

Notes: In columns 1, the dependent variable is the simulated values of x. In columns 2 , we estimate $\mathbb{E}(x|r)$ using the KPSS methodology, while in columns 3, the dependent variable is $\mathbb{E}(x|r)$ estimated using two different signal-to-noise ratios for x_s and x_n. Robust standard errors in parentheses.In the final column (4), $\mathbb{E}(x \mid r)$ is constructed using the *true* signal-to-noise ratio and the true variance of the noise term for each firm used in the data-generating process, rather than estimating these parameters from the data. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table D4: Summary Statistics: True and estimated patent value

	Obs.	Mean	Std. Dev.	10%	50%	90%
True value (x_i)	18247117	0.415	0.433	0.029	0.271	1.011
Estimated Value $\mathbb{E}(x_i|r_i)$ (One distribution)	18247117	0.413	0.241	0.085	0.412	0.740
Estimated Value $\mathbb{E}(x_i|r_i)$ (two distribution)	18247117	0.413	0.242	0.085	0.410	0.740

Note: The *simulated values* are the actual simulated values of x as described in D1. The *Estimated Value (One Distribution)* represents the estimated values of $\mathbb{E}(x|r)$ following the KPSS methodology, while the *Estimated Value (Two Distributions)* refers to the estimated values of $\mathbb{E}(x|r)$ obtained using separate signal-to-noise ratios for s-type and n-type patents.

Table D5: Correlation: True and estimated patent value

	True value (x_i)	Estimated Value $\mathbb{E}(x_i|r_i)$	
		One Distribution	Two Distribution
True value (x_i)	1.000		
Estimated Value $\mathbb{E}(x_i|r_i)$ (One distribution)	0.557	1.000	
Estimated Value $\mathbb{E}(x_i|r_i)$ (two distribution)	0.559	0.995	1.000

Note: The *simulated values* are the actual simulated values of x as described in D1. The *Estimated Value (One Distribution)* represents the estimated values of $\mathbb{E}(x|r)$ following the KPSS methodology, while the *Estimated Value (Two Distributions)* refers to the estimated values of $\mathbb{E}(x|r)$ obtained using separate signal-to-noise ratios for s-type and n-type patents.

Part 2: Appendix E

Individual regressions corresponding to Tables 4 and 6

Table E1: Patent Level Regression: Inventing Team Size

	KPSS Estimates		Large and Small team Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[Team Size: Large]	0.034***	0.025***	0.107***	0.098***	0.241***	0.232***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Log (Market cap (const. usd))		0.751***		0.760***		0.750***
		(0.001)		(0.001)		(0.001)
Constant	15.814***	-1.407***	15.865***	-1.561***	15.734***	-1.477***
	(0.001)	(0.028)	(0.001)	(0.028)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.884	15.884	15.777	15.777
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.693	0.792	0.682	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS methodology. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Patent Value: Teamsize (separate), estimated using separate distribution assumptions for patents based on inventor team size. The value is equally distributed among all patents granted on a given day. In columns (5) and (6), the dependent variable is Patent Value: Teamsize (mixed), where we further differentiate large-team patents from small-team patents using our modified formula and estimate the patent value for the days when both types of patents were granted. A patent is classified as having a "large team" if its inventor team size falls in the top quartile within its IPC class-year. Robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Table E2: Patent Level Regression: R&D Offshoring

	KPSS Estimates		Forein and Domestic Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[All Foreign Inventor]	-0.025***	-0.008***	-0.071***	-0.055***	-0.157***	-0.140***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log (Market cap (const. usd))		0.751***		0.745***		0.751***
		(0.001)		(0.001)		(0.001)
Constant	15.823***	-1.404***	15.781***	-1.313***	15.839***	-1.382***
	(0.001)	(0.028)	(0.001)	(0.029)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.773	15.773	15.822	15.822
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.676	0.771	0.682	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS method. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Pat Value: Foreign (separate), estimated using separate distribution assumptions for patents with and without U.S.-based inventors. The value is equally distributed among all patents granted on a given day. In columns (5) and (6), the dependent variable is Pat Value: Foreign (mixed), where we further differentiate between patents with only foreign inventors and those with at least one U.S.-based inventor using our modified formula and estimate the patent value for the days when both types of patents were granted. A patent is defined as "foreign" if all inventors listed on the patent are non-U.S.-based. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table E3: Patent Level Regression: Reliance on Science

