Appendix A: Proof of Proposition 1

(a) can be obtained from Equation (7). (b) can be obtained from Equation (9). (c) to (f) can be obtained from Equations (4), (10), (11), and (12), respectively.

Appendix B: Proof of Proposition 2

We can show that Proposition 2 is true by comparing Equation (4) with Equation (16).

Appendix C: Proof of Proposition 3

Proposition 3 requires Equation (7) to be larger than Equation (19), i.e.,

$$s_b{}^* = \frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}E[1+\ln(k_r L)]\right\} > \frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}(1-\delta)E\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right\} = s_c{}^*. \tag{C1}$$

The above inequality can be rewritten as

$$\delta E[1+\ln(k_r L)] > (1-\delta)\ln\left(\frac{1-\beta\delta}{1-\delta}\right). \tag{C2}$$

As shown in Figure B1, $\delta > (1-\delta)\ln\left(\frac{1}{1-\delta}\right)$. As

$$\delta E[1+\ln(k_r L)] > \delta > (1-\delta)\ln\left(\frac{1}{1-\delta}\right) > (1-\delta)\ln\left(\frac{1-\beta\delta}{1-\delta}\right),$$

Proposition 3 follows.

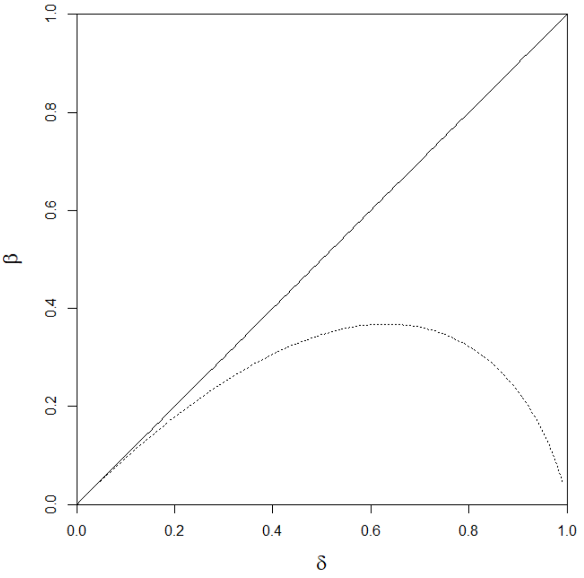


Figure B1. Plots of δ (solid) and $(1-\delta)\ln\left(\frac{1}{1-\delta}\right)$ (dot)

Appendix D: Proof of Proposition 4

Comparing Equation (21) with Equation (10), it is easy to see that the expected risk mitigation spending is increased because the multiplier in Equation (21), $\frac{1}{1-\delta}$, is greater than one. Furthermore, $\frac{E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]} > \frac{E[\ln(k_rL)]}{1+E[\ln(k_rL)]}$ because $\frac{x}{1+x}$ increases with x and $\frac{1-\beta\delta}{1-\delta} > 1$.

Appendix E: Proof of Proposition 5

Comparing Equation (22) with Equation (11), cyber insurance decreases expected mitigated loss if and only if

$$\frac{1}{k_s}\frac{1}{E[1+\ln(k_rL)]} > \frac{1}{k_s}\frac{1}{1-\beta\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}. \tag{E1}$$

If $\beta = 0$, Equation (22) becomes $\frac{1}{k_s}\frac{1}{1+E\left[\ln\left(\frac{1}{1-\delta}k_rL\right)\right]}$, which is smaller than Equation (11). If $\beta = 1$, Equation (22) becomes $\frac{1}{k_s}\frac{1}{1-\delta}\frac{1}{1+E[\ln(k_rL)]}$, which is larger than Equation (11). Differentiating Equation (22) shows that it is increasing in β:

$$\frac{\partial}{\partial\beta}\frac{1}{k_s}\frac{1}{1-\beta\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]} = \frac{\delta\left[2+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{k_s\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]^2(1-\beta\delta)^2} > 0.$$

Hence, β needs to be sufficiently small for cyber insurance to reduce expected mitigated loss.

Appendix F: Comparative Statics (with respect to *β*)

The equilibrium expected risk prevention investment, s_c^*, is, according to Equation (19),

$$\frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}(1-\delta)E\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}.$$

It decreases with β because the natural log is an increasing function. The equilibrium expected risk prevention effectiveness, $q(s_c^*)$, is, according to Equation (20),

$$1-\frac{k_r}{k_s}\frac{1}{1-\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

It decreases with β because β decreases the denominator of the second term, which increases the size of the second term, which is negative. The expected risk mitigation spending for a given breach, r_c^*, is, according to Equation (16),

$$\frac{1}{k_r}E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right].$$

It decreases with β because the natural log is an increasing function. The equilibrium expected risk mitigation spending, $[1-q(s_c{}^*)]E[r_c{}^*(L)]$, is, according to Equation (21),

$$\frac{1}{k_s}\frac{1}{1-\delta}\frac{E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

It decreases with β because β decreases $\frac{1-\beta\delta}{1-\delta}$ and the function $\frac{x}{1+x}$ is an increasing function of x. The equilibrium expected mitigated loss, $[1-q(s_c{}^*)]E\left[\left\{1-m\left(r_c{}^*(L)\right)\right\}L\right]$, is, according to Equation (22),

$$\frac{1}{k_s}\frac{1}{1-\beta\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

It increases with β because β decreases the denominator of both $\frac{1}{1-\beta\delta}$ and $\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}$. From Equation (23), we can see that the equilibrium expected firm's utility,

$$E[W_1{}^{**}] = w_0 - p - \frac{1}{k_s} - \frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}(1-\delta)E\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\},$$

increases with β because β decreases the size of the last term above, which is negative.

Appendix G: Comparative Statics (with respect to δ)

The equilibrium expected risk prevention investment, s_c^*, is, according to Equation (19),

$$\frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}(1-\delta)E\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}.$$

Differentiating the above with respect to δ gives

$$\frac{-\beta(1-\delta)-(1-\beta\delta)E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{k_s(1-\delta)(1-\beta\delta)\left(1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right)} < 0.$$

The equilibrium expected risk prevention effectiveness, $q(s_c^*)$, is, according to Equation (20),

$$1-\frac{k_r}{k_s}\frac{1}{1-\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

It decreases with δ because $q(s)$ moves in the same direction as s. The expected risk mitigation spending for a given breach, r_c^*, is, according to Equation (16),

$$\frac{1}{k_r}E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right].$$

Differentiating the above with respect to δ gives

$$\frac{1-\beta}{k_r(1-\delta)(1-\beta\delta)} > 0.$$

The equilibrium expected risk mitigation spending, $[1-q(s_c{}^*)]E[r_c{}^*(L)]$, is, according to Equation (21),

$$\frac{1}{k_s}\frac{1}{1-\delta}\frac{E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

Differentiating the above with respect to δ gives

$$\frac{E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{k_s(1-\delta)^2\left\{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}}+\frac{\frac{1-\beta}{(1-\delta)^2}}{k_s(1-\beta\delta)\left\{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}}\left\{1-\frac{E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}\right\}>0,$$

because both of the above terms are positive. The equilibrium expected mitigated loss, $[1-q(s_c{}^*)]E\left[\left\{1-m\left(r_c{}^*(L)\right)\right\}L\right]$, is according to Equation (22),

$$\frac{1}{k_s}\frac{1}{1-\beta\delta}\frac{1}{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]}.$$

Differentiating the above with respect to δ gives

$$\frac{\beta}{k_s(1-\beta\delta)^2\left\{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}}-\frac{\frac{1-\beta}{1-\delta}}{k_s(1-\beta\delta)^2\left\{1+E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\}^2},$$

which is positive if and only if

$$\beta\left(2-\delta+(1-\delta)E\left[\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right)-1>0.$$

The left-hand side above can be shown to be increasing in β. In other words, it is more likely to be positive with a large β and negative with a small β. This is consistent with Proposition 5, which suggests that for the equilibrium expected mitigated loss to decrease, β has to be sufficiently small. The left-hand side above can also be shown to be decreasing in δ, which means that it is more likely to be positive if δ is small and negative if δ is large. The equilibrium expected firm's utility is given by Equation (23):

$$w_0-p-\frac{1}{k_s}-\frac{1}{k_s}\ln\left\{\frac{k_s}{k_r}(1-\delta)E\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_rL\right)\right]\right\},$$

which is simply equal to $w_0-p-\frac{1}{k_s}-s_c^*$. In other words, it moves in an opposite direction to s_c^*. Hence, it increases with δ.

