Appendix A: Captive Customers System (CSS)

Proof of Lemma 1

In a CCS, customers who join the priority queue should satisfy $U_1 \geq U_2$. We then obtain $K(\theta)_{(\alpha)} = (1-\theta)(W_2 - W_1) + \alpha(W_2 - \bar{W})$. The functions of the priority fee and the revenue with decision variable θ and reference dependency effect parameter α are shown as follows:

$$\begin{cases} K(\theta)_{(\alpha)} = \dfrac{\rho^2\left((1-\theta)+\alpha\theta\right)}{\mu(1-\rho)(1-\theta\rho)}; \; R(\theta)_{(\alpha)} = \lambda\theta(K(\theta)_{(\alpha)}+c) + \lambda(1-\theta)c \;\; (0 \leq \theta \leq 1) \end{cases} \tag{A1}$$

Where, $\lambda\theta(K(\theta)_{(\alpha)}+c)$ and $\lambda(1-\theta)c$ indicate the revenue from priority and regular customers, respectively. By optimizing the revenue function $R(\theta)_{(\alpha)}$, subject to $0 \leq \theta \leq 1$, we obtain the following optimal results:

1) If $0 < \alpha < (1-\rho)/(2-\rho)$, we derive the following optimal results (in this paper, we use the superscript * to denote optimal solutions):

$$\begin{cases} \theta^*_{(\alpha)} = \dfrac{1}{\rho}\left(1 - \sqrt{\dfrac{1-\alpha-\rho}{1-\alpha}}\right); \; K(\theta^*)_{(\alpha)} = \dfrac{\rho}{\mu(1-\rho)}\left((1-\alpha) - \sqrt{(1-\alpha)(1-\alpha-\rho)}\right) \\ R(\theta^*)_{(\alpha)} = \dfrac{\rho\left((1-\alpha) - \sqrt{(1-\alpha)(1-\alpha-\rho)}\right)^2}{(1-\rho)(1-\alpha)} + \lambda c \end{cases} \tag{A2}$$

2) If $\alpha \geq (1-\rho)/(2-\rho)$, the corresponding optimal results are as follows:

$$\begin{cases} \theta^*_{(\alpha)} = 1; \; K(1)_{(\alpha)} = \dfrac{\rho^2}{\mu(1-\rho)^2}\alpha; \; R(1)_{(\alpha)} = \dfrac{\rho^3}{(1-\rho)^2}\alpha + \lambda c \end{cases} \tag{A3}$$

3) If $\alpha = 0$ (the existing studies), we can derive the following expression:

$$\begin{cases} \theta^*_{(0)} = \dfrac{1}{\rho}(1-\sqrt{1-\rho}); \; K(\theta^*)_{(0)} = \dfrac{\rho(1-\sqrt{1-\rho})}{\mu(1-\rho)}; \; R(\theta^*)_{(0)} = \dfrac{\rho(1-\sqrt{1-\rho})^2}{(1-\rho)} + \lambda c \end{cases} \tag{A4}$$

Proof of Managerial Insight 1

(1) We can obtain the following relationship, following equation (A2),

$$\frac{\partial \theta^*_{(\alpha)}}{\partial \alpha} = \frac{1}{2\sqrt{(1-\rho-\alpha)/(1-\alpha)}} \frac{1}{(1-\alpha)^2} > 0 \tag{A5}$$

(2) We also obtain the characteristic of the optimal priority fee, following equation (A2). That is,

$$\frac{\partial K(\theta^*)_{(\alpha)}}{\partial \alpha} = \frac{\rho}{\mu(1-\rho)}[-1 + \frac{2(1-\alpha)-\rho}{2\sqrt{(1-\alpha)(1-\alpha-\rho)}}] > 0 \tag{A6}$$

(3) For equation (A2), we denote $u = 1-\alpha$ and $z = \sqrt{(1-\alpha)^2 - (1-\alpha)\rho}$. We obtain the following:

$$\frac{\partial R(\theta^*)_{(\alpha)}}{\partial \alpha} = \frac{\rho}{(1-\rho)} \frac{2(u-z)u\left(-1+(2u-\rho)/2z\right) - (u-z)^2(-1)}{u^2} = \frac{\rho}{(1-\rho)} \frac{(\mu-z)^2}{uz} > 0 \tag{A7}$$

Therefore, the optimal results $\theta^*_{(\alpha)}$, $K(\theta^*)_{(\alpha)}$, and $R(\theta^*)_{(\alpha)}$ also increase the loss reference-dependence α .

(4) From equation (A3), we find that both $K(1)_{(\alpha)}$ and $R(1)_{(\alpha)}$ increase with α .

To summarize, we obtain the following relations for optimal revenue results:

$$R(\theta^*)_{(\alpha \geq \frac{1-\rho}{2-\rho})} > R(\theta^*)_{(0<\alpha<\frac{1-\rho}{2-\rho})} > R(\theta^*)_{(\alpha=0)} \tag{A8}$$

Proof of Lemma2

In a CCS, the social welfare (SW) function is as follows:

$$SW(\theta)_{(\alpha)} = R(\theta)_{(\alpha)} + \lambda\int_0^1 U(H)dH = R(\theta)_{(\alpha)} + \lambda\int_0^{(1-\theta)} U_2(H)dH + \lambda\int_{(1-\theta)}^1 U_1(H)dH$$
$$= \lambda\pi + \lambda W\frac{\theta^2(2\alpha-1)\rho + 2\rho\theta(1-\alpha)-1}{2(1-\theta\rho)} \quad (0<\theta<1) \tag{A9}$$

We then obtain the following optimal results:

1) If $0<\alpha<0.5$, we can obtain optimal solutions as (A10).