	KPSS Estimates		Science and Non-Science Patent day		Modified estimates on Mixed patent day	
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[Patent Science Dummy]	0.008***	0.015***	0.042***	0.049***	0.133***	0.140***
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Log (Market cap (const. usd))		0.751***		0.756***		0.751***
		(0.001)		(0.001)		(0.001)
Constant	15.818***	-1.411***	15.847***	-1.489***	15.764***	-1.458***
	(0.001)	(0.028)	(0.001)	(0.028)	(0.001)	(0.028)
Avg DV	15.820	15.820	15.857	15.857	15.796	15.796
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.680	0.778	0.687	0.786	0.681	0.779
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251

The dependent variable in columns (1) and (2) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS method. The dependent variable in columns (3) and (4) is the natural log transformation of the patent private value: Patent Value: Sci&Non-Sci (separate), estimated using separate distribution assumptions for science-based and other patents. However, the patent value is equally distributed among all patents granted on the science patent day—days when at least one science-based patent is granted. In columns (5) and (6), the dependent variable is Patent Value: Sci&Non-Sci (mixed), where we further differentiate science-based patents from non-science-based patents using our modified formula and estimate the patent value for the days when both types of patents were granted. The "science-based patents" are defined as those ranking in the top three quartiles for the number of non-patent literature (NPL) citations within a specific IPC class and year, provided they cite at least one NPL. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table E4: Patent Level Regression: Vertical Integration

	KPSS Estimates			Vetical Intgration		
	(1)	(2)	(3)	(4)	(5)	(6)
𝟙[Vertical Integration: High]	0.170***	0.050***	0.006***	0.403***	0.282***	0.130***
	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)
Log (Market cap (const. usd))		0.383***	0.728***		0.385***	0.748***
		(0.000)	(0.001)		(0.000)	(0.001)
Constant	15.891***	7.089***	-0.867***	15.885***	7.028***	-1.300***
	(0.001)	(0.010)	(0.030)	(0.001)	(0.010)	(0.031)
Avg DV	15.939	15.939	15.940	16.000	16.000	16.001
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	No	No	Yes	No	No	Yes
R^2	0.143	0.467	0.786	0.161	0.476	0.785
N	1,294,402	1,294,402	1,293,900	1,294,402	1,294,402	1,293,900

The dependent variable in columns (1), (2) and (3) is the natural log transformation of the patent private value estimated using stock market reactions following the KPSS methodology. The dependent variable in columns (4), (5) and (6) is the natural log transformation of the patent private value: Patent Value: Vertical Integration (separate), estimated using separate distribution assumptions for patents assigned to firms with high versus low vertical integration. The value is equally distributed among all patents granted on a given day. A firm is classified as highly vertically integrated if it falls in the top quartile of vertical integration within its industry-year, based on the Frésard, Hoberg, and Phillips (2020) dataset. Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1

Table E5: Regression All: Dependent variable: Forward Citation

	Forward Citation:All					Forward Citation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.1255*						
	(0.0654)						
Pat Val: KPSS (std.)		0.3288***					
		(0.0584)					
Pat Val: Teamsize (mixed, std.)			0.6390***				
			(0.0592)				
Pat Val: Foreign (mixed, std.)				0.3724***			
				(0.0592)			
Pat Val: Sci & NoSci (mixed, std.)					0.6522***		
					(0.0614)		
Pat Val: KPSS (VI, std.)						0.3321***	
						(0.0661)	
Pat Val: Vertical Integration (std.)							0.2960***
							(0.0619)
Log (Market cap (const. usd))	1.5276***	1.3963***	1.2726***	1.3792***	1.2679***	1.4936***	1.5091***
	(0.0705)	(0.0776)	(0.0769)	(0.0774)	(0.0764)	(0.0872)	(0.0869)
Constant	-25.4466***	-22.4650***	-19.6527***	-22.0768***	-19.5402***	-22.9733***	-23.3195***
	(1.5342)	(1.6980)	(1.6825)	(1.6943)	(1.6725)	(1.9476)	(1.9423)
Avg DV	14.407	14.407	14.407	14.407	14.407	15.661	15.661
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-6.90e+06	-6.90e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable *Patent Forward Cites* measures the number of forward citations received by a patent within five years of grant. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are reported in parentheses. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E6: Regression All: Dependent variable: Breakthrough Citation

