Online Appendix

Appendix A: Proof of Theorems

Proof of Theorem 1. Dispatch times can be modeled as renewal epochs with the number of dispatches in a time interval being the corresponding renewal process. From the renewal-reward theorem, the average reward per unit time equals the expected reward per renewal cycle divided by the expected time between renewal epochs. Since a fixed cost ϕ, a delay cost $Z_h(N,T)$, and a delivery cost $Z_\ell(N,T)$ are incurred in each cycle, the numerator of $E[\mathcal{C}(N,T)]$ is the expected renewal cycle cost. The time between renewal epochs is IID with mean $E[T]+1/\lambda$, as this is the time between the first order arrival and the vehicle dispatch, plus the time for the first order to arrive. This gives the denominator of $E[\mathcal{C}(N,T)]$. ■

Proof of Theorem 2. Since $H_n(t,z) = P\{Z(t)\le z, N(t)=n\}$, we can write

$$\begin{aligned}
H_n(t+\Delta t,z) &= P\{Z(t+\Delta t)\le z, N(t+\Delta t)=n\} = (1-\lambda\Delta t)P\{Z(t)\le z-nh_n\Delta t, N(t)=n\} + \\
&\quad \lambda\Delta t\int_0^z P\{Z(t)\le z-u, N(t)=n-1\}dG_n(u) + o(\Delta t)\\
&= (1-\lambda\Delta t)H_n(t,z-nh_n\Delta t) + \lambda\Delta t\int_0^z H_{n-1}(t,z-u)dG_n(u) + o(\Delta t)
\end{aligned}$$

as, in a small time Δt, the probability of no order arrival in that time is $(1-\lambda\Delta t)$, and that of one order arrival is $\lambda\Delta t$, with higher order terms such as $(\Delta t)^2$ not included. By conditioning on whether or not an event occurred between t and $t+\Delta t$ we obtain the first equation above. It is worthwhile noting that when there are no order arrivals, the only increase in $Z(t)$ is the delay cost incurred for the n orders, which is $nh_n\Delta t$. Also, if there is an arrival, and the arrival brings an additional amount of delivery time of u, then z at time $t+\Delta t$ is equivalent to $z-u$ at time t. The latter is unconditioned using the CDF $G_n(\cdot)$. Dividing the above equation by Δt we get

$$\begin{aligned}
\frac{H_n(t+\Delta t,z)-H_n(t,z)}{\Delta t} &= nh_n\left[\frac{H_n(t,z-nh_n\Delta t)-H_n(t,z)}{nh_n\Delta t}\right] - \lambda H_n(t,z-nh_n\Delta t)\\
&\quad +\lambda\int_0^z H_{n-1}(t,z-u)dG_n(u) + \frac{o(\Delta t)}{\Delta t}
\end{aligned}$$

by simply subtracting both sides by $H_n(t,z)$ before dividing by Δt. Letting $\Delta t\to 0$ such that $\frac{o(\Delta t)}{\Delta t}\to 0$, the PDE is obtained. Hence the theorem is proved. ■

Proof of Theorem 3. For both special cases we use a common result that is stated as a Lemma (without proof, which can be found in most standard texts on engineering mathematics).

LEMMA 1. *The solution to any linear first-order differential equation of the form*

$$\frac{dy(t)}{dt} + f(t)y(t) = r(t)$$

is given by

$$y(t) = e^{-g}\left[\int e^g r(t)dt + C\right],\quad g = \int f(t)dt.$$

We prove Theorem 3 by the principle of mathematical induction (PMI). First consider the special case $h_n = h$. The case $n=1$ can be verified from Equation (21). As is standard in PMI, let the case $n=k$ be true, i.e.,

$$\tilde{H}_k(t,w) = \frac{(\lambda/(hw))^{k-1}}{(k-1)!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)e^{-(\lambda+khw)t}[e^{hwt}-1]^{k-1}. \tag{26}$$

Considering the case $n=k+1$ for which, from the ODE (20), we have

$$\begin{aligned}\frac{d\tilde{H}_{k+1}(t,w)}{dt} &= -(k+1)hw\tilde{H}_{k+1}(t,w)-\lambda\tilde{H}_{k+1}(t,w)+\lambda\tilde{H}_k(t,w)\tilde{G}_{k+1}(w)\\ &= -((k+1)hw+\lambda)\tilde{H}_{k+1}(t,w)+\lambda\left[\frac{(\lambda/(hw))^{k-1}}{(k-1)!}\right.\\ &\qquad \left.\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)e^{-(\lambda+khw)t}[e^{hwt}-1]^{k-1}\right]\tilde{G}_{k+1}(w),\end{aligned}$$

where the latter equation follows by writing the case $n=k$ in Equation (26).

