Online Appendix: "Product Design Enhancement for Fashion Retailing"

Appendix A Summary of Notation

For ease of reference, we list below the basic notation used in the paper.

- a: baseline market demand parameter;
- t_d: product trendiness under the manual design;
- $\widetilde{t}$: product trendiness under the AI-assisted design; a random variable with two possible states, high at $t_u + \theta$ and low at $t_u - \theta$;
- t_u: human designer's impact on the AI-assisted design;
- θ: AI's impact on the AI-assisted design, or innovation uncertainty, where $\theta \in [0, t_u]$;
- q_d: quality improvement under the manual design;
- q_u: quality improvement under the AI-assisted design;
- p_d: product sales price if the manual design is adopted;
- p_f: product sales price if the AI-assisted design is popular;
- p_m: product markdown price if the AI-assisted design is unpopular;
- p_0: product sales price if the existing product is offered;
- c: unit production cost coefficient for product quality improvement;
- d: design cost coefficient for overall product value enhancement;
- Π_0: firm's profit when the existing product is offered;
- Π_d: firm's profit when the manual design is adopted;
- Π_u: firm's overall profit when the AI-assisted design is adopted;
- Π_f^{stage2}: firm's stage 2 profit when the AI-assisted design is popular;
- Π_m^{stage2}: firm's stage 2 profit when the AI-assisted design is unpopular;
- Π_{IE}: firm's stage 1 expected profit when the AI-assisted design is adopted and the flexible product offering strategy is used;
- Π_{II}: firm's stage 1 expected profit when the AI-assisted design is adopted and the markdown sales strategy is used.
- Π_{IA}: firm's stage 1 expected profit when the AI-assisted design is adopted and the abandon sales strategy is used.

Appendix B Mathematical Proofs

Proof of Proposition 1 We apply the backward induction method to solve this problem.

Stage 2 pricing: Given q_d determined in stage 1, the firm solves the following pricing problem:

$$\begin{aligned} \underset{p_d}{\text{maximize}} \quad & \Pi_d = (a + q_d t_d - p_d) \cdot (p_d - c q_d) - d q_d^2 t_d^2 \\ \text{subject to} \quad & p_d \geq 0, \;\; p_d - c q_d \geq 0, \;\; and \;\; a + q_d t_d - p_d \geq 0, \end{aligned} \tag{B.8}$$

where the constraints guarantee the non-negativity of the profit margin and demand. Treating q_d as given, it is straightforward to see the firm's profit is concave in the selling price. Hence, if $q_d(c - t_d) \leq a$, $p_d^* = \frac{1}{2}(a + c q_d + q_d t_d)$ is the global optimal solution and the corresponding profit is $\frac{1}{4}(a + q_d(t_d - c))^2 - d q_d^2 t_d^2$; otherwise (when $q_d(c - t_d) \geq a$), the firm sets a high price to generate zero sales and the firm's overall profit is $-d q_d^2 t_d^2$.

Stage 1 quality improvement: Knowing the optimal pricing decisions in stage 2, the firm determines the quality improvement in stage 1 to maximize its profit. Following above analysis, there exists two solutions of q_d that lead to different profit functions for the firm, i.e., option (a) $q_d(c - t_d) \leq a$ where the profit is $\frac{1}{4}(a + q_d(t_d - c))^2 - d q_d^2 t_d^2$, or option (b) $q_d(c - t_d) \geq a$ where the firm's profit function is $-d q_d^2 t_d^2$. Apparently, option (b) is always dominated. So, we focus on option (a), which leads to the following scenarios contingent upon model parameters.

- When $c \leq t_d$ and $d \leq \frac{(t_d - c)^2}{4 t_d^2}$, the problem is unbounded. That is, q_d is set to be infinite which also leads to an infinite profit.
- When $c \leq t_d$ and $d > \frac{(t_d - c)^2}{4 t_d^2}$, the objective function is concave with the optimal quality improvement $q_d^* = \frac{a(t_d - c)}{4 d t_d^2 - (t_d - c)^2}$, $p_d^* = \frac{-a(c^2 - c t_d - 2 d t_d^2)}{4 d t_d^2 - (t_d - c)^2}$, and $\Pi_d^* = \frac{a^2 d t_d^2}{4 d t_d^2 - (t_d - c)^2}$.
- When $c \geq t_d$ and $d \leq \frac{(t_d - c)^2}{4 t_d^2}$, the firm's profit is convex and decreasing in q_d. Hence, $q_d^* = 0$.
- When $c \geq t_d$ and $d > \frac{(t_d - c)^2}{4 t_d^2}$, the objective function is convex and maximized at $q_d^* = 0$. □

Proof of Lemma 1 Treating q_u as given in stage 1, two steps are needed to characterize the firm's product offering strategy. First, given a realized value of trendiness, we need to solve the corresponding pricing decision when an existing product/improved product is offered. We demonstrate the solution method for the scenario when the realized trendiness is high, i.e., $\widetilde{t} = t_u + \theta$. The case with a low value of realized trendiness can be analyzed similarly.

If the improved product is offered, then the firm solves the following maximization problem:

$$\begin{aligned} \underset{p_f}{\text{maximize}} \quad & \Pi_f^{stage\ 2} = [a + q_u(t_u + \theta) - p_f] \cdot (p_f - cq_u) \\ \text{subject to} \quad & p_f \geq 0, \;\; p_f - cq_u \geq 0, \;\; and \;\; a + q_u(t_u + \theta) - p_f \geq 0. \end{aligned} \tag{B.9}$$

Given q_u, it is straightforward that if $q_u(c - t_u - \theta) \leq a$, the constrained optimal price is $p_f^* = \frac{1}{2}[a + q_u(t_u + \theta + c)]$; otherwise (when $q_u(c - t_u - \theta) > a$), the firm sets a price high enough to generate zero sales and profit. If the existing product is offered, equivalently, $q_u = 0$. This reduces the firm's profit function to be $(a - p_0) \cdot p_0$ which is concave in p_0 with its optimal value at $p_0^* = \frac{a}{2}$ and profit at $\frac{a^2}{4}$. Comparing the firm's profit under the improved and existing product, if $c \geq t_u + \theta$, serving the existing product is a dominant strategy and the profit is $\frac{a^2}{4}$. If $c \leq t_u + \theta$, the improved product dominates.