	Breakthrough:All					Breakthrough: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0214**						
	(0.0104)						
Pat Val: KPSS (std.)		0.0767***					
		(0.0164)					
Pat Val: Teamsize (mixed, std.)			0.1267***				
			(0.0167)				
Pat Val: Foreign (mixed, std.)				0.0829***			
				(0.0164)			
Pat Val: Sci & NoSci (mixed, std.)					0.1197***		
					(0.0166)		
Pat Val: KPSS (VI, std.)						0.0678***	
						(0.0171)	
Pat Val: Vertical Integration (std.)							0.0549***
							(0.0165)
Log (Market cap (const. usd))	0.0349*	0.0042	-0.0157	0.0018	-0.0128	-0.0114	-0.0062
	(0.0194)	(0.0204)	(0.0204)	(0.0204)	(0.0204)	(0.0222)	(0.0222)
Constant	0.7046	1.4018***	1.8545***	1.4564***	1.7892***	2.2403***	2.1232***
	(0.4360)	(0.4616)	(0.4603)	(0.4611)	(0.4601)	(0.5229)	(0.5225)
Avg DV	1.283	1.283	1.283	1.283	1.283	1.295	1.295
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.52e+06	-5.52e+06	-5.52e+06	-5.52e+06	-5.52e+06	-4.95e+06	-4.95e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Top 1% of Cited Patents] equals one if a patent belongs to the top 1 percent of the forward-citation distribution among patents granted in the same year and IPC class, and zero otherwise; this variable is multiplied by 100 for ease of interpretation (Squicciarini, Dernis, & Criscuolo, 2013). The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E7: Regression All: Dependent variable: Renew

	Renewed Full Term:All					Renewed Full Term: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	-0.0009						
	(0.0013)						
Pat Val: KPSS (std.)		0.0255***					
		(0.0009)					
Pat Val: Teamsize (mixed, std.)			0.0264***				
			(0.0009)				
Pat Val: Foreign (mixed, std.)				0.0261***			
				(0.0009)			
Pat Val: Sci & NoSci (mixed, std.)					0.0252***		
					(0.0009)		
Pat Val: KPSS (VI, std.)						0.0260***	
						(0.0009)	
Pat Val: Vertical Integration (std.)							0.0257***
							(0.0009)
Log (Market cap (const. usd))	-0.0073***	-0.0191***	-0.0194***	-0.0193***	-0.0189***	-0.0189***	-0.0184***
	(0.0015)	(0.0016)	(0.0016)	(0.0016)	(0.0016)	(0.0016)	(0.0016)
Constant	0.7221***	0.9892***	0.9955***	0.9945***	0.9841***	0.9860***	0.9765***
	(0.0342)	(0.0352)	(0.0350)	(0.0351)	(0.0352)	(0.0352)	(0.0350)
Avg DV	0.540	0.540	0.540	0.540	0.540	0.540	0.540
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.91e+05	-4.90e+05	-4.90e+05	-4.90e+05	-4.90e+05	-4.88e+05	-4.88e+05
N	774,044	774,044	774,044	774,044	774,044	770,304	770,304

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Renewed (Full Term)] equals one if the patent is renewed through the full statutory term, and zero otherwise, based on observed maintenance fee payments. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E8: Regression All: Dependent variable: Litigation Citation

	Litigation:All					Litigation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0005						
	(0.0045)						
Pat Val: KPSS (std.)		0.0849***					
		(0.0131)					
Pat Val: Teamsize (mixed, std.)			0.0933***				
			(0.0129)				
Pat Val: Foreign (mixed, std.)				0.0864***			
				(0.0131)			
Pat Val: Sci & NoSci (mixed, std.)					0.0920***		
					(0.0132)		
Pat Val: KPSS (VI, std.)						0.0908***	
						(0.0146)	
Pat Val: Vertical Integration (std.)							0.0897***
							(0.0143)
Log (Market cap (const. usd))	-0.0706***	-0.1048***	-0.1080***	-0.1053***	-0.1074***	-0.1190***	-0.1179***
	(0.0158)	(0.0166)	(0.0166)	(0.0166)	(0.0166)	(0.0185)	(0.0185)
Constant	1.8465***	2.6242***	2.6970***	2.6353***	2.6835***	3.4168***	3.3959***
	(0.3490)	(0.3667)	(0.3663)	(0.3668)	(0.3662)	(0.4237)	(0.4255)
Avg DV	0.589	0.589	0.589	0.589	0.589	0.631	0.631
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.47e+06	-4.47e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Litigation] equals one if the patent is involved in at least one patent litigation event. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E9: Regression All: Dependent variable: Reassign Citation

