**Online Supplement of Paper:**

**An Incentive Mechanism for Private Parking-sharing Programs in Imperfect Information Setting**

# 1. Proofs of Theorems and Properties

## 1.1. Proof of Theorem 1

The first part of Theorem 1 follows directly from the definition of the optimal matching solution. As for the second part, note that in an optimal matching solution of Problem $P^*$ with the driver set $\{i\} \cup C$, the matching pairs for drivers $i(\in C)$ are also feasible for Problem $P^*$ with the driver set $C$. This implies that for any $C \subseteq I \setminus \{i\}$, $\hat{a}_i$ and $\hat{a}^C$,

$$\begin{aligned} p_i^{VCG}(m_i^*(\hat{a}^{\{i\} \cup C})) &= \sum_{k \in C} v_k(m_k^*(\hat{a}^C)) - \sum_{k \in C} v_k(m_k^*(\hat{a}^{\{i\} \cup C})) \\ &= V(M^*(\hat{a}^C)) - \sum_{k \in C} v_k(m_k^*(\hat{a}^{\{i\} \cup C})) \\ &\geq 0 \end{aligned} \tag{1}$$

where $\sum_{k \in C} v_k(m_k^*(\hat{a}^{\{i\} \cup C}))$ is the sum of the values of all drivers in $C$ generated by the optimal matching solution of Problem $P^*$ with the driver set $\{i\} \cup C$, which, by the definition of optimality, is less than the optimal objective value of Problem $P^*$ with the driver set $C$.

Therefore, Equation (1) holds when all drivers in $\{i\} \cup C$ truthfully report their $\hat{a}^{\{i\} \cup C} = (a_i^{K_i}, a_{k_1}^{K_{k_1}}, a_{k_2}^{K_{k_2}}, ..., a_{k_{|C|}}^{K_{k_{|C|}}})$. Finally, the system transfers the payment $p_i^{VCG}(m_i^*(\hat{a}^{\{i\} \cup C})) \geq 0$ to the matched owner. The compensation of any matched owner $j$ is greater than 0 and the sum of all owners' compensations is equal to the sum of all drivers' payments. This completes the proof of Theorem 1.

## 1.2. Proof of Property 1

Consider any $i \in I$ and $C \subseteq I \setminus \{i\}$. It is necessary to show that for any report profile $\hat{a}^{\{i\} \cup C}$, $V(M^*(\hat{a}^{\{i\} \cup C}) \geq V(M^*(\hat{a}^C))$. For this purpose, we have three cases. When driver $i$'s entry and her report $\hat{a}_i$ do not induce the system assign her a slot, there is no impact on the slot allocation among all the drivers in $C$. Given our assumption that a driver's value is zero (no travel cost saving) if she is not matched with any slot, then $V(M^*(\hat{a}^{\{i\} \cup C}) = V(M^*(\hat{a}^C))$. When driver $i$'s entry and her report $\hat{a}_i$ induces the system to assign her a slot and the slot allocation among all the drivers in $C$ is unchanged, with the assumption that a matched driver has a positive value (travel cost saving), we have $V(M^*(\hat{a}^{\{i\} \cup C}) > V(M^*(\hat{a}^C))$. Finally, when driver $i$ enters and gets a slot, and her report $\hat{a}_i$ changes the slot allocation for drivers in $C$ (i.e., $(m_k^*(\hat{a}^{\{i\} \cup C})_{k \in C}) \neq (m_k^*(\hat{a}^C)_{k \in C})$), the match $[\varnothing, (m_k^*(\hat{a}^C)_{k \in C})]$ is a feasible solution to Problem $P^*$ with the driver set $\{i\} \cup C$. Then based on the definition of optimality, we have $V(M^*(\hat{a}^{\{i\} \cup C}) \geq V(\varnothing, (m_k^*(\hat{a}^C)_{k \in C})) = V(M^*(\hat{a}^C))$. The proof of Property 1 is completed.

### 1.3. Proof of Theorem 2

For any $i \in I$, $C \subseteq I \setminus \{i\}$ and $\hat{a}^{\{i\}\cup C}$, from Equation (13) in the paper, to prove $U_i^{VCG}(\hat{a}_i, \hat{a}_{-i}, \xi) \geq 0$, it suffices to show $V(M^*(\hat{a}^{\{i\}\cup C}) \geq V(M^*(\hat{a}^C))$. Clearly, this inequality is ensured by Property 1. Theorem 2 is thus proved.