Now consider the case when the realized value of fashion is low, that is, $\widetilde{t} = t_u - \theta$. If the improved product is offered, the firm solves the following problem:

$$\begin{aligned} \underset{p_m}{\text{maximize}} \quad & \Pi_m^{stage\ 2} = [a + q_u(t_u - \theta) - p_m] \cdot (p_m - cq_u) \\ \text{subject to} \quad & p_m \geq 0, \;\; p_m - cq_u \geq 0, \;\; and \;\; a + q_u(t_u - \theta) - p_m \geq 0. \end{aligned} \tag{B.10}$$

At optimality, if $q_u(c - t_u + \theta) \leq a$, the constrained optimal price $p_m^* = \frac{1}{2}[a + q_u(t_u - \theta + c)]$; otherwise (when $q_u(c - t_u + \theta) > a$), the firm sets a price high enough to generate zero sales and profit in this stage. If the existing product is offered, the analysis has been conducted before. In this case, the optimal price is $p_0^* = \frac{a}{2}$ and the profit is $\frac{a^2}{4}$. Comparison of the firm's profit under the improved and existing product leads to the following conclusion: If $c \geq t_u - \theta$, serving the existing product is the best strategy. Otherwise, if $c \leq t_u - \theta$, serving the improved product is the optimal strategy. Combining previous analyses, we characterize the optimal pricing decisions under the AI-assisted design in three regions as follows:

- Region (a): For $c \leq t_u - \theta$, the firm offers the improved product at $p_f^* = \frac{1}{2}[a + q_u(c + t_u + \theta)]$ if realized trendiness is high, and at $p_m^* = \frac{1}{2}[a + q_u(c + t_u - \theta)]$ if realized trendiness is low. This represents the markdown sales strategy where the improved product is always offered regardless of the realization of fashion trends.

- Region (b): For $t_u - \theta < c \leq t_u + \theta$, the firm offers the improved product if realized trendiness is high at $p_f^* = \frac{1}{2}[a + q_u(c + t_u + \theta)]$ and offers the existing product if realized trendiness is low. This is the flexible offering strategy where the improved product is offered when the realized trendiness is high while the existing product is offered when the realized trendiness is low.

- Region (c): For $c \geq t_u + \theta$, the firm offers the existing product regardless of the realized value of fashion trends. In this case, the optimal price is $p_0^* = \frac{a}{2}$ and the optimal profit is $\Pi = \frac{a^2}{4}$. □

Proof of Proposition 2 We analyze the firm's optimal level of quality improvement in stage 1 in each of the aforementioned regions (in Lemma 1) separately.

- Region (a): In this region, the markdown sales strategy is used in stage 2. Accordingly, the firm's overall profit across both periods can be written as

$$\Pi_{\text{II}} = \frac{1}{8}(a + q_u(t_u + \theta - c))^2 + \frac{1}{8}(a + q_u(t_u - \theta - c))^2 - dq_u^2\left(t_u^2 + \theta^2\right),$$

  where the firm looks for a profit maximizing $q_u \geq 0$. This function is bounded only when the design cost coefficient d is sufficiently high, that is, when $d \geq I_a = \frac{(t_u - c)^2 + \theta^2}{4(t_u^2 + \theta^2)}$. Otherwise, the firm would set q_u to be infinitely high generating infinite profit, which is not an interesting case. So, we assume $d \geq I_a = \frac{(t_u - c)^2 + \theta^2}{4(t_u^2 + \theta^2)}$ when $c \leq t_u - \theta$. Under these conditions, the firm's profit is concave in q_u with a unique optimal solution at $q_u^* = q_{\text{a}} = \frac{a(t_u - c)}{-(t_u - c)^2 - \theta^2 + 4d(t_u^2 + \theta^2)}$.

- Region (b): In this region, the flexible offering strategy is used in stage 2. The firm's overall profit is

$$\Pi_{\text{IE}} = \frac{1}{8}(a + q_u(t_u + \theta - c))^2 + \frac{a^2}{8} - dq_u^2\left(t_u^2 + \theta^2\right),$$

  where the firm sets $q_u \geq 0$ to maximize the above profit function. This function is bounded only when the design cost coefficient d is again sufficiently high, that is, when $d \geq I_b = \frac{(t_u + \theta - c)^2}{8(t_u^2 + \theta^2)}$. Otherwise, the firm would again set q_u to be infinitely high and generate infinite profit. So, we assume $d \geq I_b = \frac{(t_u + \theta - c)^2}{8(t_u^2 + \theta^2)}$ when $t_u - \theta \leq c \leq t_u + \theta$. Under these conditions, the firm's profit is concave in q_u with a unique optimal solution at $q_u^* = q_{\text{b}} = \frac{a(t_u + \theta - c)}{-(t_u + \theta - c)^2 + 8d(\theta^2 + t_u^2)}$.

- Region (c): In this region, the firm knows the existing product will be offered in stage 2 regardless of the realized trendiness. Hence $q_u^* = 0$.

Taking into account all cases, the optimal solutions are as follows:

$$q_u^* = \begin{cases} q_{\mathrm{a}} & \text{if } c \leq t_u - \theta \\ q_{\mathrm{b}} & \text{if } t_u - \theta \leq c \leq t_u + \theta \\ 0 & \text{if } c \geq t_u + \theta \end{cases}. \tag{B.11}$$

The optimal profit is:

$$\Pi_u^* = \begin{cases} \frac{a^2\left(\theta^2-4d\left(\theta^2+t_u^2\right)\right)}{4(c^2-2ct_u-(4d-1)(\theta^2+t_u^2))} & \text{if } c \leq t_u - \theta \\ \frac{a^2\left(c^2-2c(\theta+t_u)+(1-16d)\theta^2+(1-16d)t_u^2+2\theta t_u\right)}{8(c^2-2c(\theta+t_u)+(1-8d)\theta^2+(1-8d)t_u^2+2\theta t_u)} & \text{if } t_u - \theta \leq c \leq t_u + \theta \quad \square \\ \frac{a^2}{4} & \text{if } c \geq t_u + \theta \end{cases} \tag{B.12}$$

Proof of Proposition 4 We show the effect of θ on the firm's optimal profit under the AI-assisted design. According to Proposition 2, the firm's profit expression depends on model parameters. When $c \geq t_u$, we consider two cases in terms of θ:

- If $\theta \leq c - t_u$, the firm offers the existing product. The optimal profit is always $\frac{a^2}{4}$, which is not affected by θ.

- If $\theta \geq c - t_u$, the firm offers the flexible offering strategy where the first order derivative of the firm's profit with respect to θ is $\frac{\partial \Pi_{\mathrm{IE}}^*}{\partial \theta} = \frac{2a^2 d(t_u+\theta-c)(c\theta+t_u(t_u-\theta))}{(c^2-2c(\theta+t_u)+(1-8d)\theta^2+(1-8d)t_u^2+2\theta t_u)^2}$, which is non-negative. Thus, the corresponding profit is increasing in θ.

In summary, when $c \geq t_u$, the firm's optimal profit is first independent of and then increases as θ increases. When $c \leq t_u$, we again consider two cases in terms of θ:

- If $\theta \leq t_u - c$, the firm offers the (II) strategy and the optimal profit is $\Pi_{\mathrm{II}}^* = \frac{a^2\left(\theta^2-4d\left(\theta^2+t_u^2\right)\right)}{4(c^2-2ct_u-(4d-1)(\theta^2+t_u^2))}$. Taking the first order derivative of Π_{II}^* with respect to θ, we have $\frac{\partial \Pi_{\mathrm{II}}^*}{\partial \theta} = -\frac{a^2(4d-1)\theta(c-t_u)^2}{2(c^2-2ct_u-(4d-1)(\theta^2+t_u^2))^2}$. It is easily observed that the effect of θ on profit depends on d. Specifically, when $d \geq 0.25$, the first order derivative is non-positive, and the optimal profit decreases as θ increases; when $d \leq 0.25$, the first order derivative is non-negative, and the optimal profit increases in θ.