Using Lemma 1, we get

$$\begin{aligned}\tilde{H}_{k+1}(t,w) &= e^{-((k+1)hw+\lambda)t}\int e^{((k+1)hw+\lambda)t}\lambda\left[\frac{(\lambda/(hw))^{k-1}}{(k-1)!}\right.\\ &\qquad\left.\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)e^{-(\lambda+khw)t}[e^{hwt}-1]^{k-1}\right]dt+C\\ &= e^{-((k+1)hw+\lambda)t}\frac{(\lambda/(hw))^k}{k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\int khwe^{hwt}[e^{hwt}-1]^{k-1}dt+C.\end{aligned}$$

Next, by noticing the integrand is the derivative of $[e^{hwt}-1]^k$, we get

$$\tilde{H}_{k+1}(t,w) = e^{-((k+1)hw+\lambda)t}\frac{(\lambda/(hw))^k}{k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)[e^{hwt}-1]^k+C.$$

Using the initial condition $\tilde{H}_{k+1}(0,w)=0$, we obtain the constant $C=0$. Therefore, the case $n=k+1$ is true.

Next we consider the special case $h_n=(n-1)h$. The case $n=1$ can be verified from Equation (21) realizing that $h_1=0$. As is standard in PMI, let the case $n=k$ be true, i.e.

$$\tilde{H}_k(t,w)=\frac{(\lambda/(hw))^{k-1}}{(2k-2)!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\sum_{i=1}^k\alpha_{k,i}e^{-(\lambda+i(i-1)hw)t}. \tag{27}$$

Consider the case $n=k+1$ for which, from the ODE (20), we have

$$\begin{aligned}\frac{d\tilde{H}_{k+1}(t,w)}{dt} &= -(k+1)hkw\tilde{H}_{k+1}(t,w)-\lambda\tilde{H}_{k+1}(t,w)+\lambda\tilde{H}_k(t,w)\tilde{G}_{k+1}(w)\\ &= -((k+1)khw+\lambda)\tilde{H}_{k+1}(t,w)+\lambda\left[\frac{(\lambda/(hw))^{k-1}}{(2k-2)!}\right.\\ &\qquad\left.\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\sum_{i=1}^k\alpha_{k,i}e^{-(\lambda+k(k-1)hw)t}\right]\tilde{G}_{k+1}(w),\end{aligned}$$

where the latter equation follows by writing the case $n=k$ in Equation (27).

Using Lemma 1, we get

$$\begin{aligned}\tilde{H}_{k+1}(t,w) &= e^{-((k+1)khw+\lambda)t}\left[\int e^{((k+1)khw+\lambda)t}\lambda\left\{\frac{(\lambda/(hw))^{k-1}}{(2k-2)!}\right.\right.\\ &\qquad\left.\left.\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\sum_{i=1}^k\alpha_{k,i}e^{-(\lambda+i(i-1)hw)t}\right\}dt+C\right]\\ &= e^{-((k+1)khw+\lambda)t}\left[\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\right.\\ &\qquad\left.\sum_{i=1}^k\alpha_{k,i}\int 2k(2k-1)hwe^{(k(k+1)-i(i-1))hwt}dt+C\right]\\ &= e^{-((k+1)khw+\lambda)t}\left[\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\right.\end{aligned}$$

$$\left.\sum_{i=1}^{k}\alpha_{k,i}\frac{2k(2k-1)}{k(k+1)-i(i-1)}e^{(k(k+1)-i(i-1))hwt}+C\right]$$
$$=e^{-((k+1)khw+\lambda)t}\left[\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\right.$$
$$\left.\sum_{i=1}^{k}\alpha_{k+1,i}e^{(k(k+1)-i(i-1))hwt}+C\right]$$