### 1.4. Proof of Theorem 3

For driver $i$'s choice of $\hat{a}_i^{K_i}$ to strategically dominate any other choice $a_i^k, 1 \leq k < K_i$, it must hold that for any given $\hat{a}^C$ $(C \subseteq I \setminus \{i\})$,

$$U_i^{VCG}(a_i^{K_i}, \hat{a}^C, \xi) \geq U_i^{VCG}(\hat{a}_i^k, \hat{a}^C, \xi), \quad 1 \leq k < K_i, \forall i \in I \tag{2}$$

Based on Equation (13) in the paper, to show that Equation (2) holds, it is sufficient to show that for any $C \subseteq I \setminus \{i\}$ and any $1 \leq k < K_i$, $u_i^{VCG}(m_i^*(a_i^{K_i}, \hat{a}^C)) \geq u_i^{VCG}(m_i^*(a_i^k, \hat{a}^C))$ holds.

Under the proposed VCG-based pricing mechanism given in Equation (10) of the paper, we have

$$\begin{aligned} u_i^{VCG}(M^*(a_i^{K_i}, \hat{a}^C)) &= V^*(M^*(a_i^{K_i}, \hat{a}^C)) - V^*(M^*(\hat{a}^C)), \text{ and} \\ u_i^{VCG}(M^*(a_i^k, \hat{a}^C)) &= V^*(M^*(a_i^k, \hat{a}^C)) - V^*(M^*(\hat{a}^C)), 1 \leq k < K_i. \end{aligned} \tag{3}$$

Based on the definition of the strategy set $A_i$ in Section 4 of the paper, we have $J_i(a_i^k) \subseteq J_i(a_i^{K_i}), 1 \leq k < K_i$, which means that driver $i$'s report $a_i^{K_i}$ leads to a larger size of feasible slot set than any other report $a_i^k, 1 \leq k < K_i$. Also, note that other drivers' feasible sets are fixed due to the given strategy (report) profile $\hat{a}^C$. Then the expanded feasible slot set of driver $i$ (weakly) raises the optimal objective value of Problem $P^*$ with the driver set $\{i\} \cup C$. Thus we have

$$V^*(M^*(a_i^{K_i}, \hat{a}^C)) \geq V^*(M^*(a_i^k, \hat{a}^C)), 1 \leq k < K_i. \tag{4}$$

Equations (3) and (4) jointly imply $u_i^{VCG}(m_i^*(a_i^{K_i}, \hat{a}^C)) \geq u_i^{VCG}(m_i^*(a_i^k, \hat{a}^C)), 1 \leq k < K_i$. We thus have Theorem 3.

### 1.5. Proof of Theorem 4

Based on Theorems 2 and 3, we know that all drivers $i \in I$ choose to truthfully report and accept the system's recommended slots if they choose to report at $\tau = 1$. Thus, it is sufficient to show that it is optimal to report at $\tau = 1$ for all drivers $i \in I$. Consider driver $i$'s choice at $\tau = \Gamma - 1$. Suppose that driver $i$ reports at $\tau = \Gamma - 1$. For any given $C_{\Gamma-1}$ $(i \notin C_{\Gamma-1})$, if she chooses to reject the system's recommendation $m_i^*(\hat{a}^{\{i\}\cup C_{\Gamma-1}})$ and wait to the next reporting time $\tau = \Gamma$, her expected net utility is $\Phi_{i,\Gamma-1}^{VCG}(\{i\} \cup C_{\Gamma-1}) = E_{C_\Gamma | C_{\Gamma-1}} \Phi\ (\{i\} \cup C_\Gamma)\ = \sum_{C_\Gamma \subseteq \mathcal{C}_\Gamma} \Pr(C_\Gamma | C_{\Gamma-1})[v_i(m_i^*(\hat{a}^{\{i\}\cup C_\Gamma})) - p_i^{VCG}(m_i^*(\hat{a}^{\{i\}\cup C_\Gamma}))]$ where $\mathcal{C}_\Gamma$ is the set of drivers who have not reported their types or have rejected all the system's recommendations before $\tau = \Gamma$ (Below we use $\mathcal{C}_\tau$ in a similar way).