- If $\theta \geq t_u - c$, again, the firm offers the flexible offering strategy where the first order derivative of the firm's profit with respect to θ is $\frac{\partial \Pi_{\mathrm{IE}}^*}{\partial \theta} = \frac{2a^2 d(t_u+\theta-c)(c\theta+t_u(t_u-\theta))}{(c^2-2c(\theta+t_u)+(1-8d)\theta^2+(1-8d)t_u^2+2\theta t_u)^2}$, which is always non-negative and the optimal profit is increasing in θ.

In summary, when $c \leq t_u$, if $d \leq 0.25$, the optimal profit always increases in θ; if $d \geq 0.25$, the optimal profit first decreases and then increases in θ with the turning point at $\theta = t_u - c$. $\square$

Proof of Proposition 5 In the base model, $t_d = t_u$. So, we use t_u to replace t_d in this proof for ease of exposition. To guarantee bounded solution and profit under both designs, we assume that $d \geq \max(d_0, d_1)$. Since the optimal solution under the AI-assisted design is model parameter dependent, we carry out the comparison following the three regions defined previously.

- Region (a) where $c \leq t_u - \theta$: In this region, the firm improves the product quality in stage 1 and then always offers the improved product regardless of realized fashion trends under the AI-assisted design. In this case, the model under the manual design is a special case of the model under the AI-assisted design when $\theta = 0$. Following Proposition 4, the firm prefers the manual design when $d \geq 0.25$ (and $\theta \leq t_u - c$ which is implied by this region) and prefers the AI-assisted design otherwise.

- Region (b1) when $t_u - \theta \leq c \leq t_u$: In this region, under the AI-assisted design, the firm adopts the flexible product offering strategy in stage 2. Taking the difference of the optimal profits under the two designs, we have:

$$\Pi^*_{\mathrm{IE}} - \Pi^*_{\mathrm{d}} = \frac{a^2[(c-t_u)^2(-c+\theta+t_u)^2 - 4dG(c)]}{8[4dt_u^2 - (t_u-c)^2][c^2 - 2c(\theta+t_u) + (1-8d)\theta^2 + (1-8d)t_u^2 + 2\theta t_u]}, \quad \text{(B.13)}$$

where $G(c) = [c^2(4\theta^2 + 3t_u^2) + 2ct_u(-4\theta^2 - 3t_u^2 + \theta t_u) + t_u^2(3\theta^2 + 3t_u^2 - 2\theta t_u)]$. Note that the denominator of the difference function is always positive due to the assumption that $d \geq \max(d_0, d_1)$. The numerator is a linear function of d with coefficient $G(c)$ that is quadratic and convex in c. Note that $G(c = t_u - \theta) = 4\theta^2$ and $G(c = t_u) = -t_u^2\theta^2$. So, the profit difference changes its sign exactly once at $\bar{\bar{d}}$ if $t_u - \theta < c \leq \bar{c}$, and is always positive regardless of d if $\bar{c} < c \leq t_u$, where $\bar{c} = \frac{t_u\left(-2\theta\left(\sqrt{\theta^2+t_u^2}-2\theta\right)+3t_u^2-\theta t_u\right)}{4\theta^2+3t_u^2}$ and $\bar{\bar{d}} = \frac{(c-t_u)^2(-c+\theta+t_u)^2}{4\theta^2(4c^2-8ct_u+3t_u^2)+8\theta t_u^2(c-t_u)+12t_u^2(c-t_u)^2}$. Hence, if $c \leq \bar{c}$, the firm's profit difference changes its sign and prefers the AI-assisted design when $d \leq \bar{\bar{d}}$ and the manual design otherwise. If $c \geq \bar{c}$, the profit difference is always positive and it is optimal to always use the AI-assisted design.

  Finally, we prove that $\bar{\bar{d}}$ increases in c on its feasible interval, $c \in [t_u - \theta, \bar{c})$. As defined earlier, $\bar{c} = \frac{t_u\left(-2\theta\left(\sqrt{\theta^2+t_u^2}-2\theta\right)+3t_u^2-\theta t_u\right)}{4\theta^2+3t_u^2}$ is the left root of the denominator of $\bar{\bar{d}}$, which is renamed as $g(c) = 4\theta^2\left(4c^2 - 8ct_u + 3t_u^2\right) + 8\theta t_u^2(c-t_u) + 12t_u^2(c-t_u)^2$. We now take the first order derivative of $\bar{\bar{d}}$ with respect to c which leads to $\frac{\partial \bar{\bar{d}}}{\partial c} = \frac{(t_u-c)(t_u+\theta-c)h(c)}{2g^2(c)}$, where $h(c) = (4\theta^2 + 3t_u^2)c^3 + (3\theta t_u^2 - 12\theta^2 t_u - 9t_u^3)c^2 + (-6\theta t_u^3 + 9\theta^2 t_u^2 + 9t_u^4)c + \theta^3 t_u^2 - \theta^2 t_u^3 + 3\theta t_u^4 - 3t_u^5$. Note that $\frac{(t_u-c)(t_u+\theta-c)}{2g^2(c)} \geq 0$. So, in order to show that $\bar{\bar{d}}$ increases in c, we only need to show that $h(c) \geq 0$ for any c in the given range. Note that $h(c)$ is a cubic function of c with $\frac{\partial h(c)}{\partial c} = (12\theta^2 + 9t_u^2)c^2 + (6\theta t_u^2 - 24\theta^2 t_u - 18t_u^3)c - 6\theta t_u^3 + 9\theta^2 t_u^2 + 9t_u^4$, which is quadratic and convex in c and its left root coincides with $\bar{c}$, the upper bound of the feasible range of c. Hence, we have $\frac{\partial h(c)}{\partial c} \geq 0$, and accordingly, $h(c)$ increases in c and $h(c)$ reaches its lowest value when $c = t_u - \theta$. Since $h(c = t_u - \theta) = 4\theta^3(t_u - \theta)(t_u + \theta) \geq 0$, this implies that $h(c) \geq 0$. Consequently, $\frac{\partial \bar{\bar{d}}}{\partial c} \geq 0$ and $\bar{\bar{d}}$ increases in c in the given range.

- Region (b2) when $t_u \leq c \leq t_u + \theta$, the firm could not profitably serve the improved product in stage 2 under manual design so will not improve at all in stage 1. However, under the AI-assisted design, the firm uses the flexible offering strategy and its profit is always higher than that under the no improvement strategy and hence higher than that under the manual design.
- Region (c) when $c \geq t_u + \theta$, the firm could not profitably serve the improved product in stage 2 under either design. So, the design choice is irrelevant in this case. □

Appendix C Model Extensions

In this section, we extend the base model by relaxing two assumptions. In §Appendix C.1, we analyze a case where human designers may have different impacts under the two design strategies. In §Appendix C.2, we consider an alternative model where the retailer does not have the flexibility to offer the original product if the newly designed product under the innovative design is unpopular. The technical proofs of propositions in this section are provided in Appendix D.