	Reassign: All					Reassign: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	0.0002						
	(0.0003)						
Pat Val: KPSS (std.)		0.0039***					
		(0.0004)					
Pat Val: Teamsize (mixed, std.)			0.0056***				
			(0.0004)				
Pat Val: Foreign (mixed, std.)				0.0036***			
				(0.0004)			
Pat Val: Sci & NoSci (mixed, std.)					0.0042***		
					(0.0004)		
Pat Val: KPSS (VI, std.)						0.0033***	
						(0.0004)	
Pat Val: Vertical Integration (std.)							0.0031***
							(0.0004)
Log (Market cap (const. usd))	0.0012**	-0.0004	-0.0011**	-0.0003	-0.0005	-0.0003	-0.0002
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
Constant	-0.0181	0.0173	0.0334***	0.0148	0.0204*	0.0691***	0.0672***
	(0.0113)	(0.0119)	(0.0118)	(0.0119)	(0.0119)	(0.0138)	(0.0138)
Avg DV	0.074	0.074	0.074	0.074	0.074	0.078	0.078
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-6.09e+04	-6.08e+04	-6.07e+04	-6.08e+04	-6.08e+04	-8.85e+04	-8.85e+04
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Reassignment] equals one if the patent experiences at least one ownership reassignment during its lifetime. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E10: Regression All: Dependent variable: Trilateral Patent Citation

	Trilateral Patent:All					Trilateral Patent:VI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (std.)	-0.0003						
	(0.0004)						
Pat Val: KPSS (std.)		0.0129***					
		(0.0007)					
Pat Val: Teamsize (mixed, std.)			0.0142***				
			(0.0008)				
Pat Val: Foreign (mixed, std.)				0.0123***			
				(0.0007)			
Pat Val: Sci & NoSci (mixed, std.)					0.0137***		
					(0.0008)		
Pat Val: KPSS (VI, std.)						0.0063***	
						(0.0007)	
Pat Val: Vertical Integration (std.)							0.0063***
							(0.0007)
Log (Market cap (const. usd))	0.0276***	0.0224***	0.0219***	0.0226***	0.0221***	0.0149***	0.0149***
	(0.0009)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)	(0.0008)
Constant	-0.5292***	-0.4114***	-0.3999***	-0.4164***	-0.4047***	-0.0422**	-0.0429**
	(0.0192)	(0.0183)	(0.0182)	(0.0183)	(0.0183)	(0.0178)	(0.0178)
Avg DV	0.209	0.209	0.209	0.209	0.209	0.210	0.210
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	No	No	No	No	No	No	No
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.92e+05	-5.91e+05	-5.91e+05	-5.91e+05	-5.91e+05	-5.16e+05	-5.16e+05
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable is $\mathbb{1}$[Trilateral Patent] equals one if the patent belongs to a triadic patent family with filings at the USPTO, EPO, and JPO, and zero otherwise. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t{-}1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E11: Regression All: Dependent variable: Forward Citation

	Forward Citation:All					Forward Citation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.1425**						
	(0.0659)						
Pat Val: KPSS (within-IPC std.)		0.4804***					
		(0.0533)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.7630***				
			(0.0549)				
Pat Val: Foreign (mixed, within-IPC std.)				0.5207***			
				(0.0537)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.7715***		
					(0.0564)		
Pat Val: KPSS (VI, within-IPC std.)						0.4082***	
						(0.0594)	
Pat Val: Vertical Integration (within-IPC std.)							0.2565***
							(0.0557)
Within-IPC demeaned market cap	1.5001***	1.3731***	1.2985***	1.3629***	1.2968***	1.3988***	1.4369***
	(0.0537)	(0.0567)	(0.0562)	(0.0566)	(0.0563)	(0.0625)	(0.0624)
Constant	-2.8561***	-2.8047***	-2.7737***	-2.8015***	-2.7666***	-1.9646***	-1.9874***
	(0.1013)	(0.1012)	(0.1015)	(0.1012)	(0.1016)	(0.1358)	(0.1359)
Avg DV	0.001	0.001	0.001	0.001	0.001	0.678	0.678
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-7.62e+06	-6.90e+06	-6.90e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable *Patent Forward Cites* measures the number of forward citations received by a patent within five years of grant. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are reported in parentheses. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E12: Regression All: Dependent variable: Breakthrough Citation

	Breakthrough:All					Breakthrough: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0231**						
	(0.0103)						
Pat Val: KPSS (within-IPC std.)		0.1048***					
		(0.0153)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.1496***				
			(0.0154)				
Pat Val: Foreign (mixed, within-IPC std.)				0.1103***			
				(0.0153)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.1441***		
					(0.0153)		
Pat Val: KPSS (VI, within-IPC std.)						0.0964***	
						(0.0152)	
Pat Val: Vertical Integration (within-IPC std.)							0.0789***
							(0.0149)
Within-IPC demeaned market cap	-0.0140	-0.0418***	-0.0536***	-0.0431***	-0.0520***	-0.0437***	-0.0389***
	(0.0130)	(0.0134)	(0.0134)	(0.0134)	(0.0134)	(0.0144)	(0.0144)
Constant	0.2432**	0.2543***	0.2593***	0.2547***	0.2599***	0.7281***	0.7279***
	(0.0983)	(0.0982)	(0.0981)	(0.0982)	(0.0981)	(0.1331)	(0.1330)
Avg DV	-0.000	-0.000	-0.000	-0.000	-0.000	0.006	0.006
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.51e+06	-5.51e+06	-5.51e+06	-5.51e+06	-5.51e+06	-4.95e+06	-4.95e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Note: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Top 1% of Cited Patents] equals one if a patent belongs to the top 1 percent of the forward-citation distribution among patents granted in the same year and IPC class, and zero otherwise; this variable is multiplied by 100 for ease of interpretation (Squicciarini, Dernis, & Criscuolo, 2013). The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E13: Regression All: Dependent variable: Renew