$$\begin{cases} \theta^*_{(\alpha)(SW)} = \dfrac{1}{\rho}(1-\sqrt{1-\rho}); \; K(\theta^*)_{(\alpha)(SW)} = \dfrac{\rho^2\left(1-(1-\alpha)(1-\sqrt{1-\rho})/\rho\right)}{\mu(1-\rho)\sqrt{1-\rho}} \\ SW(\theta^*)_{(\alpha)} = \lambda\pi + \lambda[\dfrac{(1-2\alpha)\left(1-(1-\sqrt{1-\rho})/\rho\right)(1-\sqrt{1-\rho})-\sqrt{1-\rho}}{2\sqrt{1-\rho}}]W \end{cases} \tag{A10}$$

2) If $\alpha\geq 0.5$, we can obtain optimal solutions as (A11).

$$\begin{cases} \theta^*_{(\alpha)(SW)} = 1; \; K(1)_{(\alpha)} = \dfrac{\rho^2}{\mu(1-\rho)^2}\alpha; \; SW(1)_{(\alpha)} = \lambda\pi - \lambda\dfrac{1}{2}W \end{cases} \tag{A11}$$

3) If $\alpha=0$ (i.e., no loss reference-dependence is considered), the optimal results are as follows:

$$\begin{cases} \theta^*_{(0)(SW)} = \dfrac{1}{\rho}(1-\sqrt{1-\rho}); \; K(\theta^*)_{(0)} = \dfrac{\rho(1-\sqrt{1-\rho})}{\mu(1-\rho)}; \; SW(\theta^*)_{(0)} = \lambda\pi - \lambda\dfrac{\sqrt{1-\rho}-(1-\rho)}{\rho}W \end{cases} \tag{A12}$$

Proof of Managerial Insight 2

1) From equation (A10), where $0<\alpha<0.5$, we obtain the following relationship:

$$\begin{cases} \dfrac{\partial K(\theta^*)_{(\alpha)(SW)}}{\partial\alpha} = \dfrac{\rho(1-\sqrt{1-\rho})}{\mu(1-\rho)\sqrt{1-\rho}} > 0; \; \dfrac{\partial SW(\theta^*)_{(\alpha)}}{\partial\alpha} = -\lambda[\dfrac{\left(1-(1-\sqrt{1-\rho})/\rho\right)(1-\sqrt{1-\rho})}{\sqrt{1-\rho}}]W < 0 \end{cases} \tag{A13}$$

From Eq. (A13), we find that the optimal $K(\theta^*)_{(\alpha)}$ increases with α. The optimal $SW(\theta^*)_{(\alpha)}$, however, decreases with α. From Eq. (A11), where $\alpha\geq 0.5$, we obtain $\theta^*_{(\alpha)(SW)}=1$. That is, $K(1)_{(\alpha)}$ increases with the effect of loss reference-dependence α. When $\alpha\geq 0.5$, all customers join the priority queue.

2) When $0<\alpha<0.5$, we compare the optimal SW with that of existing research to determine a relationship:

$$SW(\theta^*)_{(\alpha)} - SW(\theta^*)_{(0)} = -\lambda[\frac{\alpha\left(1-(1-\sqrt{1-\rho})/\rho\right)(1-\sqrt{1-\rho})}{\sqrt{1-\rho}}]W < 0 \tag{A14}$$

3) If $\alpha\geq 0.5$, optimal SW can be compared with that of prior research results, a relationship can be established.

$$SW(1)_{(\alpha)} - SW(\theta^*)_{(0)} = \lambda(\frac{\sqrt{1-\rho}-(1-\rho)}{\rho}-\frac{1}{2})W = \lambda(\frac{2\sqrt{1-\rho}-(2-\rho)}{2\rho})W < 0 \tag{A15}$$

Taking customer loss reference-dependence into account in a CCS is likely to result in lower social welfare, and the optimal social welfare outcomes can be summarized as follows:

$$SW(\theta^*)_{(\alpha\geq 0.5)} < SW(\theta^*)_{(0<\alpha<0.5)} < SW(\theta^*)_{(\alpha=0)} \tag{A16}$$

Appendix A: Non-Monopoly Service System (NCCS)

Proof of Possible Outcomes in an NCCS

In an NCCS, entering customers have three options: joining the priority queue, joining the regular queue, and balking. The decision should be based on where max $\{U_0, U_1, U_2\}$ is achieved. Based on the analysis, two scenarios can be distinguished: one with two separate queues and the other with only one queue.

1) If $K > \alpha(W_2 - \overline{W})(W - W_1)/(W - W_2)$) holds, the arrival rates of different queues are

$$\begin{cases} \lambda_1 = \lambda \cdot G(H \geq \dfrac{K - \alpha(W_2 - \overline{W})}{W_2 - W_1}); \lambda_2 = \lambda \cdot G(\alpha \dfrac{W_2 - \overline{W}}{W - W_2} \leq H \leq \dfrac{K - \alpha(W_2 - \overline{W})}{W_2 - W_1}); \lambda_0 = \lambda \cdot G(H \leq \alpha \dfrac{W_2 - \overline{W}}{W - W_2}) \end{cases} \tag{A17}$$

2) If $K \leq \alpha(W_2 - \overline{W})(W - W_1)/(W - W_2)$ holds, the arrival rates of different queues are

$$\{\lambda_1 = \lambda \cdot G(H \geq K/(W - W_1)); \quad \lambda_0 = \lambda \cdot G(H \leq K/(W - W_1)) \tag{A18}$$

Proof of Lemma 3: Two Queues in the System (i.e., $\theta > 0$ and $\xi > 0$)

When there are two queues in the system, we can rewrite (A18) as

$$\begin{cases} K(\theta,\xi)_{(\alpha)} = (1-\theta)(W_2 - W_1) + \alpha(W_2 - \overline{W}); \ \alpha \dfrac{\theta\rho(1-\rho)(\theta+\xi)}{[(1-\theta\rho) - (\theta+\xi)(1-\theta\rho^2)]} = (1-(\theta+\xi)). \end{cases}$$