Appendix C.1 The Model with Unequal Designer Impact

So far, we have assumed that the designer has an equal impact on both designs, i.e., $t_d = t_u$, and neither design has a (dis)advantage on product trendiness in expectation. As a result, the focus is on how uncertainty and cost structure affect the firm's decisions. When we extend the model to relax the assumption of equal trendiness, the firm's design strategy will also be impacted by the differences in the designer's ability to influence design. Like the base model, we focus on the model parameter set where the optimal quality improvement is upper bounded. That is, we assume that $d \geq \max(d_0, d_1)$. Recall from the setup of the base model that $t_d \in [t_u - \theta, t_u + \theta]$.

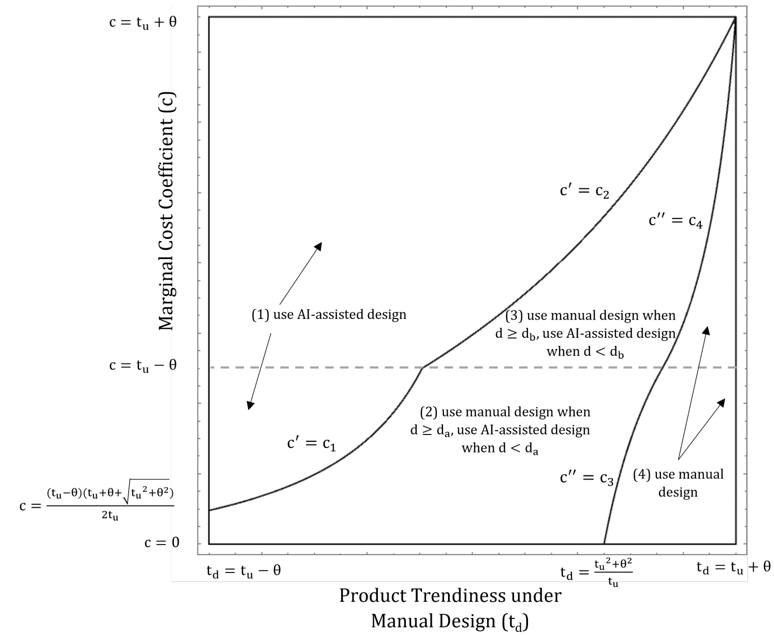

Figure 6: The Optimal Design Strategy under Unequal Designer Impact ($t_u \neq t_d$)

Otherwise, one design will always dominate the other. Also, similar to the base model, when marginal cost carries too much weight, $c \geq t_u + \theta$, the firm will not improve the product quality under either design. So, we focus on the interesting case when $c \leq t_u + \theta$. The optimal decisions are presented in the following proposition, and it is also displayed in Figure 6 on a (t_d, c) plane.

Proposition 7 (Firm's Design Strategy under Unequal Designer Impact). *There exist cut-off functions, c' and c'', where $c'' \leq c'$, such that the manual design is always preferred when marginal cost coefficient is small, i.e, $c \leq c''$ and the AI-assisted design is always favored when marginal cost coefficient is large, i.e., $c \geq c'$. For medium values of marginal cost coefficient, i.e., $c'' \leq c \leq c'$, there exists a threshold value, d', such that the firm prefers the AI-assisted design if the design cost coefficient is small, i.e., $d \leq d'$ and prefers the manual design otherwise.*[4]

This result has a number of implications. First of all, in region (1) of Figure 6 where the

[4]Note that when $0 \leq c \leq t_u - \theta$, $d' = d_a = \frac{-X_{1a}}{\theta^2(c-t_d)^2}$, $c' = c_1$, $c'' = c_3$; when $t_u - \theta \leq c < t_u + \theta$, $d' = d_b = \frac{-X_{1b}}{(c-t_d)^2(-c+\theta+t_u)^2}$, $c' = c_2$, $c'' = c_4$. Note also that $X_{1a} = 4c(t_d - t_u)(c(t_d + t_u) - 2t_d t_u) - 4\theta^2(c - t_d)^2$, $X_{1b} = c^2\left(t_d^2 - 4\left(\theta^2 + t_u^2\right)\right) - 2ct_d\left(t_d(\theta + t_u) - 4\left(\theta^2 + t_u^2\right)\right) + t_d^2\left(-3\theta^2 - 3t_u^2 + 2\theta t_u\right)$; $c_1 = \frac{t_d\left(\theta^2 + (t_u - t_d)\left(\sqrt{\theta^2 + t_u^2} + t_u\right)\right)}{\theta^2 - t_d^2 + t_u^2}$, $c_2 = \frac{t_d\left(-2\sqrt{\theta^2+t_u^2}(\theta - t_d + t_u) + t_d(\theta + t_u) - 4\left(\theta^2 + t_u^2\right)\right)}{t_d^2 - 4(\theta^2 + t_u^2)}$, $c_3 = \frac{2t_d\left(-\theta^2 + t_d t_u - t_u^2\right)}{-\theta^2 + t_d^2 - t_u^2}$, $c_4 = \frac{t_d\left(-\sqrt{2}\sqrt{\theta^2+t_u^2}(\theta - t_d + t_u) + t_d(\theta + t_u) - 2\left(\theta^2 + t_u^2\right)\right)}{t_d^2 - 2(\theta^2 + t_u^2)}$.

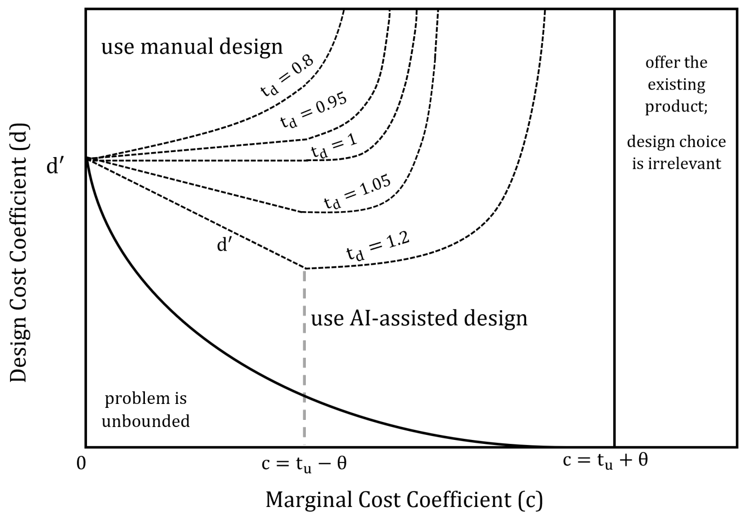

Figure 7: Numerical Plot: $t_u = 1$ and $\theta = 0.5$

marginal cost c is sufficiently high, i.e., $c \geq c'$, the firm would always adopt the AI-assisted design. The positive effect of high marginal costs on the AI-assisted design is quite consistent with the observation in the base model. However, different from the base model, if the designer's influence is higher under the manual design, i.e., when $t_d \geq \frac{t_u^2+\theta^2}{t_u}$, it will outperform the AI-assisted design given that the marginal cost coefficient is small enough when $c \leq c''$. See region (4) in Figure 6. This is not surprising since a high level t_d works in favor of the manual design.