	Renewed Full Term:All					Renewed Full Term: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	-0.0009						
	(0.0013)						
Pat Val: KPSS (within-IPC std.)		0.0235***					
		(0.0008)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0242***				
			(0.0008)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0240***			
				(0.0008)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0231***		
					(0.0008)		
Pat Val: KPSS (VI, within-IPC std.)						0.0232***	
						(0.0008)	
Pat Val: Vertical Integration (within-IPC std.)							0.0229***
							(0.0008)
Within-IPC demeaned market cap	-0.0157***	-0.0226***	-0.0228***	-0.0228***	-0.0224***	-0.0225***	-0.0222***
	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)
Constant	0.0171***	0.0177***	0.0178***	0.0177***	0.0178***	0.0192***	0.0203***
	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0039)	(0.0039)
Avg DV	0.000	0.000	0.000	0.000	0.000	-0.000	-0.000
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.88e+05	-4.87e+05	-4.87e+05	-4.87e+05	-4.87e+05	-4.84e+05	-4.84e+05
N	774,044	774,044	774,044	774,044	774,044	770,304	770,304

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E14: Regression All: Dependent variable: Litigation Citation

	Litigation:All					Litigation:VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0000						
	(0.0045)						
Pat Val: KPSS (within-IPC std.)		0.1165***					
		(0.0115)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.1239***				
			(0.0114)				
Pat Val: Foreign (mixed, within-IPC std.)				0.1177***			
				(0.0116)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.1229***		
					(0.0116)		
Pat Val: KPSS (VI, within-IPC std.)						0.1173***	
						(0.0129)	
Pat Val: Vertical Integration (within-IPC std.)							0.1124***
							(0.0127)
Within-IPC demeaned market cap	-0.1554***	-0.1866***	-0.1884***	-0.1868***	-0.1880***	-0.1954***	-0.1934***
	(0.0103)	(0.0107)	(0.0107)	(0.0107)	(0.0106)	(0.0117)	(0.0117)
Constant	-0.3638***	-0.3515***	-0.3505***	-0.3516***	-0.3497***	0.1237	0.1281
	(0.0270)	(0.0271)	(0.0271)	(0.0271)	(0.0271)	(0.0784)	(0.0785)
Avg DV	-0.001	-0.001	-0.001	-0.001	-0.001	0.033	0.033
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.94e+06	-4.47e+06	-4.47e+06
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Litigation] equals one if the patent is involved in at least one patent litigation event. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E15: Regression All: Dependent variable: Reassign Citation

	Reassign: All					Reassign: VI Sample	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	0.0002						
	(0.0003)						
Pat Val: KPSS (within-IPC std.)		0.0051***					
		(0.0003)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0067***				
			(0.0004)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0049***			
				(0.0003)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0054***		
					(0.0003)		
Pat Val: KPSS (VI, within-IPC std.)						0.0043***	
						(0.0004)	
Pat Val: Vertical Integration (within-IPC std.)							0.0040***
							(0.0004)
Within-IPC demeaned market cap	-0.0021***	-0.0035***	-0.0039***	-0.0034***	-0.0035***	-0.0036***	-0.0035***
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
Constant	-0.0636***	-0.0630***	-0.0629***	-0.0631***	-0.0630***	-0.0100***	-0.0099***
	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0025)	(0.0025)
Avg DV	-0.000	-0.000	-0.000	-0.000	-0.000	0.004	0.004
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.96e+04	-5.95e+04	-5.93e+04	-5.95e+04	-5.94e+04	-8.71e+04	-8.71e+04
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable $\mathbb{1}$[Reassignment] equals one if the patent experiences at least one ownership reassignment during its lifetime. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at $t-1$) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table E16: Regression All: Dependent variable: Trilateral Patent Citation