As $\theta + \xi > 0$, we can obtain a unique solution for $(\theta+\xi)$ as follows:

$$(\theta+\xi)_{(\alpha)} = \frac{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)] - \sqrt{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)]^2 - 4(1-\theta\rho^2)(1-\theta\rho)}}{2(1-\theta\rho^2)} \tag{A19}$$

When there are two queues in system, the priority fee and revenue functions are shown as follows:

$$\begin{cases} K(\theta,\xi)_{(\alpha)} = \dfrac{(\theta+\xi)\rho^2}{\mu(1-(\theta+\xi)\rho)(1-\theta\rho)}[(1-\theta)(\theta+\xi) + \alpha\theta]; R(\theta,\xi)_{(\alpha)} = \lambda\theta(K(\theta,\xi)_{(\alpha)} + c) + \lambda\xi c \end{cases} \tag{A20}$$

where $\lambda\theta(K(\theta,\xi)_{(\alpha)} + c)$ and $\lambda\xi c$ indicate the revenue from priority and regular customers, respectively.

2) By examining the first-order derivative of $\xi(\theta)_{(\alpha)}$ (i.e., $\xi(\theta)_{(\alpha)} = (\theta+\xi)_{(\alpha)} - \theta$) in equation (A19), we show that the derivative is a decreasing function with θ:

$$\frac{\partial\xi(\theta)_{(\alpha)}}{\partial\theta} = -\frac{\left(4(\alpha+1) + \dfrac{[2\alpha(1-\rho)\theta\rho][\alpha+1] + 2\alpha(1-\theta\rho) + 2\alpha(1-\theta\rho^2) + 2\theta\rho(1-\rho)]}{\sqrt{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)]^2 - 4(1-\theta\rho^2)(1-\theta\rho)}}\right)\rho(1-\rho)}{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)] + \sqrt{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)]^2 - 4(1-\theta\rho^2)(1-\theta\rho)}]^2} - 1 < 0 \tag{A21}$$

Thus, the derivatives of $(\theta+\xi)_{(\alpha)}$ can be easily obtained from those of $\xi(\theta)_{(\alpha)}$:

$$\partial(\theta+\xi)_{(\alpha)}/\partial\theta = \partial\xi(\theta)_{(\alpha)}/\partial\theta + 1 < 0 \tag{A22}$$

Based on the expression of $(\theta+\xi)_{(\alpha)}$, we also have the following relation:

$$\frac{\partial(\theta+\xi)_{(\alpha)}}{\partial\alpha} = \frac{(1-\rho)\theta\rho}{2(1-\theta\rho^2)}(1 - \frac{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)]}{\sqrt{[\alpha(1-\rho)\theta\rho + (2-\theta\rho-\theta\rho^2)]^2 - 4(1-\theta\rho^2)(1-\theta\rho)}}) < 0 \tag{A23}$$

That is, the overall proportion of customers (e.g., the sum of proportion of priority and regular customers) $(\theta+\xi)_{(\alpha)}$ in the service system decreases with the parameter of α.

Finally, the derivatives of $\xi(\theta)_{(\alpha)}$ can be easily obtained from those of $(\theta+\xi)_{(\alpha)}$:

$$\partial\xi(\theta)_{(\alpha)}/\partial\alpha = \partial(\theta+\xi)_{(\alpha)}/\partial\alpha - \partial\theta/\partial\alpha = \partial(\theta+\xi)_{(\alpha)}/\partial\alpha < 0 \tag{A24}$$

Proof of Proposition 1

In equation (A19), when $\alpha = 0$, we obtain $(\theta+\xi)_{(0)} = (1-\theta\rho)/(1-\theta\rho^2)$ and $\xi(\theta)_{(0)} = (\theta+\xi)_{(0)} - \theta$. If $\xi(\theta)_{(0)} = 0$, we obtain $(1-(\rho+1)\overline{\theta}_{(0)} + \overline{\theta}_{(0)}{}^2\rho^2) = 0$. Then,

$$\overline{\theta}_{(0)} = [(\rho+1) - \sqrt{(\rho+1)^2 - 4\rho^2}]/2\rho^2 \tag{A25}$$

Therefore, when $0 < \theta < \overline{\theta}_{(0)}$, both queues (priority and regular) exist in the system (i.e., $\theta > 0$ and $\xi > 0$).

Proof of Proposition 2

With the monotonicity in Eq.(A24), the maximum value of θ that satisfies $\xi(\theta)_{(\alpha)} = (\theta+\xi)_{(\alpha)} - \theta \geq 0$ needs to be the one where $\xi(\theta)_{(\alpha)} = 0$ in Eq. (A19). Let's denote it as $\bar{\theta}_{(\alpha)}$; then $\bar{\theta}_{(\alpha)}$ satisfy $\xi(\bar{\theta}_{(\alpha)}) = 0$.

We obtain the following relation based on $\xi(\bar{\theta}_{(\alpha)}) = 0$:

$$\alpha(1-\rho)\rho\bar{\theta}_{(\alpha)}^{\ 2} - (1-\bar{\theta}_{(\alpha)})[1-(\rho+1)\bar{\theta}_{(\alpha)} + \bar{\theta}_{(\alpha)}^{\ 2}\rho^2] = 0 \tag{A26}$$

Based on equation (A17), we assume that there is a function $F(\theta_{(\alpha)})$ as follows:

$$F(\theta_{(\alpha)}) = \alpha(1-\rho)\rho\theta_{(\alpha)}^{\ 2} - (1-\theta_{(\alpha)})[1-(\rho+1)\theta_{(\alpha)} + \theta_{(\alpha)}^{\ 2}\rho^2] \tag{A27}$$