In region (2) and (3) where the marginal cost is moderate when $c'' \leq c \leq c'$, we show that there exists a threshold value for the design cost coefficient, d', below which the AI-assisted design is preferred, and the manual design is favored otherwise. The effect of the design cost on the firm's design choice in the extended model is quite consistent with its effect in the base model. What is interesting here is the effect of the marginal cost c. Note that d' is a function of c and other model parameters (t_d, t_u, θ) and it could take different forms depending on the values of these parameters. Figure 7 presents a numerical plot showing how (c, t_d) affects the threshold d'.

Following Figure 7, we can make several observations. First, if $c \geq t_u - \theta$, the threshold d' always strictly increases in c. This implies that a higher marginal cost makes the AI-assisted design more likely to be dominant. This is aligned with the effect of c when $c \geq c'$, in which case AI-assisted design always dominates. However, if c is on the low side, $c \leq t_u - \theta$, the impact of c on d' is not obvious and is actually contingent on the designer's impact on different designs, t_d

and t_u. Specifically, an increase in c works in favor of a design where the designer impact is more significant. For example, when $t_d > t_u$, the manual design has a higher level of designer impact. We observe that d' always strictly decreases in c, which implies that a higher c makes the manual design more likely to be dominant. Recall from Figure 3 that when $c \leq t_u - \theta$ ($\leq t_u < t_d$), the firm will improve the product's quality and always offer the enhanced product under both design strategies. In order for the AI-assisted design to outperform the manual one, the firm needs to set a much higher quality improvement level under the AI-assisted design than the manual design. However, with the same product offering strategy under both designs, as c increases, the firm has less room to set a much higher quality improvement level under the AI-assisted design in consideration of the risk of unpopular design. Together with the higher designer impact under the manual design, it will be harder for the AI-assisted design to dominate the manual one. In other words, the manual design is more likely to dominate as c increases.

Appendix C.2 Alternative Model Without Flexible Product Offering

In the previous analysis, we have assumed that when the AI-assisted design turns out to be unpopular, the retailer has the flexibility to instead offer the original product (with no enhancement) to the market. In this section, we explore a scenario where the flexible offering strategy is unavailable. That is, when the enhanced design turns out to be a "miss," the retailer may be forced to abandon product sales. Since the flexible offering strategy only impacts the AI-assisted design, the model analysis and key results under the manual design (i.e., Proposition 1) remain the same. Under the AI-assisted design, the sequence of events is as follows. In the first stage, the firm decides to improve product quality by q_u. This decision is assumed to be made before the innovation uncertainty is revealed. In the second stage, the value of product trendiness is revealed, which can be either high or low. Under a high level of trendiness, $\widetilde{t} = t_u + \theta$, the retailer sets p_f when the enhanced product is sold at full price, which is similar to the base model; under a low level of trendiness, $\widetilde{t} = t_u - \theta$, the retailer either sets the markdown price p_m for the enhanced product or completely abandons the enhanced product and obtains a zero sales revenue. The retailer's decision can then be framed as a two-stage optimization problem, where the standard backward induction method can be used to solve for the optimal solutions. Specifically, the firm solves the second stage pricing problems, which are presented in Equations 5 and 6. Anticipating the best response pricing functions $p_f^*(q_u)$ and $p_m^*(q_u)$, the firm then sets the level of quality improvement,

q_u, to maximize the expected profit in Equation C.14:

$$\begin{aligned} \underset{q_u}{\text{maximize}} \quad & E(\Pi) = \frac{1}{2}\max\{\overbrace{[a + q_u(t_u+\theta) - p_f^*(q_u)]\cdot[p_f^*(q_u) - cq_u]}^{\text{offer enhanced product under popular design}}, 0\} \\ & + \frac{1}{2}\max\{\overbrace{[a + q_u(t_u-\theta) - p_m^*(q_u)]\cdot[p_m^*(q_u) - cq_u]}^{\text{offer enhanced product under unpopular design}}, 0\} - \overbrace{dq_u^2(t_u^2+\theta^2)}^{\text{design cost}} \\ \text{subject to} \quad & q_u \geq 0. \end{aligned} \tag{C.14}$$

The optimal decisions are formalized in the following proposition[5] and also displayed in Figure 8 on a (c, d) plane. Similar to the base model, we first impose a lower bound on the coefficient in the design cost, i.e., $d > d_2$, to ensure finite optimal level of quality improvement and profit.[6]

Proposition 8 (Optimal Quality/Product Decision under AI-Assisted Design, Without Flexible Offering). *Under the AI-assisted design and without flexible offering, the firm's optimal level of quality improvement can be described into three regions (a) - (c) as follows (see Figure 8).*

- *In region (a) where* $t_u - \theta \leq c \leq t_u$ *and* $d \geq \hat{d} = \frac{(t_u - c + \theta)\theta}{4(t_u^2+\theta^2)}$, *or* $0 \leq c \leq t_u - \theta$, *the optimal quality improvement level is* $q_u^* = q_a$. *In stage 2, the firm offers the improved product regardless of the state of product trendiness, namely, the markdown sales strategy.*
- *In region (b) where* $t_u - \theta \leq c \leq t_u + \theta$ *and* $d \leq \hat{d} = \frac{(t_u - c + \theta)\theta}{4(t_u^2+\theta^2)}$, *the optimal quality improvement level is* $q_u^* = q_b$. *In stage 2, the firm only offer the enhanced product when design is popular. The firm will not offer any product to the market when design is unpopular.*
- *In region (c) where* $t_u \leq c \leq t_u + \theta$ *and* $d \geq \hat{d} = \frac{(t_u - c + \theta)\theta}{4(t_u^2+\theta^2)}$, *it is too costly for the firm to improve product quality. So,* $q_u^* = 0$, *and the firm always offers the existing product to the market.*

The values of q_a and q_b in Proposition 8 are identical to the optimal solutions of the base model (i.e., Proposition 2). There are a few points that are noteworthy about this result. First, the firm would not pursue quality improvement when both the design and marginal production cost coefficients are high (i.e., region (c)). Second, in region (a), the design cost coefficient is high, but the marginal production cost coefficient is low. The low marginal cost allows the firm to offer the

[5]Note that the analysis of this alternative model is similar to the base model with flexible product offering. For brevity, the analysis details are not presented in the paper but are available upon request from the authors.