where the last expression for $\alpha_{k+1,i}$ is based on Equation (22), which can be rewritten as

$$\alpha_{k+1,i}=\frac{2k(2k-1)}{k(k+1)-i(i-1)}\alpha_{k,i}$$

for all $i\in\{1,2,\ldots,k\}$. We can then rewrite the above expression as

$$\tilde{H}_{k+1}(t,w)\,=\,e^{-\lambda t}\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\sum_{i=1}^{k}\alpha_{k+1,i}e^{-i(i-1)hwt}+Ce^{-((k+1)khw+\lambda)t}.$$

Using the initial condition $\tilde{H}_{k+1}(0,w)=0$, we obtain the constant C as

$$C=-\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\sum_{i=1}^{k}\alpha_{k+1,i}.$$

To derive C we state a Lemma.

Lemma 2. *If $\alpha_{n+1,i}$ is given by*

$$\alpha_{n+1,i}=\binom{2n}{n-i+1}\frac{2i-1}{n+i}(-1)^{i+1}$$

then

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=0.$$

We begin by using the result that for any $m\le 2n-1$ we have

$$\sum_{k=0}^{m}(-1)^k\binom{2n}{k}=(-1)^m\binom{2n-1}{m}$$

which is from Gradshteyn and Ryzhik (2014) and can also be proved using induction. Multiplying the above result by $(-1)^{-m}$ we get

$$\sum_{k=0}^{m}(-1)^{-m+k}\binom{2n}{k}=\binom{2n-1}{m}$$

and letting $m=n-1$ we get

$$(-1)^{n-1}\binom{2n}{0}+(-1)^{n-2}\binom{2n}{1}+(-1)^{n-3}\binom{2n}{2}+\ldots+\binom{2n}{n-1}=\binom{2n-1}{n-1},\tag{28}$$

realizing that $(-1)^r=(-1)^{-r}$. Next, rewriting the expression of Lemma 2 we get

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=\sum_{i=1}^{n+1}\binom{2n}{n-i+1}\frac{2i-1}{n+i}(-1)^{i+1}$$

which we can express as

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=\sum_{i=1}^{n+1}\binom{2n}{n-i+1}\left\{1-\frac{n-i+1}{n+i}\right\}(-1)^{i+1}=\sum_{i=1}^{n+1}(-1)^{i+1}\binom{2n}{n-i+1}-\sum_{i=1}^{n}(-1)^{i+1}\binom{2n}{n-i},$$

realizing that the second sum only continues to n, as the term is zero when $i=n+1$. Now for the first expression, making the change of variable $j=i-1$, we get

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=\sum_{j=0}^{n}(-1)^{j+2}\binom{2n}{n-j}+\sum_{i=1}^{n}(-1)^{i+2}\binom{2n}{n-i}$$

by multiplying the second sum by (-1). The term corresponding to $j=0$ is the only extra term in the first sum. So we can rewrite the above sum as

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=(-1)^2\binom{2n}{n}+2\sum_{i=1}^{n}(-1)^{i+2}\binom{2n}{n-i},$$

and when the summation is expanded from $i=n$ to $i=1$, i.e. in reverse, we get

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=(-1)^2\binom{2n}{n}+2\left\{(-1)^{n+2}\binom{2n}{0}+(-1)^{n+1}\binom{2n}{1}+(-1)^{n}\binom{2n}{2}+\ldots+(-1)^1\binom{2n}{n-1}\right\}.$$

Now, using Equation (28), we get

$$\sum_{i=1}^{n+1}\alpha_{n+1,i}=(-1)^2\binom{2n}{n}+2(-1)^3\binom{2n-1}{n-1}=\binom{2n}{n}-2\frac{(2n-1)!}{(n-1)!n!}=\binom{2n}{n}-2n\frac{(2n-1)!}{n(n-1)!n!}=0$$

and hence the Lemma is proved. ■

From Lemma 2 we have $\sum_{i=1}^{k}\alpha_{k+1,i}=-\alpha_{k+1,k+1}$, which can be used to obtain

$$C=\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\alpha_{k+1,k+1}.$$

Plugging back into $\tilde{H}_{k+1}(t,w)$ we get

$$\tilde{H}_{k+1}(t,w)=e^{-\lambda t}\frac{(\lambda/(hw))^k}{2k!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_k(w)\tilde{G}_{k+1}(w)\sum_{i=1}^{k+1}\alpha_{k+1,i}e^{-i(i-1)hwt}.$$

Therefore, the case $n=k+1$ is true.