If she chooses to accept, her net utility is

$$\varphi^{VCG}_{i,\Gamma-1}(\{i\}\cup C_{\Gamma-1}) = v_i(m^*_i(\hat{a}^{\{i\}\cup C_{\Gamma-1}})) - p^{VCG}_i(m^*_i(\hat{a}^{\{i\}\cup C_{\Gamma-1}}))$$

Therefore, if driver $i$ reports at $\tau = \Gamma - 1$, her expected net utility is $\max\{\varphi^{VCG}_{i,\Gamma-1}(\{i\}\cup C_{\Gamma-1}), \Phi^{VCG}_{i,\Gamma-1}(\{i\}\cup C_{\Gamma-1})\}$. However, if driver $i$ chooses to report at $\tau = \Gamma$, she can obtain an expected net utility of $\Phi^{VCG}_{i,\Gamma-1}(\{i\}\cup C_{\Gamma-1})$. It is clear that driver $i$'s choice of reporting at $\tau = \Gamma - 1$ is (weakly) better than reporting at $\tau = \Gamma$ for any $C_{\Gamma-1} \subseteq \mathcal{C}_{\Gamma-1}$ and any report profiles $\hat{a}^{\{i\}\cup C_{\Gamma-1}}$ and $\hat{a}^{\{i\}\cup C_{\Gamma}}$. By repeating this procedure in a step-by-step manner, we can show that for all drivers $i \in I$, reporting at $\tau$ is (weakly) better than reporting at $\tau+1$ for all $\tau \in \{1, 2, \cdots, \Gamma-1\}$. Theorem 4 is thus proved.

### 1.6. Proof of Theorem 5

We first show that for $t \in \{1, 2, \cdots, T\}$, given that drivers truthfully report, $m^*_t(a^{I_t}, J_t) = m^{DE*}_t(a^{I_t}, J_t)$. Consider $t = T$. It is clear that Problem $P^*$ is identical to Problem DE. Then we have $m^*_T(a^{I_T}, J_T) = m^{DE*}_T(a^{I_T}, J_T)$ for all pairs of $I_T$ and $J_T$. Next, consider $t = T-1$. The dynamic efficiency requires that $m^{DE*}_{T-1}(a^{I_{T-1}}, J_{T-1})$ solves

$$\max_{m_{T-1}(a^{I_{T-1}}, J_{T-1})} E_{I_T,,J_T|I_{T-1},,J_{T-1}}\left[\sum_{k=t}^{T} V(m_k(a^{I_k}, J_k))\right].$$

With the independent assumption, this optimization problem can be rewritten as

$$\max_{m_{T-1}(a^{I_{T-1}}, J_{T-1})} V(m_{T-1}(a^{I_{T-1}}, J_{T-1})) + E_{I_T,,J_T}\left[V(m^{DE*}_T(a^{I_T}, J_T))\right] \tag{5}$$

Note that the second term of the objective function in (5) is irrelevant to $m_{T-1}(a^{I_{T-1}}, J_{T-1})$. Again this implies that the solution to (5) is the same as that to problem $P^*$ for $t = T-1$ (i.e., $m^*_{T-1}(a^{I_{T-1}}, J_{T-1}) = m^{DE*}_{T-1}(a^{I_{T-1}}, J_{T-1})$). By repeating this proof procedure in a step-by-step way, we have $m^*_t(a^{I_t}, J_t) = m^{DE*}_t(a^{I_t}, J_t)$ for every $t \in \{1, 2, \cdots, T\}$.

Now we show that the static VCG-based pricing mechanism (with the system-efficient match solution) induces drivers to truthfully report for every period $t \in \{1, 2, \cdots, T\}$. For $t = T$ and all pairs of $I_T$ and $J_T$, the dynamic case is the same as the static case. Thus Theorem 3 implies that $m^*_{i,t}(\hat{a}^{I_t}, J_t)$ and $p^{VCG}_i(m^*_{i,t}(\hat{a}^{I_t}, J_t))$ induce all drivers $i \in I_T$ to report truthfully. Consider $t = T-1$. The period-T expected utility of driver $i \in I_{T-1}$ is

$$E^{i,T} = \sum_{J_T\in\mathbb{J}, I_T\in\mathbb{I}\setminus\{i\}} \Pr_{\mathbb{I}_t}(I_t)\Pr_{\mathbb{J}_t}(J_t)\left[\sum_{C_T\subseteq I_T} \Pr(i\cup C_T)\left(v_i\left(m^*_{i,t}\left(a^{i\cup C_T}, J_t\right)\right) - p^{VCG}_i\left(m^*_{i,t}\left(a^{i\cup C_T}, J_t\right)\right)\right)\right]$$

which is irrelevant to $I_{T-1}$ and $J_{T-1}$ due to the independence assumption.