[6]Note that the cut-off value d_2 takes different functions depending on the value of c. Specifically, if $0 \leq c \leq t_u - \theta$, $d_2 = I_c = \frac{(t_u-c)^2+\theta^2}{4(t_u^2+\theta^2)}$, if $t_u - \theta \leq c \leq t_u + \theta$, $d_2 = I_d = \frac{(t_u+\theta-c)^2}{8(t_u^2+\theta^2)}$ and if $c \geq t_u + \theta$, $d_2 = 0$.

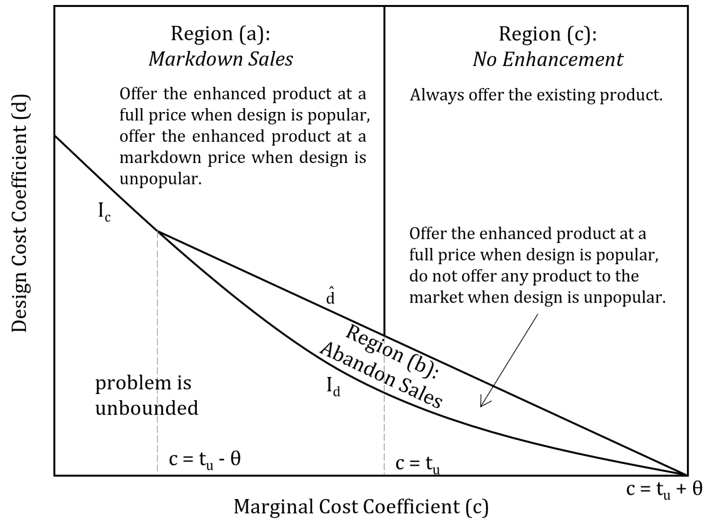

Figure 8: Optimal Quality/Product Decision under AI-Assisted Design, Without Flexible Offering

enhanced product at a markdown price, even if the design is unpopular. So, the firm will always offer the enhanced product in the second stage regardless of the realization of product trendiness. These outcomes are consistent with the base model. Now let us consider region (b), where the marginal cost coefficient is sufficiently high, and the design cost coefficient is low. In this scenario, the firm improves the quality in the first stage but then decides not to serve the market in the second stage if the enhanced product is a "miss". In expectation, this loss can be compensated by the positive gains under popular design. This is a new product offering scenario, referred to as "Abandon Sales", or "Abandonment".

Next, we compare the optimal profits under manual and AI-assisted designs. Similar to the base model, we focus on regions where the optimal solutions are bounded (i.e., the optimal value is finite), and assume an equal human designer impact, i.e., $t_d = t_u$. Recall from Propositions 1 and 8 that when both design and marginal cost coefficients are sufficiently large (i.e., region (c) in Figure 8), the firm will not enhance the product quality under either design, and the existing product will be served. In this case, the choice of design strategy is irrelevant. So, we focus on regions where the firm will make design enhancements (i.e., region (a) and (b) in Figure 8) in the following proposition.

Proposition 9 (Firm's Design Strategy for Product Enhancement, Without Flexible Offering). *There exists a threshold value, $\bar{d} = \frac{1}{4}$, such that the firm prefers the AI-assisted design if the design cost*

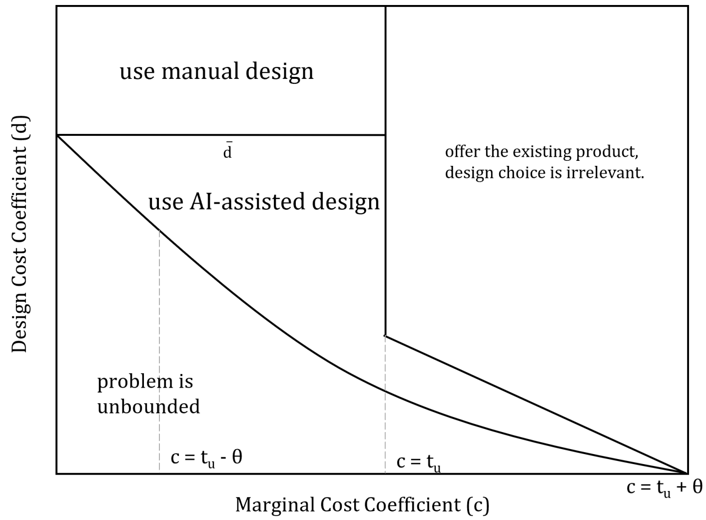

Figure 9: Firm's Design Strategy for Product Enhancement, Without Flexible Offering

coefficient is below this threshold, i.e., $d \leq \bar{d}$, and prefers the manual design otherwise. See also Figure 9.

This proposition implies that the manual design is only preferred when the design cost coefficient is sufficiently high. To get the intuition behind this result, recall that there are two product offering scenarios for the AI-assisted design: markdown sales, represented by region (a) in Figure 8; abandon sales, represented by region (b) in Figure 8. According to Proposition 8, in region (b), abandoning sales always leads to a higher profit than markdown sales do. In region (a), similar to the base model, the manual design is a special case of the AI-assisted design with $\theta = 0$. The effect of increasing θ in this region, however, is non-monotone, and the direction of change depends on the weight of the design cost. When the design cost carries sufficient weight, the optimal profit decreases as θ increases, and the firm would prefer the manual design; otherwise, the optimal profit increases as θ increases, and the firm would prefer the AI-assisted design. The effect of the design cost on the firm's design choice decisions in the alternative model is consistent with its effect in the base model. Finally, we summarize the effect of innovation uncertainty θ on the firm's design strategy in the proposition below.

Proposition 10 (**Effect of Innovation Uncertainty on the Firm's Design Strategy, Without Flexible Offering**). *As the level of innovation uncertainty, θ, increases, it is more likely for the firm to adopt the AI-assisted design.*

Our finding about the impact of innovation uncertainty on design strategy in the base model continues to hold even when the flexible offering strategy is disabled. An intuitive explanation for above result is as follows. A higher level of innovation uncertainty would give the firm more advantage when design is popular, and the "abandon sales" scenario is involved. Alternatively, according to Proposition 8 and Figure 8, as θ increases, region (b) expands, implying that the AI-assisted design is more likely to dominate.