In equation (A27), we obtain $F(0) = -1$ and $F(1) = \alpha(1-\rho)\rho > 0$. Here, the first-order derivative of $F(\theta_{(\alpha)})$ regarding $\theta_{(\alpha)}$ is shown as $\partial F(\theta_{(\alpha)})/\partial\theta_{(\alpha)} > 0$. Thus, $F(\theta_{(\alpha)})$ increases with $\theta_{(\alpha)}$, and $\bar{\theta}_{(\alpha)}$ is unique in the interval (0, 1) that satisfies $F(\bar{\theta}_{(\alpha)}) = 0$. Therefore, when $0 < \theta < \bar{\theta}_{(\alpha)}$, both queues (priority and regular) exist in the system. Based on equation (A27), we obtain

$$\frac{\partial\bar{\theta}_{(\alpha)}}{\partial\alpha} = -\frac{(1-\rho)\rho\bar{\theta}_{(\alpha)}^{\ 2} + [1-(\rho+1)\bar{\theta}_{(\alpha)} + \bar{\theta}_{(\alpha)}^{\ 2}\rho^2]}{\left((1-\bar{\theta}_{(\alpha)})[(\rho(1-\bar{\theta}_{(\alpha)}\rho) + (1-\bar{\theta}_{(\alpha)}\rho^2)] + \alpha(1-\rho)\rho 2\bar{\theta}_{(\alpha)}\right)} < 0$$

The largest value $\bar{\theta}_{(\alpha)}$ can achieve is at $\alpha = 0$; then, we obtain $1-(\rho+1)\bar{\theta}_{(0)} + \bar{\theta}_{(0)}^{\ 2}\rho^2 = 0$. This is the same form as equation (A25), which defines $\bar{\theta}_{(0)}$. Thus,

$$\bar{\theta}_{(\alpha)} = \bar{\theta}\big|_{any\ a} \leq \bar{\theta}_{(\alpha=0)} = \left((\rho+1) - \sqrt{(\rho+1)^2 - 4\rho^2}\right)\Big/2\rho^2 = \bar{\theta}_{(0)} \tag{A28}$$

With the monotonicity of equation (A24), the maximum value of α that satisfies $\xi(\theta)_{(\alpha)} > 0$ needs to be the one where $\xi(\theta)_{(\alpha)} = 0$ in equation (A19). Let us denote it by a sufficiently large positive number $\bar{\alpha} > 0$. Then, $\xi(\theta)_{(\alpha)}$ satisfies $\xi(\theta)_{(\alpha)} = 0$ when $\alpha \geq \bar{\alpha}$. By using the equations (A19) and (A24), here,

$$\bar{\alpha} = (1-\theta)[1-(\rho+1)\theta + \theta\rho^2]/(1-\rho)\rho\theta^2 > 0 \tag{A29}$$

Thus, we find that, for $\alpha \geq \bar{\alpha}$, the optimal point is achieved at $\xi^* = \xi(\theta)^*_{(\alpha)} = 0$, and $\theta^* > 0$.

Proof of Lemma 4: Priority Queue Only in the System (i.e., $\xi = 0$ and $\theta > 0$)

Priority queue only means that customers have only two choices: joining the priority queue or balking.

When $\xi = 0$, $\theta = P\left(H \geq K/(W - W_1)\right)$, we obtain $K(\theta, 0)_{(\alpha)} = (1-\theta)(W - W_1)$.

Recall that, for equation (A18), the fact of $\xi = 0$, which means $\alpha(W_2 - \bar{W})/(W - W_2) \geq [K - \alpha(W_2 - \bar{W})]/(W_2 - W_1)$. Please note that the system still provides two queue options. It is anticipated that customers do not find it beneficial to join the regular queue. Thus, if $\xi = 0$, we have the following equations in regard to waiting times:

$$\bar{W} = \frac{\theta\rho}{\mu(1-\theta\rho)}, W_1 = \frac{\theta\rho}{\mu(1-\rho\theta)}, \text{ and } W_2 = \frac{\theta\rho}{\mu(1-\theta\rho)^2}.$$

Based on the above equations, we obtain the following relation:

$$\alpha(W_2 - \bar{W})/(W - W_2) \geq (1-\theta) \Rightarrow \alpha\theta^2\rho(1-\rho) \geq (1-\theta)[(1-\theta\rho) - \theta(1-\theta\rho^2)] \tag{A30}$$

This is the same form as equation (A26), which defines $\bar{\theta}_{(\alpha)}$. Thus, $\theta \geq \bar{\theta}_{(\alpha)}$ when there is only a priority queue. This also is exactly the function to obtain the upper boundary of $\bar{\alpha}$. Therefore, when $\alpha \geq \bar{\alpha}$, we obtain $\theta \geq \bar{\theta}_{(\alpha)}$. As a result, when there are only priority customers (e.g., $\xi = 0$ and $\theta > 0$), customers need to pay the priority and entrance fees (i.e., $K(\theta, 0) + c$) to receive service in queue. The expressions for the priority fee and revenue are shown as follows:

$$\begin{cases} K(\theta,0)_{(\alpha)} = \dfrac{(1-\theta)^2\rho}{\mu(1-\rho)(1-\theta\rho)}; \quad R(\theta,0)_{(\alpha)} = \lambda\theta(K(\theta,0)+c) \qquad (\bar{\theta}_{(\alpha)} \le \theta \le 1) \end{cases} \tag{A31}$$

Proof of Lemma 5: Regular Queue Only in the System

Regular queue only in the system. By using equation (A19), when $\theta = 0$, we obtain $\xi(0)_{(\alpha)} = 1$.