	Trilateral Patent:All					Trilateral Patent:VI	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Abnormal Return × Market Value (within-IPC std.)	-0.0003						
	(0.0004)						
Pat Val: KPSS (within-IPC std.)		0.0216***					
		(0.0007)					
Pat Val: Teamsize (mixed, within-IPC std.)			0.0228***				
			(0.0007)				
Pat Val: Foreign (mixed, within-IPC std.)				0.0211***			
				(0.0007)			
Pat Val: Sci & NoSci (mixed, within-IPC std.)					0.0224***		
					(0.0007)		
Pat Val: KPSS (VI, within-IPC std.)						0.0148***	
						(0.0007)	
Pat Val: Vertical Integration (within-IPC std.)							0.0143***
							(0.0006)
Within-IPC demeaned market cap	-0.0006	-0.0064***	-0.0066***	-0.0062***	-0.0065***	-0.0122***	-0.0119***
	(0.0006)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0005)	(0.0005)
Constant	-0.1472***	-0.1450***	-0.1448***	-0.1451***	-0.1447***	0.0727***	0.0732***
	(0.0033)	(0.0034)	(0.0034)	(0.0034)	(0.0034)	(0.0048)	(0.0048)
Avg DV	0.000	0.000	0.000	0.000	0.000	0.006	0.006
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Log Likelihood	-5.96e+05	-5.94e+05	-5.94e+05	-5.94e+05	-5.94e+05	-5.18e+05	-5.18e+05
N	1,444,021	1,444,021	1,444,021	1,444,021	1,444,021	1,294,585	1,294,585

Notes: This table reports estimates from patent-level regressions where the dependent variable is 𝟙[Trilateral Patent] equals one if the patent belongs to a triadic patent family with filings at the USPTO, EPO, and JPO, and zero otherwise. The main explanatory variables are standardized measures of patent value. Column (1) includes standardized abnormal stock returns multiplied by market capitalization (at t–1) around the patent grant date. Column (2) uses standardized patent values estimated using the baseline KPSS methodology. Columns (3)–(5) use standardized patent values estimated using the generalized KPSS approach, allowing patent value distributions to vary by inventor team size, foreign-inventor status, and reliance on science, respectively. Column (6) uses standardized patent values estimated using a baseline KPSS methodology and Column (7) includes generalized KPSS approach for a sub-sample of vertical integration firms. All patent value measures are standardized to have mean zero and unit variance. Market capitalization (in constant U.S. dollars) enters in logarithmic form. Standard errors are based on a bootstrap procedure implemented using the `boottest` command. Because `boottest` does not report standard errors directly, we compute bootstrap-based standard errors from the reported confidence intervals and use these for inference. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Appendix F
Robustness Analysis

Robustness tests, repeating Tables 2–6 using alternative measures of team size, offshoring, science, and vertical integration

Table F1: Summary Statistics: Patent Level

	Obs.	Mean	Std. Dev.	10%	50%	90%
𝟙[Large Team (median)]	1444649	0.375	0.484	0.000	0.000	1.000
𝟙[2/3 Foreign]	1444649	0.123	0.329	0.000	0.000	1.000
𝟙[Science Dummy (median)]	1444649	0.160	0.367	0.000	0.000	1.000
𝟙[Vertical Integration(median)]: High	1295446	0.550	0.498	0.000	1.000	1.000
Pat Val: KPSS (const. usd)	1444638	16.117	26.003	1.450	7.433	37.405
Pat Val: Teamsize (median) (mixed)	1444638	15.764	25.539	1.415	7.233	36.686
Pat Val:2/3 Foreign (mixed)	1444638	16.120	26.050	1.443	7.428	37.425
Pat Val: Sci&NoSci (median) (mixed)	1444638	15.762	25.629	1.413	7.207	36.550
Pat Val: KPSS (Verti Ineg.)	1295089	19.941	47.919	1.609	8.346	42.862
Pat Val: Vertical Integration (median) (sep)	1295089	19.139	46.096	1.335	7.802	40.695

Notes: This table reports summary statistics at the patent level. 𝟙[Large Team (median)] equals one if the inventor team size is above the median within the same IPC class–year. 𝟙[2/3 Foreign] equals one if at least two-thirds of inventors listed on the patent are non-U.S. based. 𝟙[Science Dummy (median)] equals one if the patent is science-based, defined as having above-median non-patent literature (NPL) citations within an IPC class–year, conditional on citing at least one NPL. 𝟙[Vertical Integration (median): High] equals one if the assignee's vertical integration measure is above the median within the same industry–year, based on (Frésard, Hoberg, & Phillips, 2020). Patent value measures are expressed in constant U.S. dollars. *Pat Val: KPSS (const. USD)* reports baseline patent values estimated using the standard KPSS methodology. *Pat Val: Team Size (median) (mixed)*, *Pat Val: 2/3 Foreign (mixed)*, and *Pat Val: Sci&NoSci (median) (mixed)* report patent values estimated using the generalized KPSS approach with mixed distributions corresponding to the indicated patent characteristics. *Pat Val: KPSS (Vertical Integration)* and *Pat Val: Vertical Integration (median) (sep)* report patent values for the subsample with available vertical-integration data, using baseline KPSS and characteristic-specific (separate) signal to noise ratio, respectively.