Appendix H: Analysis with General Form $q(s)$ and $m(r)$ Functions

Proposition 1 establishes the baseline for comparison with cyber insurance. Without specifying the functional form for $q(s)$ and $m(r)$, it is not possible to obtain closed-form solutions for the base model. However, we can still evaluate the effects of cyber insurance on the different equilibrium quantities based on changes in the first-order conditions. In the base model, the first order solution is given by Equation (3). Hence, at the end of $t = 1$,

$$w_1^* = w_0 - s - r_b^*(\ell) - [1 - m(r_b^*(\ell))]\ell. \tag{H1}$$

Substituting Equation (H1) into the firm's objective function at $t = 0$, we have

$$E[W_1{}^*] = q(s)(w_0 - s) + [1 - q(s)]E[w_0 - s - r_b^*(L) - [1 - m(r_b^*(L))]L]. \tag{H2}$$

The first-order condition with respect to s is

$$\frac{\partial E[W_1{}^*]}{\partial s} = q'(s)E[r_b^*(L) + [1 - m(r_b^*(L))]L] - 1 = 0,$$

which can be simplified to

$$q'(s) = \frac{1}{E[r_b^*(L)+[1-m(r_b^*(L))]L]}. \tag{H3}$$

H.1 Robustness of Proposition 2

With cyber insurance, at $t = 1$, the first-order condition is given by Equation (15). As $m'(r)$ is decreasing in r and the right-hand side of Equation (15) is smaller than the right-hand side of Equation (3), i.e.,

$$\frac{1-\delta}{(1-\beta\delta)\ell} < \frac{1}{\ell},$$

the *ex-post* risk mitigation per incident is increased, i.e., $r_c^*(\ell) > r_b^*(\ell)$. Hence, Proposition 2 continues to hold.

H.2 Robustness of Proposition 3

By backward induction, substituting $r_c^*(\ell)$ into Equation (2), we have

$$w_1 = w_0 - s - p - r_c^*(\ell) - [1 - m(r_c^*(\ell))]\ell + \delta(r_c^*(\ell) + \beta[1 - m(r_c^*(\ell))]\ell).$$

At $t = 0$, the firm's objective function becomes

$$E[W_1{}^*] = w_0 - s - p - [1 - q(s)]E[r_c^*(L) + [1 - m(r_c^*(L))]L - \delta(r_c^*(L) + \beta[1 - m(r_c^*(L))]L)].$$

The first-order condition with respect to s is

$$\frac{\partial E[W_1{}^*]}{\partial s} = -1 + q'(s)E[r_c^*(L) + [1 - m(r_c^*(L))]L - \delta(r_c^*(L) + \beta[1 - m(r_c^*(L))]L)] = 0,$$

which can be simplified to

$$q'(s) = \frac{1}{E[r_c^*(L)+[1-m(r_c^*(L))]L-\delta(r_c^*(L)+\beta[1-m(r_c^*(L))]L)]}. \tag{H4}$$

Proposition 3 holds if and only if the right-hand side of Equation (H4) is larger than the right-hand side of Equation (H3), i.e.,

$$\frac{1}{E\left[r_c^*(L)+\left[1-m\left(r_c^*(L)\right)\right]L-\delta\left(r_c^*(L)+\beta\left[1-m\left(r_c^*(L)\right)\right]L\right)\right]}>\frac{1}{E\left[r_b^*(L)+\left[1-m\left(r_b^*(L)\right)\right]L\right]},$$

or equivalently,

$$E\left[r_c^*(L)+\left[1-m\left(r_c^*(L)\right)\right]L-\delta\left(r_c^*(L)+\beta\left[1-m\left(r_c^*(L)\right)\right]L\right)\right]<E\left[r_b^*(L)+\left[1-m\left(r_b^*(L)\right)\right]L\right], \quad \text{(H5)}$$

i.e., the net cost of the incident is lower with insurance than without it. Intuitively, this should be the case as the whole point of purchasing insurance is to minimize liability in the event of an incident. To prove this mathematically, we make use of chain rule and the first-order condition (Equation (14) = 0) to show the first derivative of the net cost of the incident with respect to β:

$$\frac{\partial}{\partial\beta}E\left[r_c^*(L)+\left[1-m\left(r_c^*(L)\right)\right]L-\delta\left(r_c^*(L)+\beta\left[1-m\left(r_c^*(L)\right)\right]L\right)\right]<0. \quad \text{(H6)}$$

When β is at its smallest possible value, i.e., $\beta=0$, Equation (15) becomes $m'(r)=\frac{1-\delta}{\ell}$. Equation (H5) becomes

$$E\left[(1-\delta)r_c^*(L)+\left[1-m\left(r_c^*(L)\right)\right]L\right]<E\left[r_b^*(L)+\left[1-m\left(r_b^*(L)\right)\right]L\right]$$

or

$$E\left[(1-\delta)\left[r_c^*(L)-r_b^*(L)\right]-\delta r_b^*(L)\right]<E\left[\left[m\left(r_c^*(L)\right)-m\left(r_b^*(L)\right)\right]L\right],$$

which is always true because

$$E\left[\left[m\left(r_c^*(L)\right)-m\left(r_b^*(L)\right)\right]L\right]=E\left[L\int_{r_b^*(L)}^{r_c^*(L)}m'(r)\,dr\right]=E\left[L\left\{\int_{r_b^*(L)}^{r_c^*(L)}\left[m'(r)-\frac{1-\delta}{L}\right]dr+\int_{r_b^*(L)}^{r_c^*(L)}\frac{1-\delta}{L}\,dr\right\}\right]=$$

$$E\left[(1-\delta)\left[r_c^*(L)-r_b^*(L)\right]+L\int_{r_b^*(L)}^{r_c^*(L)}\left[m'(r)-\frac{1-\delta}{L}\right]dr\right]>E\left[(1-\delta)\left[r_c^*(L)-r_b^*(L)\right]-\delta r_b^*(L)\right].$$

If $\beta>0$, the left-hand side of Equation (H5) will only get smaller according to Equation (H6). Hence, Equation (H5) is always true for all of the possible values of β. Therefore, Proposition 3 holds as long as $q(s)$ and $m(r)$ are concave functions. The functional form is not important.

H.3 Robustness of Proposition 4

The expected risk mitigation spending with cyber insurance is $[1-q({s_c}^*)]E[\,r_c^*(L)]$. The expected risk mitigation spending without cyber insurance is $[1-q({s_b}^*)]E[\,r_b^*(L)]$. Proposition 4 holds if and only if

$$[1-q({s_c}^*)]E[\,r_c^*(L)]>[1-q({s_b}^*)]E[\,r_b^*(L)]$$

or

$$E[\,r_c^*(L)] - E[\,r_b^*(L)] > q(s_c{}^*)E[\,r_c^*(L)] - q(s_b{}^*)E[\,r_b^*(L)],$$

which is true because

$$E[\,r_c^*(L)] - E[\,r_b^*(L)] > q(s_b{}^*)\{E[\,r_c^*(L)] - E[\,r_b^*(L)]\} > q(s_c{}^*)E[\,r_c^*(L)] - q(s_b{}^*)E[\,r_b^*(L)].$$

H.4 Robustness of Proposition 5

The equilibrium expected mitigated loss with cyber insurance is $[1 - q(s_c{}^*)]E\big[\{1 - m\big(\,r_c^*(L)\big)\}L\big]$. The equilibrium expected mitigated loss without cyber insurance is $[1 - q(s_b{}^*)]E\big[\{1 - m\big(\,r_b^*(L)\big)\}L\big]$. Cyber insurance reduces the equilibrium expected mitigated loss if and only if

$$[1 - q(s_b{}^*)]E\big[\{1 - m\big(\,r_b^*(L)\big)\}L\big] > [1 - q(s_c{}^*)]E\big[\{1 - m\big(\,r_c^*(L)\big)\}L\big]. \tag{H7}$$

Hence, we need both $1 - q(s_c{}^*)$ and $1 - m\big(\,r_c^*(L)\big)$ to be small. In other words, we need $r_c^*(L)$ and $s_c{}^*$ to be as large as possible. At $t = 1$, the first-order condition with respect to *r* is given by Equation (15):

$$m'(r) = \frac{1-\delta}{(1-\beta\delta)\ell}.$$

The right-hand side increases with β. Hence, $r_c^*(L)$ decreases with β. At $t = 0$, the first-order condition with respect to *s* is given by Equation (H4):

$$q'(s) = \frac{1}{E\big[r_c^*(L)+\big[1-m\big(r_c^*(L)\big)\big]L-\delta\big(r_c^*(L)+\beta\big[1-m\big(r_c^*(L)\big)\big]L\big)\big]}.$$

As shown in Section H.2, the right-hand side above increases with β. Hence, like $r_c^*(L)$, $s_c{}^*$ decreases with β. Assuming that it is actually possible for Equation (H7) to be satisfied at $\beta = 0$, Equation (H7) is satisfied only if β is sufficiently small.