Appendix D Mathematical Proofs for Results in the Extended Models

Proof of Proposition 7 Now we consider the case when $t_d \neq t_u$. Note that if $t_d \geq t_u + \theta$, the manual design always dominates. So, here we assume that $t_d \leq t_u + \theta$. Similarly, if $t_d \leq t_u - \theta$, then the AI-assisted design dominates. So, we assume $t_d \geq t_u - \theta$. Note also when $c \geq t_d$, the firm will not enhance the existing product under the manual design, which is not an interesting case to consider. So, in this proof, we focus on the case where $c \leq t_d$ and $t_u - \theta \leq t_d \leq t_u + \theta$. Given this, we separately consider the following cases:

- Case (1) when $t_d \leq t_u$ and $c \leq t_u - \theta$ which implies $c \leq t_d$. The profit difference between the two designs can be expressed as:

$$\Pi^*_{\mathrm{II}} - \Pi^*_{\mathrm{d}} = \frac{dX_{1a} + \theta^2(c - t_d)^2}{4\left(c^2 - 2ct_d + (1-4d)t_d^2\right)\left(c^2 - 2ct_u - (4d-1)\left(\theta^2 + t_u^2\right)\right)}, \tag{D.15}$$

  where $X_{1a} = 4c(t_d - t_u)(c(t_d + t_u) - 2t_d t_u) - 4\theta^2(c - t_d)^2$. Both terms in the denominator are linearly decreasing in d. Given $d \geq \max(d_0, d_1)$, both terms are negative, resulting with a positive denominator. The numerator is linear in d. The coefficient of d may be positive or negative, depending on X_{1a}. Given X_{1a} is a quadratic, concave function in terms of c, negative when $c = 0$ and positive when $c = t_d$, it is easy to show that for $c \in [0, t_u - \theta]$ there exists one solution to $X_{1a} = 0$, which we define as $c_1 = \frac{t_d\left(\theta^2 + (t_u - t_d)\left(\sqrt{\theta^2 + t_u^2} + t_u\right)\right)}{\theta^2 - t_d^2 + t_u^2}$. Specifically, if $c \geq c_1$, the profit difference is always positive, hence the firm always uses the AI-assisted design; otherwise if $c \leq c_1$, the manual design is preferred if $d \geq d'$, and AI-assisted design is preferred if $d \leq d'$, where $d' = d_a = \frac{-X_{1a}}{\theta^2(c - t_d)^2}$.

- Case (2) when $t_d \leq t_u$ and $t_u - \theta \leq c \leq t_d$. We compare the optimal profits:

$$\Pi^*_{\mathrm{IE}} - \Pi^*_{\mathrm{d}} = \frac{dX_{1b} + (c - t_d)^2(-c + \theta + t_u)^2}{8\left(c^2 - 2ct_d + (1-4d)t_d^2\right)\left(c^2 - 2c(\theta + t_u) + (1-8d)(\theta^2 + t_u^2) + 2\theta t_u\right)}, \tag{D.16}$$

where $X_{1b} = c^2\left(t_d^2 - 4\left(\theta^2 + t_u^2\right)\right) - 2ct_d\left(t_d(\theta + t_u) - 4\left(\theta^2 + t_u^2\right)\right) + t_d^2\left(-3\theta^2 - 3t_u^2 + 2\theta t_u\right)$. The denominator is positive for the same reasoning as in Case (1). Depending on the sign of X_{1b}, the numerator is either always positive, or changes sign exactly once as d increases. Given X_{1b} is a quadratic, concave function in terms of c, and positive when $c = t_d$, it is easy to show that for $c \in [0, t_u - \theta]$ there exists one solution to $X_{1b} = 0$, which we define as $c_2 = \frac{t_d\left(-2\sqrt{\theta^2+t_u^2}(\theta - t_d + t_u) + t_d(\theta + t_u) - 4\left(\theta^2 + t_u^2\right)\right)}{t_d^2 - 4(\theta^2 + t_u^2)}$. In this case, if $c \geq c_2$, the profit difference is always positive, hence the firm prefers the AI-assisted design; otherwise if $c \leq c_2$, the manual design is preferred if $d \geq d'$, and the AI-assisted design is preferred if $d \leq d'$, where $d' = d_b = \frac{-X_{1b}}{(c-t_d)^2(-c+\theta+t_u)^2}$.

- Case (3) when $t_d \geq t_u$ and $c \leq t_u - \theta$: The specific expression of the profit difference is the same as Equation (D.15). Given $t_d \geq t_u$, X_{1a} is always negative, which means the manual design is preferred when d is sufficiently large. Since $d \geq \max(d_0, d_1)$, if $d_0 \geq d_1$, then when $d = d_0$, the optimal profit under the AI-assisted design is finite, which is smaller than the profit under the manual design as it approaches infinity as d approaches d_0. So the manual design is preferred regardless of d if $d_0 \geq d_1$, or equivalently, if $X_{2a} \leq 0$, where $X_{2a} = \theta^2(2t_d - c) + (t_d - t_u)(c(t_d + t_u) - 2t_d t_u)$. Otherwise, if $X_{2a} \geq 0$ then when $d \leq d' = d_a$, the AI-assisted design is preferred, and when $d \geq d' = d_a$, the manual design is preferred. X_{2a} is a linear function of c, hence $c \geq c_3$, then there exists such d', otherwise if $c \leq c_3$, the manual design is always preferred, where $c_3 = \frac{2t_d\left(-\theta^2 + t_d t_u - t_u^2\right)}{-\theta^2 + t_d^2 - t_u^2}$.

- Case (4) when $t_d \geq t_u$ and $t_u - \theta \leq c \leq t_d$: In this case, the firm either uses the manual design, or use the AI-assisted design with the flexible offering strategy. According to the analysis of Equation (D.16), if $c \geq c_2$, the profit difference is positive, the AI-assisted design is preferred; if $c \leq c_2$, the manual design is preferred if d is sufficiently large. We note that $d \geq \max(d_0, d_1)$ holds, but now the expression of d_1 is different. Similar as before, if $d_0 \geq d_1$, or equivalently, if $X_{2b} \leq 0$ then the manual design dominates for any d, where $X_{2b} = c^2\left(t_d^2 - 2\left(\theta^2 + t_u^2\right)\right) - 2ct_d\left(t_d(\theta + t_u) - 2\left(\theta^2 + t_u^2\right)\right) - t_d^2(t_u - \theta)^2$. Otherwise, if $X_{2b} \geq 0$, then the AI-assisted design is preferred if $d \leq d' = d_b$, and the manual design is preferred if $d \geq d' = d_b$. Given X_{2b} is a quadratic, concave function in terms of c and positive when $c = t_d$, then there exists one root $c_4 = \frac{t_d\left(-\sqrt{2}\sqrt{\theta^2+t_u^2}(\theta - t_d + t_u) + t_d(\theta + t_u) - 2\left(\theta^2 + t_u^2\right)\right)}{t_d^2 - 2(\theta^2 + t_u^2)}$, such that if $c \geq c_4$ there exists such d', and otherwise if $c \leq c_4$ the manual design dominates for any d. □

Proof of Proposition 8 Treating q_u as given in stage 1, two steps are needed to characterize the firm's product offering strategy. First, given a realized value of fashion, we need to solve the corresponding pricing decision. We demonstrate the solution method for the scenario when the realized trendiness is high, i.e., $\widetilde{t} = t_u + \theta$. The case with a low value of realized trendiness can be analyzed similarly.