For any realized $I_{T-1}$, $J_{T-1}$ and $C \subseteq I_{T-1} \setminus \{i\}$, when drivers in $C$ report $\hat{a}^C_{T-1}$ and driver $i$ reports $\hat{a}_{i,T-1}$, driver $i$'s period-$T-1$ expected utility, denoted by $E^{i,T-1}(\hat{a}_{i,T-1}, \hat{a}^C_{T-1})$, is the expected flow utility obtained in period $T-1$ plus the expected utility in period $T$ (i.e., $E^{i,T}$):

$$E^{i,T-1}(\hat{a}_{i,T-1}, \hat{a}^C_{T-1}) = u_i(m^*_{T-1}((\hat{a}_{i,T-1}, \hat{a}^C_{T-1}), J_{T-1})) + E^{i,T}$$

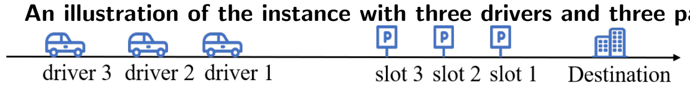
Since $E^{i,T}$ is independent of $(\hat{a}_{i,T-1}, \hat{a}^C_{T-1})$, driver $i$'s period-$T-1$ incentive to report truthfully or untruthfully relies only on the expected flow utility which is determined by the static VCG-based pricing mechanism and the optimal match solution $m^*_{T-1}((\hat{a}_{i,T-1}, \hat{a}^C_{T-1}), J_{T-1})$. Therefore, based on Theorem 3, the static VCG-based pricing mechanism induces driver $i$ to report truthfully in period $T-1$ for any $I_{T-1}$, $J_{T-1}$ and $C \subseteq I_{T-1} \setminus \{i\}$. Again, we can repeat this proof for all $t \in \{1, 2, \cdots, T-2\}$ to indicate that the static VCG-based pricing mechanism (with the system-efficient match solution) induces all drivers to report truthfully.

## 2. Nash-equilibrium strategies of an instance with three drivers

### 2.1. Instance description

In this part, we analyze drivers' reporting behaviors in Nash-equilibrium strategies under the time-based linear pricing method. In particular, an instance is given to show that the commonly observed time-based linear pricing method without considering the drivers' walking-time costs may induce drivers to misreport their departure times.

**Figure 1** **An illustration of the instance with three drivers and three parking slots**




In the instance, as illustrated in Figure 1, three drivers ($I = \{1, 2, 3\}$) are sorted in an ascending order of their driving times to the same destination along a line (i.e., $t_1 < t_2 < t_3$). Three parking slots ($J = \{1, 2, 3\}$) are sorted in an ascending order of their walking times to the destination (i.e., $t'_i(1) < t'_i(2) < t'_i(3), \forall i \in I$). According to value Equation (1) in the paper, we have $v_1(j) < v_2(j) < v_3(j)$ when the three drivers competing for the same parking slot $j$, $\forall j \in J$. When driver $i$ uses parking slot $j$ for $w_i(j)$ units of time, her payment is specified by $p_i(j) = \rho w_i(j)$, where $\rho$ is the fixed fee rate (i.e., \$ per minutes) in the time-based linear pricing method. We further suppose that the drivers have the same staying time at the destination and we have $w_i(1) < w_i(2) < w_i(3)$, $\forall i \in I$, considering the walking times from the three slots. Therefore, the utility of driver $i$ of using parking slot $j$ is

$$u_i(j) = v_i(j) - \rho w_i(j), i \in I, j \in J. \tag{6}$$

Thus, we have the relation: $u_i(1) > u_i(2) > u_i(3)$, for the same driver $i$, $i \in I$ using the three parking slots, which means that there is a diminishing preference for slots 1, 2 and 3.