H.5 Robustness of Comparative Statics (Table 2)

In Section H.4, we show that $s_c{}^*$ decreases with β. As $q(s_c{}^*)$ increases with $s_c{}^*$, it also decreases with β. In accordance with Equation (15), $r_c{}^*(\ell)$ decreases with β, i.e., $E[r_c{}^*(L)]$ decreases with β. As $1 - q(s_c{}^*)$ increases with β and $r_c{}^*(\ell)$ decreases with β, $[1 - q(s_c{}^*)]\ E[r_c{}^*(L)]$ decreases with β. As $r_c{}^*(\ell)$ decreases with β and $m\big(r_c{}^*(\ell)\big)$ increases with $r_c{}^*(\ell)$, $1 - m\big(r_c{}^*(\ell)\big)$ increases with β. As both $1 - q(s_c{}^*)$ and $1 - m\big(r_c{}^*(\ell)\big)$ increase with β, $[1 - q(s_c{}^*)]\ E\big[\{1 - m\big(r_c{}^*(L)\big)\}L\big]$ increases with β. By the chain rule and the first-order conditions, it can be shown that

$$\frac{\partial E[W_1^{**}]}{\partial \beta} = \frac{\partial}{\partial \beta}\left(w_0 - s_c{}^* - p - [1 - q(s_c{}^*)]\left\{(1-\delta)\, r_c^*(L) + (1-\beta\delta)\left[1 - m\left(r_c^*(L)\right)\right]L\right\}\right) > 0.$$

Hence, the comparative statics in Table 2 do not rely on the functional form of $q(s)$ and $m(r)$.

H.8 Robustness of Comparative Statics (Table 3)

Differentiating the denominator of Equation (H4) with respect to δ, we have

$$\frac{\partial}{\partial \delta} E\left[r_c^*(L) + \left[1 - m\left(r_c^*(L)\right)\right]L - \delta\left(r_c^*(L) + \beta\left[1 - m\left(r_c^*(L)\right)\right]L\right)\right].$$

Using chain rule and the first-order condition (Equation (14) = 0), it can be shown that the above is negative. Hence, $s_c{}^*$ decreases with δ. As $q(s_c{}^*)$ moves in the same direction as $s_c{}^*$, it also decreases with δ. As $\frac{\partial}{\partial \delta}\frac{1-\delta}{(1-\beta\delta)\ell} = -\frac{1-\beta}{(1-\beta\delta)^2\ell} < 0$, the right-hand side of Equation (15) decreases with δ and hence $r_c{}^*(\ell)$ and $E[r_c{}^*(L)]$ increase with δ. As both $1 - q(s_c{}^*)$ and $E[r_c{}^*(L)]$ increase with δ, $[1 - q(s_c{}^*)]E[r_c{}^*(L)]$ also increases with δ. The equilibrium expected mitigated loss is $[1 - q(s_c{}^*)]E\left[\left\{1 - m\left(r_c^*(L)\right)\right\}L\right]$. Its first derivative with respect to δ is

$$-q'(s_c{}^*)\frac{\partial s_c^*}{\partial \delta} E\left[\left\{1 - m\left(r_c^*(L)\right)\right\}L\right] + [1 - q(s_c{}^*)]E\left[\left\{-m'\left(r_c^*(L)\right)\frac{\partial r_c^*(L)}{\partial \delta}\right\}L\right].$$

As $\frac{\partial s_c^*}{\partial \delta} < 0$, whereas $\frac{\partial r_c^*(L)}{\partial \delta} > 0$, the overall sign of the above is unclear. By the chain rule and the first-order conditions, it can be shown that

$$\frac{\partial}{\partial \delta}\left(w_0 - s_c{}^* - p - [1 - q(s_c{}^*)]\left\{(1-\delta)\, r_c^*(L) + (1-\beta\delta)\left[1 - m\left(r_c^*(L)\right)\right]L\right\}\right) > 0.$$

Hence, the comparative statics in Table 2 do not rely on the functional form of $q(s)$ and $m(r)$.

Appendix I: Robustness of Proposition 3 in the Presence of Deductible

By backward induction, at $t = 0$, the firm's expected wealth is given by Equation (5), or

$$E[W_1{}^*] = w_0 - s - [1 - q(s)]E\left\{\frac{1}{k_r}[1 + \ln(k_r L)]\right\},$$

if $L = \ell$ is sufficiently small such that there is no claim, and

$$E[W_1{}^*] = w_0 - s - p - [1 - q(s)]E\left\{\frac{1}{k_r}(1-\delta)\left[1 + \ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right] - d\right\},$$

if $L = \ell$ is sufficiently large such that a claim is made. The first order condition associated with a sufficiently small ℓ is therefore the same as that in the base model, i.e., $\frac{\partial E[W_1{}^*]}{\partial s} = 0$, or

$$q'(s) = \frac{1}{E\left\{\frac{1}{k_r}[1+\ln(k_r L)]\right\}}, \tag{I1}$$

while the first order condition associated with a sufficiently large ℓ is

$$q'(s) = \frac{1}{E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]-d\right\}}. \tag{I2}$$

The denominator in both Equations (I1) and (I2) represent the firm final cost of the breach. Since this final cost must be smaller with cyber insurance than without it, i.e., $E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]-d\right\} < E\left\{\frac{1}{k_r}[1+\ln(k_r L)]\right\}$ for a sufficiently large ℓ, the right-hand side of Equation (I1) is smaller than the right-hand side of Equation (I2) for a sufficiently large ℓ. And since $q'(s)$ is a decreasing function of *s*, the optimal *s* is decreased for a sufficiently large ℓ. Hence, overall, cyber insurance still decreases the optimal *s*, albeit to a lesser degree. Hence, Proposition 3 continues to hold in the presence of a deductible.

Appendix J: Robustness of Proposition 3 in the Presence of An Indemnity Limit

By backward induction, at $t = 0$, the firm's expected wealth is

$$E[W_1{}^*] = w_0 - s - p - [1-q(s)]E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right\},$$

if $L = \ell$ is sufficiently small such that the uncapped claim size is smaller than i, and

$$E[W_1{}^*] = w_0 - s - p - [1-q(s)]E\left\{\frac{1}{k_r}[1+\ln(k_r L)]+i\right\},$$

if $L = \ell$ is sufficiently large such that uncapped claim size is greater than i. The first order condition associated with a sufficiently small ℓ is therefore the same as that in the cyber insurance model (Equation 18), i.e.,

$$q'(s) = \frac{1}{E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right\}}, \tag{J1}$$

while the first order condition associated with a sufficiently large ℓ is

$$q'(s) = \frac{1}{E\left\{\frac{1}{k_r}[1+\ln(k_r L)]-i\right\}}. \tag{J2}$$

We also note that Equation (J2) is almost the same as the corresponding first order condition of the base model in Equation (6), except for the presence of the term i in the denominator. The term i decreases the denominator and, hence, increases the right-hand side of Equation (J2). Since $q'(s)$ is decreasing in *s*, the overall effect is a decrease in the optimal *s*. Appendix C has already shown that Equation (J1) leads to a decrease in the optimal *s*. Since both Equation (J1) and Equation (J2) decrease the optimal *s*, overall, cyber insurance still decreases the optimal *s* and Proposition 3 continues to hold.

Appendix K: The Presence of Risk Aversion

K.1 Robustness of Proposition 3 in the Presence of Risk Aversion

By backward induction, at $t = 0$, the firm's expected utility for the base model is

$$E[u(W_1{}^*)] = q(s)u(w_0 - s) + [1 - q(s)]E\left\{u\left(w_0 - s - \frac{1}{k_r}[1 + \ln(k_r L)]\right)\right\}.$$

The first order condition is

$$\frac{\partial E[u(W_1{}^*)]}{\partial s} = q'(s)\left[u(w_0 - s) - E\left\{u\left(w_0 - s - \frac{1}{k_r}[1 + \ln(k_r L)]\right)\right\}\right] - q(s)u'(w_0 - s) - [1 - q(s)]E\left\{u'\left(w_0 - s - \frac{1}{k_r}[1 + \ln(k_r L)]\right)\right\} = 0. \tag{K1}$$

By backward induction, at $t = 0$, the firm's expected utility for the cyber insurance model is

$$E[u(W_1{}^*)] = q(s)u(w_0 - s - p) + [1 - q(s)]E\left\{u\left(w_0 - s - p - \frac{1}{k_r}(1 - \delta)\left[1 + \ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right)\right\}.$$

The first order condition is

$$\frac{\partial E[u(W_1{}^*)]}{\partial s} = q'(s)\left(u(w_0 - s - p) - E\left\{u\left(w_0 - s - p - \frac{1}{k_r}(1 - \delta)\left[1 + \ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right)\right\}\right) - q(s)u'(w_0 - s - p) - [1 - q(s)]E\left\{u'\left(w_0 - s - p - \frac{1}{k_r}(1 - \delta)\left[1 + \ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right)\right\} = 0. \tag{K2}$$

In general, both Equation (K1) and Equation (K2) can be expressed as:

$$q'(s) = \frac{q(s)u'(\text{wealth without breach}) + [1 - q(s)]E[u'(\text{wealth with breach})]}{u(\text{wealth without breach}) - E[u(\text{wealth with breach})]}, \tag{K3}$$

where wealth with breach is random. Cyber insurance's effects on the firm's wealth depends on two parts. First, it reduces the firm's wealth when there is no breach because the firm has to pay for the insurance. Second, it increases the firm's wealth when there is a breach because some of the costs are now borne by the insurer. The first effect decreases the "wealth without breach" term in Equation (K3), which increases the first term in the numerator because $u''(w) < 0$ and decreases the denominator. Overall, the first effect increases the right-hand side of Equation (K3). Because $q'(s)$ is decreasing in *s*, the first effect creates a pressure to decrease s^*.