Table F2: Patent Level Regression: Alternate Definition

	Teamsize		R&D Offshoring		Science		Vertical Integration	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
𝟙[Large Team (median)]	0.010***	0.121***						
	(0.001)	(0.001)						
𝟙[2/3 Foreign]			-0.009***	-0.064***				
			(0.002)	(0.002)				
𝟙[Science Dummy (median)]					0.017***	0.167***		
					(0.001)	(0.001)		
𝟙[Vertical Integration(median)]: High							-0.008***	0.567***
							(0.002)	(0.002)
Log (Market cap (const. usd))	0.751***	0.751***	0.751***	0.751***	0.751***	0.751***	0.729***	0.729***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Constant	-1.409***	-1.469***	-1.403***	-1.391***	-1.409***	-1.453***	-0.876***	-1.276***
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.030)	(0.030)
Avg DV	15.820	15.795	15.820	15.818	15.820	15.792	15.940	15.857
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4-digit IPC Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.778	0.778	0.778	0.778	0.778	0.778	0.786	0.800
N	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,443,251	1,293,900	1,293,900

Notes: This table reports patent-level regressions using alternate definitions of key explanatory variables and patent-value estimation methods. The dependent variable in Columns (1), (3), (5) and (7) is the natural logarithm of patent private value estimated using the baseline KPSS methodology, which assumes a common signal-to-noise ratio across all patents. The dependent variable in Columns (2), (4), (6) and (8) is the natural logarithm of patent private value estimated using the generalized KPSS methodology, which allows the signal-to-noise ratio to vary by patent type. 𝟙[Large Team (median)] equals one if the inventor team size is above the median within the same IPC class–year. 𝟙[2/3 Foreign] equals one if at least two-thirds of inventors listed on the patent are non-U.S. based. 𝟙[Science Dummy (median)] equals one if the patent is science-based, defined as having above-median non-patent literature (NPL) citations within an IPC class–year, conditional on citing at least one NPL. 𝟙[Vertical Integration (median): High] equals one if the assignee's vertical integration measure is above the median within the same industry–year, based onFrésard, Hoberg, and Phillips (2020). All specifications include year fixed effects, four-digit IPC fixed effects, and firm fixed effects. Robust standard errors are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table F3: Regression: Abnormal return as the dependent variable (Alternate Definition)

	Abnormal Return			
	(1)	(2)	(3)	(4)
Patent day	0.0261***	0.0240***	0.0247***	0.0239***
	(0.0051)	(0.0045)	(0.0047)	(0.0062)
Patent day: Large Team Pat (median)	0.0154***			
	(0.0059)			
Patent day: Foreign Invt Pat (2/3)		0.0073		
		(0.0069)		
Patent day: Science Pat (median)			0.0167**	
			(0.0069)	
Patent day: High VI Firm (median)				0.0286***
				(0.0089)
Number of Patent	0.0005	0.0000	0.0002	0.0007
	(0.0007)	(0.0004)	(0.0006)	(0.0006)
Log(Market Cap (const. usd))	-0.1597***	-0.1214***	-0.1505***	-0.1760***
	(0.0037)	(0.0039)	(0.0042)	(0.0050)
Constant	3.1375***	2.4842***	3.0247***	3.5003***
	(0.0722)	(0.0795)	(0.0839)	(0.0981)
Avg DV	0.039	0.037	0.040	0.044
Firm Fixed Effects	Yes	Yes	Yes	Yes
Business Date	Yes	Yes	Yes	Yes
R^2	0.017	0.020	0.019	0.016
N	14,163,165	7,853,417	9,919,551	9,318,888

Note: This table presents robustness checks using alternate definitions of patent characteristics. The table reports firm-level regressions of abnormal stock returns around patent grant dates. Large-team, science-based, and vertical-integration indicators are redefined using median-based thresholds within IPC class–year (or industry–year for vertical integration), while the foreign-inventor indicator equals one if at least two-thirds of inventors listed on the patent are located outside the United States. The dependent variable is the cumulative abnormal return over the three-day window following the patent grant date. The indicator *Patent day* equals one on days when a firm is granted at least one patent. All columns restrict the sample to firms that have both patent and non-patent days in the estimation period. Columns (1)–(4) further restrict the sample to firms that exhibit within-firm variation in the corresponding patent-type indicator: large-team patents (Column 1), patents with all non-U.S. inventors (Column 2), science-based patents (Column 3), and patents granted by firms with high vertical integration (Column 4). All specifications include firm fixed effects and business-date fixed effects. Control variables include the log of firm market value and the number of patents granted on the event day. Standard errors are clustered at the firm level.