Thus, we can derive the following revenue function:

$$R(0,\xi)_{(\alpha)} = \lambda\xi(0)_{(\alpha)}c = \lambda c \quad (\theta = 0). \tag{A32}$$

Proof of Proposition 3

Recall the Proofs of Lemma 3, Lemma 4, and Lemma 5. We obtain the following three outcomes:

1) $0 < \theta < \bar{\theta}_{(\alpha)}$. In this case, both queues are used by customers.
2) $\bar{\theta}_{(\alpha)} \le \theta \le 1$. In this case, $\xi(\theta)_{(\alpha)} = 0$, and only the priority queue is used by customers.
3) $\theta = 0$. In this case, $\xi(\theta)_{(\alpha)} > 0$, and only the regular queue is used by customers.

Proof of Lemma 6: The Continuity of Priority Fee and Revenue Functions

To obtain the maximum revenue value by optimizing the functions of revenues, first, we need to demonstrate the continuity of functions (i.e., equations (A20), (A31), and (A32)).

(1) The Continuity of Priority Fee Function:

First, we prove that priority fee and revenue functions are continuous at the boundary point of domain of θ when $\theta = \bar{\theta}_{(\alpha)}$. If $\theta = \bar{\theta}_{(\alpha)}$, by using equation (A31), we can derive equivalent expressions as follows:

$$(1-\bar{\theta}_{(\alpha)} + \bar{\theta}_{(\alpha)} - \bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)}) - (1-\bar{\theta}_{(\alpha)}\rho^2)\bar{\theta}_{(\alpha)}(1-\bar{\theta}_{(\alpha)}) = \alpha(1-\rho)\rho\bar{\theta}_{(\alpha)}{}^2$$

$$\Leftrightarrow (1-\bar{\theta}_{(\alpha)})^2 = \rho\bar{\theta}_{(\alpha)}[\alpha(1-\rho)\bar{\theta}_{(\alpha)} + (1-\bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)})]$$

When $\theta = \bar{\theta}_{(\alpha)}$, based on equation (A31), the priority fee function is shown as follows:

$$K(\theta,0)_{(\alpha)} = \frac{(1-\bar{\theta}_{(\alpha)})^2\rho}{\mu(1-\rho)(1-\bar{\theta}_{(\alpha)}\rho)} = \frac{[(1-\bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)}) + \alpha(1-\rho)\bar{\theta}_{(\alpha)}]\rho^2\bar{\theta}_{(\alpha)}}{\mu(1-\rho)(1-\bar{\theta}_{(\alpha)}\rho)} \tag{A33}$$

When $\theta = \bar{\theta}_{(\alpha)}$ and $\xi = 0$, based on equation (A20), the priority fee function is shown as follows:

$$K(\theta,\xi)_{(\alpha)} = \frac{\bar{\theta}_{(\alpha)}\rho^2}{\mu(1-\bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)}\rho)}(1-\bar{\theta}_{(\alpha)} + \alpha)\bar{\theta}_{(\alpha)} \tag{A34}$$

Based on equation (A26), we also can derive the following equivalent relations:

$$(1-\rho\bar{\theta}_{(\alpha)} - \bar{\theta}_{(\alpha)} + \rho^2\bar{\theta}_{(\alpha)}{}^2)(1-\bar{\theta}_{(\alpha)}) = \alpha(1-\rho)\rho\bar{\theta}_{(\alpha)}{}^2 \Leftrightarrow \left((1-\rho\bar{\theta}_{(\alpha)})^2 - (1-\rho)\bar{\theta}_{(\alpha)}\right)(1-\bar{\theta}_{(\alpha)}) = \alpha(1-\rho)\rho\bar{\theta}_{(\alpha)}{}^2$$

$$\Leftrightarrow (1-\bar{\theta}_{(\alpha)}\rho)^2(1-\bar{\theta}_{(\alpha)}) + \alpha(1-\rho)\bar{\theta}_{(\alpha)}(1-\bar{\theta}_{(\alpha)}\rho) = (1-\bar{\theta}_{(\alpha)} + \alpha)\bar{\theta}_{(\alpha)}(1-\rho)$$

Therefore, when $\theta = \bar{\theta}_{(\alpha)}$, the two functions of priority fee have the following relationship:

$$K(\theta,\xi)_{(\alpha)} = \frac{\bar{\theta}_{(\alpha)}\rho^2\left((1-\bar{\theta}_{(\alpha)} + \alpha)\bar{\theta}_{(\alpha)}\right)}{\mu(1-\bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)}\rho)} = \frac{[(1-\bar{\theta}_{(\alpha)}\rho)(1-\bar{\theta}_{(\alpha)}) + \alpha(1-\rho)\bar{\theta}_{(\alpha)}]\rho^2\bar{\theta}_{(\alpha)}}{\mu(1-\bar{\theta}_{(\alpha)}\rho)(1-\rho)} = K(\theta,0)_{(\alpha)} \tag{A35}$$

(2) The Continuity of Revenue Function:

Based on equation (A20), we know that, when there are $\theta = \bar{\theta}_{(\alpha)}$ and $\xi = 0$, the priority fee is a continuous function. Then, two revenue functions have the following relations:

$$R(\theta,\xi)_{(\alpha)} = \lambda\theta(K(\theta,\xi)+c) + \lambda\xi c = \lambda\theta(K(\theta,0)+c) = R(\theta,0)_{(\alpha)} \tag{A36}$$

Therefore, the revenue function for two queues and that for only a priority queue are continuous at the point of $\theta = \bar{\theta}_{(\alpha)}$. Next, we prove that the priority fee and revenue function are continuous at the boundary point of domain of θ when $\theta = 0$. When $\theta = 0$, the reference waiting time equals the expected waiting time of regular customers, which means $\overline{W} = W_2 = \xi\rho/\mu(1-\xi\rho)$. Recall that, in the Proof of Lemma 3, we obtain $\xi(0)_{(\alpha)} = 1$ when $\theta = 0$. Thus, when $\theta = 0$, we obtain the following relations:

$$R(\theta,\xi)_{(\alpha)} = \lambda\theta(K(\theta,\xi)_{(\alpha)} + c) + \lambda\xi c = \lambda\xi c = R(0,\xi)_{(\alpha)} = \lambda c \tag{A37}$$

Therefore, both revenue functions are continuous at the point of $\theta = 0$.