The second effect increases the "wealth with breach" term in Equation (K3), which decreases both the denominator and the numerator. If the percentage decrease in the numerator is smaller than the percentage decrease in the denominator, the overall effect on the right-hand side is positive, creating a pressure to decrease s^*. In other words, the second effect is negative if

$$\frac{[1-q(s_b^*)]|E[u'(\text{wealth with breach}_b)]-E[u'(\text{wealth with breach}_c)]|}{q(s_b^*)u'(\text{wealth without breach}_b)+[1-q(s_b^*)]u'(\text{wealth with breach}_b)} <$$

$$\frac{E[u(\text{wealth with breach}_c)]-E[u(\text{wealth with breach}_b)]}{u(\text{wealth without breach}_b)-E\{u(\text{wealth with breach}_b)\}}.$$

After some re-arrangement and applying Equation (K3),

$$\frac{[1-q(s_b^*)]|E[u'(\text{wealth with breach}_b)]-E[u'(\text{wealth with breach}_c)]|}{E[u(\text{wealth with breach}_c)]-E[u(\text{wealth with breach}_b)]} < q'(s_b^*). \tag{K4}$$

Denote $x_b = \text{wealth with breach}_b$ and $x_b + \Delta = \text{wealth with breach}_c$, Equation (K4) simplifies to:

$$\frac{\left|E\left[\int_{x_b}^{x_b+\Delta} u''(x)dx\right]\right|}{E\left[\int_{x_b}^{x_b+\Delta} u'(x)dx\right]} < \frac{q'(s_b^*)}{1-q(s_b^*)}. \tag{K5}$$

For a realistic decreasing absolute aversion function, such as $u(x) = \ln(x)$, $|u''(x)|/u'(x)$ decreases in x. Hence, Equation (K5) is more likely to hold if x_0 is sufficiently large or if Δ is sufficiently large, or both. This sufficient condition is easy to satisfy. For example, for $u(x) = \ln(x)$ and $q(s) = 1 - e^{-s}$, we have

$$\frac{\left|E\left[\int_{x_b}^{x_b+\Delta} \frac{1}{x^2}dx\right]\right|}{E\left[\int_{x_b}^{x_b+\Delta} \frac{1}{x}dx\right]} < 1.$$

As long as x_b is at least 1, i.e., the utility does not become zero as ln(1) = 0, the expression above always holds regardless of Δ. In other words, if a breach does not make the firm's utility go to zero, Equation (K5) is satisfied. Hence, overall, s^* is decreased.

K.2: Numerical Examples for Risk Aversion with Endogenous Pricing of Cyber Insurance

In the following, we provide numerical examples to show that our results continue to hold in the presence of risk aversion and endogenous pricing. The game sequence is as follows: at $t = 0$, the firm decides its risk prevention investment, s, and the insurer sets the price p simultaneously. Since there is perfect competition in the insurance market, the insurer will offer cyber insurance at the actuarially fair price, i.e., $p = [1 - q(s)]\kappa = [1 - q(s)]\delta\{r + \beta[1 - m(r)]\ell\}$. We note that despite perfect competition, the price is a function of s and therefore it can be endogenously determined.

At $t = 1$, if there is no breach, the firm does not need to do anything and there will be no change to the firm's utility. However, if there is a breach, the firm has to determine the risk mitigation spending r; some of the expenses and losses are shared with the insurer.

Because of increased complexity, the problem is not analytically tractable. Hence, we analyze it numerically. For simplicity, we set $k_s = 1, k_r = 1, w_0 = 10, L \sim Uniform(1,10), \beta = \frac{1}{10}$, and $\delta = \frac{1}{5}$. We use the log utility function, which models risk aversion and is commonly used in the literature (Pulley 1983). We again apply backward induction. For the baseline model without cyber insurance, at $t = 1$, in case of a breach, the firm will maximize the utility

$$\ln(w_0 - s - r - [1 - m(r)]\ell)$$

with respect to r. The first order condition is:

$$\frac{-1+m'(r)\ell}{w_0-s-r-[1-m(r)]\ell} = 0,$$

which is the same as Equation (3). Therefore, $r_b{}^*(\ell) = \frac{1}{k_r}\ln(k_r\ell) = \ln(\ell)$ as $k_r = 1$. At $t = 0$, the firm substitutes $r = r_b{}^*(\ell)$ into its objective function to obtain:

$$\mathrm{E}[\ln(W_1{}^*)] = q(s)\ln(w_0 - s) + [1 - q(s)]\mathrm{E}\big[\ln\big(w_0 - s - 1 - \ln(L)\big)\big].$$

With the help of a mathematical software, the equilibrium expected risk prevention investment, expected risk prevention effectiveness, expected risk mitigation spending, expected mitigated loss, and expected firm utility are, respectively, $s_b{}^* = 0.9699$, $q(s_b{}^*) = 0.6209$, $[1 - q(s_b{}^*)]E[\,r_b{}^*(L)] = 0.5908$, $[1 - q(s_b{}^*)]\mathrm{E}\big[\{1 - m\big(\,r_b{}^*(L)\big)\}L\big] = 0.3791$, and $\mathrm{E}[\ln(W_1{}^{**})] = 2.0728$.

For the cyber insurance model, at $t = 1$, the firm will maximize the utility

$$\ln\big(w_0 - s - p - r - [1 - m(r)]\ell + \delta(r + \beta[1 - m(r)]\ell)\big)$$

with respect to r. The first order condition is:

$$\frac{-(1-\delta)+(1-\delta\beta)m'(r)\ell}{w_0-s-p-r-[1-m(r)]\ell+\delta(r+\beta[1-m(r)]\ell)} = 0,$$

which is the same as Equation (15). With $k_r = 1, \beta = \frac{1}{10}$, and $\delta = \frac{1}{5}$, $r_c^* = \ln\left(\frac{49}{40}\ell\right)$. At $t = 0$, the insurer will set the competitive cyber insurance price to be:

$$p^* = [1 - q(s)] \times \frac{1}{5} \times \mathrm{E}\left\{\ln\left(\frac{49}{40}L\right) + \frac{4}{49}\right\} = 0.3686[1 - q(s)], \qquad \text{(K6)}$$

where $L \sim Uniform(1,10)$. At the same time, with $r_c^* = \ln\left(\frac{49}{40}\ell\right)$, the firm's objective function is

$$\mathrm{E}[\ln({W_1}^*)] = q(s)\ln(w_0 - s - p) + [1 - q(s)]\mathrm{E}\left\{\ln\left(w_0 - s - p - \frac{4}{5}\left[1 + \ln\left(\frac{49}{40}L\right)\right]\right)\right\}.$$

The first order condition is:

$$q'(s)\mathrm{E}\left[\ln\left(\frac{w_0 - s - p}{w_0 - s - p - \frac{4}{5}\left[1 + \ln\left(\frac{49}{40}L\right)\right]}\right)\right] = \frac{q(s)}{w_0 - s - p} + \frac{[1 - q(s)]}{\mathrm{E}\left\{\ln\left(w_0 - s - p - \frac{4}{5}\left[1 + \ln\left(\frac{49}{40}L\right)\right]\right)\right\}}.$$

It can be numerically shown that the equilibrium expected risk prevention investment, expected risk prevention effectiveness, expected risk mitigation spending, expected mitigated loss, and expected firm utility are, respectively, ${s_c}^* = 0.7995$, $q({s_c}^*) = 0.5504$, $[1 - q({s_c}^*)]E[{r_c}^*(L)] = 0.7918$, $[1 - q({s_c}^*)]\mathrm{E}\big[\{1 - m({r_c}^*(L))\}L\big] = 0.3670$, and $\mathrm{E}[\ln({W_1}^{**})] = 2.0740$.