Table F4: Signal to Noise ratio

	Baseline	Team Size	R&D Offshoring	Science	Vertical Integration	
	(1)	(2)	(3)	(4)	(5)	(6)
Patent day	0.019***				0.023***	
	(0.005)				(0.005)	
Patent day: Large Team Pat (median)		0.021***				
		(0.007)				
Patent day: No Large Team Pat (median)		0.017***				
		(0.006)				
Patent day: Foreign Invt Pat (2/3)			0.018*			
			(0.010)			
Patent day: No Foreign Invt Pat (2/3)			0.019***			
			(0.005)			
Patent day: Science Pat (median)				0.024***		
				(0.008)		
Patent day: No Science Pat (median)				0.017***		
				(0.006)		
Patent day: High VI Firm (median)						0.032***
						(0.007)
Patent day: Low VI Firm (median)						0.010
						(0.007)
Constant	-7.622***	-7.622***	-7.622***	-7.622***	-7.561***	-7.561***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Avg DV	-7.622	-7.622	-7.622	-7.622	-7.561	-7.561
Firm*Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Date FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.169	0.169	0.169	0.169	0.177	0.177
N	16,637,718	16,637,718	16,637,718	16,637,718	13,277,826	13,277,826

Notes: This table reports estimates from regressions of firm-level abnormal stock returns around patent grant dates. The dependent variable is the three-day idiosyncratic return of firm f following patent grant date d, denoted r_{fd}. All specifications include firm-by-year fixed effects and grant-date fixed effects. Column (1) and (5) implements the baseline KPSS specification, which estimates a single patent-day effect and implicitly assumes a common signal-to-noise ratio across patent types. Columns (2), (3), (4) and (6) relax this restriction by allowing patent-day effects to vary across dimensions of heterogeneity: inventor team size, R&D offshoring, reliance on basic science, and vertical integration. This table also reports robustness checks using alternate definitions of patent characteristics; large-team, science-based, and vertical-integration indicators are defined using median-based thresholds within IPC class–year (or industry–year for vertical integration), while R&D offshoring is defined based on the share of foreign inventors. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table F5: Regression: Abnormal Returns (Adjusted for Grant Probability) as the Dependent Variable

	Abnormal Return			
	(1)	(2)	(3)	(4)
Patent day	0.0694***	0.0764***	0.0637***	0.0471***
	(0.0090)	(0.0085)	(0.0089)	(0.0121)
Patent day: Large Team Pat	0.0232*			
	(0.0127)			
Patent day: Foreign Invt Pat		-0.0117		
		(0.0143)		
Patent day: Science Pat			0.0380***	
			(0.0138)	
Patent day: High VI Firm				-0.0041
				(0.0213)
Number of Patent	-0.0005	-0.0005	-0.0006	0.0004
	(0.0005)	(0.0004)	(0.0005)	(0.0006)
Log(Market Cap (const. usd))	-0.1550***	-0.1284***	-0.1624***	-0.1701***
	(0.0040)	(0.0042)	(0.0043)	(0.0069)
Constant	3.0803***	2.6293***	3.2467***	3.4282***
	(0.0778)	(0.0859)	(0.0840)	(0.1379)
Avg DV	0.043	0.042	0.045	0.042
Firm Fixed Effects	Yes	Yes	Yes	Yes
Business Date	Yes	Yes	Yes	Yes
R^2	0.017	0.020	0.018	0.016
N	11,963,634	7,532,889	10,861,460	5,479,710

Note: This table reports firm-level regressions of abnormal stock returns around patent grant dates, where returns are adjusted for the probability of patent grant. The dependent variable is the cumulative abnormal return over the three-day window following the patent grant date. On grant days, returns are scaled by the inverse of the ex-ante probability of patent success, $(1-\pi)^{-1}$. The indicator *Patent day* equals one on days when a firm is granted at least one patent. All columns restrict the sample to firms that have both patent and non-patent days in the estimation period. Columns (1)–(4) further restrict the sample to firms that exhibit within-firm variation in the corresponding patent-type indicator: large-team patents (Column 1), patents with all non-U.S. inventors (Column 2), science-based patents (Column 3), and patents granted by firms with high vertical integration (Column 4). All specifications include firm fixed effects and business-date fixed effects. Control variables include the log of firm market value and the number of patents granted on the event day. Standard errors are clustered at the firm level.

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