In summary, the revenue functions in the NCCS are continuous with $\theta \in [0,1]$.

$$\begin{cases} R(\theta,\xi)_{(\alpha)} = \dfrac{\theta(\theta+\xi)\rho^3}{(1-(\theta+\xi)\rho)(1-\theta\rho)}[(1-\theta)(\theta+\xi)+\alpha\theta] + \lambda(\theta+\xi)c \quad (0 \le \theta \le \overline{\theta}_{(\alpha)}) \\ R(0,\xi)_{(\alpha)} = \lambda\xi c = \lambda c \quad (\theta = 0) \\ R(\theta,0)_{(\alpha)} = \dfrac{\rho^2\theta(1-\theta)}{(1-\rho)(1-\theta\rho)}((1-\theta)) + \lambda\theta c \qquad (\overline{\theta}_{(\alpha)} \le \theta \le 1) \end{cases} \tag{A38}$$

Proof of Proposition 4

Recall the Proof of Lemma 1; we know that, when $\alpha \ge \overline{\alpha}$, $\xi(\theta)_{(\alpha)} = 0$.

Recall the poof of Lemma 2; we also know that, when $\alpha \ge \overline{\alpha}$, $\overline{\theta}_{(\alpha)} \le \theta \le 1$.

Thus, when $\alpha \ge \overline{\alpha}$, we have $R(\theta,\xi)_{(\alpha)} = R(\theta,0)_{(\alpha)}$, where $\overline{\theta}_{(\alpha)} \le \theta \le 1$.

1) If the entrance fee is small (e.g., $c = 0$), we use $R(\theta,0)_{(\alpha)}$ derived on θ to obtain the response function:

$$\partial R(\theta,0)_{(\alpha)}/\partial\theta = \rho^2(1-\theta)(1-3\theta+2\theta^2\rho)/(1-\rho)(1-\theta\rho)^2 \tag{A39}$$

By setting equation (A38) as equal to 0, we obtain $\theta_1 = 1$, and $\theta_2 = (3-\sqrt{9-8\rho})/4\rho$.

Thus, if $0 < \theta \le \theta_2$ holds, $\partial R(\theta,0)_{(\alpha)}/\partial\theta > 0$; if $\theta_2 < \theta \le 1$ is true, then we obtain $\partial R(\theta,0)_{(\alpha)}/\partial\theta \le 0$.

Therefore, $R(\theta,0)_{(\alpha)}$ increases with θ when $0 < \theta \le \theta_2$ but decreases with θ when $\theta_2 < \theta \le 1$.

In summary, we obtain the following results:

- Based on the previous Proof of Lemma 1, we obtain $\xi(\theta)_{(\alpha)} = 0$ when $\alpha \ge \bar{\alpha}$.
- If $\theta_2 \le \overline{\theta}_{(\alpha)}$, we obtain $\theta^* = \overline{\theta}_{(\alpha)}$, $K^* = K(\overline{\theta}_{(\alpha)},0)_{(\alpha)} > 0$, and $R(\theta,0)^*_{(\alpha)} = R(\theta = \overline{\theta}_{(\alpha)},0)_{(\alpha)}$
- If $\theta_2 > \overline{\theta}_{(\alpha)}$, we obtain $\theta^* = \theta_2$, and $K^* = K(\theta_2,0)_{(\alpha)} > 0$; here, $R(\theta,0)^*_{(\alpha)} = R(\theta = \theta_2,0)_{(\alpha)}$.

2) Similarly, if the entrance fee is large (e.g., $c > 0$), we obtain the following:

- If $0 < \tilde{\theta} \le \overline{\theta}_{(\alpha)}$, $\max f(\theta) = f(\overline{\theta}_{(\alpha)})$ when $\overline{\theta}_{(\alpha)} \le \theta \le 1$. Thus, $\partial R(\theta,0)_{(\alpha)}/\partial\theta > 0$ holds if $c > f(\overline{\theta}_{(\alpha)})$, and the maximum of $R(\theta,0)_{(\alpha)}$ will be obtained at the point of right boundary (i.e., $\theta^* = 1$).
- If $\overline{\theta}_{(\alpha)} < \tilde{\theta} \le 1$, $\max f(\theta) = f(\tilde{\theta})$ when $\overline{\theta}_{(\alpha)} \le \theta \le 1$. In this case, $\partial R(\theta,0)_{(\alpha)}/\partial\theta > 0$ holds when $c > f(\tilde{\theta})$ and the maximum of $R(\theta,0)_{(\alpha)}$ will be obtained at the point right boundary (i.e., $\theta^* = 1$).

Let us denote that there is a value $\overline{c} = \max f(\theta) = \max\{f(\overline{\theta}_{(\alpha)}), f(\tilde{\theta})\}$, where $-4 + 6\tilde{\theta} - 6\tilde{\theta}^2\rho + 2\rho + 2\tilde{\theta}^3\rho^2 = 0$.

In summary, if $c > \overline{c} = \max\{f(\overline{\theta}_{(\alpha)}), f(\tilde{\theta})\}$, $\theta^* = 1$, and $K^* = K(1,0)_{(\alpha)} = 0$.

The results above lead to the following three results:

1) If $\alpha \ge \bar{\alpha}$ and $c = 0$, then $\theta^* = \overline{\theta}_{(\alpha)}$, $\xi^* = \xi(\theta^*) = 0$ and $K^* = K(\overline{\theta}_{(\alpha)},0)_{(\alpha)} > 0$ if $\theta_2 \le \overline{\theta}_{(\alpha)}$.