Table K1 compares the equilibrium outcomes of the base model and cyber insurance model with risk aversion and competitive pricing.

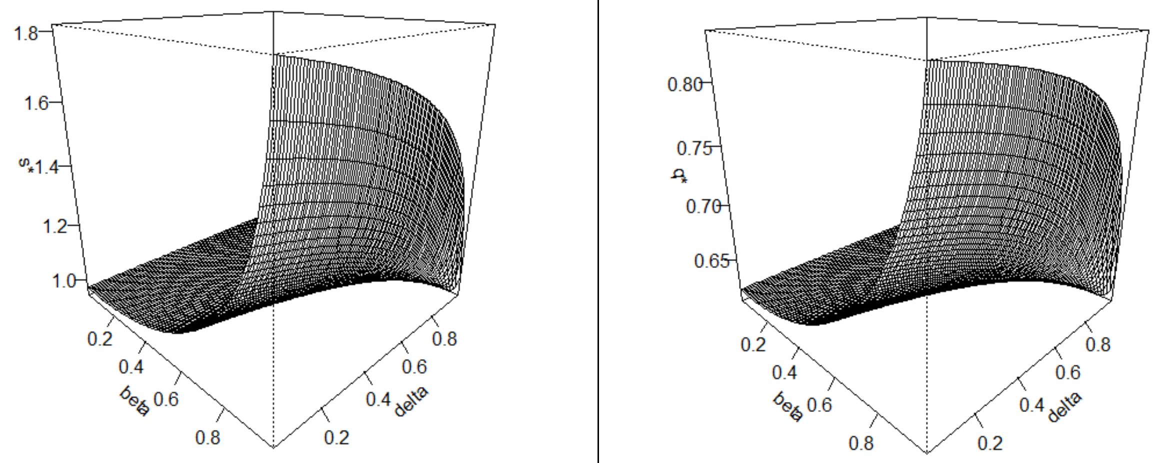
Table K1. Comparing the Equilibrium of the Base Model and the Cyber Insurance Model

	Base Model	Cyber Insurance Model	Change
Equilibrium Expected Risk Prevention Investment, s^*	0.9699	0.7995	Decreased
Equilibrium Expected Risk Prevention Effectiveness, $q(s^*)$	0.6209	0.5504	Decreased
Expected Risk Mitigation Spending for a Given Breach, $r^*(\ell)$	$\ln(\ell)$	$\ln\left(\frac{49}{40}\ell\right)$	Increased
Equilibrium Expected Risk Mitigation Spending, $[1 - q(s^*)]E[r^*(L)]$	0.5908	0.7918	Increased
Equilibrium Expected Mitigated Loss, $[1 - q(s^*)]E\big[\{1 - m(r^*(L))\}L\big]$	0.3791	0.3670	Decreased
Equilibrium Expected Firm's Utility, $\ln({W_1}^{**})$	2.0728	2.0740	Increased

Appendix L: Endogenous Pricing with Risk Aversion and Insurer-Incentivized Risk Prevention

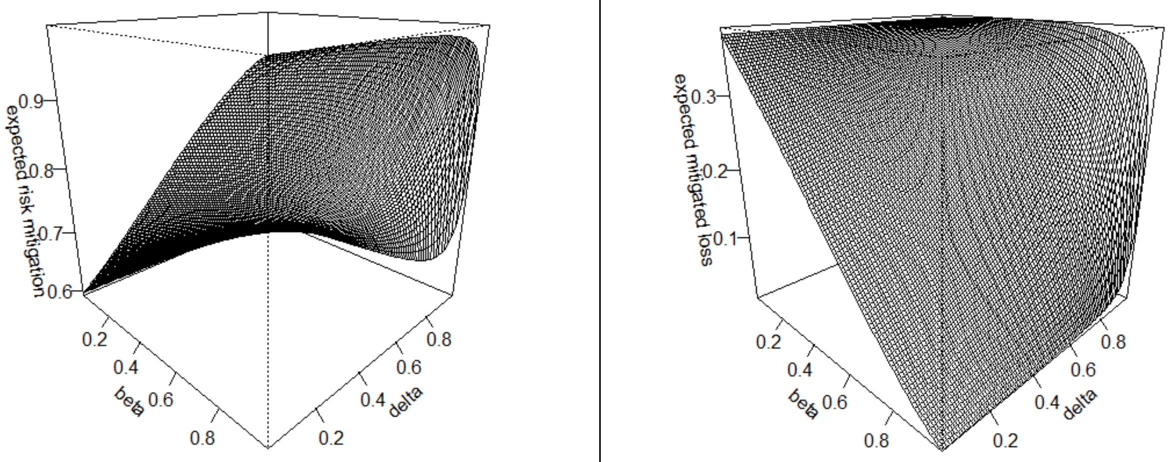
The problem presented in Section 4.4 is too complex to be solve analytically. Therefore, we address the problem using a brute force numerical approach. Specifically, we numerically solve the optimal risk prevention investment ${s_c}^*$ and compute the equilibrium values of the other variables for all $0.01 \le \beta \le 0.99$ and $0.01 \le \delta \le 0.99$ (with step size of 0.01). We set $w_0 = 10$, $k_s = 1$, $k_r = 1$, and $L \sim Uniform(1,10)$. The optimal risk prevention investment ${s_c}^*$, and the equilibrium risk prevention effectiveness $q({s_c}^*)$, expected risk mitigation spending, expected mitigated risk, expected utility, and cyber insurance price with respect to β and

δ are presented in Figure L.1. For ease of viewing, Figures L.2 and L.3 present the 2D plots with respect to β and δ, respectively. The red lines in Figures L.2 and L.3 represent the equilibrium outcomes of the base model.