### 2.2. Nash-equilibrium strategies

Now we establish the demand-reporting game of the three drivers. We suppose that the three parking slots are all feasible for the three drivers if they all report true departure times $a_i^3$, i.e., $J_i(a_i^3) = J = \{1,2,3\}$, $\forall i \in I$. Accordingly, driver $i$'s strategy set is defined as $A_i = \{a_i^1, a_i^2, a_i^3\}$, $\forall i \in I$, where $a_1^1$ and $a_1^2$ are the other two alternatives for all drivers, resulting in $J_i(a_1^1) = \{1\}$ and $J_i(a_1^2) = \{1,2\}$, respectively. As a result, we have

***Nash-equilibrium strategies.*** *With the time-based pricing method, the Nash-equilibrium strategies of the numerical example are constructed as*

$$\hat{a}^N = \begin{cases} \{a_1^3, a_2^1, a_3^1\}, \text{if } u_2(1)(1-\xi) \geq u_2(2)(3-\xi), \\ \{a_1^1, a_2^2, a_3^1\}, \text{if } u_1(1)(1-\xi) \geq 3u_1(2)(1-\xi) + 2u_1(3)\xi \text{ and } u_2(1-\xi) \leq u_2(2)(3-\xi), \\ \{a_1^3, a_2^2, a_3^1\}, \text{if } u_1(1)(1-\xi) \leq 3u_1(2)(1-\xi) + 2u_1(3)\xi \text{ and } u_2(1-\xi) \leq u_2(2)(3-\xi), \end{cases} \tag{7}$$

Let us consider the first Nash-equilibrium strategies in Equation (7), i.e., $\hat{a}^N = \{a_1^3, a_2^1, a_3^1\}$, in which driver 1 truthfully reports her departure time $a_1^3$ but the other drivers misreport their departure times as $a_2^1$ and $a_3^1$, respectively, if the expected utility $u_2(1)(1-\xi)$ is better than $u_2(2)(1-\xi)$. Note that for any equilibrium, the non-dominated strategy of driver 3 is to report $a_3^1$. With the (mis)reports in $\hat{a}^N$, the system generates an inefficient matching solution $M^*(\hat{a}^N) = ((1,2),(2,\varnothing),(3,1))$, which induces that driver 2 is not assigned any parking slots while parking slot 3 is not matched with any drivers. However, if the three drivers truthfully report their departure times, i.e., $\hat{a} = (a_1^3, a_2^3, a_3^3)$, there are two system-wide optimal matching solutions $M^*(\hat{a}) = ((1,3),(2,2),(3,1))$ and $M^{'*}(\hat{a}) = ((1,1),(2,2),(3,3)))$ with the maximum system efficiency. The system selects either one with the equal possibility. Therefore, compared with the system-wide optimal solution, Nash-equilibrium strategy $\hat{a}^N = (a_1^3, a_2^1, a_3^1)$ leads to the loss of the system efficiency. Considering the rest two Nash-equilibrium strategies in Equation (7), the second one $\hat{a}^N = \{a_1^1, a_2^2, a_3^1\}$ induces inefficient matching solutions but the last one does not.

### 2.3. Construction of Nash-equilibrium strategies

The rest part presents the construction method of the Nash-equilibrium strategies of the instance, which is used to find all Nash-equilibrium strategies in the numerical experiment of the paper.

First of all, the non-dominated strategy sets of three drivers are identified as $A_1 = \{a_1^1, a_1^2, a_1^3\}$, $A_2 = \{a_2^1, a_2^2\}$ and $A_3 = \{a_3^1\}$, respectively, according to the optimal matching model $P^*$ in the paper. As driver 3 has the only strategy $a_3^1$, thus we just need to consider the selected strategies of the other drivers from their non-dominated strategy sets $A_1$ and $A_2$, respectively. Consider driver 1, if driver 2 selects her strategy as $a_2^1$, the expected utilities of driver 1 by selecting each strategy from $A_1$ are

$$\mathbb{E}u_1(a_1^1, a_2^1, a_3^1) = u_1(1) * (1-\xi)^2, \tag{8}$$

$$\mathbb{E}u_1(a_1^2, a_2^1, a_3^1) = u_1(1) * (1-\xi)^2 + u_1(2) * (2\xi - \xi^2), \tag{9}$$

$$\mathbb{E}u_1(a_1^3, a_2^1, a_3^1) = u_1(1) * (1-\xi)^2 + u_1(2) * (2\xi - \xi^2). \tag{10}$$

Moreover, we have

$$\mathbb{E}u_1(a_1^1, a_2^1, a_3^1) \le \mathbb{E}u_1(a_1^2, a_2^1, a_3^1) = \mathbb{E}u_1(a_1^3, a_2^1, a_3^1). \tag{11}$$

The above relation implies that strategy $a_1^3$ is the best one for driver 1 if the selected strategies of the other drivers are $a_2^1$ and $a_3^1$.