Hence, by the principle of mathematical induction, Theorem 3 is proved for both special cases of $h_n=h$ and $h_n=(n-1)h$. ■

Proof of Theorem 4. We first describe how to compute $E[T(z)]$, which is denoted as $K(z)$:

$$\begin{aligned}K(z)&=E[T(z)]=\int_0^\infty P\{T(z)\geq t\}dt=\int_0^\infty P\{Z(t)\leq z\}dt\\&=\sum_{n=1}^{\infty}\int_0^\infty P\{Z(t)\leq z,N(t)=n\}dt=\sum_{n=1}^{\infty}\int_0^\infty H_n(t,z)dt.\end{aligned}$$

Taking the LST of $K(z)$, denoted by $\tilde{K}(w)$, we obtain from Theorem 3 for the special case of $h_n=h$,

$$\tilde{K}(w)=\sum_{n=1}^{\infty}\int_0^\infty\frac{(\lambda/(hw))^{n-1}}{(n-1)!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_n(w)e^{-(\lambda+nhw)t}[e^{hwt}-1]^{n-1}dt, \tag{29}$$

and for the special case of $h_n=(n-1)h$,

$$\tilde{K}(w)=\sum_{n=1}^{\infty}\int_0^\infty\frac{(\lambda/(hw))^{n-1}}{(2n-2)!}\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_n(w)\sum_{i=1}^{n}\alpha_{n,i}e^{-(\lambda+i(i-1)hw)t}dt,$$

where $\alpha_{n,i}$ can be obtained from Equation (22).

Interestingly, notice that

$$\tilde{G}_1(w)\tilde{G}_2(w)\ldots\tilde{G}_n(w)=E[e^{-w(V_1+\ldots+V_n)}],$$

and hence $\tilde{G}_1(w)\tilde{G}_2(w)\dots\tilde{G}_n(w) =$

$$E[e^{-w(nh_n+\ell)X_1}]E[e^{-w(h_{n-1}(n-1)+\ell)Y_1}]E[e^{-w(h_{n-2}(n-2)+\ell)Y_2}]\dots E[e^{-w(h_1+\ell)Y_{n-1}}]E[e^{-w\ell X_0}].$$

For the special case of $h_n = h$ we can obtain some elegant results, while the special case of $h_n = (n-1)h$ is actually rather straightforward to integrate. We first present the case $h_n = h$. From Equation (29), if we take only the terms corresponding to t alone, we obtain (for $h_n = h$)

$$\int_0^\infty e^{-(\lambda+nhw)t}[e^{hwt}-1]^{n-1}dt = \frac{1}{hw}\int_0^\infty (1+y)^{-\lambda/(hw)-(n+1)}y^{n-1}dy = \frac{B(n,\lambda/(hw)+1)}{hw},$$

where $B(\cdot,\cdot)$ is the Beta function, and the first equality is obtained by making the substitution $y = e^{hwt}-1$. Since the Beta function can be written in terms of the gamma function (i.e. $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$), we have for the case of $h_n = h$,

$$\tilde{K}(w) = \sum_{n=1}^\infty \left(\frac{\lambda}{hw}\right)^{n-1} \tilde{G}_1(w)\tilde{G}_2(w)\dots\tilde{G}_n(w)\frac{1}{hw}\frac{\Gamma(\lambda/(hw)+1)}{\Gamma(\lambda/(hw)+1+n)}, \tag{30}$$

by realizing that $\Gamma(n) = (n-1)!$, which gets cancelled. Then, for potential candidate values of z we can obtain $K(z)$ by approximating the infinite sum in Equation (30) and numerically inverting $\tilde{K}(w)$. Note that the gamma function in Equation (30) can be omitted by realizing that

$$\frac{\Gamma(\lambda/(hw)+1)}{\Gamma(\lambda/(hw)+1+n)} = \frac{1}{(\lambda/(hw)+1)(\lambda/(hw)+2)\dots(\lambda/(hw)+n)},$$

although when coding this into mathematical software packages, it may be preferable to leave it in the original gamma form itself.