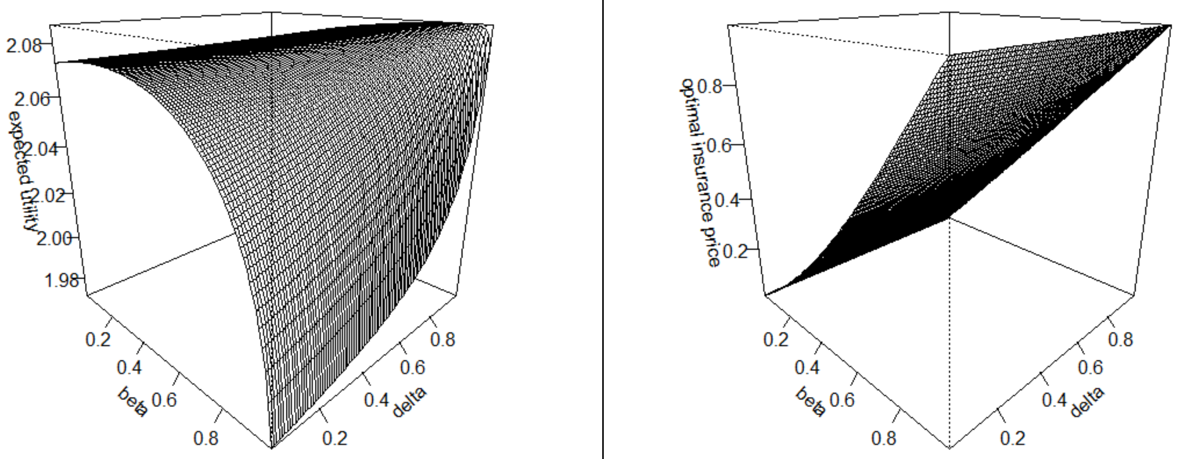
(a) Optimal Risk Prevention Investment

(b) Equilibrium Risk Prevention Effectiveness



(c) Equilibrium Expected Risk Mitigation Spending

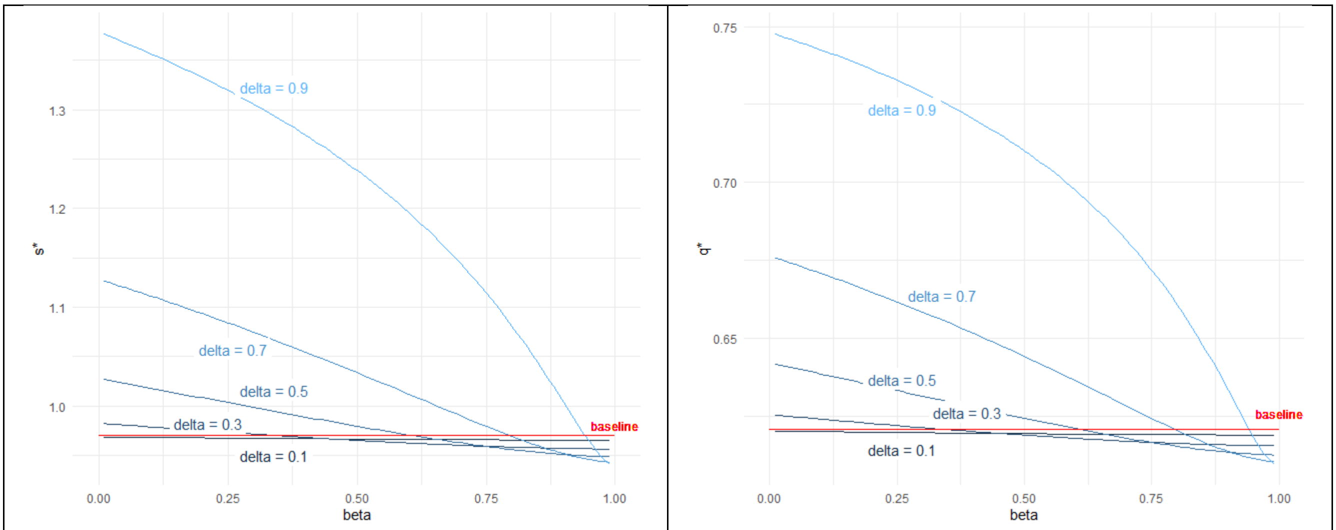
(d) Equilibrium Expected Mitigated Loss



(e) Equilibrium Expected Utility

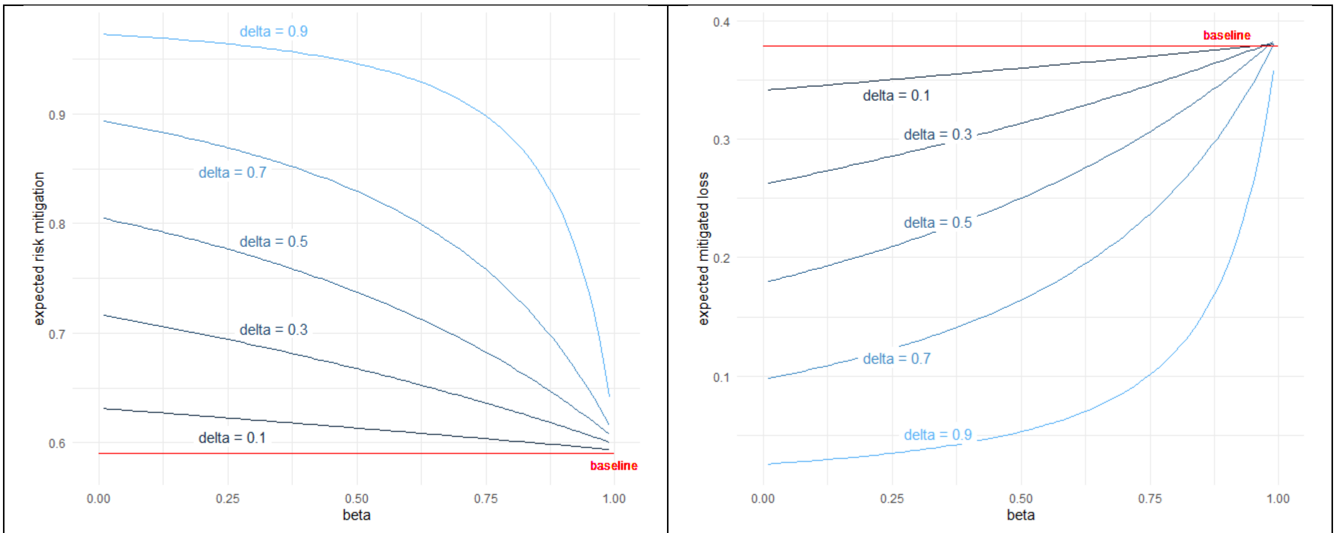
(f) Equilibrium Cyber Insurance Price

Figure L.1 Equilibrium outcomes under different β and δ



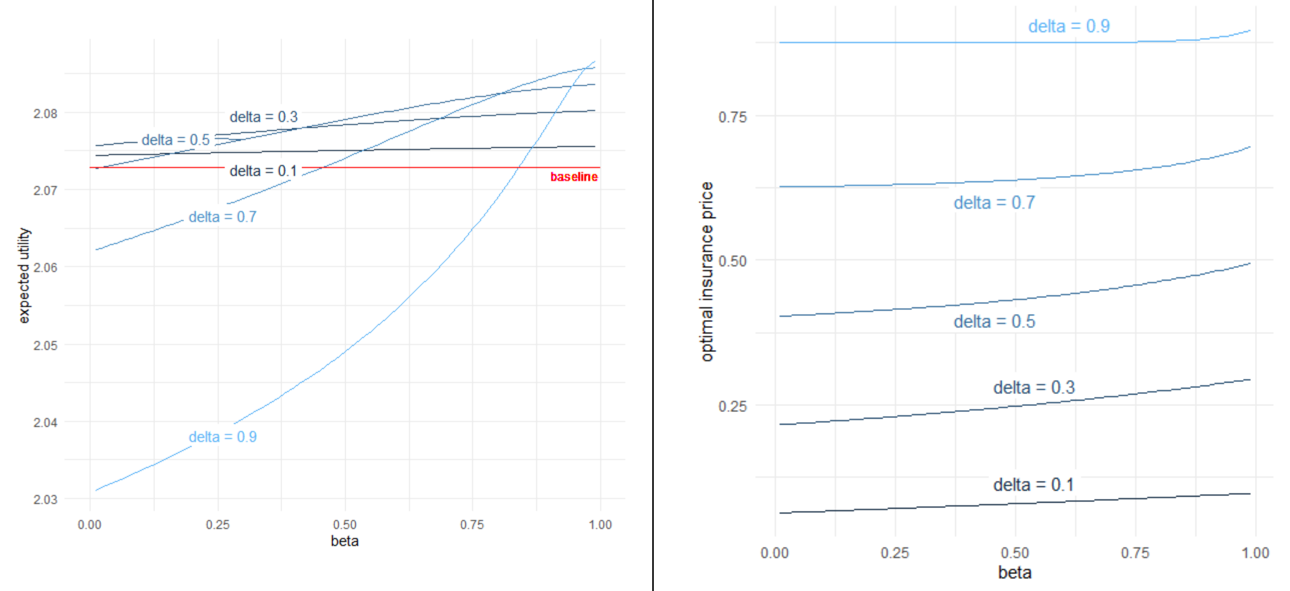
(a) Optimal Risk Prevention Investment

(b) Equilibrium Risk Prevention Effectiveness



(c) Equilibrium Expected Risk Mitigation Spending

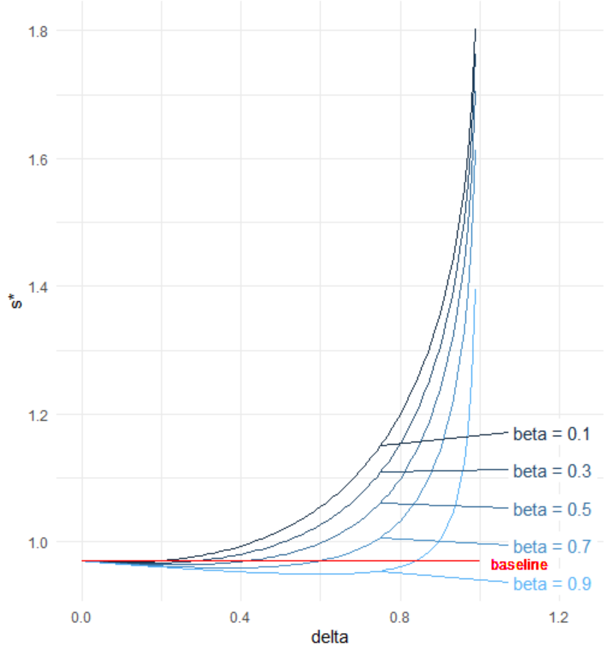
(d) Equilibrium Expected Mitigated Loss



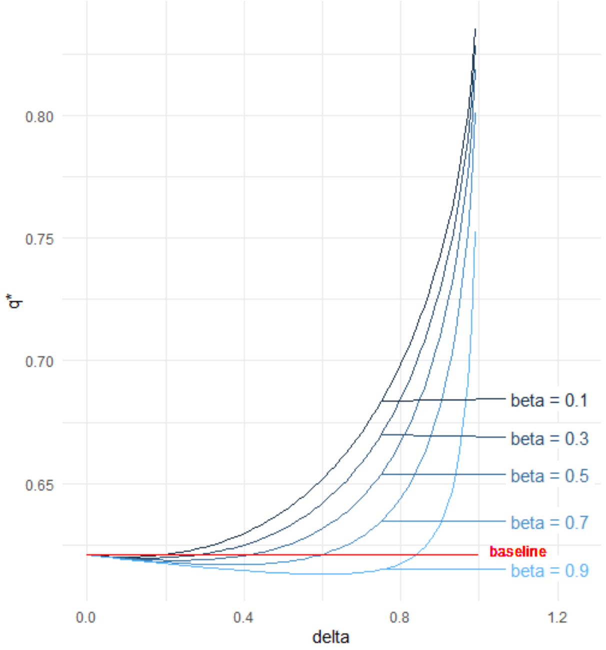
(e) Equilibrium Expected Utility

(f) Equilibrium Cyber Insurance Price

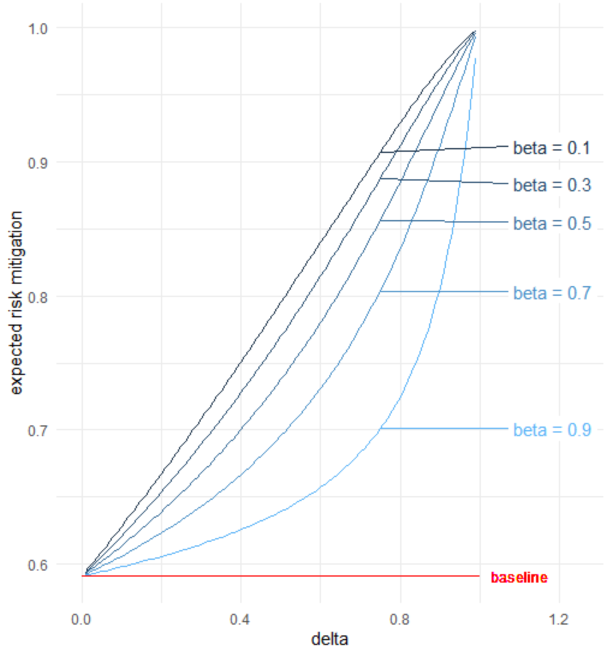
Figure L.2 Equilibrium outcomes against β



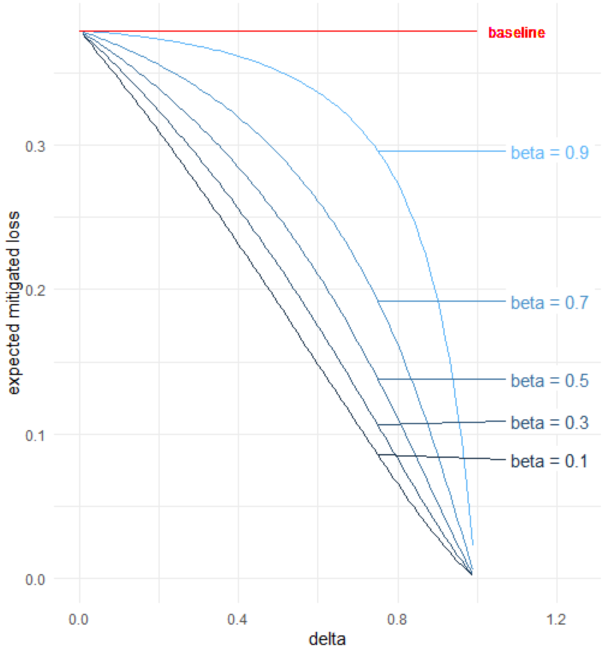
(a) Optimal Risk Prevention Investment



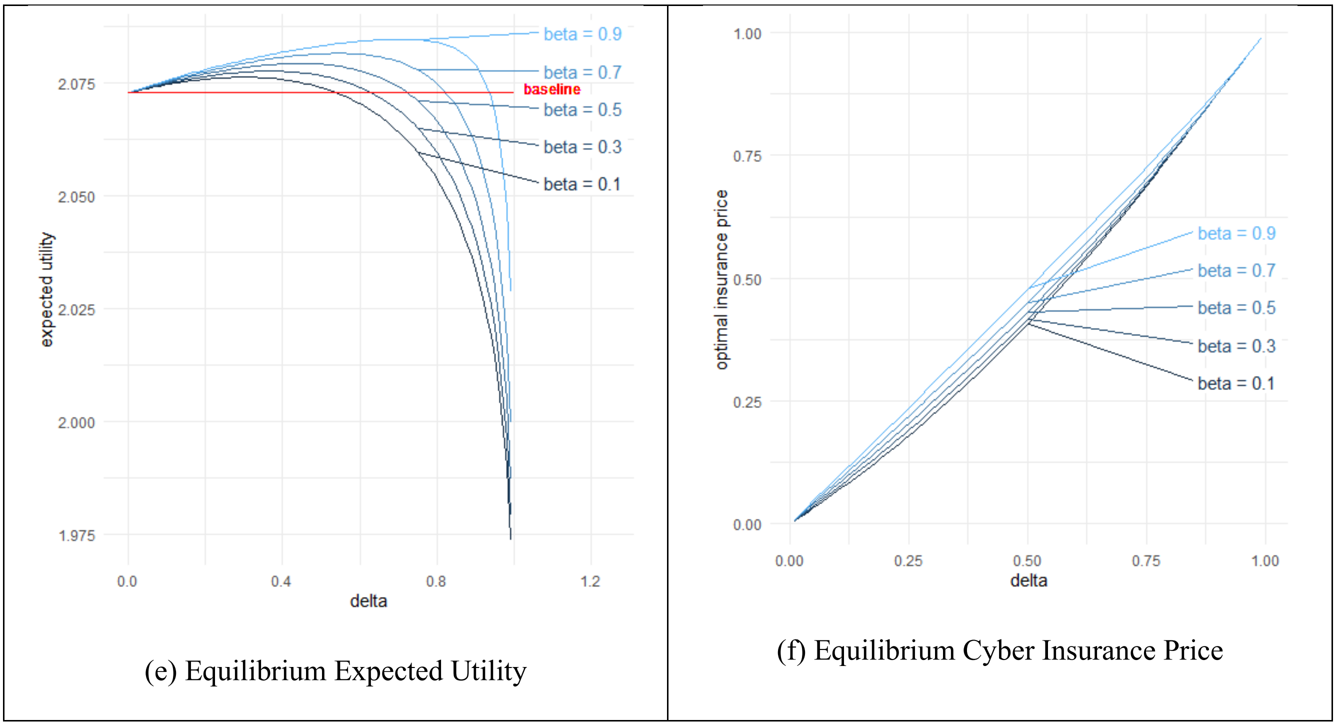
(b) Equilibrium Risk Prevention Effectiveness



(c) Equilibrium Expected Risk Mitigation Spending



(d) Equilibrium Expected Mitigated Loss



(e) Equilibrium Expected Utility

(f) Equilibrium Cyber Insurance Price

Figure L.3 Equilibrium outcomes against δ

Appendix M: The Presence of Ex-Ante Risk Mitigation

Comparing the first order conditions in Equation (27) and Equation (29), the *ex post* mitigation is increased by cyber insurance because $\frac{1-\delta}{1-\delta\beta} < 1$. Hence, Proposition 2 continues to hold.

With $m_1(r_1) = 1 - e^{-k_{r_1} r_1}$, the optimal *ex post* risk mitigation is:

$$r_1{}^* = \frac{1}{k_r}\ln\left(\frac{1-\beta\delta}{1-\delta}[1-m_0(r_0)]k_r\ell\right) \tag{M1}$$

By backward induction, substituting Equation (M1) into Equation (28), the firm's expected final wealth evaluated at $t = 0$ is

$$E[W_1{}^*] = w_0 - s - r_0 - p - [1-q(s)]E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}[1-m_0(r_0)]k_r L\right)\right]\right\}. \tag{M2}$$

At $t = 0$, the firm will maximize Equation (M2) with respect to s and r_0. The first order conditions are:

$$q'(s) = \frac{1}{E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}[1-m_0(r_0)]k_r L\right)\right]\right\}}, \tag{M3}$$

and

$$m_0{}'(r_0) = \frac{k_r}{[1-q(s)](1-\delta)}\frac{1-\beta\delta}{1-\delta}[1-m_0(r_0)]E[k_r L]. \tag{M4}$$

In the absence of cyber insurance, $\delta = 0$. Because $q'(s)$ is decreasing in s, Equation (M3) suggests that, if a local maximum exists, cyber insurance decreases risk prevention investment if:

$$E\left\{\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}[1-m_0(r_0)]k_rL\right)\right]\right\} < E\left\{\frac{1}{k_r}[1+\ln([1-m_0(r_0)]k_rL)]\right\},$$

or

$$\delta E[1+\ln([1-m_0(r_0)]k_rL)] > (1-\delta)\ln\left(\frac{1-\beta\delta}{1-\delta}\right), \tag{M5}$$

which is almost the same as Equation (C2). Because $\delta > (1-\delta)\ln\left(\frac{1}{1-\delta}\right)$, as shown in Appendix C,

$$\delta E[1+\ln([1-m_0(r_0)]k_rL)] > \delta > (1-\delta)\ln\left(\frac{1}{1-\delta}\right) > (1-\delta)\ln\left(\frac{1-\beta\delta}{1-\delta}\right).$$

Hence, Equation (M5) always holds, meaning Proposition 3 continues to hold.