Similarly, if driver 2 selects her strategy as $a_2^2$, we have

$$\mathbb{E}u_1(a_1^3, a_2^2, a_3^1) \ge \mathbb{E}u_1(a_1^2, a_2^2, a_3^1). \tag{12}$$

$$\begin{cases} \mathbb{E}u_1(a_1^3, a_2^2, a_3^1) \le \mathbb{E}u_1(a_1^1, a_2^2, a_3^1), & u_1(1) * (1-\xi) \ge 3u_2(1) * (1-\xi) + u_1(3) * \xi, \\ \mathbb{E}u_1(a_1^3, a_2^2, a_3^1) \ge \mathbb{E}u_1(a_1^1, a_2^2, a_3^1) & \text{otherwise}. \end{cases} \tag{13}$$

Note that for the strategies profile $\{a_1^2, a_2^2, a_3^1\}$ and $\{a_1^3, a_2^2, a_3^1\}$, when driver 3 does not show up, there are two optimal matching solutions to optimal matching model, (i.e., $\{m(1,1), m(2,2)\}$ and $\{m(1,2), m(2,1)\}$). The system breaks the tie by randomly selecting either $\{m(1,1), m(2,2)\}$ or $\{m(1,2), m(2,1)\}$ as the generated matching solution.

For driver 2, if driver 1 selects her strategy as $a_1^1$, $a_1^2$, or $a_1^3$, the corresponding expected utilities of driver 2 by selecting each strategy in $A_2$ are calculated as follows.

If driver 1 selects a strategy $a_1^1$, we have

$$\begin{cases} \mathbb{E}u_2(a_1^1, a_2^1, a_3^1) \ge \mathbb{E}u_2(a_1^1, a_2^2, a_3^1), & u_2(1) * (1-\xi) \ge u_2(2) * (2-\xi) \\ \mathbb{E}u_2(a_1^1, a_2^1, a_3^1) \le \mathbb{E}u_2(a_1^1, a_2^2, a_3^1). & \text{otherwise} \end{cases} \tag{14}$$

If driver 1 selects a strategy $a_1^2$, we have

$$\begin{cases} \mathbb{E}u_2(a_1^2, a_2^1, a_3^1) \ge \mathbb{E}u_2(a_1^2, a_2^2, a_3^1), & u_2(1) * (1-\xi) \ge u_2(2) * (3-\xi) \\ \mathbb{E}u_2(a_1^2, a_2^1, a_3^1) \le \mathbb{E}u_2(a_1^2, a_2^2, a_3^1). & \text{otherwise} \end{cases} \tag{15}$$

If driver 1 selects a strategy $a_1^3$, we have

$$\begin{cases} \mathbb{E}u_2(a_1^3, a_2^1, a_3^1) \ge \mathbb{E}u_2(a_1^3, a_2^2, a_3^1), & u_2(1) * (1-\xi) \ge u_2(2) * (3-\xi) \\ \mathbb{E}u_2(a_1^3, a_2^1, a_3^1) \le \mathbb{E}u_2(a_1^3, a_2^2, a_3^1). & \text{otherwise} \end{cases} \tag{16}$$

Now we deduce the Nash-equilibrium strategy profiles according to the above equations. The relations (11)-(13) mean that under some specific conditions, the stable strategy profiles for driver 1 have $\{a_1^3, a_2^1, a_3^1\}$, $\{a_1^2, a_2^1, a_3^1\}$, $\{a_1^3, a_2^2, a_3^1\}$ and $\{a_1^1, a_2^2, a_3^1\}$. Similarly, the relations (14)-(16) mean that under some certain conditions, the stable strategy profiles for driver 2 have $\{a_1^1, a_2^1, a_3^1\}$, $\{a_1^1, a_2^2, a_3^1\}$, $\{a_1^2, a_2^1, a_3^1\}$, $\{a_1^1, a_2^2, a_3^1\}$, $\{a_1^3, a_2^1, a_3^1\}$ and $\{a_1^3, a_2^2, a_3^1\}$. The Nash-equilibrium profiles are just the intersection of stable strategy profiles of driver 1 and 2. Consequently, we obtain the three Nash-equilibrium strategies in Equation (7).