Next, for the special case of $h_n = (n-1)h$ we can directly integrate the expression for $\tilde{K}(w)$ given above and get

$$\tilde{K}(w) = \sum_{n=1}^\infty \frac{(\lambda/(hw))^{n-1}}{(2n-2)!}\tilde{G}_1(w)\tilde{G}_2(w)\dots\tilde{G}_n(w)\sum_{i=1}^n \frac{\alpha_{n,i}}{\lambda+i(i-1)hw} \tag{31}$$

where $\alpha_{n,i}$ can be obtained from Equation (22).

Finally, what remains for the long-run average cost per unit time $E[\mathcal{C}(z)]$ in Equation (17) is $E[Z(T(z)+)]$, which is tricky to compute. Although we will subsequently describe an approximation for $E[Z(T(z)+)]$, we first present a bound. Observe that there are two ways the threshold z can be reached: (i) when the total delivery cost just reaches z as the delay cost keeps increasing; (ii) when an arrival that occurs at $T(z)$ causes a jump in $Z(t)$ to overshoot z. In case (i) $E[Z(T(z)+)] = z$, while in case (ii) $E[Z(T(z)+)] > z$. Hence we have

$$E[Z(T(z)+)] \geq z,$$

and we can therefore obtain a lower bound for $E[\mathcal{C}(z)]$ of

$$E[\mathcal{C}_{LB}(z)] = \frac{\phi+z}{E[T(z)]+1/\lambda} = \frac{\phi+z}{K(z)+1/\lambda}. \tag{32}$$

As an approximation for $E[Z(T(z)+)]$ in Equation (17), we use for the special case of $h_n = h$

$$E[Z(T(z)+)] \approx z + \beta(q^*)(q^*h/2+\ell)\frac{Var[Y]+\tau^2}{2\tau},$$

and for the special case of $h_n = (n-1)h$

$$E[Z(T(z)+)] \approx z + \beta(q^*)(q^*(q^*-1)h/2 + \ell)\frac{Var[Y] + \tau^2}{2\tau},$$

where the excess jump above z, on average, is calculated as the average excess life (from the well-known inspection paradox), Y corresponds to the time between any two random delivery locations, and q^* is the optimal batch size (in Section 4). Hence when $h_n = h$, we approximate $E[\mathcal{C}(z)]$ as

$$E[\mathcal{C}_{approx}(z)] = \frac{\phi + z + \beta(q^*)(q^*h/2 + \ell)\frac{Var[Y]+\tau^2}{2\tau}}{K(z) + 1/\lambda}, \tag{33}$$

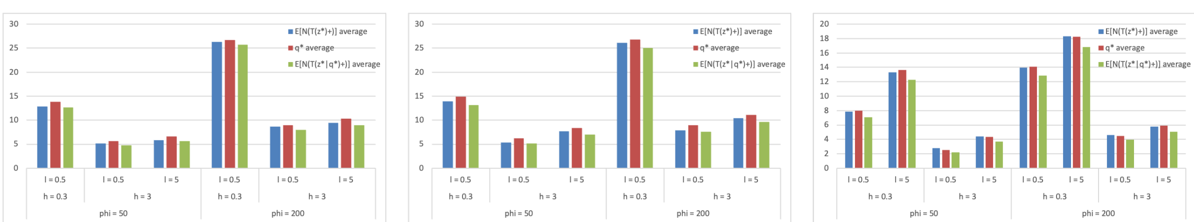
while for the special case of $h_n = (n-1)h$ we have

$$E[\mathcal{C}_{approx}(z)] = \frac{\phi + z + \beta(q^*)(q^*(q^*-1)h/2 + \ell)\frac{Var[Y]+\tau^2}{2\tau}}{K(z) + 1/\lambda}, \tag{34}$$