Because the risk prevention investment is decreased, the probability of a successful attack is increased compared to the baseline model without cyber insurance. As the *ex post* risk mitigation is also increased, the overall expected risk mitigation spending is also increased. Hence, Proposition 4 continues to hold in the presence of *ex ante* risk mitigation.

The expected mitigated loss is given by $[1-q(s)]E[\{1-m_0(r_0)\}\{1-m_1(r_1)\}L]$. Because the first factor, i.e., the probability of a successful attack, is increased, while the second factor, i.e., the expected mitigated loss in case of a breach, is decreased by cyber insurance, the overall expected mitigated loss is decreased only if the decrease in the second factor is sufficient in size relative to the increase in the first factor. This can happen if *ex post* risk mitigation spending is sufficiently increased, i.e., the right-hand side of Equation (29) is sufficiently small. This requires β to be sufficiently small. Hence, Proposition 5 continues to hold in the presence of *ex ante* risk mitigation.

Note that when $\delta = 0$, we have the first order condition with respect to r_0 for the baseline model without cyber insurance, i.e.,

$$m_0'(r_0) = \frac{k_r}{[1-q(s)]}[1-m_0(r_0)]E[k_rL]. \tag{M6}$$

Cyber insurance decreases *ex ante* risk mitigation if

$$\frac{1-\beta\delta}{(1-\delta)^2} > 1,$$

which always holds because $\beta + \delta < 2$. Hence, cyber insurance has opposite effects on *ex ante* and *ex post* risk mitigation spending. Specifically, it tends to increase *ex post* risk mitigation spending but decrease *ex ante* risk mitigation spending.

Appendix N: The Effects of a Strategic Hacker

By backward induction, in case of a successful attack at $t = 2$, the firm will maximize its utility in accordance with Equation (3). This is because the hacker's decision only affects the probability of a successful attack. It does not affect the size of the loss. Hence, the optimal *ex post* risk mitigation spending is as presented in Equation (4), i.e., $r_b{}^*(\ell) = \frac{1}{k_r}\ln(k_r\ell)$. At $t = 1$, the hacker optimizes his or her hacking effort. The first order condition for the objective function presented in Equation (30) is:

$$q_-{}'(\varepsilon) = \frac{h}{[1-q_+(s)]g\mathrm{E}\{[1-m(r_b{}^*(L))]L\}} = \frac{hk_r}{[1-q_+(s)]g}. \tag{N1}$$

Let the optimal hacking effort be $\varepsilon_b{}^*$. Equation (N1) suggests that as s increases, $\varepsilon_b{}^*$ decreases. At $t = 0$, the firm maximizes the following objective function:

$$\mathrm{E}[W_1{}^*] = \{1 - [1 - q_+(s)]q_-(\varepsilon_b{}^*)\}(w_0 - s) + [1 - q_+(s)]q_-(\varepsilon_b{}^*)\mathrm{E}\left\{w_0 - s - \frac{1}{k_r}[1 + \ln(k_rL)]\right\}, \text{ or}$$

$$\mathrm{E}[W_1{}^*] = (w_0 - s) - [1 - q_+(s)]q_-(\varepsilon_b{}^*)\mathrm{E}\left\{\frac{1}{k_r}[1 + \ln(k_rL)]\right\}. \tag{N2}$$

The only difference between Equation (N2) and Equation (5) is that the probability of a successful attack is now $[1 - q_+(s)]q_-\big(\varepsilon_b{}^*(s)\big)$, instead of $[1 - q(s)]$. The first order condition with respect to s is:

$$-\frac{\partial}{\partial s}\{[1 - q_+(s)]q_-\big(\varepsilon_b{}^*(s)\big)\} = \frac{1}{E\left\{\frac{1}{k_r}[1+\ln(k_rL)]\right\}}. \tag{N3}$$

Note that both sides of Equation (N3) are positive.

For the cyber insurance model, in case of a successful attack at $t = 2$, the firm will maximize its *ex post* utility according to Equation (14). This is because the hacker's decision does not affect the size of the loss in case of a successful attack. Because the firm's *ex post* decision is not affected by the presence of the strategic hacker, the *ex post* risk mitigation spending is still given by Equation (16), i.e., $r_c{}^*(\ell) = \frac{1}{k_r}\ln\left(\frac{1-\beta\delta}{1-\delta}k_r\ell\right)$. As the firm's *ex post* decision is not affected by the presence of the strategic hacker in either the baseline model or the cyber insurance model, Proposition 2 continues to hold.

Moving backward to $t = 1$, the hacker optimizes his hacking effort according to

$$q_-'(\varepsilon) = \frac{h}{[1-q_+(s)]g\mathrm{E}\{[1-m(r_c{}^*(L))]L\}} = \frac{h(1-\beta\delta)k_r}{[1-q_+(s)]g(1-\delta)}, \tag{N4}$$

Because $r_c{}^* > r_b{}^*$ (according to Proposition 2), the right-hand side of Equation (N4) is bigger than the right-hand side of Equation (N1). Hence, all else being equal, cyber insurance tends to discourage hacking, i.e., $\varepsilon_c{}^*(s) < \varepsilon_b{}^*(s)$, because the gain from hacking becomes less attractive.

Moving backward to $t = 0$, the firm maximizes the following objective function:

$$\mathrm{E}[W_1{}^*] = w_0 - s - p - [1 - q_+(s)]q_-\big(\varepsilon_c{}^*(s)\big)\mathrm{E}\left\{\frac{1}{k_r}(1-\delta)\left[1 + \ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right\}. \tag{N5}$$

The only difference between Equation (N5) and Equation (17) is that the probability of a successful attack is now $[1 - q_+(s)]q_-\big(\varepsilon_c{}^*(s)\big)$. The first order condition is

$$-\frac{\partial}{\partial s}\{[1 - q_+(s)]q_-\big(\varepsilon_c{}^*(s)\big)\} = \frac{1}{\mathrm{E}\left[\frac{1}{k_r}(1-\delta)\left[1+\ln\left(\frac{1-\beta\delta}{1-\delta}k_r L\right)\right]\right]}. \tag{N6}$$

Following the proof in Appendix C, the right-hand side of Equation (N6) is larger than the right-hand side of Equation (N3). As $\varepsilon_c{}^*(s) < \varepsilon_b{}^*(s)$, $q_-\big(\varepsilon_c{}^*(s)\big) < q_-\big(\varepsilon_b{}^*(s)\big)$. Hence, the left-hand side of Equation (N6) is decreased compared to that of Equation (N3). The overall effect of these changes is that the optimal s is decreased. Hence, Proposition 3 continues to hold in the presence of strategic hacking.

The expected risk mitigation spending with cyber insurance is $[1 - q_+(s_c^*)]q_-\big(\varepsilon_c{}^*(s_c^*)\big)E[\,r_c^*(L)]$. The expected risk mitigation spending without cyber insurance is $[1 - q_+(s_b^*)]q_-\big(\varepsilon_b{}^*(s_b^*)\big)E[\,r_b^*(L)]$. Since Proposition 2 holds, $E[\,r_c^*(L)] > E[\,r_b^*(L)]$. However, unlike the case without strategic interaction, we cannot guarantee that the probability of a successful attack is increased by cyber insurance, or

$$[1 - q_+(s_c^*)]q_-\big(\varepsilon_c{}^*(s_c^*)\big) \overset{?}{\gtrless} [1 - q_+(s_b^*)]q_-\big(\varepsilon_b{}^*(s_b^*)\big),$$

because $[1 - q_+(s_c^*)] > [1 - q_+(s_b^*)]$ while $q_-\big(\varepsilon_c{}^*(s_c^*)\big) < q_-\big(\varepsilon_b{}^*(s_b^*)\big)$. Whether Proposition 4 holds depends on the nature of the strategic interaction between risk prevention and hacking and the specific utility function of the hacker, which is beyond the scope of this study.