2) If $\alpha \ge \bar{\alpha}$ and $c = 0$, then $\theta^* = \theta_2 = (3-\sqrt{9-8\rho})/4\rho$, $\xi^* = \xi(\theta^*) = 0$, $K^* = K(\theta_2,0)_{(\alpha)} > 0$; here, $R(\theta,0)^*_{(\alpha)} = R(\theta = \theta_2,0)_{(\alpha)}$ if $\theta_2 \ge \overline{\theta}_{(\alpha)}$

3) If $\alpha \ge \bar{\alpha}$ and $c > \overline{c} = \max\{f(\overline{\theta}_{(\alpha)}), f(\tilde{\theta})\}$, then $\theta^* = 1$, $\xi^* = \xi(\theta^*) = 0$, and $K^* = K(\theta,0)^*_{(\alpha)} = K(1,0)_{(\alpha)} = 0$.

Proof of the Function of Social Welfare in an NCCS

(1) Social Welfare Function (two queues in the system)

Recall the Proof of Lemma 3 (two queues in the system), the expression of $SW(\theta,\xi)_{(\alpha)}$ is shown as follows:

$$\begin{aligned} SW(\theta,\xi)_{(\alpha)} &= U(\theta,\xi)_{(\alpha)} + R(\theta,\xi)_{(\alpha)} = U(\theta,\xi)_{(\alpha)} + \lambda \int_{1-(\theta+\xi)}^{(1-\theta)} U_2(H)dH + \lambda \int_{(1-\theta)}^{1} U_1(H)dH \\ &= \lambda(\theta+\xi)\pi - \frac{\rho^2(\theta+\xi)\left\{\rho\theta\left[2\xi\alpha-(\theta+\xi)(2-\theta)\right]+\left[2\xi(1-\theta)-\xi^2+2\theta-\theta^2\right]\right\}}{2(1-(\theta+\xi)\rho)(1-\theta\rho)} (0<\theta<\bar{\theta}_{(\alpha)}) \end{aligned} \quad \text{(A40)}$$

(2) Social Welfare Function (priority queue only in the system)

Recall the Proof of Lemma 4 (only priority queue in the system), the expression of $SW(\theta,0)_{(\alpha)}$ is,

$$SW(\theta,0)_{(\alpha)} = R(\theta,0)_{(\alpha)} + U(\theta,0)_{(\alpha)} = R(\theta,0)_{(\alpha)} + \lambda \int_{(1-\theta)}^{1} U_1(H)dH = \lambda\theta\pi - \frac{\theta^2\rho^2(2-\theta)}{2(1-\theta\rho)} \ (\bar{\theta}_{(\alpha)} \le \theta \le 1) \quad \text{(A41)}$$

(3) Social Welfare Function (regular queue only in the system)

Based on the Proof of Lemma 5, if only regular customers are in the queue, we obtain $\theta = 0$ and $\xi = 1$.

The expression $SW(0,1)_{(\alpha)}$ is shown as follows:

$$SW(0,1)_{(\alpha)} = R(0,1)_{(\alpha)} + U(0,1)_{(\alpha)} = \lambda c + \lambda \int_{0}^{1} U_2(H)dH = \lambda\pi - \frac{\rho^2}{2(1-\rho)} \quad \text{(A42)}$$

The function of social welfare includes revenue and customers' utility; therefore, the demarcation point $\bar{\theta}_{(\alpha)}$ also is the demarcation point of the social welfare function. As the uniqueness of $\bar{\theta}_{(\alpha)}$ is demonstrated in the Proof of Proposition 1, $\bar{\theta}_{(\alpha)}$ also is unique in the domain of θ for the function of social welfare.

Proof of Lemma 7: Continuity of the Social Welfare Function

Recall the Proof of Lemma 6; when $\theta = \bar{\theta}_{(\alpha)}$, based on equations (A40) and (A41), we obtain $SW(\theta,\xi)_{(\alpha)} = SW(\theta,0)_{(\alpha)}$. Similarly, when $\theta = 0$ and $\xi = 1$, based on equations (A41) and (A42), we obtain $SW(\theta,\xi)_{(\alpha)} = SW(0,1)_{(\alpha)}$. Therefore, the SWe functions are continuous at the points of $\theta = 0$ and $\theta = \bar{\theta}_{(\alpha)}$.

In summary, based on the proofs above, the expressions of social welfare are shown as follows:

$$\begin{cases} SW(\theta,\xi)_{(\alpha)} = \lambda(\theta+\xi)\pi \\ -\dfrac{\rho^2(\theta+\xi)}{2(1-(\theta+\xi)\rho)(1-\theta\rho)}\left\{\rho\theta\left[2\xi\alpha-(\theta+\xi)(2-\theta)\right]+\left[2\xi(1-\theta)-\xi^2+2\theta-\theta^2\right]\right\} 0 \le \theta \le \bar{\theta}_{(\alpha)}) \\ SW(\theta,0)_{(\alpha)} = \lambda\theta\pi + \rho^2\theta^2(\theta-2)/2(1-\theta\rho) \quad (\bar{\theta}_{(\alpha)} \le \theta \le 1) \\ SW(0,1)_{(\alpha)} = \lambda\pi - \rho^2/2(1-\rho) \quad (\theta = 0) \end{cases} \quad \text{(A43)}$$

When $\alpha \ge \bar{\alpha}$, the optimal point of social welfare is achieved at $\xi^* = \xi(\theta)^*_{(\alpha)} = 0$, and $\theta^* > 0$.

Proof of Proposition 5

Recall the Proof of Lemma 1; we know that, when $\alpha \ge \bar{\alpha}$, $\xi(\theta)_{(\alpha)} = 0$.

Recall the Poof of Lemma 2; we also know that, when $\alpha \ge \bar{\alpha}$, $\bar{\theta}_{(\alpha)} \le \theta \le 1$.