and hence the theorem is proved. ■

Appendix B: Cost- and Distance-Based Parameter Analysis

This section provides additional results that capture the effects associated with varying individual parameters, which motivated the discussion on policy performance in Section 6.3. We mainly focus on two metrics: long-run average cost per unit time across the three policies (ii), (iii), and (iv) (i.e. $E[\mathcal{C}(q^*, T)]$, $E[\mathcal{C}(z^*)]$, and $E[\mathcal{C}(z^*|q^*)]$), and average batch size (q^*, $E[N(T(z^*)+)]$ and $E[N(T(z^*|q^*)+)]$). The quantities $E[\mathcal{C}(z^*)]$, $E[\mathcal{C}(z^*|q^*)]$, $E[N(T(z^*)+)]$, and $E[N(T(z^*|q^*)+)]$ presented correspond to the mean values based on the 1,000 simulation runs of each problem instance. We do not present results on dispatch time, as for the two policies, this time is related to the batch size through λ. We begin by studying the effect of the cost parameters h, ℓ, and ϕ on the average batch size, i.e. $E[N(T(z^*)+)]$, q^*, and $E[N(T(z^*|q^*)+)]$ in Figure 10. The three panels correspond to TSP 1, TSP 2, and FCFS, which correspond to the three delivery mechanism parameters displayed in the first column of Table 3.

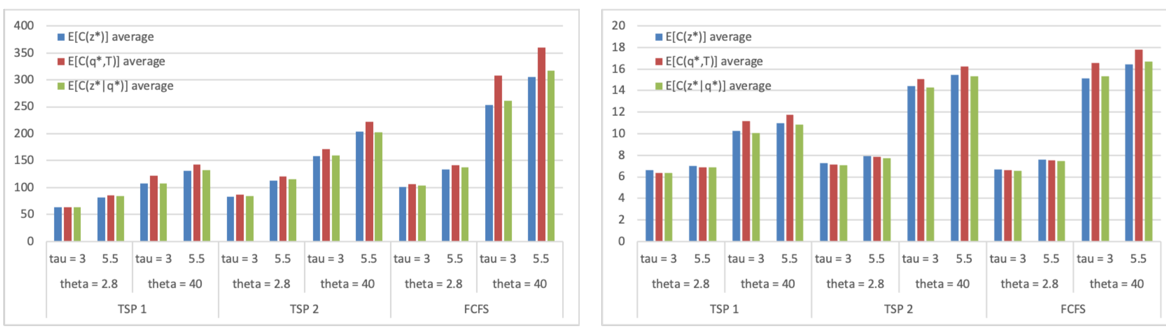


(a) TSP 1 (b) TSP 2 (c) FCFS

Figure 10 Resulting mean batch size under three policies with varying h, ℓ, and ϕ.

The average batch size displayed in Figure 10 is obtained by averaging across 32 runs corresponding to the two values of each of λ, θ, τ, $Stdev[X]$ and $Stdev[Y]$. Before analyzing the effects of the cost parameters h, ℓ, and ϕ, note that there are eight sets only in the FCFS case. This is because for the two TSP cases, as we described earlier in this subsection, we have removed the case $h = 0.3$ and $\ell = 5$ from our analysis. In all the cases, an increase in ϕ (keeping other parameters constant, irrespective of the three policies or delivery mechanisms) results in an increase in batch size. This is expected since a higher fixed cost would make it advantageous to batch more. Likewise, an increase in ℓ (the delivery cost per unit time) would also increase the batch size due to economies of scale. However, as expected, an increase in the delay cost rate h results in smaller batch sizes.

Also notice that TSP 2 results in the largest batch size, followed extremely closely by TSP 1, and then FCFS. This can be directly attributed to the $\beta(n)$ and $\gamma(n)$ values, which, for the TSP cases, are much smaller than one (the corresponding value for FCFS). Thus, waiting for a larger batch size is advantageous when routing based on the TSP, since this leads to a remarkable drop in delivery times. Next, notice that the blue, red, and green bars corresponding to $E[N(T(z^*)+)]$, q^*, and $E[N(T(z^*|q^*)+)]$, respectively, are reasonably close in size, indicating that, on average, the three policies result in a similar batch size (we will later discuss their relative magnitudes when we discuss all of the examples together). It is worthwhile pointing out that, as expected, $E[N(T(z^*|q^*)+)] < q^*$, as a dispatch always occurs at or before q^* in the cost- and quantity-based dynamic policy. We note that we did not present graphs of costs per unit time ($E[\mathcal{C}(z^*)]$ and $E[\mathcal{C}(q^*,T)]$) but, as expected, higher h, ℓ, or ϕ values imply higher long run average costs per time.