If the improved product is offered, then the firm solves the following maximization problem:

$$\begin{aligned} \underset{p_f}{\text{maximize}} \quad & \Pi_f^{stage\ 2} = [a + q_u(t_u + \theta) - p_f] \cdot (p_f - cq_u) \\ \text{subject to} \quad & p_f \geq 0, \ \ p_f - cq_u \geq 0, \ \ and \ \ a + q_u(t_u + \theta) - p_f \geq 0. \end{aligned} \tag{D.17}$$

Given q_u, it is straightforward that if $q_u(c - t_u - \theta) \leq a$, the constrained optimal price is $p_f^* = \frac{1}{2}[a + q_u(t_u + \theta + c)]$; otherwise (when $q_u(c - t_u - \theta) > a$), the firm sets a price high enough to generate zero sales and profit. The analysis for the low value of trendiness is similar. Next, we solve the stage 1 quality improvement problem. For clear exposition, we define $q_a = \frac{a(t_u - c)}{4d(t_u^2 + \theta^2) - (t_u - c)^2 - \theta^2}$ and $q_b = \frac{a(t_u + \theta - c)}{8d(t_u^2 + \theta^2) - (t_u + \theta - c)^2}$. The optimal product decisions can be characterized in three regions, depending on the marginal cost coefficient c as follows:

- When $c \leq t_u - \theta$, the markdown sales strategy is used in stage 2. Accordingly, the firm's overall profit across both periods can be written as

$$\Pi_{\text{II}} = \frac{1}{8}(a + q_u(t_u + \theta - c))^2 + \frac{1}{8}(a + q_u(t_u - \theta - c))^2 - dq_u^2\left(t_u^2 + \theta^2\right),$$

  where the firm looks for a profit maximizing $q_u \geq 0$. This function is bounded only when the design cost coefficient d is sufficiently high, that is, when $d \geq I_c = \frac{(t_u - c)^2 + \theta^2}{4(t_u^2 + \theta^2)}$. Otherwise, the firm would set q_u to be infinitely high generating infinite profit, which is not an interesting case. So, we assume $d \geq I_c = \frac{(t_u - c)^2 + \theta^2}{4(t_u^2 + \theta^2)}$ when $c \leq t_u - \theta$. Under these conditions, the firm's profit is concave in q_u with a unique optimal solution at $q_u^* = q_{\text{a}}$.

- When $t_u - \theta \leq c \leq t_u + \theta$, there exists three different product strategies. The first strategy is to keep q_u^* small and cover both markets regardless of popular/unpopular design, and the optimal profit is the same as above. The second strategy is to keep q_u^* large and cover market only when the resulting design is popular. The firm's overall profit is then

$$\Pi_{\text{IA}} = \frac{1}{8}(a + q_u(t_u + \theta - c))^2 - dq_u^2\left(t_u^2 + \theta^2\right),$$

  where the firm sets $q_u \geq 0$ to maximize the above profit function. This function is bounded only when the design cost coefficient d is again sufficiently high, that is, when $d \geq I_d = \frac{(t_u + \theta - c)^2}{8(t_u^2 + \theta^2)}$. Otherwise, the firm would again set q_u to be infinitely high and generate infinite profit. So, we assume $d \geq I_d = \frac{(t_u+\theta-c)^2}{8(t_u^2+\theta^2)}$ when $t_u - \theta \leq c \leq t_u + \theta$. Under these conditions, the firm's profit is concave in q_u with a unique optimal solution at $q_u^* = q_{\text{b}}$. The third strategy is to offer existing product regardless of the realized fashion trends. Hence $q_u^* = 0$ and the associated optimal profit is $\frac{a^2}{4}$. A comparison of these product strategies yields threshold conditions outlined in Proposition 8.

- Region (c): When $c \geq t_u + \theta$, even under the popular design, the firm could not profitably serve the market, because of high marginal cost. Hence, under this degenerate case, the optimal strategy is to offer the existing product.

□

Proof of Proposition 9 It is assumed that $t_d = t_u$. So, we use t_u to replace t_d in this proof for ease of exposition. To guarantee bounded solution and profit under both designs, we focus on regions where the optimal solutions are bounded.

- In region (a) where $t_u - \theta \leq c \leq t_u$ and $d \geq \hat{d} = \frac{(t_u-c+\theta)\theta}{4(t_u^2+\theta^2)}$, or $0 \leq c \leq t_u - \theta$, the firm uses the markdown sales strategy under the AI-assisted design. In this case, the model under the manual design is a special case of the model under the AI-assisted design when $\theta = 0$. The optimal profit under AI-assisted design is $\Pi_{\text{II}}^* = \frac{a^2\left(\theta^2-4d\left(\theta^2+t_u^2\right)\right)}{4(c^2-2ct_u-(4d-1)(\theta^2+t_u^2))}$. Taking the first order derivative of Π_{II}^* with respect to θ, we have $\frac{\partial \Pi_{\text{II}}^*}{\partial \theta} = -\frac{a^2(4d-1)\theta(c-t_u)^2}{2(c^2-2ct_u-(4d-1)(\theta^2+t_u^2))^2}$. It is easily observed that the effect of θ on profit depends on d. Specifically, when $d \geq 0.25$, the first order derivative is non-positive, and the optimal profit decreases as θ increases; when $d \leq 0.25$, the first order derivative is non-negative, and the optimal profit increases in θ. So, it is obvious that when $d \geq 0.25$, the manual design dominates; when $d \leq 0.25$, the AI-assisted design dominates.

- In region (b) where $t_u - \theta \leq c \leq t_u + \theta$ and $d \leq \hat{d} = \frac{(t_u-c+\theta)\theta}{4(t_u^2+\theta^2)}$, the firm uses the abandon sales strategy under the AI-assisted design. The threshold $\hat{d}$ monotonically decreases in c, and so $\hat{d} \leq \frac{1}{2(\frac{t_u^2}{\theta^2}+1)}$, where the right-hand side monotonically increases in θ. Given $\theta \leq t_u$, it is easy to see that $\hat{d} \leq \frac{1}{4}$. As shown above, when $d \leq \frac{1}{4}$ and when the markdown sales strategy is involved, the profit under the AI-assisted design is higher than that under the manual design. From Proposition 8, we see that the profit with the abandon sales strategy is higher than that with the markdown sales strategy in region (b). Hence, In region (b) the AI-assisted design is also more preferred than the manual design. □

Proof of Proposition 10 To prove that as uncertainty increases, it is more likely for the AI-assisted design to dominate, it suffices to show that as θ increases, region (b) in Proposition 8 becomes larger. Proposition 8 indicates that in terms of d, region (b) is bounded as follows: $\frac{(t_u+\theta-c)^2}{8(t_u^2+\theta^2)} = I_d \leq d \leq \hat{d} = \frac{(t_u-c+\theta)\theta}{4(t_u^2+\theta^2)}$. $\hat{d} - I_d = \frac{(c+\theta-t_u)(-c+\theta+t_u)}{8(\theta^2+t_u^2)}$, and its first order derivative equals $\frac{\theta\left[(c-t_u)^2+t_u^2\right]}{4(\theta^2+t_u^2)^2}$, which is non-negative. Hence, $\hat{d} - I_d$ increases as θ gets bigger. In terms of c, this region is bounded as follows: $t_u - \theta \leq c \leq t_u + \theta$. It is easy to see that this bound expands as θ increases. □