Therefore, $SW(\theta,\xi)_{(\alpha)} = SW(\theta,0)_{(\alpha)}$ when $\alpha \ge \bar{\alpha}$.

Even if we are not able to obtain an explicit expression of the optimal θ^*, we can study the asymptotic behavior of θ^*. Recall the proof of Proposition 5; we obtain the following relations:

1) When $0 < \rho \le \rho_1 = (2\lambda\pi - \sqrt{2\lambda\pi})/(2\lambda\pi - 1) < 1$, we obtain the following results:

- Based on the previous Proof of Lemma 1, we obtain $\xi(\theta)_{(\alpha)} = 0$ when $\alpha \ge \bar{\alpha}$.
- We obtain the optimal result $\theta^* = 1$, and $K^* = K(1,0)_{(\alpha)} = 0$ when $0 < \rho \le \rho_1$.

2) When the system's offered load $(2\lambda\pi-\sqrt{2\lambda\pi})/(2\lambda\pi-1)=\rho_1<\rho<1$, we obtain the following results:

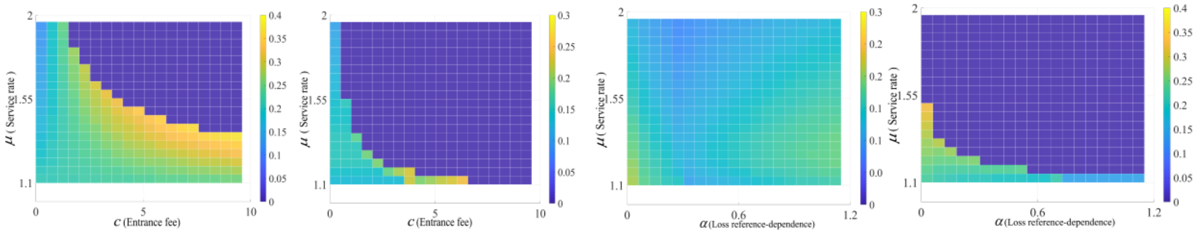
- Based on the previous Proof of Lemma 3, we obtain $\xi(\theta)_{(\alpha)}=0$ when $\alpha \geq \bar{\alpha}$.
- We obtain the unique optimal result $\theta^*=\hat{\theta}\in[\bar{\theta}_{(\alpha)},1]$, and $K^*=K(\theta^*,0)_{(\alpha)}>0$ if $\rho_1<\rho<1$, if $\hat{\theta}\geq\bar{\theta}_{(\alpha)}$.
- We obtain the unique optimal result $\theta^*=\bar{\theta}_{(\alpha)}$, and $K^*=K(\theta^*,0)_{(\alpha)}>0$ when $\rho_1<\rho<1$, if $\hat{\theta}\leq\bar{\theta}_{(\alpha)}$.

These results lead to the following two results:

1) If $\alpha \geq \bar{\alpha}$ *and* $0<\rho\leq\rho_1$, then $\theta^*=1$, $\xi^*=\xi(\theta^*)=0$ and $K^*=K(\theta,0)^*_{(\alpha)}=K(1,0)_{(\alpha)}=0$.

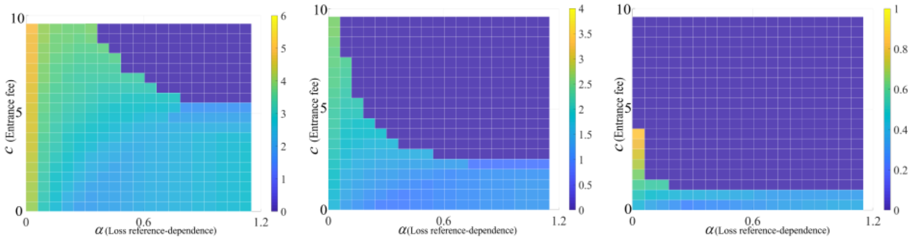
2) If $\alpha \geq \bar{\alpha}$ *and* $\rho_1<\rho<1$, then $0\leq\theta^*=\max\{\hat{\theta},\bar{\theta}_{(\alpha)}\}\leq 1$, $\xi^*=\xi(\theta^*)=0$ and $K^*=K(\theta^*,0)_{(\alpha)}>0$.

Appendix A: 2D Heat Map for the 3D Graph of Optimal Priority Fee (K^*) in NCCS



(a) $\alpha=0$ (b) $\alpha=0.6$ (a) $c=0$ (b) $c=4$

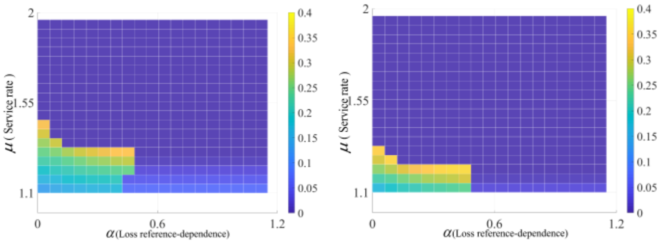
2D Heat Map: K^*as a function of c *and* μ **2D Heat Map:** K^*as a function of α *and* μ



(a) $\mu=1.1$ (b) $\mu=1.2$ (c) $\mu=1.5$

2D Heat Map: K^*as a function of α *and* c

Figure 14. 2D heat map of optimal priority fee for revenue maximization



(a) $\pi=5$ (b) $\pi=10$

2D Heat Map: K^*as a function of α *and* μ

Figure 15. 2D heat map of optimal priority fee for social welfare maximization

We provide detailed 2D heat maps that correspond to the 3D graph of K^* to offer a clearer visualization for its value variations and better understand the insights related to the optimal proportions of priority and regular customers, as observed and discussed in our manuscript.

Appendix B: Extension that Incorporates Reference-Dependent Preferences for Gain

Due to the page limits of the online supplement, we could not include all proofs for the Extension (i.e., section 6) in this document. These proofs are available upon request.