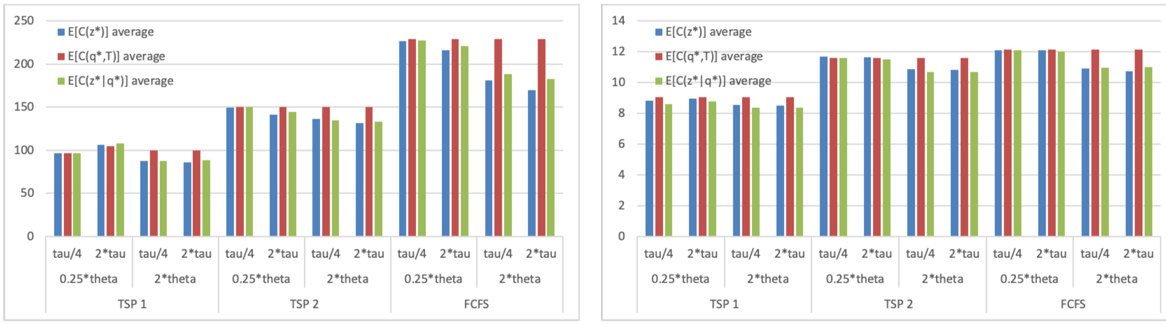


(a) $\lambda = 2.5$ (b) $\lambda = 0.1$

Figure 11 Resulting average expected cost per unit time under three policies with varying τ, θ, and delivery mechanism

Next please refer to Figure 11, where we present the expected cost per unit time averaged over possible values of h, ℓ, ϕ, $Stdev[X]$, and $Stdev[Y]$. In both the left and the right panel (where the difference in terms of inputs is λ), higher values of τ and θ (i.e. average time between delivery locations and average time from fulfilment location to a delivery location, also known as stem time) imply higher costs. This makes sense, as longer travel times imply higher delivery costs. But consider the difference between the average expected costs using the three policies (lower is better). In Figure 11(a) where λ is higher than in (b), the Z-threshold dynamic policy fares better than both the fixed batch size case and the Zq^*-threshold dynamic policy (based on both cost and batch size). However in some of the cases in Figure 11(b), the trend is reversed (although the Zq^*-threshold policy cost is lower than the static policy based on quantity, which is expected). An interesting observation is that although the arrival rate is 25 times higher in the left panel (Figure 11(a)), the costs are only about 10-15 times higher. Finally, in most cases the cost for TSP 1 is the lowest and FCFS the highest.

Instead of varying the standard deviation of the stem times and the times between delivery locations, we vary the coefficient of variation, i.e. the ratio of the standard deviation to the mean in our experiments. These results are shown in Figure 12. Note that the fixed batch size and fixed time policies only depend on



(a) $\lambda = 2.5$ (b) $\lambda = 0.1$

Figure 12 Resulting average expected cost per unit time under three policies with varying $Stdev[X]/\theta$ and $Stdev[Y]/\tau$

the mean of the travel times (unlike the dynamic policies with z-threshold). Hence, across the coefficient of variation values of 0.25 and 2 for either times X or Y, $E[\mathcal{C}(q^*,T)]$ remains a constant (so will q^*) with the only exception being when $\lambda = 2.5$ and we use TSP 1 and where some experiments had to be rejected. However, notice the considerable difference in performance. The reason the dynamic policy's expected cost $E[\mathcal{C}(z^*)]$ decreases with coefficient of variation is that when it is high, we tend to get many small values and a few extremely large ones. Grabbing the small ones until some threshold is reached is hence advantageous. However, similar to the previous figure, here too when $\lambda = 0.1$ (right panel), some situations do exist in which the dynamic policy does not perform as well as the fixed batch size policy.